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Analyzing different designs of liner shipping feeder networks: A case study

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A R T I C L E I N F O

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A B S T R A C T

This paper proposes different network structures for a real liner shipping company to serve demands between Norwegian and European continental ports. The current practice of the shipping company deploys a feeder network where all routes depart from the European port. We study the impact of using a hub-and-spoke network that is composed of mother and daughter routes with the possibility of splitting pickups and deliveries. Computational studies carried out on problem instances based on realistic data show that significant cost reductions can in several cases be obtained by including the proposed network structure.

1. Introduction

According to the national transportation plan 2018–2029 of Norway, cargo transport is expected to increase by 30% until 2029 with a strong political focus on moving cargo transportation from land to sea (Norwegian Ministry of Transport and Communications, 2017). This political choice of moving from land to sea is justified to avoid the future high pressure on roads that will require a heavy extension of the infrastructure on land. Also, there are environmental considerations behind this strategic decision as maritime shipping offers the most carbon efficient mode of transportation. World Shipping Center (2019) estimates that large container vessels release 10 grams of carbon dioxide to carry 1 ton of cargo over 1 km, whereas it is 21 and 59 grams for Diesel train and trucks, respectively. Therefore, the Norwegian short sea sector can expect increased demand in the upcoming years for both shipping between Norwegian ports and also between Norwegian and European ports.

This paper presents a case study for a liner shipping company that transports goods between Norwegian ports and the European continental port at Rotterdam. The shipping company serves the major ports of Norway and accommodates different container types (from standard containers of twenty-foot equivalent unit (TEU) and forty-foot equivalent unit to specialized containers). There is a given weekly demand to and from each Norwegian port and the continental port. Since the demand among Norwegian ports is so small, it can be considered as negligible.

The shipping company operates in liner operation mode (see the classification of Lawrence (1972)) for which a schedule establishes fixed vessel routes to ensure a regular service frequency (typically each week) for served ports. This shipping mode is often compared to public transportation services by bus or train. Timetables and prices are planned well in advance, from several weeks to months, and are in general open to the public in order to simplify advanced planning for customers (Haralambides, 2007). There are three different routes ensured by the shipping company that all depart from Europe, and each of which visits a subset of Norwegian

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ports before going back to Europe. In total, there are currently 21 Norwegian ports served by a fleet of five vessels with capacities between 700 and 900 TEUs. To ensure a weekly service, two routes require two vessels each because their total sailing times are about ten days, and the third route is covered by only one vessel because its duration is seven days. The current operations are known in the literature as feeder network, which consists of a hub port (Rotterdam) and many feeder ports (located at Norway), and the transportation of containers can go both ways between hubs and ports with no transshipment (Meng et al., 2014).

This work aims to find the best network design for the considered case company by determining the optimal fleet of vessels to be deployed in terms of number and size, and the route to be sailed for each vessel so that all demands are fulfilled in a weekly service. The main contribution is to study different network structures and their impact on operational costs. For this purpose, this study is not limited to the existing network structure of the shipping company, i.e. feeder network, but also proposes to consider a hub-and-spoke network design. This network consists of a heterogeneous fleet of mother and daughter vessels where mother vessels sail one or many mother routes that link selected Norwegian ports to Europe, and daughter vessels that sail daughter routes to serve the remaining Norwegian ports. Such a network design is inspired by the current practice of global large liner shipping companies, for which several deployed routes are interconnected with transshipment ports, and a container can be transshipped as many times as necessary to reach its final destination. In our study, the introduction of a daughter route in the network is optional depending on the operational costs of the whole system. This means that, in this paper, the hub-and-spoke network is a generalization of the feeder networks. In a case where a daughter route is deployed, it must be connected to a mother route for cargo transshipment. The structure of a daughter route can be either a simple cycle or a butterfly route. Clearly, this analysis is not only limited to the Norwegian case study, but can be applied to any shipping company that operates a feeder service connecting a major international port hub to ports located in a specific region. For example, the shipping company Unifeeder ensures, among others, a feeder service line between Amsterdam and east coast of Sweden, and the shipping company Mannlines covers a feeder line between Rotterdam, Klaipeda, Riga, and St. Petersburg.

The remainder of this paper is organized as follows. Section 2 provides a literature review. Section 3 presents the different liner shipping network structures considered in this case study. Section 4 introduces the mathematical models for the network structures, while Section 5 describes the label setting algorithm for generating non-dominated candidate routes. Section 6 presents the computational study, and Section 7 gives concluding remarks.

2. Literature review

Most studies related to Liner Shipping Network Design (LSND) mainly focus on deep sea shipping, where cargoes are transported between continents and can be transshipped at multiple ports for effective utilization of the vessel capacity. Agarwal and Ergun (2008) present an LSND, where the company can deploy a heterogeneous fleet of vessels to provide weekly services. In their work, a synchronization between routes is required because transshipment is only allowed when two routes visit the same port on the same day. However, the transshipment cost is not taken into consideration despite its importance in the total operational cost. The problem is difficult to solve, so solution approaches based on a greedy algorithm, column generation, and Benders decomposition are developed and compared. Álvarez (2009) extends the latter research to include transshipment costs, non-simple routes, and speed decisions. The model maximizes cargo revenues while considering costs on deployed vessels, fuel costs, handling costs, and the penalty of forfeited cargo. The problem is solved using a combined tabu search and column generation based algorithm. However, it should be noticed that the transshipment cost is inappropriately computed because the model considers only one transshipment cost for non-simple routes. Reinhardt and Pisinger (2011) consider transshipment costs for both simple and butterfly routes. The proposed model can be exactly solved using a branch-and-cut algorithm for instances up to 10 ports and three vessels.

Hub-and-spoke networks are variants of the LSND, where transshipment is only possible at specific ports, called hubs. Typically, larger ships are deployed on routes visiting the hubs, while smaller ships are deployed on feeder routes visiting spoke ports. Zheng et al. (2015) restrict the transshipment of cargo originating from or destined to feeder ports to at most two times. Furthermore, the model includes transit time restrictions on the cargo path from its origin to its destination.

Brouer et al., 2014 discuss the potential of applying operations research to create cost-effective and energy-efficient solutions to the LSND problem. The authors present a benchmark set of instances based on real-life data collected from the largest liner-shipping company in the world. The problem allows split delivery and pickup of containers among different ships, and cargo transshipment is allowed as many times as required. The problem is formulated using a compact mathematical model that cannot solve small problem instances. Thus, a tabu search combined with a heuristic column generation is proposed. Karsten et al. (2017) extend the latter matheuristic to include transit time restrictions on cargo flows. The main component of the extended matheuristic is to iteratively solve an integer program for a simple service as a moving operator in a large neighborhood search. In addition to transit time restrictions, Karsten et al. (2017) optimize the speed on each sailing leg.

Thun et al. (2016) attempt to analyze the effect of different route structures on the LSND problem. Here, the number of port visits is generalized to include one port visit as in simple cycles, two port visits as in butterfly cycles, and many port visits for a general route structure. Due to the complexity of the problem, the developed branch-and-price method is tested on instances that allow a maximum of two visits per port. The experiments show that cost reductions can be obtained by adopting advanced route structures.

The feeder network design is a particular case of the hub-and-spoke problem where the cargoes are transported between hubs and feeder ports without transshipment. Fagerholt (2004) addresses this problem to find optimal weekly routes such that operational costs are minimized. A two-stage solution method is proposed where, in the first stage, all feasible routes are generated and, in the second stage, a mathematical model is solved to select a subset of routes. Wang and Meng (2014) include transit time restrictions and solve the problem using a column generation based heuristic. Santini et al. (2018) also consider transit time and add speed optimization at each leg of routes. Balakrishnan and Karsten (2017) propose a model for which rejection of demand is allowed, and the number of transshipments for each container is limited.

Holm et al. (2018) introduce a novel concept for short sea shipping where transshipments of cargoes are performed at candidate locations at sea, called ocean hubs. Containers between Europe and Norway are transported with large vessels and transshipped at ocean hubs to small vessels that sail feeder routes. Because the transshipment is performed at sea, mother and daughter vessels need to be synchronized so that they meet at the same ocean hub at the same time. The problem is formulated as a mixed integer programming model and tested on a small real case study with nine ports.

This paper compares the feeder network design and hub-and-spoke network design for short sea liner shipping. Compared to most studies, any Norwegian port can be used for transshipment in the hub-and-spoke design. Moreover, we analyze different route structures for daughter routes and allow for split pickups and deliveries of cargoes. The proposed network structures are modeled with path-flow formulations and solved either exactly or by a simple yet effective heuristic approach. Since the demand is from and to the continental port, the developed mathematical models are based on the pre-generated routes and do not have to track the cargo flow explicitly. These models are, therefore, less complex than the one proposed by Brouer et al. (2014) for the general form of the LSND problem. Indeed, the computational study shows that a commercial solver can solve the proposed models for small problem size in a reasonable computational time.

3. Network structures

In this section, we present the network structures under consideration illustrated with examples based on the case company.

3.1. Variant 1: Mother routes only

In the first network structure, which corresponds to the current practice of the case company, any route originates from the continental port and serves all or a subset of ports located alongside the Norwegian coastline. Such a route is referred to as *mother route* and is sailed by so-called *mother vessels*. To maintain a weekly service frequency, each port has to be visited once a week. Thus, it is required that the number of mother vessels deployed on a mother route is equal to the number of weeks to complete that route (rounded up to the nearest integer). For instance, if a cycle takes ten days, one vessel cannot ensure a weekly service. Therefore, two vessels have to be deployed where the cycles of the first and second vessels start at day 1 and 8, respectively.

The shipping company can deploy a heterogeneous fleet of mother vessels, where the capacity of each vessel needs to be sufficiently high to accommodate the cargoes to be carried along its route. Since the flow of cargoes to be transported from the Norwegian ports to the continental port is higher than the reverse flow, we assume that a mother vessel starts by serving the northernmost port of its route and visits the other ports along its southbound journey. This results in a better average transit time per unit of cargo compared to adapting port visits during the northbound journey. Nonetheless, we should precise that an even better average transit time can be obtained by considering both northbound and southbound journey visits for Norwegian ports. With this generalization, the resulting mother routes are slightly longer than the one with the southbound journeys only. The operational cost in such a system would become higher because of the additional sailing and port costs for the ports visited during the northbound journey.

We denote this network structure where the system is composed of only mother routes as *Variant 1*, and it is illustrated in Fig. 1 for a subset of Norwegian ports served by the case company. The shown routes represent a possible solution of Variant 1, where the first visited port is always the northernmost port of the related route, i.e. Orkanger for mother route 1 and Bergen for mother route 2. The other ports of each route are visited during the southbound journey. The cargoes to be unloaded to and loaded from a port are handled during the same port visit.

3.2. Variant 2: Hub-and-spoke system with combined mother and daughter routes

The second network structure considers the possibility to include *daughter routes* (and daughter vessels) to the liner shipping network. Different mother routes can be deployed to serve lines between the continental European port and the selected Norwegian ports. Daughter vessels then serve the remaining ports on a feeder route basis. A daughter route is connected to a mother route at a specific Norwegian port that is used for cargo transshipment. In this configuration, a Norwegian port can be served by either a mother route or a daughter route. The network structure is, therefore, a generalization of alternatives discussed in Section 3.1. The aim is to study the impact of considering daughter vessels within the logistics system.

We propose two different alternatives for this network structure in the next two subsections.

3.2.1. Variant 2a: Only simple cycles as daughter routes

In Variant 2a, a daughter route can only be a *simple cycle*, where each port is visited one time, and the route starts and ends at the transshipment port. The total duration of each route must not exceed one week so that a weekly service is ensured for all visited ports.

Fig. 2 shows a mother route that visits Ålesund and Bergen. Two simple daughter routes use Ålesund port for the transshipment. The first route serves Averøy and Orkanger, and the second route serves Måløy only. There is also a third daughter route that uses Bergen as a transshipment port and serves Husnes, Haugesund, and Tananger.

3.2.2. Variant 2b: Daughter routes can be both simple cycles and butterfly routes

In Variant 2b, a daughter route can be either a simple cycle or a *butterfly* route. A butterfly route basically consists of two simple cycles served by the same vessel. The departure and arrival port of each simple cycle is the transshipment port, and the total duration

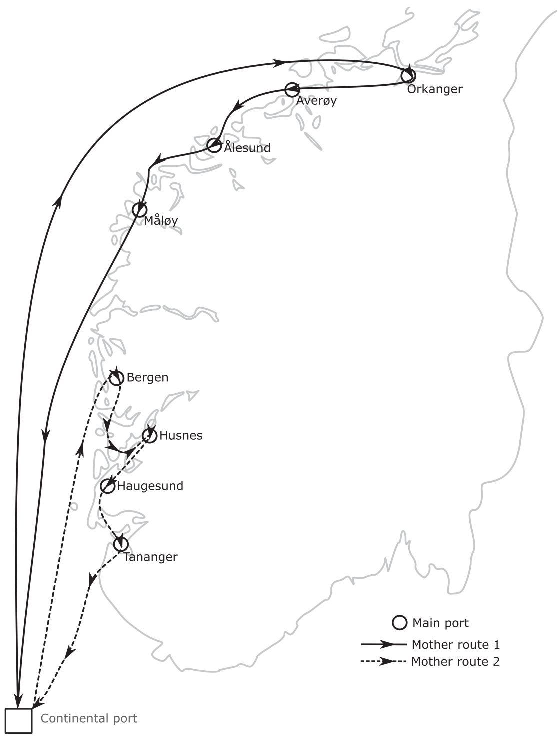

Fig. 1. Variant 1 with only mother vessels.

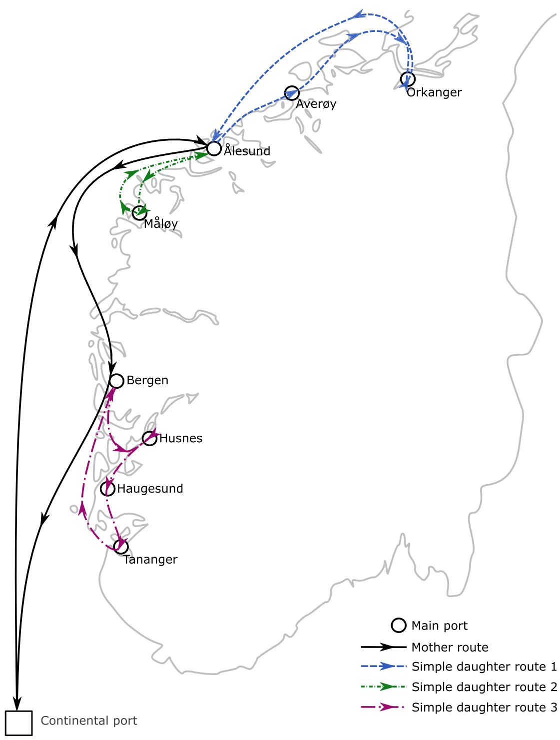

Fig. 2. Variant 2a with mother routes and simple daughter routes.

of the butterfly route must not exceed one week. Compared to the simple cycle, the butterfly structure has the advantage of better usage of the vessel capacity. Indeed, after completing the first loop, the vessel is fully unloaded at the transshipment port and loaded with the cargoes of the second cycle, whereas the same combination of visited ports could be infeasible with a simple route structure

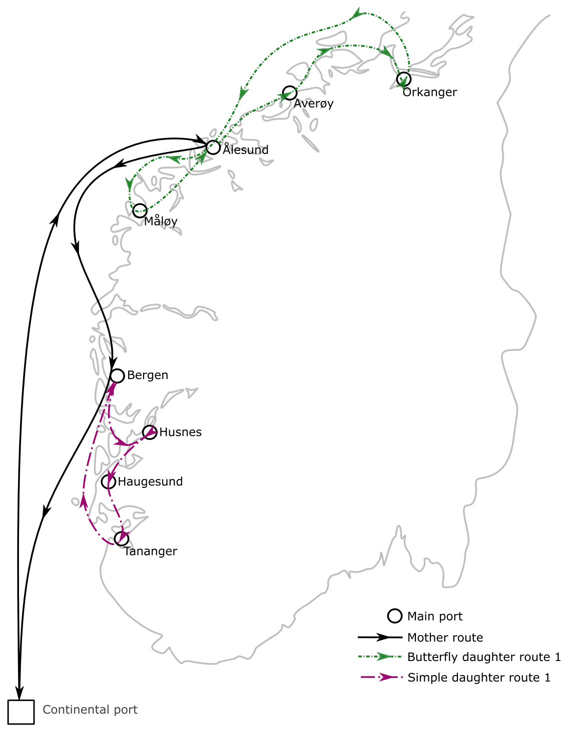

Fig. 3. Variant 2b with mother routes and simple/butterfly daughter routes.

due to the vessel capacity restriction.

In Fig. 3, the daughter routes comprise one butterfly route and one simple cycle. The butterfly route originates from Ålesund and serves Averøy and Orkanger before going back to the transshipment port. Then, the same vessel goes to Måløy and completes its route at Ålesund. The total duration of this route is less than one week to ensure a weekly service for all visited ports. The simple cycle originates from Bergen and serves Husnes, Haugesund, and Tananger.

4. Mathematical models

In this section, we present path-based formulations for the network structures presented in Section 3.

4.1. Model for Variant 1: Mother routes only

The mathematical model for Variant 1, denoted hereafter by $V1$, requires as an input a set of feasible mother routes. Each route contains the necessary information for the model, including the set of ports included, the number of vessels sailing the route so that a weekly service is ensured for all visited ports, and the total operational cost. The latter is composed of the costs of port calls, time charter for vessels sailing the route, the bunker cost, and the cargo handling cost. The notation and decision variables required by $V1$ are defined as follows.

Sets

$\mathcal{P}$ set of Norwegian ports, indexed by p,

$\mathcal{R}^M$ set of mother routes, indexed by r,

$\mathcal{R}_p^M$ set of mother routes that include port p, indexed by r.

Parameters

C_r^M total cost of using a fleet of mother vessels that takes mother route r; this cost considers the number of vessels required to ensure a weekly frequency and includes port calls, time charter, bunker, and cargo handling costs.

Decision variables

x_r a binary variable that takes the value 1 if mother route r is selected, and 0 otherwise.

Then, $V1$ can be read as follows.

$$(V1):\ \min \sum_{r\in\mathcal{R}^M} C_r^M x_r \tag{1}$$

subject to:

$$\sum_{r\in\mathcal{R}_p^M} x_r \geqslant 1, \quad p \in \mathcal{P}, \tag{2}$$

$$x_r \in \{0, 1\}, \quad r \in \mathcal{R}^M. \tag{3}$$

The objective function (1) minimizes the total costs of the whole network system. Constraints (2) ensure that all ports are served, while Constraints (3) are the binary requirements for the variables.

4.2. Model for Variant 2: Combined mother and daughter routes

The input of the model of Variant 2 is a set of mother and daughter routes. Depending on the type of daughter routes provided (simple or butterfly), the model can be applied for either Variant 2a or 2b.

In contrast to Variant 1, the transported cargoes of a given mother route is unknown because, in this variant, a transshipment can occur with a daughter route, meaning that in addition to cargoes of the mother route's ports, the cargoes of ports served by a connected daughter route have to be transported by the mother route. Also, this variant allows split pickups and deliveries where demand to and from a Norwegian port can be shared by more than one mother route visiting that port. Therefore, the model has to compute the maximum transported cargo on a selected mother route. This amount has an impact on *(i)* the capacity (and hence the cost) of the mother vessel(s) to deploy and *(ii)* the cargo handling time of the related mother route (and hence how many mother vessels to sail this route for a weekly service). As a consequence, for a selected mother route, the model must track the vessel type to use and the number of vessels to deploy.

First, the following sets and parameters are defined.

Sets

$\mathcal{K}$ set of mother vessel types, indexed by k,

$\mathcal{R}^D$ set of daughter routes, indexed by d,

$\mathcal{R}_{rp}^D$ set of daughter routes that can be linked to mother route r using transshipment port p,

$\mathcal{P}_r$ set of Norwegian ports visited by mother route r,

$\mathcal{P}_{rp}^-$ set of Norwegian ports that are visited before p in mother route r,

$\mathcal{D}^\mathcal{R}$ set of pairs of mother and daughter routes that can be linked, $\mathcal{D}^\mathcal{R} = \{(r, d), r \in \mathcal{R}^M, p \in \mathcal{P}_r, d \in \mathcal{R}_{rp}^D\}$,

$\mathcal{D}_p^\mathcal{R}$ subset of $\mathcal{D}^\mathcal{R}$ for tuples of mother and daughter routes that use port p for transshipment.

Parameters

C_k^{MH} weekly time charter cost of a mother vessel of type k,

C_{rk}^{RM} operational cost of mother route r using vessel type k; this cost includes port calls, bunker and cargo handling costs,

C_d^D total cost of daughter route d; the cost includes the weekly time charter of daughter vessel deployed on this route, bunker, port calls, cargo handling, and transshipment costs,

U_k capacity of mother vessels of type k (in TEU),

D_p cargo to deliver to port p,

P_p cargo to pick up from port p,

L_d^- total cargoes to deliver to ports served by daughter route d,

L_d^+ total cargoes to pick up from ports served by daughter route d,

H_E cargo handling rate at the continental port (in TEU per hour),

H_p cargo handling rate at port p (in TEU per hour),

S_r sailing time of mother route r (in hours),

N the maximum number of daughter vessels that can do the transshipment at a port.

The total cargoes L_d^+ and L_d^- served by daughter route d must be loaded and unloaded onto and off of mother vessels at the transshipment port and at the continental port, respectively. The transshipment cost included in C_d^D is related to the handling cost of these cargoes at both ports. Also, the time to handle these cargoes must be included in the total duration of the mother route, which is dynamically computed by the mathematical model $V2$ as will be developed in the following.

As mentioned above, model $V2$ is valid for the network structure with mother and daughter routes for both Variants 2a and 2b. Furthermore, $V2$ allows the possibility to split pickup and delivery for a port served by a mother vessel, and not a daughter vessel. For example, a demand destined to a port can be shared by two mother routes. It is also possible for a port to receive the cargo from Rotterdam by one mother route, while the cargo from this port to Rotterdam is served by another mother route.

Decision variables

x_{rk} a binary variable that takes value 1 if vessel type k is used to sail mother route r, and 0 otherwise,
y_{rk} a positive integer variable that indicates the number of mother vessels of type k used on mother route r,
q_r a non-negative continuous variable for the amount of cargo carried on mother route r including cargoes of daughter routes linked to r,
t_{rk} total sailing time and cargo handling time for mother route r with vessel type k,
z_{rd} a binary variable that takes value 1 if daughter route d and mother route r are linked, and 0 otherwise,
a_p a binary variable that takes value 1 if port p is served by a mother route, and 0 otherwise,
b^-_{rp} delivered cargoes at port p by mother route r,
b^+_{rp} picked up cargoes at port p by mother route r,

The model is formulated as follows:

$$(V2):\ \min \sum_{r\in\mathcal{R}^M}\sum_{k\in\mathcal{K}}(C^{MH}_k y_{rk} + C^{RM}_{rk}x_{rk}) + \sum_{(r,d)\in\mathcal{D}^{\mathcal{R}}} C^D_d z_{rd} \tag{4}$$

subject to:

$$\sum_{k\in\mathcal{K}} x_{rk} \leqslant 1, \quad r\in\mathcal{R}^M, \tag{5}$$

$$\sum_{r\in\mathcal{R}^M_p}\sum_{k\in\mathcal{K}} x_{rk} + \sum_{(r,d)\in\mathcal{D}^{\mathcal{R}}_p} z_{rd} \geqslant 1, \quad p\in\mathcal{P}, \tag{6}$$

$$N\sum_{k\in\mathcal{K}} x_{rk} \geqslant \sum_{(r,d)\in\mathcal{D}^{\mathcal{R}}} z_{rd}, \quad r\in\mathcal{R}^M, \tag{7}$$

$$q_r \geqslant \sum_{p'\in\mathcal{P}_r} b^-_{rp'} + \sum_{(r,d)\in\mathcal{D}^{\mathcal{R}}} L^-_d z_{rd} + \sum_{p'\in\mathcal{P}^-_{rp}}\left(b^+_{rp'} - b^-_{rp'} + \sum_{(r,d)\in\mathcal{D}^{\mathcal{R}}_{p'}}(L^+_d - L^-_d)z_{rd}\right), \quad r\in\mathcal{R}^M, p\in\mathcal{P}_r, \tag{8}$$

$$a_p \leqslant \sum_{r\in\mathcal{R}^M_p}\sum_{k\in\mathcal{K}} x_{rk}, \quad p\in\mathcal{P}, \tag{9}$$

$$a_p \geqslant \sum_{k\in\mathcal{K}} x_{rk}, \quad p\in\mathcal{P}, r\in\mathcal{R}^M_p, \tag{10}$$

$$\sum_{r\in\mathcal{R}^M_p} b^-_{rp} = D_p a_p, \quad p\in\mathcal{P}, \tag{11}$$

$$\sum_{r\in\mathcal{R}^M_p} b^+_{rp} = P_p a_p, \quad p\in\mathcal{P}, \tag{12}$$

$$\sum_{k\in\mathcal{K}} U_k x_{rk} \geqslant q_r, \quad r\in\mathcal{R}^M, \tag{13}$$

$$t_{rk} \geqslant S_r x_{rk} + \sum_{p\in\mathcal{P}_r}\left(\frac{b^+_{rp}+b^-_{rp}}{H_E} + \frac{b^+_{rp}+b^-_{rp}}{H_p}\right) + \sum_{p\in\mathcal{P}_r}\sum_{(r,d)\in\mathcal{D}^{\mathcal{R}}_p}\left(\frac{L^+_d+L^-_d}{H_E} + \frac{L^+_d+L^-_d}{H_p}\right)z_{rd}, \quad r\in\mathcal{R}^M, k\in\mathcal{K}, \tag{14}$$

$$y_{rk} \geqslant \frac{t_{rk}}{168}, \quad r\in\mathcal{R}^M, k\in\mathcal{K}, \tag{15}$$

$$x_{rk}\in\{0,1\}, \quad r\in\mathcal{R}^M, k\in\mathcal{K}, \tag{16}$$

$$y_{rk}\in\mathbb{N}, \quad r\in\mathcal{R}^M, k\in\mathcal{K}, \tag{17}$$

$$z_{rd}\in\{0,1\}, \quad (r,d)\in\mathcal{D}^{\mathcal{R}}, \tag{18}$$

$$a_p\in\{0,1\}, \quad p\in\mathcal{P}, \tag{19}$$

$$t_{rk}\geqslant 0, \quad r\in\mathcal{R}^M, k\in\mathcal{K}, \tag{20}$$

$$b_{rp}^{+}, b_{rp}^{-} \in \mathbb{N}, \quad r \in \mathcal{K}, p \in \mathcal{P}_r. \tag{21}$$

The objective function (4) minimizes the total weekly costs of the fleet of mother and daughter vessels used for the shipping network. The model decides which mother routes to sail and the number and size of mother vessels to be deployed on each selected mother route. The related operational cost is reflected in the first term of (4). The second term of (4) computes the operational cost of the daughter routes deployed in the network.

Constraints (5) enforce that for any mother route only one vessel type can be deployed. Constraints (6) state that all ports have to be served by either a mother route or a daughter route. Constraints (7) ensure that a daughter route can be selected only when a connected mother route is used. Constraints (8) compute the required vessel capacity to sail a mother route r. This computation considers the initial cargo to transport to all ports served by r and, in addition, the cargo of ports to be served by connected and selected daughter routes. The order of visited ports is important in (8) to count the delivered and picked up cargoes at each port and to correctly compute the required vessel capacity. If port p is served by a mother route, a_p is enforced to be equal to 1 by Constraints (9) and (10). In that case, variables b_{rp}^{+} and b_{rp}^{-} make it possible splitting pickups and deliveries, respectively. Their sum must be equal to the delivered and picked up cargoes as stated by Constraints (11) and (12), respectively.

Constraints (13) select the vessel type of appropriate capacity to sail mother route r. Constraints (14) compute the total duration of the selected mother routes (in hours), which includes the sailing time of the route and the cargo handling time at the origin and destination ports. More precisely, the second term of the right-hand side of Constraints (14) captures the handling time of the partial or total cargo delivered to (or unloaded at) port p using route r, while the last term computes the handling time for transshipped cargo at port p. By computing the total sailing time, the number of required mother vessels to be deployed on each route for a weekly service can be obtained as stated in Constraints (15), where 168 is the number of hours per week. Finally, Constraints (16)–(21) define the domain of the different decision variables.

4.3. Model for Variant 2 with no split delivery

Model $V2$ requires additional decision variables and constraints to allow split pickup and delivery at the transshipment ports, which has an impact on the performance of the model. Therefore, we propose a simplified version, denoted hereafter by $V3$, where no splitting is allowed.

$$(V3): \ \min (4)$$

subject to:

$$q_r \geqslant \sum_{p' \in \mathcal{P}_r} D_{p'} \sum_{k \in \mathcal{K}} x_{rk} + \sum_{(r,d) \in \mathcal{D}^{\mathcal{R}}} L_d^{-} z_{rd} + \sum_{p' \in \mathcal{P}_{rp}^{-}} \left((P_{p'} - D_{p'}) \sum_{k \in \mathcal{K}} x_{rk} + \sum_{(r,d) \in \mathcal{D}_{p'}^{\mathcal{R}}} (L_d^{+} - L_d^{-}) z_{rd} \right), \quad r \in \mathcal{R}^M, p \in \mathcal{P}_r, \tag{22}$$

$$t_{rk} \geqslant S_r x_{rk} + \sum_{p \in \mathcal{P}_r} \left(\frac{P_p + D_p}{H_E} + \frac{P_p + D_p}{H_p} \right) x_{rk} + \sum_{p \in \mathcal{P}_r} \sum_{(r,d) \in \mathcal{D}_p^{\mathcal{R}}} \left(\frac{L_d^{+} + L_d^{-}}{H_E} + \frac{L_d^{+} + L_d^{-}}{H_p} \right) z_{rd}, \quad r \in \mathcal{R}^M, k \in \mathcal{K}, \tag{23}$$

(5), (6), (7), (13), (15), (16), (17), (18), (20).

Model $V3$ is the same as $V2$, except for the computation of the total handled cargo, and the total sailing and cargo handling time of the selected mother routes. The pickup and delivery decision variables, b_{rp}^{+} and b_{rp}^{-}, are replaced with input parameters, P_p and D_p, as shown in Constraints (22) and (23).

5. Label setting algorithm for route generation

The presented mathematical models for the different network structures are based on sets of valid mother and daughter routes. These sets can be pre-generated using a shortest path label setting algorithm, which is widely used in the literature for similar problems, e.g. Irnich (2008), and is based on a dynamic programming approach to restricting the generated routes to only feasible and non-dominated routes.

5.1. Generation of mother and daughter routes

The label setting algorithm operates in sequential stages, where each stage corresponds to a partial route. A decision is made for each stage whether it is feasible and non-dominated or not.

The starting port of all mother routes is the continental port. The next candidates correspond to going from the starting port to any of the Norwegian ports. Then, each such candidate is further extended to include only south-bound port visits since only south-bound journeys are allowed, as previously explained. Furthermore, any candidate can be extended to go back to the continental main port, and a complete route is obtained (provided it respects the restriction regarding the maximum capacity among available mother vessels). The extension process is repeated in a similar way until no further port can be visited. By doing so, feasible mother routes are easily generated because only south-bound journeys are permitted and there is no time restriction on mother routes. At this point, it is worth mentioning that to compute the total cost C_r^M of mother route r required by $V1$, the route completion time is first computed according to sailing time and handled cargo on *r*. Then, the number of mother vessels that are deployed on *r* is set to the number of weeks the completion of the route takes. However, this information is not needed for $V2$ and $V3$ because the number of mother vessels is decided by the model.

The daughter routes are generated in a similar way. A daughter route can start at any Norwegian port and can be extended to any other Norwegian port. The starting port represents the transshipment port where mother and daughter routes are connected to deliver and pick up cargoes. In contrast to a mother route, a daughter route corresponds to one daughter vessel and, consequently, has both time and capacity restrictions. This means that the daughter routes have to be completed within one week, and the number of containers on board a daughter vessel cannot exceed its capacity at any time, e.g., a 100-TEU daughter vessel cannot sail a route requiring a capacity of 105 TEUs. In addition, dominance criteria are included to limit the number of routes generated. If two partial routes have the same current port and the same ports visited, but in a different order, one partial route could dominate the other one. First, the maximum number of containers on board at any time for both partial routes is checked. Indeed, although two routes include the same ports, the order of visits has an impact on the containers on board. This is because the demand differs from one port to another, and the loaded cargo of a given port can be greater or lower than the unloaded cargo. Second, the bunker costs of both partial routes are computed. Hence, when a partial route has a lower *(i)* maximum number of containers on board and *(ii)* bunker cost, it dominates the other partial route. By applying these steps, the result is a set of feasible and non-dominated simple routes. To generate butterfly daughter routes, it is sufficient to merge two simple routes having a total duration less or equal to one week.

To illustrate the dominance between two partial daughter routes, we consider an example with four ports A, B, C, and D, where A is the transshipment port. The demands to B, C, and D are 35, 20, and 20 TEUs, respectively. The demands to transport from B, C, and D are 30, 50, and 15 TEUs, respectively. For the partial route A-B-C-D, the number of containers on board the vessel after leaving each port is 75, 70, 100, and 95, respectively. Thus, the maximum number of containers on board is 100. Conversely, for the partial route A-C-B-D, the number of containers on board after visiting each port is 75, 105, 100, and 95 TEUs, respectively. The resulting maximum number of containers on board is 105. If in addition, the bunker cost of the first partial route is lower than the bunker cost of the second partial route, then the first partial route dominates the second one.

5.2. Route generation-based heuristic

Even though a label setting algorithm is used for route generation, the number of routes increases exponentially as the number of ports increases. Therefore, it can be required to reduce the set of routes given as input to the mathematical model in order to solve larger instances.

We do this by exploiting the following geographical characteristic of our case study problem, where the ports are located along a coastline (more or less along a vertical line). Most likely, the most cost-efficient routes have ports that are close to each other and go only in one direction rather than ones that “zig-zag” up and down along the coastline. Hence, whether it is for mother or daughter routes, a partial route is extended up to only the *e*-th nearest ports in south-bound direction from the current port or returns to the initial port. Here, *e* is an input parameter to the procedure, which controls the size of the considered daughter routes and, thus, the number of routes generated in the process. Also, an additional step is added for the daughter route generation so that a complete route is duplicated as many times as the number of visited ports, where each duplication has a different transshipment port among the visited ports.

Fig. 4 illustrates an example of a partial route starting from Averøy, which is here the transshipment port, and extended to southern ports. Because the parameter *e* is set to 2, the partial route Averøy-Ålesund-Måløy can be extended to Bergen or Husnes (which are the two ports closest to Måløy), but not to Haugesund and Tananger. The extension to Husnes is represented in Fig. 4. Furthermore, the partial route Averøy-Ålesund-Måløy-Husnes is duplicated to have three other partial routes with the order of ports, but different transshipment ports: Ålesund, Måløy and Husnes.

6. Computational study

The aim of the computational study is to compare the different proposed liner shipping network structures. More precisely, we investigate whether an advanced network design than the feeder network could be beneficial for the shipping company, in which transshipment between mother and daughter routes, as well as splitting deliveries and pickups, are allowed. The experiments are performed on various problem instances based on realistic data. The mathematical models were implemented using OPL script language and solved with the commercial solver Cplex 12.9. To generate mother and daughter routes, the label setting algorithm has been implemented using Visual Studio C++. The routes were generated on a Lenovo laptop operating Windows 10 with Intel i7 processor and 8 GB RAM. To solve the instances, a computer cluster was used on a node with Lenovo M5 having 2 × Intel E5-2643v3 processor and 512 GB RAM.

6.1. Test instances

Fig. 5 shows a map with the 21 Norwegian ports served by the case company. This set of ports constitutes the basis of test instances from which different classes are derived. In all, there are three classes, each of which comprises four instances. First, Class A is constructed such that the total number of ports (including the continental port) is either 10 or 11 ports. The two first instances, A1 and A2, include ports that are rather located in the south and middle of Norway, i.e. from Tananger to Orkanger; whereas the ports of instances A3 and A4 are scattered throughout the Norwegian coastline and include the northernmost served port, Stokmarknes. The

Fig. 4. A partial route generated from Averøy with the heuristic for $e = 2$.

purpose of this class is to measure the performance of the different model variants. The second class, B, consists of medium-sized instances with 15 ports. These ports are randomly selected among the ports served by the company. Instances B1, B2 and B3 include ports that are dispersed through the Norwegian coastline (from Egersund to Stokmarknes). However, instance B4 has Gjemnes (located in the middle of Norway) as the northernmost port to be served.

Lastly, Class C represents the largest instances that are built with up to 22 ports served by the company. Instance C2 contains genuine data. The served ports and the demand from/to the continental port are provided/estimated by the case company. Instance C1 is built using the same data of C2, but excluding two of the ports. It is introduced to study the impact of the number of ports on the CPU time for the route generation and solver. Instances C3 and C4 are similar to C2 but assume that there is a future increase in demand. The demand for C2 is randomly increased so that the total demand of C3 and C4 represents an increase of about 30% and 85%, respectively. A summary of the different instances is given in Table 1.

The case company currently deploys (on time charter) a fleet of vessels with capacities from around 700 to 900 TEUs. In our study, we consider a heterogeneous fleet of mother vessels with extended sizes to test whether the company could benefit from deploying larger ships. Indeed, the model decides the optimal number and size of mother vessels to deploy among the candidate vessel types with capacities of 700, 900, 1100, and 1500 TEUs, respectively. Similarly, the fleet of daughter vessels is chosen among a set of candidate vessel types with capacities of 100, 200, and 300 TEUs, respectively. The weekly time charter (TC) cost (in USD) and fuel consumption rates (in tonnes/hour) of each vessel type are reported in Table 2 (Hamburg Index, 2019). Based on data for similar container vessels, the sailing speed is assumed to be the same for all vessels and estimated to be 12 knots. The bunker price is set to 600 USD per tonne for low sulfur marine gas oil. This price is roughly based on the average market price in Rotterdam (Bunker, 2019).

The fixed cost related to a port call is established from the average of several ports' fees in Norway and estimated to be 100 USD for all daughter vessels, and 200 USD for all mother vessels. Further, the cargo handling rate in Norwegian ports is assumed fixed to 15 TEU/hour. This rate is slightly higher for the continental port, which is 20 TEU/hour. The cost to handle one container at any port is set to 30 USD/TEU.

6.2. Results for Class A instances

The first experiment is performed on instances of Class A and aims to test the performance of the different network structure variants. Since the problem size of this class is relatively small, all models were tested by considering all non-dominated routes (`All routes`) and compared with the route generation-based heuristic for $e = 1$ and $e = 2$. The results related to route generation are depicted in Table 3, where `# MR` and `# DR` report the number of mother and daughter routes generated, respectively. The CPU time in seconds required to generate routes for each configuration is also reported. As can be seen from Table 3, the total number of routes for

Fig. 5. Map of the Norwegian ports served by the liner shipping company.

Table 1
Test instances with ports included and demand to (Southbound) and from (Northbound) Rotterdam.

Instance	# ports	Included ports	Total weekly demand (TEU)	
			Northbound	Southbound
A1	10	Håvik, Haugesund, Husnes, Svelgen, Ikornnes, Ålesund, Molde, Gjemnes, Orkanger	572	498
A2	10	Tananger, Haugesund, Bergen, Florø, Måløy, Ålesund, Molde, Gjemnes, Orkanger	698	806
A3	11	Egersund, Tananger, Haugesund, Bergen, Måløy, Molde, Gjemnes, Orkanger, Salten, Stokmarknes	606	698
A4	11	Tananger, Haugesund, Bergen, Florø, Måløy, Ålesund, Molde, Glomfjord, Salten, Stokmarknes	733	846
B1	15	Egersund, Tananger, Haugesund, Bergen, Høyanger, Svelgen, Hareid, Ålesund, Sunndalsøra, Averøy, Orkanger, Glomfjord, Salten, Stokmarknes	810	863
B2	15	Egersund, Haugesund, Bergen, Høyanger, Måløy, Hareid, Ikornnes, Molde, Sunndalsøra, Gjemnes, Averøy, Glomfjord, Salten, Stokmarknes	585	688
B3	15	Egersund, Håvik, Husnes, Bergen, Høyanger, Florø, Svelgen, Måløy, Sunndalsøra, Gjemnes, Averøy, Orkanger, Glomfjord, Salten	530	544
B4	15	Egersund, Tananger, Håvik, Haugesund, Husnes, Bergen, Florø, Svelgen, Måløy, Ikornnes, Ålesund, Molde, Sunndalsøra, Gjemnes	831	894
C1	20	All ports except Ikorness and Ålesund	885	965
C2	22	All ports	1089	1135
C3	22	All ports	1449	1460
C4	22	All ports	1926	2200

each variant depends on the number of ports, the combination between mother and daughter routes, and the structure (simple or butterfly) of daughter routes. The number of daughter routes grows particularly fast if we include butterfly routes (Variant 2b), which can be counteracted by applying the heuristic restriction to the generation process. Overall, the route generation procedure is very efficient for this problem size as it takes at most 0.24 s.

Table 2
Candidate vessel types for mother and daughter vessels.

Capacity (TEU)	TC (USD)	Fuel cons. (ton/hr)
	Mother vessels	
700	49,000	0.50
900	56,000	0.58
1100	63,000	0.65
1500	70,000	0.73
	Daughter vessels	
100	25,000	0.23
200	30,000	0.28
300	35,000	0.33

Table 3
Results for the route generation of Class A instances.

		Variant 1	Variant 2a			Variant 2b		
		All routes	All routes	$e = 1$	$e = 2$	All routes	$e = 1$	$e = 2$
A1	# MR	511	511	45	221	511	45	221
	# DR	–	1440	116	907	21,239	191	4588
	CPU (s)	0.01	0.07	0.01	0.03	0.24	0.01	0.07
A2	# MR	511	511	45	221	511	45	221
	# DR	–	794	80	446	9147	137	1561
	CPU (s)	0.01	0.09	0.01	0.03	0.23	0.01	0.03
A3	# MR	1023	1023	55	364	1023	55	364
	# DR	–	1420	158	968	7086	245	2190
	CPU (s)	0.01	0.06	0.02	0.06	0.12	0.01	0.05
A4	# MR	1023	1023	55	364	1023	55	364
	# DR	–	1130	101	609	4893	166	1301
	CPU (s)	0.01	0.03	0.01	0.03	0.08	0.02	0.06

Table 4
Solution cost and CPU time for CPLEX, and fleet of optimal solution for instances of Class A.

		Variant 1	Variant 2a			Variant 2b		
		All routes	All routes	$e = 1$	$e = 2$	All routes	$e = 1$	$e = 2$
A1	Cost (USD)	142,160	142,160	142,160	142,160	142,160	142,160	142,160
	CPU (s)	0.03	198.3	0.4	23.9	882.5	0.4	37.4
	Moth. Vess.	2 × 700	2 × 700	2 × 700	2 × 700	2 × 700	2 × 700	2 × 700
	Daugh. Vess.	–	-	–	–	–	–	–
A2	Cost (USD)	161,868	161,868	161,868	161,868	161,868	161,868	161,868
	CPU (s)	0.03	212.8	0.5	26.0	757.4	0.5	38.6
	Moth. Vess.	2 × 900	2 × 900	2 × 900	2 × 900	2 × 900	2 × 900	2 × 900
	Daugh. Vess.	–	–	–	–	–	–	–
A3	Cost (USD)	227,970	200,073	204,462	204,462	196,198	204,462	204,662
	CPU (s)	0.1	2031.2	2.0	140.3	3516.7	1.4	162.9
	Moth. Vess.	1 × 700; 2 × 700	2 × 700	2 × 900	2 × 900	2 × 700	2 × 900	2 × 900
	Daugh. Vess.	–	1 × 100	1 × 100	1 × 100	1 × 100;	1 × 100	1 × 100
A4	Cost (USD)	228,180	200,875	208,221	208,221	200,875	208,221	208,179
	CPU (s)	0.2	2407.7	0.9	187.2	3027.5	1.0	175.1
	Moth. Vess.	1 × 700; 2 × 700	2 × 900	2 × 900	2 × 900	2 × 900	2 × 900	2 × 900
	Daugh. Vess.	–	1 × 100	1 × 200	1 × 200	1 × 100	1 × 200	1 × 100

Table 4 summarizes the obtained results from solving the models to optimality, where `Cost (USD) is` related to the total costs of the optimal solution and `CPU (s)` gives the required computational time (in seconds) for CPLEX to solve the corresponding model. The next two rows (`Moth.Vess` and `Daugh.Vess`) provide the fleet of used mother and daughter vessels of the solution, e.g. `1 × 700; 2 × 700` means that the solution includes two mother routes where the first route requires one 700-TEU vessel (the duration of this route is less than or equal to one week) and the second route requires two 700-TEU vessels (the duration of this route is between one and two weeks).

In this experiment, Variant 1 is compared against Variants 2a and 2b to study the impact of introducing daughter vessels on the network structure. The heuristic-based route generation is tested for different values of the neighborhood parameter, $e = 1$ and $e = 2$. Since the problem size of instances of Class A is small, model $V2$ is tested with all possible non-dominated mother and daughter routes for both Variants 2a and 2b. As can be seen from Table 4, optimal solutions could be obtained in a reasonable computational time. For instance, the average CPU time of Variant 2b with all possible routes is about 2046 s. When it comes to operational costs, it can be observed that the same solutions were obtained for instances A1 and A2 with all variants. The same fleet of mother vessels was optimal for transporting the cargoes of the whole system, meaning that there is no benefit from using daughter vessels. Conversely, instances A3 and A4 reveal that operational costs can be reduced by 13.8% when daughter vessels are introduced. The cost reduction for instance A4 is mainly due to reducing the fleet of mother vessels and replacing it with larger vessels, i.e. using two 900-TEU vessels instead of three 700-TEU vessels.

An interesting result can be also observed for instance A3 as the fleet of mother vessels is reduced from a total of three 700-TEU vessels to only two for Variants 2a and 2b when all routes are considered. The solution for Variant 2a reveals that the optimal mother route sails from Tananger to Salten, while the daughter route uses Tananger port for transshipment and serves Egersund and Stokmarknes, which are the southernmost and northernmost ports of this instance, respectively. By doing so, the demand is better distributed between the different routes and lower mother vessel capacities are sufficient. Also, the operational cost can be further reduced when butterfly routes are considered as it can be seen from the result of A3 with Variant 2b. The mother route obtained with Variant 2b is shorter than the mother route of the solution of Variant 2a, which is translated in a reduced bunker cost. This is possible due to the introduction of the butterfly route structure that allows for a better use of daughter vessels. However, we should precise that this is the only instance for which a better cost than Variant 2a is obtained.

Furthermore, Table 4 shows that the CPU time required to solve $V2$ drastically increases as the number of routes increases. The CPU time required for instance A3 to solve Variant 2b goes from 1 s when routes are generated with $e = 1$ to about 3517 s when all non-dominated routes are considered. However, the heuristic-based route generation cannot guarantee optimal solutions. For example, the solutions found for instances A3 and A4 with Variants 2a and 2b are not optimal, but do still represent interesting solutions as the operational cost is reduced by around 10% compared to Variant 1.

6.3. *Results for Class B and C instances*

The second experiment is related to the Class B and C instances. The results of the route generation are shown in Table 5. In this experiment, Variants 2a and 2b are restricted to be tested with the route generation-based heuristic and $e = 1$ only. Although the route generation procedure shows a good performance when generating all routes even for large instances (for example, all mother and butterfly daughter routes for C2, which are more than 5.5 million routes, were generated in about 425 s), the complexity of the model makes $V2$ intractable to solve with $e > 1$ for these problem sizes. This means that for Variants 2a and 2b, better solutions might exist.

Table 6 displays the results on the instances from Classes B and C. In general, the required computational time to solve the different instances is reasonable, except for C3 with Variant 1. Also, it can be seen that the same costs are obtained for Variants 2a and 2b for all instances of Classes B and C, meaning that there is no benefit from introducing advanced route structures in this case.

Table 6 shows there is a significant cost reduction when introducing daughter vessels for instances B2, and B4. Compared to Variant 1, the cost can be reduced by up to 7.8%. A similar observation applies for instances C1 and C3 where the reduction rates are 2.0% and 6.9%, respectively. For all these instances, the reduction is made possible by reducing the fleet of mother vessels and introducing daughter vessels. The resulting network has a “relay” structure where a mother route reaches a port at the middle or north of Norway, and then a daughter route is used for northernmost ports. To illustrate, Fig. 6 shows the solution of instance C3 obtained with both Variants 2a and 2b.

As mentioned in Section 3.1, Variant 1 represents the current practice for the case company. Furthermore, given that *(i)* instances C2, C3, and C4 include only the ports served by the company, and *(ii)* their solutions for Variant 1 are composed of a fleet of 700-TEU and 900-TEU vessels (which corresponds to the actual vessel sizes of the company), these solutions can be considered as reference solutions. As it can be seen from Table 6, no cost improvement is possible from introducing daughter vessels and/or split deliveries and pickups for instance C2, whereas cost reductions are obtained with Variants 2a and 2b for instances C3 and C4. Thus, depending on the demand, the company could decrease the operational costs by considering advanced network structure.

One interesting observation from Table 6 is related to the solutions of instances B1 and C4. The costs obtained with both Variants 2a and 2b are lower than the cost of Variant 1 although no daughter route is used. When analyzing these solutions, the difference comes from allowing split pickups and deliveries in $V2$, which, in this case, contributes to cost reduction. It should be noted that the solution of instance C3 also includes a split pickup and delivery. As illustrated in Fig. 6, both mother routes 1 and 2 serve the ports at Hareid, Ikornness, and Ålesund, meaning that cargoes of these ports are split between the two mother routes.

Another interesting observation from the results of Classes A, B and C is that solutions with daughter vessels are related to instances including ports located in the northern part of Norway. Indeed, the solutions of instances A3 and A4 include daughter routes

Table 5
Results for the route generation of Class B and C instances.

		Variant 1 All routes	Variant 2a $e = 1$	Variant 2b $e = 1$
B1	# MR	16,383	105	105
	# DR	–	435	749
	CPU (s)	0.25	0.03	0.03
B2	# MR	16,383	105	105
	# DR	–	1607	2626
	CPU (s)	0.20	0.07	0.07
B3	# MR	16,383	105	105
	# DR	–	2342	4088
	CPU (s)	0.17	0.06	0.16
B4	# MR	16,383	105	105
	# DR	–	295	620
	CPU (s)	0.23	0.02	0.02
C1	# MR	524,287	190	190
	# DR	–	1403	2,586
	CPU (s)	9.67	0.08	0.11
C2	# MR	2,097,151	231	231
	# DR	–	1225	2,322
	CPU (s)	40.34	0.11	0.25
C3	# MR	2,097,031	231	231
	# DR	–	636	1106
	CPU (s)	46.45	0.06	0.09
C4	# MR	1,716,691	231	231
	# DR	–	209	337
	CPU (s)	36.58	0.04	0.04

that go further to the north to serve Stokmarknes port, whereas mother routes are sufficient for instances A1 and A2, as these include ports located only in the south and middle part of Norway. The same remark is also applicable when the ports and solutions of instance B4 are compared to the other instances of Class B. In terms of costs, it is more beneficial to use daughter routes to serve the most northern ports with longer sailing distances compared to mother routes, which require additional mother vessels with higher charter cost and fuel consumption.

6.4. Effects of allowing split pickups and deliveries

Table 7 further studies the effect of allowing split pickups and deliveries at ports visited by mother routes for both versions of Variant 2. The results are reported for only Class C to study the impact on the current problem size operated by the company. For instances C1, C3, and C4, there is a cost decrease when splitting is allowed. It can be considered as marginal for instance C1, but it is significant for instances C3 and C4, which is 5.2% and 5.7%, respectively. Using a fleet of smaller mother vessels contributes to having lower operational costs, i.e. 700-TEU and 900-TEU vessels are sufficient to transport all cargoes and offer lower costs compared to 1100-TEU and 1500-TEU vessels. Upon analyzing the solution of instance C3 with no split (model $V3$), the first mother route serves the ports from Salten to Ålesund, then goes back to the continental port. The maximum number of containers on board of this route is 683 TEUs, which can be served by a 700-TEU vessel. The second route starts with the Norwegian ports at Ikornnes, Hareid, and Måløy, then serves the ports in South before arriving at the continental port. The maximum number of containers on board for this route is 963 TEUs that requires an 1100-TEU vessel. However, when split pickups and deliveries are allowed (model $V2$), the ports at Hareid, Ikornness, and Ålesund are served by both Mother route 1 and Mother route 2, as shown in Fig. 6. For example, the cargo to deliver to Ålesund consists of 282 TEUs. Mother route 1 transports 190 TEUs, and Mother route 2 transports 92 TEUs for this delivery. Conversely, the demand of Hareid is delivered by Mother route 2 and picked up by Mother route 1. By doing so, cargoes are better distributed between the mother routes. Indeed, the maximum number of containers on board is 675 and 900 TEUs for Mother route 1 and Mother route 2, respectively. Consequently, 700-TEU and 900-TEU vessels are sufficient to transport the whole demand of instance C3.

7. Conclusion

This paper studied different network structures to offer a cost-effective liner network shipping design. The case study is related to a liner shipping company that ensures the cargo transportation between a European continental port and Norwegian ports with a

Table 6
Results for the Class B and C instances.

		Variant 1 All routes	Variant 2a $e = 1$	Variant 2b $e = 1$
B1	Cost (USD)	274,215	238,570	238,570
	CPU (s)	9.6	9.1	10.5
	Moth. Vess.	3 × 1100	1 × 700; 2 × 700	1 × 700; 2 × 700
	Daugh. Vess.	–	–	–
B2	Cost (USD)	233,370	215,098	215,098
	CPU (s)	0.6	9.3	10.8
	Moth. Vess.	1 × 700; 2 × 700	2 × 900	2 × 900
	Daugh. Vess.	–	1 × 200	1 × 200
B3	Cost (USD)	158,060	158,060	158,060
	CPU (s)	0.5	5.6	6.9
	Moth. Vess.	2 × 700	2 × 700	2 × 700
	Daugh. Vess.	–	–	–
B4	Cost (USD)	212,100	206,669	206,669
	CPU (s)	0.8	10.1	20.9
	Moth. Vess.	1 × 700; 2 × 700	2 × 1100	2 × 1100
	Daugh. Vess.	–	1 × 100	1 × 100
C1	Cost (USD)	276,672	271,451	271,451
	CPU (s)	1674.7	74.5	85.3
	Moth. Vess.	3 × 1100 –	1 × 700; 2 × 700	1 × 700; 2 × 700
	Daugh. Vess.	–	1 × 100	1 × 100
C2	Cost (USD)	316,426	294,510	294,510
	CPU (s)	212.8	126.3	145.7
	Moth. Vess.	2 × 700; 2 × 900	2 × 700; 2 × 700	2 × 700; 2 × 700
	Daugh. Vess.	–	–	–
C3	Cost (USD)	365,247	344,048	344,048
	CPU (s)	78,679.0	239.7	503.3
	Moth. Vess.	3 × 700; 2 × 900	2 × 700; 2 × 900	2 × 700; 2 × 900
	Daugh. Vess.	–	1 × 100	1 × 100
C4	Cost (USD)	451,305	444,963	444,963
	CPU (s)	195.1	1656.9	2024.0
	Moth. Vess.	2 × 700; 2 × 700; 2 × 900	2 × 700; 2 × 700; 2 × 900	2 × 700; 2 × 700; 2 × 900
	Daugh. Vess.	–	–	–

weekly service frequency. Both feeder network design and hub-and-spoke network design were studied. Simple and butterfly daughter routes, as well as the possibility of splitting pickups and deliveries at the ports were considered in the hub-and-spoke network design. The proposed approach to solve the different versions of the problem consists of two steps. In the first step, all non-dominated (mother and daughter) candidate routes are generated using a label setting algorithm. In the second step, the routes are given as an input to the mathematical models. Moreover, a heuristic that generates a limited number of routes is developed to solve large problem sizes.

The computational study was carried out on different problem instances derived from realistic data in collaboration with the case company. The results showed that the current practice of the liner shipping company based on using only mother routes can be improved by introducing daughter routes and split pickups and deliveries, which lead to a significant cost decrease. Also, the daughter routes can be limited to simple cycles as there is only limited benefit from extending them to butterfly routes in this case study. Thus, a shipping company can select the best design between feeder network and hub-and-spoke network by using the developed solution method as a decision support tool.

Even though hub-and-spoke showed to offer interesting results for some cases in terms of operational costs, the results are mainly limited by the heuristic performance. Indeed, as shown in the computational study, better results could be obtained and, therefore, a more advanced heuristic approach should be developed, especially when additional ports are included. Optional cargo is also another aspect to consider for the development of an advanced heuristic. In this mode, it is possible to accept or reject some cargo that is

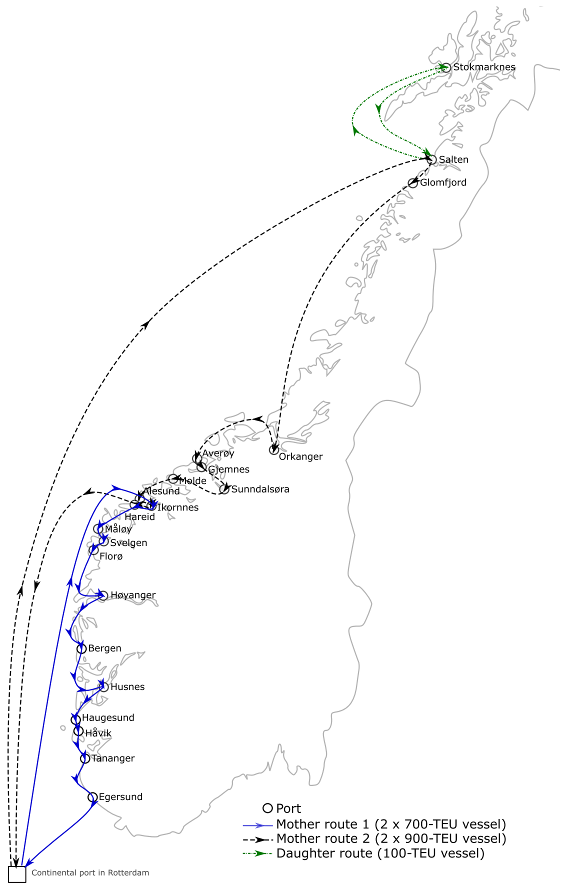

Fig. 6. Mother and daughter routes of the obtained solution for instance C3 with Variants 2a and 2b.

Table 7
Comparison between split (model $V2$) and no split pickups and deliveries (model $V3$) on Class C instances.

		Variant 2a (e = 1)		Variant 2b (e = 1)	
		No split	Split	No split	Split
C1	Cost (USD)	272,335	271,451	272,335	271,451
	CPU (s)	14.4	74.5	16.0	85.3
	Moth. Vess.	1 × 700;	1 × 700;	1 × 700;	1 × 700;
		2 × 700	2 × 700	2 × 700	2 × 900
	Daugh. Vess.	1 × 200	1 × 100	1 × 200	1 × 100
C2	Cost (USD)	294,510	294,510	294,510	294,510
	CPU (s)	18.2	126.3	21.3	145.7
	Moth. Vess.	1 × 700;	1 × 700;	1 × 700;	1 × 700;
		2 × 700	2 × 700	2 × 700	2 × 700
	Daugh. Vess.	–	–	–	–
C3	Cost (USD)	361,798	344,048	361,798	344,048
	CPU (s)	15.3	239.7	16.6	503.3
	Moth. Vess.	2 × 700;	2 × 700;	2 × 700;	2 × 700;
		2 × 1100	2 × 900	2 × 1100	2 × 900
	Daugh. Vess.	1 × 100	1 × 100	1 × 100	1 × 100
C4	Cost (USD)	470,331	444,963	470,331	444,963
	CPU (s)	5.4	1656.9	6.3	2024.0
	Moth. Vess.	2 × 1100;	2 × 700;	2 × 1100;	2 × 700;
		3 × 1500	2 × 700;	3 × 1500	2 × 700;
			2 × 900		2 × 900
	Daugh. Vess.	–	–	–	–

considered optional. The problem then becomes to maximize the revenue of transported cargo. The problem can also be extended to consider more than one continental hub with a general demand pair pattern between ports. In this case, the developed mathematical models should be extended to consider the additional problem assumption, and an advanced solution method should be developed to solve large problems.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at https://doi.org/10.1016/j.tre.2020.101839.

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