Reliability assessment combining importance resampling and the cross entropy method☆

Ivar Bjerkebæk *, Håkon Toftaker

Department of Energy Systems, SINTEF Energy Research, Trondheim, Norway

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ABSTRACT

In the ongoing transition towards a sustainable energy system, the electric power system increases in complexity and must adapt to rapid changes and new uncertainties. Thus, development and application of appropriate probabilistic methods is all the more important. Reliability analysis based on Monte Carlo simulation (MCS) has become increasingly popular, and its strength lies in the ability to account for a large number of random variables, general stochastic processes and assessing the probability distribution of the output. However, power system reliability is governed by relatively rare interruption events which poses a fundamental challenge to MCS. This paper presents a variance reduction technique for Monte Carlo based reliability analysis which combines resampling and the cross entropy (CE) method. The motivation for the work is to mitigate the computational burden related to rare event sampling, while at the same time preserving the flexibility of MCS by introducing few assumptions on the stochastic model. The method is demonstrated on a synthetic test system and gives a speedup of about 10 times compared to a crude simulation.

1. Introduction

The modern power system is facing increased uncertainty and variation in both production and load patterns due to integration of intermittent renewable energy resources and new power intensive technologies such as electrification of transport and process industry. At the same time, digital solutions for continuous monitoring and control offer possibilities for smarter and more flexible system operation. Having robust and flexible methods for reliability evaluation is a necessary prerequisite to succeed in maintaining a rational level of reliability in a developing and increasingly complex power system. Current techniques for power system reliability analysis can be classified in two main categories [1]; analytical methods and Monte Carlo simulation (MCS). Monte Carlo techniques are usually computationally faster and less cumbersome to implement for systems with complex generation and demand patterns. However, analytical methods may have an advantage in dealing with rare events for systems subject to simpler operating conditions. Monte Carlo techniques can again be divided in non-sequential simulation and sequential simulation. In the non-sequential approach, samples of the system state are drawn from a stationary distribution, while the sequential approach simulates the system state in a chronological manner. In addition, there exists hybrid approaches [2]. Monte Carlo simulation has proven to be extremely useful in dealing with large and complex systems with a high number of random variables, and sequential Monte Carlo simulation (SMCS) is the natural candidate for modelling time dependent systems. The SMCS approach has been used to model various time dependent, stochastic mechanisms influencing power system reliability such as generation from wind power [3–5], EV charging [6], component failures linked to adverse weather [7], and failure mechanisms linked to time development of technical condition [8].

Power system reliability analysis by SMCS poses a major computational challenge, and the computational burden is usually substantially higher than for non-sequential methods. The high computational cost can be explained by two factors. The first challenge relates to rare event sampling. In this work, we limit the analysis to consider permanent component outages in the transmission grid. Interruptions caused by such outages are rare, but can have severe consequences. This makes it necessary to draw a large number of samples to estimate reliability indices with sufficient precision since most samples will not contain an interruption event. The second challenge is related to contingency analysis, i.e. determining the system response to failure states which usually involves power flow, or optimal power flow (OPF) calculations.

These calculations can be time consuming and may exhibit challenging convergence properties, especially for large systems. Challenges related to contingency analysis applies to both Monte Carlo based and analytical reliability analysis.

Variance reduction (VR) techniques can be applied to accelerate the convergence of Monte Carlo simulations. VR techniques have in common that they exploit some prior knowledge of the system to reduce the variance of the Monte Carlo estimator [9]. Different VR techniques which have been applied to power system reliability analysis include Latin hypercube sampling [10], subset simulation [11], and the cross entropy (CE) algorithm. The CE algorithm or CE method, first described in [12], has proven to achieve great variance reduction for many types of systems. The CE method is based on importance sampling and uses an iterative procedure to search for an importance sampling distribution (ISD) which is close to the theoretically optimal distribution. Variations of the CE method has been adapted to different simulation models for reliability evaluation. Notably it has been applied to sequential simulations [3,5,13]. Other applications include non-sequential models for composite transmission and generation systems [14], generation adequacy [15], and later a non-sequential model for generating adequacy in multi-area power system with correlated wind power generation [16]. Apart from reliability evaluation, the CE method has also been adapted to other probabilistic power system calculations such as probabilistic load flow [17] and security constrained optimal power flow [18].

The CE method must be tailored to search for a close-to-optimal ISD for a specific type of parametric distribution. To the best of the author's knowledge, previous applications of the CE method to SMCS have assumed that the component states follow a two-state Markov-process. That is, a component which can be in either a functioning state or a failure state where the times spent in each state are exponential random variables with some mean time to failure (MTTF) and mean time to repair (MTTR). Since a Markov-process is memoryless by definition [19], this puts considerable restrictions on how time dependent influencing factors can be included. One of the main strengths of SMCS is that in addition to obtaining expected values, the distribution of the reliability index can be estimated. A caveat with the CE method and importance sampling in general is that although expected values of reliability indices can be estimated with high accuracy, the distribution of the reliability indices gets distorted. The motivation for assessing the output distribution is to gain information about the consequence of power interruptions beyond the expectation value.

This paper addresses the abovementioned drawbacks with the CE method applied to time dependent systems, namely the limited ability to model general time dependent stochastic processes, and the distortion of the probability distribution. The main contributions of the work are:

(i) A novel cross entropy technique based on importance resampling which does not rely on information about the underlying stochastic processes and thus can be applied to a general set of reliability models.

(ii) A demonstration of how the cumulative distribution (CDF) of the target function can be restored after importance sampling with the CE method.

The remainder of the article is structured as follows: Section 2.1 describes the sequential Monte Carlo Simulation, Section 2.2 explains the resampling technique, 2.3 gives a short introduction to the CE method, and 2.4 explains how these concepts are combined in the proposed method. Simulation results are presented and discussed in Section 3.
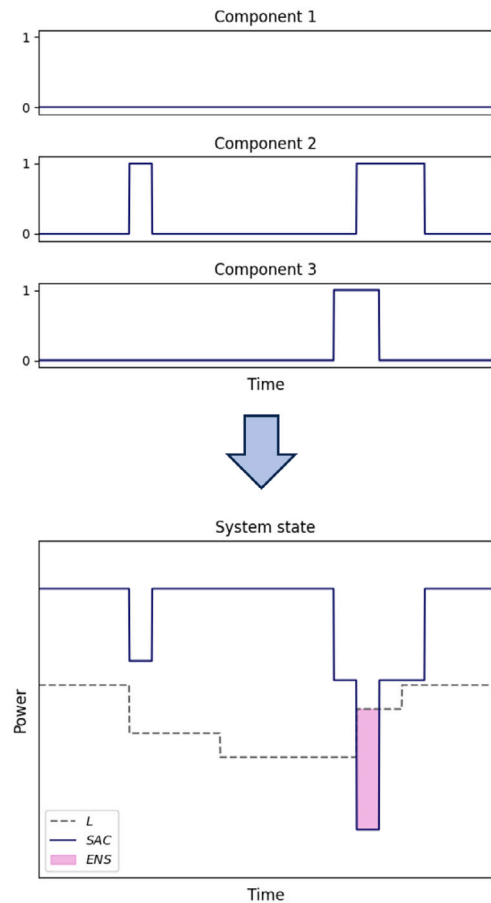

Fig. 1. Illustration of SMCS for a hypothetical system with 3 components. The component states are combined to generate a time series of the system's available capacity (SAC) in the bottom figure. The interrupted power is the difference between SAC and the load *L*. Energy not supplied (ENS) is the time integral of interrupted power.

2. Method

2.1. Sequential Monte Carlo simulation

The following subsection briefly describes the main principles of the SMCS which was implemented. The simulation is divided in two main processes, namely random sampling of component states and consequence analysis of the resulting contingencies. The *contingency state* is represented by a binary stochastic vector $\mathbf{X}(t) = [X_1(t), \dots, X_n(t)]$, where each variable $X_i \in \{0, 1\}$ represents the state of a power system component. A component is functioning when $X_i = 0$ and is out of operation when $X_i = 1$. The evolution of the contingency state is simulated chronologically by drawing the time to failure (TTF) and time to repair (TTR) from the respective probability distributions (PDFs) for each component. When the TTF or TTR is drawn, the time is incremented and the simulation stops when the time exceeds the simulation period *T* which is set in advance. The component states are assumed to be conditionally independent, therefore each component is simulated in parallel to improve the run time. In addition, only samples which contain a component failure are stored in memory. The evolution of the component states is illustrated in Fig. 1. In principle the sojourn times, TTF and TTR, can be random variables with arbitrary distributions, and generally the contingency state follows a semi-Markov process [19]. In the special case where sojourn times are exponential random variables, the contingency state follows a Markov process.

The overall system state is defined by a contingency state and an *operating state*. The operating state is specified by a combination of

load demand $\mathbf{L}(t)$ and generation $\mathbf{G}(t)$ on all buses in the system. The operating state also includes the topological state (breaker positions etc.). For each combination of operating state and contingency state we calculate the amount of interrupted power P_{interr}, and we use the methodology for contingency analysis by [20,21] for this purpose. The contingency analysis is based on an AC-OPF which minimizes the costs of power generation and load shedding.

2.2. Resampling

The cross entropy method we use is based on a resampling technique which increases the possible number of outcomes for the system evolution. Each sample is a trajectory of the contingency state $\mathbf{J} = [J_1, \ldots, J_n]$, where J_i is the trajectory of a single component state. Component state trajectories are illustrated in Fig. 1. The component state trajectory can be represented mathematically as a collection of tuples

$$J_i = \left[(x_{i0}, t_{i0}), (x_{i1}, t_{i1}), \ldots \right], \tag{1}$$

where $x_{ij} \in \{0, 1\}$ are the component states, and $t_{ij} \in \{0, T\}$ are the corresponding transition times from one state to the next. Note that the relation to the contingency state vector is $X_i(t_{ij}) = x_{ij}$. Having sampled N state trajectories $J_{i1}, \ldots, J_{iN}$, for a component i, we can write the empirical distribution [22] for the trajectory as

$$\hat{f}_i(j_i) = \frac{1}{N} \sum_{k=1}^{N} I\{j_i = J_{ik}\}. \tag{2}$$

where I is the indicator function, and the hat notation is used to distinguish it from the true PDF of the trajectory, f. Notice that the true distribution f is continuous, while the empirical counterpart (2) is a discrete distribution. The empirical distribution can also be expressed as a product

$$\hat{f}_i(j_i) = \prod_{k=1}^{N} N^{-I\{j_i = J_{ik}\}}, \quad j_i \in \{J_{ik}\}. \tag{3}$$

(3) is not normalized unless we specify that the sample space consists of the observed trajectories only. The advantage of rewriting (2) to a product will be apparent when we later solve the *cross entropy problem* in (12) which involves the logarithm of $\hat{f}$.

The idea of using resampling is to exploit the fact that the components are statistically independent to construct a joint distribution for the system as the product of the marginal distributions, that is

$$\hat{f}(\mathbf{j}) = \prod_{i=1}^{n} \hat{f}_i(j_i). \tag{4}$$

Note that $\hat{f}$ is not an empirical distribution in the normal sense because the sample space contains contingency state trajectories which are not present in the non-permuted set of component state trajectories. Having simulated N state trajectories for n components, $\hat{f}$ is defined on a state space consisting of all N^n permutations of the component state trajectories. A random permutation of component trajectories is obtained by resampling from (4). Let $\mathbf{J}^\dagger$ denote a resampled system state trajectory, that is

$$\begin{aligned} \mathbf{J}^\dagger &= [J_1^\dagger, \ldots, J_n^\dagger], \\ J_i^\dagger &\in \{J_{ik}\}, \quad k = 1, \ldots, N, \\ \mathbf{J}^\dagger &\sim \hat{f}(\mathbf{j}). \end{aligned} \tag{5}$$

The permutation principle is shown in Fig. 2. To provide an example, the topmost system trajectory in Fig. 2 corresponds to the vector $\mathbf{J}^\dagger = [j_{11}, j_{21}]$, the second topmost trajectory corresponds to $\mathbf{J}^\dagger = [j_{11}, j_{22}]$ and so on. The hypothesis is that all the possible permutations of the component trajectories contain more information about the system than the original samples without permutation. However, it is not feasible to estimate reliability indices by summing the contribution from all permutations since the number of permutations is extremely large for any relevant number of N. Therefore we want to use importance resampling to sample the "important" permutations more frequently.
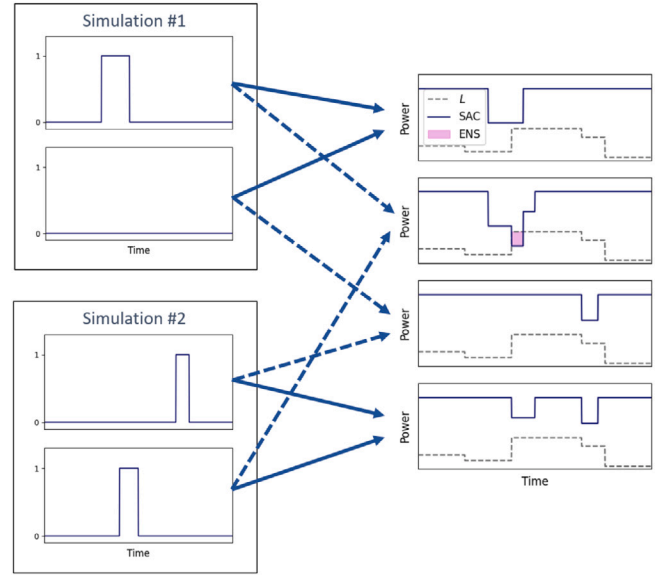

Fig. 2. Permutation of component state trajectories (left) to form new system state time series (right). The example illustrates $N = 2$ simulations of a hypothetical system with $n = 2$ components. The number of possible permutations is $N^n = 4$.

2.3. Importance sampling and the CE method

To aid the following discussion we introduce some concepts and notation related to the general problem of Monte Carlo integration. Let $\mathbf{X}$ be a random vector with PDF $f(\mathbf{x})$

$$\mathbf{X} \sim f(\mathbf{x}). \tag{6}$$

Suppose we want to compute the expected value θ of some *target function* $h(\mathbf{X})$.

$$\theta \equiv E[h(\mathbf{X})] = \int h(\mathbf{x}) \, f(\mathbf{x}) \, d^n x. \tag{7}$$

Monte Carlo integration is the method of approximating θ by random sampling. We sample N independent random vectors from f and approximate θ by the sample average

$$\hat{\theta} = \frac{1}{N} \sum_{j=1}^{N} h(\mathbf{X}_j). \tag{8}$$

$\hat{\theta}$ is also called the *crude* Monte Carlo estimate.

Importance sampling [9] is a variance reduction technique where samples are drawn from a distribution g, different from the true distribution f. The importance sampling estimator can be found by rewriting the expected value in (7) as

$$\theta = \int h(\mathbf{x}) \frac{f(\mathbf{x})}{g(\mathbf{x})} g(\mathbf{x}) = E_g \left[h(\mathbf{X}) \frac{f(\mathbf{X})}{g(\mathbf{X})} \right], \tag{9}$$

where E_g denotes the expected value under g. Thus, the importance sampling estimator is

$$\tilde{\theta} = \frac{1}{N} \sum_{j=1}^{N} h(\mathbf{X}_j) \frac{f(\mathbf{X}_j)}{g(\mathbf{X}_j)}. \tag{10}$$

For a smart choice of ISD, the variance of $\tilde{\theta}$ can be much smaller than that of the crude estimate, and finding a suitable ISD is the main challenge of importance sampling. In fact there exists a theoretically optimal ISD, g^*, resulting in identically zero variance of $\tilde{\theta}$. The optimal ISD is given by

$$g^*(\mathbf{x}) = \frac{h(\mathbf{x}) \, f(\mathbf{x})}{\theta}. \tag{11}$$
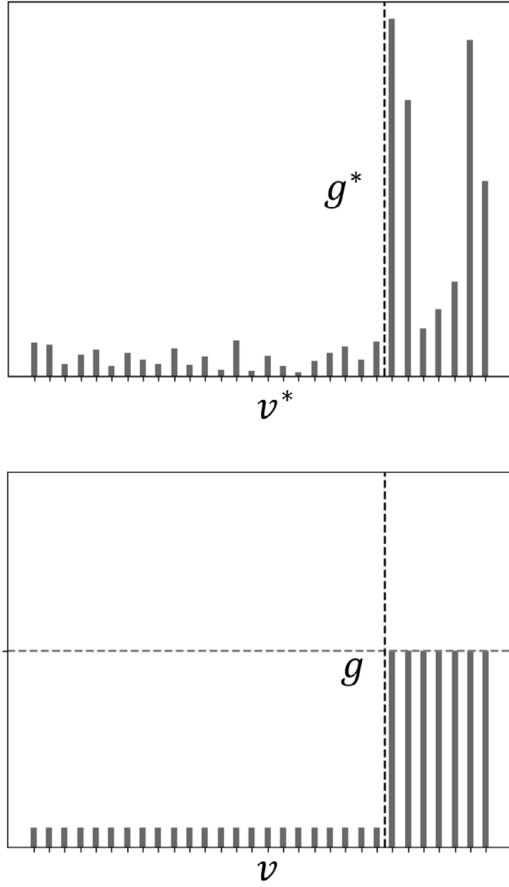
Fig. 3. Top: Hypothetical sketch of the theoretically optimal ISD g^*. Bottom: A close-to-optimal ISD g formed by dividing the sample space in two, and forcing uniform likelihood in each subspace.

This can easily be verified by inserting g^* into (10). The problem is of course that we do not know the value of θ in advance.

The cross entropy algorithm [9,12,23] presents an automatic method to find an ISD that is as close as possible to g^* under the constraint that the ISD must belong to the same type of parametric distribution as f. This is done by minimizing the *Kullback–Leibler distance*, also called cross entropy, between the ISD and g^*. It can be shown [23] that an unbiased estimator for the parameters that minimize the cross entropy is obtained by solving the maximization program

$$\arg\max_{\mathbf{v}} \sum_{j=1}^{N} h(\mathbf{X}_j)\, W(\mathbf{X}_j; \mathbf{u}, \mathbf{w})\, \ln f(\mathbf{X}_j; \mathbf{v}). \tag{12}$$

where $\mathbf{u}$ is the parameter vector for the reference distribution, $f = f(\mathbf{x}; \mathbf{u})$, $\mathbf{v}$ are the parameters for the ISD, $g = f(\mathbf{x}; \mathbf{v})$, and $\mathbf{X}_j \sim f(\mathbf{x}; \mathbf{w})$ where $\mathbf{w}$ is arbitrary. W denotes the likelihood ratio as $W(\mathbf{X}; \mathbf{u}, \mathbf{w}) \equiv \frac{f(\mathbf{X}; \mathbf{u})}{f(\mathbf{X}; \mathbf{w})}$.

2.4. Combining resampling and the CE method

In the following subsection we introduce a novel method combining resampling and the CE algorithm, and a solution of the cross entropy problem (12) will be derived. The derivation was originally presented in the first author's masters thesis [24]. The idea of using importance resampling was inspired from previous applications of importance resampling to estimate bootstrap distribution tails [25,26], but bootstrap resampling is fundamentally different from the proposed sampling method where observations are permuted. The resampling

distribution $\hat{f}$ defined in (4) is a multivariate finite support discrete distribution. Each marginal distribution $\hat{f}_i$ is parameterized by N probability weights, and thus the joint distribution $\hat{f}$ has $n \times N$ parameters. Although the solution to the cross entropy problem (12) is known for finite support discrete distributions, there are far too many parameters v_{ik} that needs to be estimated in this case. Since N is the number of initial simulations, the number of parameters increases with the sample size which is clearly an unfavourable property. Therefore we introduce an alternative parametrization such that each marginal distribution is specified by a single parameter. We do this by dividing all observed component state trajectories J_i defined in (1) in two sub-spaces, and then constrain the ISD to have uniform likelihood in each subspace. The first subspace Ω_1 contains all observed trajectories that visit the failure state at some point, and all other observed trajectories belong to the complementing subspace $\Omega_0 = \Omega_1^c$. The hypothesis is that trajectories that visit the failure state are more important when estimating reliability indices, and thus should have a higher likelihood under the theoretically optimal ISD on average. This idea is illustrated in Fig. 3. Now, a formal derivation of the CE solution for this parametrization follows, and for the sake of clarity we repeat that n denotes the number of components, and N denotes the number of samples. The resampling distribution can be written as

$$\hat{f}(\mathbf{j}; \mathbf{u}) = \prod_{i=1}^{n} \prod_{k=0}^{1} u_{ik}^{I\{j_{ik} \in \Omega_{ik}\}}. \tag{13}$$

Note that when the value of all parameters is $u_{ik} = N^{-1}$, this reduces to the same distribution as defined in (4). Inserting the expression for $\hat{f}$ into (12), the CE problem reads

$$\arg\max_{\mathbf{v}} \sum_{l=1}^{M} h(\mathbf{J}_l^\dagger)\, W(\mathbf{J}_l^\dagger; \mathbf{u}, \mathbf{w}) \sum_{i=1}^{n} \sum_{k=0}^{1} I\{J_{li}^\dagger \in \Omega_{ik}\}\, \ln v_{ik}, \tag{14}$$

where M is the number of resampled system trajectories, which may be smaller or larger than N. Problem (14) can be solved by Lagrange multipliers. The maximization problem (14) is subject to the n constraints that the marginal distributions $\hat{f}_i$ must be normalized. The normalization criteria can be expressed as

$$\sum_{k=0}^{1} N_{ik}\, v_{ik} - 1 = 0, \tag{15}$$

where N_{ik} denotes the number of trajectories in each subspace $N_{i0} + N_{i1} = N$. From (14) and (15), the Lagrange function can be defined as

$$\mathcal{L}(\mathbf{v}, \lambda_1, \dots, \lambda_n) \equiv$$
$$\sum_{l=1}^{M} h(\mathbf{J}_l^\dagger)\, W(\mathbf{J}_l^\dagger; \mathbf{u}, \mathbf{w}) \sum_{i=1}^{n} \sum_{k=0}^{1} I\{J_{li}^\dagger \in \Omega_{ik}\}\, \ln v_{ik}$$
$$- \sum_{i=1}^{n} \lambda_i \left(\sum_{i=0}^{1} N_{ik}\, v_{ik} - 1 \right) \tag{16}$$

The solution of the CE problem is given by the critical point of the Lagrange function.

$$\frac{\partial \mathcal{L}}{\partial v_{ik}} = 0, \tag{17}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_i} = 0. \tag{18}$$

From (17) we find that

$$v_{ik} = \frac{1}{\lambda_i N_{ik}} \sum_{l=1}^{M} h(\mathbf{J}_l^\dagger)\, W(\mathbf{J}_l^\dagger; \mathbf{u}, \mathbf{w})\, I\{J_{li}^\dagger \in \Omega_{ik}\}. \tag{19}$$

And from (18) it follows by inserting (19) that

$$\lambda_i = \lambda = \sum_{l=1}^{M} h(\mathbf{J}_l^\dagger)\, W(\mathbf{J}_l^\dagger; \mathbf{u}, \mathbf{w}). \tag{20}$$

Combining (19) and (20) we finally obtain the estimator for the optimal parameters

$$v_{ik} = \frac{1}{N_{ik}} \frac{\sum_{l=1}^{M} h(\mathbf{J}_l^\dagger) \, W(\mathbf{J}_l^\dagger; \mathbf{u}, \mathbf{w}) \, I\{J_{li}^\dagger \in \Omega_{ik}\}}{\sum_{l=1}^{M} h(\mathbf{J}_l^\dagger) \, W(\mathbf{J}_l^\dagger; \mathbf{u}, \mathbf{w})}. \tag{21}$$

Note that there is only one free parameter per component, since when the value of one of the two parameters is set, the other one follows from normalization according to (15).

In the following, we describe how this result was implemented in a CE algorithm to estimate expected energy not supplied (EENS). Energy not supplied is the time integral of interrupted power P_{interr} over a given period, and EENS is as the name suggests the expected value of ENS [20]. In this paper, all values of ENS and EENS are annual values. As target function we use $h(\mathbf{J}) = I\{\text{ENS}(\mathbf{J}) > 0\}$. Note that the expected value of h gives the probability of non-zero ENS, $E[I\{\text{ENS}(\mathbf{J}) > 0\}] = P(\text{ENS} > 0)$, which is a rare event. The following enumerated steps describe how the CE algorithm was implemented using the solution of the CE problem in (21). The reader is directed to Refs. [9,23] for a comprehensive explanation and motivation of the iterative procedure described in the steps 0–3. The portion of samples ρ with non-zero ENS in each iteration is used to determine when the algorithm has converged,

0. (Preparation). Sample N state trajectories J_i for each power system component i via SMCS as described in Section 2.1. These trajectories form the resampling distribution $\hat{f}$ defined in (4). Set the iteration count $t \leftarrow 0$, and choose a moderately small value for the parameter ρ, e.g. $\rho \leftarrow 0.05$. Initialize the parameter vector to the same value as the reference parameters $v_{ik}^{(0)} \leftarrow u_{ik} = N^{-1}$.

1. Resample M system state trajectories according to $\hat{f}(\mathbf{j}; \mathbf{v}^{(t)})$ and estimate the parameter vector by

$$v_{ik}^{(t+1)} = \frac{1}{N_{ik}} \frac{\sum_{l=1}^{M} h(\mathbf{J}_l^\dagger) \, W(\mathbf{J}_l^\dagger; \mathbf{u}, \mathbf{v}^{(t)}) \, I\{J_{li}^\dagger \in \Omega_{ik}\}}{\sum_{l=1}^{M} h(\mathbf{J}_l^\dagger) \, W(\mathbf{J}_l^\dagger; \mathbf{u}, \mathbf{v}^{(t)})}, \tag{22}$$

Update the parameter vector to

$$v_{ik}^{(t+1)} \leftarrow \alpha \, v_{ik}^{(t+1)} + (1 - \alpha) \, v_{ik}^{(t)}, \tag{23}$$

where $\alpha < 1$ is a tuning parameter which improves convergence by limiting the increments in parameter values.

2. Determine the portion $\rho^{(t)}$ of the M resampled system trajectories that give nonzero ENS. If $\rho^{(t)} \geq \rho$, the CE algorithm has converged and we proceed to step 3. If not, increment the iteration count $t \leftarrow t + 1$ and repeat steps 1–2.

3. Use the final parameter vector to construct the ISD, $f(\mathbf{x}; \mathbf{v})$, and use importance resampling to estimate the system EENS.

In addition to estimating EENS, we estimate the cumulative distribution of ENS. Assume that we have resampled K system trajectories $\mathbf{J}_1^\dagger, \ldots, \mathbf{J}_K^\dagger$ in step 3, and calculated the corresponding energy not supplied by contingency analysis $\text{ENS}_1, \ldots, \text{ENS}_K$. This set of ENS values is not representative for the system since these originate from system trajectories sampled from the ISD, and thus give a distorted ENS distribution. However, we can evaluate the cumulative distribution on chosen points by importance sampling with the same set of ENS values. This is done as follows. For a value x of ENS, estimate $P(ENS \leq x)$ by

$$\tilde{P}(\text{ENS} \leq x) = \frac{1}{K} \sum_{l=1}^{K} I\{\text{ENS}(\mathbf{J}_l) \leq x\} \, W(\mathbf{J}_l; \mathbf{u}, \mathbf{v}). \tag{24}$$

The estimator in (24) should have lower variance than the corresponding crude estimator since the CE algorithm finds an ISD that better resolves the part of sample space where ENS > 0. This approach is not limited to a specific importance sampling scheme. Note that this calculation can be performed efficiently on a computer since we only need to evaluate the identity function and calculate the inner product of two vectors. Thus the cumulative distribution can be evaluated for a range of values x with a low computational cost.
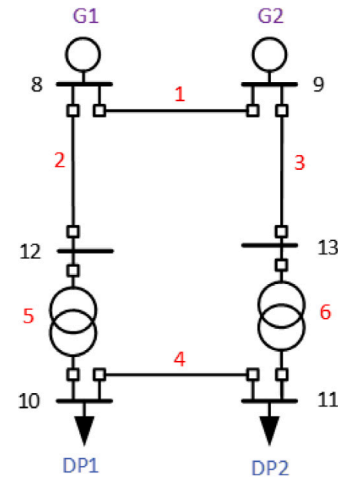

Fig. 4. Single-line diagram of the test system. The system is a 132/66 kV meshed transmission grid with: 6 bus bars, 2 transformers, 4 lines, 2 generators, and 2 delivery points (load points).

Table 1
Comparison of performance for the CE method and crude Monte Carlo (CMC).

	EENS [kW h] (std. error [%])	$P(\text{ENS} > 0)$	N	CPU time [s]
CE	1250.4 (4.7)	$6.28 \cdot 10^{-4}$	$2 \cdot 10^5$	41.1
CMC	1195.1 (4.8)	$6.21 \cdot 10^{-4}$	$3 \cdot 10^6$	398.2
Analytical	1262.9	–	–	–

3. Results and discussion

The method is demonstrated on a reliable 6-bus test system shown in Fig. 4. Failure rates and repair rates are chosen from [27], which are in turn obtained from the Norwegian standardized system for collection and calculation of disturbance and reliability data [28]. The test system is described in greater detail, and is openly available at [29]. The component states are modelled by a two-state Markov process with MTTF and MTTR set to the inverse of the failure and repair rate. The Markov process is chosen because the analytical EENS value can be calculated by integrating the known stationary distribution [21], using the common two-state Markov model also improves reproducibility of results. To simplify validation and comparison with the analytical method, the study is limited to consider a single operating state of demand and generation. As the sampling procedure only applies to component states, scale up to multiple operating states should be unproblematic. The stop criterion used for the simulation is that the relative standard error β of EENS must be lower than 5% and a crude simulation was run for comparison of performance. The relative standard error is given by $\beta = \hat{\sigma}/\hat{\theta}$, where $\hat{\sigma}$ is empirical standard error and $\hat{\theta}$ is the EENS estimator. The results are reported in Table 1. When using the CE method, the same number of samples N was used both in the initial simulation stage, and for resampling. The analytical result is calculated using the methodology from [20]. The convergence plots for the two simulations is shown in Fig. 5, and the estimated cumulative distributions are shown in Fig. 6.

Compared to the crude simulation, the sample size needed to reach sufficient accuracy, i.e. that the relative standard error is less than 5%, is reduced by 93% with the CE approach. And the total run time of the CE simulation is 10% of the crude simulation. The cumulative distributions obtained with the CE method and the crude simulation are hard to distinguish, which shows that the CE method can be used to estimate distribution tails with significantly fewer samples. And in the case of ENS, the tail is the only interesting part of the distribution since most system trajectories are mapped to zero. The CDF shows that
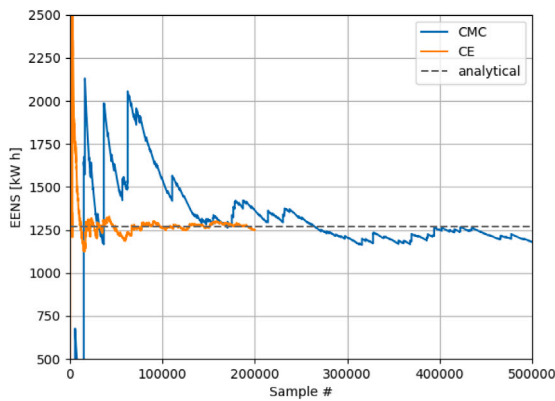
Fig. 5. Convergence plot of EENS for the crude estimator and the CE estimator. The analytical value is plotted for reference. The x-axis is truncated at $N = 5 \cdot 10^5$, but the crude simulation converged at $N = 3 \cdot 10^6$.
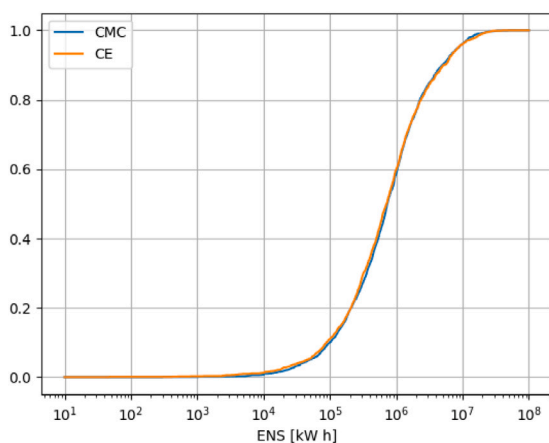

Fig. 6. Estimated cumulative distribution for ENS, using the CE method and CMC. The graphs show the conditional probability given nonzero ENS.

the distribution of ENS is highly skewed, and resembles a log-normal distribution. The median nonzero ENS value obtained from the CDF is 0.71 GW h, which is 2–3 orders of magnitude larger than the EENS. And the 95th percentile is 6.8 GW h, i.e. 3–4 orders of magnitude larger than EENS. The expected value holds limited information about the distribution of ENS since the distribution is skewed and a major part of the probability is at 0, which means the probability of experiencing ENS values close to EENS is often close to zero. This demonstrates how reliability indices in terms of expectation values give limited information on system risk on their own since they do not properly communicate the vulnerability to severe, and possibly catastrophic interruptions. In this case double outage of transformers.

Since the purpose of this article is to present a new computational method, the test case was kept relatively simple and the proposed method was demonstrated on a small test system with constant load demand. However, the method is applicable to systems with variable load and production resulting in a large number of operating states. This is especially relevant for reliability analysis of power systems with a high share of intermittent renewable energy production. The Monte Carlo simulation also has potential to include stochastic load and production in an importance sampling scheme to efficiently capture the contribution from operating states associated with high system risk. As mentioned, the proposed CE resampling scheme does not assume a specific stochastic process governing component failures and can therefore be used to assess the system risk due to component failures dependent on influencing factors such as weather, load, or technical condition and maintenance strategies.

Computational issues were encountered in initial experiments to apply the proposed CE resampling method to larger, reliable test systems. To elaborate, the suggested cross entropy scheme is limited to search within the space of distributions with independent sample paths. With increasing dimension two issues arise, first this set of distributions may not contain a good ISD and second, it becomes difficult to estimate the optimal parameters **v** and thus obtain convergence with the CE algorithm. Both issues can lead to high variance of the importance sampling estimator due to extreme skewness of the resulting distribution of the likelihood ratio. This problem is sometimes referred to as *degeneracy of the importance sampling estimator* [9]. The authors suggest that further development of the proposed method should focus on enhancing the convergence of the CE algorithm for larger systems.

4. Concluding remarks

This paper has presented a novel method for calculating reliability indices via SMCS which significantly increases the accuracy of the Monte Carlo estimator, and alleviates the computational cost related to rare event simulation. The only assumption required to use the suggested variance reduction technique is that component states are conditionally independent. The reliability analysis is extended by assessing the cumulative distribution of ENS. The cumulative distribution gives information about the risk of severe interruption events, and this information is not contained in conventional reliability indices. The CDF is recovered from the distorted distribution resulting from importance sampling.

CRediT authorship contribution statement

Ivar Bjerkebæk: Conceptualization, Methodology, Software, Visualization, Writing – original draft, Writing – review & editing. **Håkon Toftaker:** Conceptualization, Methodology, Software, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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