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Optimal preventive policies for parallel systems using Markov decision process: application to an offshore power plant

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ABSTRACT

This work proposes a Markov Decision Process (MDP) model for identifying windows of opportunities to perform preventive maintenance for multi-unit parallel systems subject to a varying demand. The main contribution lies in proposing: (i) a reward function that does not depend on maintenance costs, which are typically difficult to assess and classify; and (ii) a new metric for prevention.

By optimizing the capacity utilization rate and the decision flexibility, which is denoted in terms of standby units, for a set of typical operational scenarios, the optimal opportunities for preventive interventions are identified within the respective prevention ranges, in relation to an offshore power plant (case study). The sequential decision problem is solved using the Value Iteration algorithm to obtain the optimal long-term policies.

As a result, a backlog management decision-support solution is developed, using a low-cost computational model, which provides scenario-dependent preventive policies and promotes the integration of operations with maintenance, being easy to implement, maintain and communicate with stakeholders.

1. Introduction

A key problem in the operation of complex engineering systems, such as aircrafts, ships, and offshore platforms, is the coordination of long-term production and preventive maintenance. This is particularly problematic in adverse operational environments such as oilfield operations, where logistical aspects have a major impact on total repair times and operational costs. Moreover, in these environments it is often difficult to assess and classify maintenance costs, either resulting from preventive or corrective actions. Equipment damage and downtime often incur substantial economic losses in the offshore environment, not only due to setup and repair costs, but also resulting from production disruptions. Offshore production systems typically consist of parallel machinery systems with some degree of redundancy and, as such, they may be operated under different policies whose control actions normally include: (i) maintain the current activity level until the next decision epoch (*i.e.*, wait); (ii) activate a standby component and increase activity; (iii) deactivate a component and place it on standby; and (iv) release a standby component for preventive maintenance.

Faced with the problem of influencing the behavior of a probabilistic system, the operator must be able to make decisions under uncertainty, *i.e.*, find a sequence of actions that drive the system to perform optimally with respect to some predetermined performance criteria. The probabilistic nature of these systems arises from variations driven by: (i) the supply-demand relationship (*i.e.*, operation stress); (ii) the system/component characteristics (*e.g.*, reliability); and (iii) the failure and degradation mechanisms. Making decisions in such an environment can benefit from the use of decision-support tools that systematically adjust the prevention levels, aiming to reduce the risk of failures and production losses.

In the design of maintenance decision-support tools, Dekker (1996) regards the modeling of system degradation and the occurrence of failure events as essential components of these approaches to understand how they are influenced by the maintenance regime, *e.g.*, a given Operations and Maintenance (O&M) policy. An optimal O&M policy should optimize production while mitigating system degradation. Thus, in that context, preventive maintenance actions (*e.g.*, inspections, testing, adjustments, cleaning, and lubrication)

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become an important part of the maintenance work that can extend the system's useful life.

In this work we propose a new preventive policy generation model for multi-unit parallel systems under a varying demand. Our model uses the capacity utilization rate to compose a measure of system's performance. This allows one to coordinate production and maintenance without an overly sensitive dependence to uncertain cost estimates.

One objective of this work is the development of a simple model, easy to implement, use and communicate, and capable of promoting O&M integration in the offshore operational environment. In other words, a model that can provide optimal long-term preventive policies to the operator/maintainer by answering the question: *Under what operating conditions of a parallel production-system is a preventive action optimal in the long run?*

The proposed approach uses the Markov Decision Process (MDP) as a mathematical framework for modeling sequential decision-making with partially random outcomes. Our model is developed based on a case study, which refers to the main power generation system of a floating, production, storage and offloading unit (FPSO) and the state-space definition is based on the observable operating conditions of the parallel machinery system (*i.e.*, k-out-of-n configurations). Since the system performance is to be measured in average terms, the expected average reward is chosen as the optimization criterion, and the optimal policies are obtained by a dynamic programming formulation using the Value Iteration algorithm.

This paper contributes to the engineering practice by combining existing models and methodologies to generate optimal preventive policies for a smooth long-term operation of parallel machinery systems. Concretely, the contributions of the work are:

- A new MDP-based methodology for identifying windows of opportunity to perform preventive maintenance that, unlike other works, yields appropriate ranges of prevention to be considered by the operator, and which avoids complicated cost estimation by optimizing the capacity utilization rate.
- The improved interpretability intrinsic to the proposed MDP model that facilitates the communication of the resulting decision policy to the stakeholders.
- The low dimensionality of the model that promotes easy model design and maintenance, and relatively low computational cost to calculate optimal decision policies.
- A demonstration of the good features of the proposed methodology, by means of an application to the synthesis of decision policies for a real-world offshore power plant.

More specifically, the paper investigates the relationship between failure/repair rate combinations (*i.e.*, scenario severity) with prevention levels. The approach suggests an alternative value-function, which optimizes the capacity utilization rate combined with decision flexibility, instead of cost minimization. The work makes its case in a maintenance backlog management problem, based on data from an actual system and the interpretations thereof. The same offshore power plant of the case study was also considered by Machado et al. (2014) and Perera et al. (2015). The expected benefits of the approach are: (i) an indirect optimization of the total operating costs; (ii) a regularization of the capacity utilization rate of similar systems (*e.g.*, fleet management); and (iii) a reduction of energy consumption and the respective climate emissions of these engines.

The article is organized as follows. In Section 2 we discuss some related solution approaches and position our model in relation to the literature. In Section 3 we present the problem statement, the solution approach and the proposed MDP model for the synthesis of optimal O&M policies. In Section 3.1, we present a case study based on real-world offshore power plant and discuss the obtained results. Finally, in Section 5 we present the conclusions and final considerations.

2. Related works

In the literature on MDP, several Markov decision models are applied to condition-based maintenance (CBM), where a system's condition/state scale is considered with respect to a set of related available actions.

Stengos and Thomas (1980), for example, consider identical blast furnaces and, by using MDP, they find the cost-related optimal policy for the case of two units. One of the results is that a specific cycle should be followed to reduce the probability of both units failing simultaneously.

Chan and Asgarpoor (2006) present a method to find an optimal maintenance policy using an 8-state Markov model with two actions: "do nothing" and "do maintenance" with respect to the optimal preventive interval.

Amari et al. (2006) provide a generic procedure to obtain optimal inspection schedules and decisions for k-out-of-n load-sharing systems in a cost-effective condition-based approach, using a 6-state condition scale with 4 actions: "no action," "minor maintenance," "preventive maintenance," and "corrective maintenance."

Ossai et al. (2016) develop a 6-state Markov model for components of wind turbines with a survival function, using the Weibull distribution to establish the impacts of component maintenance on down time and failure risks. Grillo et al. (2016) present a method based on MDP to optimally schedule energy storage devices using a 14-state Markov model minimizing the costs and publishing decision-support tables. Aghezzaf et al. (2007), for example, aim to find an integrated preventive strategy that meets the demand, while minimizing the expected sum of production and maintenance costs.

Chen and Trivedi (2005) present a semi-Markov decision process (SMDP) approach to optimize preventive intervals, considering three types of decisions: (0) no action is taken; (1) minimal maintenance is performed; and (2) major maintenance is performed. Wu and Zhao (2010) also applied SMDP to optimize preventive intervals related to wind turbine gearboxes representing deterioration in 7 states, 4 different actions and using the policy iteration (PI) algorithm in a cost-effective related approach.

Our approach aims to find a preventive O&M strategy that satisfies the demand and optimizes the capacity utilization rate in the long-run. Since the amount of prevention is a key decision in terms of maintenance optimization, our approach synchronizes production with preventive activities. According to Vatn (2018), the coordination between production and maintenance is among the crucial aspects of the approaches that belong to the so-called Industry 4.0.

The main similarities of our model with respect to those mentioned above are: (i) we investigate the relationship between time-to-failure and time-to-repair as Chan and Asgarpoor (2006); (ii) we aim to generate standardized policies for decision support in different scenarios as Grillo et al. (2016); (iii) we search for integrated O&M policies as Aghezzaf et al. (2007); (iv) our model comprises a small set of related actions as Chen and Trivedi (2005), Chan and Asgarpoor (2006), Amari et al. (2006) and Wu and Zhao (2010).

The distinct features of our model are: (i) the proposal to optimize the rate of capacity utilization combined with decision flexibility as a way to mitigate risk of failure; (ii) the proposed prevention metric with lower/upper bounds; and (iii) the simplicity and ease of communication with stakeholders.

3. Problem scope and solution approach

3.1. Problem statement

A major Oil & Gas operator is experiencing a significant increase in the maintenance backlog related to the power generation systems of its FPSO fleet. Although a condition-monitoring system is available, providing diagnostics and prognostics for each of the turbo-generators, the system has not been integrated towards a preventive operation of

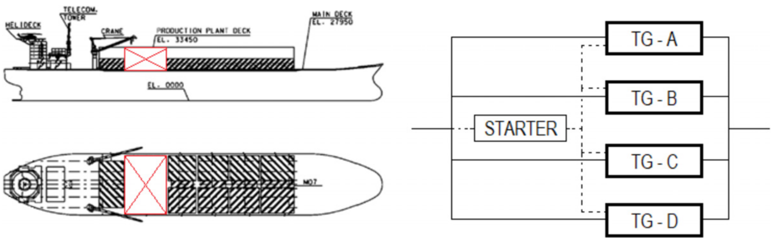

Fig. 1. Situation of the FPSO's main power generation system.

Table 1
Failure and repair estimates.

Failure estimates	λ [/h]	MTBF [h]
TG-A (Case study)	0.004807	208.0
Aero-derivative gas turbine (OREDA)	0.002212	452.0
One failure per month	0.001369	730.0
One failure per year	0.000114	8760.0
Repair estimates	**μ [/h]**	**MTTR [h]**
Short repair/inspection	0.125000	8.0
Preventive repair 1 (Case study)	0.045300	22.1
Preventive repair 2	0.041666	24.0
Corrective repair 1 (Case study)	0.025100	39.8
Mid-life (Case study)	0.013888	72.0
Overhaul (Case study)	0.004629	216.0

Source: Petrobras; Perera et al. (2015); Rausand and Høyland (2004); and Technology and Society (2015).

the parallel machines. After a series of meetings, it was decided that standard stationary policies should be identified and prescribed by the headquarter's turbo-machinery experts, aiming to coordinate and synchronize production with preventive activities. Considering a set of typical operating scenarios, based on a chosen case and also on statistics (*e.g.*, failure/repair rates) from the technical literature, optimal opportunities for preventive maintenance should be prescribed in accordance with the appropriate prevention range.

The offshore power plant under study is located on the deck of an FPSO operating in Campos Basin, off the coast of Rio de Janeiro, Brazil (see Fig. 1). The system contains 4 identical parallel turbo-generators (TG-A/B/C/D) consisting of aero-derivative gas turbine engines with a nominal capacity of 25000 kW, coupled with electric generators with a nominal capacity of 28750 kVA. The range of required grid load is from 35 to 55 MW which dictates the operation of 2 or 3 generators, allowing different operating policies.

Failure and repair rates were estimated by Perera et al. (2015), from a set of condition monitoring data (*e.g.*, failure and repair events) consisting of 22596 operating hours of a selected gas turbine engine (TG-A). Data from the preventive maintenance plan was also considered. Table 1 presents the failure and repair estimates used in this case study.

The system is operated according to a cold standby strategy, assuming that the standby components are protected from the stress associated with operation so that no component fails before its activation, as in (Peiravi et al., 2019). Regarding the switching system, a starting failure probability is also considered in our model as a constant value.

The offshore machinery technician typically makes the control decisions and takes actions empirically. However what she/he cannot know for certain is: which action, among the available ones, in a given situation, is optimal in a long-term perspective. More specifically, in which decision epoch a standby component (*i.e.*, a turbo-generator) *should* be released for preventive maintenance.

3.2. Solution methodology

According to Puterman (1994), MDP, also referred to as stochastic dynamic programs or stochastic control problems, can model sequential decision-making problems. The approach assumes the Markov property, that is, the effect of an action on a state depends solely on the action and the current state of the system. The problem is to choose, prior to the first decision epoch, a policy to maximize a function of a reward sequence which reflects the decision maker's inter-temporal trade-offs.

A Markov decision process is a tuple (S, A, p, r), where:

- S is a set of states for the process to visit, called state space;
- A is a set of actions that can be executed at different decision epochs;
- $p : S \times A \times S \mapsto [0, 1]$ is a function that returns the probability of the system's transition to a given state $j \in S$, given that the process is in a state $i \in S$ and the decision-maker implements the action $a \in A$. Each function evaluation is denoted as $p(j|i, a)$;
- $r : S \times A \mapsto R$ is a function that gives the cost (or reward) of choosing an action $a \in A$ when the process is in a state $i \in S$.

At a given decision epoch, the decision maker observes the system in $s \in S$, and may choose an action $a \in A_s$ from the set of feasible actions in state s. Let $A = \cup_{s \in S} A_s$ and assume that S and A_s do not vary with the time t. As a result of choosing an action a in state s in decision epoch t, two things happen: (i) the decision-maker receives a reward $r(s, a)$; and (ii) the system state at the next decision epoch is determined by the probability distribution $p(\cdot|s, a)$.

Let the real-valued function $r(s, a)$ denote the reward received by the decision-maker for taking action $a \in A_s$ at system state $s \in S$. When positive, $r(s, a)$ may be regarded as an income, otherwise as a cost. One requirement is that its value or expected value is known before choosing an action, another is that it is not affected by future actions. According to Puterman (1994) a policy or strategy provides the decision-maker with a prescription for choosing actions in any possible state, whilst a decision rule specifies the action to be chosen at a particular decision epoch, *i.e.*, a policy is a sequence of decision rules and decision makers seek policies which are optimal in some context.

Three classical MDP solution methods are: (i) policy iteration (PI); (ii) value iteration (VI); and (iii) linear programming (LP). According to Dekker et al. (2008), the VI algorithm can be faster than the PI algorithm if the transition dynamic matrix is sparse and only few transitions are possible.

Further, the VI algorithm is relatively easy to implement (Hernández-Lerma, 1989) and better suited for discrete solutions (Dreyfus, 1956). For these desirable features, we choose the dynamic programming formulation and the VI algorithm to generate optimal preventive policies for a long-term operation of parallel machinery systems. The basic nomenclature used in the MDP model is presented in Table 2.

The VI algorithm induces a Markov process and finds, by iteratively updating the value of every state in a fixed order, the sequence of actions that yields the best outcome of the value function. In the present approach we use the average reward criterion, *i.e.*, without discount factor, so it is necessary to determine when to stop calculating successive approximations.

The convergence/stopping criterion is based on the span semi-norm $sp(v^{n+1} - v^n)$, which is defined as $sp(v) = \max_{s \in S} v(s) - \min_{s \in S} v(s)$ for all $v \in V$. This is a measure of how close a vector is to being constant.

As described by (Puterman, 1994, 364), Algorithm 1 finds a stationary ϵ-optimal policy $(d_\epsilon)^\infty$ and an approximation to its gain. Puterman (1994) also shows that the algorithm produces iterates that

Table 2
MDP basic nomenclature.

Symbol	Description
S	Set of system states s
A_s	Set of available actions a in state $s \in S$
j	A given destination state
$d(s)$	Action chosen by decision rule d in state s
d^{∞}	Stationary policy which uses decision rule d in every period
$d_{\epsilon}(s)$	Optimal decision in state s with respect to the tolerance ϵ
π	Policy $(d_1, d_2, \ldots, d_{N-1})$; $N \leq \infty$
π^*	Optimal policy
ϵ	Error tolerance (stopping criteria)
$sp(\cdot)$	Span semi-norm $sp(v^{n+1} - v^n)$
argmax	Subset of elements at which the maximum of a function is obtained
n	Iteration index
N	Number of iterations
V	Set of values $v^n(s)$ with n denoting the iteration number
v^{π}	Expected total reward under policy π
$v^0(s)$	Value at the iteration 0, $v^0(s) \in V$
$v^{n+1}(s)$	Value of state s at iteration $n+1$
$r(s,a)$	Reward for choosing action a in state s
$p(j\|s,a)$	Probability that the system occupies state j at time $t+1$ when action a is chosen in state s at time t
$g^{\pi}(s)$	Gain or expected average reward of policy π

Table 3
Model nomenclature.

Symbol	Description
λ	Component failure rate estimate
μ_1	Minor repair rate estimate (preventive)
μ_2	Major repair rate estimate (corrective)
α	Standby utility parameter - Range [0.5:1.5]
β	Starting failure probability - Range [0:0.15]
$Prev$	Prevention factor array [1:10]
m	Severity ratio (λ/μ_1)
s_t	State vector $s_t = (x_1, x_2, x_3, x_4)$
x_1	Number of components in operation
x_2	Number of components in standby
x_3	Number of components in preventive maintenance
x_4	Number of components in corrective maintenance
u_s	Utility of state s
L	System current load (power demanded by the FPSO)
l_s	Current component load (power generated per component), $l_s \leq L/x_1$
$lact$	Component activation load - Range [10:17]
lt	Component target load - Range [22:27]
$lmin$	Component minimum load - Range [10:25]
$E(s)$	Expected sojourn time in state s

Algorithm 1 Value Iteration.

Require: an MDP $M = (S, A, p, r)$
1: Select $v^0 \in V$, specify $\epsilon > 0$ and set $n = 0$.
2: **for** $s \in S$ **do**
3: Compute $v^{n+1}(s)$ by $v^{n+1}(s) = \max_{a \in A_s} \left\{ r(s,a) + \sum_{j \in S} p(j|s,a) v^n(j) \right\}$
4: **end for**
5: **if** $sp(v^{n+1} - v^n) < \epsilon$ **then**
6: Go to step 8, otherwise increment n by 1 and return to step 2
7: **end if**
8: **for** $s \in S$ **do**
9: Choose $d_{\epsilon}(s) \in \arg\max_{a \in A_s} = \left\{ r(s,a) + \sum_{j \in S} p(j|s,a) v^n(j) \right\}$
10: **end for**
11: **return** $\pi^* = (d_{\epsilon}(s) : s \in S)$, an optimal policy

converge to the optimal value function, which thereby yield an optimal decision policy π^*.

3.3. Proposed model

In this section, we present the development of the MDP model for the offshore power plant under study. The nomenclature used in the model appears in Table 3.

The states and transitions at component-level are presented in Fig. 2 where solid arrows represent the transitions due to events (*e.g.*, failures and repair completions), whereas dotted arrows represent the transitions governed by control actions which are coded as: (1) "Wait;" (2) "Activate;" (3) "Deactivate;" and (4) "Do preventive."

Although stochastic, the system is observable by means of a state vector $s_t = (x_1, x_2, x_3, x_4)$ containing the number of components in each of the possible states (see Fig. 2). The quantity x_1 denotes the number of components in "Operating State" (OP), x_2 denotes the number of components in "Standby" (STB), whereas x_3 and x_4 denote the number of components in "Preventive Repair" (PRV) and "Corrective Repair" (CRT) respectively.

If the decision is to wait until the next decision epoch, transitions may occur by chance, either due to a failure or a repair completion. Failure of a component causes a transition from (OP) to (CRT), while a repair brings the respective component to (STB). By choosing to deactivate a component, a transition from (OP) to (STB) occurs, and the decision to do preventive, which brings a standby component to (PRV) is of special interest in this approach.

In order to develop a model considering only the relevant states and transitions to represent the continuous and normal operation of the system of interest, a procedure has been adopted as follows:

1. Define, with the stakeholders, the normal operating conditions, decision rules, action sets and limits;
2. From the full operative state (*i.e.*, all components operating), add states and transitions towards the least operative states, such that a strongly connected graph is obtained, *i.e.*, an irreducible Markov chain;
3. Simulate all the plausible transitions, adding new states when necessary, according to the combinations of failure and repair completion events;
4. Collect data in order to estimate the transition probabilities;

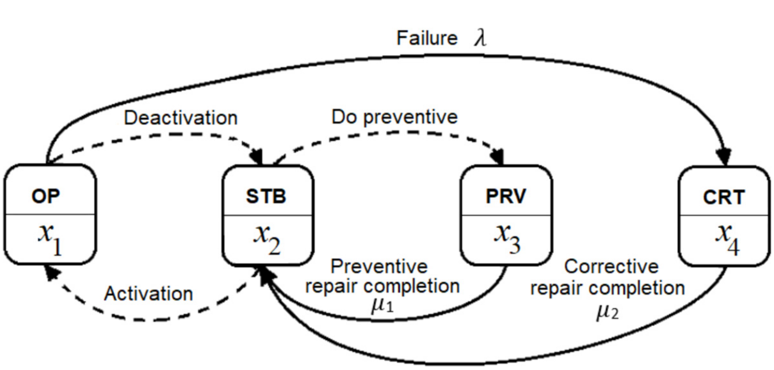

Fig. 2. States and transitions at component-level. The system state $s = (x_1, x_2, x_3, x_4)$ is defined by the number of components in operation (x_1, OP), in standby (x_2, STB), undergoing preventive maintenance (x_3, PRV), and undergoing corrective repair (x_4, CRT). Solid lines correspond to controllable actions, whereas dashed lines indicate uncontrollable events.

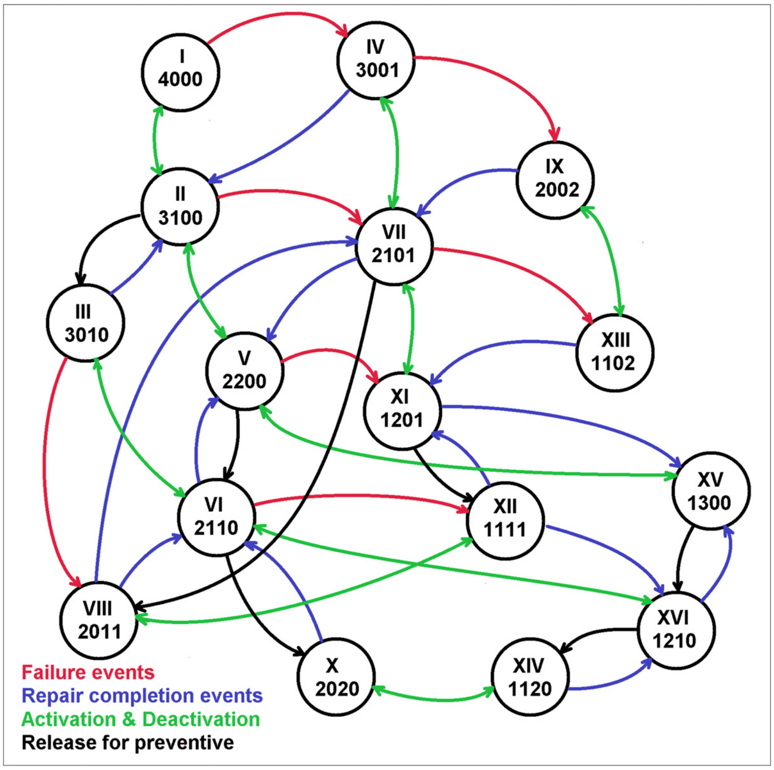

Fig. 3. States and transitions at system-level. Labeled from I to XVI, each state relates to a vector (x_1, x_2, x_3, x_4) with the number of components in operation, standby, preventive maintenance, and corrective repair respectively.

5. Test the resulting Markov chain, decision rules and rewards.

As a result of applying the above procedure, a 16-state Markov chain evolved as presented in Fig. 3. The model assumptions are: (i) discrete state- and action-spaces; (ii) rewards and transition probabilities are stationary and bounded, *i.e.*, $r(s,a) \leq M < \infty$ and $P(j|s,a) \leq 1, \forall a \in A, s, j \in S$; (iii) failure and repair rates are constant and equal for all identical components; (iv) the components time-to-failure/repair follows an exponential distribution; (v) failures are independent; (vi) the third consecutive component failure, resulting from independent causes, is blocked assuming that one repair completion, preventive or corrective, will happen previously; (vii) the system is maintained under perfect repair (*i.e.*, as good as new); and (viii) repair starts immediately after failure occurs.

In Fig. 3 we present the states and transitions at system-level. The states $S = \{I, II, III, \dots, XVI\}$ are labeled with roman numerals, and encoded with a tuple of 4 digits which represent the $x_1, \dots, x_4$ values. Failure events/transitions are represented by red unidirectional arrows, whereas activation and deactivation related transitions are represented by green bidirectional arrows. Repair completion events are represented by blue unidirectional arrows, and the release of a component for preventive maintenance is indicated by a black unidirectional arrow. From the full operative state (I-4000), for example, when a failure occurs the next system state will be (IV-3001) and a corrective repair starts. A repair completion at state (IV-3001), for example, triggers the system to transition to state (II-3100) and so on.

3.3.1. *Transition probabilities*

The transition probabilities $p(j|s,a)$ are derived from the system's failure and repair rates (see Table 1) according to the available actions/transitions in each state. When the action (1) "Wait" is chosen from the states (I-4000), (II-3100) or (V-2200) for example, the system will remain in the current state until a failure occurs. Thus, assuming $s = (x_1, x_2, x_3, x_4)$ as the current state, we have:

$$p(j|s,1) = 1, \qquad j \rightarrow (x_1 - 1, x_2, x_3, x_4 + 1) \tag{1}$$

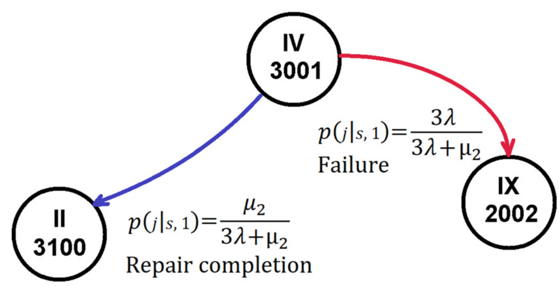

Fig. 4. Transition probabilities from state IV under action (1) "Wait."

On the other hand, action (2) "Activate" is made probabilistic by assigning a value β denoting the starting failure probability, whereas actions (3) "Deactivate" or (4) "Do Preventive" result in a deterministic and instantaneous transitions.

From states where there are components under repair, *i.e.*, x_3 and/or $x_4 \neq 0$, the action (1) "Wait" results in three possible events (*i.e.*, a failure, a preventive or a corrective repair completion), with transition probabilities satisfying, respectively:

$$p(j|s,1) = \frac{x_1\lambda}{x_1\lambda + x_3\mu_1 + x_4\mu_2}, \qquad j \rightarrow (x_1 - 1, x_2, x_3, x_4 + 1) \tag{2}$$

$$p(j|s,1) = \frac{x_3\mu_1}{x_1\lambda + x_3\mu_1 + x_4\mu_2}, \qquad j \rightarrow (x_1, x_2 + 1, x_3 - 1, x_4) \tag{3}$$

$$p(j|s,1) = \frac{x_4\mu_2}{x_1\lambda + x_3\mu_1 + x_4\mu_2}, \qquad j \rightarrow (x_1, x_2 + 1, x_3, x_4 - 1) \tag{4}$$

One example of the derivation of transition probabilities is presented in Fig. 4, considering state IV under action (1) "Wait."

On the other hand, the actions (2) "Activate," (3) "Deactivate," and (4) "Do Preventive," result in transition probabilities satisfying, respectively:

$$p(j|s,2) = 1 - \beta, \qquad j \rightarrow (x_1 + 1, x_2 - 1, x_3, x_4) \tag{5}$$

$$p(j|s,3) = 1, \qquad j \rightarrow (x_1 - 1, x_2 + 1, x_3, x_4) \tag{6}$$

Table 4
Action availability scheme.

Actions	I 4000	II 3100	III 3010	IV 3001	V 2200	VI 2110	VII 2101	VIII 2011
$1 - Wait$	1	1	1	1	1	1	1	1
$2 - Activate$	0	1	0	0	1	1	1	0
$3 - Deactivate$	1	1	1	1	1	1	1	1
$4 - Do\ preventive$	0	1	0	0	1	1	1	0
Actions	**IX 2002**	**X 2020**	**XI 1201**	**XII 1111**	**XIII 1102**	**XIV 1120**	**XV 1300**	**XVI 1210**
$1 - Wait$	1	1	1	1	1	1	1	1
$2 - Activate$	0	0	1	1	1	1	1	1
$3 - Deactivate$	1	1	0	0	0	0	0	0
$4 - Do\ Preventive$	0	0	1	0	0	0	1	1

"0" denotes a non-feasible action

$$p(j|s,4) = 1, \qquad j \rightarrow (x_1, x_2 - 1, x_3 + 1, x_4) \tag{7}$$

The action availability scheme, *i.e.*, what actions are feasible in each state, is presented in Table 4.

3.3.2. Decision rules and rewards

The main factor composing the decision rules and rewards is the system current demand L (*i.e.*, the power drawn by the FPSO). The supplied power never exceeds L and it is proportional to the current load carried by each generator such that $l_s \leq L/x_1$, and the rewards are proportional to the utility of the states. Let u_s denote the utility of a state s which is computed as:

$$u_s = \frac{L}{x_1 \cdot lt} + \alpha \cdot x_2, \forall s \in S \tag{8}$$

where lt denotes the target load for one component, x_1 is the number of components in operation, α denotes the standby utility parameter, and x_2 the number of units in standby.

The first term represents the capacity utilization rate and is given as the ratio between the total demand (L) and the total dispatched capacity ($x_1 \cdot lt$), since x_1 is the number of components in operation and lt is the target load for one component. Ideally, this ratio should approach 1, *i.e.*, 100%, and the model aims for dispatching the minimum capacity to satisfy the demand. Notice that the above equation is well defined because at least one turbo-generator is in operating mode, that is, $x_1 \geq 1$ in all reachable system states (see Fig. 3).

The second term represents the importance, given by the operator, to the chance of carrying out preventive actions, *i.e.*, the utility of having standby components. It is given as the multiplication of the standby utility parameter α with the number of standby components x_2 in state s.

It can be seen from Table 4 that, normally, there are more feasible actions in states with some standby component ($x_2 > 0$). Regarding the target load lt, since the turbo-generators share the load equally, it may be adjusted, for example, to the best efficient point (BEP). In the application the target load, lt was adjusted to the unit nominal capacity.

Let $E(s)$ denote the expected sojourn time in state s when action (1) "Wait" is chosen, which is computed by:

$$E(s) = \frac{1}{x_1\lambda + x_3\mu_1 + x_4\mu_2}, \forall s \in S \tag{9}$$

Considering equations (8) and (9), the action sets and the normal operating conditions/limits, the decision rules and rewards may be defined as presented in Table 5.

It is important to notice that the current component load varies with the state and demand. Since the power supplied never exceeds the demand L in any given state, and the parallel components share the load equally, we have $l_s \cdot x_1 \leq L$.

Table 5
Decision rules and rewards.

Action choice	Constraints	Rewards	Otherwise
$1 - Wait$	$lmin < l_s \leq lt$	$r(s,1) = u_s E(s)$	0
$2 - Activate$	$x_2 > 0$ and $l_s \geq lact$	$r(s,2) = u_s$	0
$3 - Deactivate$	$(lact - l_s) \geq 0$	$r(s,3) = u_s$	0
$4 - Do\ Preventive$	$x_2 \geq 1$ and $(x_3 + x_4) < 2$	$r(s,4) = u_s(Prev/m)$	0

Maintenance capacity is limited to 2 simultaneous repair jobs.

Table 6
Main results (scenario-dependent prevention ranges).

Scenario - MTBF/MTTR(prev./corr.)[h]	Prevention range [Lower-Upper]	Comp. time [sec]	No. iterations @55MW
S1 - 8760 / (24/72)	5 - 7	107.19	1493
S2 - 8760 / (72/216)	6 - 6	77.72	1058
S3 - 730 / (8/24)	2 - 3	37.72	328
S4 - 730 / (8/72)	1 - 3	34.05	382
S5 - 730 / (24/72)	2 - 5	27.07	297
S6 - 730 / (24/216)	1 - 4	76.91	851
S7 - 730 / (22.1/39.8)	3 - 5	19.02	179
S8 - 452 / (22.1/39.8)	3 - 6	14.74	166
S9 - 208 / (22.1/39.8)	1 - 6	13.28	136

As can be seen from Table 5, when action (1) "Wait" is chosen, the sojourn time is multiplied by the state utility. Actions (2) "Activate" and (3) "Deactivate" are rewarded by the current state utility only, and action (4) "Do Preventive" is rewarded considering the prevention factor, $Prev$, which is the level of prevention to be chosen by the operator as a precaution to failure.

Doing preventive (action 4) is the most important decision in our approach. However, not being a natural attitude, the adoption of preventive actions depends on identifying a suitable O&M policy. This is achieved by defining: (i) an appropriate scale for prevention, *i.e.*, ($Prev$); and (ii) a proper reward to the precautionary attitude, otherwise such an action would have no logical appeal.

In summary, the objective is to find the policy that maximizes the sequence of rewards, which is computed recursively by Algorithm 1, subject to: (i) the Markov chain topology; (ii) the decision rules/rewards and action sets; and (iii) the imposed prevention factor scale. Once the VI algorithm converges, the optimal policy is obtained. By changing the MDP *environment*, optimal preventive opportunities are allowed to emerge.

In each run of the application, the problem is solved for each demand level in the interval $[35, 36, 37, \ldots, 55]$ MW, forming a policy chart with 21 columns (optimal policies) and 16 rows (system states) for each $Prev$ value. Fig. 5 presents an example of a policy chart for $Prev = 5$. It is

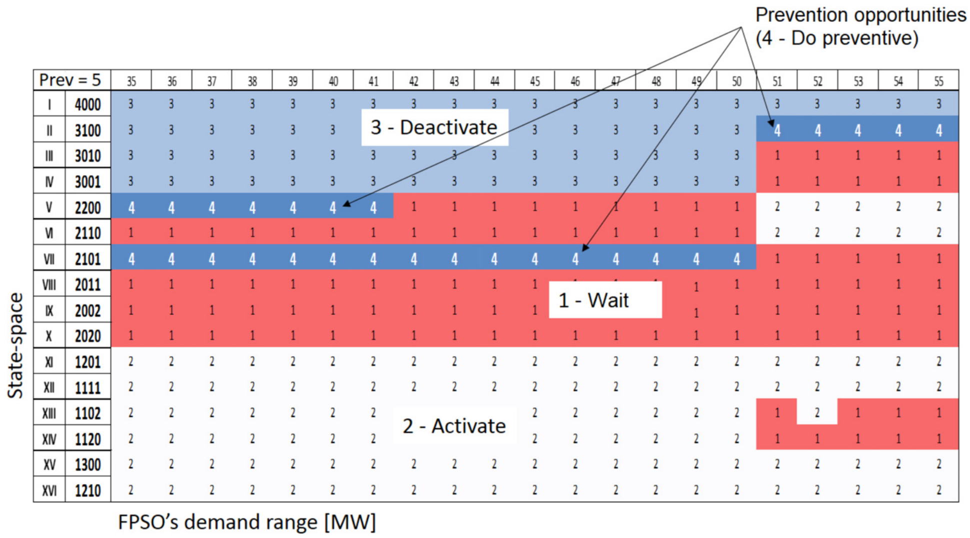

Fig. 5. Example of a policy chart for Scenario S8 and with $Prev = 5$. ($1 = Wait$ in red, $2 = Activate$ in white, $3 = Deactivate$ in light blue, and $4 = Do\ preventive$ in dark blue). When generating, for a given scenario, a set of policy charts in a prevention scale with $Prev \in [1 : 10]$, for example, it is possible to observe the emergence and growth of preventive opportunity windows (in dark blue filled with number 4) indicating the respective prevention ranges (see Table 7).

Table 7
Prevention range identification.

$[Prev]$	Scenario S7 [%]	Scenario S8 [%]	Scenario S9 [%]
10	25.2	25.2	32.0
9	25.2	25.2	32.0
8	25.2	25.2	32.0
7	25.2	25.2	32.0
6	25.2	**25.2**	**32.0**
5	**25.2**	21.8	21.1
4	3.4	14.3	21.1
3	**3.4**	**14.3**	21.1
2	0.0	0.0	17.7
1	0.0	0.0	**15.6**

Upper and lower bounds for prevention in "bold".

worth noting that the preventive action is only available in 7 of the 16 states, as presented in Table 4.

In Fig. 5, the opportunities for preventive maintenance appear as fringes in dark blue filled with number 4 in state (V-2200) from demand 35 to 41 MW, in state (VII-2101) from demand 35 to 50 MW, and in state (II-3100) from demand 51 to 55 MW. With regards to the capacity utilization rate defined in Eq. (8), the term $(\frac{L}{x_1 \cdot lt})$ varies between 0.4 and 2.2 with an average of 1.2 and reaches 1 between states V and X from 48 to 52 MW (see Fig. 5), *i.e.*, the target region of the solution-space, meaning that the power-generation system is supplying the demand. When the capacity utilization is greater than 1, it means that the FPSO's auxiliary power generator is activated to meet the power deficit. The term $(\alpha \cdot x_2)$ varies from 0 to 3.

Fig. 6 presents the gain and span evolution using Algorithm 1. As our model has only positive rewards, the calculated gain $v_n(s)$ is monotonic and bounded by the optimal policy gain. The span, which is the stopping criterion, is driven towards ϵ as the algorithm approaches convergence.

Because the main power generation system does not fulfill its function with only one turbo-generator in operation, only the first ten states are considered for decision support in the present case study. This is similar to the case of a four-engine aircraft that needs at least two engines running to keep the flight.

4. Case study

To demonstrate the use of the proposed MDP model, in generating preventive policies, experiments were carried out on a set of representative scenarios, so that, when observing the progressive appearance of preventive opportunities, in the policy charts, the corresponding prevention ranges are identified (see Table 6). From the point where no changes are observed in the policies, as the prevention factor, $Prev$ increases/decreases, we assume the respective upper and lower bounds for prevention. In summary, these results indicate when, and to what extent, the preventive opportunities (*i.e.*, associated cost and downtime) are worthwhile in terms of mitigating the risks of system degradation/failure.

The computational experiments were implemented in Matlab running in a 2.90 GHz CPU, with processor Intel Core i5-2310 and 4 GB of RAM in a 64-bit operating system. The experiments were performed with some default parameters such as: Activation load, $lact = 15$ MW; Target load per component, $lt = 25$ MW; Minimum load per component, $lmin = 12$ MW; Power range, $L \in [35 : 55]$; Standby utility, $\alpha = 0.9$; Starting failure probability, $\beta = 0.05$; Error tolerance, $\epsilon = 0.05$; and a maximum number of 3000 iterations.

4.1. Application results

The results, *i.e.*, a set of policy charts (similar to Fig. 5), for each prevention level for a given scenario is referenced in Tables 6 and 7 and a brief sensitivity evaluation is provided in Figs. 7 and 8.

Appendix A presents the transition probabilities and state utilities computed for the base-case (Scenario S8) at 45 MW, for verification purposes.

The MDP model has generated, for each of the nine scenarios, different ranges for prevention, indicating its sensitivity to the combinations of failure and repair rates. Notice that by maintaining the repair rates and increasing only the failure rate (*i.e.*, reducing the MTBF), for example, the recommended prevention ranges increase as presented in Table 7, where the upper and lower bounds appear in bold.

The percentage values in Table 7 refer to the proportional appearance (*i.e.*, area on the policy chart) of the preventive actions in relation to a total of 147 situations where it might be available (*i.e.*, 7 states in 21 demand levels). For Scenario S9 (the most severe one), for example, even for $Prev = 1$ (*i.e.*, no reward for prevention), the MDP model still "recommends" some level of prevention. For Scenario S8 and S9, with $Prev = 6$ and above, for example, no changes are observed in the respective optimal policies, indicating the upper bound for prevention.

Another important result observed in the series of experiments is that the preventive opportunities emerged progressively from low- and high-power demands, towards the center of the demand range. In low demands it is quite obvious since there is plenty of room for prevention but, in high demands, it appears that the recommended relief is aimed at

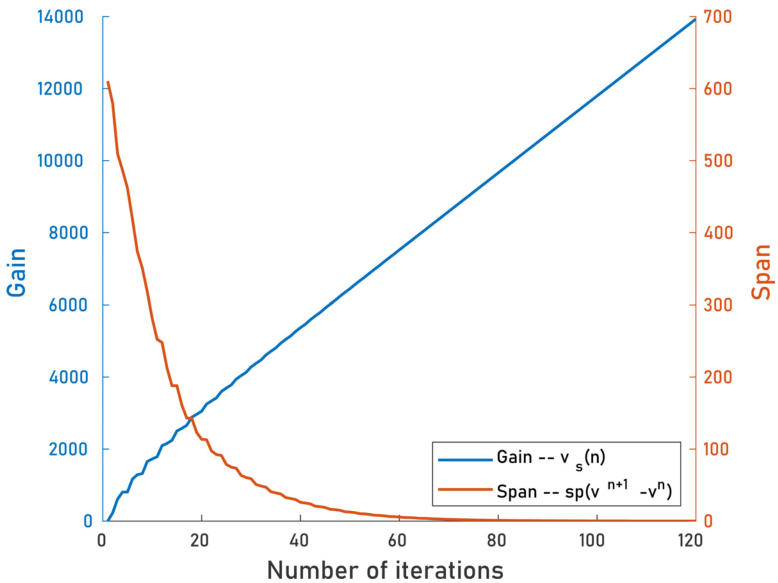

Fig. 6. Convergence in a single run @45MW of Algorithm 1.

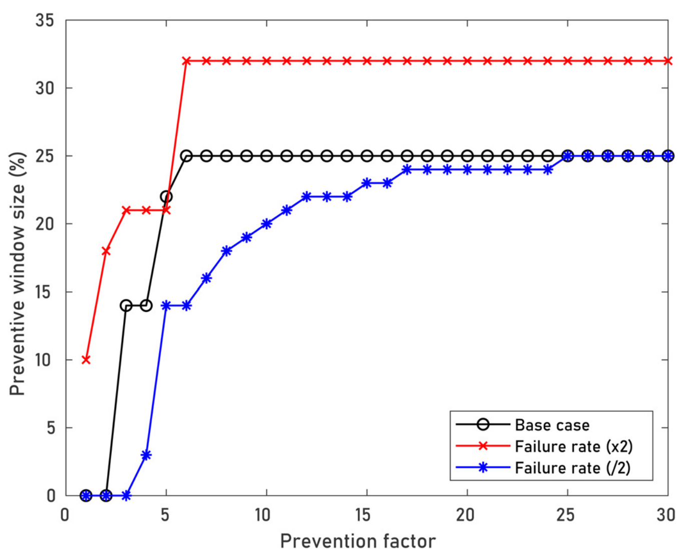

Fig. 7. Effect of varying the failure rate on the preventive windows size. $\mu_1 = 0.0453$; $\mu_2 = 0.0251$; $\alpha = 0.9$; $\beta = 0.05$; $\lambda = [0.001106; 0.004424]$.

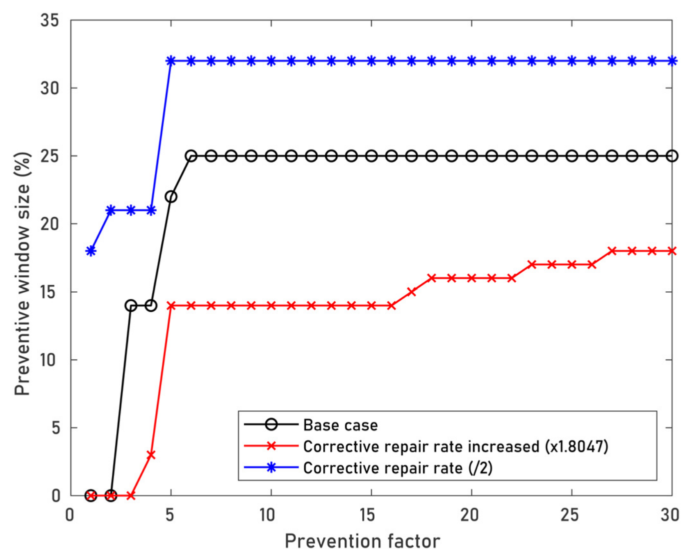

Fig. 8. Effect of varying the repair rate on the preventive windows. $\lambda = 0.002212$; $\mu_1 = 0.0453$; $\alpha = 0.9$; $\beta = 0.05$; $\mu_2 = [0.0125; 0.0453]$.

protecting the system against catastrophic failures. A sensitivity analysis is performed (see Figs. 7 and 8) to examine the effect on the optimal prevention levels by varying: i) the failure rate; and ii) the corrective repair rate.

4.2. Discussion

The proposed MDP model is employed as a decision support solution for maintenance backlog management related to an offshore power plant. Based on a case study and a set of operating scenarios, we demonstrated that the model can generate the optimal policies, prescribing upper and lower bounds for prevention. As expected, two thresholds appeared in the solution space, as shown in the policy chart of Fig. 5, clearly defining three regions. A *deactivation* region at the top (in light blue filled with number 3) and an *activation* one at the bottom (in white filled with number 2). Between these two regions (in red filled with number 1), is where the *preventive windows* normally emerged as fringes in dark blue filled with number 4.

Complementary experiments with increased starting failure probabilities, *i.e.,* $\beta > 0.05$ *resulted in less deactivations compared with the base-case, demonstrating consistency.*

In summary, the methodology indicated the proper levels of prevention by finding the answer for the main question of this study, *i.e., Under what operating conditions of a parallel production-system is a preventive action optimal in the long run?*

As it can be seen in Fig. 7, the effect of varying the failure rate has a significant impact on the size of the preventive windows. An increase in the failure rate causes an increase in the corresponding prevention levels. In Fig. 8 a similar behavior can be observed, but in the opposite direction, with regards to the corrective repair rates. In this case, the maximum corrective repair rate was limited to the preventive one (*i.e.,* $\mu_2 \leq \mu_1$) for coherence.

Uncertainty aspects on failure rates could be taken into account in the model by considering a semi-Markov formalism (Chen and Trivedi, 2005; Wu and Zhao, 2010) to allow probability distributions other than the exponential.

The experimental results indicated that optimal preventive opportunities may be non-intuitive. Notice, for example, that the preventive actions appeared only in the states (II-3100, V-2200 and VII-2101) and no preventive actions appeared in the state (VI-2110). It happened in all the experiments, which can be an effect of the chain topology or an emergent property of the system.

The model solved the problem in reduced computation time possibly due to its low dimensionality (*i.e.*, discrete and small state- and action-space).

Notice that the MDP model proposed in the paper aims to serve as a decision-support tool for the operator concerning the long-term performance of the power-plant, not as a detailed model for making automatic/autonomous decisions. This means that a KPI analysis cannot be carried out solely based on the recommended actions, but additionally would have to factor in the decisions taken by the operator and the exogenous inputs in the actual system. In order to investigate the impact on KPIs, we simulated the optimal decision policy considering: (i) a randomized demand, L and failure events; (ii) Scenario S8; and (iii) $Prev = 5$, in a period of 180 decision epochs, *i.e.*, time units. From these simulations, a set of KPIs resulted as follows:

- Total capacity utilization rate = 84.4%;
- Total demand fulfillment = 98.0%;
- Average unit load, $l_s = 21.1$ MW (equivalent to average fuel flow);
- Availability (probability that the demand is satisfied) = 91.1%;
- Backlog resolution (main objective) = 147 of 360 maintenance man time units; and
- Maintenance capacity utilization = 51.9%.

These suggested KPIs are to be considered for tuning the model in a future implementation.

The model can be easily communicated to stakeholders and possesses "*what if*" analysis capabilities, which may promote insights regarding a preventive and smooth operation of similar parallel machinery systems. In combination with condition-monitoring information, the approach can promote O&M integration since it allows maintenance personnel to choose which machine should be given priority for preventive maintenance at each decision epoch. From the headquarter's perspective, a set of policy charts can be published in a policy and procedures manual in order to coordinate and integrate operation with maintenance.

5. Final considerations

This work proposed a methodology based on the Markov decision process for a preventive operation and maintenance of parallel machinery systems subjected to a varying demand. The related stochastic process is modeled, and the relationship between operating scenario severity (*i.e.*, failure and repair rate combinations) and prevention levels is investigated. A prevention metric is proposed allowing the identification of the proper prevention range, according to the operating scenario. Among the expected benefits of the approach, we highlight: (i) an indirect optimization of the total operating costs; (ii) a regularization of the capacity utilization rate of similar systems (*e.g.*, fleet management); and (iii) a reduction of the total energy consumption/emissions of the gas-turbine engines. Finally, the proposed model can be used for planning or training purpose, with promising applications in many offshore production systems (*e.g.*, power generation, water injection, gas compression, *etc.*) since most of them are designed as 4-unit parallel systems with 33% redundancy.

As for future work we can mention: (i) include time-demand curves in the algorithm and provide policies for a predefined planning horizon (*e.g.*, for demand peaks, offloading, etc.); (ii) extend the model capabilities (*e.g.*, allowing intermediate degradation states and non-identical components); (iii) study a semi-Markov formalism to allow probability distributions other than the exponential; and (iv) test different value-functions.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A

The appendix presents the transition probabilities and state utilities computed for Scenario S8 with $L = 45$ MW and $Prev = 2$ for verification (Appendix A.1 and Appendix A.2).

Appendix A	Lambda	Mu_1	Mu_2	Beta	Load
	0.002212	0.0453	0.0251	0.05	45.00
Origin state $s = s_t$	**Decision**	**Event**	**Destination state $j = s_{t+1}$**	**Transition prob. p(j|s, a)**	**State Utility u(s)**
I-4000	1 - Do nothing	Failure	IV-3001	1.0000	0.00
	3 - Deactivate	Deactivation	II-3100	1.0000	0.45
II-3100	1 - Do nothing	Failure	VII-2101	1.0000	1.50
	2 - Activate	Start	I-4000	0.9500	1.50
		Starting failure	II-3100	0.0500	1.50
	3 - Deactivate	Deactivation	V-2200	1.0000	1.50
	4 - Do preventive	Prevention	III-3010	1.0000	3.00
III-3010	1 - Do nothing	Preventive repair completion	II-3100	0.8722	0.60
		Failure	VIII-2011	0.1278	0.60
	3 - Deactivate	Deactivation	VI-2110	1.0000	0.60
IV-3001	1 - Do nothing	Corrective repair completion	II-3100	0.7909	0.60
		Failure	IX-2002	0.2091	0.60
	3 - Deactivate	Deactivation	VII-2101	1.0000	0.60
V-2200	1 - Do nothing	Failure	XI-1201	1.0000	2.70
	2 - Activate	Start	II-3100	0.9500	2.70
		Starting failure	V-2200	0.0500	2.70
	3 - Deactivate	Deactivation	XV-1300	1.0000	0.00
	4 - Do preventive	Prevention	VI-2110	1.0000	5.40
VI-2110	1 - Do nothing	Preventive repair completion	V-2200	0.9110	1.80
		Failure	XII-1111	0.0890	1.80
	2 - Activate	Start	III-3010	0.9500	1.80
		Starting failure	VI-2110	0.0500	1.80
	3 - Deactivate	Deactivation	XVI-1210	1.0000	0.00
	4 - Do preventive	Prevention	X-2020	1.0000	3.60
VII-2101	1 - Do nothing	Corrective repair completion	V-2200	0.8502	1.80
		Failure	XIII-1102	0.1498	1.80
	2 - Activate	Start	IV-3001	0.9500	1.80
		Starting failure	VII-2101	0.0500	1.80
	3 - Deactivate	Deactivation	XI-1201	1.0000	0.00
	4 - Do preventive	Prevention	VIII-2011	1.0000	3.60
VIII-2011	1 - Do nothing	Corrective repair completion	VI-2110	0.3565	0.90
		Preventive repair completion	VII-2101	0.6435	0.90
	3 - Deactivate	Deactivation	XII-1111	1.0000	0.00

Appendix A.1. Transition probabilities and state utilities computed for Scenario 8; L=45 MW; and $Prev$=2.

Appendix A.2. Transition probabilities and state utilities computed for Scenario 8; L=45 MW; and *Prev*=2.

Origin state $s = s_t$	Decision	Event	Destination state $j = s_{t+1}$	Transition prob. $p(j\|s, a)$	State Utility $u(s)$
IX-2002	1 - Do nothing	LOOP	IX-2002	0.9498	0.90
		Corrective repair completion	VII-2101	0.0502	0.90
	3 - Deactivate	Deactivation	XIII-1102	1.0000	0.00
X-2020	1 - Do nothing	LOOP	X-2020	0.9094	0.90
		Preventive repair completion	VI-2110	0.0906	0.90
	3 - Deactivate	Deactivation	XIV-1120	1.0000	0.00
XI-1201	1 - Do nothing	LOOP	XI-1201	0.9749	0.00
		Corrective repair completion	XV-1300	0.0251	0.00
	2 - Activate	Start	VII-2101	0.9500	0.00
		Starting failure	XI-1201	0.0500	0.00
	4 - Do preventive	Prevention	XII-1111	1.0000	0.00
XII-1111	1 - Do nothing	Preventive repair completion	XI-1201	0.6435	0.00
		Corrective repair completion	XVI-1210	0.3565	0.00
	2 - Activate	Start	VIII-2011	0.9500	0.00
		Starting failure	XII-1111	0.0500	0.00
XIII-1102	1 - Do nothing	LOOP	XIII-1102	0.9498	0.00
		Corrective repair completion	XI-1201	0.0502	0.00
	2 - Activate	Start	IX-2002	0.9500	0.00
		Starting failure	XIII-1102	0.0500	0.00
XIV-1120	1 - Do nothing	LOOP	XIV-1120	0.9094	0.00
		Preventive repair completion	XVI-1210	0.0906	0.00
	2 - Activate	Start	X-2020	0.9500	0.00
		Starting failure	XIV-1120	0.0500	0.00
XV-1300	1 - Do nothing	LOOP	XV-1300	1.0000	0.00
	2 - Activate	Start	V-2200	0.9500	0.00
		Starting failure	XV-1300	0.0500	0.00
	4 - Do preventive	Prevention	XVI-1210	1.0000	0.00
XVI-1210	1 - Do nothing	LOOP	XVI-1210	0.9547	0.00
		Preventive repair completion	XV-1300	0.0453	0.00
	2 - Activate	Start	VI-2110	0.9500	0.00
		Starting failure	XVI-1210	0.0500	0.00
	4 - Do preventive	Prevention	XIV-1120	1.0000	0.00

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