

1 **Title:**

2 **Modeling VO<sub>2</sub> on-kinetics based on intensity-dependent Delayed Adjustment and Loss**  
3 **of Efficiency (DALE)**

4 Running title: Time to revise the 3-phase model for VO<sub>2</sub> on-kinetics?

5 Øyvind Gløersen<sup>3,4,5\*</sup>, Alessandro L Colosio<sup>1,2</sup>, Jan Boone<sup>2</sup>, Dag Kristian Dysthe<sup>4</sup>, Anders  
6 Malthé-Sørensen<sup>4</sup>, Carlo Capelli<sup>1</sup>, Silvia Pogliaghi<sup>1</sup>

7 <sup>1</sup>Department of Neurosciences, Biomedicine and Movement Sciences, University of Verona,  
8 Via Casorati 43, Verona, 37131, Italy.

9 <sup>2</sup>Department of Movement and Sports Sciences, Ghent University, Watersportlaan 2, Ghent,  
10 Belgium.

11 <sup>3</sup>Department of Physical Performance, Norwegian School of Sport Sciences, Sognsveien 220,  
12 Oslo, Norway.

13 <sup>4</sup>Department of Physics, University of Oslo, Oslo, Norway

14 <sup>5</sup>Smart Sensors and Microsystems, SINTEF Digital, Oslo, Norway

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17 Corresponding Author: Øyvind Gløersen, PhD. ORCID code: 0000-0002-5424-6183

18 Institution: SINTEF Digital, Forskningsveien 1, Oslo, Norway

19 Telephone number: +47 93060009;

20 Email addresses: oyvind.gloersen@sintef.no

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22

23 **Abstract**

24 This study presents and evaluates a new mathematical model of  $\dot{V}O_2$  on-kinetics, with the  
25 following properties: (i) a progressively slower primary phase following the size-principle of  
26 motor unit recruitment, explaining the delayed  $\dot{V}O_2$  steady state seen in the heavy exercise  
27 intensity domain, and (ii) a severe-domain slow component modelled as a time-dependent  
28 decrease in efficiency. Breath-by-breath  $\dot{V}O_2$  measurements from eight subjects performing  
29 step cycling transitions, in the moderate, heavy and severe exercise domains, were fitted to  
30 the conventional 3-phase model and the new model. Model performance was evaluated with a  
31 residual analysis and by comparing Bayesian (BIC) and corrected Akaike (AICc) information  
32 criteria. The residual analysis showed no systematic deviations, except perhaps for the initial  
33 part of the primary phase. BIC favored the new model, being 9.3 (SD 7.1) lower than the  
34 conventional model while AICc was similar between models. Compared to the conventional  
35 3-phase model, the proposed model distinguishes between the kinetic adaptations in the  
36 heavy and severe domains by predicting a delayed steady state  $\dot{V}O_2$  in the heavy and no  
37 steady state  $\dot{V}O_2$  in the severe domain. This allows to determine when stable oxygen costs of  
38 exercise are attainable and it also represents a first step in defining time-dependent oxygen  
39 costs when stable energy conversion efficiency is not attainable.

40

41 *Keywords:*

42 Oxidative metabolism,  $VO_2$  kinetics, Primary component, Slow component, Modelling, Loss  
43 of efficiency, Work economy

## 44 **New & Noteworthy**

45 We propose and assess a new minimalistic integrated model for the  $\dot{V}O_2$  on-kinetics, inspired  
46 by the currently available best evidence of the underlying mechanisms. We show that the  
47 model provides a similar fit as the conventionally used 3-phase model, even though a stricter  
48 data fitting method is used for the proposed model. The proposed model clarifies  
49 misconceptions related to the  $\dot{V}O_2$  slow component's behaviour, by clearly predicting that  
50 steady-state  $\dot{V}O_2$  is attainable in the moderate and heavy exercise intensity domains.  
51 Furthermore, the model opens new possibilities for assessing oxygen cost during severe  
52 intensity exercise without the fallible assumption of time-constant energy-conversion  
53 efficiency.

## 54 **Introduction**

55 The current model for the oxygen uptake ( $\dot{V}O_2$ ) kinetics to a step increase in energy demand,  
56 termed the 3-phase model, predicts that  $\dot{V}O_2$  measured at the mouth adjusts to increased  
57 external work rate in three phases. The first phase, termed the cardio-pulmonary phase, is  
58 attributed to increased blood flow through the lungs caused by increased heart rate and  
59 increased venous return due to the muscle pump (1-3). The second phase, termed the primary  
60 phase, is attributed to increased  $\dot{V}O_2$  in the working muscles. Because of the blood's transit  
61 time through the venous system, the onset of this phase is delayed by 15-30 s when measured  
62 at the level of the mouth, i.e. the time delay (1, 2). During the primary phase,  $\dot{V}O_2$  follows a  
63 first order exponential pattern. The third phase is dependent on exercise intensity: for  
64 moderate intensity exercise, it is the attainment of a steady state  $\dot{V}O_2$ ; for heavy intensity  
65 exercise, the third phase includes an additional, delayed increase in  $\dot{V}O_2$  before steady state is  
66 attained; for severe intensity exercise,  $\dot{V}O_2$  will continue to increase until it reaches  $\dot{V}O_{2\max}$   
67 and/or the subject reaches exhaustion (4). Despite the difference in behavior between the  
68 heavy and severe exercise intensity domains, the third phase is commonly modelled with a  
69 single delayed exponential function (Although there is a lack of consensus on how this phase  
70 should be quantified (5)). Compared to the primary phase, the onset delay of the third phase  
71 is minutes rather than seconds (1–3 min), and the time constant is substantially larger (6).  
72 Accordingly, this phase is termed the  $\dot{V}O_2$  slow component (5).

73 In the past 30 years, the 3-phase modeling of  $\dot{V}O_2$  kinetics during a step response has offered  
74 an invaluable non-invasive means to evaluate the dynamic behavior of  $\dot{V}O_2$ , which again has  
75 facilitated characterization of the limiting factors of  $O_2$  delivery and utilization (3, 7).

76 However, the connection between the 3-phase model's mathematical form and the underlying  
77 physiological mechanisms is in fact not obvious (8). For the primary component – considered  
78 to reflect  $\dot{V}O_2$  within the working muscles – the first order exponential model is underpinned  
79 by observations at the whole muscle level (8). However, there is evidence that the single first  
80 order exponential behavior observed at the whole muscle level arises from a sum of non-  
81 uniform responses in the muscle cells it is comprised of (8-11). Specifically, fast-twitch  
82 glycolytic fibers exhibit slower  $\dot{V}O_2$  kinetics and lower energy conversion efficiency than  
83 slow-twitch fibers (9). It follows from Henneman's recruitment principle (12) that one should  
84 expect to see a slowing (and possibly an increased amplitude) of the whole-muscle  $\dot{V}O_2$   
85 kinetics as recruitment of fast-twitch fibers becomes significant, which occurs at an exercise  
86 intensity of 50–60% of  $\dot{V}O_{2\max}$  (13). Indeed, these theoretical predictions align well with data  
87 from studies comparing  $\dot{V}O_2$  kinetics in the moderate and heavy exercise intensity domains  
88 (14). Within the 3-phase model, the above-described behavior is typically accounted for by  
89 the  $\dot{V}O_2$  slow component. However, if slowing of the time constant and the increase in overall  
90 gain from the moderate to the heavy domain are in fact the results of an increased  
91 contribution of glycolytic fibers with increasing work rate, it would be more appropriate to  
92 call this phenomenon a “delayed steady state” and to ascribe it to the primary phase of  $\dot{V}O_2$   
93 kinetics. The reason for this is that it represents the same physiological mechanism, i.e. the  
94 working muscles' inertia to attain a steady rate of oxidative phosphorylation (15).

95 In contrast, the failure to reach a steady-state  $\dot{V}O_2$  observed in the severe exercise intensity  
96 domain cannot be explained by the mechanism outlined above only. The mechanisms  
97 underlying this inability to attain steady-state  $\dot{V}O_2$  are still debated, however, it appears that  
98 the majority (about 85%) of the increase in  $\dot{V}O_2$  observed over time arises from the working  
99 muscles (16, 17). This behavior only appears when the subject exceeds a critical work rate  
100 (4), and is accompanied by the inability to maintain a stable intra-cellular biochemical  
101 environment in the working muscles (16). These changes in the biochemical environment  
102 lead to a gradual reduction in the energy-conversion efficiency of the myocytes, resulting in  
103 the inability to attain steady-state  $\dot{V}O_2$  even if the work rate is constant. As such, this loss of  
104 efficiency developing over time may be distinct and independent from the “delayed steady  
105 state” described earlier (15). It would therefore be preferable if the term “ $\dot{V}O_2$  slow  
106 component” was reserved for the behavior outlined in this paragraph, which is normally only  
107 observed in the severe exercise intensity domain.

108 The framework described above is corroborated by recent findings from our group (15, 18)  
109 and others (19), suggesting that, in the heavy domain, a contribution of fast twitch fibers may  
110 suffice to explain the observed slowing of the  $\dot{V}O_2$  kinetics and, possibly, a concomitant  
111 increase in gain, without the need for increased recruitment over time as an explanatory  
112 mechanism. Moreover, in the heavy domain, the so-called  $\dot{V}O_2$  slow component may not be  
113 the result of a loss of efficiency manifesting over time; rather, it may be the result of a slower  
114 up-regulation of the activity of rate-limiting enzymes and provision of adequate substrates. In  
115 the severe domain of exercise, however, the bioenergetic approach proposed by Colosio et al.  
116 (15, 18) verified the existence of a true slow component of  $\dot{V}O_2$  (i.e. a loss of efficiency over  
117 time), which develops in unison with increased muscle activation. Accordingly,  
118 characterization of the  $\dot{V}O_2$  response in this domain should include a delayed onset term,  
119 similar to the third phase of the traditional 3-phase modeling of  $\dot{V}O_2$  kinetics.

120 In summary, the traditional successive delayed exponential modeling of the  $\dot{V}O_2$ -on kinetics  
121 is being questioned (15, 20-22) on the grounds of recent evidence that supports a domain-  
122 dependent modeling of the primary component, and modeling of the slow component as a  
123 gradually developing change in energy conversion efficiency. A critical re-visitation of the  
124 current fitting strategies may contribute to further our understanding of the possibly distinct  
125 physiological determinants of the adjustment of oxidative metabolism at exercise onset in the  
126 different domains of exercise. Accordingly, the aim of this study is to propose and to test the  
127 performance of a new minimal model for the on-kinetics of the primary and slow components  
128 of  $\dot{V}O_2$  kinetics. Specifically, the model should satisfy two key points. First, the progressively  
129 slower  $\dot{V}O_2$  kinetics at the mouth should be explained by recruitment of muscle fiber  
130 populations with different first order kinetics, whilst the kinetics of each fiber population  
131 remain unchanged across intensity domains. Second, the slow component should represent an  
132 inability to maintain stable energy conversion efficiency in the severe exercise intensity  
133 domain. We will test the model, termed Delayed adjustment and Loss of Efficiency (DALE),  
134 on step responses into the moderate, heavy and severe exercise intensity domains, and  
135 compare the model's performance to the conventional 3-phase model.

## 136 **Methods**

### 137 Mathematical model

138 *Fiber-type dependency of  $\dot{V}O_2$  kinetics in the working muscles*

139 The DALE model assumes two fiber populations, corresponding to slow twitch (st) and fast  
 140 twitch (ft) fibers. Both fiber populations follow first order kinetics, but with different time  
 141 constants ( $\tau_{st}$  and  $\tau_{ft}$ ). Furthermore, the model assumes that the two fiber populations are  
 142 activated in a strictly sequential manner, with no fast twitch activation prior to full activation  
 143 of the slow twitch population. Hence, the exercise intensity corresponding to full activation  
 144 of slow twitch fibers marks the point where  $\dot{V}O_2$  kinetics in the working muscles will start to  
 145 become progressively slower. In the current study, this threshold (known to occur at  
 146 intensities 50–60% of  $\dot{V}O_{2\max}$  (13)). is termed  $\dot{V}O_{2,st,max}$ . Following Henneman's size  
 147 principle for motor unit recruitment (12), no further activation of slow twitch fibers can occur  
 148 at exercise intensities above  $\dot{V}O_{2,st,max}$ , and any additional increase in intensity will lead to  
 149 activation of the ft-fiber population. Letting  $A_{st}$  and  $A_{ft}$  represent the attractor for  $\dot{V}O_2$  (after  
 150 subtracting baseline  $\dot{V}O_2$ ) in each fiber population, this is described by the two differential  
 151 equations

$$\tau_{st} \cdot \dot{V}O_{2,st} = A_{st} - \dot{V}O_{2,st}, \quad (1)$$

$$\tau_{ft} \cdot \dot{V}O_{2,ft} = A_{ft} - \dot{V}O_{2,ft}, \quad (2)$$

154

152 where  $\dot{V}O_2$  is the time derivative of  $\dot{V}O_2$ ,  $A_{st}$  is bounded upwards to  $\dot{V}O_{2,st,max}$ , and  $A_{ft}$  is zero  
 153 if  $\dot{V}O_2 \leq \dot{V}O_{2,st,max}$ .

155 *Time dependent energy conversion efficiency during constant work rate exercise*

156 To accommodate the gradual loss of energy conversion efficiency (16) observed during  
 157 constant work rate exercise in the severe (4) and (possibly) heavy exercise intensity domains,  
 158  $A_{ft}$  is allowed to be time dependent in these domains. Specifically, the model allows for a  
 159 time delayed linear increase in  $A_{ft}$ :

$$A_{ft}(t) = A_{0,ft} + \min\{0, \dot{A} \cdot (t - t_{d,sc})\}, \quad (3)$$

160

161 where  $t_{d,sc}$  is the onset of the linear increase, and  $\dot{A}$  is the rate of change ( $\text{mL} \cdot \text{min}^{-2}$ ) in  $A_{ft}(t)$   
 162 for  $t \geq t_{d,sc}$ .

163 *Analytical solution during a step-increase in work rate*

164 In the case of a step increase in work rate, Eqs. (1) and (2) are linear inhomogeneous  
 165 differential equations with closed form analytical solutions. Due to the delayed onset of the  
 166 linear increase term in Eq (3), Eq. (2) must be solved using  $A_{ft}(t) = A_{0,ft}$  for  $t < t_{d,sc}$  and  $A_{ft}(t) =$

167  $A_{0,ft} + \dot{A}(t-t_{d,sc})$  for  $t \geq t_{d,sc}$ , and the two solutions are stitched together by requiring continuity  
 168 at  $t = t_{d,sc}$ . Furthermore, we include a delayed onset ( $t_{d,p}$ ) of the primary phase due to the  
 169 venous transport time, by setting the initial conditions to be  $\dot{V}O_{2,st}(t = t_{d,p}) = \dot{V}O_{2,ft}(t = t_{d,p}) =$   
 170 0. In this case, the differential equations have solutions:

$$\dot{V}O_{2,st}(t) = A_{st} \left( 1 - e^{-\frac{t-t_{d,p}}{\tau_{st}}} \right), \quad (4)$$

$$\dot{V}O_{2,ft}(t) = A_{0,ft} \left( 1 - e^{-\frac{t-t_{d,p}}{\tau_{ft}}} \right) + \begin{cases} 0, & \text{if } t < t_{d,sc} \\ \dot{A} \left( t - t_{d,sc} - \tau_{ft} \left( e^{-\frac{t-t_{d,sc}}{\tau_{ft}}} - 1 \right) \right), & \text{if } t \geq t_{d,sc}. \end{cases} \quad (5)$$

171 Finally,  $\dot{V}O_2$  measured at the mouth is given by

$$\dot{V}O_2(t) = \dot{V}O_{2,st}(t) + \dot{V}O_{2,ft}(t) + \dot{V}O_{2,baseline}, \quad (6)$$

172 where  $\dot{V}O_{2,baseline}$  is the oxygen uptake directly prior to the step increase.

### 173 Experimental data

174 The model was tested on a dataset from a recently published study (15), which comprised  
 175 three step-increases in power output during ergometer cycling. The dataset included eight  
 176 active males (age 25 years (SD 2), body mass 74 kg (SD 10), height 181 cm (SD 5),  $\dot{V}O_{2,peak}$   
 177  $49 \text{ ml} \cdot \text{min}^{-1} \cdot \text{kg}^{-1}$  (SD 3)), and the data collection was approved by the University of Verona  
 178 Ethics Committee for Research on Human Subjects. The detailed description of the  
 179 experimental protocol can be found in the original paper (15). In brief, work rate during the  
 180 three different step-increases were set individually for each participant to ensure that they  
 181 corresponded to moderate, heavy and severe exercise intensities (4). The moderate-, heavy-  
 182 and severe-domain boundaries were established from a ramp incremental exercise test to  
 183 exhaustion, with the metabolic intensity at the gas exchange threshold (GET) defining the  
 184 boundary between moderate and heavy, and the respiratory compensation point (RCP)  
 185 defining the boundary between heavy and severe (23).

186 The constant work rate steps were defined to be 80% of GET, midpoint between GET and  
 187 RCP, and 60%  $\Delta$  between GET and  $\dot{V}O_{2,peak}$  for moderate, heavy and severe, respectively.  
 188 These metabolic intensities were translated into constant external loads by left-shifting the  
 189  $\dot{V}O_2/W$  relationship from the incremental test, to account for the mean response time. In turn,  
 190 the mean response time was determined based on  $\dot{V}O_2$  data as a function of time from the  
 191 ramp incremental exercise, as the intersection point of the line fitting the baseline  $\dot{V}O_2$  data  
 192 and the line fitting the incremental portion of the  $\dot{V}O_2$  up to GET (24). The step increases in

193 work rate followed three minutes of 20 W cycling, and the step was repeated three times with  
194 durations 3, 6 and 9 minutes (i.e. a total of 9 step responses for each participant, 3 to each  
195 domain). Pulmonary gas exchange and ventilation were measured breath-by-breath  
196 throughout each constant load trial using a metabolic cart (Jaeger Oxycon Pro, Viasys  
197 Healthcare GmbH, Höchberg, Germany). Gas exchange measurements that were aberrant  
198 were removed using a 3 SD cutoff from the local mean, before the measurements were  
199 linearly resampled to 1 s intervals and then decimated to 5 s averages.  $\dot{V}O_2$  during the  
200 constant load trials was averaged over the three trials (3, 6 and 9 minutes) in each intensity  
201 domain. GET and RCP were determined with ventilator technique based on inspection of the  
202 fractional concentration of end-tidal  $O_2$  and  $CO_2$  and the ventilatory equivalents for  $O_2$  and  
203  $CO_2$  during the ramp trial (25), and was assessed by three blinded expert reviewers.

#### 204 Data fitting

205 In line with the assumption that  $\tau$  is constant for each fiber population, measurements of  $\dot{V}O_2$   
206 during the constant load trials were fitted to Eqs (4) and (5) using least squares optimization  
207 to all trials (moderate, heavy, severe) simultaneously. Specifically, the optimization variables  
208  $\tau_{st}$  and  $\tau_{ft}$  were constants across all three exercise intensity domains, while the remaining  
209 variables ( $t_{d,p}$ ,  $t_{d,sc}$ ,  $\dot{A}$  and  $A$ ) were domain-specific (i.e. these parameters could take different  
210 values in different domains). This resulted in 12 optimization variables for the DALE model,  
211 since  $t_{d,sc}$  and  $\dot{A}$  are not included in the moderate domain.  $\dot{V}O_{2,st,max}$  was set a priori to  $\dot{V}O_2$  at  
212 GET. The data fitting was accomplished using the Trust Region algorithm implemented in  
213 Matlab's Curve fitting toolbox, using initial values and bounds as specified in Table 1. The  
214 conventional 3-phase model was fitted to the data using the same optimization algorithm  
215 (Trust Region algorithm, initial values and bounds specified in Table 1). In line with previous  
216 studies in the field (7, 26, 27), all optimization variables were free to vary between intensity  
217 domains. This resulted in 15 optimization variables for the conventional model (three in  
218 moderate, six in heavy and severe).

219 \*\*\* Table 1

220 Table 1: Initial values (IV) and bounds (LB: lower bound, UB: upper bound) for the model  
221 parameters used in the data fitting procedure.  $A_{3-5}$  represents average  $\dot{V}O_2$  measured from 3-5  
222 minutes during the step that the model was fitted to. The Trust-region algorithm was used,  
223 with a tolerance on the function value of  $10^{-9}$ .



	<b>Parameter</b>	<b>Unit</b>	<b>IV</b>	<b>LB</b>	<b>UB</b>
DALE model	$t_{d,p}$	s	10	0	20
	$\tau_{st}$	s	20	10	40
	$\tau_{ft}$	s	45	20	90
	$t_{d,sc}$	s	90	0	180
	$\dot{A}$	$\text{mL}\cdot\text{min}^{-2}$	0	-60	150
	$A$	$\text{L}\cdot\text{min}^{-1}$	$A_{3-5}$	$0.8\cdot A_{3-5}$	$1.2\cdot A_{3-5}$
Conventional model	$t_{d,p}$	s	10	0	20
	$\tau_p$	s	25	5	90
	$A_p$	$\text{L}\cdot\text{min}^{-1}$	$A_{3-5}$	$0.5\cdot A_{3-5}$	$1.5\cdot A_{3-5}$
	$t_{d,sc}$	s	90	0	180
	$\tau_{sc}$	s	100	50	1000
	$A_{sc}$	$\text{L}\cdot\text{min}^{-1}$	0	0	$0.5\cdot A_{3-5}$

224

225 \*\*\* End table 1

226

## 227 Statistics

228 Plots of model residuals versus time were prepared to check if there were periods where the  
 229 model did not provide a good fit. The residuals at each time point were averaged over all  
 230 participants and displayed with 95 % confidence intervals (CI). The hypothesis that the slow  
 231 component exists only in the severe intensity domains was assessed by checking if the model  
 232 parameter  $\dot{A}$  was non-zero in the severe domain, but not different from zero in the heavy  
 233 domain, using two one-sample  $t$ -tests. Significance level was  $\alpha = 0.05$ .

234 Finally, the DALE model's overall performance compared to the 3-phase model was  
 235 evaluated using the corrected Akaike information criterion (AICc, that allows to select the  
 236 model that minimizes information loss) and the Bayesian information criterion (BIC, that  
 237 allows to select the model that generated the observed data) (28). Differences in AICc and  
 238 BIC were reported as mean (SD) across all participants. Furthermore, differences in AICc  
 239 were interpreted using Akaike weight (29), which represents the probability that either model  
 240 minimizes information loss, and differences in BIC were interpreted using the  
 241 recommendations of Kass and Raftery (30) (<2: Not worth more than a bare mention, 2-6:  
 242 Positive, 6-10: Strong, >10: Very strong).

243 **Results**

244 The absolute intensity at which the constant-load trials were conducted was 129 W (SD 27),  
 245 209 W (SD 29) and 267 W (SD 38) for the moderate, heavy and severe domain respectively.  
 246 An example of the individual fitting results in all three exercise intensity domains is shown in  
 247 Figure 1. This participant had a  $\tau_{ft}$  that was 38 s longer than  $\tau_{st}$ ; a distinct linear increase in  
 248  $\dot{V}O_2$  ( $\dot{A} = 126 \text{ mL}\cdot\text{min}^{-2}$ ) in the severe trial, and a relatively small linear increase in  $\dot{V}O_2$  ( $\dot{A} =$   
 249  $24 \text{ mL}\cdot\text{min}^{-2}$ ) in the heavy trial. The model coefficients for all participants, for both the  
 250 DALE and conventional models, are shown in Table 2. As seen from the table,  $\tau_{ft}$  was 19 s  
 251 (SD 26) longer than  $\tau_{st}$ . The parameter  $\dot{A}$  was  $16 \text{ mL}\cdot\text{min}^{-2}$  (SD 29) during the heavy trial,  
 252 which was not different from zero ( $P = 0.17$ ). However, two of the participants did show a  
 253 linearly increasing  $\dot{V}O_2$  in the heavy domain. In contrast,  $\dot{A}$  was  $88 \text{ mL}\cdot\text{min}^{-2}$  (SD 42) during  
 254 the severe trial ( $P < 0.001$ ), and all participants showed a distinct increase in  $\dot{V}O_2$  following  
 255 the primary phase.

256

257 \*\*\* Table 2

258 Table 2: Model coefficients for the DALE-model and the conventional 3-phase model. All  
 259 values are presented as mean (SD). Entries marked *clamped* indicates that the value was fixed  
 260 to the value in the column to the left of the entry, according to the sequential fitting strategy.  
 261 NA: parameter not applicable to this domain. Parameter definitions in footnote.

	Unit	Moderate	Heavy	Severe	
DALE model	$t_{d,p}$	[s]	11.5 (3.0)	13.8 (1.7)	12.4 (1.9)
	$A_0$	[L·min <sup>-1</sup> ]	1.30 (0.27)	2.08 (0.41)	2.55 (0.34)
	$\tau_{st}$	[s]		28 (8)	
	$\tau_{ft}$	[s]		47 (24)	
	$\dot{A}$	[mL·min <sup>-2</sup> ]	NA	16 (29)	88 (42)
	$t_{d,sc}$	[s]	NA	72 (72)	118 (43)
Conventional	$t_{d,p}$	[s]	12.7 (3.0)	13.0 (2.2)	13.2 (1.0)
	$\tau_p$	[s]	26 (9)	30 (7)	29 (5)
	$A_p$	[L·min <sup>-1</sup> ]	1.30 (0.27)	2.04 (0.42)	2.47 (0.35)
	$t_{d,sc}$	[s]	NA	144 (32)	137 (38)
	$\tau_{sc}$	[s]	NA	455 (449)	306 (260)

$A_{sc}$	[L·min <sup>-1</sup> ]	NA	0.38 (0.37)	0.74 (0.35)
$\dot{V}O_{2,baseline}$	[L·min <sup>-1</sup> ]	0.68 (0.04)	0.71 (0.07)	0.70 (0.05)

262 **Parameters common for both models:**  $t_{d,p}$  = time delay of primary phase,  $t_{d,sc}$  = time delay of slow  
263 component. **Parameters in DALE model:**  $A_0$  = initial  $\dot{V}O_2$  amplitude (at  $t = 0$ ),  $\tau_{st}$  = time constant of  
264 slow twitch fiber population,  $\tau_{ft}$  = time constant of fast twitch fiber population,  $\dot{A}$  = change in  $\dot{V}O_2$   
265 amplitude with time, **Parameters in conventional model:**  $\tau_p$  = time constant of primary phase,  $A_p$  =  
266 amplitude of primary phase,  $\tau_{sc}$  = time constant of slow component phase,  $A_{sc}$  = amplitude of slow  
267 component phase

268 \*\*\* End table 2

269

### 270 *Residual analysis and comparison to the 3-phase model*

271 The model residuals averaged over participants are shown in Figure 2. By visual inspection,  
272 the residuals appear randomly distributed with constant variance, except perhaps for the  
273 initial ~90 s. In this region, corresponding to the primary phase, both the DALE model and  
274 the conventional 3-phase model showed signs of small systematic deviations. For the  
275 between-model comparisons: AICc was similar for both models, being 1.8 (SD 7.1) lower for  
276 the conventional model. The Akaike weights were 0.36 for DALE and 0.64 for the  
277 conventional model, meaning that there was no clear evidence in favor of either model in  
278 minimizing information loss. BIC favored the DALE model, being 9.3 (SD 7.1) lower than  
279 the 3-phase model, which is considered “strong” evidence in favor of DALE being the model  
280 that generated the experimental data (30).

### 281 **Discussion**

282 The aim of this study was to propose and assess a model for the on-kinetics of the primary  
283 and slow components of  $\dot{V}O_2$ , with the following two requirements: (i) a progressively slower  
284 primary phase (or delayed  $\dot{V}O_2$  adjustment) attributed to the size-principle of motor unit  
285 recruitment, and (ii) a “true” slow component attributed to a gradually decreasing energy  
286 conversion efficiency with time. For the first point, we found that  $\tau_{st}$  and  $\tau_{ft}$  were 28 and 47 s,  
287 respectively, which adheres well with the difference in  $\tau$  between fast and slow twitch fibers  
288 observed during in-vivo experiments on humans (9). For the latter point, the proposed  
289 (DALE) model showed no systematic deviations from experimental data and provided a  
290 similar fit as the conventional delayed exponential formulation.

291 The DALE model is an important alternative to the conventional 3-phase model because it  
292 establishes a clear distinction between the delayed steady-state  $\dot{V}O_2$  typically observed in the

293 heavy domain, which we argue should be ascribed to the primary component (15), and the  
294 inability to achieve a steady state  $\dot{V}O_2$  typically seen in the severe domain (16). Although the  
295 mechanisms underlying the proposed model have been previously reported in the literature  
296 and other studies have presented models that accommodate them in part (10, 14, 31), this is  
297 the first comprehensive model tested on experimental data in the moderate, heavy and severe  
298 exercise intensity domains. The results of the experimental evaluation, which was conducted  
299 using a fitting strategy that is stricter than the conventionally used fitting strategy where all  
300 variables are free to vary, largely support the proposed model. Importantly, systematic  
301 deviations between model and experimental data were observed only during the primary  
302 phase and were also present when using the conventional 3-phase model. These deviations  
303 are most likely modulations of pulmonary  $\dot{V}O_2$  by circulatory and ventilatory dynamics (32),  
304 which suggests that model refinements of the early portion of the  $\dot{V}O_2$  adjustment might be  
305 appropriate.

#### 306 *Dependency of primary phase kinetics on muscle fiber type*

307 The values for  $\tau_{st}$  and  $\tau_{ft}$  (28 and 47 s, respectively) found in this study adhere well with  
308 studies on the muscle level (33 and 55 s, Krstrup et al. (9)). They are also close to the results  
309 from studies investigating the primary time constant during steps from elevated baselines.  
310 Specifically, Wilkerson and Jones (11) reported  $\tau_p = 26$  s and 46 s following a step increase in  
311 workload from unloaded to moderate and heavy intensities, respectively. In consonance with  
312 the model proposed in the current study, they interpreted this blunting of the primary phase  
313 kinetics as resulting from a shift from recruitment of primarily slow twitch fibers to primarily  
314 fast twitch fibers. Further, Pringle et al. (33) investigated the relationship between muscle  
315 fiber type distribution and  $\dot{V}O_2$  pulmonary kinetics in a diverse group of subjects. When  
316 comparing groups with different fiber composition, they found differences in  $\tau_p$  during steps  
317 to the heavy and severe domains, where the group with primarily slow twitch fibers showed  
318 faster kinetics (shorter  $\tau_p$ ) than the group with primarily fast twitch fibers. They also found a  
319 tendency for the primary phase time constant to lengthen more in subjects with a low fraction  
320 of slow twitch fibers. Interestingly, there was no relationship between fiber type distribution  
321 and  $\tau_p$  in steps to the moderate domain. All these findings agree with the DALE model, with  
322 the latter finding depending on the model assumption of no fast twitch fiber activation in the  
323 moderate intensity domain. This assumption, however, is challenged by the findings of  
324 Brittain et al. (34), who tested different steps within the moderate intensity domain (from 20  
325 W to 50% of the lactate threshold and from 50 to 90% of the lactate threshold). They reported

326 an increase in the primary time constant in the higher compared to lower ranges of the  
327 moderate domain. In contrast, Spencer et al. (35), did not find differences in the primary time  
328 constant within the moderate domain during steps of different amplitude but from the same  
329 baseline. However, if the findings of Brittain et al. (34) are representative, this could imply  
330 that the DALE model's assumption of no fast twitch activation within the moderate domain  
331 (and consequently, unchanged  $\tau_p$ ) is too simplistic. Alternatively, it can imply that fibers  
332 activated by motor units higher in the recruitment hierarchy have systematically slower  
333 kinetics, even if they are slow twitch fibers. Assessing fiber type activation directly is  
334 experimentally challenging, however, studies on glycogen depletion in different fiber types  
335 indicate a predominant, but not exclusive, activation of slow twitch fibers within the  
336 moderate domain (36). Taken together, these findings indicate that there is a shift in  
337 activation from slow twitch to fast twitch fibers at the boundary between moderate and heavy  
338 exercise intensity, but that the shift is probably not wholly abrupt. The DALE model can be  
339 modified to accommodate a gradual shift in fiber type recruitment, but this comes at the cost  
340 of increased model complexity (at least one additional model parameter). The importance of  
341 including a gradual shift in the model depends on how abrupt the shift in recruitment is: if the  
342 shift occurs over a small fraction of aerobically sustained workloads (say, 90-110% of GET),  
343 then modifying the model will have trivial consequences. However, the modification will be  
344 significant if the shift in recruitment progresses over a relatively large fraction of aerobically  
345 sustained workloads. Further studies on the relationship between muscle fiber type activation  
346 and the threshold separating moderate from heavy exercise would be useful to understand if,  
347 and how, the model should be modified. If the shift is close to abrupt, we maintain that the  
348 DALE model is appropriate as an idealized representation of the underlying mechanisms.

349 The assumption of homogenous kinetics within each muscle fiber population (slow- or fast  
350 twitch) should also be critically evaluated. Within-population heterogeneity undoubtedly  
351 exists, and the assumption of homogeneity is a simplification. The important question is if it  
352 is a reasonable simplification. This can be taken to mean that  $\tau_{st}$  and  $\tau_{ft}$  determined by fitting  
353 the DALE model closely resembles the average  $\tau$  of each population. On the grounds of  
354 numerical simulations with models that include within-population heterogeneity  
355 (supplemental material S1, <https://doi.org/10.6084/m9.figshare.19434983.v1>), we contend  
356 that the assumption of homogeneity is likely to be appropriate. However, both above-  
357 mentioned assumptions (abrupt shift in recruitment and within-population homogeneity)  
358 would benefit from more investigations, both numerical and experimental.

359 *Fiber dependent energy conversion efficiency*

360 A topic that was not highlighted in the current study is the difference in energy conversion  
361 efficiency between slow- and fast twitch fibers. In vitro studies on muscle fibers with  
362 different myosin isoforms show that fast twitch fibers have a higher ATP cost of tension (37,  
363 38), and probably also a higher mechanical efficiency at contraction velocities associated  
364 with ergometer cycling at ~80 rpm (39, 40). These findings, combined with several studies  
365 designed to investigate the relationship between muscle fiber type activation and  $\dot{V}O_2$   
366 kinetics (see review by Jones et al. (41)), have led to the conclusion that the  $\dot{V}O_2$  slow  
367 component is intrinsically linked to recruitment of fast twitch fibers. The DALE model  
368 disagrees only as a matter of terminology. Specifically, in the heavy exercise intensity  
369 domain, the DALE model predicts a delayed steady state, and allows for an increased  $\dot{V}O_2$   
370 gain compared to the moderate domain. The difference from the conventional 3-phase model  
371 is that we consider these effects to originate from the shift from anaerobic to aerobic ATP  
372 production within each muscle fiber, which is slower in fast twitch than slow twitch fibers  
373 (9). On this basis, even if the steady state is delayed and the  $\dot{V}O_2$  gain might be higher than in  
374 the moderate domain, we argue that this phenomenon should be considered a “delayed  
375 primary phase”. A further development of the model could be to include fiber-specific gains,  
376 where the slow twitch fiber gain is determined from steps to the moderate exercise intensity  
377 domain, and the fast twitch fiber gain is determined during steps from an elevated baseline at  
378 the transition between moderate and heavy. According to most studies in the literature, one  
379 should expect a higher gain for the fast twitch compared to slow twitch model component  
380 (16). The findings of Spencer et al. (35) indicate that a higher gain might be apparent already  
381 within the moderate domain. This finding can suggest fast twitch activation within the  
382 moderate domain, which would violate one of our model assumptions.

383 *Slow component as a delayed, gradual change in energy conversion efficiency*

384 We observed no systematic deviations from the prediction of a linearly increasing  $\dot{V}O_2$  after  
385 120 s in the severe domain. However, there were no systematic deviations using the  
386 conventional delayed exponential representation either. Hence, the residual analysis alone  
387 does not give a decisive answer for which of the models is more appropriate. The delayed  
388 exponential formulation was originally based on the presumption of a progressive recruitment  
389 of fast twitch muscle fibers (4, 6, 17, 42, 43). However, the  $\dot{V}O_2$  slow component has been  
390 shown to emerge without progressive muscle fiber recruitment (44-46), which erodes the

391 argument of a causal relationship between progressive recruitment of fast twitch fibers and  
392 the  $\dot{V}O_2$  slow component. Rather, the evidence points towards a gradual loss of energy  
393 conversion efficiency that is mainly attributed to an increased cost of ATP per force  
394 generation (13, 16, 47, 48) in the active muscle fibers. Consequently, it is more likely that the  
395 progressive iEMG activation seen during severe intensity constant load exercise is an effect  
396 of a gradually declining energy conversion efficiency in active muscle fibers – requiring  
397 additional neural stimulation to maintain a stable work rate – and not the cause of the  $\dot{V}O_2$   
398 slow component per se (48). Therefore, we contend that a linearly increasing gain with time  
399 is more appropriate than a delayed exponential function as a mechanistically based,  
400 minimalistic, integrated model for the  $\dot{V}O_2$  slow component in the severe intensity domain.

401 Although most participants had kinetics consistent with our expectations (i.e. attaining a sub-  
402 maximal steady state  $\dot{V}O_2$  in heavy and a gradually increasing  $\dot{V}O_2$  in severe), a few  
403 individuals deviated from this behavior. Specifically, two participants did not reach steady  
404 state  $\dot{V}O_2$  in heavy. Furthermore, although all participants showed a gradually increasing  $\dot{V}O_2$   
405 in severe, with six of eight subjects attaining  $\dot{V}O_{2,max}$  at the end of the 9 minute trial, one  
406 individual only showed a slight increase in  $\dot{V}O_2$  and ended just above his RCP at the end of  
407 the 9 minute trial. These deviations might be explained by inappropriate exercise intensity  
408 (i.e. prescribed power output) during the constant load trials, causing some participants to  
409 exercise close to the borders between moderate and heavy, and between heavy and severe.  
410 There are several studies showing that  $\dot{V}O_2$  kinetics measured close to the heavy/severe  
411 border might not always be consistent with conventional expectations (49, 50), and that a  
412 gradual increase in  $\dot{V}O_2$  might be observed even in the moderate domain if given enough  
413 time to develop (51). To summarize, even though most participants in the current study  
414 followed the expected behavior, we recognize that there is some dispute about the qualitative  
415 behavior of  $\dot{V}O_2$  kinetics, particularly when exercising close to boundaries between intensity  
416 domains.

#### 417 *Perspective*

418 Recent mechanistic evidence leads us to propose a revision of the conventional 3-phase  
419 mathematical modeling of the  $\dot{V}O_2$  on-response. The revised model, termed DALE (Delayed  
420 Adjustment and Loss of Efficiency), was shown to be valid across exercise intensity domains,  
421 and had a fitting performance comparable to (or better than) that of the conventional delayed  
422 exponential representation. From a mechanistic perspective, the DALE model's distinction

423 between a delayed steady state and a true slow component may allow to clarify common  
424 misapprehensions and caveats when interpreting pulmonary  $\dot{V}O_2$  kinetics. Specifically, the  
425 DALE model allows assessment of the overall metabolic energy cost of exercise/work  
426 economy in the moderate and heavy domains of exercise, where a steady state, though a  
427 delayed one, is attainable. Moreover, the time dependent gain included in the DALE model  
428 might be used to estimate the cost of severe intensity exercise.

429 In summary, the DALE model opens the possibility of assessing the function of different  
430 muscle fiber types, the domain-specific response to specific interventions (e.g. glycogen  
431 manipulations, fatigue, etc.) and it might also provide the first steps for a modeling framework  
432 where oxygen cost in all exercise domains can be accurately predicted.

### 433 **Supplemental material**

434 S1: “Effects of violating the DALE model’s assumptions”,  
435 <https://doi.org/10.6084/m9.figshare.19434983.v1>



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## Figures

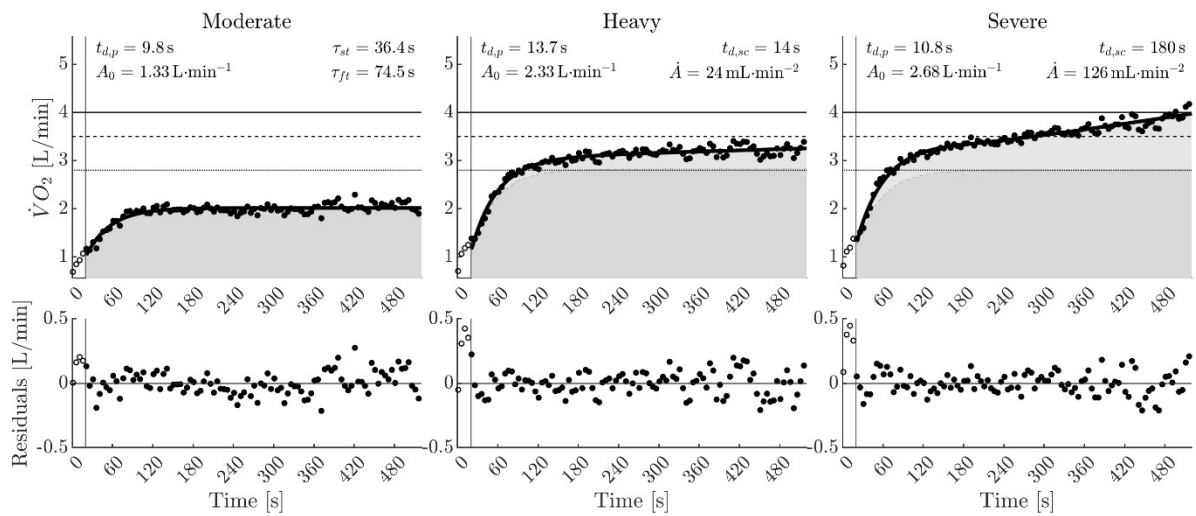


Figure 1: Model fit in the moderate, heavy and severe domains for an example subject. Top row:  $\dot{V}O_2$  vs time; bottom row: model residuals vs time. Measurements before 20 s are marked with open symbols and are not included in the analysis because they are confounded by the cardiopulmonary phase. The optimal parameters for each fit are shown in the upper left corner of each panel. Explanation of model parameters:  $t_{d,p}$  = time delay of primary phase,  $t_{d,sc}$  = time delay of slow component,  $\tau_{st}$  = time constant of slow twitch fiber population,  $\tau_{ft}$  = time constant of fast twitch fiber population,  $A_0$  = initial  $\dot{V}O_2$  amplitude (at  $t = 0$ ),  $\dot{A}$  = change in  $\dot{V}O_2$  amplitude with time. The dotted, dashed and solid horizontal lines indicate GET, RCP and  $\dot{V}O_{2 \max}$ , respectively.

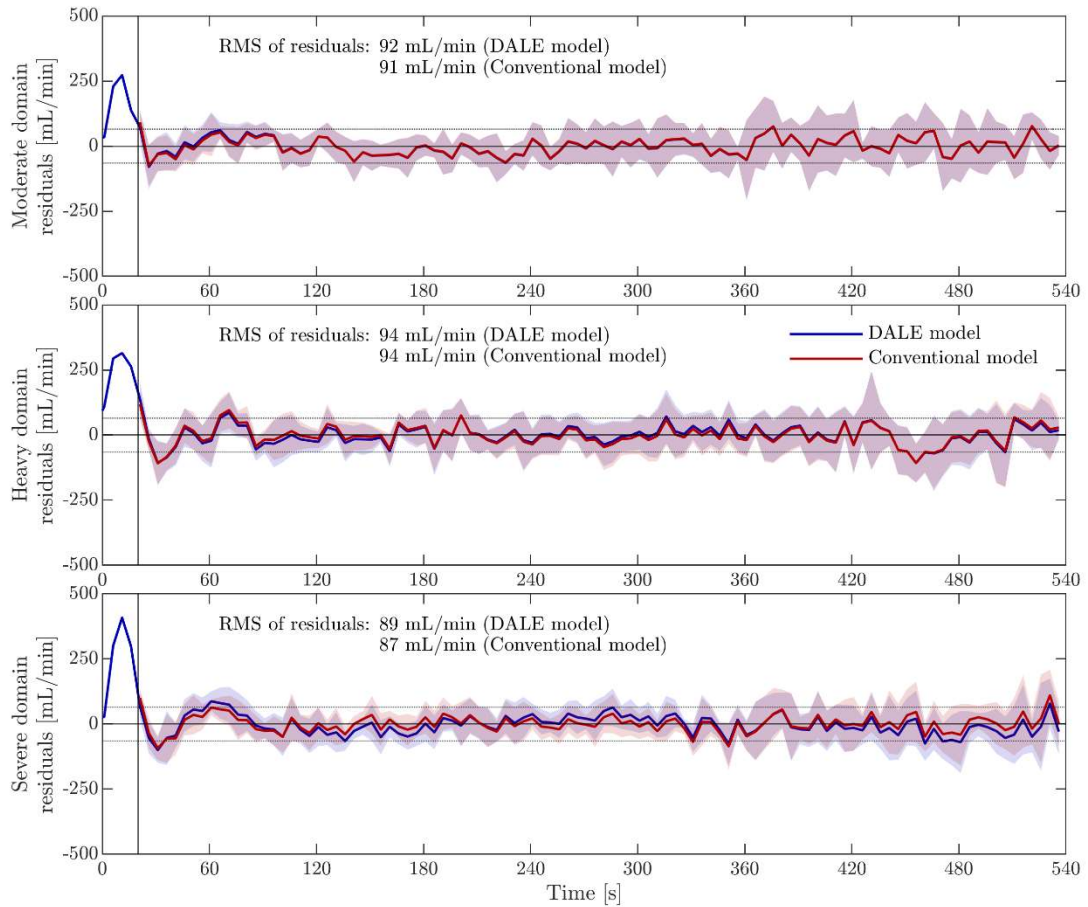


Figure 2: Residuals averaged over all participants at each time point, with 95% CI represented by the shaded areas. Blue lines represent the DALE model, red lines represent the conventional 3-phase model. The dashed horizontal lines indicate typical measurement variability, quantified as the standard deviation of  $\dot{V}O_2$  measurements during the last minute of unloaded cycling ( $65 \text{ mL} \cdot \text{min}^{-1}$ ). It appears to be small systematic deviations during the primary phase ( $20 \text{ s} < t < 120 \text{ s}$ ), but no systematic deviations for  $t \geq 120 \text{ s}$ . The vertical black line is at  $t = 20 \text{ s}$  and marks the presumed end of the cardiopulmonary phase. The root mean square (RMS) of the residuals is also reported within each panel.