symmetry

MDPI

Article

Multiple-Attribute Decision Making Based on Intuitionistic Hesitant Fuzzy Connection Set Environment

Wajid Ali [1,*], Tanzeela Shaheen [1], Iftikhar Ul Haq [1], Hamza Ghazanfar Toor [2], Faraz Akram [2], Saeid Jafari [3], Md. Zia Uddin [4] and Mohammad Mehedi Hassan [5]

1 Department of Mathematics, Air University, E-9, Islamabad 44000, Pakistan
2 Biomedical Engineering Department, Riphah International University, Islamabad 46000, Pakistan
3 Mathematical and Physical Science Foundation, 4200 Slagelse, Denmark
4 Software and Service Innovation, SINTEF Digital, 0373 Oslo, Norway
5 Information Systems Department, College of Computer and Information Sciences, King Saud University, Riyadh 11543, Saudi Arabia
* Correspondence: wajidali00258@gmail.com

Abstract: The intuitionistic hesitant fuzzy set (IHFS) is an enriched version of hesitant fuzzy sets (HFSs) that can cover both fuzzy sets (FSs) and intuitionistic fuzzy sets (IFSs). By assigning membership and non-membership grades as subsets of [0, 1], the IHFS can model and handle situations more proficiently. Another related theory is the theory of set pair analysis (SPA), which considers both certainties and uncertainties as a cohesive system and represents them from three aspects: identity, discrepancy, and contrary. In this article, we explore the suitability of combining the IHFS and SPA theories in multi-attribute decision making (MADM) and present the hybrid model named intuitionistic hesitant fuzzy connection number set (IHCS). To facilitate the design of a novel MADM algorithm, we first develop several averaging and geometric aggregation operators on IHCS. Finally, we highlight the benefits of our proposed work, including a comparative examination of the recommended models with a few current models to demonstrate the practicality of an ideal decision in practice. Additionally, we provide a graphical interpretation of the devised attempt to exhibit the consistency and efficiency of our approach.

Keywords: intuitionistic fuzzy set; hesitant fuzzy set; power aggregation operators; connection number; set pair analysis theory

Citation: Ali, W.; Shaheen, T.; Haq, I.U.; Toor, H.G.; Akram, F.; Jafari, S.; Uddin, M.Z.; Hassan, M.M. Multiple-Attribute Decision Making Based on Intuitionistic Hesitant Fuzzy Connection Set Environment. *Symmetry* **2023**, *15*, 778. https://doi.org/10.3390/sym15030778

Academic Editor: Hsien-Chung Wu

Received: 26 February 2023
Revised: 9 March 2023
Accepted: 17 March 2023
Published: 22 March 2023

1. Introduction

The technique for picking the most acceptable alternative based on the provided criteria is known as decision making (DM). Various DM procedures are originated via one feature only, but most often, these are dependent upon multiple attributes. The area of DM is then called multiple-attribute DM (MADM). MADM is a highly crucial testing domain, which helps choose the correct option associated with several leading features [1–3]. Mainly, the DM uses crisp figures to describe the preferences concerning the choices in usual MADM problems. Nevertheless, due to the inadequacy of statistics, shortage of time, and lack of data and quality values, fuzzy values may be used to specify the preferences exclusively for the attribute values.

The idea of fuzzy sets (FSs) was introduced by Zadeh [4] in 1965 and, after they have come into existence, several researchers have applied them to solve various problems and developed symmetrical decision models [5–7]. Every fuzzy set has a pair of each component with a function of membership, where this function provides a membership grade in $[0, 1]$. Later, several extensions of this fuzzy set have been proposed by the researchers. Atanassov [8,9] proposed an intuitionistic fuzzy set (IFS) in 1986. The IFS expresses ambiguous and complex information with the aid of the membership as well as non-membership grades, where the sum of both grades cannot exceed 1. The IFS has been getting a lot of attention since its initiation. For instance, Garg and Arora [10] stated a decision-making algorithm for the DMPs ("decision-making problems") with "interval-valued intuitionistic fuzzy soft set" features. Zeng et al. [11] defined the generalized probabilistic ordered operators for DMPs. Eom and Lee [12] elaborated the intuitionistic fuzzy theta-compact space. In Refs. [13,14], the authors have defined an application of the intuitionistic fuzzy pairs to solve the different DMPs. Bustince and Burillo [15] and Park et al. [16] defined the correlation coefficients for an interval-valued IFS. Atanassov [17] stated some novel topological operators over the IFS. In terms of ranking the different pairs of IFSs, several researchers have defined various measures such as the accuracy function [18], similarity measures [19], and grey relational analysis [20]-based algorithms for DMPs. Jiang et al. [21] defined the distance measures based on the transformation techniques of IFSs and their application to the "pattern recognition" problems. In [22,23], the authors have defined some generalized aggregation operators to aggregate the different pairs of IFSs and their applications to the DMPs.

In the above-mentioned literature, all the studies considered the input as a real number. However, to address the uncertainties in a more generalized manner, Torra [24,25] presented the concept of the hesitant fuzzy set (HFS). An HFS agrees to the membership grade holding a set of possible values of the interval from 0 to 1. An HFS is also an expanded form of the FS. The idea of the HFS is broadly applied in several complications. Most scholars performed a critical investigation on HF data accumulation procedures and their effects in DMPs. For instance, Mahmood et al. [26] defined the aggregation operators of the pairs of the bipolar-valued hesitant fuzzy set to solve the DMPs. Mockor et al. [27] explored the relationship of HFSs with some generalized form of fuzzy sets. Alcantud [28] ranked the different pairs of HFSs for multi-agent decisions. Zhang et al. [29] presented an extended LINMAP method with HFS information for green-supplier selection problems. Xia et al. [30] stated some operators for HFSs. Shaheen et al. [31] defined the concept of hesitant fuzzy rough sets. Sun and Ouyang [32] stated a TOPSIS-based decision-making algorithm for the DMPs. Ni et al. [33] stated a projection method for DMPs using the dual HFS. Liu and Sun [34] proposed generalized power operators for DMPs using the pairs of HFSs. Tahir et al. [35] described the concept of an IHFS ("intuitionistic hesitant fuzzy set") which is a fusion of the IFS and the HFS. In the IHFS, the grades reflect in the form of a set of possible values from $[0,1]$. Admittedly, the IHFS has developed as a potent tool for explaining the fuzziness of the DM complexities.

Aside from the aforementioned ideas, Zhao [36] suggested another approach, recognized as set pair analysis (SPA), to cope with uncertainty. This concept harmonizes the system of assurance and apprehension in a specific evaluation. The most important part of this concept is the CN ("connection number"); it is split into three kinds of senses: identity, inconsistency, and opposition. The fundamental properties of the CNs are defined by Jiang et al. [37]. Later, several researchers showed interest in explaining DM difficulties using set pair analysis (SPA) [38–40]. However, Kumar and Garg [41,42] outlined the different formulae of connection numbers to resolve the DM problems applying the TOPSIS method. Zhang [43] developed the relationship between fuzzy sets and the theory of SPA. After considering the relationship between both theories, Hu et al. [44] defined a dynamic stochastic multiple-attribute DM course. Jia et al. [45] determined the distance measures for MADM based on CNs.

The two foremost steps involved in MADM are, firstly, the aggregation phase to determine the total value of the selected option by combining the given performance values of all the attributes through the suitable integration process; and, secondly, the manipulation phase for choosing the best option. In this paper [46], the authors employ many approaches to support the MD process, such as a combination of the DEA window analysis with the Malmquist index approach to assess the efficiency of the cybersecurity industry, and a framework based on multicriteria decision making (MCDM) is provided that integrates the spherical fuzzy analytical hierarchical process (SF-AHP) and the grey complex proportional assessment (G-COPRAS), in which spherical fuzzy sets and grey numbers are used to express the ambiguous linguistic evaluation statements of experts [47]. For the aggregation phase, during the last twenty years, experts have worked hard to develop aggregation operators, used to tackle DM difficulties. For example, Xu and Yager [48] established a few fundamental geometric aggregation (GA) operators named the IF-WGA operator and IF-OWG operator, which are applied for DM based upon the intuitionistic fuzzy set. Zhang [49] established some aggregation operators and aggregated the hesitant fuzzy environment. Yager [50] demonstrated a power aggregation operator and confirmed its role in MADM. Ahmmad et al. [51] used the Aczel–Alsina Average aggregation operator for medical diagnosis. Mehmood et al. [52] produced the Hamacher Choquet-integral aggregation operators for ranking. Moreover, Tahir et al. [35] established some AOs for IHFSs and PHFSs for decision making.

The above-described aggregation operators are extensively applied to aggregate the preference values for getting the best results. However, during the research, these operators exceeded the association between the data provided. Through research, we have explored the idea that the CN of the theory of SPA can be applied extensively for MADM. Considering this aspect, we define the intuitionistic hesitant fuzzy connection number set (IHCS) by merging the prominent features of the IHFS and the SPA, and the applications of fuzzy logic in different fields should also be mentioned in the literature review. The related works were suggested as follows [53–56]. In addition, this article establishes a sequence of aggregation operators. The inspiration and attainments of this paper are presented below:

1. The origination of some PA operators, such as the IHCPA, IHCPWA, IHCPOWA, IHCPHA, IHCPG, IHCPWG, IHCPOWG, and IHCPHG operators, and verifying their properties;
2. Proposing a novel MADM technique that involves the developed operations;
3. Furnishing specific mathematical examples to validate the consistency and supremacy of the presented approach.

The rest of the article is organized as follows: In Section 2, we briefly overview the concept of the HFS and SPA theory. In Section 3, we defined the concept of IHFCNS ("intuitionistic hesitant fuzzy connection number set") and stated its properties. In Section 4, we proposed an MADM algorithm with the stated operators to address the DMPs. To validate the performance of the proposed algorithm, a numerical example is given in Section 5. Finally, a conclusion is drawn in Section 6. The detailed flow chart of the proposed study is shown in Figure 1.

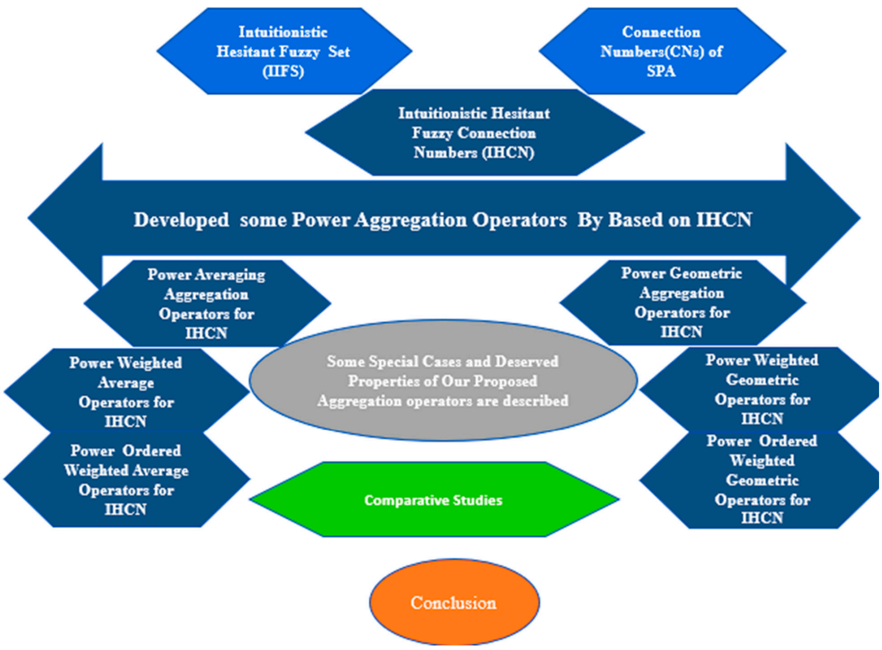

Figure 1. Flow chart of the manuscript.

2. Preliminaries

This section includes a few fundamental concepts related to the present studies on the intuitionistic hesitant fuzzy set and the theory of SPA.

Definition 1 ([35]). *The supporting mappings for an IHFS are ϕ, ψ: $\mathcal{F} \rightarrow [0,1]$. With the aid of these mappings, IHFS $\mathcal{Z}_i$ over $\mathcal{F}$ is described as below:*

$$\mathcal{Z}_i = \{(\varsigma_i, \phi_{\mathcal{Z}}(\varsigma_i), \psi_{\mathcal{Z}}(\varsigma_i)) | \varsigma_i \in \mathcal{F}\} \tag{1}$$

where $0 \leq max(\phi_{\mathcal{Z}}(\varsigma_i)) + max(\psi_{\mathcal{Z}}(\varsigma_i)) \leq 1$. $\phi_{\mathcal{Z}}(\varsigma_i)$ and $\psi_{\mathcal{Z}}(\varsigma_i)$ are collections of certain amounts in $[0,\ 1]$, representing the membership grades and non-membership grades of the component $\varsigma_i \in \mathcal{F}$.

Remark 1.

1. *For each $\varsigma_i \in \mathcal{F}$, $\mathcal{Z}_i = (\phi_{\mathcal{Z}}(\varsigma_i), \psi_{\mathcal{Z}}(\varsigma_i))$ is the IHFE.*
2. *It is assumed that the largest element will be repeated to make the lengths of the two IHFEs the same in the IHFSs.*
3. *The elements in the IHFSs will be arranged in ascending order for comparison purposes.*

Definition 2 ([35]). *For IHFS $\mathcal{Z} = (\phi_{\mathcal{Z}}, \psi_{\mathcal{Z}})$, the score function ($\mathcal{S}c(\mathcal{Z})$) as well as accuracy function ($\mathcal{T}(\mathcal{Z})$) are expressed as*:

$$\mathcal{S}c(\mathcal{Z}) = \frac{\mathcal{S}c(\phi_{\mathcal{Z}}) - \mathcal{S}c(\psi_{\mathcal{Z}})}{2}, \mathcal{S}c(\mathcal{Z}) \in [-1,\ 1]$$

$$\mathcal{T}(\mathcal{Z}) = \frac{\mathcal{S}c(\phi_{\mathcal{Z}}) + \mathcal{S}c(\psi_{\mathcal{Z}})}{2}, \mathcal{T}(\mathcal{Z}) \in [0,\ 1]$$

where $\mathcal{S}c(\phi_{\mathcal{Z}}) = \frac{sum\ of\ all\ elements\ in\ (\phi_{\mathcal{Z}})}{order\ of\ (\phi_{\mathcal{Z}})}$ and $\mathcal{S}c(\psi_{\mathcal{Z}}) = \frac{sum\ of\ all\ elements\ in\ (\psi_{\mathcal{Z}})}{order\ of\ (\psi_{\mathcal{Z}})}$.

Definition 3 ([35]). *An IHFS $\mathcal{Z}_1$ will be preferred over $\mathcal{Z}_2$, denoted by $\mathcal{Z}_1 > \mathcal{Z}_2$, based upon the following criteria*:

(i) $\mathcal{S}c(\mathcal{Z}_1) > \mathcal{S}c(\mathcal{Z}_2)$;
(ii) $\mathcal{S}c(\mathcal{Z}_1) = \mathcal{S}c(\mathcal{Z}_2)$ and $\mathcal{T}(\mathcal{Z}_1) > \mathcal{T}(\mathcal{Z}_2)$

Definition 4 ([35]). *The intuitionistic hesitant fuzzy power average and geometric operators are denoted and defined, respectively, as follows*:

$$IHFPA(\mathcal{Z}_1\mathcal{Z}_2 \ldots, \mathcal{Z}_n)$$

$$= \bigcup_{\substack{h_i \varepsilon \phi_i \\ f_i \varepsilon \psi_i}} \left(1 - \prod_{i=1}^{n}\left(1 - (h_i)^{\frac{(1+T(\mathcal{Z}_i))}{\sum_{i=1}^{n}(1+T(\mathcal{Z}_i))}}\right), \prod_{i=1}^{n}(f_i)^{\frac{(1+T(\mathcal{Z}_i))}{\sum_{i=1}^{n}(1+T(\mathcal{Z}_i))}}\right)$$

and

$$IHFPG\ (\mathcal{Z}_1\mathcal{Z}_2 \ldots, \mathcal{Z}_n)$$

$$= \bigcup_{\substack{h_i \varepsilon \phi_i \\ f_i \varepsilon \psi_i}} \left(\prod_{i=1}^{n}(h_i)^{\frac{(1+T(\mathcal{Z}_i))}{\sum_{i=1}^{n}(1+T(\mathcal{Z}_i))}}, 1 - \prod_{i=1}^{n}(1 - (f_i)^{\frac{(1+T(\mathcal{Z}_i))}{\sum_{i=1}^{n}(1+T(\mathcal{Z}_i))}})\right)$$

where

$$T(\mathcal{Z}_i) = \bigcup_{\substack{\phi_i \varepsilon \mathcal{Z}_i \\ \psi_i \varepsilon \mathcal{Z}_i}} \left(\sum_{\substack{i=1 \\ i \neq j}}^{n} Sup(\mathcal{Z}_i, \mathcal{Z}_j)\right)$$

The theory of SPA is based on a pair of sets that are interrelated and unified under a certain construction. The examples may include the system and surroundings, the system and engineering, military problems, state security, etc. SPA analyzes the common attributes and establishes a connection degree ($\mathcal{J} = \alpha + \beta i + \gamma j$) of two such sets according to the given settings. The theory is further extended for more than two sets.

Definition 5 ([36]). *The connection degree is defined as*

$$\mathcal{J} = \frac{\boldsymbol{S}}{\boldsymbol{N}} + \frac{\boldsymbol{F}}{\boldsymbol{N}}\boldsymbol{i} + \frac{\boldsymbol{P}}{\boldsymbol{N}}\boldsymbol{j}$$

$$\mathcal{J} = \alpha + \beta i + \gamma \tag{2}$$

where $\boldsymbol{N}$ represents "all attributes", $\boldsymbol{S}$ represents "identity attributes", $\boldsymbol{P}$ represents "contrary attributes", and $\boldsymbol{F} = \boldsymbol{N} - \boldsymbol{S} - \boldsymbol{P}$ represents the amount of "mutual attributes" of the two sets that are neither identity nor contrary. $\alpha = \frac{\boldsymbol{S}}{\boldsymbol{N}}, \beta = \frac{\boldsymbol{F}}{\boldsymbol{N}}, \gamma = \frac{\boldsymbol{P}}{\boldsymbol{N}}$ are, respectively, called the identity degree, discrepancy degree, and contrary degree of the two sets. According to the given circumstances, i (contrary degree) and j (discrepancy degree) can be specified from $[-1, 1]$. Clearly, $\alpha + \beta + \gamma = 1$.

3. Proposed IHCS and Aggregation Operators

This portion includes the establishment of a new concept by combining the IHFS and CN of the theory of SPA, called the intuitionistic hesitant fuzzy connection number set (IHCS). Based on this definition, a sequence of new aggregation operators is developed. These proposed operators and their properties have been analyzed and their suitability is portrayed in some newly developed algorithm.

Definition 6. *An intuitionistic hesitant fuzzy CN set (IHCS) corresponding to IHFS* $\mathcal{Z} = \{(\varsigma_i, \phi(\varsigma_i), \psi(\varsigma_i))|\varsigma_i \in \mathcal{F}\}$ *is denoted and defined as*

$$\mathcal{J}_{\mathcal{Z}} = \{(\varsigma_i, \alpha_{\mathcal{Z}}(\varsigma_i) + \beta_{\mathcal{Z}}(\varsigma_i)i + \gamma_{\mathcal{Z}}(\varsigma_i)j|\varsigma_i \in \mathcal{F}\} \tag{3}$$

where $\alpha_{\mathcal{Z}}(\varsigma_i) = \frac{1}{m}\sum_{l=1}^{m}(\{h_l(1-f_l)\})$, $\beta_{\mathcal{Z}}(\varsigma_i) = \frac{1}{m}\sum_{l=1}^{m}(\{1 - h_l(1-f_l) - f_l(1-h_l)\})$, and $\gamma_{\mathcal{Z}}(\varsigma_i) = \frac{1}{m}\sum_{l=1}^{m}(\{f_l(1-h_l)\})$, and $h_l \in \phi_{\mathcal{Z}}(\varsigma_i)$ and $f_l \in \psi_{\mathcal{Z}}(\varsigma_i)$ are arbitrary elements of the membership and non-membership sets. Here, $\alpha_{\mathcal{Z}}(\varsigma_i)$, $\beta_{\mathcal{Z}}(\varsigma_i)$, and $\gamma_{\mathcal{Z}}(\varsigma_i)$ signify the "identity," "discrepancy", and "contrary" degrees.

For a particular $k, \alpha_{\mathcal{Z}}(\varsigma_k) + \beta_{\mathcal{Z}}(\varsigma_k)i + \gamma_{\mathcal{Z}}(\varsigma_k)j$, this represents the intuitionistic hesitant fuzzy connection number IHCN. Without loss of generality, it will be written as $\alpha_k + \beta_k i + \gamma_k j$.

Remark 2.

1. *For each* $\varsigma_i \in \mathcal{F}, \mathcal{J}_{\mathcal{Z}} = \alpha_{\mathcal{Z}}(\varsigma_i) + \beta_{\mathcal{Z}}(\varsigma_i)i + \gamma_{\mathcal{Z}}(\varsigma_i)j$ *will be considered as an intuitionistic hesitant fuzzy connection number element.*
2. *Throughout the article, the lengths of the IHFEs will be kept similar by repeating the maximum value in the smaller one.*
3. h_l, f_l $(l = 1, 2, \ldots, m)$ *represents the IHFEs in ascending order.*
4. *The components of the IHCS are considered as IHCEs.*

Definition 7. *For two IHCEs* $\mathcal{J}_1 = \alpha_1 + \beta_1 i + \gamma_1 j$ *and* $\mathcal{J}_2 = \alpha_2 + \beta_2 i + \gamma_2(\varsigma_i)j$*, we have*

i. $\mathcal{J}_1(\varsigma_i) = \mathcal{J}_2(.\varsigma_i) \Leftrightarrow \alpha_1(\varsigma_i) = \alpha_2(\varsigma_i), \beta_1(\varsigma_i) = \beta_1(\varsigma_i), \gamma_1(\varsigma_i) = \gamma_2(\varsigma_i);$
ii. $\mathcal{J}_1(\varsigma_i) \leq \mathcal{J}_2(\varsigma_i)\ if\ \alpha_1(\varsigma_i) \leq \alpha_2(\varsigma_i), \gamma_1(\varsigma_i) \geq \gamma_2(\varsigma_i);$
iii. $\mathcal{J}_1(\varsigma_i)^c = \gamma_1(\varsigma_i) + \beta_1(\varsigma_i)i + \alpha_1(\varsigma_i)j$ represents the complement of the IHCE $\mathcal{J}_1(\varsigma_i)$.

Definition 8. *Let* $\mathcal{J}_1 = \alpha_1 + \beta_1 i + \gamma_1 j$ *and* $\mathcal{J}_2 = \alpha_2 + \beta_2 i + \gamma_2 j$ *be two IHCEs; then,*

i. $\mathcal{J}_1 \bigoplus \mathcal{J}_2 = (1 - (1-\alpha_1)(1-\alpha_2)) + ((\beta_1 + \gamma_1)(\beta_2 + \gamma_2) - \beta_1\beta_2)i + (\beta_1\beta_2)j;$

ii. $\mathcal{J}_1 \otimes \mathcal{J}_2 = \alpha_1\alpha_2 + (1-(1-\beta_1)(1-\beta_2))i + ((\alpha_1+\gamma_1)(\alpha_2+\gamma_2) - \alpha_1\alpha_2)j$;

iii. $\mathcal{J}_1{}^{\lambda} = a_1{}^{\lambda} + \left(1-(1-b_1)^{\lambda}\right)i + \left((a_1+c_1)^{\lambda} - a_1^{\lambda}\right)j, \lambda > 0$.

It is straightforward to prove that these operators generate IHCEs.

Definition 9. *For a group of IHCEs $\mathcal{J}_i (i = 1, 2, \ldots, n)$, the power averaging operator IHCPA: $\mathcal{J}^n \rightarrow \mathcal{J}$ is defined as*

$$IHCPA(\mathcal{J}_1, \mathcal{J}_2, \ldots \mathcal{J}_n) = \frac{\underset{i=1}{\overset{n}{\oplus}} (1+T(\mathcal{J}_i))\mathcal{J}_i}{\sum_{i=1}^{n}(1+T(\mathcal{J}_i))} \tag{4}$$

$$\begin{aligned} &= \left(1 - \prod_{i=1}^{n}\left(1-(\alpha_i)^{\frac{(1+T(\mathcal{J}_i)))}{\sum_{i=1}^{n}(1+T(\mathcal{J}_i))}}\right)\right) \\ &\quad + \left(\prod_{i=1}^{n}(\beta_i+\gamma_i)^{\frac{(1+T(\mathcal{J}_i))}{\sum_{i=1}^{n}(1+T(\mathcal{J}_i))}} - \prod_{i=1}^{n}(\beta_i)^{\frac{(1+T(\mathcal{J}_i))}{\sum_{i=1}^{n}(1+T(\mathcal{J}_i))}}\right)i \\ &\quad + \left(\prod_{i=1}^{n}(\beta_i)^{\frac{(1+T(\mathcal{J}_i))}{\sum_{i=1}^{n}(1+T(\mathcal{J}_i))}}\right) \end{aligned} \tag{5}$$

wherever $T(\mathcal{J}_i)$ is the Sup of the ith biggest IHCE ($\mathcal{J}_i$) by all the other IHCEs, that is,

$$T(\mathcal{J}_i) = \sum_{\substack{j=1 \\ j \neq i}}^{n} Sup(\mathcal{J}_i, \mathcal{J}_j)$$

Here, $Sup(\mathcal{J}_i, \mathcal{J}_j)$ is the Sup for $\mathcal{J}_i$ from $\mathcal{J}_j$, and it is calculated by:

$$Sup(\mathcal{J}_i, \mathcal{J}_j) = 1 - d(\mathcal{J}_i, \mathcal{J}_j)$$

Sup satisfies the given properties:

i. $Sup\ (\mathcal{J}_i, \mathcal{J}_j) \in [0, 1]$;

ii. $Sup\ (\mathcal{J}_i, \mathcal{J}_j) = Sup\ (\mathcal{J}_j, \mathcal{J}_i)$;

iii. $Sup\ (\mathcal{J}_i, \mathcal{J}_j) \geq Sup\ (\mathcal{J}_s, \mathcal{J}_t), if\ d(\mathcal{J}_i - \mathcal{J}_j) < d(\mathcal{J}_s - \mathcal{J}_t)$ where d is distance.

The support (Sup) amount is a similarity indicator.

Definition 10. *Let $\mathcal{J}_i$ be a group of IHCEs and $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ by the weight vector of $\mathcal{J}_i$, $\omega_i > 0$, and $\sum_{i=1}^{n}\omega_i = 1$. The power weight averaging operator IHCPWA: $\mathcal{J}^n \rightarrow \mathcal{J}$ is defined as*

$$IHCPWA(\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_n) = \frac{\underset{i=1}{\overset{n}{\oplus}} (\omega_i(1+T(\mathcal{J}_i)\mathcal{J}_i)}{\sum_{i=1}^{n}\omega_i(1+T(\mathcal{J}_i))}$$

$$\begin{aligned} &= 1 - \prod_{i=1}^{n}\left(1-(\alpha_i)^{\frac{\omega_i(1+T(\mathcal{J}_i))}{\sum_{i=1}^{n}\omega_i(1+T(\mathcal{J}_i))}}\right) \\ &\quad + \left(\prod_{i=1}^{n}(\beta_i+\gamma_i)^{\frac{\omega_i(1+T(\mathcal{J}_i))}{\sum_{i=1}^{n}\omega_i(1+T(\mathcal{J}_i))}} - \prod_{i=1}^{n}(\beta_i)^{\frac{\omega_i(1+T(\mathcal{J}_i))}{\sum_{i=1}^{n}\omega_i(1+T(\mathcal{J}_i))}}\right)i \\ &\quad + \left(\prod_{i=1}^{n}(\beta_i)^{\frac{\omega_i(1+T(\mathcal{J}_i))}{\sum_{i=1}^{n}\omega_i(1+T(\mathcal{J}_i))}}\right)j \end{aligned} \tag{6}$$

The following properties can be easily justified for the IHCPWA operators.

Property 1: (Idempotency) *Suppose $\mathcal{J}_i$ $(i = 1, 2, 3 \cdots, n)$ are $\mathcal{J}_i = \mathcal{J}$ for each i ; then,*

$$IHCPWA(\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_n) = \mathcal{J}$$

Property 2: (Boundedness) *Suppose $\mathcal{J}_i$ is a collection of IHCEs, and permits*

$$\mathcal{J}^- = \min_i \mathcal{J}_i, \mathcal{J}^+ = \max_i \mathcal{J}_i (i = 1, 2, \ldots, n)$$

then

$$\mathcal{J}^- \leq IHCPWA(\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_n) \leq \mathcal{J}^+$$

Property 3: (Monotonicity) *Suppose $\mathcal{J}_i$ and $\mathcal{J}'_i$ are collections of IHCEs; if $\mathcal{J}_i \leq \mathcal{J}'_i$ for all i , then*

$$IHCPWA(\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_n) \leq IHCPWA\left(\mathcal{J}'_1, \mathcal{J}'_2, \ldots, \mathcal{J}'_n\right)$$

Definition 11. *Let $\mathcal{J}_i$ be a group of IHCEs; the power order weight averaging operator of dimension "n", related with the weight vector $\omega = (\omega_1, \omega_2, \ldots, \omega_i)^T$ such that $\omega_i > 0$ and $\sum_{i=1}^n \omega_i = 1$, IHCPOWA: $\mathcal{J}^n \to \mathcal{J}$, is defined as*

$$\begin{aligned} IHCPOWA(\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_n) &= \frac{\bigoplus_{i=1}^{n} (\omega_i (1 + T(\mathcal{J}_{\sigma(i)})\mathcal{J}_{\sigma(i)}))}{\sum_{i=1}^n \omega_i (1 + T(\mathcal{J}_{\sigma(i)}))} \\ &= 1 - \prod_{i=1}^n \left(1 - \left(\alpha_{\sigma(i)}\right)^{\frac{\omega_i(1+T(\mathcal{J}_{\sigma(i)}))}{\sum_{i=1}^n \omega_i(1+T(\mathcal{J}_{\sigma(i)}))}}\right) \\ &\quad + \left(\prod_{i=1}^n \left(\beta_{\sigma(i)} + \gamma_{\sigma(i)}\right)^{\frac{\omega_i(1+T(\mathcal{J}_{\sigma(i)}))}{\sum_{i=1}^n \omega_i(1+T(\mathcal{J}_{\sigma(i)}))}} \right. \\ &\quad \left. - \prod_{i=1}^n \left(\beta_{\sigma(i)}\right)^{\frac{\omega_i(1+T(\mathcal{J}_{\sigma(i)}))}{\sum_{i=1}^n \omega_i(1+T(\mathcal{J}_{\sigma(i)}))}}\right) i \\ &\quad + \left(\prod_{i=1}^n \left(\beta_{\sigma(i)}\right)^{\frac{\omega_i(1+T(\mathcal{J}_{\sigma(i)}))}{\sum_{i=1}^n \omega_i(1+T(\mathcal{J}_{\sigma(i)}))}}\right) j \end{aligned} \tag{7}$$

where $\sigma(1), \sigma(2), \ldots, \sigma(n)$ indicates a permutation of $(1, 2, \ldots, n)$, in which $\mathcal{J}_{\sigma(i-1)} \geq \mathcal{J}_{\sigma(i)}$, $\omega_i (i = 1, 2, \ldots, n)$ is a group of weights so that

$$\omega_i = g\left(\frac{R_i}{TV}\right) - g\left(\frac{R_{i-1}}{TV}\right), R_i = \sum_{i=1}^n V_{\sigma(i)}, TV = \sum_{i=1}^n V_{\sigma(i)}, V_{\sigma(i.)} = 1. + T\left(\mathcal{J}_{\sigma(i)}\right)$$

and $T\left(\mathcal{J}_{\sigma(j)}\right)$ implies the Sup of the jth largest IHCE $T\left(\mathcal{J}_{\sigma(j)}\right)$ by all the other (IHCEs), that is,

$$T\left(\mathcal{J}_{\sigma(j)}\right) = \sum_{\substack{i=1 \\ i \neq j}}^n Sup\left(\mathcal{J}_{\sigma(j)}, \mathcal{J}_{\sigma(i)}\right)$$

where $\sum_{\substack{i=1 \\ i \neq j}}^n Sup\left(\mathcal{J}_{\sigma(j)}, \mathcal{J}_{\sigma(i)}\right)$ shows the Sup of the jth is the biggest IHCE $\mathcal{J}_{\sigma(j)}$, for the ith largest IHCE $\mathcal{J}_{\sigma(i)}$.

Some properties of the IHCPOWA operator are as follows:

Property 4: (Idempotency) *Suppose $\mathcal{J}_i (i = 1, 2, 3 \cdots, n)$ are $\mathcal{J}_i = \mathcal{J}$ for every i; then,*

$$\text{IHC}POWA(\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_n) = \mathcal{J}$$

Property 5: (Boundedness) *Let $\mathcal{J}_i$ be a group of IHCEs and allow*

$$\mathcal{J}^- = \min_i \mathcal{J}_i, \quad \mathcal{J}^+ = \max_i \mathcal{J}_i \ (i = 1, 2, \ldots, n)$$

then

$$\mathcal{J}^- \leq IHCPOWA\big((\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_n) \leq \mathcal{J}^+$$

Property 6: (Monotonicity) *Let $\mathcal{J}_i$ and $\mathcal{J}'_i$ be two set of IHCEs; if $\mathcal{J}_i \leq \mathcal{J}'_i$ for all i , then*

$$IHCPOWA(\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_n) \leq IHCPOWA\big(\mathcal{J}'_1, \mathcal{J}'_2, \ldots, \mathcal{J}'_n\big)$$

Property 7: (Commutativity) *Let $\mathcal{J}_i$ and $\mathcal{J}'_i$ be two set of IHCEs; if $\mathcal{J}_i \leq \mathcal{J}'_i$ for all i, then*

$$IHCPOWA(\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_n) = IHCPOWA\big(\mathcal{J}'_1, \mathcal{J}'_2, \ldots, \mathcal{J}'_n\big)$$

where $\mathcal{J}'_j$ is a permutation of $\mathcal{J}_j$.

Definition 12. *Suppose $\mathcal{J}_i$ is group of IHCEs; the power hybrid averaging operator IHCPHA: $\mathcal{J}^n \to \mathcal{J}$ is defined as*

$$IHCNPHA(\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_n) = \frac{\underset{i=1}{\overset{n}{\oplus}} \left(\omega_i (1 + T(\dot{\mathcal{J}}_{\sigma(i)}) \dot{\mathcal{J}}_{\sigma(i)})\right)}{\sum_{i=1}^{n} \omega_i \left(1 + T\left(\dot{\mathcal{J}}_{\sigma(i)}\right)\right)}$$

$$\begin{aligned} &= 1 - \prod_{i=1}^{n} \left(1 - \left(\dot{\alpha}_{\sigma(i)}\right)^{\frac{\omega_i(1+T(\dot{\mathcal{J}}_{\sigma(i)}))}{\sum_{i=1}^{n}\omega_i(1+T(\dot{\mathcal{J}}_{\sigma(i)}))}}\right) \\ &\quad + \left(\prod_{i=1}^{n} \left(\dot{\beta}_{\sigma(i)} + \dot{\gamma}_{\sigma(i)}\right)^{\frac{\omega_i(1+T(\dot{\mathcal{J}}_{\sigma(i)}))}{\sum_{i=1}^{n}\omega_i(1+T(\dot{\mathcal{J}}_{\sigma(i)}))}} \right. \\ &\quad \left. - \prod_{i=1}^{n} \left(\dot{\beta}_{\sigma(i)}\right)^{\frac{\omega_i(1+T(\dot{\mathcal{J}}_{\sigma(i)}))}{\sum_{i=1}^{n}\omega_i(1+T(\dot{\mathcal{J}}_{\sigma(i)}))}}\right) i \\ &\quad + \left(\prod_{i=1}^{n} \left(\dot{\beta}_{\sigma(i)}\right)^{\frac{\omega_i(1+T(\dot{\mathcal{J}}_{\sigma(i)}))}{\sum_{i=1}^{n}\omega_i(1+T(\dot{\mathcal{J}}_{\sigma(i)}))}}\right) j \end{aligned} \tag{8}$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_i)^T$ is a represented weight vector, where $\omega_i \in [0,\ 1]$ and $\sum_{i=1}^{n} \omega_i = 1$ and $\mathcal{J}_{\sigma(i)}$ is the ith largest objects in the IHCE arguments $\dot{\mathcal{J}}_i$ ($\dot{\mathcal{J}} = (n\omega_i)$ $\mathcal{J}_i,\ i = 1, 2, \ldots, n$), $\omega = (\omega_1, \omega_2,\ \ldots\ , \omega_n)$ is the weighting vector of the IHCE arguments $\mathcal{J}_i (i = 1, 2, \ldots, n)$, $\omega_i \in [0,\ 1]$, and $\sum_{i=1}^{n} \omega_i = 1$ with ω_i being a group such that

$$\omega_i = g\left(\frac{R_i}{TV}\right) - g\left(\frac{R_{i-1}}{TV}\right), R_i = \sum_{i=1}^{j} V_{\sigma(i)}, TV = \sum_{i=1}^{n} V_{\sigma(i)}, V_{\sigma(i)} = 1 + T\left(\dot{\mathcal{J}}_{\sigma(i)}\right)$$

and $T\left(\dot{\mathcal{J}}_{\sigma(i)}\right)$ is the Sup of the jth biggest IHCEs $\dot{\mathcal{J}}_{\sigma(i)}$ by all the other (IHCEs), that is,

$$T\left(\dot{\mathcal{J}}_{\sigma(i)}\right) = \sum_{\substack{i=1 \\ i \neq j}}^{n} Sup\left(\dot{\mathcal{J}}_{\sigma(j)}, \mathcal{J}_{\sigma(i)}\right)$$

where $\sum_{\substack{i=1\\ i\neq j}}^{n} Sup\left(\dot{\mathcal{J}}_{\sigma(j)}, \mathcal{J}_{\sigma(i)}\right)$ shows the Sup of the jth biggest IHCE $\dot{\mathcal{J}}_{\sigma(j)}$, for the ith biggest IHCE $\dot{\mathcal{J}}_{\sigma(i)}$. Specifically, the IHCPHA is reduced to the IHCPWA operator if $\omega = \left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)^T$ and the IHCPHA is reduced to the IHCPOWA operator if $\omega = \left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)$.

Definition 13. *Suppose $\mathcal{J}_i$ is family of IHCEs; the power geometric operator IHCPG: $\mathcal{J}^n \to \mathcal{J}$ is defined as*

$$IHCPG(\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_n) = \underset{i=1}{\overset{n}{\otimes}} (\mathcal{J})^{\frac{1+T(\mathcal{J}_i)}{\sum_{i=1}^{n}(1+T(\mathcal{J}_i))}} \tag{9}$$

$$\begin{aligned} &= \prod_{i=1}^{n} (\alpha_i)^{\frac{(1+T(\mathcal{J}_i))}{\sum_{i=1}^{n}(1+T(\mathcal{J}_i))}} \\ &\quad + \left(1 - \prod_{i=1}^{n} (1-\beta_i)^{\frac{(1+T(\mathcal{J}_i))}{\sum_{i=1}^{n}(1+T(\mathcal{J}_i))}}\right) i \\ &\quad + \left(\prod_{i=1}^{n} (\alpha_i+\gamma_i)^{\frac{(1+T(\mathcal{J}_i))}{\sum_{i=1}^{n}(1+T(\mathcal{J}_i))}} - \prod_{i=1}^{n} (\alpha_i)^{\frac{(1+T(\mathcal{J}_i))}{\sum_{i=1}^{n}(1+T(\mathcal{J}_i))}}\right) j \end{aligned} \tag{10}$$

where

$$T(\mathcal{J}_i) = \sum_{\substack{j=1\\ i\neq j}}^{n} Sup\ (\mathcal{J}_i, \mathcal{J}_j)$$

Definition 14. *Let $\mathcal{J}_i$ be a group of IHCEs and $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ be the weight vector of $\mathcal{J}_i$, $\omega_i > 0$ and $\sum_{i=1}^{n} \omega_i = 1$. The power weight geometric operator IHCPWG: $\mathcal{J}^n \to \mathcal{J}$ is defined as*

$$IHCPWG(\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_n) = \underset{i=1}{\overset{n}{\otimes}} (\mathcal{J})^{\frac{\omega_i(1+T(\mathcal{J}_i))}{\sum_{i=1}^{n}\omega_i(1+T(\mathcal{J}_i))}}$$

$$\begin{aligned} &= \prod_{i=1}^{n} (\alpha_i)^{\frac{\omega_i(1+T(\mathcal{J}_i))}{\sum_{i=1}^{n}\omega_i(1+T(\mathcal{J}_i))}} + \\ &\left(1 - \prod_{i=1}^{n} (1-\beta_i)^{\frac{\omega_i(1+T(\mathcal{J}_i))}{\sum_{i=1}^{n}\omega_i(1+T(\mathcal{J}_i))}}\right) i \\ &+ \left(\prod_{i=1}^{n} (\alpha_i+\gamma_i)^{\frac{\omega_i(1+T(\mathcal{J}_i))}{\sum_{i=1}^{n}\omega_i(1+T(\mathcal{J}_i))}} - \prod_{i=1}^{n} (\alpha_i)^{\frac{\omega_i(1+T(\mathcal{J}_i))}{\sum_{i=1}^{n}\omega_i(1+T(\mathcal{J}_i))}}\right) j \end{aligned} \tag{11}$$

The following characteristics can be easily proven for the IHCPWG operator.

Property 8: (Idempotency) *When $\mathcal{J}_i(i = 1,2,3\cdots,n)$ are $\mathcal{J}_i = \mathcal{J}$ for each i, then*

$$IHCPWG(\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_n) = \mathcal{J}$$

Property 9: (Boundedness) *Suppose $\mathcal{J}_i$ is a group of IHCEs, and permits*

$$\mathcal{J}^- = \min_i \mathcal{J}_i, \mathcal{J}^+ = \max_i \mathcal{J}_i\ (i = 1,2,\ldots,n)$$

then

$$\mathcal{J}^- \leq IHCPWG(\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_n) \leq \mathcal{J}^+$$

Property 10: (Monotonicity) *Let $\mathcal{J}_i$ and $\mathcal{J}'_i$ be two sets of IHCEs; if $\mathcal{J}_i \leq \mathcal{J}'_i$ for all i, then*

$$IHCPWG(\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_n) \leq IHCPWG(\mathcal{J}'_1, \mathcal{J}'_2, \ldots, \mathcal{J}'_n)$$

Definition 15. *Let $\mathcal{J}_i$ be group of IHCEs; the power order weight geometric operator of dimension 'n', related with the weight vector $\omega = (\omega_1, \omega_2, \ldots, \omega_i)^T$ such that $\omega_i > 0$ and $\sum_{i=1}^{n} \omega_i = 1$, IHCPOWG: $\mathcal{J}^n \to \mathcal{J}$, is defined as*

$$IHCPOWG(\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_n) = \underset{i=1}{\overset{n}{\otimes}} \left(\mathcal{J}_{\sigma(i)}\right)^{\frac{\omega_i(1+T(\mathcal{J}_{\sigma(i)}))}{\sum_{i=1}^{n}\omega_i(1+T(\mathcal{J}_{\sigma(i)}))}}$$

$$\begin{aligned} = \prod_{i=1}^{n}\left(\left(\alpha_{\sigma(i)}\right)^{\frac{\omega_i(1+T(\mathcal{J}_{\sigma(i)}))}{\sum_{i=1}^{n}\omega_i(1+T(\mathcal{J}_{\sigma(i)}))}}\right) \\ +\left(1-\prod_{i=1}^{n}\left(1-\beta_{\sigma(i)}\right)^{\frac{\omega_i(1+T(\mathcal{J}_{\sigma(i)}))}{\sum_{i=1}^{n}\omega_i(1+T(\mathcal{J}_{\sigma(i)}))}}\right)i \\ +\left(\prod_{i=1}^{n}\left(\alpha_{\sigma(i)}+\gamma_{\sigma(i)}\right)^{\frac{\omega_i(1+T(\mathcal{J}_{\sigma(i)}))}{\sum_{i=1}^{n}\omega_i(1+T(\mathcal{J}_{\sigma(i)}))}} \right. \\ \left. -\prod_{i=1}^{n}\left(\alpha_{\sigma(i)}\right)^{\frac{\omega_i(1+T(\mathcal{J}_{\sigma(i)}))}{\sum_{i=1}^{n}\omega_i(1+T(\mathcal{J}_{\sigma(i)}))}}\right)j \end{aligned} \tag{12}$$

where $\sigma(1), \sigma(2), \ldots, \sigma(n)$ indicates a permutation of $(1, 2, \ldots, n)$, where $\mathcal{J}_{\sigma(i-1)} \geq \mathcal{J}_{\sigma(i)}$, and $\omega_i (i = 1, 2, \ldots, n)$ is group of weights so that

$$\omega_i = g\left(\frac{R_i}{TV}\right) - g\left(\frac{R_{i-1}}{TV}\right), R_j = \sum_{i=1}^{j} V_{\sigma(i)}, TV = \sum_{i=1}^{n} V_{\sigma(i)}, V_{\sigma(i.)} = 1. + T\left(\mathcal{J}_{\sigma(i)}\right)$$

and $T\left(\mathcal{J}_{\sigma(i)}\right)$ implies the Sup of the ith biggest IHCE $\mathcal{J}_{\sigma(i)}$ by all the other (IHCEs), that is,

$$T\left(\mathcal{J}_{\sigma(i)}\right) = \sum_{\substack{j=1 \\ i \neq j}}^{n} Sup\left(\mathcal{J}_{\sigma(i)}, \mathcal{J}_{\sigma(j)}\right)$$

where $\sum_{\substack{j=1 \\ i \neq j}}^{n} Sup\left(\mathcal{J}_{\sigma(i)}, \mathcal{J}_{\sigma(j)}\right)$ shows the Sup of the ith is the biggest IHCE $\mathcal{J}_{\sigma(i)}$, for the jth largest IHCE $\mathcal{J}_{\sigma(j)}$.

Some properties of the IHCPOWG operator are as follows:

Property 11: (Idempotency) *Let $\mathcal{J}_i$ $(i = 1, 2, 3 \cdots, n)$ be $\mathcal{J}_i = \mathcal{J}$ for every i; then,*

$$IHCPOWG(\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_n) = \mathcal{J}$$

Property 12: (Boundedness) *Suppose $\mathcal{J}_i$ is a group of IHCEs, and permits*

$$\mathcal{J}^- = \min_i \mathcal{J}_i, \quad \mathcal{J}^+ = \max_i \mathcal{J}_i (i = 1, 2, \ldots, n)$$

then

$$\mathcal{J}^- \leq IHCPOWG(\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_n) \leq \mathcal{J}^+$$

Property 13: (Monotonicity) *Let $\mathcal{J}_i$ and $\mathcal{J}'_i$ be two sets of IHCEs; if $\mathcal{J}_i \leq \mathcal{J}'_i$ for all i, then*

$$IHCPOWG(\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_n) \leq IHCPOWG(\mathcal{J}'_1, \mathcal{J}'_2, \ldots, \mathcal{J}'_n)$$

Property 14: (Commutativity) *Let $\mathcal{J}_i$ and $\mathcal{J}'_i$ be two sets of IHCEs; if $\mathcal{J}_i \leq \mathcal{J}'_i$ for all i, then*

$$IHCPOWG(\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_n) = IHCPOWG(\mathcal{J}'_1, \mathcal{J}'_2, \ldots, \mathcal{J}'_n)$$

where $\mathcal{J}'_j$ is a permutation of $\mathcal{J}_j$.

Definition 16. *Suppose$\mathcal{J}_i$ is group of IHCEs; the power hybrid geometric operator of objects 'n' IHCPHG: $\mathcal{J}^n \to \mathcal{J}$ is defined as*

$$IHCPHG(\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_n) = \underset{i=1}{\overset{n}{\otimes}} \left(\dot{\mathcal{J}}_{\sigma(i)}\right)^{\frac{\omega_i(1+T(\dot{\mathcal{J}}_{\sigma(i)}))}{\sum_{i=1}^{n}\omega_i(1+T(\dot{\mathcal{J}}_{\sigma(i)}))}}$$

$$\begin{aligned} = &\prod_{i=1}^{n}\left(\dot{\alpha}_{\sigma(i)}\right)^{\frac{\omega_i(1+T(\dot{\mathcal{J}}_{\sigma(i)}))}{\sum_{i=1}^{n}\omega_i(1+T(\dot{\mathcal{J}}_{\sigma(i)}))}} \\ &+\left(1-\prod_{i=1}^{n}\left(1-\dot{\beta}_{\sigma(i)}\right)^{\frac{\omega_i(1+T(\dot{\mathcal{J}}_{\sigma(i)}))}{\sum_{i=1}^{n}\omega_i(1+T(\dot{\mathcal{J}}_{\sigma(i)}))}}\right)i \\ &+\left(\prod_{i=1}^{n}\left(\dot{\alpha}_{\sigma(i)}+\dot{\gamma}_{\sigma(i)}\right)^{\frac{\omega_i(1+T(\dot{\mathcal{J}}_{\sigma(i)}))}{\sum_{i=1}^{n}\omega_i(1+T(\dot{\mathcal{J}}_{\sigma(i)}))}}\right. \\ &\left.-\prod_{i=1}^{n}\left(\dot{\alpha}_{\sigma(i)}\right)^{\frac{\omega_i(1+T(\dot{\mathcal{J}}_{\sigma(i)}))}{\sum_{i=1}^{n}\omega_i(1+T(\dot{\mathcal{J}}_{\sigma(i)}))}}\right)j \end{aligned} \quad (13)$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_i)^T$ is a represented weight vector, where $\omega_i \in [0,1]$ and $\sum_{i=1}^{n}\omega_i = 1$, and $\mathcal{J}_{\sigma(i)}$ is the ith largest object in the IHCEs arguments $\dot{\mathcal{J}}_i$ ($\dot{\mathcal{J}} = (n\omega_i)\,\mathcal{J}_i,\ i = 1,2,\ldots,n$), $\omega = (\omega_1, \omega_2,\ \ldots\ , \omega_n)$ is the weighting vector of the IHCE influences $\mathcal{J}_i (i = 1,2,\ldots,n)$, $\omega_i \in [0,1]$. $\sum_{i=1}^{n}\omega_i = 1$, as well as ω_i, is a group where

$$\omega_i = g\left(\frac{R_i}{TV}\right) - g\left(\frac{R_{i-1}}{TV}\right), R_i = \sum_{i=1}^{j} V_{\sigma(i)}, TV = \sum_{i=1}^{n} V_{\sigma(i)}, V_{\sigma(i)} = 1 + T\left(\dot{\mathcal{J}}_{\sigma(i)}\right)$$

and $T\left(\dot{\mathcal{J}}_{\sigma(i)}\right)$ is the Sup of the jth biggest IHCEs $\dot{\mathcal{J}}_{\sigma(i)}$ by all the other IHCEs, that is,

$$T\left(\dot{\mathcal{J}}_{\sigma(i)}\right) = \sum_{\substack{j=1 \\ i \neq j}}^{n} Sup\left(\dot{\mathcal{J}}_{\sigma(i)}, \mathcal{J}_{\sigma(j)}\right)$$

where $\sum_{\substack{j=1 \\ i \neq j}}^{n} Sup\left(\dot{\mathcal{J}}_{\sigma(i)}, \mathcal{J}_{\sigma(j)}\right)$ shows the Sup of the jth largest IHCE $\dot{\mathcal{J}}_{\sigma(j)}$, for the ith biggest IHCE $\dot{\mathcal{J}}_{\sigma(i)}$. Specifically, the IHCPHG is reduced to the IHCPWG operator if $\omega = \left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)^T$ and the IHCPHG is reduced to the IHCNPOWG operator if $\omega = \left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)$.

Theorem 1. *The IHCPA operator aggregates the "n" IHCEs and again generates IHCE.*

Proof 1. *The proof is straightforward.* □

Remark 3. *Similarly, the IHCPWA, IHCPOWA, IHCPHA, IHCPG IHCPWG, IHCPOWG, and IHCPHG also generate the IHCE.*

4. Proposed MADM Algorithm Based on IHCS

This section includes the execution of the proposed operators to DMPs through IHF data. The proposed concepts are applied to denote the MADM challenges for the promising computation of developing technology companies with IHF data. Suppose $\mathfrak{H} = \{\mathfrak{H}_1, \mathfrak{H}_2, \ldots, \mathfrak{H}_m\}$ is the distinctive collection of alternatives, and $\mathfrak{G} = \{\mathfrak{G}_1, \mathfrak{G}_2, \ldots, \mathfrak{G}_n\}$ is the collection of attributes. Suppose $\omega = (\omega_t)(t = 1, 2, \ldots n)$ is the weight vector of the attributes, such that $\omega_t \geq 0$, $\sum_{t=1}^{n} \omega_t = 1$. Therefore, both the IHCPWA and IHCWG operators are applied to the MADM problems for the possible calculation of emerging technology commercialization by the IHF environment.

Step 1. Arrange all data about the alternative in matrix $\mathcal{M} = \left(\check{Y}_{it}\right)_{m\times n}$ in such a way:

$$\mathcal{M} = \begin{array}{c} \\ \mathfrak{H}_1 \\ \mathfrak{H}_2 \\ \vdots \\ \mathfrak{H}_m \end{array} \begin{array}{c} \begin{array}{cccc} \mathfrak{G}_1 & \mathfrak{G}_2 & \cdots & \mathfrak{G}_n \end{array} \\ \begin{pmatrix} \check{Y}_{11} & \check{Y}_{12} & \cdots & \check{Y}_{1n} \\ \check{Y}_{21} & \check{Y}_{22} & & \check{Y}_{2n} \\ \vdots & \ddots & \ddots & \vdots \\ \check{Y}_{m1} & \check{Y}_{m2} & \cdots & \check{Y}_{mn} \end{pmatrix} \end{array}$$

Step 2. Formulate the IHCEs $\mathcal{J}_{it} = \alpha_{it} + \beta_{it}i + \gamma_{it}j$ from the IHFEs given in $\check{Y}_{it}$, by using Definition 6.

Step 3. Compute the supports:

$$Sup(\mathcal{J}_{it}, \mathcal{J}_{ik}) = 1 - d(\mathcal{J}_{it}, \mathcal{J}_{ik}), (t, k = 1, 2, \ldots, n) \tag{14}$$

where

$$d(\mathcal{J}_{it}, \mathcal{J}_{ik}) = \frac{1}{3}(|\alpha_{it} - \alpha_{ik}| + |\beta_{it} - \beta_{ik}| + |\gamma_{it} - \gamma_{ik}|) \tag{15}$$

Step 4. Utilize the weights ω_j of attribute $\mathfrak{G}_j$ to calculate the weighted Sup T $(\mathcal{J}_{it})$ of the IHCEs $\mathcal{J}_{it}$ by other IHCEs $\mathcal{J}_{ik}$ $(t, k = 1, 2, \ldots, n, t \neq k)$

$$T(\mathcal{J}_{it}) = \sum_{\substack{k=1 \\ k \neq j}}^{n} \omega_j Sup(\mathcal{J}_{it}, \mathcal{J}_{ik}) \tag{16}$$

and calculate the weight ξ_{it} connected with the IHCEs $\mathcal{J}_{it}$, $(i = 1, 2, \ldots, n, t = 1, 2, \ldots, m)$

$$\xi_{it} = \frac{\omega_t((1 + T(\mathcal{J}_{it}))}{\sum_{i=1}^{n} \omega_t(1 + T(\mathcal{J}_{it}))} \tag{17}$$

where $\xi_{it} \geq 0$ and $\sum_{i=1}^{n} \xi_{it} = 1$.

Step 5. Use the decision data, the IHCPWA operator, and IHCPWG operators

$$\mathcal{J}_t = IHCPWA(\mathcal{J}_{i1}, \mathcal{J}_{i2}, \ldots, \mathcal{J}_{in}) = \frac{\bigoplus_{i=1}^{n} (\omega_i(1 + T(\mathcal{J}_i)\mathcal{J}_i)}{\sum_{i=1}^{n} \omega_i(1 + T(\mathcal{J}_i))}$$

$$= 1 - \prod_{i=1}^{n}\left(1 - (\alpha_{it})^{\xi_{it}}\right) + \left(\prod_{i=1}^{n}(\beta_{it} + \gamma_{it})^{\xi_{it}} - \prod_{i=1}^{n}(\beta_{it})^{\xi_{it}}\right)i + \left(\prod_{i=1}^{n}(\beta_{it})^{\xi_{it}}\right)j \tag{18}$$

and

$$IHCPWG_{\omega}(\mathcal{J}_{t1}, \mathcal{J}_{t2}, \ldots, \mathcal{J}_{tn}) = \underset{i=1}{\overset{n}{\otimes}} (\mathcal{J})^{\xi_{it}}$$

$$= \prod_{i=1}^{n}(\alpha_{it})^{\xi_{it}} + \left(1 - \prod_{i=1}^{n}(1-\beta_{it})^{\xi_{it}}\right)i + \left(\prod_{i=1}^{n}(\alpha_{it}+\gamma_{it})^{\xi_{it}} - \prod_{i=1}^{n}(\alpha_{it})^{\xi_{it}}\right)j \quad (19)$$

to obtain the complete preference elements $\mathcal{J}_i$ of $\mathfrak{H}_t(t = 1, 2, \ldots, m)$.

Step 6. Calculate the scores $Sc(\mathcal{J}_i)$ for the total IHCEs $\mathcal{J}_i$ to rank the $\mathfrak{H}_i$ each, then choose the best one(s).

$$Sc(\mathcal{J}_i) = \frac{1 + (\alpha_i - \gamma_i)(1 + \beta_i)}{2} \quad (20)$$

Step 7. Rank the alternative $\mathfrak{H}_i$ values and select the biggest, considering $(\mathcal{J}_i)$ $(i = 1, 2, \ldots, n)$.

5. Illustrative Example

In this section, we illustrate a mathematical problem to express the potential evaluation of companies of emerging technology by IHF information, demonstrating the method suggested in this paper. In a state of Pakistan, the government wants to contest an election and to overcome fraud and make it fair. The government decided to use technology for this election, so the government decided to develop an electronic voting machine (EVM). To complete this task, the government invites some electronics companies and configures five attributes for selecting the best company for the effort—"project cost" $(\mathfrak{G}_1)$,"completion time" $(\mathfrak{G}_2)$, "technical capability" $(\mathfrak{G}_3)$, "business status" $(\mathfrak{G}_4)$, and "company background" $(\mathfrak{G}_5)$—and allocate the weights of the relative importance of every attribute. The weight vector relating to it has been given as $\omega = (0.25, 0.3, 0.1, 0.2, 0.15)^T$ based on the decision maker's preferences. The four electronics companies have taken in this structure of the alternatives, namely, "Noor Manufacturers Ltd." $(\mathfrak{H}_1)$, "F&I Electronics Ltd." $(\mathfrak{H}_2)$, "Pakistan Software Company" $(\mathfrak{H}_3)$, and "Sial Group of Technological Companies" $(\mathfrak{H}_4)$, who are interested in these tasks. The major purpose of the task is to select the best company for the project. The steps for the suggested technique are executed as follows:

Step 1: The specialist has calculated all alternatives for the distinct attributes based on the IHFSs as given in Table 1.

Table 1. IHF information corresponding to attributes.

$\mathfrak{G}_1$	$\mathfrak{G}_2$	$\mathfrak{G}_3$	$\mathfrak{G}_4$	$\mathfrak{G}_5$
$(\{0.2, 0.1\}, \{0.1, 0.4\})$	$(\{0.1, 0.2\}, \{0.6, 0.3\})$	$(\{0.1, 0.3\}, \{0.5, 0.1\})$	$(\{0.2, 0.4\}, \{0.1, 0.2\})$	$(\{0.2, 0.1\}, \{0.5, 0.1\})$
$(\{0.1, 0.0\}, \{0.2, 0.2\})$	$(\{0.0, 0.1\}, \{0.1, 0.2\})$	$(\{0.1, 0.1\}, \{0.1, 0.3\})$	$(\{0.1, 0.2\}, \{0.1, 0.3\})$	$(\{0.1, 0.4\}, \{0.1, 0.2\})$
$(\{0.3, 0.2\}, \{0.2, 0.1\})$	$(\{0.1, 0.2\}, \{0.0, 0.1\})$	$(\{0.2, 0.5\}, \{0.2, 0.1\})$	$(\{0.0, 0.6\}, \{0.2, 0.1\})$	$(\{0.2, 0.2\}, \{0.5, 0.2\})$
$(\{0.3, 0.1\}, \{0.2, 0.5\})$	$(\{0.3, 0.5\}, \{0.1, 0.1\})$	$(\{0.1, 0.0\}, \{0.1, 0.2\})$	$(\{0.3, 0.2\}, \{0.2, 0.2\})$	$(\{0.3, 0.5\}, \{0.2, 0.2\})$

Step 2: The IHCE for every IHFS is calculated by using Definition 6, and thus, the calculated results are summarized in Table 2.

Table 2. Developed IHCEs.

$\mathfrak{G}_1$	$\mathfrak{G}_2$	$\mathfrak{G}_3$	$\mathfrak{G}_4$	$\mathfrak{G}_5$
$0.12 + 0.66i + 0.22j$	$0.09 + 0.52i + 0.39j$	$0.16 + 0.58i + 0.26j$	$0.25 + 0.65i + 0.1j$	$0.095 + 0.66i + 0.245j$
$0.04 + 0.77i + 0.19j$	$0.04 + 0.82i + 0.14j$	$0.08 + 0.74i + 0.18j$	$0.115 + 0.72i + 0.165j$	$0.205 + 0.69i + 0.105j$
$0.21 + 0.68i + 0.11j$	$0.14 + 0.82i + 0.04j$	$0.305 + 0.59i + 0.105j$	$0.27 + 0.61i + 0.12j$	$0.13 + 0.59i + 0.28j$
$0.145 + 0.56i + 0.295j$	$0.36 + 0.58i + 0.06j$	$0.045 + 0.81i + 0.145j$	$0.2 + 0.65i + 0.15j$	$0.32 + 0.56i + 0.12j$

Now, we apply the proposed operators to show the possible assessment of the emerging technology companies of the participants developing technology enterprises.
Step 3: Using Equation (14), we calculate the supports $Sup\ (\mathcal{J}_{kt}, \mathcal{J}_{kl})$ among the IHCEs. $(k = 1,2,3,4; t,l = 1,2,3,4,5)$
Step 4: Evaluate the weights $\xi_{it}(i = 1,2,3,4, t = 1,2,3,4,5)$ by applying Equation (17) that are associated with the IHCEs, that include $R = \gamma_{4\times 5}$

$$\xi = \begin{bmatrix} 0.266868\ 0.34021\ 0.075676\ 0.18803\ 0.129215 \\ 0.2630690.3455470.0748770.1910870.125421 \\ 0.2662250.338625\ 0.07573\ \ 0.19356\ \ 0.12686 \\ 0.2608040.3431850.0745490.1925590.128904 \end{bmatrix}$$

Step 5: Utilize the IHCPWA operators and IHCPWG operators given in Equations (18) and (19), respectively, to obtain all IHCEs $\mathcal{J}_{it}(i = 1,2,3,4, t = 1,2,3,4,5)$ of the emerging enterprises $\mathfrak{H}_i$. The aggregating values are reflected in Table 3.

Table 3. Aggregate the finding of the developing enterprises by IHCPWA and IHCPWG operators.

	IHFCNPWA	IHFCNPWG
$\mathfrak{H}_1$	$0.136176 + 0.262893i + 0.600931j$	$0.123867 + 0.609392i + 0.266741$
$\mathfrak{H}_2$	$0.079838 + 0.156161i + 0.76399j$	$0.063278 + 0.770114i + 0.166608j$
$\mathfrak{H}_3$	$0.1972 + 0.113411i + 0.68939j$	$0.185977 + 0.712337i + 0.101686j$
$\mathfrak{H}_4$	$0.251833 + 0.148609i + 0.599558j$	$0.213909 + 0.610781i + 0.17531j$

Step 6: By Equation (20), the values obtained by applying the score function are produced in Table 4.

Table 4. Results after applying score function.

	IHCPWA	IHCPWG
$\mathfrak{H}_1$	0.206531	0.38503
$\mathfrak{H}_2$	0.1045	0.408546
$\mathfrak{H}_3$	0.225995	0.572167
$\mathfrak{H}_4$	0.3003	0.531087

Step 7: Agreeing to the score values given in Table 4 and evaluating the formula of score functions, the rank of the emerging technology companies is stated in Table 5. Recall that this idea " $>$ " indicates "preference."

Table 5. Ranking of the best attributes.

	Ranking
IHCPWA	$\mathfrak{H}_4 > \mathfrak{H}_3 > \mathfrak{H}_1 > \mathfrak{H}_2$
IHCPWG	$\mathfrak{H}_3 > \mathfrak{H}_4 > \mathfrak{H}_2 > \mathfrak{H}_1$

Comparative Study

Now, we will evaluate the execution of the suggested method with the current operators, power averaging aggregation, and power geometric aggregation based on intuitionistic hesitant fuzzy data as proposed by Tahir et al. [35]. We will consider the data of Example 1 for this comparative study, and the outcomes are presented in Table 6.

Table 6. Comparative study with some existing approach discussed.

Methods		Ranking
Tahir et al. [35]	IHFPWA	$\mathfrak{H}_3 > \mathfrak{H}_4 > \mathfrak{H}_2 > \mathfrak{H}_1$
	IHFPWG	$\mathfrak{H}_3 > \mathfrak{H}_4 > \mathfrak{H}_1 > \mathfrak{H}_2$
Proposed method	IHCPWA	$\mathfrak{H}_4 > \mathfrak{H}_3 > \mathfrak{H}_1 > \mathfrak{H}_2$
	IHCPWG	$\mathfrak{H}_3 > \mathfrak{H}_4 > \mathfrak{H}_2 > \mathfrak{H}_1$

By comparison, we obtain the existing approach and the proposed method and their ranking results in Table 6 and Figure 2, which show the geometrical representation that will be helpful for understanding this comparative analysis.

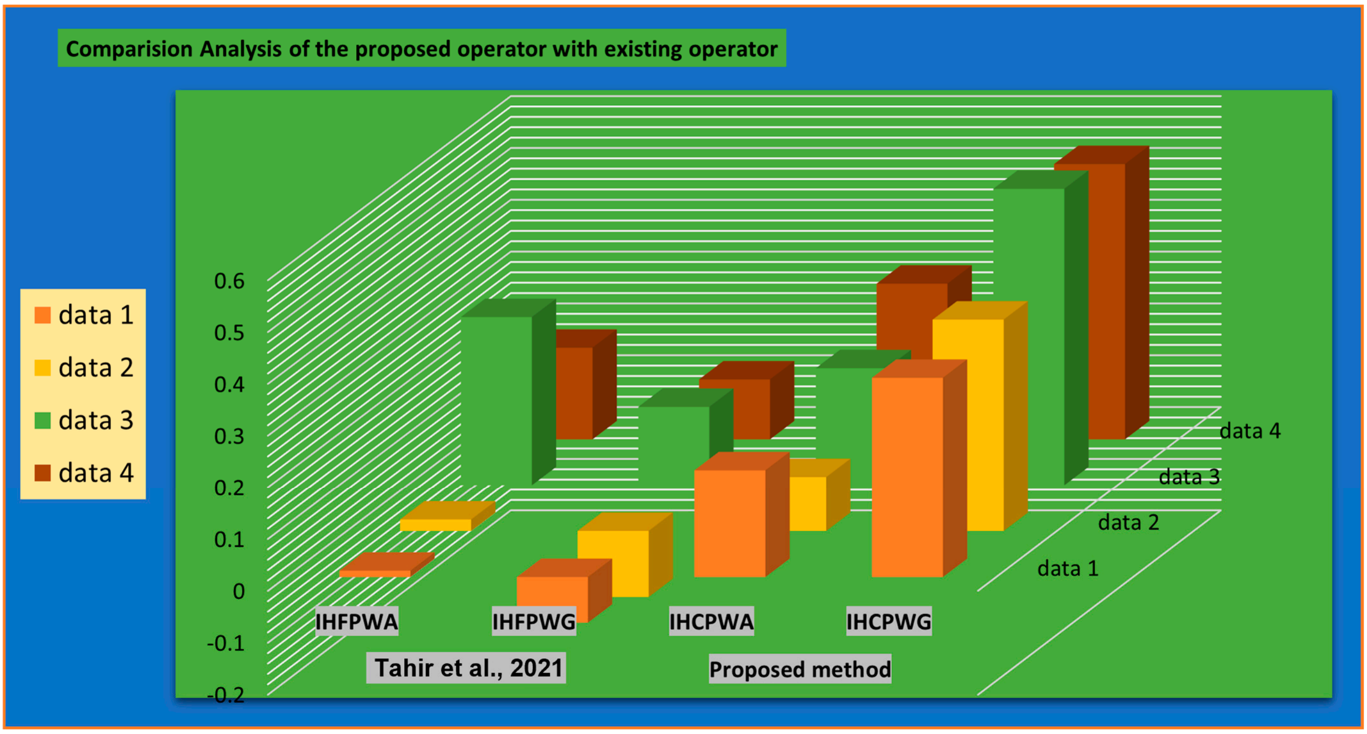

Figure 2. Graphical illustration of the developed and existing operator. Tahir et al. [35].

The advantages and benefits of the proposed notion defined in this paper is a generalized form of the CN defined for the IFS. If we have a singleton set, then the IHFS reduces to the IFS and Definition 6 for the IHFCS reduces to the IFCNS. Therefore, the IHCS is capable of coping with the ambiguity and vagueness of the environment.

6. Conclusions

We have proposed an extension of the intuitionistic hesitant fuzzy set IHFS by integrating it with the CN of SPA, resulting in the intuitionistic hesitant fuzzy connection number set (IHCS). This development has filled certain existing gaps in MADM by facilitating the design of a novel MADM algorithm. The primary contribution of this manuscript is the presentation of novel aggregation operators and basic operational laws of the IHCNs. We explored the concept of power aggregation operators and developed a series of novel operators, including the IHCPA, IHCPG, IHCPWA, IHCPWG, IHCPOWA, IHCPOWG, IHCPHA, and IHCPHG, and verified their novelty with the deserved properties. We validated the proposed methodology by applying it to a real-life problem and demonstrating its feasibility and competency. Additionally, we conducted a comparison analysis of our approach with some existing AO models to establish its validity, authenticity, and effectiveness, and presented a graphical interpretation of the proposed approach. These operators and techniques have numerous applications, including networking analysis, risk assessment, cognitive science, and other areas in uncertain situations. We plan to construct more broadly applicable information metrics to comprehend the information that affects our daily life and investigate our novel techniques in the context of multi-criteria development in the fuzzy environment. We also intend to examine the idea behind our suggested methods within the perspective of square root fuzzy information [57] and ongoing research using a Pythagorean fuzzy system [58].

Author Contributions: Conceptualization, W.A., T.S., I.U.H., H.G.T., F.A., S.J., M.Z.U. and M.M.H.; methodology, W.A. and T.S.; software, W.A.; validation, W.A., T.S., I.U.H., H.G.T., F.A., S.J., M.Z.U. and M.M.H.; formal analysis, W.A. and T.S.; investigation, W.A., T.S., I.U.H., H.G.T., F.A. and S.J.; resources, M.Z.U. and M.M.H.; data curation, W.A., T.S. and I.U.H.; writing—original draft preparation, W.A., T.S., I.U.H., H.G.T., F.A. and S.J.; writing—review and editing, M.Z.U. and M.M.H.; visualization, W.A.; supervision, W.A., T.S. and I.U.H.; funding acquisition, M.M.H. All authors have read and agreed to the published version of the manuscript.

Funding: The authors are grateful to King Saud University, Riyadh, Saudi Arabia for funding this work through Researchers Supporting Project Number (RSP2023R18).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Zhang, F.; Ma, W.; Ma, H. Dynamic Chaotic Multi-Attribute Group Decision Making under Weighted T-Spherical Fuzzy Soft Rough Sets. *Symmetry* **2023**, *15*, 307. [CrossRef]
2. Senapati, T. Approaches to multi-attribute decision-making based on picture fuzzy Aczel–Alsina average aggregation operators. *Comput. Appl. Math.* **2022**, *41*, 1–19. [CrossRef]
3. Ullah, K.; Ali, Z.; Jan, N.; Mahmood, T.; Maqsood, S. Multi-Attribute Decision Making Based on Averaging Aggregation Operators for Picture Hesitant Fuzzy Sets. *Tech. J.* **2018**, *23*, 84–95.
4. Zadeh, L.A. Fuzzy sets. *Inf. Control.* **2018**, *8*, 338–353. [CrossRef]
5. Sarfraz, M.; Ullah, K.; Akram, M.; Pamucar, D.; Božanić, D. Prioritized Aggregation Operators for Intuitionistic Fuzzy Information Based on Aczel–Alsina T-Norm and T-Conorm and Their Applications in Group Decision-Making. *Symmetry* **2022**, *14*, 2655. [CrossRef]
6. Jin, H.; Hussain, A.; Ullah, K.; Javed, A. Novel Complex Pythagorean Fuzzy Sets under Aczel–Alsina Operators and Their Application in Multi-Attribute Decision Making. *Symmetry* **2022**, *15*, 68. [CrossRef]
7. Shaheen, T.; Ali, M.I.; Toor, H. Why do we need q-rung orthopair fuzzy sets? Some evidence established via mass assignment. *Int. J. Intell. Syst.* **2021**, *36*, 5493–5505. [CrossRef]
8. Atanassov, K.T. New operations defined over the intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **1994**, *61*, 137–142. [CrossRef]
9. Atanassov, K.T. On the concept of intuitionistic fuzzy sets. In *On Intuitionistic Fuzzy Sets Theory*; Springer: Berlin/Heidelberg, Germany, 2012; pp. 1–16.
10. Garg, H.; Arora, R. A nonlinear-programming methodology for multi-attribute decision-making problem with interval-valued intuitionistic fuzzy soft sets information. *Appl. Intell.* **2018**, *48*, 2031–2046. [CrossRef]
11. Zeng, S.; Su, W.; Zhang, C. Intuitionistic fuzzy generalized probabilistic ordered weighted averaging operator and its application to group decision making. *Technol. Econ. Dev. Econ.* **2016**, *22*, 177–193. [CrossRef]
12. Eom, Y.S.; Lee, S.J. Intuitionistic Fuzzy Theta-Compact Spaces. *Int. J. Fuzzy Log. Intell. Syst.* **2013**, *13*, 224–230. [CrossRef]
13. Liu, P.; Mahmood, T.; Khan, Q. Multi-attribute decision-making based on prioritized aggregation operator under hesitant intuitionistic fuzzy linguistic environment. *Symmetry* **2017**, *9*, 270. [CrossRef]
14. Alcantud, J.C.R.; Khameneh, A.Z.; Kilicman, A. Aggregation of infinite chains of intuitionistic fuzzy sets and their application to choices with temporal intuitionistic fuzzy information. *Inf. Sci.* **2020**, *514*, 106–117. [CrossRef]
15. Bustince, H.; Burillo, P. Correlation of interval-valued intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **1995**, *74*, 237–244. [CrossRef]
16. Park, D.G.; Kwun, Y.C.; Park, J.H.; Park, I.Y. Correlation coefficient of interval-valued intuitionistic fuzzy sets and its application to multiple attribute group decision making problems. *Math. Comput. Model.* **2009**, *50*, 1279–1293. [CrossRef]
17. Atanassov, K.T. New Topological Operator Over Intuitionistic Fuzzy Sets. *J. Comput. Cogn. Eng.* **2022**, *1*, 94–102.
18. Ye, J. Multicriteria fuzzy decision-making method based on a novel accuracy function under interval-valued intuitionistic fuzzy environment. *Expert Syst. Appl.* **2009**, *36*, 6899–6902. [CrossRef]

19. Fei, L.; Wang, H.; Chen, L.; Deng, Y. A new vector valued similarity measure for intuitionistic fuzzy sets based on OWA operators. *Iran. J. Fuzzy Syst.* **2019**, *16*, 113–126.
20. Wei, G.W. Gray relational analysis method for intuitionistic fuzzy multiple attribute decision making. *Expert Syst. Appl.* **2011**, *38*, 11671–11677. [CrossRef]
21. Jiang, Q.; Jin, X.; Lee, S.J.; Yao, S. A new similarity/distance measure between intuitionistic fuzzy sets based on the transformed isosceles triangles and its applications to pattern recognition. *Expert Syst. Appl.* **2019**, *116*, 439–453. [CrossRef]
22. Zhao, H.; Xu, Z.; Ni, M.; Liu, S. Generalized aggregation operators for intuitionistic fuzzy sets. *Int. J. Intell. Syst.* **2010**, *25*, 1–30. [CrossRef]
23. Senapati, T.; Chen, G.; Mesiar, R.; Yager, R.R. Intuitionistic fuzzy geometric aggregation operators in the framework of Aczel-Alsina triangular norms and their application to multiple attribute decision making. *Expert Syst. Appl.* **2023**, *212*, 118832. [CrossRef]
24. Torra, V. Hesitant fuzzy sets. *Int. J. Intell. Syst.* **2010**, *25*, 529–539. [CrossRef]
25. Torra, V.; Narukawa, Y. On hesitant fuzzy sets and decision. In Proceedings of the 2009 IEEE International Conference on Fuzzy Systems, Jeju Island, Republic of Korea, 20–24 2009 August; pp. 1378–1382.
26. Mahmood, T.; Ullah, K.; Jan, N.; Deli, I.; Khan, Q. Some aggregation operators for bipolar-valued hesitant fuzzy information based on Einstein operational laws. *J. Eng. Appl. Sci.* **2017**, 36.
27. Močkoř, J.; Hýnar, D. On unification of methods in theories of fuzzy sets, hesitant fuzzy set, fuzzy soft sets and intuitionistic fuzzy sets. *Mathematics* **2021**, *9*, 447. [CrossRef]
28. Alcantud, J.C.R. Ranked hesitant fuzzy sets for multi-criteria multi-agent decisions. *Expert Syst. Appl.* **2022**, *209*, 118276. [CrossRef]
29. Zhang, N.; Zhou, Q.; Wei, G. Research on Green Supplier Selection Based on Hesitant Fuzzy Set and Extended LINMAP Method. *Int. J. Fuzzy Syst.* **2022**, *24*, 3057–3066. [CrossRef]
30. Xia, M.; Xu, Z.; Chen, N. Some hesitant fuzzy aggregation operators with their application in group decision making. *Group Decis. Negot.* **2013**, *22*, 259–279. [CrossRef]
31. Shaheen, T.; Ali, M.I.; Shabir, M. Generalized hesitant fuzzy rough sets (GHFRS) and their application in risk analysis. *Soft Comput.* **2020**, *24*, 14005–14017. [CrossRef]
32. Sun, Q.; Ouyang, J. Hesitant fuzzy multi-attribute decision making based on TOPSIS with entropy-weighted method. *Manag. Sci. Eng.* **2015**, *9*, 1–6.
33. Ni, Y.; Zhao, H.; Xu, Z.; Wang, Z. Multiple attribute decision-making method based on projection model for dual hesitant fuzzy set. *Fuzzy Optim. Decis. Mak.* **2022**, *21*, 263–289. [CrossRef]
34. Liu, J.; Sun, M. Generalized power average operator of hesitant fuzzy numbers and its application in multiple attribute decision making. *J. Comput. Inf. Syst.* **2013**, *9*, 3051–3058.
35. Mahmood, T.; Ali, W.; Ali, Z.; Chinram, R. Power Aggregation Operators and Similarity Measures Based on Improved Intuitionistic Hesitant Fuzzy Sets and their Applications to Multiple Attribute Decision Making. *Comput. Model. Eng. Sci.* **2021**, *126*, 1165–1187. [CrossRef]
36. Zhao, K.Q. *Set pair and set pair analysis-a new concept and systematic analysis method*; National Conference on System Theory and Regional Planning: Baotou, China, 1989; Volume 5.
37. Jiang, Y.L.; Xu, C.F.; Yao, Y.; Zhao, K.Q. Systems information in set pair analysis and its applications. In Proceedings of the 2004 International Conference on Machine Learning and Cybernetics (IEEE Cat. No. 04EX826), Shanghai, China, 26–29 August 2004; Volume 3, p. 171.
38. ChangJian, W. Application of the set pair analysis theory in multiple attribute decision-making. *J. Mech. Strength* **2007**, *6*, 1009–1012.
39. Lü, W.S.; Zhang, B. Set pair analysis method of containing target constraint mixed interval multi-attribute decision-making. In *Applied Mechanics and Materials*; Trans Tech Publications Ltd.: Stafa-Zurich, Switzerland, 2012; Volume 226, pp. 2222–2226.
40. Xie, Z.; Zhang, F.; Cheng, J.; Li, L. Fuzzy multi-attribute decision making methods based on improved set pair analysis. In Proceedings of the 2013 Sixth International Symposium on Computational Intelligence and Design, Hangzhou, China, 28–29 October 2013; Volume 2, pp. 386–389.
41. Kumar, K.; Garg, H. TOPSIS method based on the connection number of set pair analysis under interval-valued intuitionistic fuzzy set environment. *Comput. Appl. Math.* **2018**, *37*, 1319–1329. [CrossRef]
42. Kumar, K.; Garg, H. Connection number of set pair analysis based TOPSIS method on intuitionistic fuzzy sets and their application to decision making. *Appl. Intell.* **2018**, *48*, 2112–2119. [CrossRef]
43. Shen, Q.; Huang, X.; Liu, Y.; Jiang, Y.; Zhao, K. Multiattribute decision making based on the binary connection number in set pair analysis under an interval-valued intuitionistic fuzzy set environment. *Soft Comput.* **2020**, *24*, 7801–7809. [CrossRef]
44. Hu, J.; Yang, L. Dynamic stochastic multi-criteria decision-making method based on cumulative prospect theory and set pair analysis. *Syst. Eng. Procedia* **2011**, *1*, 432–439. [CrossRef]
45. Liu, J.B.; Malik, M.A.; Ayub, N.; Siddiqui, H.M.A. Distance Measures for Multiple-Attributes Decision-Making Based on Connection Numbers of Set Pair Analysis with Dual Hesitant Fuzzy Sets. *IEEE Access* **2020**, *8*, 9172–9184. [CrossRef]
46. Wang, C.-N.; Yang, F.-C.; Vo, N.T.M.; Nguyen, V.T.T. Wireless communications for data security: Efficiency assessment of cybersecurity industry—A promising application for UAVs. *Drones* **2022**, *6*, 363. [CrossRef]
47. Dang, T.-T.; Nguyen, N.-A.; Nguyen, V.-T.; Dang, L.-T. A two-stage multi-criteria supplier selection model for sustainable automotive supply chain under uncertainty. *Axioms* **2022**, *11*, 228. [CrossRef]

48. Xu, Z.; Yager, R.R. Some geometric aggregation operators based on intuitionistic fuzzy sets. *Int. J. Gen. Syst.* **2006**, *35*, 417–433. [CrossRef]
49. Zhang, Z. Hesitant fuzzy power aggregation operators and their application to multiple attribute group decision making. *Inf. Sci.* **2013**, *234*, 150–181. [CrossRef]
50. Yager, R.R. The power average operator. *IEEE Trans. Syst. Man Cybern.-Part A Syst. Hum.* **2001**, *31*, 724–731. [CrossRef]
51. Ahmmad, J.; Mahmood, T.; Mehmood, N.; Urawong, K.; Chinram, R. Intuitionistic Fuzzy Rough Aczel-Alsina Average Aggregation Operators and Their Applications in Medical Diagnoses. *Symmetry* **2022**, *14*, 2537. [CrossRef]
52. Mahmood, T.; Ali, Z.; Baupradist, S.; Chinram, R. TOPSIS Method Based on Hamacher Choquet-Integral Aggregation Operators for Atanassov-Intuitionistic Fuzzy Sets and Their Applications in Decision-Making. *Axioms* **2022**, *11*, 715. [CrossRef]
53. Peng, F.; Wang, Y.; Xuan, H.; Nguyen, T.V.T. Efficient road traffic anti-collision warning system based on fuzzy nonlinear programming. *Int. J. Syst. Assur. Eng. Manag.* **2022**, *13*, 456–461. [CrossRef]
54. Huynh, N.T.; Nguyen, T.V.; Nguyen, Q.M. Optimum Design for the Magnification Mechanisms Employing Fuzzy Logic-ANFIS. *CMC-COMPUTERS MATERIALS CONTINUA* **2022**, *73*, 5961–5983. [CrossRef]
55. Nguyen, T.V.T.; Huynh, N.-T.; Vu, N.-C.; Kieu, V.N.D.; Huang, S.-C. Optimizing compliant gripper mechanism design by employing an effective bi-algorithm: Fuzzy logic and ANFIS. *Microsyst. Technol.* **2021**, *27*, 3389–3412. [CrossRef]
56. Patel, U.D.; Radenović, S. An Application to Nonlinear Fractional Differential Equation via α-Γ F-Fuzzy Contractive Mappings in a Fuzzy Metric Space. *Mathematics* **2022**, *10*, 2831. [CrossRef]
57. Radenovic, S.; Ali, W.; Shaheen, T.; ul Haq, I.; Akram, F.; Toor, H. Multiple Attribute Decision-Making Based on Bonferroni Mean Operators under Square Root Fuzzy Set Environment. *J. Comput. Cogn. Eng.* **2022**. [CrossRef]
58. Ul Haq, I.; Shaheen, T.; Ali, W.; Senapati, T. A Novel SIR Approach to Closeness Coefficient-Based MAGDM Problems Using Pythagorean Fuzzy Aczel–Alsina Aggregation Operators for Investment Policy. *Discret. Dyn. Nat. Soc.* **2022**, *2022*, 5172679. [CrossRef]