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Report

Solution of the Economic Dispatch Problem by Spatial decomposition

With application to the power market simulator Fansi

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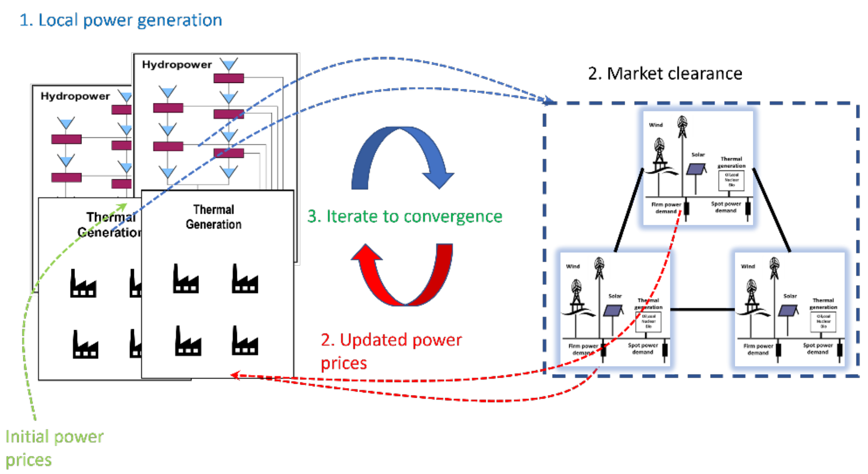
Ole Martin Hansen



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ABSTRACT

The overall solution time for the power market simulator FanSi is too high for practical use by power producers, transmission system operators and regulators. The FanSi model formulates optimisation problems in the form of Linear Programming (LP) problems to solve the economic dispatch problem (EDP). We apply the decomposition technique Lagrangian Relaxation (LR) on the EDP with the goal of reducing computation time and obtaining high quality results. The relaxation of the power balance constraints gives separate subproblems for hydropower and thermal power in geographically separated areas, and one market problem. A dual problem is solved and provides Lagrangian multipliers to the subproblems. A bundle method is used to solve the nondifferentiable dual problem. Our results are obtained from a test case with a detailed description of the Northern European power system. We report on the solution quality and speed of the decomposed EDP by comparison with the solution of the LP-problem. Our results show that the solution from the LR underestimates the system costs, the dual solution is shown to provide area power prices with some inaccuracy which is reflected in the solutions of the subproblems. The speed of the decomposed problem relative to the LP-problem varies depending on the EDP to be solved.

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SINTEF

1 Introduction

This report is written as part of the Rakett project. The project is an "Innovation project for the Industrial Sector" financed by partners in the industrial sector and the research council of Norway.

The players in the Nordic power market, i.e. producers, transmission system operators and regulators use computer models to plan for the best possible utilization of the system and energy resources. The computer models give results such as power prices, power production, dispatch etc. for different weather scenarios. This is important to avoid emptying reservoirs which may result in curtailment of electricity or to avoid too cautious operation which may result in unnecessary spillage and lost power production.

The future power system will have more non-controllable renewable power sources such as wind and solar and a stronger coupling to the continental Europe. The Fansi model, developed in the SOVN-project, was developed with methods suitable for analysis of the future power system. The results from the Fansi model are promising but comes at the cost of a high computational burden.

The FanSi model formulates mathematical optimisation problems of a large power system with detailed description of hydropower, for example the Nordic power system. In addition, the optimisation problem has a long time-horizon stretching over many timesteps, for example 8760 hours of a year. A know technique to reduce calculation time is to divide the large optimisation problem into many smaller optimisation problems, for example one can split the problem into smaller geographical areas and/or in time. This requires the coordination of the solutions from the smaller problems into a complete solution of the problem. This allows for reduced calculation time and large-scale parallel processing which in turn will reduce calculation time.

The project will investigate decomposition techniques to reduce the computational burden, and thus make the model usable for analyses or operational use. The decomposition technique used for spatial decomposition in this report is Lagrangian Relaxation (LR). LR can be used to compute a lower bound on system costs for minimisation problems. A review of LR can be found in [1] . LR is a known technique which can be used for solving the hydrothermal scheduling problem for large power systems [2], [3]. LR is attractive due to the high potential for parallelisation, it allows for splitting the large regional hydrothermal scheduling problem in one market subproblem, several separable local hydro subproblems and several local thermal subproblems. The dual problem is nondifferentiable and can be solved by using different algorithms, such as the subgradient or Bundle method [4]. The LR subproblems can be formulated as Linear Programming (LP) problems. In LP problems all functional relationships are linear. LR allows for solving all subproblems in parallel and has potential for reducing the calculation time.

The scheduling problems under study are deterministic optimisation problems for the operation of hydrothermal power systems dominated by hydropower. The objective is to minimise the operational costs of the system for a given period and future expected system costs. The future expected system costs are provided by a stochastic optimisation model FanSi, used for long-term planning of hydrothermal power systems [5]. This model includes a detailed description of hydropower, all relevant physical attributes of the market are represented and the uncertainty in weather and exogenous prices are considered.

The scheduling problem is formulated in two ways, the full LP-problem and the decomposed problem by LR. This allows for comparing the solution provided by LR against the solution from the LP-problem. The solution from the LR will provide a lower bound on the operational system costs.

The case studied is the Northern European power system with a focus on the computational performance and result quality from the decomposed problem by LR. The computational performance is the solution time. The result quality is the operational system costs and the marginal cost of electricity. Further, the solution of the primal state variables of the LR and the marginal value of water from the solution of the subproblems are

reported. The solution of the LP-problem is used as reference when reporting performance and results quality of the decomposed problem by LR.
In this work we consider scheduling problems spanning 24 to 168 hours with hourly resolution, and later scenario problems consisting of multiple weeks with lengths 26-52 weeks and several intraweek time-steps.

2 Prototype model in Rakett

The Rakett project "inherited" a multi-market model of electricity for large hydro-thermal power systems from the Pribas project [6]. This prototype model formulates optimisation problems in the form of LP or Mixed Integer Programming (MIP) problems. The model provides estimates of the marginal cost of electricity, different types of reserve capacity and the operational costs of the power system.

Due to the computational demanding MIP problems the model includes computational alleviations by two convex relaxations of the MIP problem. The convex relaxations are Linear Relaxation and Lagrangian Relaxation [7]. This computer code was developed in Julia/JuMP, a high level and high-performance programming language, which makes the model particularly attractive for further development in research projects.

In the Rakett project the code in Julia/JuMP inherited from the PRIBAS project will be used as a starting point and developed further according to project goals.

3 FanSi – a stochastic optimisation model for the electricity market

The FanSi prototype model is a stochastic optimisation model used for long term planning of hydrothermal power systems, where the objective is to minimise system costs. The FanSi model can simulate large hydrothermal power systems with a detailed description of hydropower and individual water values for all hydroelectric reservoirs of the system and high shares of intermittent energy source such as wind- and solar power. The operation of each individual hydroelectric reservoir is based on the result of formal stochastic optimisation in which all relevant physical attributes of the market are represented. Weekly decisions are determined by solving scenario fan problems considering uncertainty in weather and exogenous market prices. The overall scheduling problem is obtained by solving a sequence of scenario fan problems spanning a chosen period.

The main challenge of the FanSi model is the calculation time. Reducing the calculation time is a prerequisite for the model to be applicable by the model users. The FanSi model builds and solves optimisation problems in the form of LP-problems. The goal of the Rakett project is to investigate the potential reduction in calculation time by applying decomposition techniques to the LP-problems from the FanSi model. Therefore, the optimisation problems decomposed by LR in this report are similar to the LP-problems built by the FanSi model.

3.1 The scenario fan problems

The FanSi model builds and solves scenario fan problems. The scenario fan is decomposed in a weekly first stage (master) problem and a multi-week second stage (scenario) problem by Benders' decomposition. The second stage problem consists of a set of deterministic scenarios where the valuation of water at the end of the scenario horizon is an input parameter to the model. The scenario fan has a rolling horizon.

The master problem is solved and the state variables from the solution are passed on as input to the second stage problem. The scenario problems are solved to provide a set of linear restrictions on the future system costs function of the master problem in the form of Benders' cuts which in turn improves the problem solution. The master problem solution is improved iteratively by adding Benders' cuts and the iteration process is terminated when the solution has converged to be within a specified tolerance. The scenario fan is illustrated in Figure 1. In the illustration the first stage problem is the weekly decision problem for week 1 and the second stage problem is a set of scenarios beginning at week 2 and ending at week N+1.

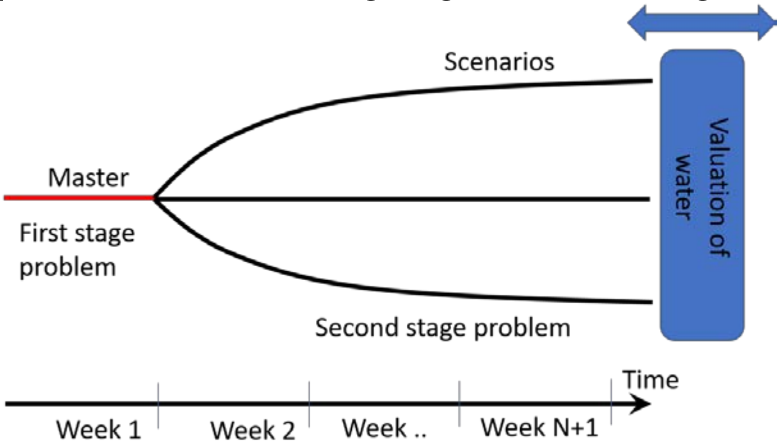

Figure 1: Illustration of the FanSi scenario fan. The first stage (master) problem is a weekly decision problem, the multi-week second stage (scenario) problem consists of many scenarios and the valuation of water at the scenario end.

The coupling point of the Benders' decomposition is between the first and second stage problems.

3.2 Size of the FanSi scenario problems

The scenario problems included in the second stage problem of the FanSi model will depend on the size of the power system being studied and the parameters set to control the scenario time horizon and the scenario time resolution. Using a Statnett dataset covering the full Nordic power system with 1235 reservoirs. The LP problem includes power balances for each time step and each area, hydro balances for each reservoir and each time step.

Table 1 shows the LP problem size for a full Nordic system for a scenario containing 260 timesteps (52 weeks and 5 intraweek timesteps) and 1560 timesteps (52 weeks and 30 intraweek timesteps).

Timesteps	**260 (52 x 5)**	**1560 (52 x 30)**
Variables	$2{,}223 \times 10^6$	$13{,}399 \times 10^6$
Constraints	$0{,}331 \times 10^6$	$1{,}984 \times 10^6$
Elements in A matrix	$4{,}096 \times 10^6$	$24{,}891 \times 10^6$

Table 1: The LP problem size for a full Nordic system for a scenario containing 260 timesteps (52 weeks and 5 intraweek timesteps) and 1560 timesteps (52 weeks and 30 intraweek timesteps).

The solution times of these large scenario LP problems are reported to be 294 seconds (260 timesteps) and 17216 seconds (1560 timesteps) using the CPLEX solver with the dual simplex algorithm.

4 Spatial decomposition

The optimisation problems of the hydro-thermal power system span a large geographical area and many time steps. The geographical area can be the Nordic power system with a detailed description of hydro power. The power system contains many separate price areas and cascaded water ways. To reduce the solution time of the optimisation problem it is possible to split the problem into smaller LP problems by decomposition techniques. The optimisation problem can be decomposed both spatially and temporally. Temporally, Benders' decomposition in combination with parallel processing can be applied to accelerate the solution process [8], [9]. Spatially, Lagrangian decomposition or Lagrangian Relaxation can be applied. In [2] and [7] Lagrangian Relaxation is applied on short term hydrothermal scheduling problems for large power systems where hydropower and thermal power is solved separately in local problems. The work [3] compares Lagrangian decomposition of large-scale stochastic hydrothermal unit commitment problems in space and per scenario.

In the referenced works the LR is applied on MIP problems. In this report we focus on the spatial decomposition of LP problems into LP subproblems. The subproblems are solved separately and iterative coordination is required to obtain the complete solution.

4.1 Lagrangian Relaxation

The FanSi model is a stochastic optimisation model considering uncertainty in weather and exogenous prices which is revealed by solving the deterministic scenario problems. In this report deterministic optimisation problems like the master problem or the scenario problems of the FanSi model are formulated and solved.

The FanSi model enables the study of large hydrothermal power systems spanning a large geographical area, such as the Nordic or the Northern European power systems. The system is divided in local subsystems or price areas. The price areas are interconnected by transmission power lines. The detailed hydropower is modelled by a number of modules, each with a reservoir and a power station. The water from each module is released in three possible water ways, *discharge*, *bypass* and *spillage*, including module coupling topology in cascade watercourses. The water balance equations is respected per time step. Each reservoir has physical storable and non-storable inflow time series from historical records. The hydropower generation is described by linear discharge-production curves. The valuation of water is set at the end of the scheduling horizon by a linear cut description. The thermal power is modelled with associated production capacity limits and the marginal cost of production. For illustrative purposes a single area of the power system is shown in Figure 2. Where the power generators are connected to a busbar and the generated power meets the power demand. The power demand has both elastic and inelastic components. Each area is connected to other areas by power transmission lines for power exchange. Utilisation of the transmission lines is bounded by exchange capacity limits.

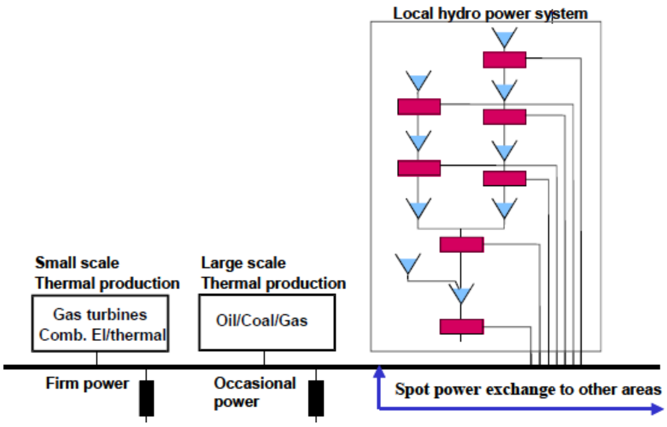

Figure 2: Model setup of a hydrothermal power system in a price area.

The optimisation problems are formulated as LP problems. The objective is to minimise the system costs of operation while respecting the power system constraints for a specified period (e.g., 1 week, 52 weeks). The power system constraints include power balance constraints for each area. Hydropower constraints include intra week reservoir balances per load period per reservoir, constraints on lower and upper reservoir volumes, constraints on discharge and bypass. Thermal power is constrained by lower and upper generation limits.

$$Z = \min\ (Cost\ of\ thermal\ generation\ +\ future\ expected\ value\ of\ water) \tag{1a}$$

s.t.

$$Power\ balance\ constraints \tag{1b}$$
$$Hydropower\ constraints \tag{1c}$$
$$Thermal\ power\ constraints \tag{1d}$$

4.2 Decomposition by Lagrangian Relaxation

The LR described in the following follows the formulation in [7]. Some noteworthy differences are that in this report we focus on decomposition of the long-term hydrothermal scheduling problem which is originally formulated as a large LP-problem. Different types of reserve capacity are not included in the modelling. We solve the scheduling problem with a long time-horizon of up to 3 years and a coarse time resolution (weekly or daily time steps). Exchange on transmission lines is bounded by capacity limits.

The problem described in section 4.1 can be decomposed by Lagrangian Relaxation (LR), by relaxing the power balance constraints. The power balance constraints and Benders cuts are introduced into the objective function by adding terms of the form λ(z - p):

$$L(x,\lambda) = Z +\ \lambda(z - p) \tag{2a}$$

s.t.

$$Hydropower\ constraints \tag{2b}$$
$$Thermal\ power\ constraints \tag{2c}$$

Where λ is the Lagrangian multiplier, a nonnegative weight that penalise deviation from the power balance constraint. Lagrangian multipliers are introduced per time-step and price area. This LR allows for decomposed LP problems in separate LP subproblems and a dual problem. The *subproblems* are the hydropower problems, the thermal power problems, and a market problem. The objective function value of the decomposed problem by LR is the sum of the objective function values of the subproblems.

4.2.1 The market problem

The hydro and thermal generation costs are calculated in separate problems. Thus, the market problem is a minimisation problem of the remaining system costs for a given set of Lagrangian multipliers and the future costs of hydro operation. The market problem minimise the system costs related to area aggregated production per time step:

$$Z_M =\ \min(Remaining\ system\ costs\ + \alpha + \boldsymbol{\lambda_c}(\beta_c - \alpha)) \tag{3a}$$

$$s.t.$$

$$Power\ balance\ constraints \tag{3b}$$

The Lagrangian multipliers $\boldsymbol{\lambda_c}$ is associated with the relaxation of the Benders cuts.

4.2.2 The hydro problems

The hydro problems are maximisation of revenue problems for the areas with hydro production:

$$Z_H = \max(Generation\ revenue\ + \alpha + \boldsymbol{\lambda_c \pi v}) \tag{4a}$$

$$s.t.$$

$$Hydro\ constraints \tag{4b}$$

The parts of the Benders cuts are calculated in the equations (6) and (8).

4.2.3 The thermal problems

The thermal problems are maximisation of revenue problems for the areas with thermal production:

$$Z_T = \max(Generation\ revenue) \tag{5a}$$

$$s.t.$$

$$Thermal\ constraints \tag{5b}$$

4.2.4 The dual problem

The dual problem maximises the Lagrangian L with respect to the Lagrangian multipliers λ.

$$Z_D = \max_{\lambda} L(x, \lambda) = \max_{\lambda}(\min_{x}(system\ costs) + \lambda(z - p)) \tag{6a}$$

$$s.t.$$

$$Constraints \tag{6b}$$

The dual problem in (6) is nondifferentiable and concave and is solved by the bundle method. Alternatives to the bundle method are the cutting plane method and subgradient methods. The bundle method is chosen because it is robust, stable and has good convergence properties [4]. It has a quadratic objective function constrained by linear cuts. The cuts are obtained from the solution of the subproblems. The constraints matrix increases as new cuts are added to the problem with a corresponding increase in solution time. The dual problem is described in detail in [7].

The objective function value obtained from the LR is the sum of the market problem, the thermal problems, and the hydro problems:

$$Z_{LR} = Z_M + \sum_{Hydro\ areas} -Z_H + \sum_{Thermal\ areas} -Z_T \tag{7}$$

Notice the negative sign of the objective values from the hydro and thermal problems. The negative sign is necessary for the hydro and thermal problems because they are formulated for maximizing profit.

In Figure 3 a visualisation of the LR solution procedure is shown. The dual problem proposes values for the Lagrangian multipliers λ to the subproblems. For the given multiplier values the subproblems are solved and the sum area production per time step and price area are returned. The production mismatches between the solution of the market problem and the thermal power and the hydropower problems provides estimates for the subgradients constraining the Lagrangian multipliers of the dual problem. A new set of cuts are added, and the dual problem solved again which in turn provides a new set of values for the subproblems. The objective function value of the decomposed problem by LR in (7) is iteratively improved. Each iteration provides a lower bound for the objective function value. The iteration process is terminated according to a stopping criterion.

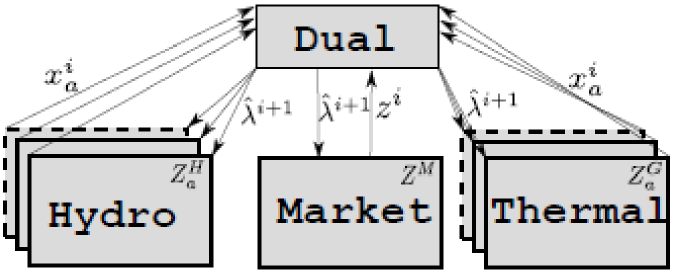

Figure 3: LR solution procedure.

5 Results

The results presented in the following are obtained by using the Julia/JuMP code developed in the PRIBAS project. Some modifications were done to accommodate the model to build FanSi master and scenario LP-problems.

5.1 The hydrothermal power system

The hydrothermal power system under study is defined in the dataset called HydroCen_LowEmission and is used in several SINTEF projects such as PRIBAS and HydroCen. It contains the North European power system, divided in 57 price areas. The dataset contains 58 yearly inflow scenarios, and the intraweek time resolution is 3 hours in 56 load periods. The hydropower system contains 1093 hydroelectric reservoirs with corresponding multi-segment PQ-curves. The dataset uses a total of 94 yearly inflow time series and 57 wind time series. No calibration has been performed on the dataset after performing changes in for example the transmission capacity.

5.2 Warm starting the solution process

The solution time of LP problems can be significantly reduced by inserting a near-optimal solution basis as a starting point. For example, the overall scheduling problem for a power system can be obtained by solving weekly decision problems in sequence over one year and for many weather scenarios. Here we can use the solution from the first LP problem as a starting point for the next LP problem if these problems are built with an equal LP structure and size. For example, the solution from week 1 can be the start basis for week 2 etc. The LP builder routines of the Julia/JuMP code inherited from the PRIBAS project are carefully made to exploit this time reducing measure. The use of a near-optimal solution basis as a starting point is referred to as a *warm start*, or *warm stared* problems.

5.3 Weekly LP problems

The applied Julia/JuMP code can be set up to solve the weekly decision problem with hourly time resolution. At the beginning of the optimisation period the initial reservoir volumes for all hydroelectric reservoirs sets the system state. At the end of the week a future cost function is provided by Benders cuts provided by the FanSi model. The included constraints per timestep is listed in Table 2.

Constraints per timestep
Hydro balance
Power balance
Minimum and maximum bypass
Minimum and maximum discharge
Up and down ramping on discharge
Minimum and maximum generation on thermal units
Start-up costs for thermal generation units
Shut-down costs for thermal generation units
Up and down ramping on flow on transmission lines

Table 2: Included constraints per timestep.

It is possible to build LP-problems with varying size by defining the number of hours included in the optimisation problem, for example one can define an optimisation problem spanning 24, 48, 72 hours etc., coupled to the future cost function at the end of the week. This is a convenient way to studying the connection between LP problem size and computation time. The LP problem size for increasing number of hours in the optimisation problem is shown in Figure 4.

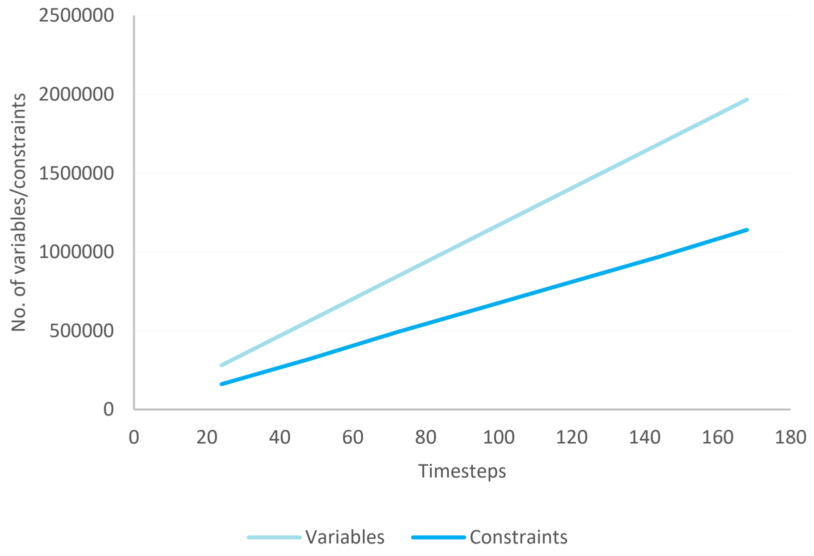

Figure 4: Size of the weekly problem with varying number timesteps in the LP problem.

The LP solution time for increasing LP problem size is show in Figure 5. The LP problems are solved without warm start for the dual and primal simplex algorithms of the Cplex solver. The black curve shows the solution time for the dual simplex algorithm and the red curve shows the solution time for the primal simplex algorithm. The dual algorithm is faster than the primal algorithm for the five largest LP-problems. Notice the logarithmic scale of the vertical axis.

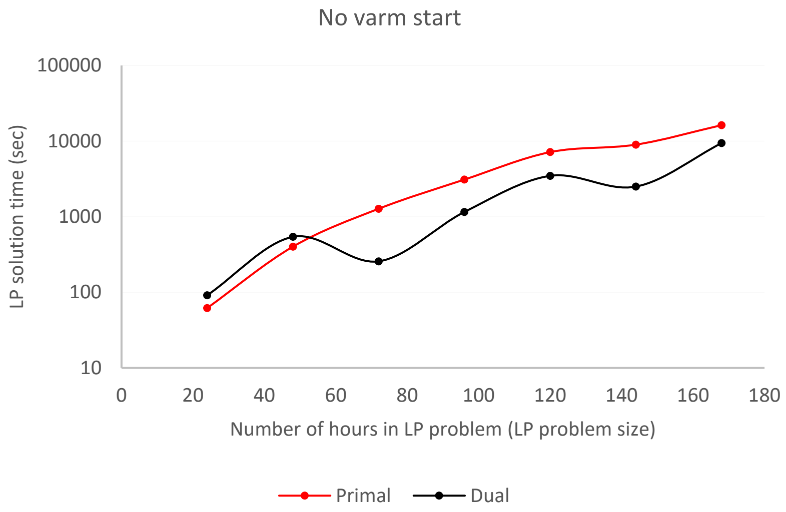

Figure 5: The LP solution time without warm start for varying LP-problem size. The black curve shows the results from the Dual Simplex method and the red curve shows the results from the Primal Simplex method. The vertical axis has a logarithmic scale.

The LP solution time for increasing LP problem size with warm start is shown in Figure 6. The LP problems are solved with warm start for the dual and primal algorithms of the Cplex solver. The warm started LP-problems use a near-optimal solution basis obtained from the solution of a similar LP-problem with different values of inflow, inelastic demand and wind power. The black curve shows the solution time for the dual simplex algorithm and the red curve shows the solution time for the primal simplex algorithm. The dual algorithm is faster than the primal algorithm. The warm started LP problems are solved significantly faster than the non-warm started LP problems.

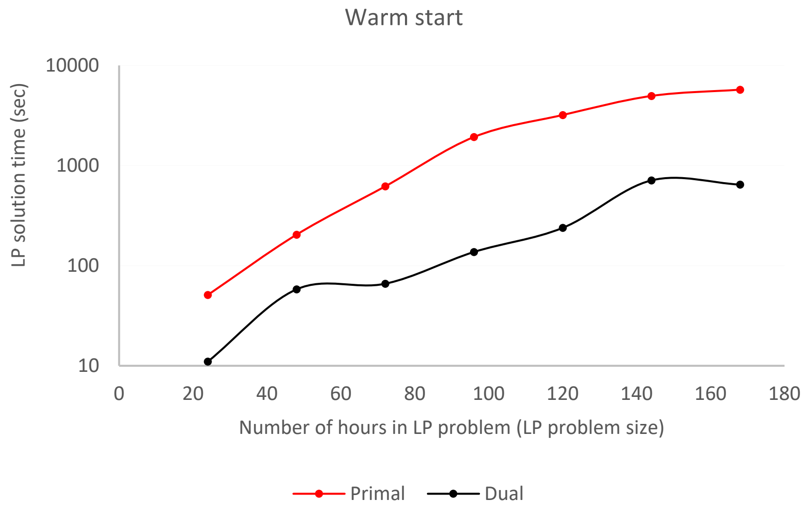

Figure 6: The LP solution time with warm start for varying LP-problem size. The black curve shows the results from the Dual Simplex method and the red curve shows the results from the Primal Simplex method. The vertical axis has a logarithmic scale.

Note that the comparisons between the dual and primal simplex algorithms shown in Figure 5 and Figure 6 are made for a specific problem structure. The outcome of such comparisons will typically depend on problem characteristics, such as the ratio between constraints and variables and the density of the A matrix.

5.4 Weekly problems solved by LR

In this section we present results from the decomposed LP-problem by LR. The decomposed problem has many objective function values, one from the dual problem and many subproblems (one market problem, plus several thermal problems and hydro problems). We compare the sum of the market problem and all thermal and hydro subproblems Z_{LR} from equation (7) to the objective function value of the full LP problem Z from equation (1a). If these values are equal, the decomposed problem has fully converged. The objective function value of the dual problem Z_D from equation (6a) forms the lower bound and is used in the stopping criterion of the iteration process.

The *objective function gap* ΔZ_{stop}^i is the difference between the objective function value from the solution of the dual problem Z_D^i and the sum of the objective function values from the solutions of the sub-problems Z_{LR}^i

$$\Delta Z_{stop}^i = Z_D^i - Z_{LR}^i, \quad (8)$$

where the index *i* refers to the iteration number. When $\Delta Z_{stop}^i < \delta$ for 3 consecutive iterations the iteration process is terminated, δ is a defined stopping criterion.

The *convergence gap* is the difference between the objective values from the solution of the full LP problem Z (obtained in advance) and the decomposed problem by LR Z_{LR}^i or the dual problem Z_D^i.

$$\Delta Z_c^i = \mathrm{Z} - Z_{LR/D}^i \quad (9)$$

During the interation process of the LR the solution is gradually improved, thus the convergence gap ΔZ_c^i decreases per interation. The convergence gap measures the gap (or distance) from optimality and is a quality indicator for the solution from the decomposed problem. The decomposed problem by LR provides a lower bound on the objective function value, therefore the convergence gap will be larger than zero at termination.

The calculation times presented are the solution times reported by the Cplex solver for the optimisation problems.

5.4.1 24 hours problem

We start by looking at an optimisation problem spanning 24 hours. The initial Lagrangian multipliers λ are set to zero, which must be considered a poor starting point, since the actual prices most of the time are greater than zero. The stopping criterion δ=10 €.

The LP problem has 281 000 variables and 161 000 constraints. The solution time is 62 seconds using primal simplex (91.5 seconds for dual simplex) algorithm without warm start. With warm start the solution time is 11 second for dual simplex (51 seconds for primal simplex) algorithm.

Figure 7 the evolution of the convergence gap per iteration is shown. Notice the algorithmic scale on the vertical axis. The black curve shows the convergence gap for the dual problem, and the red curve shows the convergence gap for the subproblems. The black curve is not continuous due to the removal of negative values when using the logarithmic scale in the figure. For illustrative purposes, the logarithmic scale is preferred because the convergence gap spans many orders of magnitude (10^9 - 10^3). The convergence gap is reduced per iteration, but from around iteration 300 and onwards the convergence gap is minimally reduced. The LR iteration process is terminated after 4970 iterations with a convergence gap ΔZ_c^i= 38822838 - 38818267 = 4571 k€.

Figure 8 shows the LR solution time and the dual problem solution time per iteration number, the vertical axis scale is logarithmic. The total solution time was 433 329 seconds (~5 days). The solution time increases exponentially with increasing iteration number. The solution time for the dual problem is shown by the red dashed line and the total solution time for LR problem is shown by the black line. The difference between these curves is decreasing with increasing iteration number and eventually the dual problem dominates the solution time. The dual problem is computationally expensive compared to the subproblems. As mentioned in Section 4.2.4, the dual problem is formulated according to the Bundle method, leading to a quadratic optimisation problem which gradually increases in size because new cuts are added for each iteration. The exact formulation of the dual problem is provided in [7]. It is likely that the computational time for solving the dual problem can be significantly improved by techniques such as cut relaxation and/or linearization of the quadratic objective, but such techniques were not tested in this work.

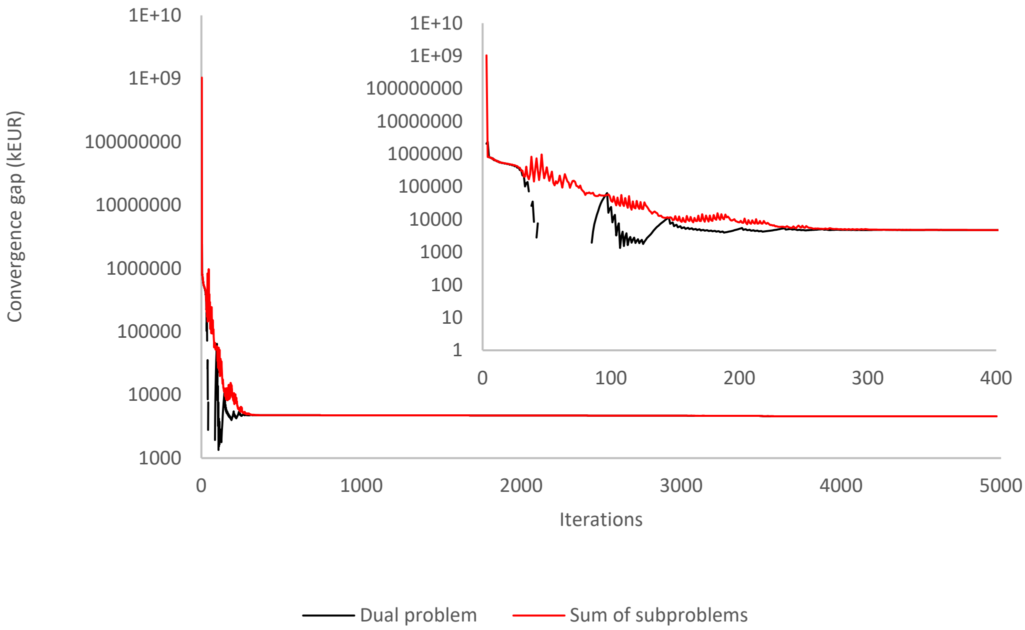

Figure 7: The evolution of the convergence gap for the dual problem and the LR per iteration. The embedded figure shows the evolution of the convergence gap for the first 400 iterations.

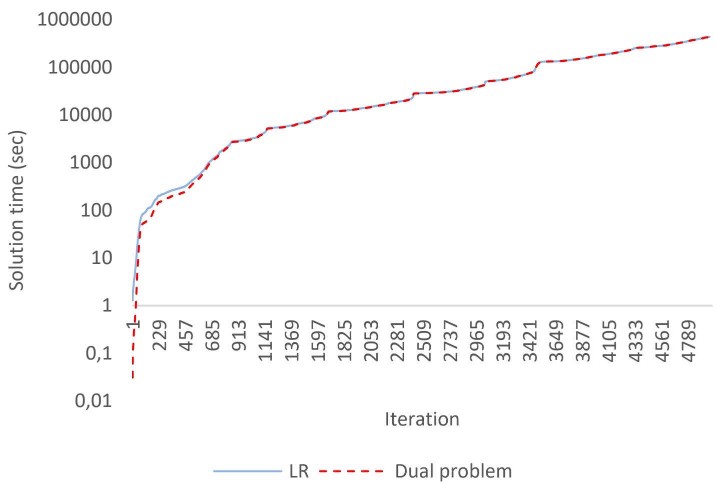

Figure 8: LR solution time per iteration. The solution time of the dual problem is also shown.

Figure 9 shows the power prices in areas 6 (Vestsyd) and 20 (Danmark-vest).

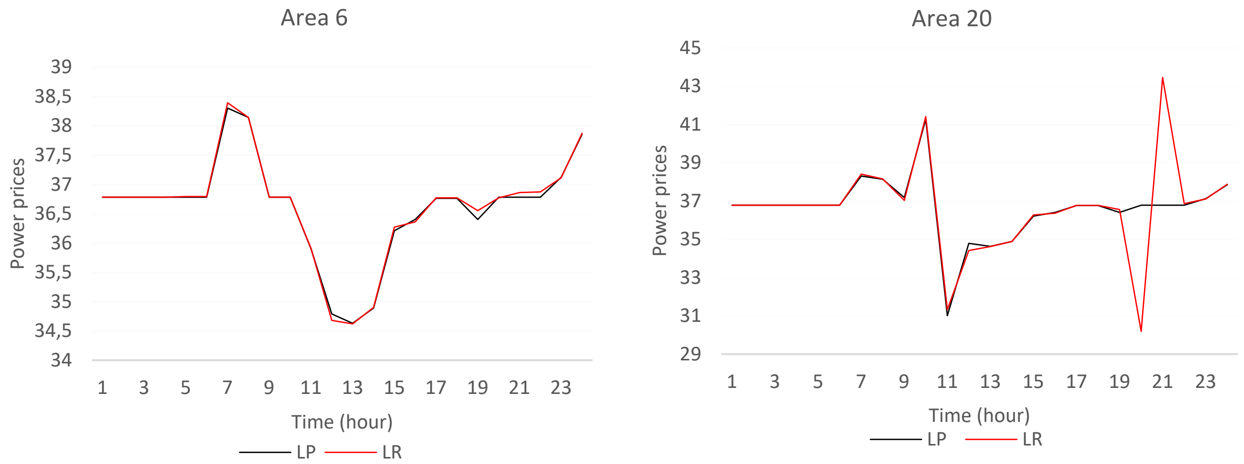

Figure 9: The area power prices (€/MWh) in areas 6 (Vestsyd) and 20 (Danmark-vest) obtained from the solution of the LP and the LR.

The area power prices from LR and LP show only small differences in area 6 (Vestsyd). In area 20 (Danmark-vest) the area power prices from LR and LP show significant differences in the hours 20-21. In hour 20 the area power prices are 36.8 €/MWh (LP) and 30.20 €/MWh (LR) a deviation of (36.8-30.2)/36.8*100=18%. In hour 21 the deviation is 18.1%.

5.4.2 Results discussion

The results from the LR solutions show that there is a convergence gap at termination with the defined stopping criterion δ=10. The solution time is very long (433 329 seconds), the solution improvement is minimal after iteration ~300 and most of the solution time is spent solving the dual problem. The solution times for LR were higher than LP (62 seconds without warm start).

The consequence on power prices was shown for the areas 6 Vestsyd and 20 Danmark-vest. Power price deviations of 18% were found in area 20 Danmark-vest when comparing the LR and LP solutions.

From these observations it is necessary to stop the LR iteration process earlier by defining a stopping criterion δ which allows for a higher objective function gap. This will reduce computation time, but the consequence on the result quality must be considered.

5.5 Weekly problems solved by LR with relaxed stopping criterion

Considering the results from the previous section, the stopping criterion is set to δ=1000 € to reduce computation time. In this section we will examine the consequence of relaxing the stopping criterion δ. The consequence on the objective function value, the computation time and the area power prices are reported. In the previous section the initial Lagrange multipliers λ (area prices) were set to zero. In this section the LR process is started with initial Lagrangian multipliers λ. The initial Lagrangian multipliers λ are obtained from the solution of the optimisation problem for a previous week, these initial values will speed up the iteration process. The results shown are obtained from the solutions of single runs.

We further consider two simplex algorithms for solving LP subproblems: The dual simplex and the primal simplex.

5.5.1 LR solution times

In Figure 10 the LR and LP solution times are compared for varying problem size. The smallest optimisation problem spans 24 hours and the largest spans 168 hours. Only the solution time reported by the Cplex solver is considered, the time used for building and updating the optimisation problems are not included. The solution time is the sum of time spent solving the subproblems and the dual problem, solved in series (no parallel processing). The LP solution time is shown in black, while the LR solution times are shown in light and dark blue. The LR solution time is lower for the larger optimisation problems which include 120-168 hours. The primal simplex algorithm is faster than the dual simplex algorithm.

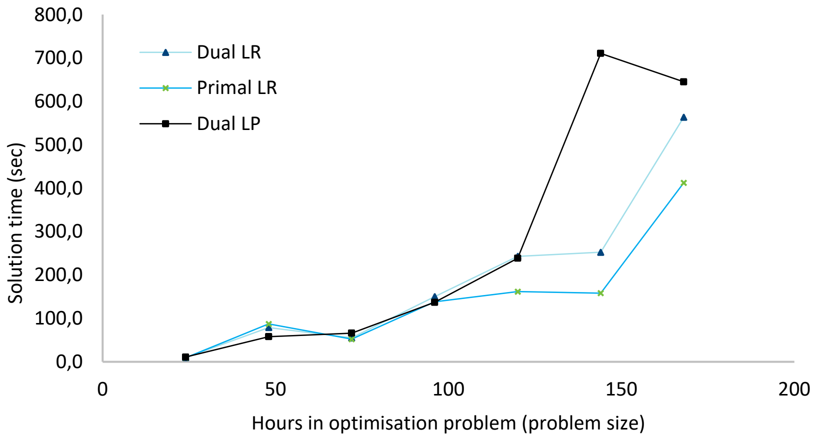

Figure 10: Solution time for problems solved by LR using the dual or primal simplex algorithm and the LP solution time for the dual simplex algorithm.

5.5.2 Solution time with parallel processing of LR

The LR subproblems, shown in the blue ellipse in Figure 13 can easily be solved in parallel.

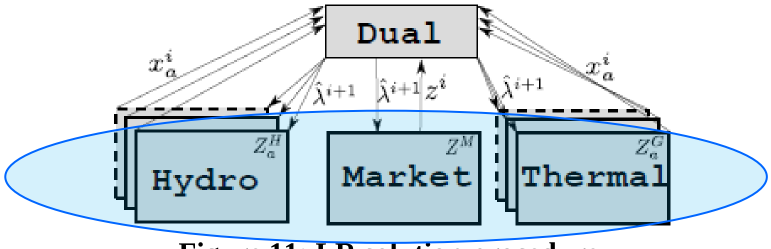

Figure 11: LR solution procedure.

The dual problem provides Lagrangian multiplier to the subproblems and the subproblems return estimated Lagrangian multiplier subgradients through multiplier mismatches. The subproblem with the highest solution time plus the solution time of the dual problem will define the solution time of a fully parallelised problem solved by LR. These estimated solution times for a parallelised LR solution procedure are shown in Figure 12. The dark blue curve shows the LP solution time, and the light blue curve shows the estimated solution time for the parallelised LR.

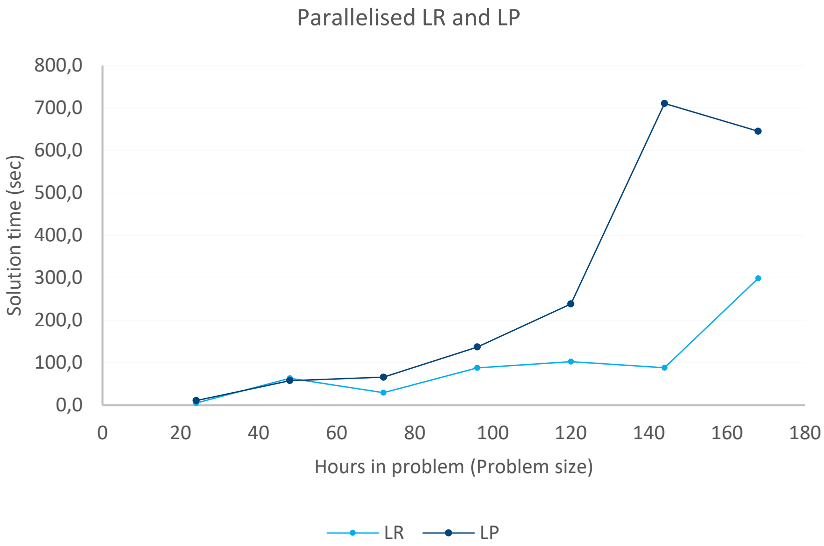

Figure 12: Estimated solution times for the problems solved by parallelised LR and problems solved by LP.

The parallelised LR reduced the calculation time for the three largest problems (120,144,168) hours by approximately (60, 60, 100) seconds comparing to the LR problem run in series.

Figure 13 shows the number of iterations before the stopping criterion is reached and LR iteration process is terminated for varying problem size.

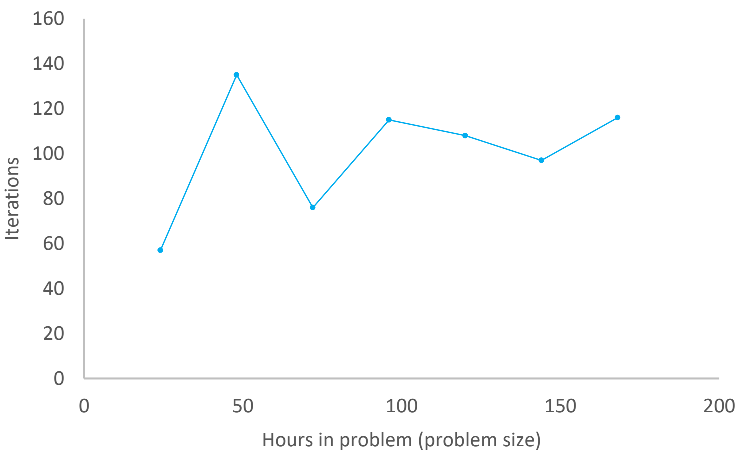

Figure 13: Number of iterations before the LR is terminated for varying problem size.

5.5.3 Results

We investigate the solution from LR procedure for the problem with 168 hours. In Figure 14 the convergence gap versus the iteration number of the LR iteration process is shown. In the embedded figure we zoom in on the vertical axis to see the values of the convergence gap better. The gap is approximately 20000 k€ after 116 iterations, meaning that LR underestimates the system costs with about 20 000 k€ for the defined stopping criterion.

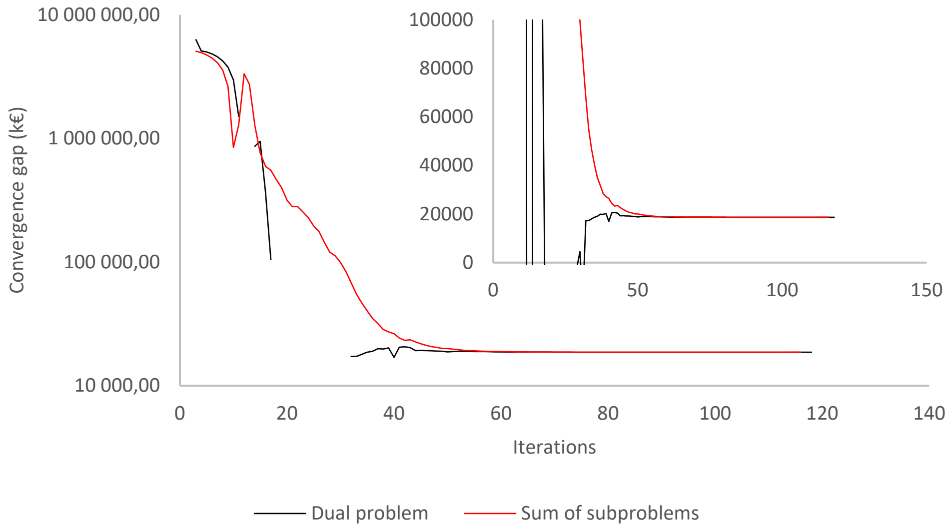

Figure 14: The evolution of the convergence gap for the dual problem and the LR per iteration. The embedded figure shows the evolution of the convergence gap with a linear scale on the vertical axis.

The hourly area prices for area 1 and area 8, Ostland and Norgemidt, obtained by the LR and LP solutions are shown in Figure 15. The green (red) curve shows the prices obtained by LR (LP). The dotted blue curve shows the start prices used as initial Lagrangian multipliers λ for the LR. The LR prices are in between the start prices and the LP prices.

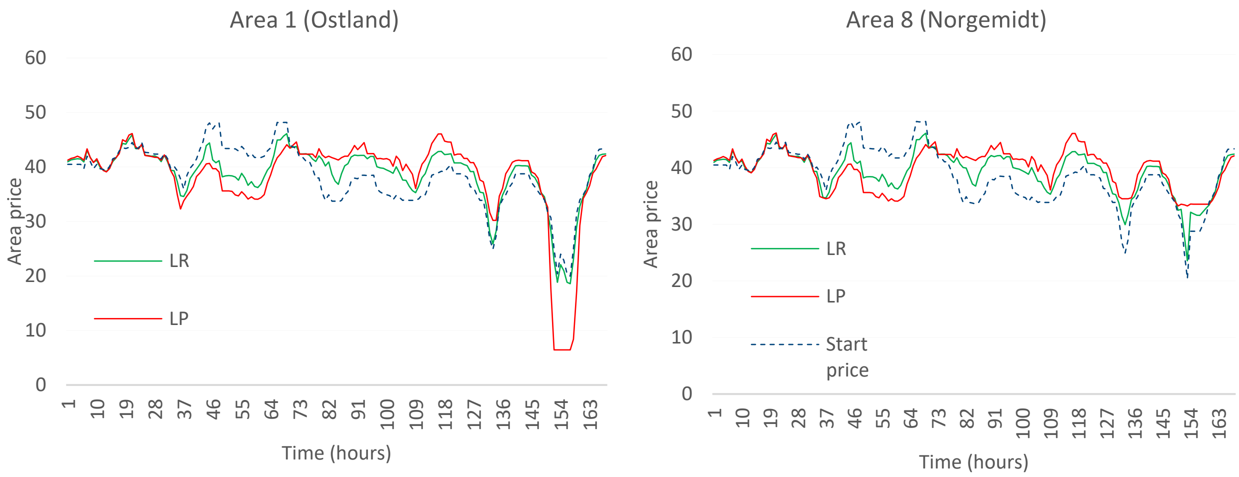

Figure 15: Area prices obtained by the LR solution and the LP solution, and the initial prices.

The prices obtained from the LR solution show significant deviations from the LP solutions. The LR results are between the start prices and the LP results, which may indicate that the convergence criterion is too loose, and that further iterations would reduce the gap between LR and LP prices.

5.5.4 Results discussion

The solution times for LR are lower than the LP solution times when the stopping criterion δ=1 k€ (with initial values for the Lagrangian multipliers λ). The primal simplex method is faster than the dual simplex method. For the largest problem with 168 hours the solution time is reduced by approximately 50% using parallelised LR compared to LP.

The results from LR shows that there is a convergence gap at termination with the defined stopping criterion. The consequence on power prices was shown for two areas. Significant power price deviations were found when comparing the LR and LP solutions. From these observations LR does not converge to give high quality objective values or power prices.

5.6 Scenario LP problems

The scenario problems solved in Fansi normally have a time horizon of 52 - 156 weeks. The scenario fan problem solved in Fansi is illustrated in Figure 1. The time resolution is user specified with the highest time resolution being hourly and the lowest time resolution being weekly. Typically, most of the solution time of the scenario fan simulator is spent solving the scenario problems due to the larger LP problem size compared to the master problem.

The future costs function at the scenario end is described by Benders cuts provided by the FanSi model. The scenario problem does not include start-up and shut-down costs for thermal generation or ramping constraints as the weekly problem. The constraints to variables ratio for the weekly problem is 0.566 (1 114 000 constraints /1 967 000 variables) and for the scenario problem it is 0.366 (1 326 785 constraints/3 624 713 variables). The scenario problem has a lower constraints to variables ratio than the weekly problem.

The number of variables and constraints in the scenario problems with variable scenario problem length is shown in Figure 16, where the number of intraweek time steps is 7. The problem size increases with increasing scenario problem length.

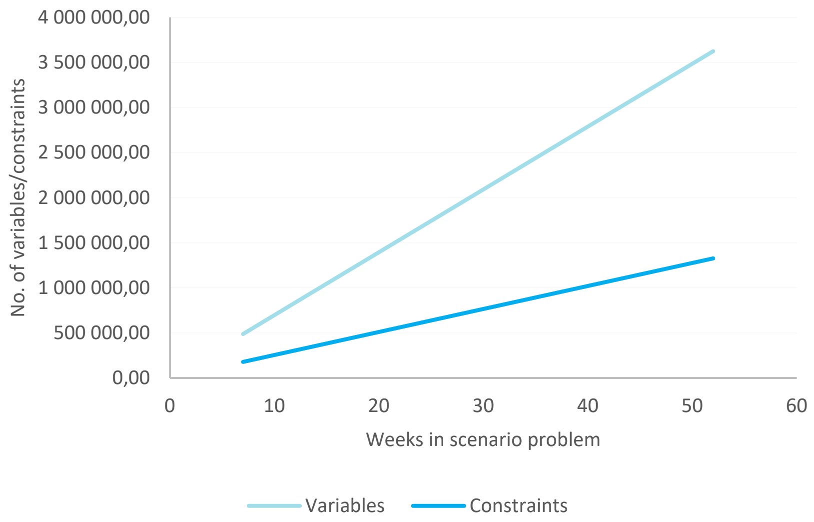

Figure 16: LP problem size for varying scenario problem lengths.

The mean solution time without warm start versus the number of weeks in the scenario problem is shown in Figure 17. The error bars are statistical and represents one standard deviation from the mean. The results are obtained from 10 runs from 10 weather years 1961-1970 for each problem size, all the scenarios start at week 1 with varying scenario lengths of 7, 13 ,26 and 52 weeks. The fastest of the dual simplex and the barrier algorithm of Cplex is used (concurrent optimization). The solution time increases with problem size. For example, a doubling of the problem size, referring to the points 26 weeks and 52 weeks, increases the solution time from 152 seconds to 409 seconds.

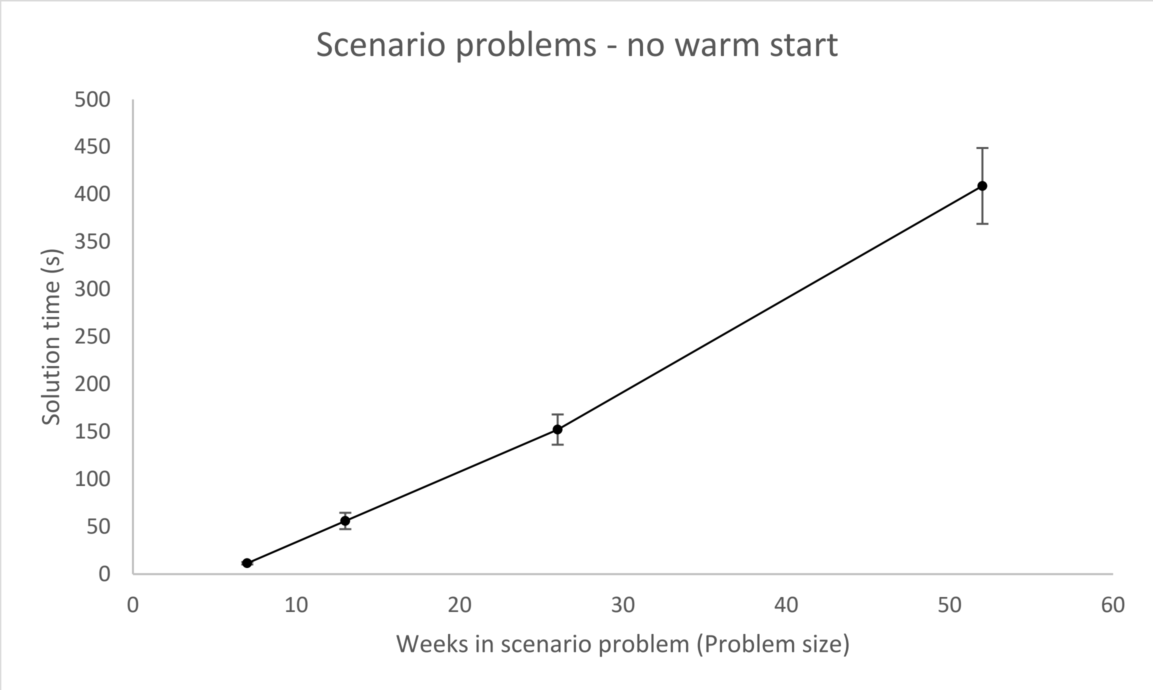

Figure 17: Mean solution time for varying problem size without warm start. The error bars are statistical and shows one standard deviation from the mean.

5.7 Scenario LR problems

Here we investigate the result quality of the solution from the LR applied on the scenario problems, we report the corresponding solution time and the LR convergence properties are presented. We compare results from using LR on scenario problems with the results obtained from the scenario LP problems in section 5.6

The weather scenario is 1962. The prices obtained from the LR solution of the weather scenario 1961 are used as starting prices.

5.7.1 Scenario problem length of 52 weeks with 1 timestep per week

The following results are obtained by solving the scenario problem by LR.

5.7.1.1 Convergence

In Figure 19 the evolution of the convergence gap per iteration is shown, Equation (9). The convergence gap is shown for the dual problem in black and the sum of subproblems in red. The black curve is not continuous due to the removal of negative values of the convergence gap ΔZ_c^i when using the logarithmic scale in the figure, see equation (9). The objective function value from the full LP is Z=1 291 961 908 206 k€. This value is very large due to the use of slack variables with high penalty costs in the problem constraints. If the constraints are not met, the slack variables ensure feasible solutions with added penalty costs. The slack variable coefficient is 10^5 k€ per unit. Slack variables are added to the constraints on bypass and discharge. These constraints are added for all timesteps on all hydro modules. The gap is approximately 46434 k€ (3.6E-6 %) at termination after 357 iterations, thus the true objective is underestimated by 46434 k€ for the defined stopping criterion.

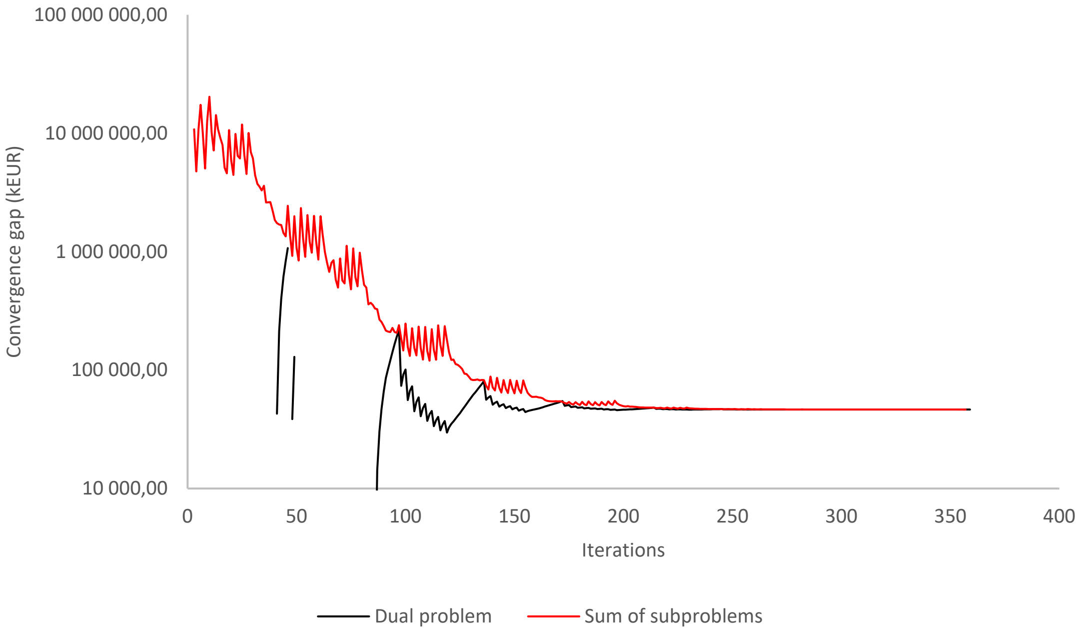

Figure 18: The convergence gap versus the iteration number of the LR iteration process.

In Figure 20 the evolution of the objective function gap versus the iteration number of the LR iteration process is shown. The objective function gap is defined in Equation (8) and is the difference between the two curves shown in Figure 19. The objective function gap is used to terminate the LR solution process. The stopping criterion is set to δ=1 k€ and by visual inspection of Figure 20 the iteration process terminates when the objective function gap is approximately 1 k€ at iteration 357.

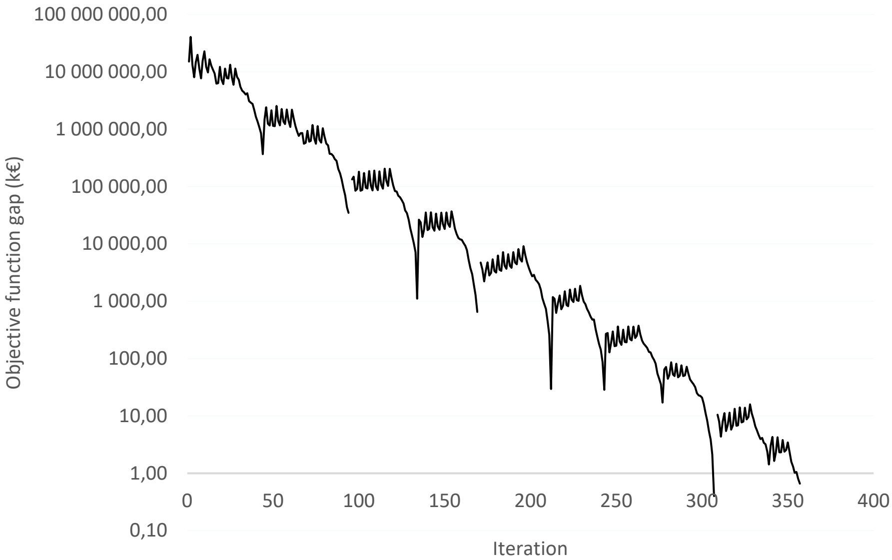

Figure 19: The objective function gap versus the iteration number for the LR solution process.

The evolution of the Lagrangian multiplier obtained from the dual problem and sent to the subproblem is shown in Figure 21. The Lagrangian multiplier represents the area price for the LR subproblem for the area Ostland in week 1. The area price for week 1 has stabilised after approximately 150 iterations.

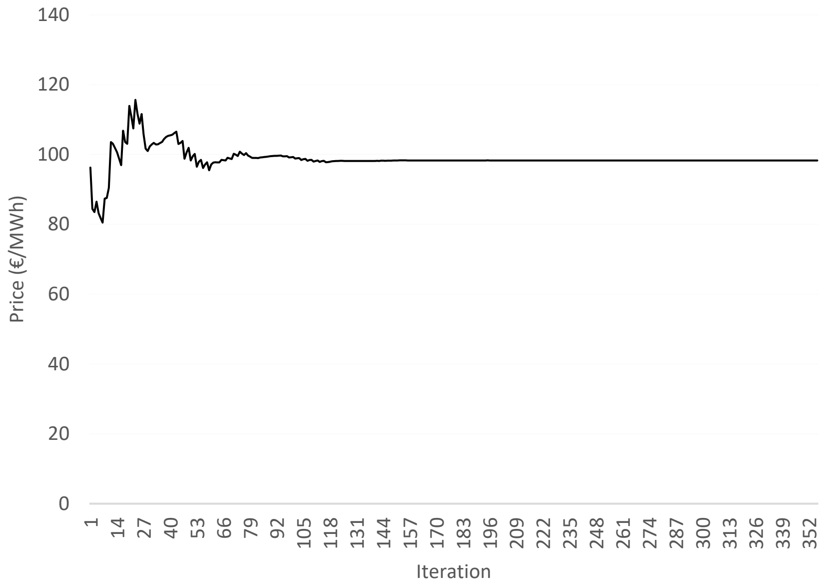

Figure 20: The evolution of the price per iteration for the area Ostland.

The evolution of the marginal water values and the reservoir volumes from the LR iteration process for three selected reservoirs in week 1 are shown in Figure 22 and Figure 23. For these reservoirs the marginal water values and the reservoir volumes appear stable after approximately 100 iterations.

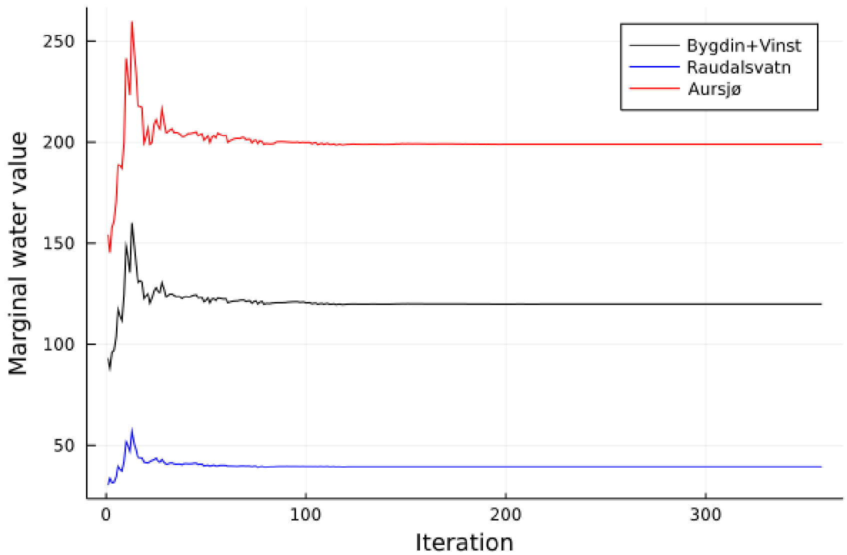

Figure 21: The evolution of the marginal water values per iteration for three selected reservoirs in week 1.

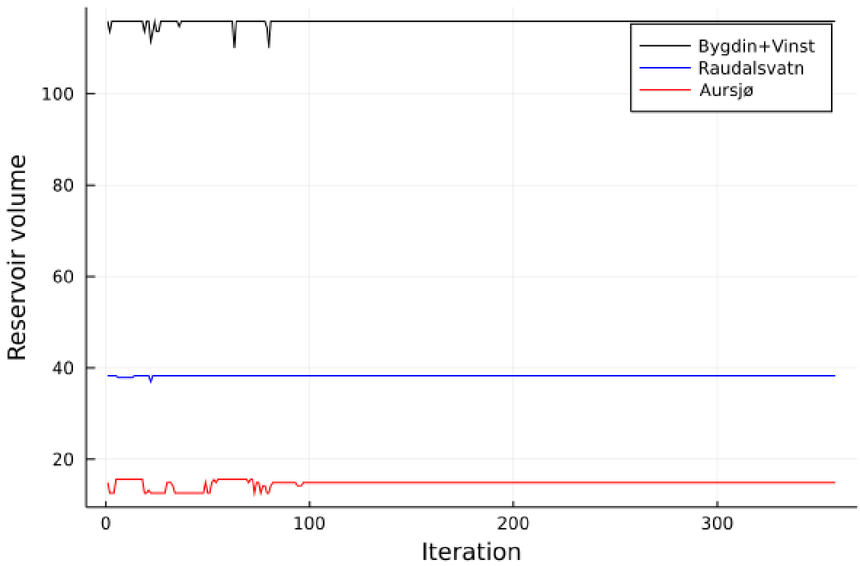

Figure 22: The evolution of the reservoir volumes per iteration for three selected reservoirs in week 1.

5.7.1.2 Solution time

The accumulated solution time of the problem solved by LR as a function of the iteration number is shown in Figure 24. The red curve shows the solution time for LR sub-problems run in series, the green curve shows the solution time for LR sub-problems run in parallel and the black dashed line shows the solution time of the dual problem. The solution time is significantly reduced when sub-problems are run in parallel. The solution time of the dual problem is the major contributor to the overall solution time.

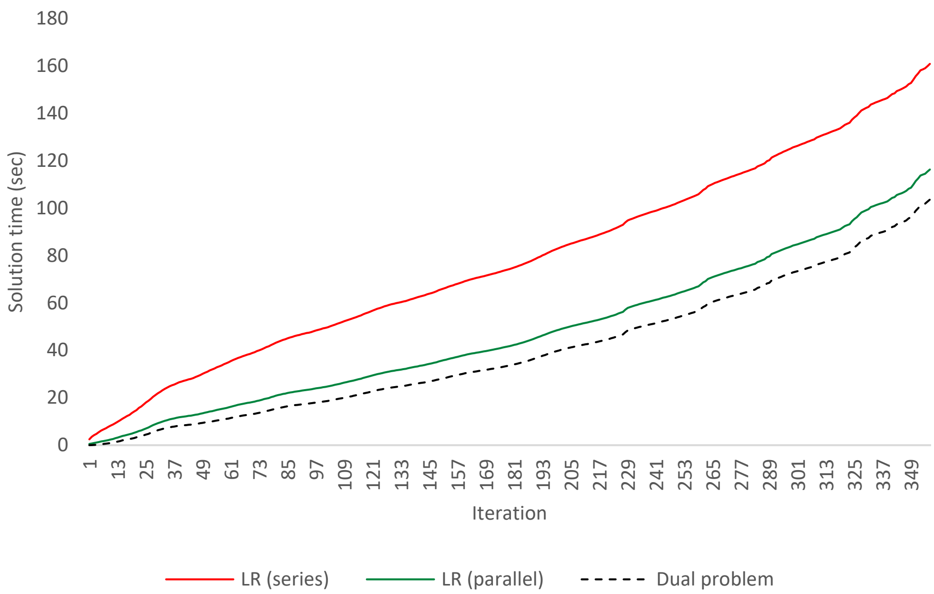

Figure 23: The accumulated solution time per iteration.

5.7.1.3 Sub-problem results

In this section we compare the reservoir volumes and marginal water values from the solution of the LR sub-problems from the last iteration before termination and compare them to the solution from the LP-problem.

Figure 25 shows a scatter plot of the marginal water values for the reservoirs in the Ostland area at the first timestep of the scenario problem. The black points are taken from the solution of the LP-problem and the red points are taken from the solution of the LR sub-problem formulated for the Ostland area. The area power price in Ostland is 78.24 €/MWh from the LP solution and 98.30 €/MWh from the LR solution. Therefore, the marginal water values obtained from the LR are consistently ~25 % higher (98.30/78.24).

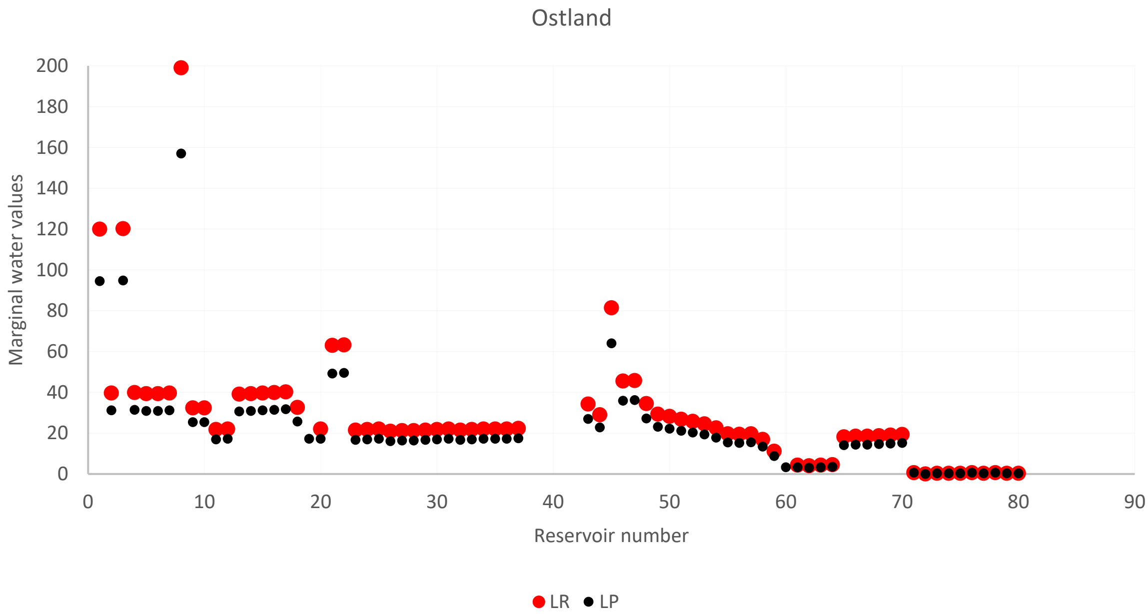

Figure 24: Scatter plot of the marginal water values for all reservoirs in the Ostland area for the first timestep of the scenario problem. The results are obtained from the LP solution and the solution of the LR-subproblem for area Ostland.

The reservoir volumes after the first timestep in the Ostland area is shown in the scatter plot of Figure 26. The red points show the reservoir volumes obtained from the solution of the LR subproblem for the area Ostland and the black points show the reservoir volumes from the solution of the LP-problem. Most of the red and black points overlap and have equal reservoir volume, but there are some exceptions, for example reservoirs 56 and 80 have large differences.

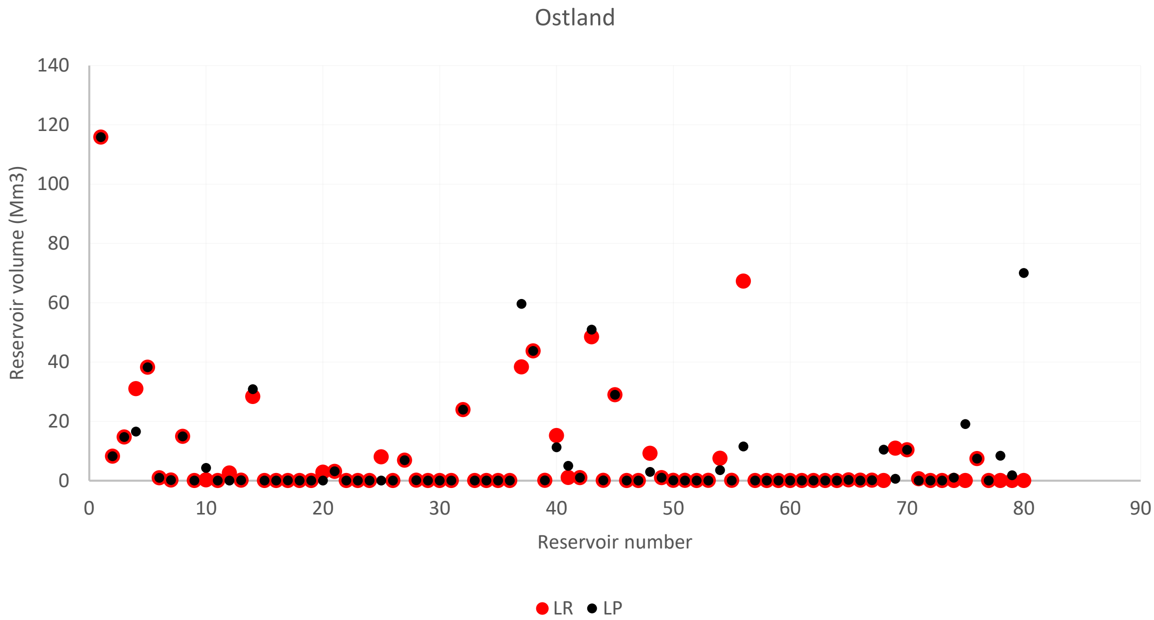

Figure 25: Reservoir volumes in the Ostland area after the first timestep of the scenario problem from the solution of the LP-problem and the LR subproblem for the Ostland area.

5.7.2 Scenario problem length of 52 weeks with 7 timesteps per week

The following results are obtained by solving the scenario problem by LR. The scenario problem contains 52 weeks with 7 intraweek time steps.

5.7.2.1 Convergence

The objective function value of the full LP-problem is 1 291 970 441 752 k€. This value is very large due to penalty costs as explained in section 5.7.1.1. The evolution of the convergence gap of the LR solution process is shown in Figure 27. The red curve shows the convergence gap for the sum of the sub-problems and the black curve shows the convergence gap of the dual problem. The convergence gap is 462 215 k€ (3.5E-5 %) at termination. The convergence gap seems to stabilise after approximately 200 iterations. The solution is still improving after 200 iterations, but much slower.

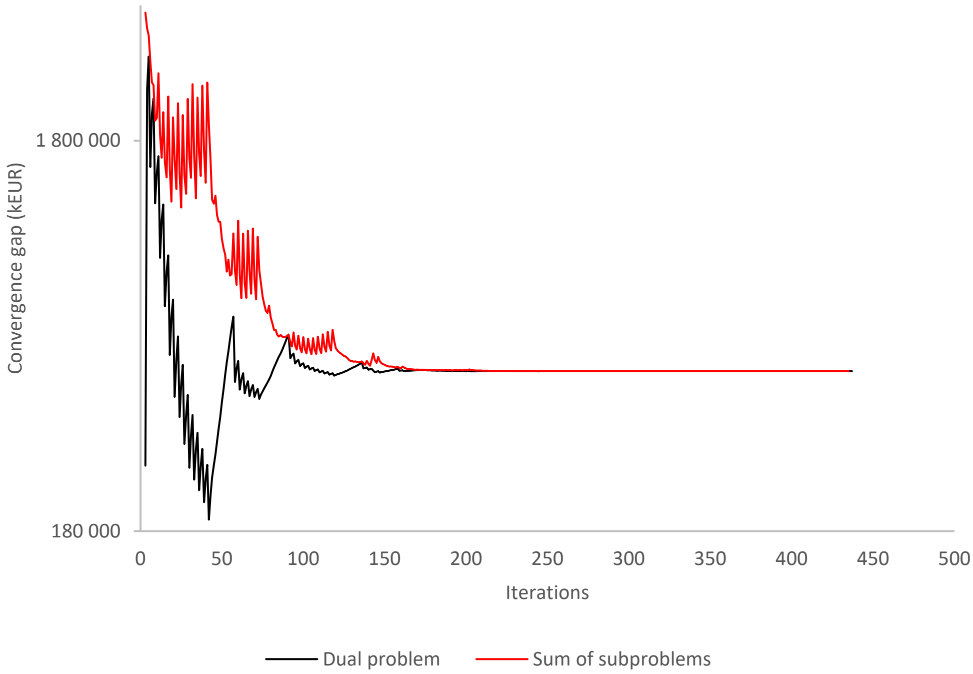

Figure 26: The evolution of the convergence gap of the LR solution process.

The evolution of the objective function gap is shown in Figure 28. The stopping criterion is reached in 435 iterations where the process is terminated.

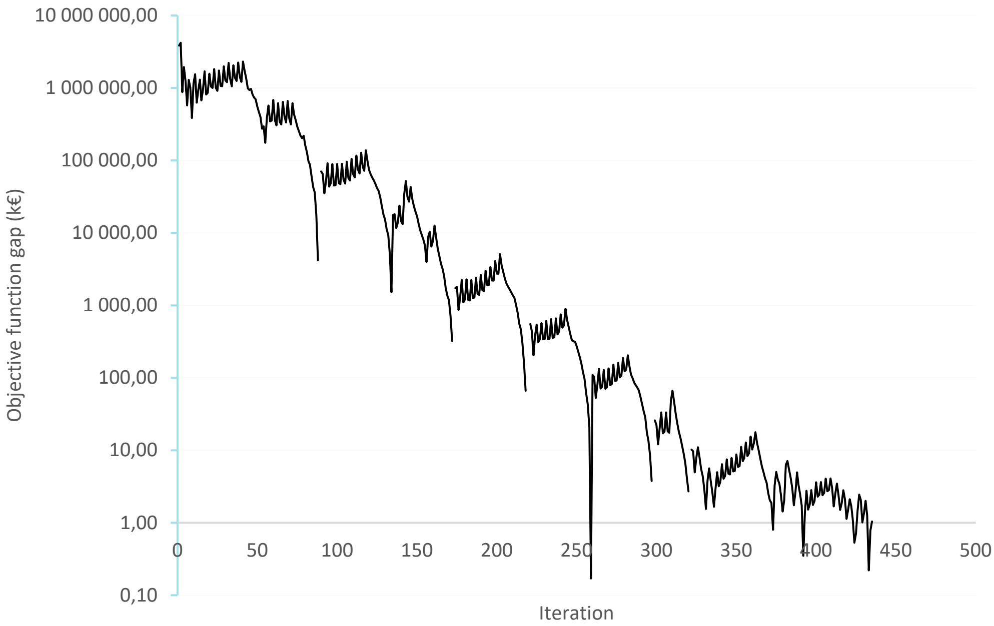

Figure 27: The evolution of the objective function gap of the LR solution process.

5.7.2.2 Solution time

The accumulated solution time of the LR solution process as a function of the iteration number is shown in Figure 29. The red (green) curve shows the accumulated solution time for LR sub-problems run in series (parallel), and the black dashed line shows the accumulated solution time of the dual problem. The accumulated dual problem solution time increases rapidly from around iterations 350 and on-wards. Stopping the LR iteration process after 200 iterations would reduce the simulation time to approximately 1000 second if LR is run in parallel and 2000 if LR is run in series. From Figure 28 the objective function gap is over 1000 k€ after 200 iterations which means allowing a stopping criterion δ>1000 k€.

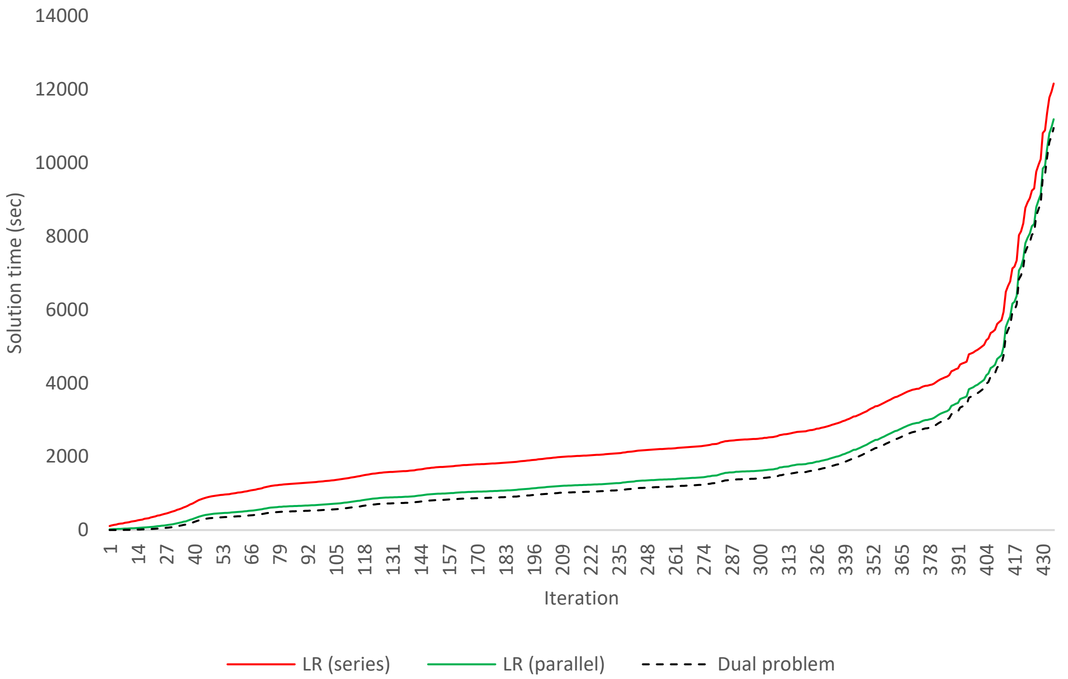

Figure 28: The accumulated solution time for the LR solution process versus the iteration number.

5.7.2.3 Results

In the FanSi model, Benders cuts are obtained from the solution of the scenario problems by extracting the dual values of the reservoir balance restrictions (marginal water values) and the corresponding reservoir volumes of the first time-step, in addition to the objective function value. The marginal water values with corresponding reservoir volumes and the objective function values from the LR and the LP solutions are compared to determine the quality of the solution of the scenario problem by LR.

In Figure 30 a scatter plot of the marginal water values for all reservoirs in price area Ostland is shown. The area price obtained from the LP solution is 187.95 €/MWh and the area price from the dual problem is 122.97 €/MWh in the LR solution. Therefore, the marginal water values from the LP-solution are higher than the values from the LR solution. The price ratio between the LP and LR solution 188/123 is approximately 1.5 and will define the ratio between the LP and LR marginal water values.

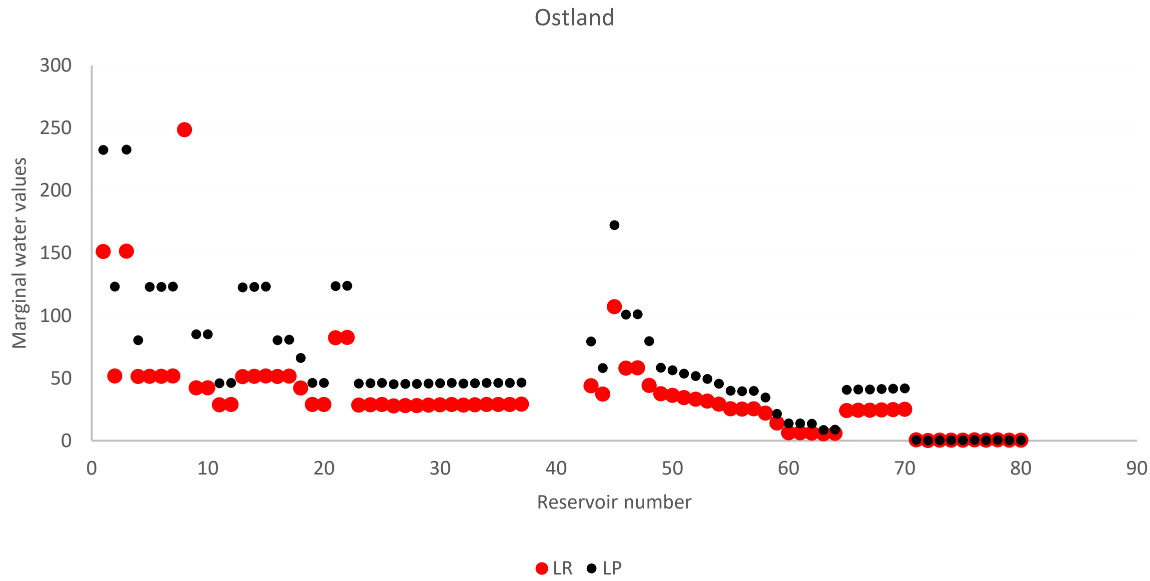

Figure 29: Scatter plot of the marginal water values for all reservoirs in price area Ostland.

The scatter plot in Figure 31 shows the reservoir volumes of all reservoirs in price area Ostland. Most of the red and black markers overlap, but there are some noteworthy exceptions. For example, reservoir 1 has a much lower volume in the LR solution than the LP solution. The volume has been halved over one timestep due to the lower water value in the LR.

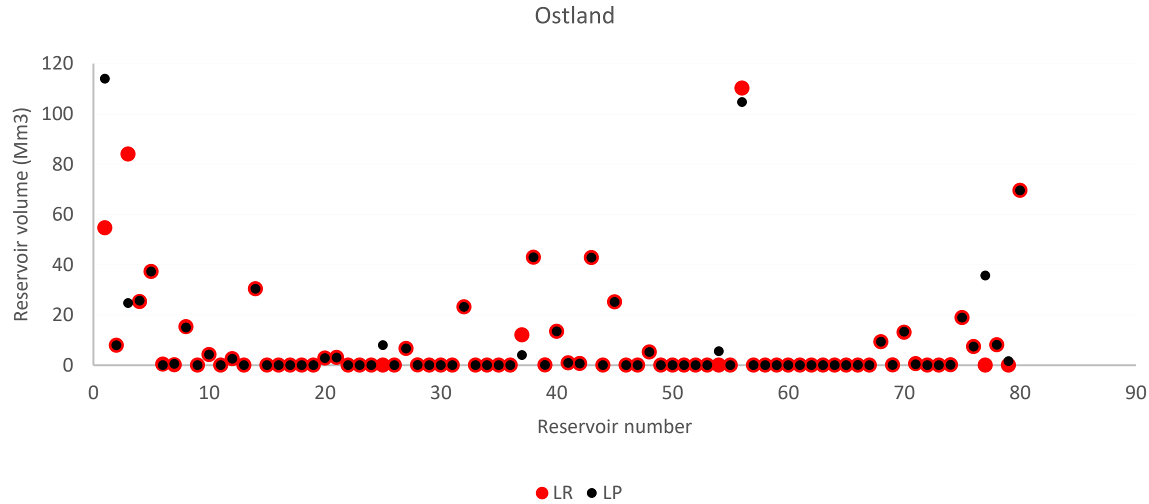

Figure 30: Scatter plot of the reservoir volumes for the area Ostland.

5.7.3 Comparing results from several scenarios

The results in this section are taken from the solution of several LR and LP runs with scenario lengths 26 weeks and 52 weeks where both have 7 intraweek time steps. The results are obtained from the LP and LR solutions for the scenarios. The number of scenarios for each configuration is listed in the Table 2. The scenarios are run sequentially, starting from the year 1961.

In Table 2 statistical estimates for the mean μ and standard deviation σ of the solution times of the scenario problems solved by LR for two scenario lengths. The subscripts to the statistics are *ser*, *par* and *dual*. They indicate whether the LR solution time is obtained from runs in series (*ser*) or parallel (*par*). Subscript *dual* indicate the solution time for the dual problem. The stopping criterion is δ=1 k€. The LR solution times are higher than the LP solution times.

Table 3: Statistical estimates of the solution time of the scenario problems solved by LR.

Length	Intraweek time steps	Number of scenarios	Solution time LR		
			$\mu_{ser}\ (\sigma_{ser})$	$\mu_{par}(\sigma_{par})$	$\mu_{dual}(\sigma_{dual})$
26	7	10	1161 (383)	927 (353)	811 (333)
52	7	7	13436 (4876)	12535 (4834)	12276 (4854)

5.7.3.1 Results from the scenario problems with length 52 weeks

There following results are from the solution of the scenario problems with length 52 weeks and 7 intraweek timesteps. The mean marginal water values for all reservoirs in the area Ostland is shown in the scatter plot of Figure 32. The red points show the results from the solution of the LR, and the black point are results from the solution of the LP-problem. The water values from the solution of the LR are lower than those from the LP solution.

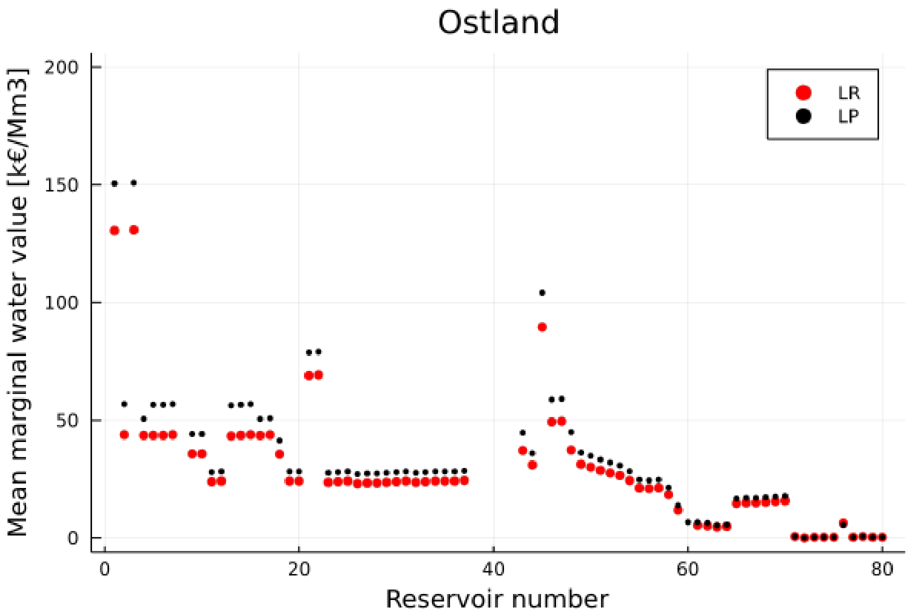

Figure 31: The mean marginal water values for all reservoirs of the Ostland area.

The mean reservoir volume for all reservoirs in the area Ostland is shown in Figure 33. The red points are from the solution of the LR and the black point are from the solution of the LP-problem. Most of the red and black markers overlap, but there are some noteworthy exceptions. One would expect that the underestimation of the marginal water value from the LR would lead to lower reservoir content, but for example, reservoir 3 and 58 has a higher volume in the LR solution than the LP solution. We did not conduct a deeper study of the LP and LR solutions, which would be necessary to explain these deviations.

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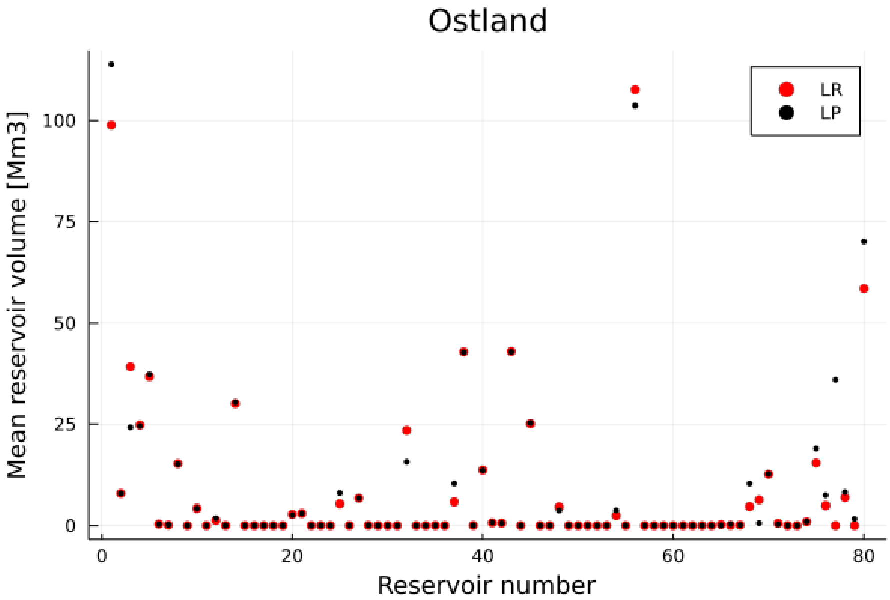

Figure 32: The mean reservoir volumes from the Ostland area, comparing the two solutions.

5.7.3.2 Results discussion

The statistical results from the solutions of the scenario problems by LR were presented. The mean solution time obtained from LR are higher than the LP solution time. It is possible to reduce the solution time significantly by allowing a higher value for the stopping criterion δ, but the task of finding a robust value for the stopping criterion which reduces the solution time has not been done. It is a difficult task with competing objectives, minimising the solution time versus the result quality. In addition, the scenarios can be parametrised in numerous ways which adds further complexity to the task.

The ratio between the mean water values, for the first time-step in area 1, from the LP and LR solutions are approximately 1.15, which means the mean area price differ by 15%. The mean reservoir volumes show differences for several reservoirs in area 1.

In the FanSi model the generation of Benders cuts are obtained from the solution of the scenario LP-problem. The differences shown for the mean water values, mean reservoir volumes and mean objective function values, will results in significantly different Benders cuts from the full LP and the solution by LR.

5.7.4 Varying the penalty parameters of the dual problem

As mentioned in Section 4.2.4, the dual problem is a quadratic optimisation problem, where the quadratic terms appear in the objective. These are scaled using a penalty parameter, as discussed below. Tuning the penalty parameter of the dual problem can change the convergence properties of the LR, and even lead the the algorithm to converge at a non-optimal solution. Here we report the evolution of the convergence gap per iteration, the area price and the penalty parameter per iteration for three configurations of the penalty parameters. The configurations are as shown in Table 3. The scenario problem lengths are 52 weeks with one time step per week. The scenario year is 1962.

Table 4: Three configurations for the penalty parameters which influence the problem solution from LR.

Nullstep	Decent step	Configuration
0.8	2.0	1
0.5	1.5	2
0.95	2.5	3

The dual problem maximises a quadratic objective function:

$$\max_{z,\lambda} z - \frac{1}{2\mu^k}\left\|\lambda - \lambda^k\right\|^2 \quad (10)$$

s.t. constraints from (6b).

Here k is the current iteration, μ^k is the penalty parameter. λ^k is the stable set of multipliers from previous iteration. The μ^k penalty parameter and the λ^k multipliers are updated in the LR solution procedure following the details described in [4]. For each iteration, if the multipliers λ proposed by the dual problem provide a sufficient increase on the sum of the objective functions of the subproblems of equation (7), the iteration step is classified as a descent step. If they do not increase the sum of the objective functions the iteration step is classified as a null step. The penalty parameters is updated per iteration by:

$$\mu^k = K * \mu^k \quad (11)$$

where K takes the values from Table 3 depending on the step type (descent- or null step).

5.7.4.1 Coefficient config 1

The evolution of the convergence gap of the LR solution process is shown in Figure 34. The red curve shows the convergence gap for the sum of the sub-problems and the black curve shows the convergence gap of the dual problem. The convergence gap is 46 434 k€ at termination. The convergence gap seems to stabilise after approximately 200 iterations. The solution is still improving after 200 iterations, but much slower.

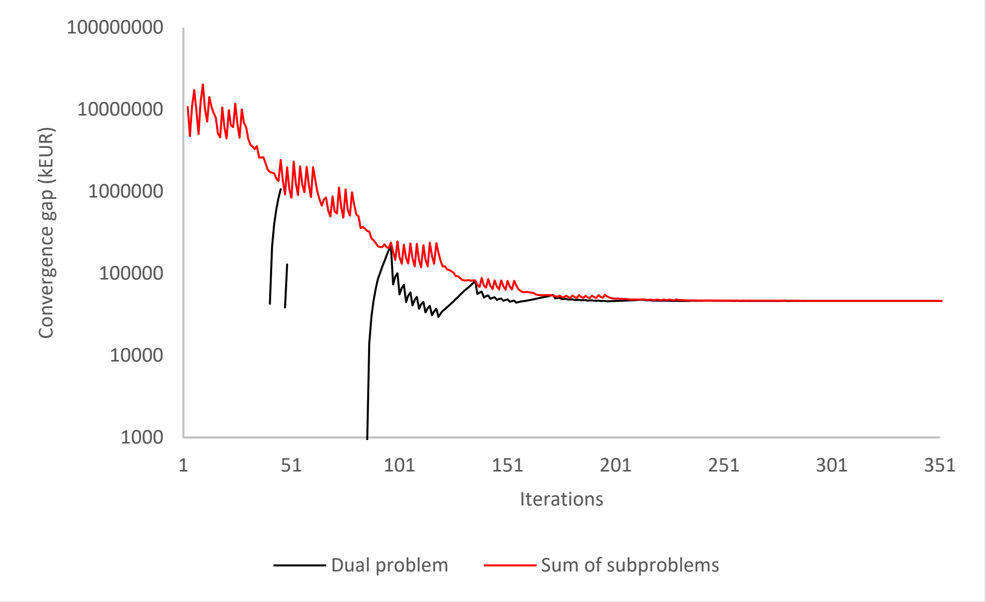

Figure 33: The evolution of the convergence gap of the LR solution process for parameter values from configuration 1.

In Figure 35 the evolution of the power price for the first time-step in the Ostland area is shown. The power price has stabilised at 98.30 €/MWh after approximately 141 iterations. The power price from the LP-solution is 78.24 €/MWh.

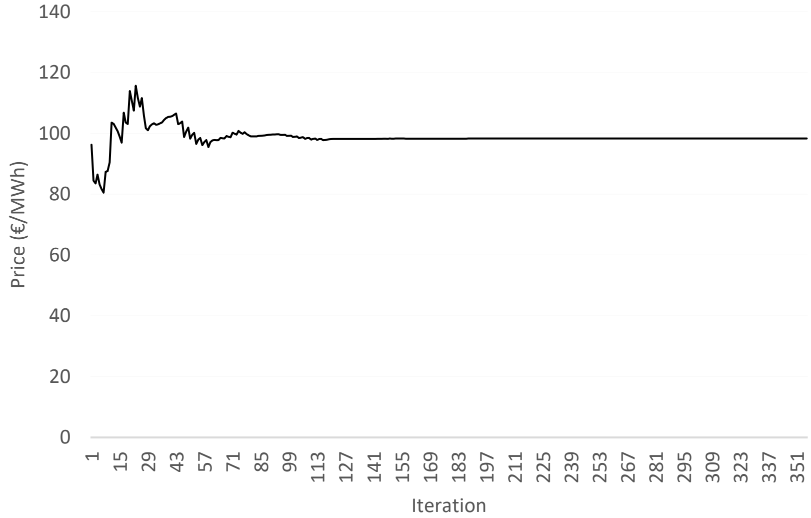

Figure 34: The evolution of the power price for the Ostland area for the first time-step.

The evolution of the penalty values $1/\mu^k$ is categorised by null steps and descent steps. The null steps increase the inverse penalty parameter, and the descent steps decrease the inverse penalty. In Figure 36 a series of null steps can be identified; the first series of null steps are between iteration 27-43 with increasing inverse penalty. The wiggly lines following from iteration 44-59 are a mix of null steps and descent steps. The penalty impact of the null steps is smaller than the impact from descent steps, as seen in the list of parameters in Table 3. The number of null steps is higher than the number descent steps in the iteration process. For example the number of steps required to span 5 orders of magnitude for the parameters in configuration 1 in Table 3 is 0.8^(number of null steps) = 10^5 and 2^(number of descent steps)=10^5. Calculating the number of consecutive steps to traverse 5 orders of magnitude for each configuration give, null steps = 52 and descent steps = 17. This should be kept in mind when performing parameter tuning.

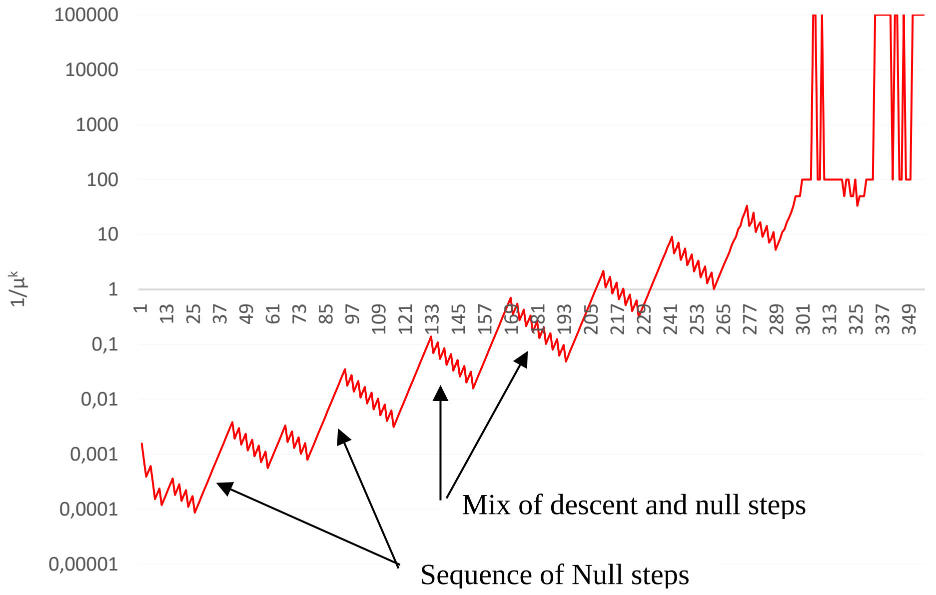

Figure 35: The evolution of the inverse penalty parameter per iteration.

A figure combining the evolution of the objective function gap and the evolution of the inverse penalty per iteration is shown in Figure 37. This illustrates the effect of the penalty on the objective function gap.

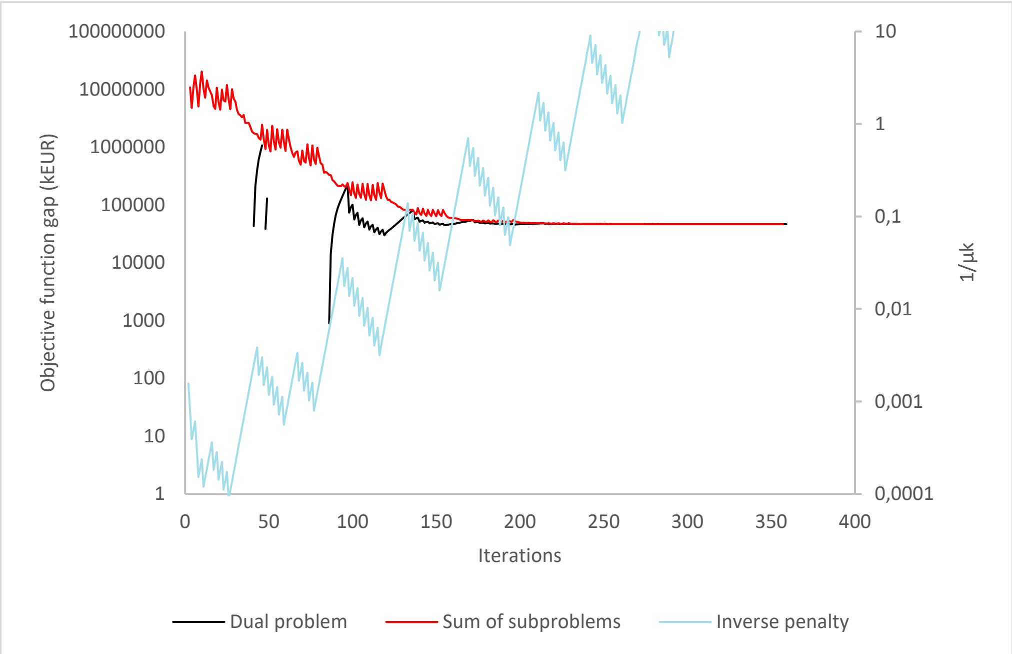

Figure 36: The evolution of the convergence gap and the inverse penalty of the LR solution process for parameter values from configuration 1.

5.7.4.2 Coefficient config 2

The evolution of the convergence gap of the LR solution process for coefficient configuration 2 is shown in Figure 38. The red curve shows the convergence gap for the sum of the sub-problems and the black curve shows the convergence gap of the dual problem. The convergence gap is 1 936 431 k€ at termination. The convergence gap seems to stabilise after approximately 20 iterations. The solution is still improving after 20 iterations, but much slower.

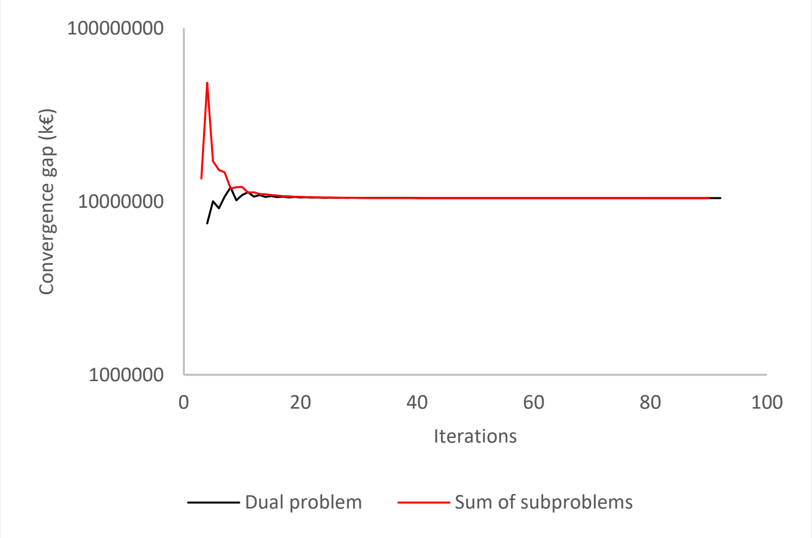

Figure 37: The evolution of the convergence gap of the LR solution process for parameter values from configuration 2.

In Figure 39 the evolution of the power price for the first time-step in the Ostland area is shown. The power price has stabilised at approximately 40 €/MWh after ~20 iterations. The power price from the LP-solution is 78.24 €/MWh.

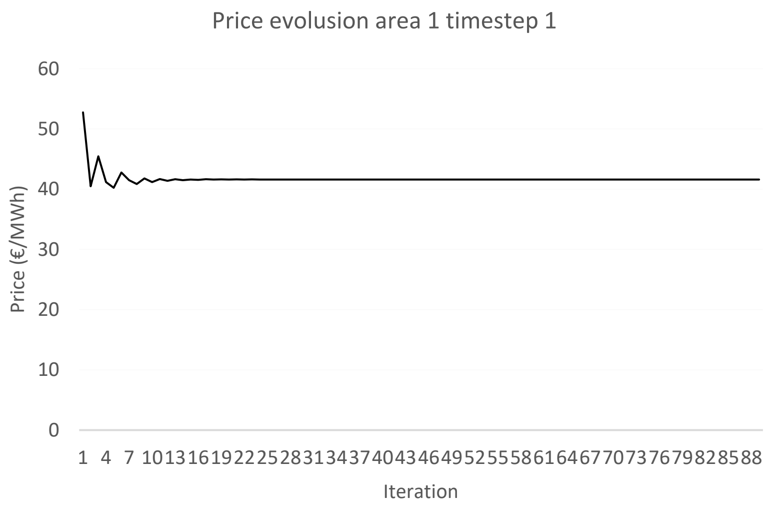

Figure 38: The evolution of the power price for the Ostland area for the first time-step.

The evolution of the inverse penalty values $1/\mu^k$ is for the LR iteration process is shown in Figure 40. The null steps increase the inverse penalty values, and the descent steps decrease the inverse penalty. The penalty values are updated as shown in Table 3 for configuration 2. The evolution of the inverse penalty values for configuration 2 is increasing faster than for configuration 1. This allows the LR iteration process to reach the termination point quicker, but with a higher convergence gap and poorer result quality.

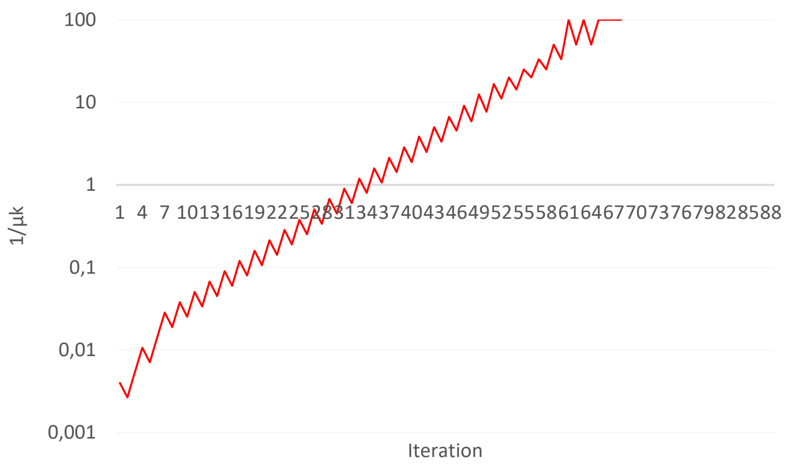

Figure 39: The evolution of the inverse penalty parameter per iteration.

5.7.4.3 Coefficient config 3

The evolution of the convergence gap of the LR solution process for coefficient configuration 3 is shown in Figure 41. The red curve shows the convergence gap for the sum of the sub-problems and the black curve shows the convergence gap of the dual problem. The convergence gap is 106942 k€ at the stopping point.

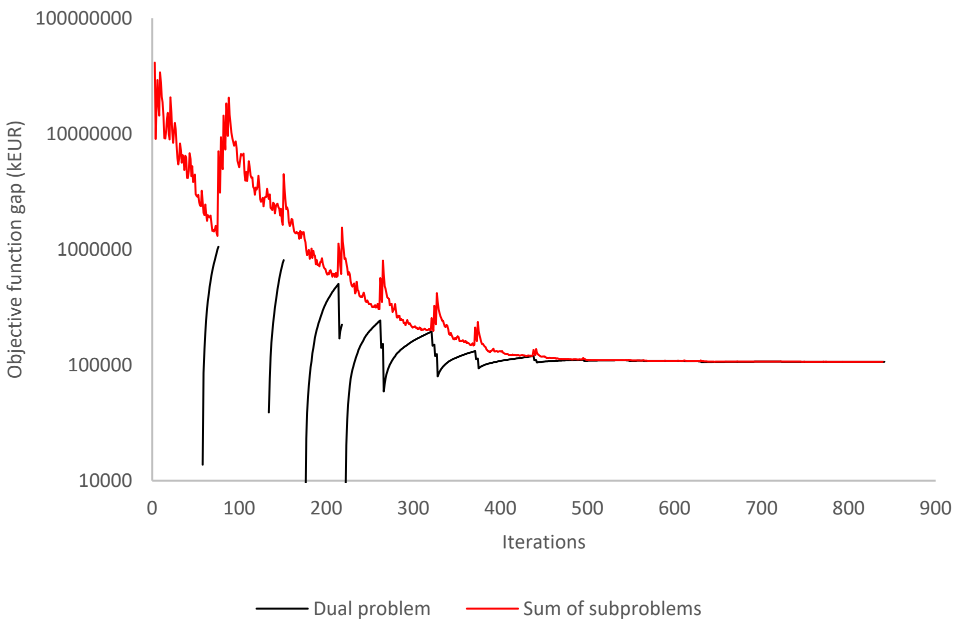

Figure 40: The evolution of the convergence gap of the LR solution process for parameter values from configuration 3.

In Figure 42 the evolution of the power price for the first time-step in the Ostland area is shown. The power price has stabilised at approximately 80 €/MWh after ~200 iterations. The power price from the LP-solution is 78.24 €/MWh.

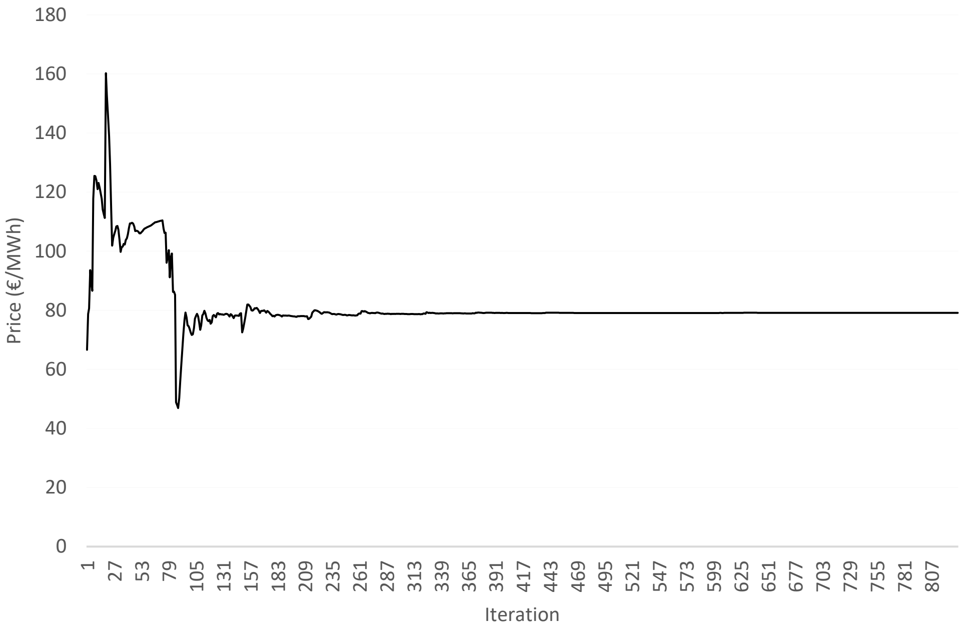

Figure 41: The evolution of the power price for the Ostland area for the first time-step.

The evolution of the inverse penalty values $1/\mu^k$ is for the LR iteration process is shown in Figure 43. The null steps increase the inverse penalty values, and the descent steps decrease the inverse penalty. The penalty values are updated as shown in Table 3 for configuration 3. The evolution of the inverse penalty values for configuration 3 is increasing slower than for configuration 1. This increases the time before the iteration process can be terminated.

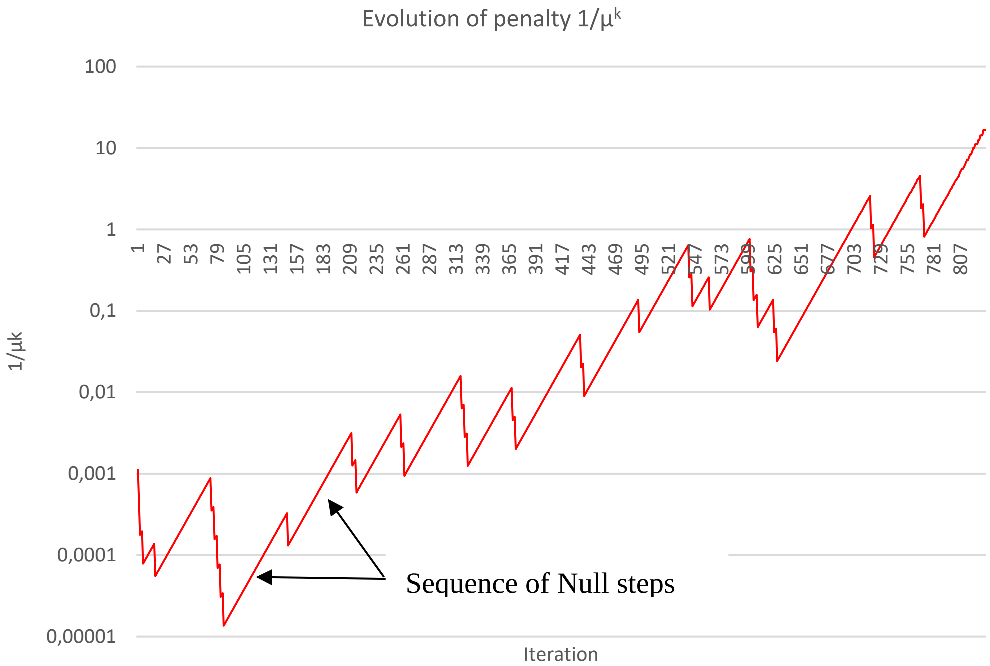

Figure 42: The evolution of the inverse penalty parameter per iteration.

5.7.4.4 Comparing prices

The power prices per week for areas 1, 2, 3, 4, 7, 8, 9, 10 and 11 are shown in Figure 44 for configurations 1 and 3 and compared to the full LP-solution. The prices from the problem solved by LR (red and black curves) are not equal to the full LP-solution. The power prices obtained from configuration 3 are closer to the power prices from the LP-solution than the ones from configuration 1.

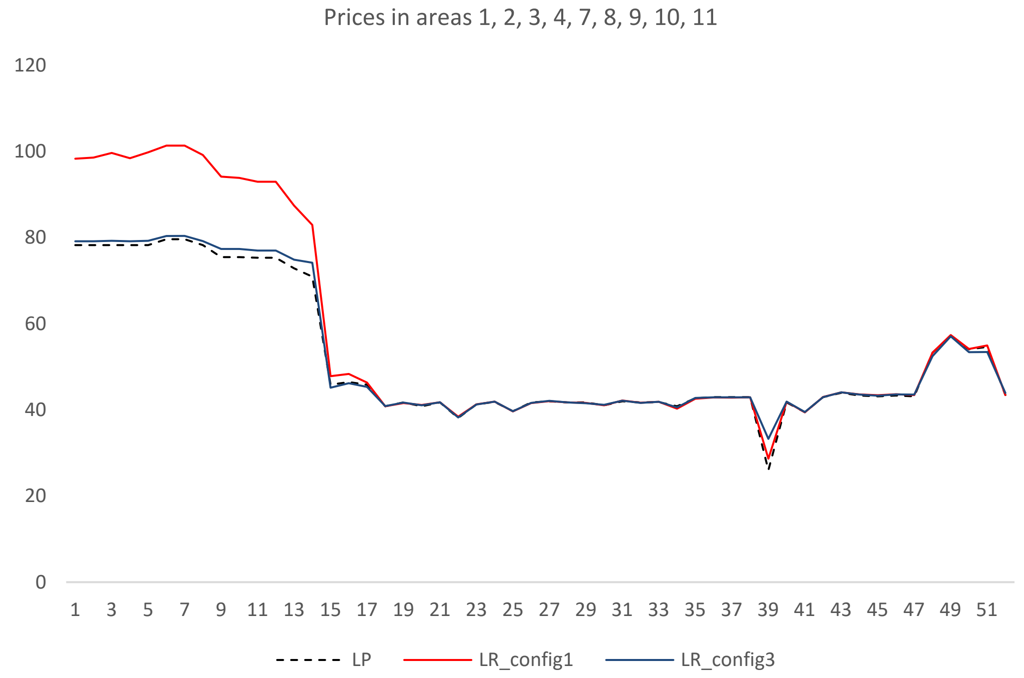

Figure 43: Comparing the power prices for areas 1, 2, 3, 4, 7, 8, 9, 10 and 11 for configurations 1 and 3 to the full LP-solution.

The power prices per week for areas areas 12, 13, 14, 16, 17, 18, 38, 39, 40, 41 and 57 are shown in Figure 45 for configurations 1 and 3 and compared to the full LP-solution. The prices from the problem solved by LR (red and black curves) are not equal to the full LP-solution.

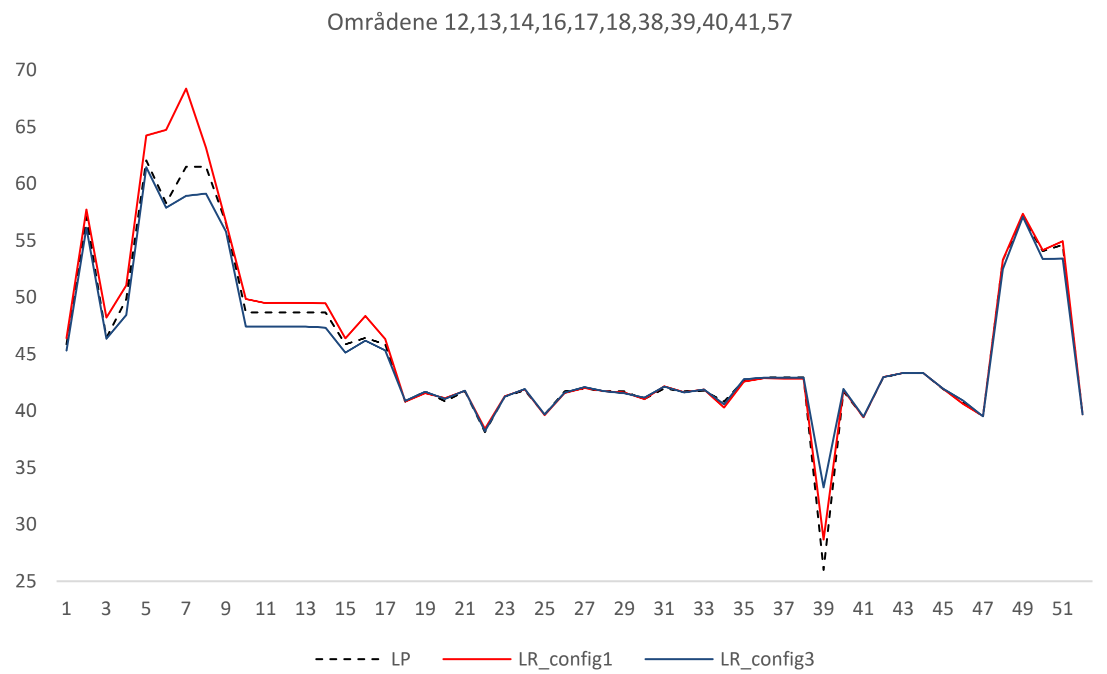

Figure 44: Comparing the power prices for areas 12, 13, 14, 16, 17, 18, 38, 39, 40, 41 and 57 for configurations 1 and 3 to the full LP-solution.

5.7.4.5 Results discussion

The results from varying the penalty parameters of the dual problem of the LR according to the configurations listed in Table 3 has been shown. These parameter configurations resulted in different convergence properties and result quality. The parameters values can be studied and tuned to improve the convergence properties and results quality. However, we will not perform such a study in this work, due to significant variations of the scenario problems. For example, the scenario problems vary between datasets due to the level of details for hydropower and because the model users are allowed to define time resolution and scenario length.

We find that the choice of penalty parameter significantly impacts the results of the LR algorithm. Further work could go deeper into understanding the relationship between high objective function value, numerical stability and appropriate penalty parameters for the particular type of problems studied here.

6 Discussion and conclusions

Results from spatial decomposition by Lagrangian Relaxation on the economic dispatch problem as formulated within the Fansi model was presented. The LP problems are solved with the objective of minimising system costs of the Northern European hydro-thermal power system. The solutions of spatial decomposition by LR and the full LP-problem was compared. Specifically, we compared the result quality and solution time. For result quality we focused on convergence gap of the total system costs, the power prices, the marginal water values, and the reservoir content.

First, we studied the weekly optimisation problems. For these problems we found that the primal simplex method was faster than the dual simplex method for solving the hydropower sub-problems within the LR algorithm. Using near optimal initial values to warm start the LR solution process reduced the solution time. Solving the weekly optimisation problem with parallelised solution of the LR sub-problems reduced solution time by approximately 50% compared to the solution time of the full LP. The LR algorithm was warm started by initial values, and we used a stopping criterion of 1 k€. The stopping criterion yielded a convergence gap at termination and significant power price deviations compared to the solution of the full LP-problem. Due to the relaxation of the power balance when applying LR, the resulting power balances are not necessarily respected, and an additional primal recovery phase is required to obtain valid primary results, such as dispatch decisions.

The multiweek scenario problems were studied and statistical results from the solutions of the scenario problems by LR were presented. Similarly to the weekly master problem, the scenario problems are deterministic LP problems, but their structure differs from the master problem structure. The mean solution time obtained from LR are higher than the LP solution time. It is possible to reduce the solution time significantly by allowing a higher value for the stopping criterion δ, but the task of finding a robust value for the stopping criterion which reduced the solution time has not been done. It is a difficult task with competing objectives, minimising the solution time versus the result quality. In addition, the scenarios can be parametrised in numerous ways which adds further complexity to the task.

When comparing detailed results obtained from the solution from LR and LP-problem significant differences were found. Mean water value differences of 15 % and significant mean reservoir differences for several reservoirs (first time-step for area 1). The consequence for FanSi is significantly different Benders cuts in master problem.

Finally, we presented results from three penalty parameter configurations of the dual problem of the LR. These parameter configurations resulted in different convergence properties and result quality.

A conclusion from our tests of using LR both on the master and scenario problems formulated by Fansi is that the obtained result quality of the solution found by LR is not high enough. Thus, based on the presented results, we will not further pursue the use of LR decomposition within the Rakett project. However, we recognize that improvements are likely, and list below a set of possible avenues for further studies:

- The solution of the dual problem (quadratic LP) was found to be a bottleneck with respect to computational time. Techniques such as cut relaxation and linearization of the quadratic terms in the objective functions could be further explored.
- We chose the Bundle method to update the Lagrangian multipliers. This method is known to be accurate, but computationally slow. Faster methods, such as the subgradient method, could be tested.
- The hydropower subproblems were divided based on price areas, with large variations in the number of hydropower reservoirs within each area. A finer division to ensure a more balanced sizes of the hydropower subproblems would likely lead to better performance when applying parallel processing.
- The details related to updating the Bundle method's penalty parameter were not fully explored. This is a complex area where problem-specific knowledge is needed. For example, the relationship between the numerical scaling of the optimisation problem, the objective value and the size of the penalty could be further explored. Moreover, the heuristic rules for choosing between null and descent-steps could be revisited.

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