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Report

Primod – A fundamental short-term model for power system analyses and multi-market price forecasting

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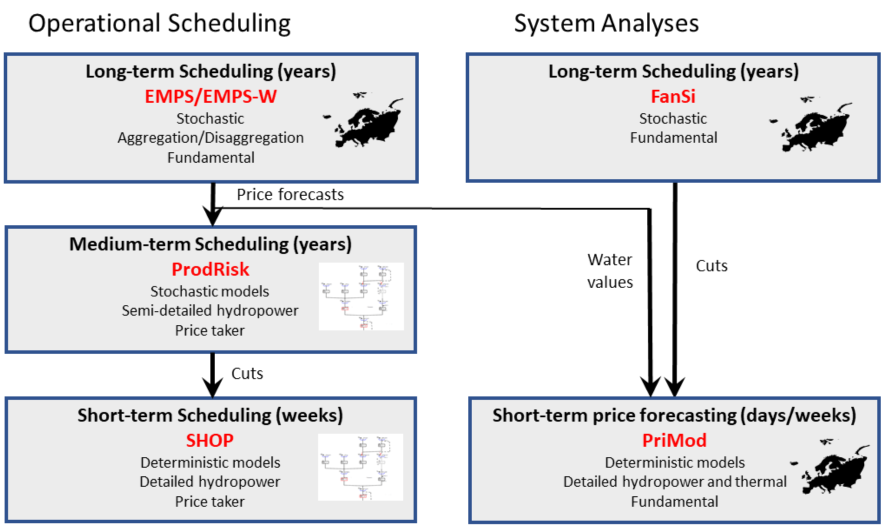

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ABSTRACT

This report documents a short-term fundamental market model named Primod which was developed within the research project "Pricing Balancing Services in the Future Nordic Power Market" (PRIBAS). The Primod model forms, together with the existing EMPS and FanSi long-term models, a toolchain suited for comprehensive studies of costs and marginal costs of secure provision of electricity within Nordic power system.

The report explains the context within Primod can be applied and elaborates on the mathematical formulation. Finally, a case study describing how the model can be used to find costs and marginal costs of balancing energy is presented.

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APPENDICES

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1 Introduction

Primod is a short-term hydrothermal scheduling model developed within the research project "Pricing Balancing Services in the Future Nordic Power Market" (PRIBAS).

Primod is based on linear programming (LP) and mixed integer linear programming (MIP) for solving the unit commitment and least cost dispatch problem for a short-time horizon, provided exogenously given long-term valuation of water in hydropower storages. It has been developed in a high-level language (Python/Pyomo) to facilitate rapid prototyping and experimentation with new functionalities. The model comprises co-optimization of electricity and reserve capacity and a separate module for studying the costs and marginal costs of handling system imbalances.

In this report we describe the rationale for this type of model, discuss its natural place in the scheduling toolchain, and elaborate on the detailed representation of the physical system and the underlying assumptions.

1.1 The PRIBAS Project

The PRIBAS project was conducted 2017-2021 with the primary goal to develop and verify a *model concept* able to compute marginal costs for all physical electricity products in the Nordic power market. The model concept should allow detailed modeling of different types of reserve capacity as well as balancing energy. The concept should also be suited to assess how the different market products and corresponding market clearing sequences impact system operation and costs.

Early on we acknowledged that the ambitious goals set for the model concept, does not allow for one single model, but rather a model toolchain. The Nordic system comprise a large amount of hydropower reservoirs with significant storage capacity. Constructing a single model both covering the long-term horizon needed to a) compute strategies for hydropower operation, and b) realistically treat the short-term aspects needed when computing the cost of procuring reserves and activating balancing energy is a computationally prohibitive task. Consequently, we ended up with a toolchain discussed in Section 1.3.

1.2 Motivation

The Nordic power market is in transition, both in terms of technologies used for power generation and market structures. Binding targets exist for renewable power generation, as well as decisions to decommission nuclear generation capacity. Thus, the overall share of intermittent generation will continue to grow, and consequently, the need for flexibility and controllability both in production and demand will increase.

In this context there is a need for long-term forecasts of prices[1] and operational[2] costs associated with all electricity products, including energy and different types of reserve capacity and balancing energy,

[1] In this report the term "prices" refers to the marginal cost of delivering the product.
[2] By the term "operational cost" we refer to dispatch and commitment costs seen by the model.

in order to make robust and correct investment decisions, e.g., related to building new cables to the European continent and upgrading and expanding the hydropower system. Fundamental market models have and will play an important role in providing consistent forecasts of power system costs and prices. Currently used fundamental market models for hydro-thermal systems typically only concern the product *energy*, assume that all uncertainty is revealed in weekly steps, and that all functional relationships are linear. These assumptions will be significantly challenged in the future European power market. The computation of realistic cost for balancing services, which also include the products *reserve capacity* and *balancing energy*, requires a much higher degree of details, e.g., in the representation of uncertainty and description of the physical system.

1.3 The Scheduling Hierarchy

In hydro-thermal power systems with large reservoir capacities, the planning for optimal utilization of the resources/generation scheduling/ the scheduling process is divided into long-term scheduling, seasonal scheduling, and short-term scheduling, with a suitable coupling between the levels. This is done to be able to find the optimal usage of water in the long-term taking into account the stochastic nature of inflow and other weather-related uncertainties, and to use this strategy when making detailed and realistic scheduling plans for the short-term horizon.

The left side of Figure 1 illustrate SINTEF's traditional scheduling toolchain used by many players in the Nordic power market [1]. Due to the ongoing changes in the Nordic and European power system described in Section 1.2, new model tools are necessary to capture the effects of more short-term variations in both demand and production. A more detailed long-term model (The Scenario Fan Simulator - FanSi) was developed at SINTEF 2013-2016 through the project "Stochastic optimisation model for Scandinavia with individual water values and grid restrictions" (SOVN). This model optimizes each individual reservoir without using the aggregation/disaggregation principle of the EMPS model [2]. This allows the model to find an optimal strategy for each individual reservoir represented by Benders cuts for each week (without "refining" the solution through a mid-term model). The project "Models for Aggregation and Disaggregation" (MAD) used a new method for aggregation/disaggregation to achieve better operation and utilization of the Nordic power system considering the new challenges addressed above [3]. This also allows the model (EMPS-W) to calculate water values for each individual reservoir. Both FanSi and EMPS-W can provide end-of-horizon valuation of hydropower storages to model Primod model, as illustrated in Figure 1. This figure illustrates how the Primod model fits into the existing scheduling toolchain developed and maintained by SINTEF Energy Research. As indicated in the Figure 1, Primod can be run together with EMPS-W or FanSi for the purpose of system analyses (right side in figure). The toolchain for operational scheduling (left side in figure) is further elaborated in [1].

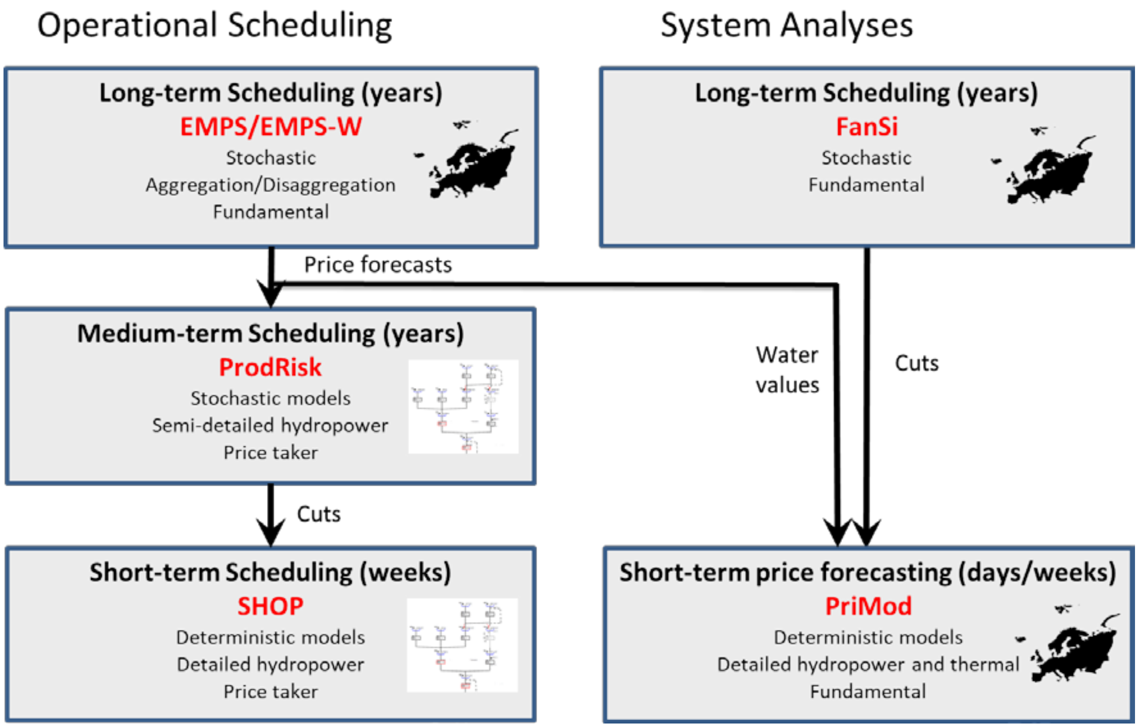

Figure 1 The scheduling toolchain developed and maintained by SINTEF Energy Research.

1.4 Market Context

The Primod model is primarily intended for analyses of the hydro-dominated Nordic power market. A thorough description of this market can be found in other SINTEF reports, such as [1], [4], [5]. Below we briefly describe the market context within which Primod was born.

After the deregulation of the power market in Norway in 1991, the power system has been market based. Today, energy is traded mostly through the day-ahead market, but with increasing volumes traded closer to the hour of delivery in the intraday market. This trend is a result of an increased share of the power production originating from unpredictable energy sources like wind and solar. Energy is also traded through bilateral contracts between power producers and consumers. The balance between supply and demand is largely secured in the day-ahead market.

Since it is not possible to perfectly predict the weather and the system state for the next day, and since the cleared day-ahead volumes may not be feasible when considering physical operation, there will be a need to adjust the schedules. Consequently, balancing services are needed to secure the balance between supply and demand at real-time operation. More specifically, balancing services are needed to handle:

1. **Outages** of power system components (power plants, transmission facilities, etc.). Such events are hard to predict and may cause severe system disturbances.
2. **Weather dependent exogenous factors** (impacting intermittent generation and temperature-dependent demand). Although forecasting methods continue to improve, weather forecast errors will always exist.

3. **Demand forecast errors**. As for point 2) forecasting errors for electricity demand will always exist, leaving a need for balancing power. Parts of the demand is temperature-dependent and thus points 2 and 3 are linked.
4. **Structural imbalances** caused by the market design. These are imbalances that are due to the discrete time resolution of day-ahead and intraday markets. Obligations may change in large steps in between consecutive hours, whereas load and intermittent production changes are continuous. Both the granularity (hourly time resolution) and time difference between market closure and real-time operation leaves behind a need for balancing services.
5. **Congestions** in the power grid that are not explicitly seen by the day-ahead and intraday markets. These are treated by use of manually regulated reserves.

In the Nordic system, the transmission system operators (TSOs) are responsible for matching supply and demand of electricity in real time and for secure system operation subject to the possible occurrence of the above-mentioned events. Thus, the TSOs need to procure reserve capacity to ensure the availability of balancing energy. The reserve procurement is mostly market-based (pay-as-cleared), and these markets are mostly cleared prior to the day-ahead market clearing.

The system balancing has traditionally been a national task, but as the Nordic power system is tighter integrated to the North-European power system through HVDC cables, the European power markets are also becoming more integrated. Many studies show that there is a large cost saving potential from exchanging reserve and balancing resources between countries, and between the Nordic power system and Northern-Europe [6], [7]. Studies also show that having a simultaneous clearing of the spot market and reserve capacity markets is optimal [7].

1.5 Computational Environment

Primod is programmed in Python using the open-source and Python-based optimization modeling language Pyomo [8]. The optimization problems can be solved by using any third-party optimization solver that Pyomo has a interface to. In our research we have used CPLEX.

2 Integrating Primod in a Toolchain

Traditionally, cost and price forecasts in the Nordic market has been conducted using long-term hydrothermal scheduling models. Long-term models cover a long scheduling horizon with multi-dimensional stochastic processes and thus need to compromise on the time resolution and the level of technical detail represented. With a tool like Primod one can take a closer look at selected (or representative) days or weeks to study the impact of such details. However, with a much shorter time horizon, Primod rely on proper valuation of stored water in hydropower reservoirs from a longer-term model. In this Section we describe how Primod's part in the model toolchain.

2.1 Basic Data Description

Primod runs on similar data set as the long-term programs EMPS-W and FanSi, which we refer to as an *EMPS dataset* in the following. Most of the input data are on the HDF5 format provided with version 10 of the EMPS model. We briefly comment on the major classes and use of data below.

Transmission system
The transmission system is described as a transportation model, with a defined topology and maximum transfer and possibly a loss factor for each link connecting two price areas (or 'bidding zones'). The physical properties of the grid (resistance, reactance, etc.) are not represented. Primod allows for constraining flow changes between time steps (ramping) and the possibility to procure capacity for the exchange of reserves.

Price boundaries
There is a defined price floor and price roof. We do not allow negative power prices, and curtailment of demand should be the last resort.

Time series of weather-related data
Weather related data (inflow to hydropower modules, wind power, solar power, temperatures) are provided as time series.

Time series of exogenous market prices
The system boundary, defining which parts of the European power system to include, is flexible. At the boundary, price series to exogenous markets could be provided.

Demand
Demand is described as price-inflexible (with a curtailment cost) and price-flexible.

Hydropower
The hydropower is described by modules, comprising one station and one reservoir. The hydropower station will often comprise many hydropower units, but these are not individually modelled in the EMPS dataset. The station's production function is described by a power-to-discharge curve (PQ-curve), as illustrated in Figure 2. These curves are adjusted for the actual head before used.

In the optimization performed in the EMPS and FanSi models, the concave approximation of the PQ-curve, shown by pink dotted lines to the left in Figure 2, represents the production function. A weakness with this formulation is the possibility for low discharges at best efficiency. As elaborated in Section 4.2, Primod improves this modelling by introducing a binary commitment variable per hydropower station with an associated minimum power (Pmin) and discharge (Qmin), as illustrated to the right in Figure 2.

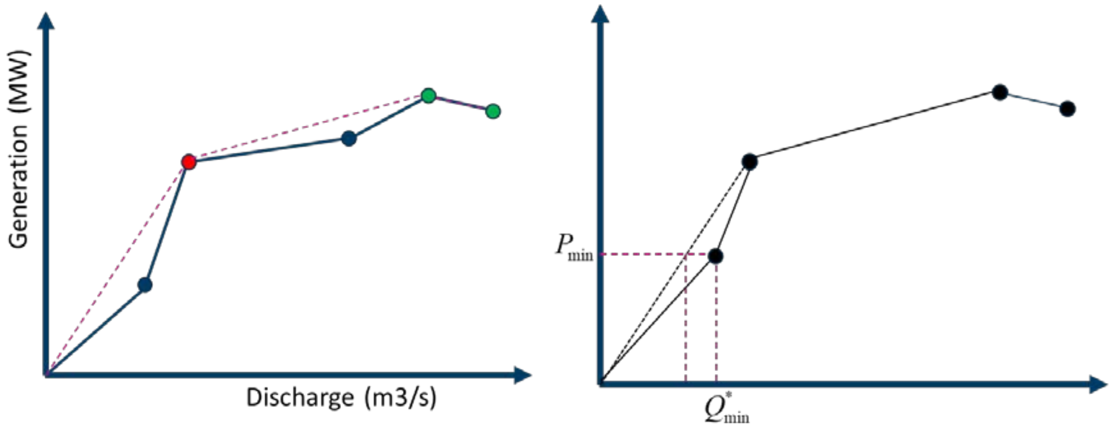

Figure 2 Illustration of production function. Concave approximation used in optimization in EMPS and Fansi (left), and adjustment made in Primod (right).

Thermal generators
Thermal generators are represented by their marginal cost and maximum capacity. In the EMPS model it is possible to apply linearized start-up costs according to the modelling in [9]. Primod improves this modelling to incorporate binary unit commitment, ramping constraints and minimum up- and down-times, as described in Section 4.3.

Decision stages
The long-term models apply decision stages of one week, and treats all decisions stages within the scheduling period in sequence. At the beginning of each week all information for that week is perfectly known and the optimal system operation can be found. The decisions are typically found by formulating and solving an LP problem for the entire week. The future beyond that week is still uncertain (due to uncertainty in weather), and the water left in the reservoirs at the end of the week is valuated with respect to this uncertain future.

2.2 Interpolation in Cuts

Primod was initially developed to optimize the unit commitment and dispatch for one week, assuming a pre-computed end-of-horizon-valuation of stored water provided by water values or Benders cuts (or just *cuts*). The resulting optimization problems soon turned out to be too large to solve within reasonable computation times.

The splitting of a deterministic week problem into several smaller subproblems can be arranged, e.g., by Benders decomposition, which requires iterations. Some, or possibly all, of the computational benefit from the splitting may be lost in the added time introduced by the iterations. Moreover, the complexity of the code increases when introducing a decomposition scheme.

In the development of Primod we followed a second approach, namely the interpolation in cost functions to facilitate decomposition of the weekly optimization problem into smaller subproblems (day). Our primary motivation for doing so is to save computation time and to avoid the added complexity with facilitating Benders decomposition.

An interesting side-effect is that the deterministic structure of the week problem is broken down to daily deterministic problems. We will also discuss this point.

Assume that the weekly deterministic optimization problem is described as:

$$\begin{aligned}&\min c^T x + \alpha_{t+1}\\ &s.t.\\ &Ax = b\\ &\alpha_{t+1} + \pi_{t+1}\tilde{x} \geq \beta_{t+1}\end{aligned} \quad (1.1)$$

Where c is the cost vector, x the vector of decision variables, A the constraint matrix, b the constraint right-hand sides, $\tilde{x}$ the state variables (a subset of x), α the ECF being constrained by Benders cuts, and π and β are the Benders cut coefficients.

We split the weekly optimization problem into daily problems in (1.2). The index *d* denotes day and ND means number of days within the week. Interpolating in the consecutive expected cost functions (ECF), as illustrated in Figure 3.

$$\begin{aligned}&\min c_d^T x_d + \frac{ND-d}{ND}\alpha_t + \frac{d}{ND}\alpha_{t+1}\\ &s.t.\\ &Ax_d = b_d\\ &\alpha_t + \pi_t\tilde{x}_d \geq \beta_t\\ &\alpha_{t+1} + \pi_{t+1}\tilde{x}_d \geq \beta_{t+1}\end{aligned} \quad (1.2)$$

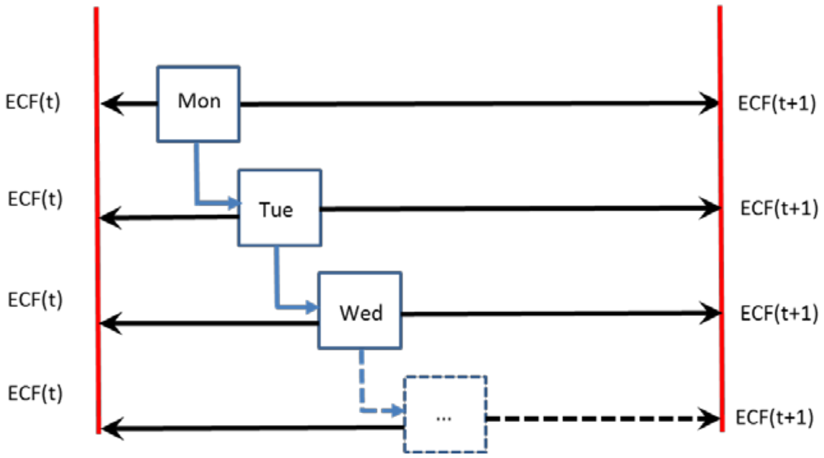

Figure 3 Illustration of rolling horizon towards an interpolated end-valuation.

By solving daily (deterministic) decision problems rather than weekly we expect the decisions to differ. Some examples:

1) Consider a reservoir with a high reservoir level at the beginning of the week and a relatively high water value (or cut coefficient). If all inflow is stored the coming week, spillage will occur. If one perfectly knows the inflow throughout the week, it is possible to schedule the system to reach the maximum reservoir limit without spilling. If one knows the inflow for the next day only, and inflow for the rest of the week is uncertain, simulating day-by-day as in Figure 3 will typically lead to spillage.
2) Consider a system where the water value increases sharply from one week to the next. The optimal weekly decisions are based on the high end-of-week water value, so that there are

incentives to store water even in the first days of the week. In contrast, interpolation will give a smoother transition between the weeks, giving incentives to produce in the beginning of the week.

3) The ECF primarily values hydropower reservoir volumes. Thus, the state of all generation technologies and system components with time-linking constraints is not captured in the ECF. This becomes more of a concern with shorter time horizon. In particular we find that a time horizon of one day is too short to properly valuate slow-ramping thermal power plants with high start-up costs.

Based on these points, we should expect the weekly optimization to be 'more optimal' than the daily. That is, the simulated system cost should be lower in the weekly than the daily optimization.

2.2.1 Impact on Computation Time

As mentioned in Section 1.5, we used Python/Pyomo to handle data and build the optimization problems. A thorough discussion on computation time soon leads to complexities such as warm start, constraint relaxation, and efficiency in model building. We will not address these details here, but point to a simple comparison between the two approaches.

We tested the time spent by the optimization solver when solving the weekly and daily problems. The average time used on a single week problem and 7 day problems is shown in Figure 4. A computational speedup of 3.4 and 6.3 is found for the a small 4 area test system and a full Nordic system (CES), respectively.

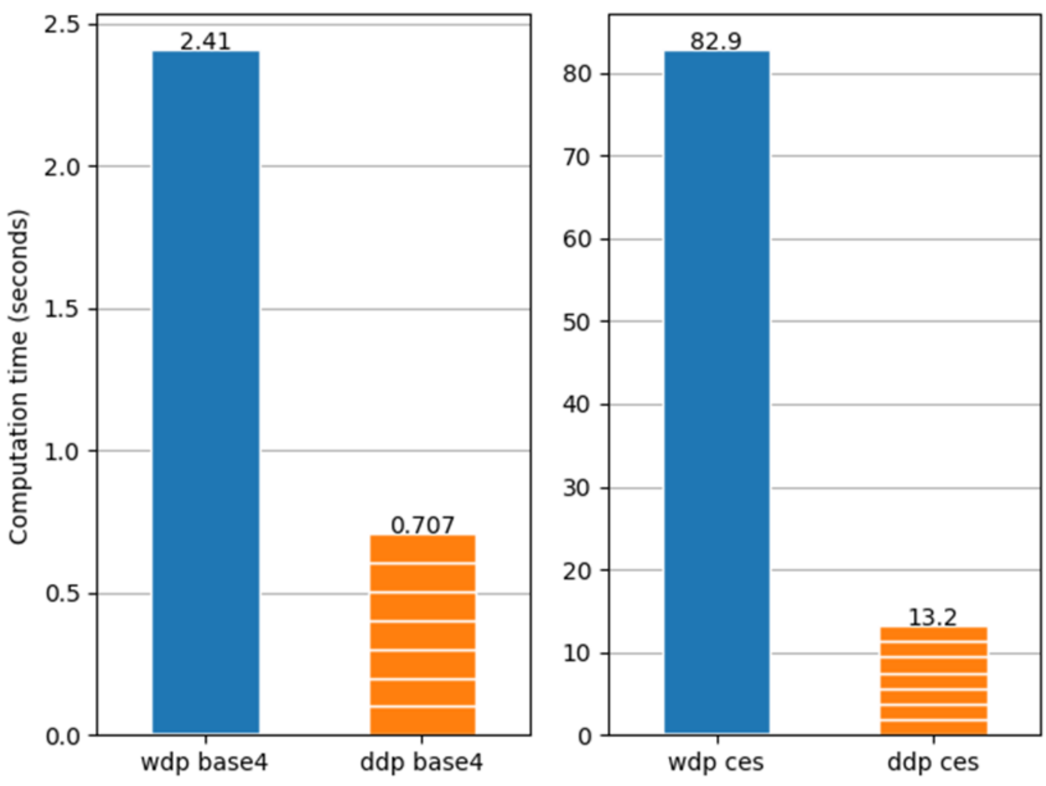

Figure 4 Average time (in seconds) spent by optimization solver (CPLEX) solving the weekly and 7 daily decision problems for a 4-area test system (left) the large-scale CES system (right).

As expected, we found that the additional computational burden involved with data handling and model building when solving the 7 daily problems compared to one weekly problem plays an important role. Some of this overhead should be possible to eliminate if using a high-performance computer language.

2.3 Representative Days

With the added functionality and finer time resolution in the Primod model also comes the increased computation times. Thus, when used for analyses, we recommend selecting a set of representative days or day sequences to simulate rather than all days in all scenarios. This is illustrated with the magnifiers in Figure 5.

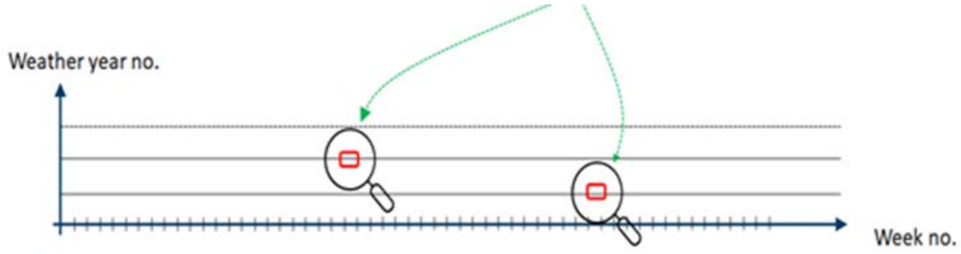

Figure 5 Detailed studies of representative days.

Finding an appropriate set of representative days will depend on the scope of the analysis at hand. In some cases one wants to study a typical day in a specific season. In other cases one wants to resemble a full simulation (covering all days in all scenarios) with representative days.

We will not provide detailed recommendations on how to pick representative days, as there are no obviously correct answers to this. A challenging point is the selection of which weather scenario to use. Keep in mind that a weather scenario for the Nordic system typically contains hundreds of individual time series for inflow, wind speeds, temperatures, etc. For example, which type of statistical properties should one look for and in which type of series?

3 Basic Model Description

The basic design of the Primod model is described in this section. Note that the term reserve capacity is somewhat loosely used to cover many types of reserve capacities with different properties.

3.1 Basic Design

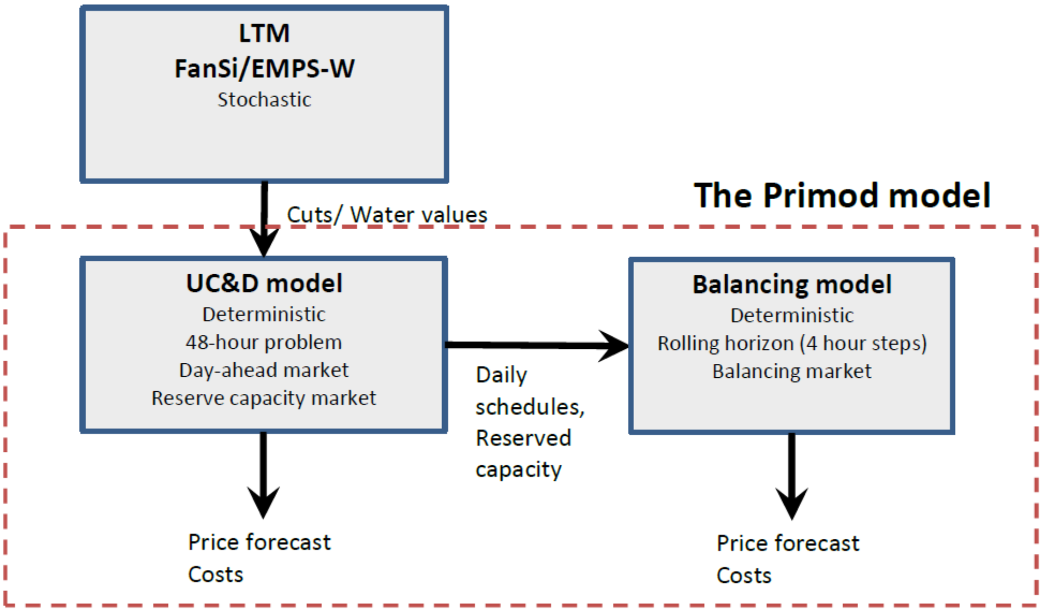

Figure 6 The basic design of the Primod model concept.

As explained above, the Primod model uses the strategy from a long-term model and compute the daily unit commitment and dispatch (UC&D) based on much of the same data description (EMPS dataset) and system boundary, but with additional technical details and a finer time resolution. The Primod model can be seen to some extent resemble the markets for day-ahead energy, reserve capacity, and balancing energy. The model takes the view of a central dispatch process where the cheapest available generation resources are scheduled to deliver electricity to the consumers with the highest willingness to pay. The underlying assumption is that all producers are risk-neutral price-takers and that the market is 'perfect', i.e., without any market power or lack of information. A schematic representation of the model and the information flow received from the long-term models is illustrated in Figure 6.

Primod consists of two stages:

1. **UC&D stage**: Simultaneous scheduling of energy and reserve capacity for the next day based on a forecast of the considered stochastic variable(s).
2. **Balancing stage**: Energy balancing for the next day using the procured resources.

The model built in the UC&D stage is solved repeatedly using a rolling horizon approach, as explained in 2.2. The model has a simultaneous dispatch of energy and reserve capacity provided an exogenously defined reserve requirement. This is in contrast with today's sequential clearing of reserve procurement and day-ahead energy in the Nordics. This sequential process is complicated with some types of reserves cleared before and some after the day-ahead market, as described in [5]. Due to the complexity, we decided to co-optimize the two products in Primod.

In the UC&D stage, the cheapest resources are allocated to cover the demand for energy and reserve capacity for the next day based on one single forecast for the stochastic variables like wind power production and demand, i.e., the model is deterministic. From a modelling point of view, the UC&D does not see the outcomes of imbalances in the Balancing stage and may therefore wrongly predict the actual need for balancing energy. Representing the imbalances as a stochastic variable in the UC&D could make the formulation more robust by providing a connection between the two steps. However, this would require accurate forecasts of the reserve demand, which in practice is challenging to obtain. The basic problem formulation is then a deterministic optimization problem with a 48-hour time horizon. The time resolution for the first 24 hours can for example be 15 minutes, and an hourly time resolution is used for the remaining 24 hours. Figure 7 illustrates the rolling horizon process of the Primod model. The red dots are the decision points. The decisions from the first 24 hours are stored (marked by the dark grey boxes in the figure). The last 24 hours (marked by the light grey box in the figure) serve the purpose of valuating the short-term state variables that are not considered by the long-term strategy. These are the thermal and hydropower unit commitment status variables, and the power and water flows subject to ramping constraints. After the 48-hour problem is solved, the solution is passed on to the Balancing problem, and the model steps forward in time to solve the next 48-hour problem illustrated by the dotted box in Figure 7.

Outputs from UC&D stage:

1) Day-ahead operational costs and unit commitments
2) Marginal costs on energy and reserves. These marginal costs can be found as dual values from an LP problem obtained by a) fixing MIP variables and re-solving LP-relaxation, or b) solving the LP-relaxation of the MIP problem. Both options are implemented in Primod.

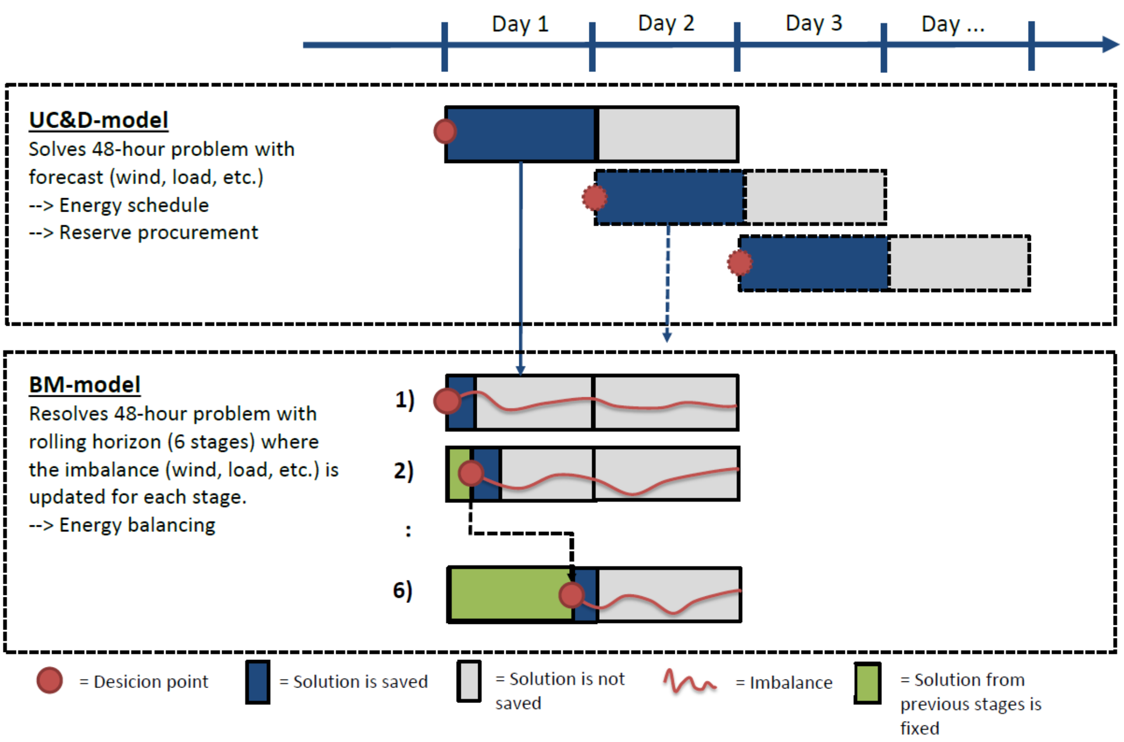

Figure 7 Illustration of the rolling horizon process.

The dispatch from the UC&D problem is fixed before entering the Balancing stage, so that the scheduled energy and reserve capacity are considered when optimizing the treatment of imbalances. In the Balancing stage the procured reserves are activated to respond to the imposed imbalances. We assume imbalances are predicted well for 4 hours ahead. The Balancing model then steps forward in time using rolling planning with a 4-hour step, as illustrated in Figure 7. For each step, the deviation in wind power production and load forecasts are updated (the red lines in the figure), and the solution from the last 4 hours is stored (the green areas in the figure). The activation of balancing energy is in the Nordic market done on an hourly basis (to become 15 min), and using shorter time steps.

Outputs from the Balancing stage:
1) Balancing costs
2) Marginal costs for balancing the system

The unit commitment and dispatch stage can be run as a stand-alone model, while the Balancing model must be run subsequent to the UC&D model. Section 4 describes the mathematical formulation of the UC&D model.

3.2 Information Flow

Figure 8 illustrates what information is updated for each day and each week in the Primod model. The figure also show what information from the UC&D solution is passed on to the Balancing Model (BM) stages and to the next day in the UC&D problem. Note that the end values for reservoir filling, water release and thermal production is sent from the BM back to the UC&D problem. For each stage in the BM, imbalances are updated.

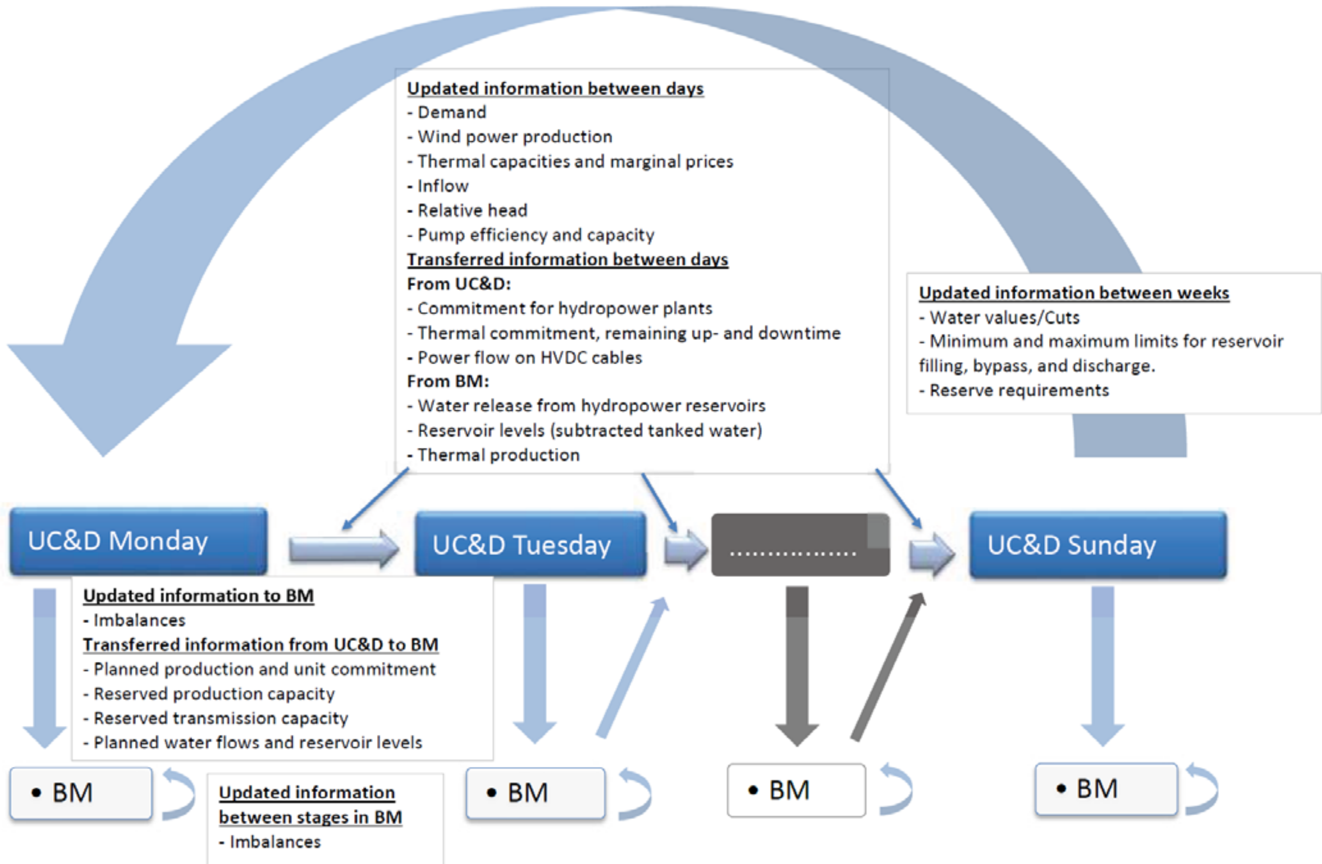

Figure 8 Information flow between days, weeks, and model stages in the Primod model concept.

3.3 Initial Values

When solving the model with several days in sequence, the initial values are taken from the end-values of the previous problem. When solving the first daily problem, several initial values must be defined:

- Initial reservoir levels: These can be provided from the Fansi or EMPS-W models or calculated based on a user-given percentage filling.
- Commitment for hydropower plants: These are initialized to zero in the model (because of low start-up costs).
- Thermal initial commitment and production level, initial downtime and uptime: These are defined in an input file, as defined in the Primod user manual [10].
- Initial flows on HVDC cables are omitted by skipping the ramping constraint for the first time-interval when the first daily problem is solved.
- Initial water releases from hydropower reservoirs are omitted by skipping the ramping constraint for the first time-interval when the first daily problem is solved.

To avoid the definition of certain state variables (such as initial HVDC flows and releases) we omit the corresponding time-linking constraint in the first time-interval. This is a simplification which contributes to underestimating the cost of operation. Making such simplifications should be weighed against the difficulty of obtaining reasonable initial values.

Defining reasonable initial commitments for thermal power plants can be important because of high start-up costs. One method for obtaining initial commitments, is to run the model for one day and use the commitments for the end of the day as initial values. Due to the importance (and cost) of the time-linking constraints for most thermal units, we do not recommend fully relaxing these in the first time-interval.

4 The Unit Commitment and Dispatch Stage (UC&D)

The mathematical formulation of the two-day optimization problem in the UC&D stage is presented in the following. To ease the formulation, but without loss of generality, we omit the conversion between power and energy by assuming a time-step length of one hour. The UC&D model can be solved as a MIP-problem with commitment variables as binary variables, or the unit commitment variables can be relaxed to make the problem linear. The problem formulation is the same with both approaches. All variables in the problem (except the tunnelling variable q_{hk}^{U} and the future cost function α) are non-negative. The nomenclature is found in Appendix A.1.

The full optimization problem is defined in the following, in a similar fashion as [11] and [12]. An approach to decompose the problem by use of Lagrangian relaxation is presented in [11].

4.1 Objective

The objective in (1) is to minimize the system costs associated with unit commitment and dispatch of the system over a two-day period and the expected cost α of operating the system in the future. As Figure 6 show, and as explained in chapter 1.3, the Primod model uses a pre-defined strategy from a long-term model. This strategy can either be in the format of cuts from FanSi of individual water values from EMPS-W. If cuts from FanSi is used, the term α represents the expected cost of operating the system in the future and is described in (16). If individual water values from the EMPS-W model is used, the term α represents the value of the stored water, and is described in (17)-(19). The cost elements represented in the objective function are the start-up cost ($C_g^S w_{gk}$ and c_{hk}^S) of thermal units and hydropower stations, generation p_{gk} from thermal units at a marginal cost C_{gk}^G, curtailment y_{ak}^E of price-inelastic demand at cost C_a^E, relaxation of the up (y_{ak}^{R+}) and down (y_{ak}^{R-}) reserve requirements at cost C_a^R, and meeting the price-elastic demand y_{dk}^D with value C_{dk}^D. The start-up cost for hydropower stations and thermal units are modelled differently in the objective function due to the use of one binary variable describing the hydropower plant commitment, and three binary variables describing the start-up, commitment and shut-down of thermal power plants. To ensure that the model prioritizes the waterway for discharge before bypass and bypass before spillage, a small penalty for using these waterways was added to the cost function where the penalty for spilling C^{S*} is marginally larger than the penalty for bypass C^{B*}.

To ensure model feasibility with varying initial values and data descriptions, penalty variables are associated with many constraints and boundaries. A large penalty, C^B and C^D respectively, for violating the minimum bypass and discharge constraints are added to the cost function. To relax the constraints on minimum reservoir filling, an option to tank water at a high cost C^T to the reservoir in the first time-step was introduced. The tanked water is subtracted before the end-valuation of the stored water, and it is not included in the end-reservoir fillings passed on as initial values to the next day. This was done because the model should not have an incentive to use this water for future generation. A small benefit b^R for procuring reserve capacity ensures that all available resources are allocated. The future expected operating cost or value of stored water is interpolated between α_t and α_{t+1} with the fraction γ in (1), and these are constrained by cuts in (16) or the individual water values in (17)-(19).

$$Z = min \sum_{k\in\mathcal{K}} \Bigg(\sum_{g\in\mathcal{G}} (C^G_{gk} p_{gk} + C^S_g w_{gk}) - \sum_{d\in\mathcal{D}} C^D_{dk} y^D_{dk} + \sum_{a\in\mathcal{A}} C^E_a y^E_{ak} + C^R_a (y^{R+}_{ak} + y^{R-}_{ak}) + \sum_{h\in\mathcal{H}} (C^{B^*} q^B_{hk} + C^{S^*} q^S_{hk} + C^B q^{Bviol}_{hk} + C^D q^{Dviol}_{hk} + C^T q^{Tank}_{hk} + c^S_{hk}) \Bigg) + b^R + \gamma\alpha_t + (1-\gamma)\alpha_{t+1} \tag{1}$$

4.2 Hydropower Constraints

The modelling of the hydropower system is based on hydropower modules. A module consists of one reservoir and one power station and is connected to other modules through the three main waterways discharge (q^D), bypass (q^B), and spillage (q^S). Some hydropower reservoirs are connected by hydraulic couplings/tunnels to transfer water between the reservoirs (q^U), and some cascades has pumping capacity to pump water (q^P) from downstream reservoirs to upstream reservoirs. Each module has a set of modules ω_h from which it receives water through one or more of the waterways. Figure 9 illustrates how these waterways relates to the reservoir. The relationship between reservoir volume and reservoir height/head and discharge and power production respectively are described using a piece-wise linear function, as illustrated by the two boxes in the figure.

Several constraints are used in the model to handle the operation of a hydropower module. Constraint (2) balances the reservoir volume (v_{hk}) at each time step with release decisions (q^R), spillage, tunnelling, pumping and regulated inflow (I^R_{hk}). (3) balances the reservoir release with unregulated inflow (I^U_{hk}), discharge and bypass as shown by the black dot in the Figure 9. The reservoir volumes (4), discharge (5) and bypass (6) variables are often subject to seasonal variations in both lower ($\underline{V}_h$, $\underline{Q}^D_h$, $\underline{Q}^B_h$) and upper ($\overline{V}_h$, $\overline{Q}^D_h$, $\overline{Q}^B_h$) boundaries to ensure that reservoirs and watercourses are operated in a sustainable manner. The upper boundaries on reservoir and discharge are limited by the upper physical capacities of the reservoir and the discharge tunnel. The pumping and tunnelling are subject to physical limitations of the pump $\overline{Q}^P_h$ and the tunnel ($\underline{Q}^U_h$, $\overline{Q}^U_h$) in (7) and (8) respectively. Primod allows hydraulic couplings of type 200 or 300 described in the EMPS dataset. These are reservoirs linked by a tunnel or canal with or without regulated hatches. The flow in the tunnels is only limited by the tunnel capacity in the Primod model. There is only one flow variable associated with each tunnel, with a defined positive direction from the sending reservoir to the receiving reservoir. The capacity of the tunnel is between $\underline{Q}^U_h$ and $\overline{Q}^U_h$, where $\underline{Q}^U_h \leq 0$. Some rivers have releases in consecutive periods constrained by a maximum allowed raping rate ($\Delta_{Q^R_h}$) as in (9). Travelling times in water courses are not considered in the current model, but is a natural expansion to further limit the flexibility in the hydropower system.

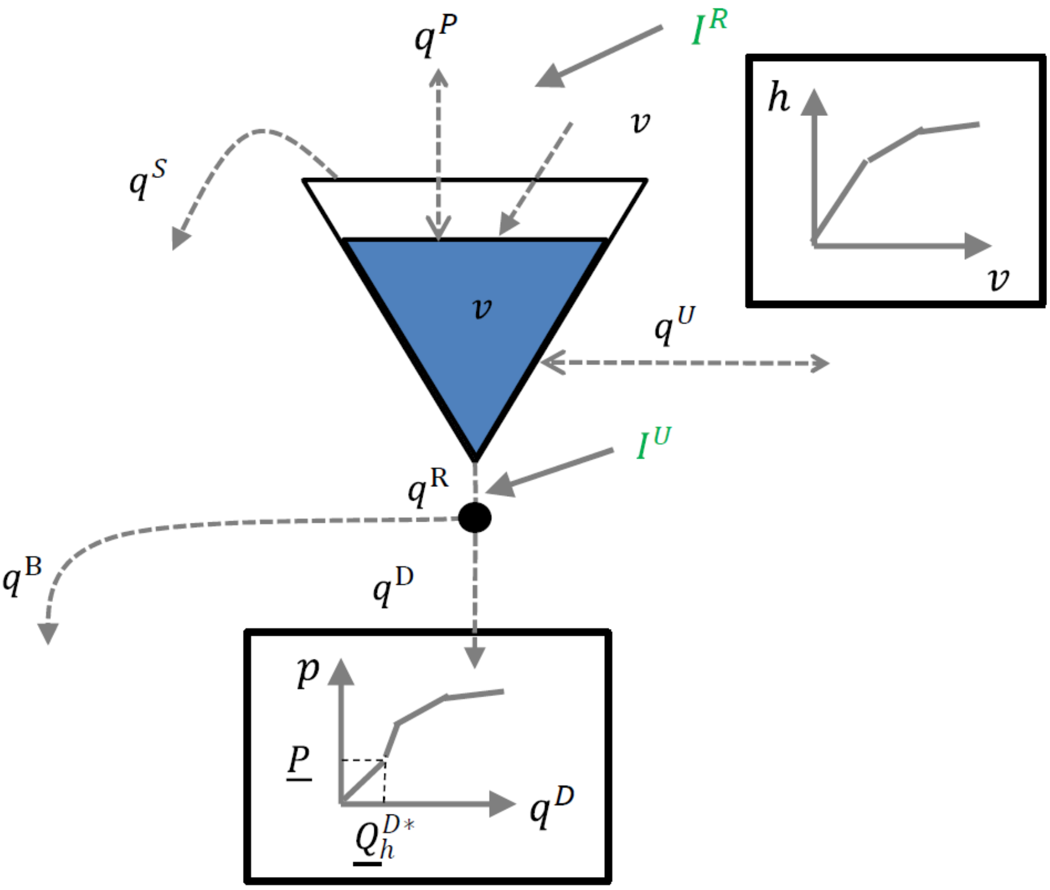

Figure 9 The hydropower module forms the building block of the hydropower system.

$$
\begin{aligned}
v_{hk} - v_{h,k-1} + \Gamma_k\left(q_{hk}^R + q_{hk}^S + q_{hk}^P + q_{hk}^U - q_h^T\right) \\
- \Gamma_k\left(\sum_{j\in\omega_h^D} q_{hk}^D + \sum_{j\in\omega_h^B} q_{hk}^B + \sum_{j\in\omega_h^S} q_{hk}^S + \sum_{j\in\omega_h^P} q_{hk}^P + \sum_{j\in\omega_h^U} q_{hk}^U\right) = I_{hk}^R
\end{aligned} \tag{2}
$$

$$\Gamma_k\left(q_{hk}^B + q_{hk}^D - q_{hk}^R\right) = I_{hk}^U \quad \forall h,k \tag{3}$$

$$\underline{V}_h \le v_{hk} \le \overline{V}_h \quad \forall h,k \tag{4}$$

$$\underline{Q}_h^D \le q_{hk}^D + q_{hk}^{Dviol} \le \overline{Q}_h^D \quad \forall h,k \tag{5}$$

$$\underline{Q}_h^B \le q_{hk}^B + q_{hk}^{Bviol} \le \overline{Q}_h^B \quad \forall h,k \tag{6}$$

$$0 \le q_{hk}^P \le \overline{Q}_h^P \quad \forall h,k \tag{7}$$

$$\underline{Q}_h^U \le q_{hk}^U \le \overline{Q}_h^U \quad \forall h,k \tag{8}$$

$$-\Delta_{Q_h^R} \leq q_{hk}^R - q_{h,k-1}^R \leq \Delta_{Q_h^R} \quad \forall h, k \tag{9}$$

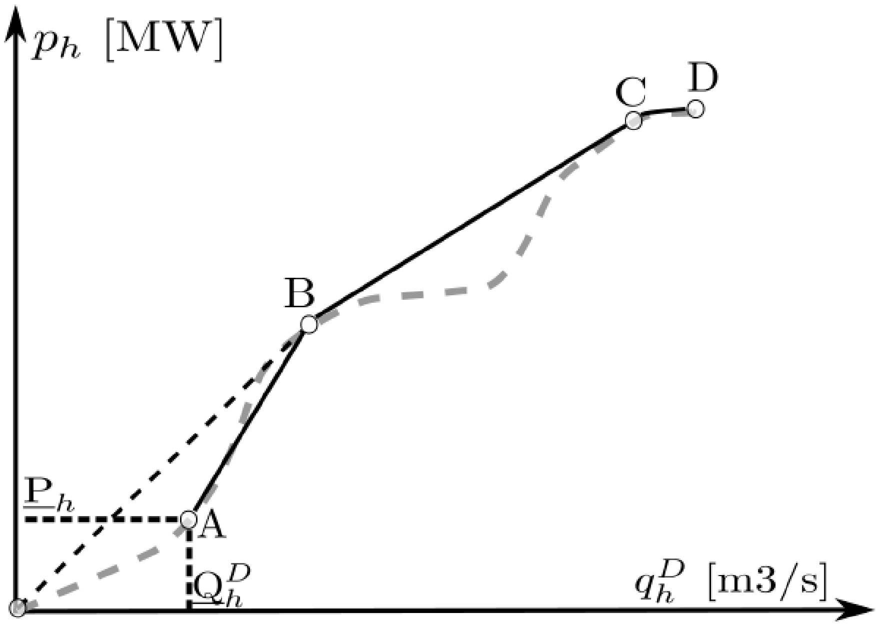

Figure 10 Illustration of the relationship between discharge and power output for a hydropower station [11].

Equations (10)-(14) constrain the operation of the hydropower station. In practice a hydropower station comprises many units (or aggregates), and for fine precisions in the calculations, the individual units should be represented, as detailed in [13]. For large-scale systems typically modelled with the Primod model, obtaining detailed data is difficult and a unit-based approach will also significantly increase computational complexity. An approximate curve representing the power output as a function of station discharge (PQ curve) is therefore used instead, as explained in the following. A station with several units will have a best efficiency point for each combination of units. This is illustrated in Figure 10 where the output from two units loaded in sequence is shown as the grey-dotted line with best efficiency points B and C. A linear approximation of the PQ-curve in Figure 10 uses the points B, C and D, which is a good approximation when the units are operated at their best efficiency points B and C. However, if the station must run on low output, e.g., close to point A, to deliver reserve capacity or to meet a minimum discharge requirement, the power output is overestimated with the linear approach. To reflect this, we introduce a minimum discharge ($\underline{Q}_h^{D*}$) and power output ($\underline{P}_h$) and model the station's power output as in (10)-(12). This corresponds to the curve defined by the points A, B, C and D. The PQ-curve is scaled according to the actual head at the beginning of the day. This is a simplification, assuming that the relative head J_h will vary little during the day, which is typically the case for the many high-head stations in Norway.

The unit commitment of the hydropower station is controlled by the binary variable u_{hk}, indicating if the station is running. The power output from a station h above its minimum generation level is described as a piecewise linear and concave functional relationship of station discharge. The discharge variable is segmented in $\mathcal{N}_h$ segments as shown in (10), where the use of each segment n is limited by a maximum limit ($\overline{Q}_{nhk}^D$) in (11). The power output is described in (12), where η_{hk} represents the efficiency (MW/m3/s) per discharge segment n. Only a selection of hydropower stations is modelled using the binary variable u_{hk}. Remaining stations are modelled without this binary variable, and other versions of equation (10)-(12) apply. For these stations, equation (10) and (12) lose the term with the binary variable, and the binary variable is removed from equation (11). Spinning reserves can only be provided by stations represented with binary variables, both upwards (r_{hk}^{+}) in (13) and downwards (r_{hk}^{-}) in (14). Start-up cost of these stations are represented by variable c_{hk}^{S} in (15) according to a cost C_h^S per start-up.

$$q_{hk}^{D} = u_{hk}\underline{Q}_h^{D*} + \sum_{n\in\mathcal{N}_h} q_{nhk}^{D} \quad \forall h,k \tag{10}$$

$$0 \le q_{nhk}^{D} \le u_{hk}\overline{Q}_{nhk}^{D} \quad \forall n,h,k \tag{11}$$

$$p_{hk} = u_{hk}\underline{P}_h + \sum_{n\in\mathcal{N}_h} J_h \eta_{nh} q_{nhk}^{D} \quad \forall h,k \tag{12}$$

$$p_{hk} + r_{hk}^{+} \le u_{hk}\overline{P}_h \quad \forall h,k \tag{13}$$

$$u_{hk}\underline{P}_h \le p_{hk} - r_{hk}^{-} \quad \forall h,k \tag{14}$$

$$c_{hk}^{S} \ge C_h^S\left(u_{hk} - u_{h,k-1}\right) \quad \forall h,k \tag{15}$$

The reservoir level (subtracted the tanked water) at the end of each two-day problem is valuated either by the Future expected cost function represented by Benders cuts in (16) or by individual water values in (17)-(19).

$$\alpha_{t+1} + \sum_{h\epsilon\mathcal{H}} \pi_{hc}\left(v_{hk} - \Gamma_k q_h^T\right) \ge \beta_c \qquad \forall c \in C_t, k = |\mathcal{K}| \tag{16}$$

The value of one additional unit of water is decreasing with increasing reservoir volume due to the increased risk of spillage, and therefore the reservoir is divided into $|\mathcal{W}|$ (typically 50) reservoir segments with different water values WV_{hwt}. Since the values decrease with increasing volumes, the volumes per reservoir segment v_{hw} is "filled up" from the bottom (the most valuable segment is filled up first) in equation (18).

$$v_{hw} \le \frac{\overline{V}_h}{|\mathcal{W}|}, \quad \forall h \tag{17}$$

$$v_{hk} - \Gamma_k q_h^T = \sum_{w\in\mathcal{W}} v_{hw} \quad \forall h, k = |\mathcal{K}| \tag{18}$$

$$\alpha_{t+1} = \sum_{h\epsilon\mathcal{H}} \sum_{w\in\mathcal{W}} WV_{hwt} v_{hw} \tag{19}$$

4.3 Thermal Constraints

The thermal generation units are limited by the minimum generation, ramping rates (up/down) as well as requirements for minimum up- and down-time. This extension of the model is presented and discussed in [14]. This modelling involves three binary decision variables per thermal unit where u_{gk}, w_{gk} and z_{gk} indicating the online/commitment status as well as start-up and shut-down decisions of thermal generating unit g. The two following equations (20) and (21) provides the logical connection between these three variables.

$$u_{g,k-1} - u_{gk} + w_{gk} - z_{gk} = 0 \qquad \forall g, k \tag{20}$$

$$w_{gk} + z_{gk} \leq 1 \qquad \forall g, k \tag{21}$$

Thermal power plants have four different operational modes [14]; the start-up phase, shut-down phase, production phase and hot stand-by phase [15]. The start-up and shut-down phases impose a delay when the plant is in transition between the minimum power output and zero power output. This delay is not considered here. In this model formulation, we only consider the production phase. When a thermal power unit is started, it must produce between the minimum power output $\underline{p}_{gk}^{G}$ and the maximum start-up ramping limit $\Delta\overline{p}_{g}^{G*}$. In the subsequent operational hours, the power plant can ramp up and down within the maximum ramping rates $\Delta\overline{p}_{g}^{G}$ and $\Delta\underline{p}_{g}^{G}$. The power plant can shut down from a production level at maximum $\Delta\underline{p}_{g}^{G*}$ [16], [17]. This is handled by (22). We do not model the transient start-up and shut-down phases of the thermal power plants, so both the start-up and the shut-down ramping rate must be $\geq \underline{p}_{gk}^{G}$. These ramping limits are in MW/timestep and not in MW/hour, to ensure that the power plants can switch between zero production and a feasible production (above minimum) between two consecutive timesteps. The production is limited by a minimum and maximum power level, as shown in (23).

$$-u_{gk}\Delta\underline{p}_{g}^{G} - z_{gk}\Delta\underline{p}_{g}^{G*} \leq p_{gk} - p_{g,k-1} \leq u_{g,k-1}\Delta\overline{p}_{g}^{G} + w_{gk}\Delta\overline{p}_{g}^{G*} \qquad \forall g, k \tag{22}$$

$$u_{gk}\underline{p}_{gk}^{G} \leq p_{gk} \leq u_{gk}\overline{p}_{gk}^{G} \qquad \forall g, k \tag{23}$$

If the thermal power plant is pre-defined a reserve capacity provider, the following equations apply to ensure that the reserved capacity is within the minimum and maximum production limits (24)-(25), and to ensure that the reserved capacity can be activated within the raping rates of the power plant (26)-(27) [18].

The parameter τ in the latter equations ensure that the capacity can be activated within the maximum response time of the rotating reserve capacity. If the response time is 2 minutes, the power plant can only ramp 1/30 of the ramping limit $\Delta\overline{p}$. This parameter is by default set to one hour in the model. When the power plant is started up, it can adjust the initial production to deliver reserve capacity down to the minimum production (constrained by (25)) and up to the maximum allowed ramping in the start-up phase (constrained by (26)). The maximum allowed ramping rate $\Delta\underline{p}_{g}^{G*}$ when the plant is shut down is not included in (27), because the power plant cannot provide reserve capacity when it shuts down and the online status is zero (as given by (24) and (25)). These equations assume that activating reserve capacity in the previous time step does not affect the ability to deliver reserve capacity in the current time step. This can be a reasonable assumption if the duration time of the activation is small compared to the time resolution of the model.

$$p_{gk} + r_{gk}^{+} \leq u_{gk}\overline{p}_{gk}^{G} \qquad \forall g \in \mathcal{G}^{R}, k \tag{24}$$

$$u_{gk}\underline{p}_{gk}^{G} \leq p_{gk} - r_{gk}^{-} \qquad \forall g \in \mathcal{G}^{R}, k \tag{25}$$

$$\tau\left(p_{gk} - p_{g,k-1}\right) + r_{gk}^{+} \leq \tau u_{g,k-1}\Delta\overline{p}_{g}^{G} + \tau w_{gk}\Delta\overline{p}_{g}^{G*} \qquad \forall g \in \mathcal{G}^{R}, k \tag{26}$$

$$-\tau\left(p_{gk} - p_{g,k-1}\right) + r_{gk}^{-} \leq \tau u_{gk}\Delta\underline{p}_{g}^{G} \qquad \forall g \in \mathcal{G}^{R}, k \tag{27}$$

The thermal power plants area also subject to minimum uptime and downtime constraints [16], [17]. (28)-(30) handle the minimum uptime T_{g}^{U}, while (31)-(33) handle the minimum downtime T_{g}^{D}.

$$\sum_{k \in T_g^{Ui}} (1 - u_{gk}) = 0 \qquad \forall g \tag{28}$$

$$\sum_{t=k}^{k+T_g^U-1} u_{gt} \geq T_g^U w_{gk} \qquad \forall g, k = T_g^{Ui} + 1, \dots, |\mathcal{K}| - T_g^U + 1 \tag{29}$$

$$\sum_{t=k}^{|\mathcal{K}|} (u_{gt} - w_{gk}) \geq 0 \qquad \forall g, k = |\mathcal{K}| - T_g^U + 2, \dots, |\mathcal{K}| \tag{30}$$

$$\sum_{k \in T_g^{Di}} u_{gk} = 0 \qquad \forall g \tag{31}$$

$$\sum_{t=k}^{k+T_g^D-1} (1 - u_{gt}) \geq T_g^D z_{gk} \qquad \forall g, k = T_g^{Di} + 1, \dots, |\mathcal{K}| - T_g^D + 1 \tag{32}$$

$$\sum_{t=k}^{|\mathcal{K}|} (1 - u_{gt} - w_{gk}) \geq 0 \qquad \forall g, k = |\mathcal{K}| - T_g^D + 2, \dots, |\mathcal{K}| \tag{33}$$

4.4 System-Wide Constraints

Power balances for each price area in each time step are provided in (34). Thermal and hydropower generations are scheduled to meet the net load, i.e., the demand (D_{ak}) subtracted the wind (and solar) power (P_{ak}), while allowing power exchange (f) with neighbouring price areas. The price roof and price floor in the model is given by the ability to curtail (y_{ak}^E) energy at a high cost and dump (d_{ak}) power at zero cost, respectively.

$$\sum_{h \epsilon \mathcal{H}_a} p_{hk} - \eta_h^P q_{hk}^P + \sum_{g \epsilon \mathcal{G}_a} p_{gk} - \sum_{d \epsilon \mathcal{D}_a} y_{dk}^D + \sum_{\ell:(a,b) \epsilon \mathcal{L}_a} [(1 - \zeta_\ell) f_{bak} - f_{abk}] + y_{ak}^E - d_{ak} = D_{ak} - P_{ak} \qquad \forall a, k \tag{34}$$

The transmission system is described by a set of connections $\ell \in \mathcal{L}$, and the subset of connections $\mathcal{L}_a$ is associated with each price area a. We let $\mathcal{L} = \mathcal{L}^{AC} \cup \mathcal{L}^{DC}$ comprise both AC ($\mathcal{L}^{AC}$) and HVDC ($\mathcal{L}^{DC}$) connections. Each connection $\ell: (a, b)$ has two directional flow variables: f_{ab} and f_{ba}. Exchange of up- (f_{abk}^+) and downregulating (f_{abk}^-) reserve capacity can be allocated each AC connection, according to (35), (36) and (37), bounded by the transmission capacity (F). Exchange of reserves are not allowed on HVDC connections in (37). The transmission losses depend linearly on the flows by a loss fraction (ζ_ℓ) in (34). Ramping limits on HVDC connections between areas are constrained by a maximum ramping rate (Δ_ℓ) in (38). The exchange of reserve capacity is limited to a fraction ϕ of the total transmission capacity in (39).

$$0 \leq f_{abk} + f_{abk}^+ \leq F_{ab} \qquad \forall \ell: (a, b) \in \mathcal{L}^{AC} \tag{35}$$

$$0 \leq f_{bak} + f_{abk}^- \leq F_{ba} \qquad \forall \ell: (a, b) \in \mathcal{L}^{AC} \tag{36}$$

$$0 \leq f_{abk} \leq F_{ab} \qquad \forall \ell: (a, b) \in \mathcal{L}^{DC} \tag{37}$$

$$-\Delta_\ell \leq (f_{abk} - f_{bak}) - \left(f_{ab,k-1} - f_{ba,k-1}\right) \leq \Delta_\ell \quad \forall \ell: (a,b) \in \mathcal{L}^{DC} \tag{38}$$

$$0 \leq f^{+}_{abk}, f^{-}_{abk} \leq \phi F_{ab} \quad \forall \ell: (a,b) \in \mathcal{L}^{AC} \tag{39}$$

The model considers spinning reserve capacity as a generic product, resembling the joint requirement for FCR and aFRR reserve capacity. Reserve requirements for spinning upward (R^{+}_{m}) and downward (R^{-}_{m}) reserves are defined per group m of price areas in (40) and (41). The reserve requirements are exogenously given. A predefined collection of hydropower plants ($\mathcal{H}^{R}_{a}$) and thermal power plants ($\mathcal{G}^{R}_{a}$) are allowed to deliver spinning reserve capacity. We allow for relaxation of the requirements in (40) and (41) through variables y^{R+}_{ak} and y^{R-}_{ak}, respectively. By default the cost C^{R}_{a} of using y^{R+}_{ak} and y^{R-}_{ak} is marginally lower than the energy rationing cost C^{E}_{a} in (1), to ensure that (40) and (41) are relaxed (by using y^{R+}_{ak} and/or y^{R-}_{ak}) before rationing price-inelastic demand. The set $\mathcal{A}^{R}_{m}$ comprises all areas/zones in a group m and the set $\mathcal{L}^{R}_{a}$ comprises all lines connecting from zone a in reserve group m to another reserve group.

$$\sum_{a \in \mathcal{A}^{R}_{m}} \left(\sum_{h \epsilon \mathcal{H}^{R}_{a}} r^{+}_{hk} + \sum_{g \epsilon \mathcal{G}^{R}_{a}} r^{+}_{gk} + y^{R+}_{ak} + \sum_{\ell:(a,b) \epsilon \mathcal{L}^{R}_{a}} (f^{+}_{bak} - f^{+}_{abk}) \right) \geq R^{+}_{m} \quad \forall m, k \tag{40}$$

$$\sum_{a \in \mathcal{A}^{R}_{m}} \left(\sum_{h \epsilon \mathcal{H}^{R}_{a}} r^{-}_{hk} + \sum_{g \epsilon \mathcal{G}^{R}_{a}} r^{-}_{gk} + y^{R-}_{ak} + \sum_{\ell:(a,b) \epsilon \mathcal{L}^{R}_{a}} (f^{-}_{bak} - f^{-}_{abk}) \right) \geq R^{-}_{m} \quad \forall m, k \tag{41}$$

$$b^{R} = \sum_{a \in \mathcal{A}^{R}_{m}} \left(\sum_{h \epsilon \mathcal{H}^{R}_{a}} r^{+}_{hk} + \sum_{g \epsilon \mathcal{G}^{R}_{a}} r^{+}_{gk} + \sum_{h \epsilon \mathcal{H}^{R}_{a}} r^{-}_{hk} + \sum_{g \epsilon \mathcal{G}^{R}_{a}} r^{-}_{gk} \right) B^{R} \tag{42}$$

A small marginal cost B^{R} for procuring reserve capacity was introduced to ensure that the model allocates all available reserve capacity in periods with abundancy of this resource in (42). This is important because the UC&D does not consider the cost of activating the procured reserve capacity, and this ensures the cheapest reserve capacity is available for activation in the subsequent Balancing stage.

4.5 Finding Results

Primod can be used for a wide range of tasks, where the user primarily will be interested in one or more of the following types of results:

- Marginal costs (or prices) for energy, reserve capacity (and balancing energy, see Section 5)
- Total system cost
- Specific results from the optimization, e.g., discharge from a hydropower plant or power flow between price areas.

The marginal costs of energy and reserve capacity are found as the dual values of equations (34) and (40)-(41), respectively. The user can choose to either solve the model with non-relaxed commitment variables (MIP formulation), or to solve the relaxed model. If the UC&D model is solved with non-relaxed commitment variables, these variables are fixed, and the model is resolved to obtain the dual values.

The total cost in (1) comprises both the here and now costs and the future expected costs (or value of water) expressed by α. If the model user is interested in comparing costs obtained from the model, care should be taken in comparing both the here and now costs, but also the future expected cost (or value of the water) at the end of the day. Therefore, both the here and now costs and the future expected costs (or value of the water) are provided as a model result. The future expected costs (or value of water) are calculated based on cuts/water values from the end of the previous week and the end of the current week in equations (16) or (19). These values are weighted in the cost function according to which day of the week is solved. It is this weighted value that is stored.

Because a two-day problem is solved, the total cost obtained directly from (1) is the cost of operating the system in a two-day period. Normally, the model user is only interested in the results from the first day, and only simulation results from the first 24-hour solution is saved. All cost elements stored are costs for running the system for the first day. The weighted alfa value is also re-calculated based on the reservoir levels at the end of the first day.

Detailed simulation results are written to HDF5 files documented in [10].

4.6 Case Studies

The Primod code has been developed and matured over the last 3 years. In this period, multiple students' theses have used the prevailing code as a starting point for further investigations of specific topics within the general field of power market modelling and analyses. These works are listed below.

In addition, SINTEF has carried out two detailed case studies for the purpose of getting of verifying the model behaviour on large-scale case studies.

4.6.1 M.Sc. Theses

1) In the M.Sc. thesis "*Optimizing Weekly Hydropower Scheduling in a Future Power System - Development of a Deterministic Short-Term Hydro-Thermal Scheduling Model*" the Primod model code was enhanced with transmission ramping constraints, start-up and shut-down of thermal units and used in a rolling horizon setup to simulate power system operation [19].
2) In the M.Sc. thesis "*Hydro-Thermal Multi-Market Optimization– Economic Surplus*" constraints to schedule up- and down-regulation reserve capacity were implemented in the Primod model. The impact of different allocation methods and reserve volumes were investigated. In addition, a tool for analysing economic surplus was developed [20].
3) In the M.Sc. thesis "*Flow-Based Market Coupling in Short-Term Hydro-Thermal Scheduling*" flow-based market coupling constraints were implemented in Primod and the effects on power prices and system operation was investigated for a small-scale test system [21].
4) In the M.Sc. thesis "*Combining Mathematical Programming and Machine Learning in Electricity Price Forecasting*" it was investigated how the complexity of the Primod model could be reduced to save computational time by exploiting patterns in the input data [22]. A framework based on Machine Learning for reducing the problem size was presented. This work formed the basis of a research article presented in [23].
5) In the M.Sc. thesis "*Demand Response in a Short-Term Hydro-Thermal Multi-Market Model*" the impact of price-based demand response was investigated by implementing gradual adaption of consumption, load shifting and peak shaving/clipping in Primod [24]. The gradual adaption adjusted the consumption prior to solving the Primod model based on the price and consumption in the previous solution.

4.6.2 Case Study #1 – Multi-Market Price Forecasting

The Primod model was used in a study on multimarket price forecasting carried out within the Norwegian Research Centre for Hydropower Technology (HydroCen), work package 3 "Market and Services", activity 1 "Future market structures and prices". The focus of the work was to simulate and compare prices for energy and procurement of reserve capacity using different modelling tools and functionalities assuming a specific 2030 system scenario (the HydroCen Low Emission scenario documented in [25]). Two different models were used: The EMPS model and Primod. Primod was run in the UC&D mode.

This task pinpointed the complexity of forecasting prices for procuring reserve capacity, as well as the impact of reserve procurement on energy prices. It was found that the Primod model provided more reasonable estimates for reserve procurement prices than the EMPS model. The work illustrates how Primod can be integrated in a toolchain to perform analyses of a large-scale system and highlights some important experiences to consider when conducting studies like this.

The work resulted in a report rapport "Multimarket modelling – application of different models to HydroCen Low Emission scenario" [26] and a conference paper presented at EEM 2020, "Impact on hydropower plant income from participating in reserve capacity markets" [27].

4.6.3 Case Study #2 – Assessing the Benefits of Exchanging Reserve Capacity

In this work the benefits of allowing spinning reserve capacity to be exchanged between bidding zones and countries within the Nordic power market was investigated. Again, the HydroCen Low Emission Scenario documented in [25] was used as a basis for the analyses, resembling a 2030 scenario of the Northern European System.

First, a set of representative days were selected to span the set of different daily optimization problems to be solved by Primod. Four different seasons (represented by weeks 9, 20, 31 and 45) for three different inflow years (dry, normal and wet) were considered. The inflow years were categorized according to their accumulated annual inflow to all Nordic hydropower reservoirs.

Next, the long-term scheduling model FanSi [2] was run for the three particular inflow years, constructing Benders cuts to be used for each case simulated by Primod. Finally, Primod was run for each of the (4x3) representative days considering exchange of reserves between countries and between bidding zones. The fraction of maximum transmission capacity set aside for reserve exchange was varied between 0-15 %. Primod was run in UC&D mode using both the MIP and the LP formulations.

Detailed results describing the tests are documented in [12]. In more general notes, the key findings are as follows:

1. The benefit of reserve exchange is substantial. For this particular study, it was found that the average daily economic benefit was 290 k€ and 102 k€ for reserve exchange between bidding zones and countries, respectively.
2. The benefit consistently increases with increasing share of transmission capacity mad available for exchange of reserves. The marginal increase typically starts to decline when the fraction of transmission capacity reaches 10-15 %.

3. The case study clearly demonstrated the importance of using a MIP formulation in Primod for such studies. Using an LP formulation tended to substantially underestimate the economic benefits of reserve exchange.

5 Balancing Model (BM)

The Balancing Model (BM) was developed in the last year of the PRIBAS project and has therefore been tested and verified to a lesser degree than the UC&D model. As an assumption, we limit the further discussion to imbalances caused by deviations in the net load.

In the following we use the term *BM stage* to refer to the sequential runs of the BM, as illustrated in Figure 7. In the BM stage, the unit and station commitments and energy schedules from the UC&D stage are fixed, and all generators have a commitment status and a scheduled "set point". The reserves have been procured and the imbalance are gradually revealed in the BM stage. The BM only imposes imbalances in price areas where reserve capacity is allocated. The major decisions in this stage are which hydropower stations and thermal units should be used in the balancing and how much each of them should deviate from their dispatched set-points. The imbalances are observed as *deviations* in net load. The BM is formulated using a "delta formulation" to clearly differentiate between the solution from the UC&D stage, and the deviations from the planned dispatch caused by the imbalances in the BM stage. The regulation margins for each station and generator in the BM stage are given by the scheduled set point p^S, and the reserved capacities $\hat{r}^+$ and $\hat{r}^-$. If ramping limits like (**26**) and (**27**) are not binding, the reserved capacities are equal to the available capacities between the set point and the minimum and maximum capacity, as illustrated in Figure 11.

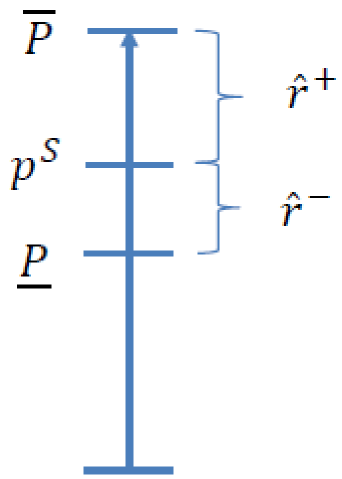

Figure 11 Generator scheduled generation and reserved capacities in the Balancing stage.

The Balancing stage consist in re-planning the same time period as for the UC&D stage, but with exact information on the realized net load. The BM stage is solved with a rolling horizon that steps forward in time in 4-hour steps. For each step, the imbalances are updated for the remaining period, and results from the previous 4-hours are stored and this solution is fixed. Consequently, 6 balancing problems are solved to cover the first 24-hours of the two-day problem. The main output from the BM is which units and stations contribute to the balancing of the system, the marginal costs of providing balancing energy, and the additional cost when deviating from scheduled generator set points. The BM is formulated as an LP problem. The BM problem size is considerably smaller than for the UC&D problem, mainly because the commitment is known and only the price areas with reserve requirements, and power plants delivering reserve capacity, need to be represented in detail in the model.

5.1 Imbalances

Imbalances are defined as the deviations from forecasts. The solution from the UC&D stage is based on a single forecast, as discussed in 3.1. This can lead to challenging balancing situations, depending on the magnitudes and distributions of imbalances. Since we have no knowledge about the imbalances in the UC&D stage, it is important that the magnitude of the exogenously given reserve requirements in the UC&D stage is harmonized with the imbalances. This harmonization must be taken care of by the modeller since there is no feedback from the imbalance stage to the UC&D stage. Note that this is different in other types of models (stochastic) where reserves are procured with knowledge about the statistical properties of the imbalances [16].

The BM receives time series of imbalances for different parameters (wind, load, availability, etc.) in the energy balancing stage. Such values can be obtained from history (when backtesting) or from a statistical model. The imbalances are revealed sequentially when performing a rolling-horizon operation through the 24 hours. The process is illustrated in Figure 12. The black solid-drawn line is the original forecast used in the UC&D problem.

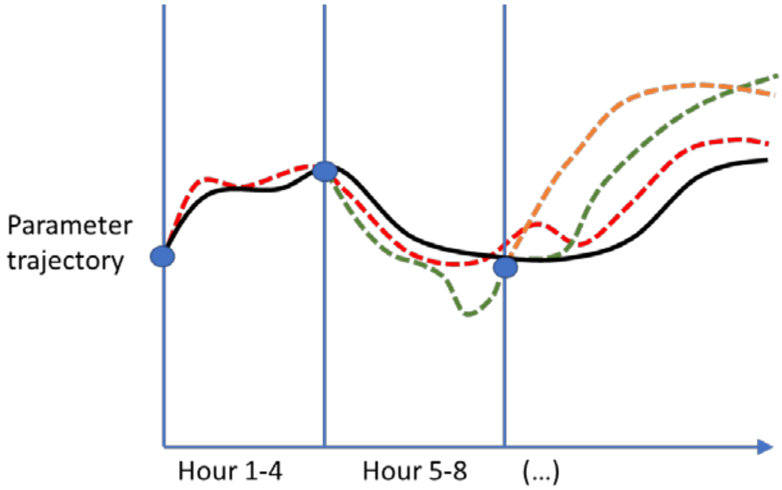

Figure 12 Updated imbalance information revealed in a rolling-horizon manner.

5.2 Assumptions

In formulating the BM model, we strive to reduce model size as much as possible. Essentially, the BM model solves a similar type of problem as the UC&D, but with certain variables fixed according to their UC&D decisions. Thus, we can simplify the BM model and a list of assumptions and model simplifications made is provided below:

- Due to the connections in watercourse topologies and the valuation of stored water, all hydropower reservoirs should be represented and valuated in the BM to correctly capture the cost of balancing. Hydropower modules in areas not considered in the BM, are only represented in the valuation of water. In order not to limit the dynamics in the watercourse, all hydropower modules in areas with reserve requirements should basically be able to change their volume, discharge, and bypass. These modules are found in the set $\mathcal{H}^B$.
- The BM differentiate hydropower modules that a) delivers reserve capacity, b) are located downstream to reservoirs of type a) with small reservoirs that are not valuated and c) are large enough to be valuated, but are not defined within the subset of units delivering reserve

capacity. Type a) can change their discharge and production as a response to the imbalances that occur. Type b) are represented by the set $\mathcal{H}^S$ and can change their discharge and thus production, and contribute to the balancing, if a power plant directly upstream increases or decreases the discharge. This is further explained in 5.5. These hydropower modules also need to be able to change the discharge in order not to get problems with their reservoir balances if capacity is activated upstream. Output from type c) hydropower modules is treated as fixed in the BM model and cannot contribute to the balancing.

- Spillage is not included in the BM. If small reservoirs are full and dispatched at maximum bypass and maximum discharge from the UC&D solution, they cannot receive more water in the Balancing stage unless spillage is included. This can become a bottleneck for increasing production (and discharge) upstream the river cascade if there are no other reservoirs downstream that can store the extra discharged water. This is a drawback when omitting the waterway for spillage in the BM, and spillage is therefore considered included in future version of the model.
- Deviations in pumping and tunnelling are not included in the balancing problem. Thus, these decision variables can be interpreted as fixed in the Balancing stage.
- Bypass violation q_{hk}^{Bviol} and discharge violation q_{hk}^{Dviol} from the UC&D stage is also passed on to the Balancing stage as state variables. This to avoid taking the cost of violating constraints for bypass and discharge flow twice (both in the UC&D stage and then again in the BM).
- The reservoir fillings from the UC&D are not, for the same reason, subtracted the tanking. All reservoir volumes subtracted the tanked volumes are valuated in the BM, as in the UC&D.
- Only spinning reserve capacity is considered in the current version of the model, and the reserved capacity on each power plant from the UC&D stage is thus dictating the capability of adjusting production in the BM stage. Note that the model can be easily expanded to also include non-spinning reserve capacity.
- Even though the model considers spinning reserve capacity, the reserves are activated for the entire time step length. If an hourly time resolution is applied, the reserves are activated during the whole hour. This assumption is considered to be reasonable, since both the UC&D and BM model has a default time resolution of 15 minutes, which is the maximum activation time for secondary (spinning) reserves.
- The rest time of reserve capacity is the minimum time after the capacity is activated before it can be activated again. We do not consider rest time in this Balancing model and assume that reserved capacity can be activated in consecutive time steps.
- The BM model includes ramping constraints on production from thermal power plants and on release from hydropower stations to ensure that activation of reserved capacity does not violate these limits. When including ramping constraints, the BM might find a solution in one Balancing stage that fulfil these constraints, but when the solution for the 4-hour stage is fixed and the imbalance is updated, the next Balancing stage might not be able to find a solution. This can happen e.g., if a hydropower station has increased the discharge to upregulate in one Balancing stage. If the imbalance switch direction in the next Balancing stage, the hydropower station is forced to reduce the discharge and production, and the model can become infeasible if the change in discharge is too large.
- Minimum up- and downtime of thermal power plants are irrelevant when considering spinning reserves and a fixed unit commitment. These are already considered in the UC&D stage.
- Cable ramping is not included in the Balancing model because reserve capacity and balancing energy is studied within the Nordic region (a synchronous area). We assume that the power flow on cables cannot be changed in the Balancing problem.
- We consider the power balance per reserve group m and not per price area as in the UC&D model. There is a set $\mathcal{A}_m^R$ of price areas per reserve group m, and the power balance is a sum over all price areas in a reserve group.
- We assume no transmission limits within a reserve group, i.e., power flow on lines internal to the reserve group are omitted from the Balancing problem.
- Due to the absence of some transmission lines and reserve capacity requirements, the Balancing problem is less restricted than the UC&D problem. This can result in production

being shifted to cheaper production units in the Balancing problem. To ensure all activation of reserve capacity is a response to imbalances, constraints are added to ensure that each activation is in the opposite direction of the imbalance.

- Power flow from a reserve group to connected areas with no reserve demand is fixed and is therefore not included in a "delta description" of the Balancing problem.

5.3 Mathematical Formulation

The mathematical formulation of the optimization problem in the BM stage is presented in the following. Many decisions from the UC&D stage are fixed in the BM stage, such as unit commitment, reserve allocation in both generation and transmission system, generator set points, water flows, reservoir fillings etc. These variables are ***state variables***, passing information from the UC&D stage to the Balancing stage. The state variables are marked with a "hat" in the following formulation, i.e., they are fixed variables from the UC&D solution. The nomenclature is found in Appendix A.2. We assume an hourly time resolution to omit the conversion between power (MW) and energy (MWh) in the formulation. The Balancing problem is an LP-problem since the unit commitment has already been solved. When solving the 6 stages in the Balancing problem, all decision variables (except the deviation in the expected future profit function $\Delta\alpha_t$ and $\Delta\alpha_{t+1}$) from the first stage solution are fixed for the time steps (the first 4 hours) in this stage before moving forward and solving the next stage in the rolling horizon problem. In the actual BM, we sample imbalances in net load. Extending the imbalances to include imbalances in wind power production, load etc. will be easy to implement if data is available. In this description we aggregate the imbalances for each reserve group for brevity. Most variables in the problem are non-negative, except the future cost function α, the reduction in bypass Δq_{hk}^{B-}, and the extra (expensive) balancing energy for downregulation b_{mk}^{-}.

5.3.1 Objective

The objective (**43**) of the Balancing problem is to minimize the cost of dispatch changes for controllable generators to balance net load changes for the remaining time horizon. The costs associated with balancing the system is the marginal cost C_{gk}^{G} of deviating in thermal production Δp_{gk} and the deviation in value $\Delta\alpha$ of the stored water at the end of the two-day problem. We associate a small cost C^B with positive and negative deviations from the planned bypass ($\Delta q_{hk}^{B+}, \Delta q_{hk}^{B-}$) to ensure reasonable results, and a large cost C^R for buying extra balancing resources for upregulation b_{mk}^{+} and downregulation b_{mk}^{-}. The bypass deviation variables, and the extra balancing resources, are split into a negative and a positive part to avoid using absolute values in the problem formulation. The objective is subject to several constraints presented in the following chapters 5.3.2-5.3.4.

$$Z = \min \sum_{k\in\mathcal{K}} \left(\sum_{g\in\mathcal{G}} C_{gk}^{G}\Delta p_{gk} + \sum_{h\in\mathcal{H}} C^{B}(\Delta q_{hk}^{B+} - \Delta q_{hk}^{B-}) + \sum_{m\in\mathcal{M}} C^{R}(b_{mk}^{+} - b_{mk}^{-}) \right) + \gamma\Delta\alpha_t + (1-\gamma)\Delta\alpha_{t+1} \tag{43}$$

5.3.2 Hydropower Constraints

The deviation in Future Profit Function $\Delta\alpha_{t+1}$ for the end of the studied week t is defined as the FCF α_{t+1}^{b} based on the end-reservoirs v_{hk}^{b} from the Balancing stage subtracted the FCF $\hat{\alpha}_{t+1}$ based on the end-reservoirs $\hat{v}_{hk}$ from the UC&D stage in (45). The reservoir fillings $\hat{v}_{hk}$ from the UC&D stage may include tanked water and hence the reservoir volumes v_{hk}^{b} in the BM stage also needs the opportunity to include tanked water. The planned tanked water must also be subtracted in the valuation of water in the BM stage given in (**44**). Note that the reservoir volumes in areas not considered in the BM also must be included in this constraint to get the correct valuation.

$$\alpha_{t+1}^{b} + \sum_{h \epsilon \mathcal{H}^B} \pi_{hc}\left(v_{hk}^{b} - \Gamma_k \hat{q}_h^T\right) + \sum_{h \epsilon \mathcal{H} - \mathcal{H}^B} \pi_{hc}\left(\hat{v}_{hk} - \Gamma_k \hat{q}_h^T\right) \geq \beta_c \quad \forall c \in C_t, k = |\mathcal{K}| \tag{44}$$

$$\Delta\alpha_{t+1} = \alpha_{t+1}^{b} - \hat{\alpha}_{t+1} \tag{45}$$

If individual water values from the EMPS-W model are used for the end-valuation of stored water, the deviation in value $\Delta\alpha$ is constrained by (**46**)-(**48**). The planned volumes $\hat{v}_{hw}$ per reservoir segment from the UC&D stage does not include the tanked water, and the planned tanking must be subtracted from the end-reservoir volume in the BM stage in the valuation. The tanking must also be subtracted because the marginal value of the water decreases with increased reservoir volume, and the value of the volume difference will therefor depend on the total reservoir level. This valuation method is not tested for the BM.

$$v_{hw}^{b} \leq \frac{\overline{V}_h}{|\mathcal{W}|}, \quad \forall h \in \mathcal{H}^B \tag{46}$$

$$v_{hk}^{b} - \Gamma_k \hat{q}_h^T = \sum_{w \in \mathcal{W}} v_{hw}^{b} \quad \forall h \in \mathcal{H}^B, k = |\mathcal{K}| \tag{47}$$

$$\Delta\alpha_{t+1} = \sum_{h \epsilon \mathcal{H}^B} \sum_{w \in \mathcal{W}} WV_{hwt}\left(v_{hw}^{b} - \hat{v}_{hw}\right) \tag{48}$$

The reservoir balance for the BM stage is given in (**49**) where the deviation in reservoir volumes must equal the deviation in water flows to and from the reservoir. Initial reservoir volumes in the Balancing problem equals the initial reservoir in the UC&D problem. The constraints for the first timestep are not presented here (but are they included in the model). Equation (**50**)-(**52**) ensure that the deviations in reservoir volume, discharge and bypass are within the minimum and maximum limits.

$$v_{hk}^{b} - v_{hk-1}^{b} + \left(\Delta q_{hk}^{D} + \Delta q_{hk}^{B+} + \Delta q_{hk}^{B-}\right) - \sum_{j \in \omega_h^D} \Delta q_{jk}^{D} - \sum_{j \in \omega_h^B} \left(\Delta q_{jk}^{B+} + \Delta q_{jk}^{B-}\right) = \hat{v}_{hk} - \hat{v}_{hk-1} \quad \forall h \in \mathcal{H}^B, k \tag{49}$$

$$\underline{V}_h \leq v_{hk}^{b} \leq \overline{V}_h \quad \forall h \in \mathcal{H}^B, k \tag{50}$$

$$\underline{Q}_h^D \leq \Delta q_{hk}^{D} + \hat{q}_{hk}^{D} + \hat{q}_{hk}^{D,viol} \leq \overline{Q}_h^D \quad \forall h \in \mathcal{H}^B, k \tag{51}$$

$$\underline{Q}_h^B \leq \Delta q_{hk}^{B+} + \Delta q_{hk}^{B-} + \hat{q}_{hk}^{B} + \hat{q}_{hk}^{B,viol} \leq \overline{Q}_h^B \quad \forall h \in \mathcal{H}^B, k \tag{52}$$

The following constraint (53) was added to avoid a negative deviation in discharge Δq_{hk}^{D} for hydropower modules not delivering reserves. (53) is valid for all hydropower modules not delivering reserves for all time steps with one exception. If the hydropower module has a small reservoir capacity ($\leq$ 0.1 Mm^3) (not valuated) constraint (54a) or constraint (54a) applies. If the small hydropower reservoir belongs to a reserve group with a positive imbalance (downregulation), constraint (54a) is active. If upstream modules reduce their discharge to reduce their production and downregulate, downstream modules with little reservoir capacity must also reduce their discharge consequently. When these modules receive less water from upstream modules, they might not be able to maintain the planned discharge. We therefore constrain the change in discharge at downstream modules with small reservoirs to be exactly equal the reduced inflow. This prevents the model from making a cost neutral decision in one balancing stage that will cause problems in the next balancing stage. To correctly valuate the water that is held back, the power production at these modules also needs to be reduced and included in the balancing, as will be further addressed in chapter 5.5. If there is a negative imbalance (upregulation), the hydropower modules with small reservoirs cannot increase their discharges more than the upstream modules, and equation (54b) replaces (54a). This is to prevent these small modules from delivering more upward balancing energy than necessary.

$$0 \le \Delta q_{hk}^{D} \qquad \forall h \in \mathcal{H}_a - \mathcal{H}_a^R - \mathcal{H}_a^S, k \tag{53}$$

$$\Delta q_{hk}^{D} = \sum_{j \in \omega_h^D} \Delta q_{jk}^{D} \qquad \forall h \in \mathcal{H}_a^S, \Delta P_{mk}^{+} + \Delta P_{mk}^{-} \ge 0 \tag{54a}$$

$$0 \le \Delta q_{hk}^{D} \le \sum_{j \in \omega_h^D} \Delta q_{jk}^{D} \qquad \forall h \in \mathcal{H}_a^S, \Delta P_{mk}^{+} + \Delta P_{mk}^{-} \ge 0 \tag{54b}$$

The change in production from hydropower stations delivering reserves must be accurately represented by the PQ-curve in (**55**)-(**57**) to capture the variations in cost/prices. The hydropower stations in $\mathcal{H}^S$ are not represented with binary variables, and (**55**)-(**57**) are slightly modified in the model, but not explicitly stated here.

$$\Delta q_{hk}^{D} + \hat{q}_{hk}^{D} - \sum_{n \in N_h} q_{nhk}^{D,b} - \hat{u}_{hk} \underline{Q}_h^{D*} = 0 \quad \forall h \in \mathcal{H}_a^R \cap \mathcal{H}_a^S, k \tag{55}$$

$$0 \le q_{nhk}^{D,b} \le \hat{u}_{hk} \overline{Q}_{nhk}^{D} \quad \forall n, h \in \mathcal{H}_a^R \cap \mathcal{H}_a^S, k \tag{56}$$

$$\Delta p_{hk} = \hat{u}_{hk} \underline{P}_h + \sum_{n \in \mathcal{N}_h} \eta_{nh} q_{nhk}^{D,b} J_h - \hat{p}_{hk} \quad \forall h \in \mathcal{H}_a^R \cap \mathcal{H}_a^S, k \tag{57}$$

The planned release q_{hk}^{R} must enter the ramping constraint on release from hydropower plants in the Balancing stage in equation (**58**) to ensure that activation of reserve capacity does not violate the allowed ramping on release. The change in bypass $\Delta q_{hk}^{B+} + \Delta q_{hk}^{B-}$ and discharge Δq_{hk}^{D} equals the change in release in the Balancing stage. Note that the BM might find a solution in one Balancing stage that fulfil this constraint, but when the solution for the 4-hour block is fixed and the imbalance is updated, the next Balancing stage might not be able to find a solution. This can happen if a hydropower station has increased the discharge to upregulate in one Balancing stage. If the imbalance switch direction in the next balancing stage, the hydropower station must reduce the discharge and production (according to equation (**60**) below), and the model can become infeasible if the change in discharge is too large.

$$\begin{aligned} -\Delta_{Q_h^R} \le \left(\Delta q_{hk}^{D} + \Delta q_{hk}^{B+} + \Delta q_{hk}^{B-}\right) - \left(\Delta q_{h,k-1}^{D} + \Delta q_{h,k-1}^{B+} + \Delta q_{h,k-1}^{B-}\right) \\ + \left(\hat{q}_{hk}^{R} - \hat{q}_{h,k-1}^{R}\right) \le \Delta_{Q_h^R} \quad \forall h \in \mathcal{H}^B, k \end{aligned} \tag{58}$$

The balancing energy from each hydropower station is limited to the reserved capacity on the station in (59).

$$-\hat{r}_{hk}^{-} \le \Delta p_{hk} \le \hat{r}_{hk}^{+} \quad \forall h \in \mathcal{H}_a^R \cap \mathcal{H}_a^S, k \tag{59}$$

The imbalance is split into a positive part ΔP_{mk}^{+} representing a positive imbalance in e.g., wind power production and a negative part ΔP_{mk}^{-} representing reduced production. This division was made to easily limit the deviation in production in (60) and (63). Constraint (60) ensures deviations in production is only a response to the imposed imbalances. This is explained in more detail in chapter 5.2. The small values δ were introduced to give reasonable dual values of the power balances in (64). If the deviations in production are limited by the size of the imbalance, increasing a particular imbalance by a small amount in the power balance can give a dual value of zero.

$$-\Delta P^{+}_{mk} - \delta \leq \Delta p_{hk} \leq -\Delta P^{-}_{mk} + \delta \quad \forall m, h \in \mathcal{H}^{R}_{a}, k \tag{60}$$

When exchanging reserve capacity and balancing energy between groups, this constraint must be modified to consider the imbalances in neighbouring areas and/or the reserved transmission capacity, as balancing energy in one group can be activated as a response to an imbalance in a neighbouring group. An imbalance netting should be performed in advance of the BM. This is not treated in this report.

5.3.3 Thermal Constraints

Equation (**26**)-(**27**) in the UC&D problem ensures that the reserved capacity can be activated i.e., that the production can deviate from the planned set point within the ramping limits of the thermal unit. However, if reserve capacity is activated in one timestep, this might affect the ability to activate reserve capacity in the subsequent time step within the ramping limits. This will depend on the activation time compared to the time step length of the model, and on the rest time of the power plant. We do not consider rest time in this Balancing model. If the activation time of reserves is assumed to be equal to the length of the model time steps, the activation of balancing energy in one time step will limit the ability to activate reserve capacity in the opposite direction in the subsequent time step. This is illustrated in Figure 13, where the planned production is the blue line, and the reserved capacity for upward and downward regulation is limited by the ramping limits shown by the red lines. Note that the decisions in this figure are discrete, and that the lines represent transitions between the decisions. If downward reserve capacity is activated in time step $k + 1$, and upward reserve capacity is activated in the subsequent step $k + 2$, the transition between the production levels in $k + 1$ and $k + 2$ will imply a ramping rate that is too high. The power trajectory for this activation of balancing energy is represented by the green dashed line.

In this case, ramping constraints, together with the planned production, must enter the BM stage to ensure that ramping limits are complied with when activating reserves. Equation **(61**) ensures that the change in actual production between two consecutive timesteps does not exceed the ramping limits. This reduces a unit's ability to shift fast from delivering up regulation to down regulation and vice versa. Ramping constraints in the Balancing problem can therefore prevent reserved capacity from being activated, and this should maybe have been considered in the planning process when allocating reserve capacity in the UC&D stage. This constraint is however unnecessary if the duration time of the activation is small compared to the time resolution of the model, and it can be assumed that the thermal unit is back at its planned production at the beginning of each time step.

$$\begin{aligned} -\hat{u}_{gk}\Delta \underline{p}^{G}_{g} - \hat{z}_{gk}\Delta \underline{p}^{G*}_{g} &\leq \left(\Delta p_{gk} - \Delta p_{g,k-1}\right) + (\hat{p}_{gk} - \hat{p}_{gk-1}) \\ &\leq \hat{u}_{g,k-1}\Delta \overline{p}^{G}_{g} + \widehat{w}_{gk}\Delta \overline{p}^{G*}_{g} \qquad \forall g \in \mathcal{G}^{R}_{a}, k \end{aligned} \tag{61}$$

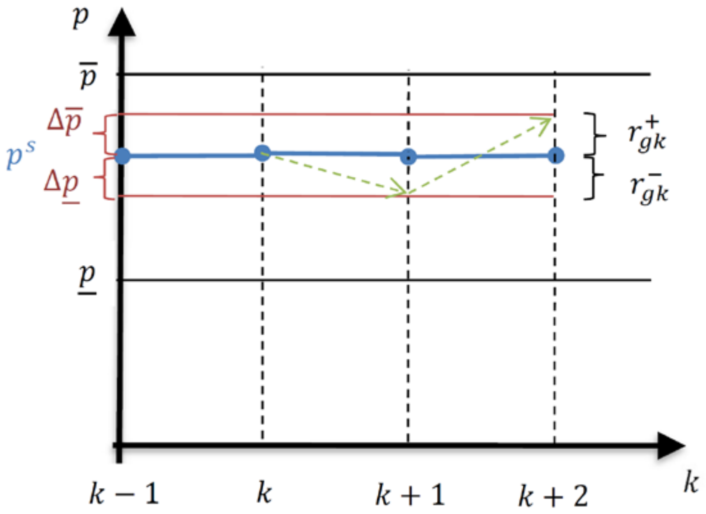

Figure 13 The green dashed line represents a possible power trajectory if reserve capacity is activated in consecutive time steps. This activation violates the ramping rate and is handled by equation (61)**.**

The balancing energy from each thermal power plant is limited to the reserved capacity in (**62**).

$$-\hat{r}_{gk}^{-} \leq \Delta p_{gk} \leq \hat{r}_{gk}^{+} \quad \forall g \in \mathcal{G}_a^R, k \tag{62}$$

Equation (**63**) ensures that all deviations in production is a response to an imbalance, and that production is not "shifted" from one power plant to another.

$$-\Delta P_{mk}^{+} - \delta \leq \Delta p_{gk} \leq -\Delta P_{mk}^{-} + \delta \quad \forall m, g \in \mathcal{G}_a^R, k \tag{63}$$

If exchanging reserve capacity and balancing energy between groups is allowed in the model, care must be taken to allow the deviation in production to also be a response to an imbalance in a neighbouring area. As with the corresponding constraint for hydropower, constraint (**63**) must also be modified to consider the imbalances in neighbouring areas and/or the reserved transmission capacity.

5.3.4 System-Wide Constraints

The power balance for all reserve groups m are given by (**64**). The balancing energy provided by the hydropower stations and thermal generating units plus the net change in import must equal the sum of imbalances in all areas in the reserve group. The model also has the option to buy expensive balancing resources through b_{mk}^{+} and b_{mk}^{-} to meet this constraint.

$$\sum_{a \in \mathcal{A}_m^R} \left[\sum_{h \in \mathcal{H}_a^R} \Delta p_{hk} + \sum_{g \in \mathcal{G}_a^R} \Delta p_{gk} + \sum_{\ell:(a,b) \epsilon \mathcal{L}_a^R} (1 - \zeta_\ell) \Delta f_{bak} - \Delta f_{abk} \right] + b_{mk}^{+} + b_{mk}^{-} = -\Delta P_{mk}^{+} - \Delta P_{mk}^{-} \quad \forall m, k \tag{64}$$

The change in flow Δf on power lines connecting areas in different reserve groups are limited by the reserved capacity on the lines f^+ from the UC&D stage in (**65**). f_{abk}^{+} is the reserved capacity for increasing flow on the power line from area a in one reserve group to area b in another reserve group. The flow on the power line from a to b can increase both due to a positive imbalance in a or due to a negative imbalance in b. Because of this, the reserved capacities f_{abk}^{-} from the UC&D stage is not used in the Balancing problem because f_{bak}^{-} and f_{abk}^{+} are identical.

$$0 \leq \Delta f_{abk} \leq \hat{f}_{abk}^{+} \quad \forall\, \ell : (a, b) \in \mathcal{L}_a^R \tag{65}$$

5.4 Finding Results

The BM stage in Primod can be used to obtain:
- Marginal costs (prices) for balancing energy.
- Cost of balancing the system.
- Specific results from the optimization, e.g., where the balancing energy is activated.

The BM solves a LP problem, so the balancing prices are found as the dual values of equation (64). The cost of balancing the system in (43) represent the costs of deviating from the planned system operation. The cost includes both the deviation in the here and now costs and the deviation in the future expected costs (or value of water) expressed by $\Delta\alpha$. Because the BM steps forward with 4-hour steps, the two-day problem is solved six times. To obtain the correct cost of balancing the system for the studied day, the cost of balancing the system for each 4-hour period together with the final deviation in future expected costs (or value of water) for the end of this day (based on reservoir fillings from the end of balancing stage 6) is given as output from the model.

5.5 Pricing of Balancing Energy

The balancing price is obtained from the BM by taking the dual value of the power balance. If thermal power plants are providing the balancing energy, the price is straight-forward. When the system is upregulated, the price of upregulating with one more unit will be equal to the marginal cost of the power plant delivering this extra unit. If the system is downregulated, the price of increasing the production with one more unit is equal to the marginal cost of the power plant increasing its production.

This is not as straight-forward if hydropower plants are providing the balancing energy, due to the watercourse topologies and the valuation of stored water. Hydropower is valuated according to its expected marginal value of water (per reservoir). To capture this value in the balancing problem, all hydropower reservoirs must be represented and valuated. If a hydropower module regulates up its production, more water than planned in the UC&D from the corresponding reservoir will travel down to the downstream reservoir or to the ocean and lose energy potential and thus economic value. This dynamic is illustrated in Figure 14. The upper module 1 has a large reservoir and a water value of 6 €/m^3. The middle module has no storage capacity and hence no water value. The bottom module has a large reservoir and a water value of 2.5 €/m^3. All modules have a power station with an energy equivalent (in MW/m3), as indicated in the figure. When water is sent from module 1 to module 3, and modules 1 and 2 are producing, the cost of this production is (6.0 €/m^3 – 2.5 €/m^3) / (2.5 MW/m^3 + 1.0 MW/m^3) = 1.0 €/MW.

If reserve capacity is allocated to module 1, and this module reduces its discharge and production to downregulate, the reservoir volume in module 1 increases. The reservoir volume in module 3 will then decrease, since the upstream inflow to this reservoir will be lower than scheduled. The value of the displaced water will increase with 3.5 €/m^3. If the BM only considers that the production at module 1 decreases, then this production will be valuated too high, with 3.5 €/m^3 / 2.5 MW/m^3 = 1.4 €/MW. Since module 2 has no reservoir capacity it will also have to reduce its discharge and production for module 1 to be able to downregulate. Both reductions in production must then be included in the BM, to get a correct cost of balancing. The marginal cost of down regulating will then be 3.5 €/m^3 / (2.5 MW/m^3 + 1.0 MW/m^3) = 1.0 €/MW. The same applies if the system is up regulated.

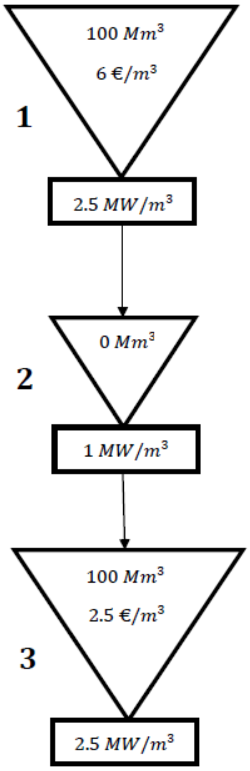

Figure 14 A small cascade of hydropower modules illustrating the cost mechanism of regulating the production at the upper hydropower module.

Hydropower modules with small reservoirs (reservoirs that are not assigned an explicit water value) must be able to change their discharge and production as the upstream module changes its discharge when reserve capacity is activated, to obtain reasonable balancing prices. The modules also need to adjust their discharge according to the incoming discharge to avoid infeasibilities to occur in their reservoir balances. But even though these modules can change their discharge, sometimes they are at full discharge or at minimum discharge, and if the reservoir cannot handle the deviation in incoming water, the waterway for bypass must be used to maintain the reservoir balance. Changing the planned bypass in the BM has a cost, and this can therefore give higher balancing prices.

5.6 Challenges

The BM solves a cost minimization problem and will seek the less costly solution within the defined constraints. If simplifications are made in the BM, the model may find a cheaper dispatch than in the UC&D model. If more expensive production must be dispatched to fulfil the reserve requirements in the UC&D, there may be a possibility for the BM to change the dispatch to a less costly solution. This could be achieved by regulating down the production at some stations, and to regulate up the production at other stations. Constraints are therefore included to ensure that all deviations in production is a response to an imbalance.

The BM is solved with a rolling horizon with six stages, as explained in Figure 7 and the introduction to chapter 5. When the Balancing problem is solved in total six times, and where an increasing part of the problem solution is fixed as the model steps forward, there is a risk that the model may end up in an infeasible situation if an unfortunate decision is taken in a previous stage. This can also occur if the model can make a cost neutral decision in one balancing stage, that will cause problems in the next balancing stage. One example is a reservoir with no production capacity that lies downstream to a regulating power station, as illustrated in Figure 15. This reservoir can increase the discharge to a downstream reservoir without affecting the cost function. If this reservoir is close to its minimum limit, its discharge from one solution can lead to this reservoir exactly reaching its bottom limit. When part of this solution is fixed, and the imbalance is updated, the reservoir might end up violating the minimum limit if it receives less water from the upstream reservoir when the balancing situation changes. This can lead to a more costly solution, or the model can become infeasible. This might have been avoided if another decision was made in the previous stage. The model's ability to find different solutions that are cost neutral should be more restricted. This challenge could be solved by adding penalty variables to ensure feasibility or even treating future imbalances as stochastic.

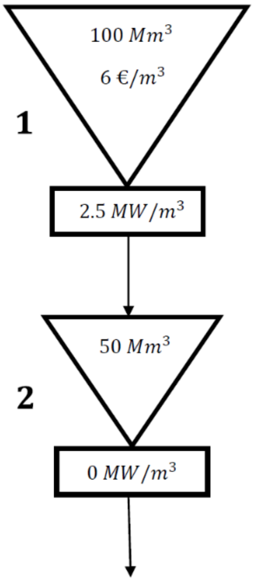

Figure 15 A small cascade of hydropower modules illustrating how the bottom reservoir can experience infeasibility when regulating the production at the upper hydropower module.

Due to the design choice not to check if activation of the reserved capacity at hydropower stations is possible in the UC&D model, there is a risk that activation of reserved capacity can lead to problems in the Balancing stage. Some of the situations that may occur in the BM are listed below. These situations can lead to more expensive regulation elsewhere, or in some cases model infeasibility. We underline that the BM model in its current version needs to be enhanced to be robust towards such situations.

1) The hydropower module delivering upward reserve capacity does not have sufficient water to increase the production.
2) The hydropower module delivering downward reserve capacity does not have sufficient reservoir capacity to store water that is held back when decreasing production.
3) Hydropower modules delivering reserve capacity that are connected to downstream reservoirs:
    a) If the downstream reservoir is almost empty, it may depend on upstream water to maintain the reservoir balance and at the same time sustain the scheduled water flows. This might prevent down regulation reserves from being activated upstream.

b) If the receiving reservoir is close to full, it may not have room to store the extra water it receives if the upstream module increases the production. If the downstream module cannot increase the discharge, this can prevent the upstream hydropower station from regulating up the production. The downstream module can increase the bypass, but this has a cost in the model, and will lead to a higher balancing price/cost.

6 Case Study

A case study using both model stages was performed to demonstrate the use of the BM and present results on reserve and balancing prices. The study was done using the HydroCen Low Emission Scenario describing a future scenario of the Northern European power system for year 2030 [25]. The model setup and reserve requirements are much the same as in the Case Study #2 presented briefly in chapter 4.6.3, but in this study we focus on marginal costs rather than costs, and we also include the BM to obtain marginal costs for balancing energy.

6.1 Case study setup

This case study is limited to one selected week, week 9, with weather data from a dry inflow year, 1969. Cuts from the FanSi model [2] is used as end-valuation of stored water in this study. The initial reservoir levels for week 9 are also obtained from FanSi. Primod is run with reserve requirements for the Nordic counties, see Table 1, with requirements per price areas/bidding zone and without the possibility to exchange reserve capacity between price areas. The model was run with hourly time resolution, start-up costs and minimum production on thermal power plants and selected hydropower stations and relaxed start-up binary variables for both thermal power plants and hydropower stations. The start-up cost for each hydropower module is estimated by a linear function of maximum production capacity, P: Start-up cost = 100 + 0.2P €. Minimum production is estimated from the production-discharge curve (PQ-curve) for each module, where the minimum generation is set to 50 % of the best efficiency point, and a 20% reduction in efficiency when operating at minimum generation (point ’A’ in Figure 10) compared to the best efficiency. To limit the number of binary variables, only the 292 modules with more than 20 MW installed capacity and 2 Mm^3 storage capacity were represented with relaxed binary commitment variables. These modules can deliver spinning reserve capacity. Selected gas and bio power plants in SE3, SE4, Finland and DK2 are reserve capacity providers in areas with limited hydropower resources. Random generated, normal distributed imbalances (limited by the size of reserve requirements to avoid infeasibilities) was applied to each price area in the BM. The imbalances were updated between each stage in the BM.

Table 1 Spinning reserve requirements per bidding zone.

Country	Bidding zone	Requirement [MW]
Norway	NO1	167
	NO2	229
	NO3	83
	NO4	146
	NO5	167
Sum		**792**
Sweden	SE1	216
	SE2	157
	SE3	354
	SE4	197
Sum		**924**
Finland	FI	**335**
Denmark	DK2	**356**
Sum		**2407**

6.2 Results

6.2.1 Balancing in Thermal Area (DK2)

The energy price, and upward and downward reserve capacity prices for DK2 are shown in Figure 16[3]. In almost every hour there is a price for procuring reserve capacity. A price for downward reserve capacity indicates that one or more power plants must run above the set point that would have been optimal without reserve procurement constraints. This typically happens if the marginal cost of the plant is higher than the energy price. When the energy price is high, the price for reserving downward capacity goes to zero because the production increases and there is an abundance of downward capacity. A price for upward capacity indicates that one or more power plants produce below the set point that would have been optimal without reserve procurement constraints even though the marginal cost of the plant(s) is below the energy price. The production must instead come from more expensive sources (other power plants or import). In DK2, the price for reserving upward capacity is never zero for the studied week.

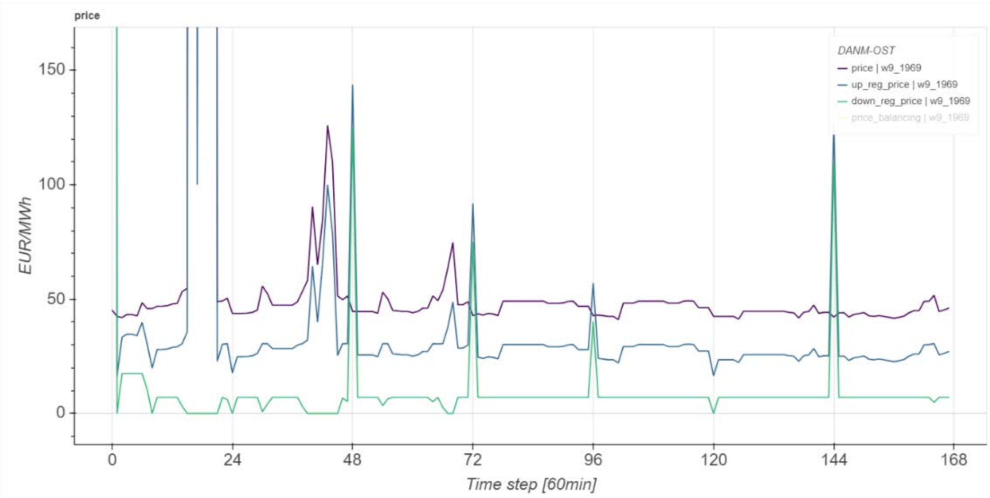

Figure 16 The energy price, and price for reserving upward and downward reserve capacity in DK2 for week 9 in 1969.

[3] The spikes seen in the shift between days is most likely due to an inconsistency in coupling states between the simulated sequences. We did not spend time on further investigate the actual reason.

DK2 is the only price area in the Nordic without any hydropower plants, and only thermal power plants (bio- and gas-fired) are reserve capacity providers in our scenario. The marginal costs for these power plants are presented in Table 2. All these power plants are dispatched in week 9 to deliver reserve capacity. Plant 10, 11 and 13 have low marginal costs and are producing at maximum capacity and delivering downward reserve capacity. Plant 12 have marginally more expensive production than plant 11 and is delivering both upward and downward reserve capacity. The same applies for plant 15. The more expensive power plants are mainly producing at minimum capacity and providing upward reserve capacity.

Table 2 The marginal cost (€/MWh) for thermal power plants delivering reserve capacity in DK2

Thermal power plant no. (DK2)	Marginal cost (€/MWh)
10	17.86
11	18.49
12	18.96
13	8.57
15	26.00
17	50.00
18	56.53
19	90.66

The dual value of the power balance (64) in the Balancing problem represents the balancing price, and is presented in Figure 17 together with the energy price. The imposed imbalance in DK2 is shown in Figure 18. A negative imbalance causes upregulation and vice versa. There are only five unique balancing prices; 56.53, 26.00, 18.96, 50.00 and 90.66 €/MWh, and these prices are the marginal costs of thermal power plant 18, 15, 12, 17 and 19. A balancing price below 30 €/MWh occurs when there is up- or downregulation provided by plant 12 and plant 15. Plant 10, 11 and 13 are not activated because it is more beneficial to reduce the production at power plants with higher marginal costs. Balancing prices at 50.00 €/MWh occurs when there is a large negative imbalance, and plant 17 must contribute with the upregulation. When the energy price is high, power plants with high marginal costs (18 and 19) produces above minimum production and can regulate the production down. If a positive imbalance occurs, these power plants are the most beneficial to regulate down, and the balancing price reaches 56.53 and 90.66 €/MWh.

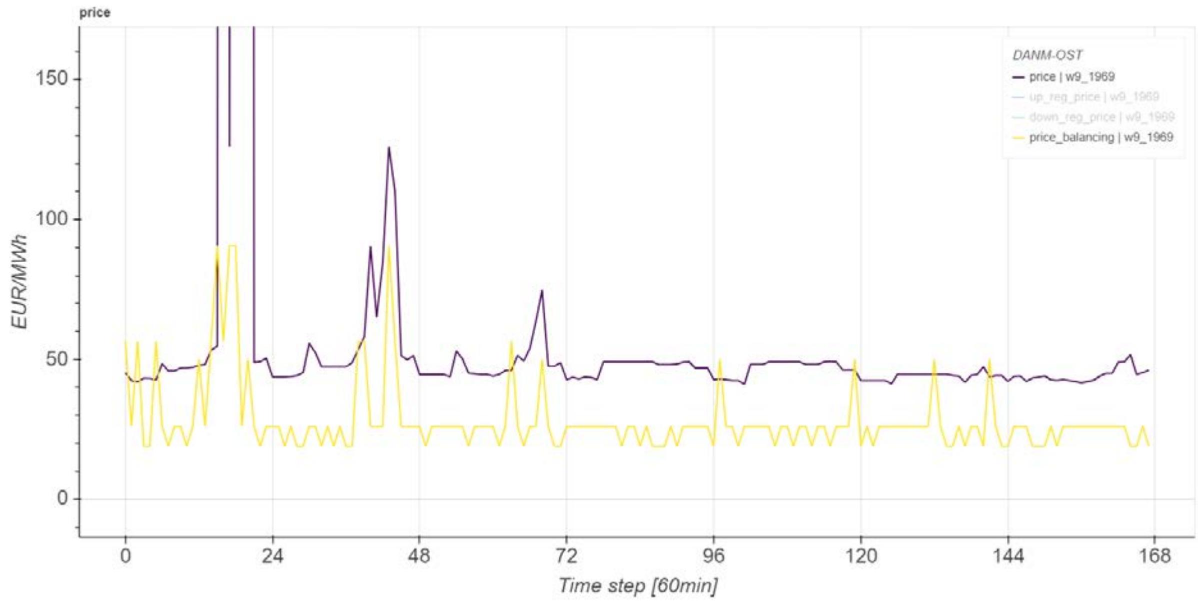

Figure 17 The energy price and the balancing price in DK2 for week 9 in 1969.

We see that the balancing price is mostly below the energy price, even though there is upregulation. This is because the energy price (obtained when solving the UC&D problem) reflects the cost of procuring reserve capacity. This implies that reserve capacity is allocated power plants with marginal costs below the energy price.

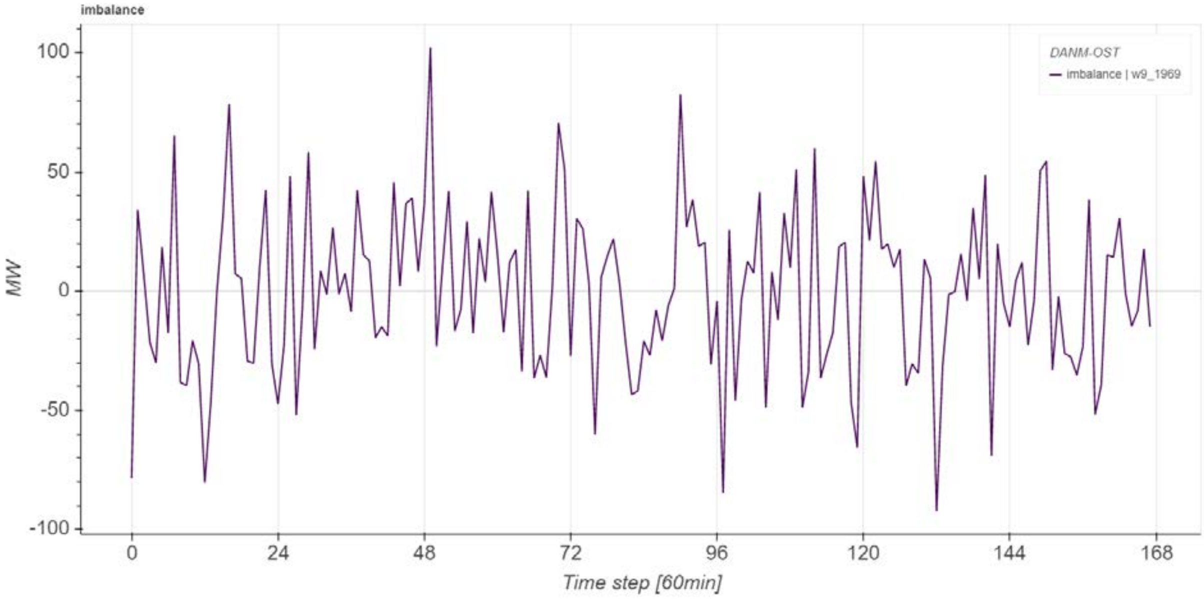

Figure 18 The imposed imbalance in DK2 for week 9 in 1969.

6.2.2 Balancing in Hydro Area (NO2)

The energy price, and upward and downward reserve capacity prices for area Telemark in price area NO2 are shown in Figure 19. The reserve demand for upward and downward capacity in this price area is 229 MW, and the marginal prices of these products show that most of the time there is an abundance of available capacity. There are many (40-50) hydropower stations providing reserve capacity in NO2. Week 9 is a winter week with high demand, and many power plants are producing at a high level and the downward reserve capacity can be allocated for free. The price for downward reserve capacity is 0 €/MW during the whole week. The same applies for the upward reserve capacity most of the time, but when the energy price is high, there is a price for reserving upward capacity. Hydropower stations have their best efficiency point (point 'B' in Figure 10) below maximum production, and stations can therefore produce at best efficiency while providing upward reserve capacity. When the energy price increases many of these power plants will produce at a higher level, and the available upward capacity decreases. This lead to a marginal price for reserving one more unit of upward reserve capacity, because one or more power plants must produce below their optimal dispatch.

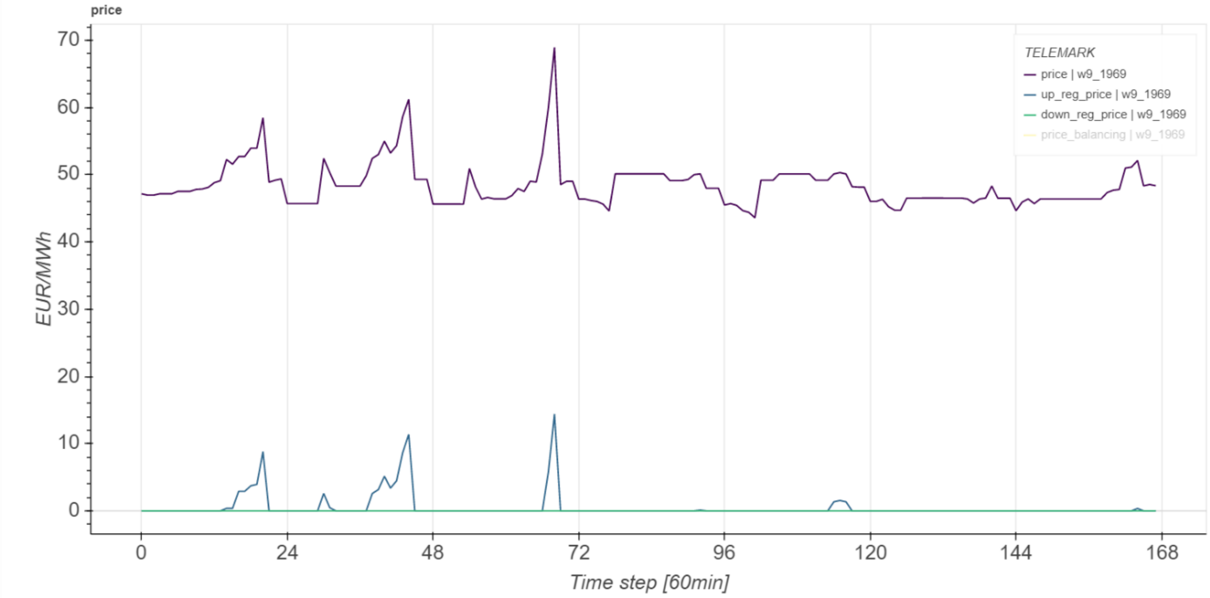

Figure 19 The energy price, and price for reserving upward and downward reserve capacity in NO2 for week 9 in 1969.

The balancing price for NO2 is presented in Figure 20 together with the energy price for the area Telemark and the area Sorland in NO2. The imposed imbalance is shown in Figure 21. The balancing prices in NO2 differ more than for DK2, both because of more power plants contributing to the balancing and because the pricing of balancing energy from hydropower is more complex, as partly described in 5.5 and 5.6. The price area NO2 consists of four modelled areas with transmission lines (and losses) between them. This can give differences in energy price between these areas, as seen in Figure 20. When activating reserve capacity in the BM, the transmission lines between these areas are ignored, and these four areas are seen as one large area (reserve group) with no internal bottlenecks. Our results show that Sorland has a lower energy price, and Telemark has a higher energy price in NO2. Telemark therefore provides a higher portion of downward reserve capacity, while Sorland provides most of the upward reserve capacity. Downregulation will take place where the production can be reduced to achieve the highest reduction in costs, while upregulation will take place where production can be increased at the lowest cost. This will result in mostly upward reserve capacity being activated in Sorland, and downward capacity being activated in Telemark. The marginal cost of increasing the production with one unit (the balancing price) can therefore be higher when activating downward reserve capacity than when activating upward reserve capacity. In this example we ignored the transmission grid bottlenecks in the BM while respecting (some of) them in the UC&D. This is a simplification that provided some counterintuitive results because the upregulation can have a lower marginal cost than the downregulation.

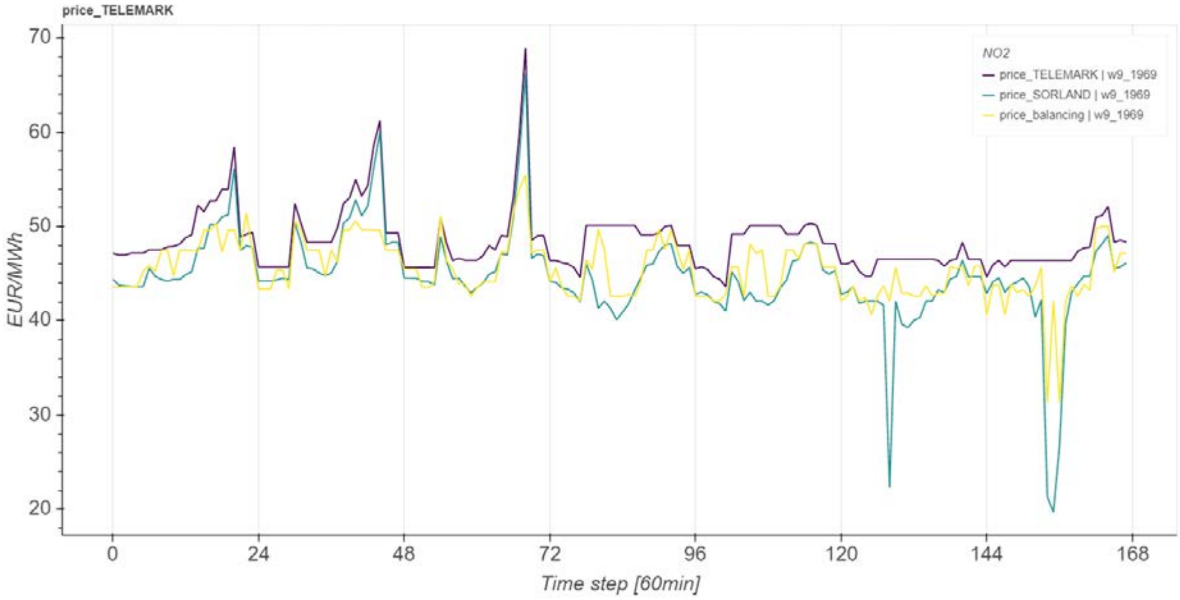

Figure 20 The energy price in Telemark and Sorland in NO2 together with the balancing price in NO2 for week 9 in 1969.

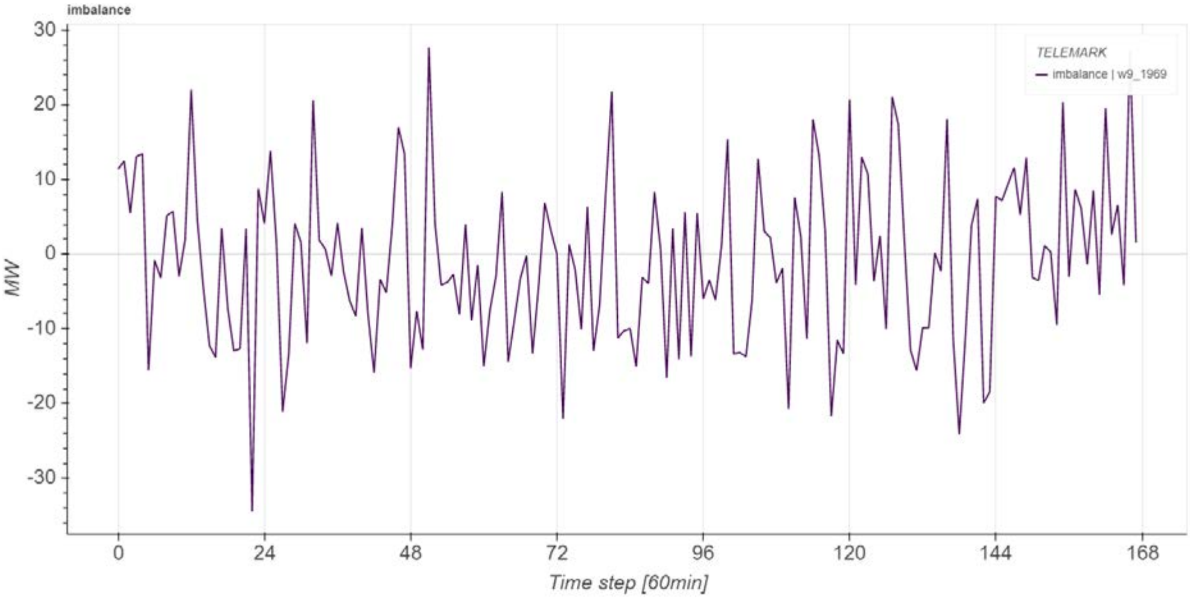

Figure 21 The imposed imbalance in NO2 for week 9 in 1969.

We will look more closely at the activation of balancing energy at three hydropower stations, 10532 (Dynjafoss) in area Sorland, and 7836 (Frøystul) and 7845 (Hjartdøla) in area Telemark. These modules set the balancing price in several hours. Dynjafoss station is connected to a small reservoir of 2.0 Mm^3 and sends water to two stations with zero reservoir capacity downstream before it ends up in a larger reservoir. Frøystul is connected to a large reservoir (> 1000 Mm^3) with no upstream reservoirs, but with many small reservoirs in cascade downstream. Dynjafoss has a medium-sized reservoir of 61 Mm^3 and is situated in the same watercourse as Frøystul, but at the lower part of the watercourse. It sends water directly to a regulated reservoir. The topology for each hydropower module can be found in appendix A.3.

Figure 22 show the planned hydropower production from Dynjafoss station from the UC&D model together with the adjusted hydropower production from the BM. The limits for upward and downward regulation are also plotted, and they coincide with the maximum and minimum production of the power station. During the weekend, these limits drop because the power plant is not fully committed (start-up variable < 1). Dynjafoss station is both delivering upward and downward reserve capacity but is only activated for upward regulation in the BM. Dynjafoss lies in Sorland where the energy price is lowest, and as a result, the cheapest activation of upward reserve capacity is found here. When Dynjafoss regulates up the production, the production also increases at the downstream modules as they receive more water. Dynjafoss is setting the balancing price in many hours, and the price deviates between 40 €/MW and 45 €/MW depending on the production level and which PQ-curve segment (efficiency) is used for increasing the production.

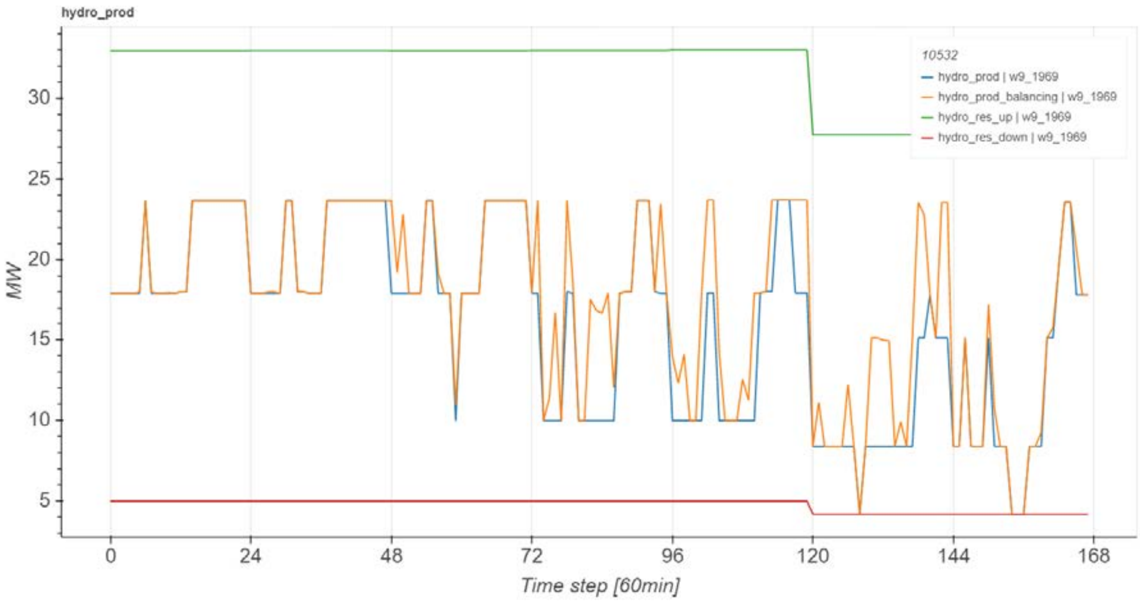

Figure 22 The planned hydropower production and the adjusted hydropower production from Dynjafoss station in NO2.

The planned hydropower production from Frøystul station is presented in Figure 23 together with the limits for upward and downward regulation and the adjusted production when reserve capacity is activated. This hydropower station also delivers bot upward and downward reserve capacity, but is only used for downward regulation in the BM. Frøystul lies in Telemark where the energy price is higher, and it is optimal to reduce the production where the most costly resources are dispatched. The power station is connected to four downstream stations with very small reservoirs, so when the production is decreased and less water travels down the watercourse, all these four power stations must also reduce the production, as explained in 5.5. This string of hydropower stations set the balancing price in some hours, with prices ranging from 40 €/MW to 50 €/MW. The lowest balancing prices occur when the production is low, and the production is regulated down. Then the power station is at a high efficiency. The highest balancing prices are found when downward reserve capacity is activated at high production levels at lower efficiencies.

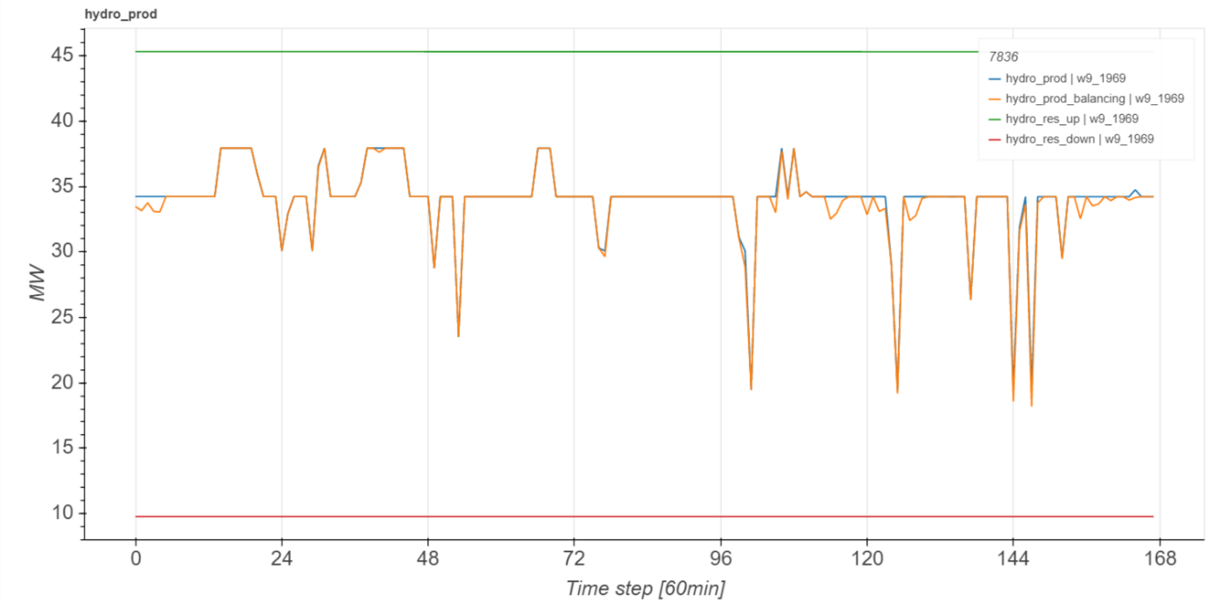

Figure 23 The planned hydropower production and the adjusted hydropower production from Frøystul station in NO2.

Figure 24 show the planned and adjusted hydropower production at Hjartdøla station. The limits for upward and downward regulation are also shown. This power station is activated for both upward and downward regulation and sets the balancing price at 47 €/MW or 50 €/MW depending on the level of the planned production.

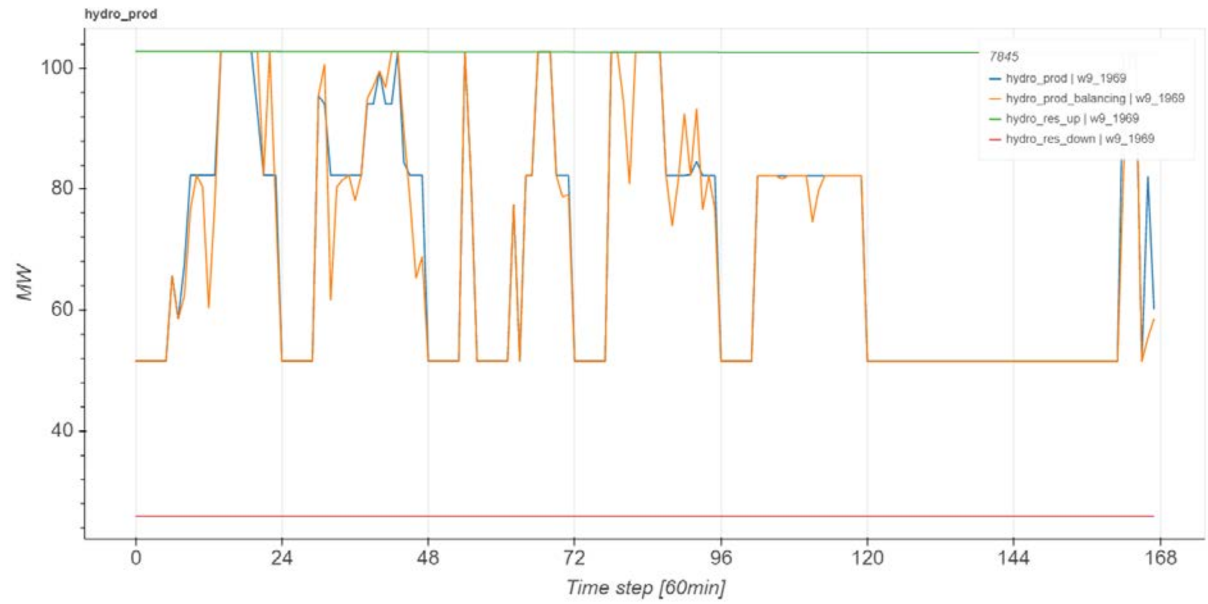

Figure 24 The planned hydropower production and the adjusted hydropower production from Hjartdøla station in NO2.

6.3 Discussion

We have demonstrated above that the UC&D and BM gives reasonable and interpretable results for marginal costs of energy and reserve capacity, allocation of reserve capacity, activation of reserve capacity and balancing prices when imposing imbalances to both thermal- and hydro-dominated price area.

We acknowledge that the BM has some limitations due to simplifications made in the modelling. The presented example in 6.2.2 clearly demonstrates the importance of harmonized transmission grid modelling across the UC&D and BM models. Alternative methods exist for the design of the BM that might give a better and/or faster model. Each 4-hour Balancing stage could be defined as one single isolated problem, with a customized end-valuation to obtain smaller problems sizes. However, the end-valuation of water for each 4-hour stage is not straight-forward to obtain. Another approach could be to only impose an imbalance for the first 4 hours, and not for the whole model horizon. Having imbalances for the whole period can give a different solution than only seeing imbalances for the first 4 hours. Seeing imbalances beyond the 4 hours is more in line with reality where e.g., weather forecasts are updated, and you get more reliable information about uncertain parameters as you move closer to real-time. The most ideal approach would be to have stochastic imbalances for the remaining period, but this would increase the complexity and computation time of the model.

7 Summary

This report has described the short-term hydrothermal scheduling model Primod, developed within research project "Pricing Balancing Services in the Future Nordic Power Market" (PRIBAS). The Primod model is a prototype model for hydro-thermal power systems designed to be adapted to the expected properties of a future power system. Primod consists of a Unit Commitment & Dispatch model co-optimizing electricity and reserve capacity and a Balancing Model simulating the costs and marginal costs of handling exogenously defined system imbalances. The report discussed and documented the mathematical formulation for both the UC&D and BM models. Moreover, a case study demonstrating the use of Primod, focusing on results from the BM model, was presented.

The Primod model should be seen as a component in a comprehensive scheduling toolchain suited for hydro-dominated power systems. The availability of such tools implemented in high-level and flexible programming languages opens for detailed studies of both system operation and market design. We underline that the model has significant room for future research, extension and improvement. Important functionalities such as detailed transmission grid representation and demand response are not represented in the current version.

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A Appendix

A.1 Nomenclature Unit Commitment & Dispatch

Sets:

$\mathcal{K}$	Set of time steps
$\mathcal{A}$	Set of price areas
$\mathcal{H}$	Set of hydropower modules
$\mathcal{G}$	Set of thermal generating units
$\mathcal{D}$	Set of price-elastic demand

$\mathcal{L}$	Set of connections
$\mathcal{L}^{AC}$	Set of connections with AC power lines
$\mathcal{L}^{DC}$	Set of connections with HVDC cables
$\mathcal{N}_h$	Set of segments on the PQ-curve to hydropower module h
$\omega_h^D, \omega_h^B, \omega_h^S$	Set of hydropower modules with discharge, bypass and spillage to module h
ω_h^U, ω_h^P	Set of hydropower modules with tunnelling and pumping to module h
$\mathcal{C}$	Set of cuts (maximum iterations)
$\mathcal{W}$	Set of water value segments
$\mathcal{M}$	Set of reserve groups
WP_a	Set of wind parks per price area a
$\mathcal{A}_m^R$	Set of price areas in reserve group m
$\mathcal{H}^R, \mathcal{G}^R$	Set of all hydropower modules and thermal generators delivering reserve capacity
$\mathcal{L}_a^R$	Set of connections from price area a in reserve group m to another reserve group

Parameters:

Hydropower:

C^{B^*}	Bypass penalty (10^3mu/m^3/s) [4]
C^{S^*}	Spillage penalty (10^3mu/m^3/s)
C^B	Cost for bypass violation (10^3mu/m^3/s)
C^D	Cost for discharge violation (10^3mu/m^3/s)
C^T	Cost for tanking (10^3mu/m^3/s)
C_h^S	Start-up cost for hydropower module h in time step k (10^3mu/MW)
I_{hk}^R	Regulated inflow to hydropower module h in time step k (Mm^3)
I_{hk}^U	Unregulated inflow to hydropower module h in time step k (Mm^3)
$\underline{V}_h, \overline{V}_h$	Minimum and maximum allowed reservoir level for hydropower module h (Mm^3)
$\underline{Q}_h^D, \overline{Q}_h^D$	Minimum and maximum discharge for plant h (m^3/s)
$\underline{Q}_h^B, \overline{Q}_h^B$	Minimum and maximum bypass for plant h (m^3/s)
$\underline{Q}_h^U, \overline{Q}_h^U$	Minimum and maximum capacity of hydraulic coupling/tunnel to and from hydro reservoir h (m^3/s)
$\underline{P}_h, \overline{P}_h$	Minimum and maximum production for hydropower plant h (MW)
$\overline{Q}_h^P$	Maximum pumping capacity for plant h (m^3/s)
η_h^P	Pumping power/efficiency for plant h (MW/m^3/s)
$\overline{Q}_{nhk}^D$	Maximum discharge volume per PQ-curve segment n for hydropower plant h (m^3/s)
$\underline{Q}_h^{D*}$	Minimum discharge corresponding to minimum production for hydropower plants $h \in \mathcal{H}^R$ (m^3/s)
η_{nh}	Generation efficiency per PQ-curve segment n for hydro plant h (MW/m^3/s)
J_h	Relative head (h_h/h_{h0}), refers to initial reservoir level, for hydro plant h
Γ_k	Conversion factor between water flow (m^3/s) and water volume (Mm^3)

Thermal power:

C_{gk}^G	Marginal cost for thermal unit g in time step k (10^3mu/MWh)
C_g^S	Start-up cost for thermal unit g in time step k (10^3mu)

[4] mu stand for "monetary unit"

$\overline{p}_{gk}^{G}$	Maximum capacity for thermal unit g in time step k (MW)
$\underline{p}_{gk}^{G}$	Minimum capacity for thermal unit g in time step k (MW)
$\Delta\underline{p}_{g}^{G}, \Delta\overline{p}_{g}^{G}$	Maximum ramp-down and ramp-up rate for thermal plant g (MW/h)
$\Delta\underline{p}_{g}^{G*}, \Delta\overline{p}_{g}^{G*}$	Maximum ramp-down and ramp-up rate for thermal plant g when shut down or started up (MW/time step)
u_g^0	Initial commitment status of thermal plant g
T_g^U	Minimum number of hours that unit g must be started up (h)
T_g^D	Minimum number of hours that unit g must be shut down (h)
T_g^{U0}	Minimum number of hours that unit g has been online prior to the study horizon (h)
T_g^{D0}	Minimum number of hours that unit g has been offline prior to the study horizon (h)
T_g^{Ui}	Minimum number of hours that unit g must be initially online due to the minimum uptime constraint (h). $T_g^{Ui} = Min\{\lvert\mathcal{K}\rvert, [T_g^U - T_g^{U0}]u_g^0\}$
T_g^{Di}	Minimum number of hours that unit g must be initially offline due to the minimum downtime constraint (h). $T_g^{Di} = Min\{\lvert\mathcal{K}\rvert, [T_g^D - T_g^{D0}][1 - u_g^0]\}$

Coupling to LTM:
Cuts from FanSi:

β_c	Constant value in cost function (intersect) for cut c (10^3mu)
π_{ht}	Cut coefficient for hydropower module h for week t (10^3mu/Mm^3)

Individual water values from EMPS-W:

WV_{hwt}	Water value segment w for reservoir h for the end of week t (10^3mu/Mm^3)

System:

D_{ak}	Price inelastic demand in price area a in time step k (MW)
P_{ak}	Wind (and solar) power production in price area a in time step k (MW)
R_m^+, R_m^-	Reserve requirements for upward and downward spinning reserves per group m (MW)
C_a^R	Cost of relaxing reserve requirement for price area a (10^3mu/MW/h)
B^R	Small income for procuring reserving reserve capacity (10^3mu/MW/h)
C_{dk}^D	Cost of price-elastic demand d in time step k (10^3mu/MWh)
C_a^E	Cost of curtailment for price area a (10^3mu)
F_{ab}	Maximum transmission capacity from price area a to price area b (MW)
ζ_ℓ	Transmission loss fraction on transmission line ℓ ([0,1])
ϕ	Fraction of transmission capacity allowed for reserve exchange
τ	Maximum activation time of rotating reserve capacity (fraction of model time step)
Δ_ℓ	Maximum ramping rate on HVDC cable ℓ (MW/h)

<u>*Variables:*</u>
Hydropower:

v_{hk}	Reservoir volume for hydropower module h in time step k (Mm^3)
q_{hk}^R	Release from hydropower module h in time step k (m^3/s)
q_{hk}^D	Discharge from hydropower module h in time step k (m^3/s)
q_{nhk}^D	Discharge per PQ-curve segment n for hydro plant h in time step k (m3/s)
q_{hk}^B	Bypass from hydropower module h in time step k (m^3/s)

q_{hk}^S	Spillage from hydropower module h in time step k (m^3/s)
q_{hk}^U	Tunnelling from hydropower module h in time step k (m^3/s)
q_{hk}^P	Pumping from hydropower module h in time step k (m^3/s)
q_h^T	Tanking to hydropower reservoir h (m^3/s)
q_{hk}^{Bviol}	Bypass violation from hydropower module h in time step k (m^3/s)
q_{hk}^{Dviol}	Discharge violation from hydropower module h in time step k (m^3/s)
p_{hk}	Production from hydropower module h in time step k (MW)
u_{hk}	Unit commitment status for hydropower plant h (binary or [0,1])
r_{hk}^-, r_{hk}^+	Reserved down- and upward capacity on hydropower plant h in time step k (MW)
α_t	Future expected cost function or value of stored water for the end of week t (10^{3}mu)
v_{hw}	Volume per reservoir level w for hydropower module h in the last time step (Mm^3)

Thermal power:

p_{gk}	Thermal power generation/consumption (MW)
u_{gk}	Unit commitment for thermal unit g for time step k (binary or [0,1])
w_{gk}	Start-up variable for thermal unit g for time step k (binary or [0,1])
z_{gk}	Shut-down variable for thermal unit g for time step k (binary or [0,1])
r_{gk}^-, r_{gk}^+	Reserved downward and upward capacity on thermal unit g in time step k (MW)

System:

y_{dk}^D	Price-elastic demand d in time step k (MW)
d_{ak}	Dump power at zero cost for price area a in time step k (MW)
y_{ak}^E	Curtailment for price area a in time step k (MW)
y_{ak}^{R+}, y_{ak}^{R-}	Relaxation of reserve requirement for upward and downward reserves (MW)
b^R	Benefit for procuring reserve capacity (10^{3}mu)
f_{abk}	Flow from price area a to price area b in time step k (MW)
f_{abk}^+, f_{abk}^-	Reserved capacity on transmission line from price area a to price area b for exchange of upward and downward reserve capacity in time step k (MW)
	Reserved capacity on transmission line from price area a to price area b for exchange of downward reserve capacity in time step k (MW)

A.2 Nomenclature Balancing Model

Sets:

$\mathcal{H}^B$	Set of hydropower modules in price areas with reserve requirements (areas in $\mathcal{A}^R$)
$\mathcal{H}^S$	Set of hydropower modules with reservoir capacity $\leq 0.1\ Mm^3$ downstream to reservoirs in $\mathcal{H}^R$

Parameters:

ΔP_m^-	(≤ 0) negative imbalance for reserve group m (MW)
ΔP_m^+	(≥ 0) positive imbalance for reserve group m (MW)
C^R	Cost of buying expensive balancing resources (10^{3}mu/MW)
δ	Small value (10^{-6}) (MW)

Variables:

α_t^b	Future expected cost function or value of stored water for the end of week t (10^{3}mu)
$\Delta\alpha_t$	Deviation in Future Profit Function for week t (10^3 mu)
Δp_{gk}	Deviation in production from thermal plant g in time step k (MWh)

Δq_{hk}^{B+}	Positive deviation (increase) in bypass for hydro plant h in time step k (m^3/s)
Δq_{hk}^{B-}	Negative deviation (decrease) in bypass for hydro plant h in time step k (m^3/s)
Δq_{hk}^{D}	Deviation in discharge for hydro plant h in time step k (m^3/s)
v_{hk}^{b}	Reservoir filling for plant h in time step k (Mm3)
Δp_{hk}	Deviation in production from hydro plant h in time step k (MW)
$q_{nhk}^{D,b}$	Discharge per PQ-curve segment n for hydro plant h in time step k (m^3/s)
b_{mk}^{+}, b_{mk}^{-}	Expensive positive and negative balancing energy (MW)
Δf_{abk}	Deviation in flow from area a in reserve group m to area b in another reserve group in time step k (MW)

A.3 Hydropower modules in case study

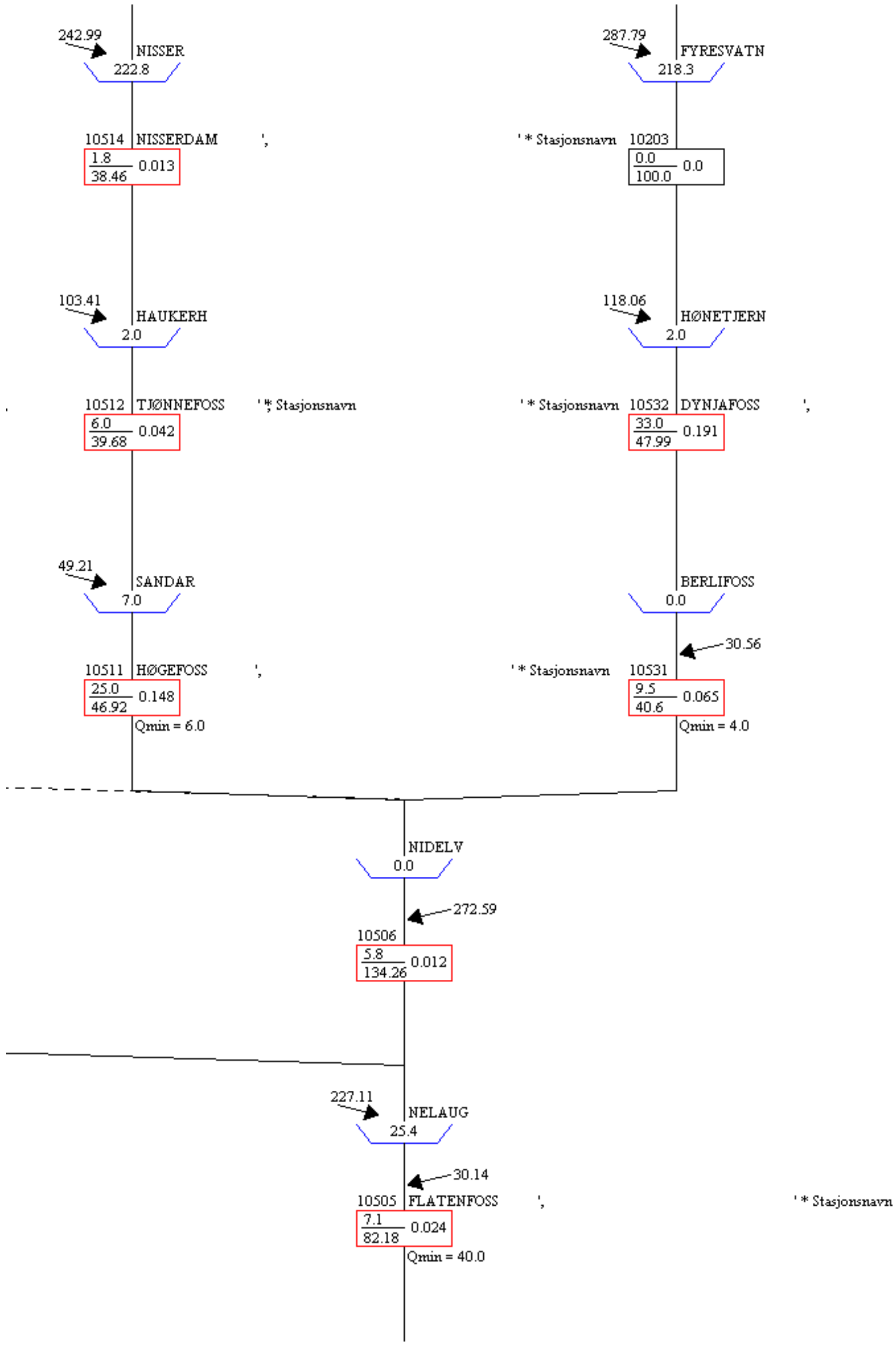

Figure 25 Dynjafoss hydropower station and surrounding modules

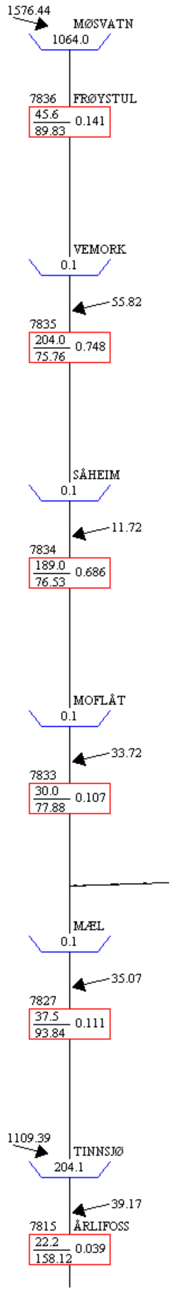

Figure 26 Frøystul hydropower station and downstream modules

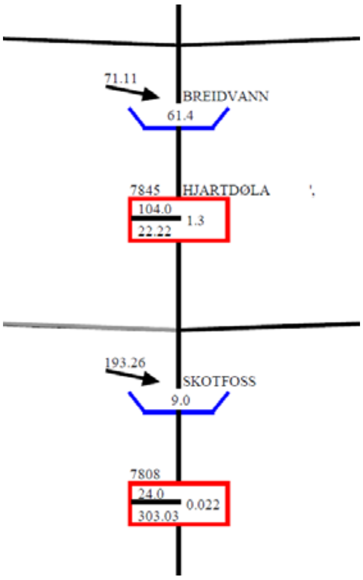

Figure 27 Hjartdøla hydropower station and its downstream module

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