Considering Head Variations in a Linear Model for Optimal Hydro Scheduling

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ABSTRACT

Linear models are very often used for hydropower optimization, both because this often represents a good approximation to the problem and because powerful software is generally available. In general, however, it is difficult to properly take into account the effects of variations in head in linear optimization algorithms. This is because head variations create non-linear conditions, and may lead to non-convexity. Even in cases where head variation is small compared to nominal head, it may be an important factor to consider when optimizing reservoir management. The problem can be solved exactly for a single reservoir, but for water-courses with cascaded plants and reservoirs simplifications have to be made.

This paper describes a solution where head sensitivity coefficients are introduced in a linear stochastic optimization model in a somewhat heuristic manner. The model in question is based on stochastic dual dynamic programming (SDDP) (Pereira 1989), is mainly used for mid-term scheduling in hydro systems, and requires linear conditions. The head coefficients are an extension of the model that entail awarding extra storage in reservoirs with plants that benefit from added head. In addition to presenting the solution technique and a theoretical background, cases are presented that show positive effects of introducing these coefficients in the form of simulated income and reservoir management.

1 INTRODUCTION

Maintaining optimal head is one of many considerations when operating a hydropower system. Finding the optimal head is however not a trivial problem, since it has to be weighed against other considerations such as risk of overflow, operating at best possible turbine efficiency and generating when prices are highest, all taking into account an uncertain future inflow and market price for generated power. The relative importance of considering head variations depends on the characteristics of a particular system, but it is nearly always a factor that one would prefer to take into consideration if possible. An optimization model that does not in some way take head variations and their effects on generation into account will generally produce a less than optimal strategy, perhaps even an unacceptable strategy for operating some hydropower systems.

Hydropower optimization models are often linear models, because this generally gives a good approximation to the problem and allows use of very efficient linear optimization algorithms. Introducing variable head creates non-linearities, and may lead to non-convexity. So in order to use linear optimization algorithms the effects of head variations are generally either ignored, or an approximate methodology such as the one described in this paper is introduced. A more precise solution is possible for systems without cascaded reservoirs, but for the general case approximations are necessary.

SINTEF Energy Research has developed the ProdRisk model for scheduling in hydropower systems based on stochastic dual dynamic programming (SDDP, see (Pereira 1989)) in combination with stochastic dynamic programming (Gjelsvik *et al.* 1999). SDDP requires linear conditions or at least convexity. The model is mainly designed for mid-term scheduling in hydropower systems, but it may also include optimal hedging for a defined utility function in the optimization problem (Mo *et al.* 2001). The present paper describes correction coefficients for variable head, in the form of head sensitivities, that have been implemented and tested in this model.

2 MATHEMATICAL DESCRIPTION

The variable head correction used in this paper is based upon an expansion around a nominal reservoir operation schedule. Release is considered fixed, and sensitivities of economic gain with respect to small changes in reservoir levels are calculated and added to the cost function, to approximately account for head variations. This section describes this expansion.

Consider a hydropower system with n reservoirs. The study period is divided into T time intervals of length one week each. (In the description of the power market, a finer subdivision may be added.) We focus on the i-th reservoir, with a storage of V_t^i at the end of week t and a water surface elevation of h_t^i, referred to sea level, say. We assume that there is a power plant with output P_t^i immediately downstream of this reservoir. If there is a reservoir below this plant, let j be its number and h_t^j its water surface elevation. In general, we then assume that the outlet of plant i in reservoir j is submerged, so that there is pressure coupling.

We assume that generated power P_t^i depends linearly on water head:

$$P_t^i = f^i\left(u_t^i\right)\frac{h_t^i - h_t^j}{h_o^i} \tag{1}$$

where u_t^i is water release and h_0^i the nominal head for plant i. The function f^i describes how the output varies with the release at nominal head. We then obtain:

$$\frac{\partial P_t^i}{\partial h_t^i} = \frac{P_t^i}{h_t^i - h_j^j} \text{ and } \frac{\partial P_t^i}{\partial h_t^j} = -\frac{P_t^i}{h_t^i - h_t^j} \tag{2}$$

We now consider the situation where the volume V_t^i is changed by an amount ΔV_t^i, without changing the release u_t^i (the change can be thought of as being brought about by a different operation at earlier stages). One effect of this is that the power generation in power plant i changes. We obtain:

$$\frac{\partial P_t^i}{\partial V_t^i} = \frac{\partial P_t^i}{\partial h_t^i}\frac{\partial h_t^i}{\partial V_t^i} = \frac{1}{A_t^i}\frac{P_t^i}{h_t^i - h_t^j} \tag{3}$$

where A_t^i is the current surface area of reservoir i at time t.

For the influence of the downstream reservoir j, we obtain correspondingly, for plant i:

$$\frac{\partial P_t^i}{\partial V_t^j} = \begin{cases} -\dfrac{1}{A_t^j}\dfrac{P_t^i}{h_t^i - h_t^j} & \text{when the outlet is submerged} \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Let the prevailing market price of power be λ_t. We take λ_t as the marginal value of changed generation. As seen from power plant *i* the change of cost (negative of change in income) due to the changes in reservoir volumes upstream and downstream is:

$$\lambda_t \frac{P_t^i}{h_t^i - h_t^j}\left[-\frac{\Delta V_t^i}{A_t^i} + \frac{\Delta V_t^j}{A_t^j}\right] \tag{5}$$

The first term has a negative sign, because an increase in reservoir content increases head and reduces cost. The positive sign of the second term is due to reduced generation and increased cost when the tailwater elevation is increased. The second term is zero if there is no reservoir downstream, or if the outlet is not submerged.

We now assume that we have available a nominal reservoir operation schedule with nominal values of $\{P_t^i\}$, $\{\lambda_t\}$, $\{V_t^i\}$, $\{h_t^i\}$ and $\{A_t^i\}$ for all *t* and *i*. Using the above formulas, we may then approximately account for the cost change due to variable head by use of extra cost terms containing ΔV:

$$\sum_{t=1}^{T}\sum_{i=1}^{n} c_t^i \Delta V_t^i \tag{6}$$

where the c_t^i-coefficients follow directly from (5).

Formula (5) contributes to both c_t^i and c_t^j. Using (5) and collecting terms, we find:

$$c_t^i = \lambda_t \left[-\frac{P_t^i}{h_t^i - h_t^j} + \frac{P_t^k}{h_t^k - h_t^i}\right]\frac{1}{A_t^i} \tag{7}$$

to be used in (6). Here *k* is the index of the power plant immediately above reservoir *i*, if there is any. As expected an increase in reservoir storage decreases cost associated with the downstream plant (*i*), but increases cost in the upstream plant (*k*). A *c*-coefficient may be positive or negative. As seen from (7), this also depends on the nominal generations P_t^i and P_t^k.

The basic interval of one week may have time-varying prices. In that case, the term $\lambda_t P_t^i$ is replaced by the corresponding sum over suitable subintervals within the week.

In the above, we have assumed the reservoir levels h_t^i to be constant over the week. This may be a good approximation in many cases, at least for relatively large reservoirs. We have chosen, however, to use mean reservoir levels, taken as the mean of the values at the beginning of the week and at the end. Thus the value of ΔV_t^i at the end of week *t* affects the generation in both week *t* and *t*+1. In this case (5) is replaced by

$$\frac{1}{2}\lambda_t \frac{P_t^i}{h_t^i - h_t^j}\left[-\frac{\Delta V_t^i}{A_t^i} + \frac{\Delta V_t^j}{A_t^j}\right] + \frac{1}{2}\lambda_{t+1}\frac{P_{t+1}^i}{h_{t+1}^i - h_{t+1}^j}\left[-\frac{\Delta V_t^i}{A_t^i} + \frac{\Delta V_t^j}{A_t^j}\right] \tag{8}$$

and (7) has to be changed correspondingly.

Use of the sensitivities derived above has been implemented in the model ProdRisk, as mentioned in the Introduction. This model uses hyperplanes to represent the expected future cost functions. The model states are the reservoir contents and inflow states. For this to work, the cost functions must be convex, and all state dependency must be contained in the hyperplanes. So it is not possible in general to have different model coefficients for various system states. Therefore the mean values of the sensitivity coefficients calculated above are used, where the mean is taken over the various inflow scenarios.

As indicated above, calculations are carried out in two steps. First, the scheduling program is run without head coefficients. From the releases and reservoir and price trajectories obtained from this run, a full set of head sensitivities $\{c_t^i\}$ is calculated for each inflow scenario. For each week, the sensitivities are averaged over the different inflow scenarios. The mean values of the sensitivities are then used to perturb the cost function according to (6) in a second run. The second run is then generally considered as giving the final schedule. Repeated recalculation of the head sensitivities based on rerunning the program with the last calculated sensitivities is possible, but the sensitivities could in principle oscillate between each calculation. Tests indicate, however, that results may converge after a few repetitions of the recalculation procedure. Furthermore, recalculation does not seem necessary for a good result.

3 SAMPLE CASES

In this section two cases are presented to illustrate the effects of introducing head sensitivity coefficients in an SDDP-based hydro optimization model. The effects of head coefficients have been tested by comparing simulated system behaviour based on strategies calculated with and without head coefficients respectively. Simulations are conducted using historical inflow statistics available for the respective systems, for these cases 50 – 60 years of inflow. We have mainly compared economic results, generated energy and reservoir operation.

Testing a strategy by simulating in this way may be a bit risky, and there are some pitfalls to be avoided. 50 – 60 years of inflow statistics is not enough to be sure of avoiding sampling errors, but enough that we feel relatively comfortable with the results.. The ProdRisk model will calculate a strategy, and simulate operation, from a given initial reservoir storage. The model depends on the EOPS model (Flatabø *et al.* 1998) for valuating reservoir contents at the end of the planning horizon. The EOPS model is based on a combination of stochastic dynamic programming to calculate marginal water values for an aggregate hydro model and a rule-based logic for disaggregation. In order to avoid effects of which initial reservoir contents are used, and avoid having to valuate end point reservoir contents when comparing generation and economic results, the model has been run with a planning horizon of 3 years for these tests and results extracted from the second year. Ideally a longer planning period could have been used, e.g. 5 years with results extracted from year 3 or 4, but for these cases 3 years has proved sufficient. For the second year initial reservoir storage is very nearly equal to final reservoir storage on average for the simulated historical scenarios, and the effects of end point storage valuation are not dominating.

3.1 CASE 1

Case 1 consists of two separate rivers with hydro systems sharing a common market, illustrated in Figure 1. The mean annual generation from the two rivers is about 2200 GWh.

What typifies this system is that there are two major reservoirs with plants that have considerable variations in head, reservoirs 2 and 6. The largest plant is the one below reservoir 4, with close to 75% of the total generation. But reservoir 4 is small and is in the model operated more or less along a fixed planned storage level every week. Of the other plants, the one below reservoir 6 provides about 20% of the total generation.

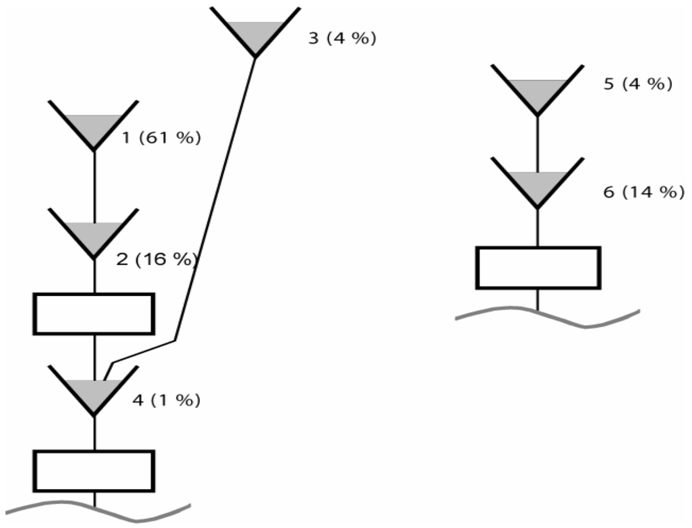

Figure 1 The Case 1 hydro system. Numbers in parenthesis show each reservoir's share of the total reservoir volume. The boxes indicate plants. Reservoir 2 has a maximum head variation of 30 meters, almost 60% of nominal head. For reservoir 6 maximum variation is almost 50 meters and nominal head over 900 meters.

We have run the model for this system with and without head coefficients. The model shows an increase in income when head coefficients are included of 6.4 NOK/MWh on average, about US$1/MWh at present exchange rates, referring to generation *without* head coefficients. Average market prices are 194 NOK/MWh. Generation increases by about 1% on average. This is achieved without any significant increase in spilled water.

Including head coefficients changes the strategy for operation of reservoirs considerably, as shown in Figure 2 and Figure 3. It is mainly these changes that affect generation and income. Since reservoir 4 has a fairly fixed operation, the main contribution to increased generation and improved results with head coefficients comes from changed storage in reservoirs 1, 2, 5 and 6.

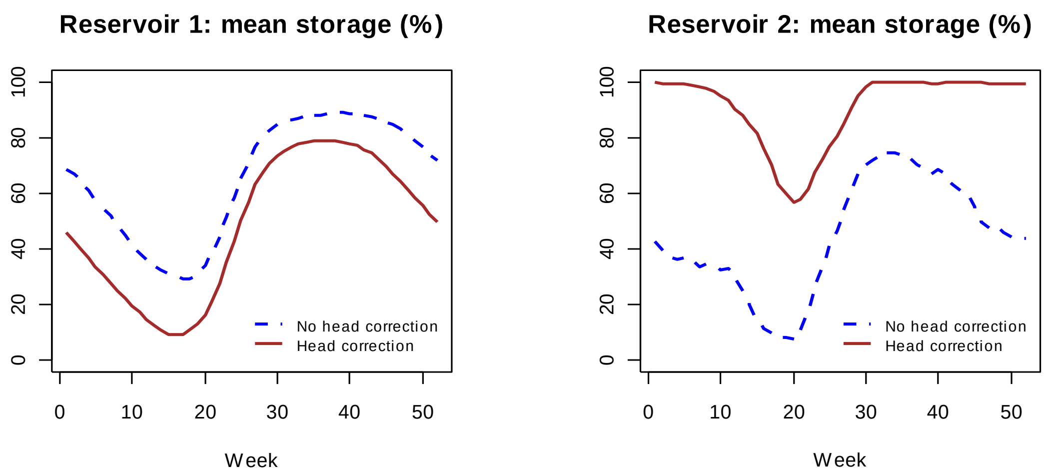

Figure 2 Simulated mean storage in reservoirs 1 and 2. Head considerations cause depletion of reservoir 1 in order to increase head above the plant below reservoir 2.

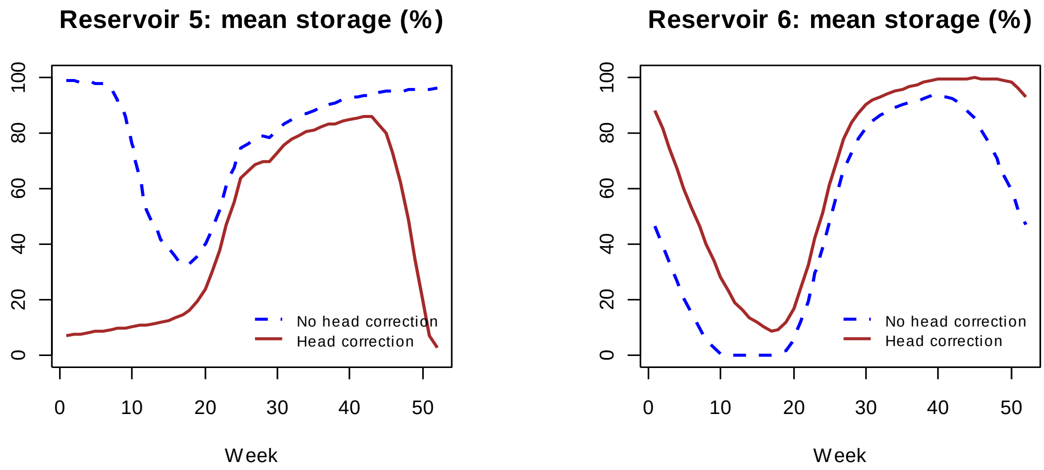

Figure 3 Simulated mean storage in reservoirs 5 and 6. Head considerations cause depletion of reservoir 5 in order to increase head above the plant below reservoir 6.

3.2 CASE 2

Case 2 also consists of two separate rivers, illustrated in Figure 4. The mean annual generation from the system is about 3270 GWh. There are four plants with significant variations in head (5, 7, 8 and 10).

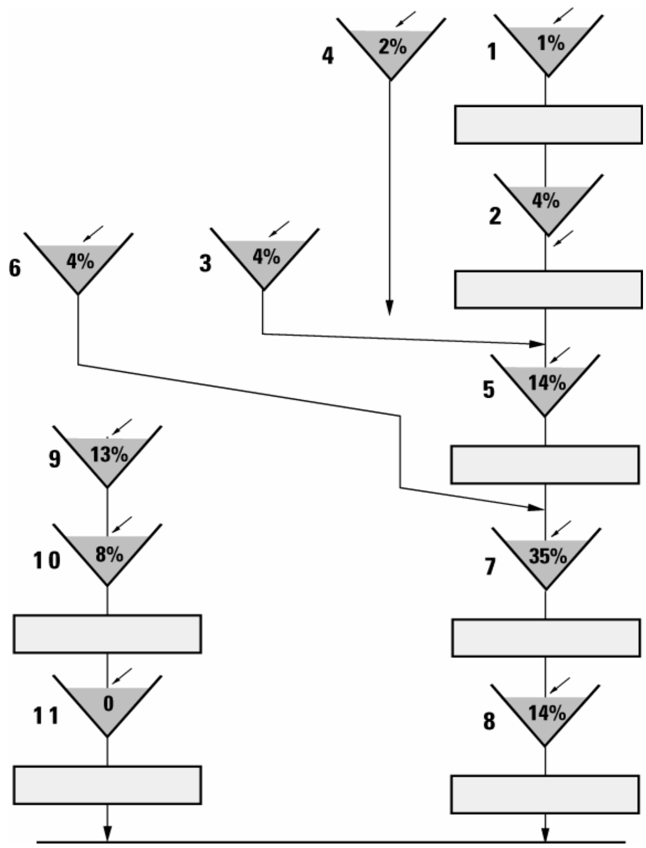

Figure 4 The Case 2 hydro system. Numbers in reservoir symbols show each reservoir's share of the total reservoir volume. Boxes indicate plants. Reservoirs 5, 7, 8 and 10 have a maximum head variation of 45, 70, 17 and 26 meters respectively. Nominal head for all these plants is in the area of 300 – 350 meters.

Run with and without head coefficients, the model shows an increase in income when head coefficients are included of 1 NOK/MWh on average referring to generation without head coefficients. Average market prices are 123 NOK/MWh. Generation increases by about 1% on average, despite an increase in spilled water.

Once more head coefficients change the strategy for operation of reservoirs considerably, as shown in the following figures. Note particularly the increased storage in reservoir 8, shown in Figure 7. Storage increases dramatically here, and this leads to a doubling of spilled water from this reservoir. Despite this, average annual generation from the plant below 8 is virtually the same in the two cases. However, the real optimum may well be somewhere in between the two solutions.

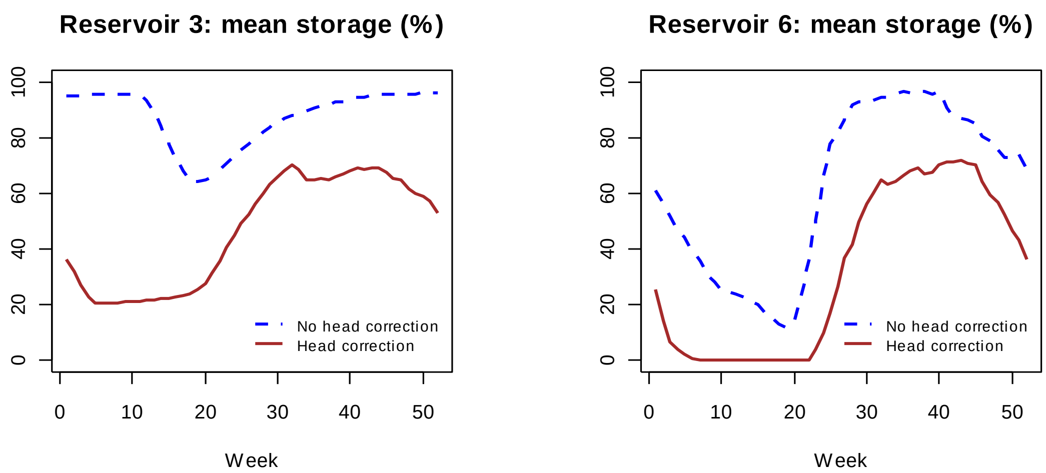

Figure 5 Simulated mean storage of reservoirs 3 and 6. These are reservoirs without plants immediately below, so they are depleted in the case with head coefficients in order to increase head above plants further downstream.

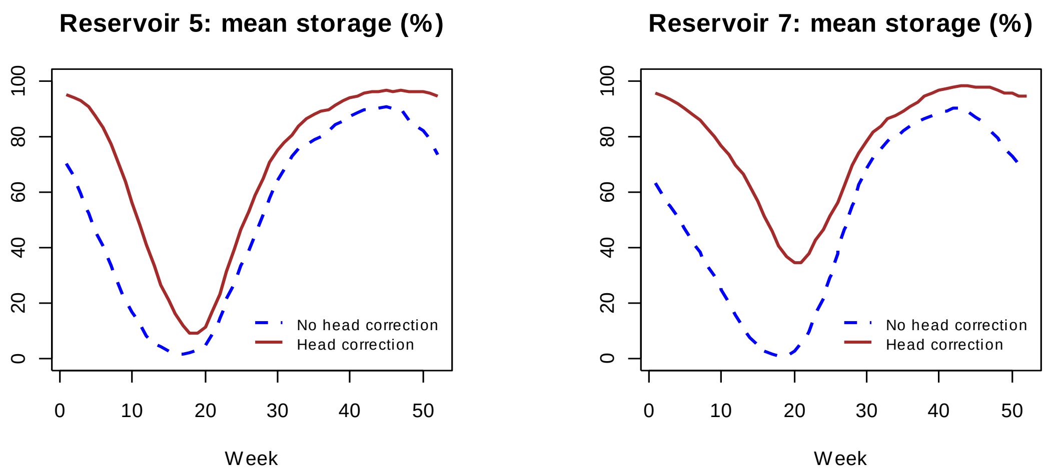

Figure 6 Simulated mean storage of reservoirs 5 and 7. These reservoirs have plants, and head variation is significant.

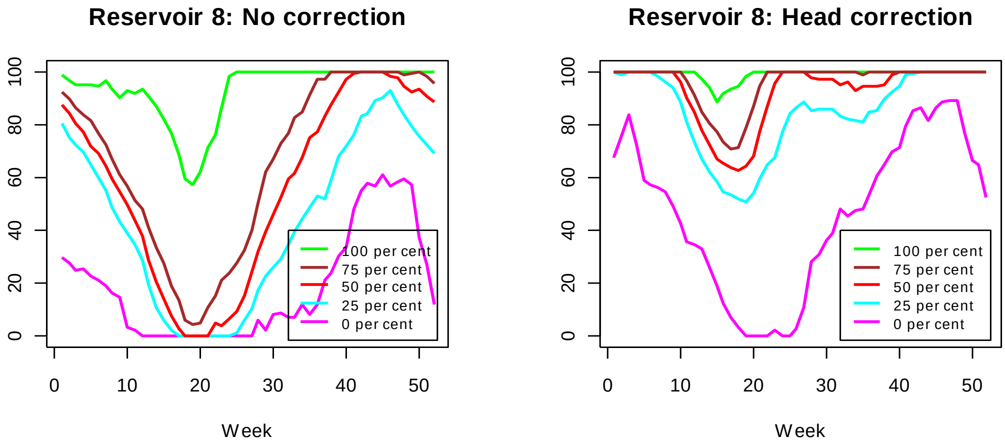

Figure 7 Simulated storage in reservoir 8, shown as percentiles from 50 simulated scenarios. Note the dramatic increase in storage when head correction coefficients are included.

4 CONCLUSION

We have calculated head sensitivity coefficients for the hydro scheduling problem and established a heuristic method of including them in a linear stochastic optimization model. This helps the model take head considerations into account when establishing hydropower strategy, while at the same time preserving the problem's convexity and linear properties.

The cases presented in Section 3, along with numerous other cases run, indicate that the calculated head sensitivity coefficients improve hydro strategy (although it can be argued that the starting point, ignoring head variations completely, is poor for comparison). These cases by themselves do not constitute proof that the coefficients provide a general solution to the problem of head in such models. Among possible sources of errors when comparing results are:

- sampling errors
- simplifications in valuating end point reservoir contents (which we have tried to avoid by providing for virtually equal initial and end point reservoir contents)
- other errors and simplifications in the model, e.g. statistical errors in stochastic models used to describe inflow and price.

However, virtually all cases run indicate that a step has been taken in the right direction. The head coefficients generally provide for a strategy that gives better economic results, higher expected generation and a qualitative change in the strategy that generally makes the model more acceptable to hydro system owners (avoids some clearly unwise strategies that lack of head optimization may give). So this seems to be an approach suited to improving linear stochastic optimization models that otherwise lack head considerations in their objective functions.

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