

Effects of scaling laws on the combustion and NO_x characteristics of hydrogen burners

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Abstract

The effect of constant velocity and constant residence time scaling on the local nitric oxide (NO_x) emissions and flame characteristics of complex partial premixed hydrogen burners were investigated numerically and theoretically. A previously developed and validated computational fluid dynamic (CFD) model was employed to conduct in total 11 simulations at various burner scales ranging from a base case of 10 kW to an up-scaled burner design at 500 kW. The flame characteristics were investigated by means of a novel CFD based regime diagram and compared to Damköhler and Karlovitz numbers obtained from scaling theory. The flame is at laboratory scale mainly characterized by the thin reaction zone regime. Employing constant velocity scaling was predicted to overall decrease the Karlovitz number, which causes the combustion to appear partially in the corrugated flamelet regime and at scales exceeding 250 kW also in the wrinkled flamelet regime. Constant residence time scaling on the other hand leads overall to a combustion with constant Damköhler numbers. However, for a constant Karlovitz number close to unity was observed for a significant part of the flame-sheet, which leads in this flame regions to a variable Damköhler number. Both investigated scaling principles lead to an increase of the overall NO_x emissions, with constant velocity scaling resulting in the highest emissions. This is mainly attributed to the larger volumes and longer

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residence times of the flame and immediate post flame region compared to constant residence time scaling. The total NO_x formation in the inner recirculation zone, on the other hand, is lower for constant velocity scaling and is found to be dominated by the local oxygen atom (O) and hydroxyl (OH) concentration. Constant velocity scaling causes a breakup of the inner recirculation zone at the 500 kW scale, which leads to a fundamentally different flow field and causes the flame to impinge onto the combustion chamber wall, whereas constant residence time scaling maintains the inner recirculation zone at all investigated scales. The breakup of the recirculation zone is attributed to the different effect of the scaling principles on the velocity to length scale ratio and momentum of the annular jet flow.

Keywords: scaling, NO_x emission, hydrogen, partially premixed burner, combustion regime diagram

1

2 **Nomenclature**

3 **Symbols**

4 A, A', B₁, B₂, C, C' markers

5 D diameter (m)

6 K proportionality constant (-)

7 k turbulent kinetic energy ($\text{m}^2 \text{s}^{-2}$)

8 L_{IRZ} inner recirculation zone length(m)

9 l length scale (m)

10 l' turbulent length scale (m)

- 11 Q thermal input (W)
- 12 S_L laminar flame speed (m s^{-1})
- 13 U velocity (m s^{-1})
- 14 u cartesian velocity component (m s^{-1})
- 15 u' turbulent velocity scale (m s^{-1})
- 16 Y mass fraction (-)
- 17 y spacial coordinate (m)

18 **Greek**

- 19 β scaling factor (-)
- 20 δ_L laminar flame thickness (m)
- 21 ϵ dissipation rate ($\text{m}^2 \text{s}^3$)
- 22 η_k Kolmogorov length scale (m)
- 23 ν kinematic viscosity ($\text{m}^2 \text{s}^{-1}$)
- 24 ρ density (kg m^{-3})
- 25 τ_c chemical time scale (s)
- 26 τ_k Kolmogorov time scale (s)
- 27 τ_T turbulent time scale (s)

28 **Dimensionless groups**

29 Da Damköhler number

30 Ka Karlovitz number

31 Re Reynolds number

32 **Subscript**

33 0 burner characteristic

34 ax axial direction

35 rt constant residence time

36 v constant velocity

37 **Superscript**

38 * scaled

39 eq chemical equilibrium

40 **Abbreviations**

41 CFD computational fluid dynamic

42 EDC eddy dissipation concept

43 FL flame

44 IPF immediate post flame

45 IRZ inner recirculation zone

46 MILD moderate or intense low-oxygen dilution

47 NO_x nitric oxide

48 ORZ outer recirculation zone

49 PDF probability density function

50 PF post flame

51 PPBB partially premixed bluff body

52 RANS Reynolds averaged Navier-Stokes

53 WJ wall jet

54 **1. Introduction**

55 Increasingly stringent regulations on emissions from stationary power and heat production has
56 motivated the development of various low emission technologies, including carbon capture and storage
57 units, flexi-fuel units enabling the use of non-carbon containing fuels such as hydrogen and recently
58 ammonia [1], and various low and ultra low NO_x burners over the last decades. For the latter, dry
59 low emission (DLE) combustion has traditionally been the most common used technology to reduce
60 NO_x emissions [2]. DLE combustion is dependent on highly controlled mixing of air and fuel to
61 achieve lean premixed combustion, thereby reducing the flame temperature and hence reducing NO_x
62 formation. Due to the reduced temperatures, special considerations have to be made, to ensure an
63 environment that at the same time does not promote unacceptable levels of unburnt hydrocarbons
64 and carbon monoxide (CO), as well as combustion instabilities. When considering high hydrogen
65 content fuels, other issues come into play. Higher burning velocities and temperatures need to be
66 carefully controlled by design optimization to avoid flashbacks. As a result, burners with complex
67 flow regimes have been proposed. This involves for example swirl burners [3], flame-sheet burners
68 [4], micro-mixing burners [5] and recently partially premixed bluff body burners [6, 7]. Common

69 for these burners are highly optimized flows and complex designs in order to obtain optimum low
70 emission, yet efficient operation.

71 The high cost associated with the development process of such complex burners motivates exper-
72 iments and numerical simulations at laboratory scale. This requires scaling of the burner geometry
73 by employing scaling laws that preserve the burner characteristics at different burner sizes. However,
74 the large number of scaling parameters found through similarity theory, many of them mutually
75 incompatible, make a complete scaling practically impossible. Scaling problems are therefore treated
76 by the use of partial scaling, which has been reviewed in detail by Spalding [8] and Beér [9]. Thus a
77 variety of different scaling laws can be found in literature, the majority of which have been developed
78 for “simple” axisymmetric, turbulent, jet flames, which provide a well-defined flow field [10]. How-
79 ever, for industrial burners typically only two scaling laws are considered, namely constant velocity
80 (i.e., the characteristic burner velocity, U_0 , is kept constant while increasing the burner dimensions)
81 and constant residence time scaling (i.e., the ratio between the characteristic burner length scale and
82 velocity scale, D_0/U_0 , is kept constant while increasing the burner dimensions) [11]. Both approaches
83 are based on the basic global equation for the thermal input, Q :

$$Q = K \rho_0 U_0 D_0^2, \quad (1)$$

84 where K is a proportionality constant and ρ_0 the inlet fluid density. They, furthermore, demand
85 geometrical similarity, hence all dimensions can be derived from the scaled characteristic burner
86 length scale, D_0^* , and assume that the Reynolds and Froude number are sufficiently large, so that
87 the burner flow is turbulent and momentum controlled (i.e., buoyancy effects are neglected) at all
88 relevant scales. By keeping the characteristic velocity, U_0 , constant and considering equation (1) the
89 scaled burner diameter for constant velocity scaling can be derived from the following relation:

$$\frac{D_{0,v}^*}{D_0} \propto \left(\frac{Q_v^*}{Q} \right)^{1/2}. \quad (2)$$

90 The objective of constant residence time scaling is to preserve the convective timescale, which rep-
91 resents the residence time for simple flames, by maintaining the ratio D_0/U_0 constant and hence

92 preserving in theory the macro-mixing characteristics of a burner [11, 12]. By obeying this require-
93 ment together with equation (1), constant residence time scaling can be described as:

$$\frac{D_{0,rt}^*}{D_0} \propto \left(\frac{Q_{rt}^*}{Q} \right)^{1/3} . \quad (3)$$

94 The effect of these two scaling laws on the macro- and micro-mixing is further elaborated in ap-
95 pendix A.

96 The first studies concerning the scalability of NO_x emissions aimed to develop scaling laws based
97 on equilibrium conditions for temperature and oxygen atom (O) concentration such as the study
98 by Lavoie and Schlander [13] and the asymptotic analysis by Peters [14]. However, flame stretch
99 can lead to a significant departure from equilibrium conditions, as has been shown by Drake and
100 Blint [15], as well as by Barlow and Carter [16, 17]. Various scaling models have, therefore, been
101 proposed that consider non-equilibrium conditions, which led to the finding that NO_x emissions can
102 be characterized by a negative one-half power dependency on the flame Damköhler number (i.e.,
103 the ratio of the characteristic time scale for macro-mixing to the characteristic chemical time scale)
104 [18–21]. Szego et al. [22] suggested that, under certain conditions, all parameters affecting global
105 NO_x emissions can be characterized by a global residence time and furnace temperature as proposed
106 by Turns et al. [23, 24], following the hypothesis that a majority of the NO_x emissions are formed
107 in large and nearly homogeneous eddies [25]. The scaling law proposed by Røkke et al. [26] is one of
108 few studies regarding NO_x emissions from partially premixed flames and was re-evaluated by Santos
109 and Costa [27] for turbulent diffusion flames. Weber [12] concluded the effect of flue gas entrainment,
110 from internal and external flue gas recirculation, and the radiation heat loss in the post flame need
111 to be included in order to make the correlation by Røkke et al. [26] applicable to industrial burners.

112 An important work regarding constant velocity scaling was conducted in the Scaling-400 project
113 [28] which led to an extensive data set for the NO_x emission performance of swirl burners in the range
114 of 30 kW to 12 MW thermal input. The project was later extended to constant residence time scaling
115 by means of numerical simulations [29]. Several studies of pulverized coal burners [30–32] as well

116 as gas burners [29, 33, 34] compared constant velocity scaling and constant residence time scaling
117 with each other. It is worth noticing that there is no consensus on one of these two scaling laws
118 being superior over the the other. Smart and Van Kamp [31] found that neither constant velocity
119 nor constant residence time scaling were able to preserve flame structures and thermochemical fields
120 adequately when scaling a pulverised coal burner. Furthermore, their study indicated the existence
121 of a minimum scale which is still representative for a full-scale burner and showed that both scaling
122 laws lead to a weakening of the inner recirculation zone.

123 The work by Weber and Mancini [35] provides the most recent overview on scaling of large scale
124 industrial flames. They emphasise that: “the scaling issue disappeared from the research agenda”
125 for almost two decades, which left many questions unanswered. This is especially true for partially
126 premixed flames, which are addressed in the present paper. Furthermore, only a limited number
127 of studies is found in the literature that investigate local scaling effects in different burner regions
128 and treat these regions individually [36, 37]. It is however expected that scaling of modern burners,
129 following the constant velocity or the constant residence time approach, will exhibit local scaling
130 effects due to the complex nature of the burner designs, which is not adequately represented by
131 a single length and velocity scale [36]. The objective of the present work is to develop a deeper
132 understanding of the scaling effects on the flame structure and NO_x formation in different regions
133 of complex modern burners. The study case presented is a partially premixed bluff body (PPBB)
134 burner which consists of inner and outer recirculation zones, stagnation points, staged fuel injection
135 and varying degree of partial premixing. Hence, the burner shares many of the characteristics found
136 in complex industrial burners, but still allows the understanding of general scaling effects. The impact
137 of different scaling laws on the combustion regimes and NO_x emissions performance of this complex
138 burner are investigated with hydrogen as fuel. Using hydrogen as fuel has the double advantage of
139 focussing on a single NO_x formation route and to relieve the computational effort. Besides, hydrogen
140 is foreseen to be a dominant fuel in the low carbon society needed to achieve the global climate
141 change targets. The scope comprises furthermore the identification of potentially critical thermal

142 loads that may limit the scalability of the burner, which have for example been found for the scaling
143 of pulverized coal [30] and natural gas burners [29]. The present work addresses the lack of research
144 regarding scaling of complex burners and aims to fill the knowledge-gap by investigating scaling
145 effects through the analysis of local combustion regimes and NO_x formation in different regions of
146 the flames.

147 The scaling effects are investigated based on a set of 11 computational fluid dynamic (CFD)
148 simulations at different scales up to a thermal load of 500 kW. The paper presents first the impact
149 of the scaling approaches on the combustion characteristics in general. This is done by means
150 of combustion regime diagrams combined with hexagon binning and the analysis of the progress
151 variable source term distribution. In addition to the CFD based results theoretical considerations
152 are presented. Finally the work focuses on NO_x emissions and scrutinizes the contribution of different
153 flow regions to the NO_x formation.

154 **2. Methodology**

155 *2.1. Burner design*

156 The present work is based on the scaling of the PPBB burner, developed by Spangelo et al. [38].
157 This burner is intended for the use in boilers and furnaces that typically operate at pressures close
158 to atmospheric and with approximately 3% excess air. An illustration of the burner can be seen in
159 figure 1. The PPBB burner employs a frustum shaped conical bluff body to stabilise the flame and
160 allows for the dilution of the fuel-air mixture by internally recirculated flue gas. Fuel is partially
161 premixed via jets in an accelerating cross-flow. The degree of premixing can be adjusted via eight
162 primary and four secondary fuel ports. The primary fuel ports are located upstream of the burner
163 throat in a converging burner section formed by the burner housing. The secondary fuel ports are
164 located downstream of the burner throat. Primary and secondary fuel ports are, in angular direction,
165 offset to each other such that the secondary fuel ports are located in between every other pair of

166 primary fuel ports. The burner was in recent years investigated experimentally by Dutka et al.
167 [6, 39–41] and numerically by Meraner et al. [42, 43] for the combustion of hydrogen and hydrogen
168 enriched fuels. Further details on the burners operational characteristics can be found therein.

169 *2.2. Numerical methods*

170 The numerical model that was employed for the simulations of the PPBB burner has been devel-
171 oped and validated against experimental data in previous work [42, 43]. A detailed description as well
172 as a discussion on modelling uncertainties can be found in the corresponding publications. Hence,
173 only a short summary is given here. The model was developed with the simulation of larger scales
174 in mind. Reducing computational costs have therefore been given a high priority, which resulted
175 in a model based on steady state, incompressible, Reynolds averaged (RANS) governing equations.
176 Another important aspect of choosing a relatively inexpensive numerical model is the possibility to
177 cover a wider range of burner scales. Steady state RANS simulations are order of magnitudes less
178 computationally expensive than for example large eddy simulations. This is not only due to the lower
179 special resolution needed for RANS simulations, but also due to the possibility to apply the steady
180 state assumption and to utilize the periodicity of the burner (i.e., reduce the simulation to one quar-
181 ter of the domain). The computational costs are further reduced by invoking non-equilibrium wall
182 functions and by employing a post processing approach for the NO_x calculation, i.e. the NO_x cal-
183 culation is decoupled from the combustion kinetics and based on a “frozen” combustion simulation.
184 The eddy-dissipation concept (EDC) [44, 45] in combination with a detailed combustion mechanism
185 for the hydrogen oxidation by Li et al. [46], containing 9 species and 19 reversible reactions, was
186 used to model the turbulent combustion process. A crucial property of the EDC is that its applica-
187 bility is not restricted to certain combustion regimes. RANS-EDC based models have already been
188 successfully applied to comparable burner configurations [47, 48]. Based on these studies, a tendency
189 to over-predict temperatures can be expected. The discrete ordinates radiation model was employed
190 to account for thermal radiation. The diffusive mass flux was calculated based on the dilute approx-

191 imation (i.e., Fick's law) for turbulent flows. A grid independency study, comprising meshes ranging
192 from 3.5 M to 14.4 M cells, was conducted for the burner at 100 kW and 500 kW scales assuming
193 that it is valid for the intermediate scales with lower Reynolds numbers as well.

194 The only deviation from the original model setup presented by Meraner et al. [43] are the
195 thermal boundary conditions for the lateral surfaces of the bluff body. These were originally modelled
196 adiabatic. The present study has, however, shown that this unrealistic assumption leads, at certain
197 scales, to a flame flashback within the boundary layer. The lateral bluff body walls have therefore
198 been modelled with a constant temperature of 293 K, corresponding the air inlet temperature. A
199 sensitivity analysis has shown that this change does not affect the global NO_x emissions.

200 **3. Results and discussion**

201 This section scrutinizes initially the impact of the two applied scaling laws on the burner charac-
202 teristics, when the burner is scaled from laboratory scale to 50 kW and 250 kW respectively. This
203 is followed by an analysis of NO_x emissions at different scales and parameters that are relevant for
204 the NO_x formation. Finally the simulation of the PPBB burner with a thermal input of 500 kW,
205 scaled using the constant velocity scaling approach, is analysed since it reveals a special case of a
206 fundamentally different flow structure compared to all other cases.

207 *3.1. Combustion regime and flame characteristics*

208 Regime diagrams have historically been developed based on the interaction of homogeneous and
209 isotropic frozen (i.e., unaffected by heat release) turbulence with a premixed flame. In order to utilize
210 them for the partially premixed burners, special considerations need to be made. Meraner et al.[43]
211 proposes to extract data from CFD simulations on a predefined iso-surface that is close enough to
212 the flame, so that the mixture composition is representative for the combustion, but at a distance
213 where the turbulence is not yet heavily affected by the heat release. This is achieved by defining
214 an iso-surface of 5% of the maximum heat release rate, which is then clipped using a normalized

215 progress variable (i.e., $(Y_{HO_2} + Y_{H_2O})/(Y_{HO_2} + Y_{H_2O})^{eq}$) of 0.5 as an upper limit to ensure that the
216 data is collected on the reactants side of the flame. The data obtained on this surface is then used
217 to categorize the flame by means of regime diagrams. Utilizing such scatter data provides a more
218 refined insight into the burner characteristics compared to the traditional approach of describing the
219 entire burner by a single point in the regime diagram, especially for combustion in burners which
220 likely occurs in a multi regime mode. Representing the burner by a scatter plot furthermore allows
221 the visualization of dependencies between local conditions, such as the equivalence ratio, and the
222 combustion regimes. However, the overlapping of data points in densely populated scatter plots can
223 make it difficult to identify the most representative regimes. Alternatively, hexagon binning [49], a
224 form of bivariate histogram, can be utilized to assign a more accurate weight to different combustion
225 regimes. This approach is employed in Figure 2 which shows the modified turbulent combustion
226 diagram based on Peters [50] for five different CFD simulations. Here, each visible bin contains at
227 least one data point and the colour assigned to it indicates the volume fraction that is represented
228 by the bin. The volume calculations are based on the volume of the cells that are intersected by
229 the predefined iso-surface. The global representation of the burner is marked by a single circular
230 marker. The turbulent length scale, l' , and the turbulent velocity scale, u' , for this point are based
231 on the area weighted average conditions in the burner throat. The laminar flame speed, S_L , and
232 laminar flame thickness, δ_L , were calculated in the open-source software Cantera [51] based on the
233 global equivalence ratio of the burner. In addition the effect of scaling on the flame characteristics
234 in a global context, based on scaling theory (see appendix A) is outlined by a red and orange line
235 respectively.

236 At laboratory scale most of the flame falls into the thin reaction zone (i.e., $1 < Ka < 100$) and the
237 corrugated flamelet (i.e., $Ka < 1$ and $u'/S_L > 1$) regimes, with the Karlovitz number, Ka , defined
238 as the ratio between the characteristic chemical time scale to the Kolmogorov (i.e., micro-mixing)
239 time scale. From previous work [43] we know that the two distinct regions seen at laboratory scale in
240 figure 2 between $Da = 1$ and $Ka = 1$, with a volume fraction exceeding 2%, are attributed to the

241 primary and secondary fuel streams at a equivalence ratios close to stoichiometry. The primary fuel
242 stream is characterized by lower length scale ratios compared to the secondary fuel stream. Fuel lean
243 flame regions fall mainly within $Ka \leq 100$ and $Da \leq 1$ where the Karlovitz number increases
244 with decreasing equivalence ratio due to the decreasing flame speed.

245 For constant velocity scaling, both the global representation of the burner as well as the peak
246 values of the hexbin plot follow the line for the theoretical scaling. At larger scales, parts of the
247 flame cross into the wrinkled flame regime (i.e., $u'/S_L < 1$). It can furthermore be seen that
248 the regions assigned to the primary and secondary fuel ports are less distinct from each other with
249 increasing thermal input and collapse to the same location at the 250 kW scale. The same is true
250 for constant residence time scaling, where they collapse as well at 250 kW. However, the overall
251 trend for constant residence time scaling is different. The global representation of the burner and the
252 overall distribution shown in the hexbin plot follow the theoretical scaling with a constant Damköhler
253 number relatively close. The peak values in the hexbin plot, however, appear to lie on an iso-line for
254 the Karlovitz number close to unity. This is important for smaller burner scales where temperature
255 and NO_x formation are more affected by micro-mixing compared to large scales where macro-mixing
256 and hence the Damköhler dependency dominates [35].

257 Figure 3 shows the scatter data of the 250 kW configuration coloured by the local equivalence
258 ratio. The data points for the scatter plots are sorted by their distance to the stoichiometric mixture
259 fraction. Hence, data points closer to the stoichiometric mixture overlay points that are further
260 away from stoichiometry, independent on which side (i.e., fuel lean or rich) they are located. For
261 both scaling approaches, a “flare” of lean data points can be seen that is leading towards larger
262 Karlovitz numbers, due to the reduced flame speed at lean mixture fractions. Constant residence
263 time scaling preserves the two distinct bands with a stoichiometric equivalence ratio, which have
264 been identified by Meraner et al. [43] for the base case. Constant velocity scaling, on the other hand,
265 shows a wider scatter of the stoichiometric mixture in the regime diagram and less distinct bands.
266 Generally a stronger separation of lean, rich and stoichiometric mixtures within the regime diagram

267 can be seen for constant residence time scaling, where rich equivalence ratios show the largest and
268 stoichiometric equivalence ratio the least variation. Note, the data points that follow a constant
269 Karlovitz number show little variation and cannot be attributed to a certain equivalence ratio. An
270 important parameter for the flame speed and thickness is the dilution by entrained combustion
271 products, which cannot be identified based on the equivalence ratio only. However, mixtures close to
272 the stoichiometric equivalence ratio respectively are expected to show less variation in the amount of
273 entrained products, while mixture that are further away are expected to show more variation, which
274 translates in a wider spread in flame speed and thickness.

275 Figure 2 and figure 3 are per definition restricted to the flame leading edge. Figure 4, on the
276 other hand, provides an overview on the combustion process in the whole domain by integrating the
277 progress variable source term (i.e., $R_{\text{HO}_2} + R_{\text{H}_2\text{O}}$) along the axial and radial coordinates. This allows
278 the visualization of its probability density distribution in a two dimensional space, defined by the
279 angular coordinate and the local equivalence ratio. Figure 4 shows results from the same simulations
280 as figure 2. The fuel port location of $+/- 22.5^\circ$ for the primary and 0° for the secondary fuel is
281 for all cases recognizable by peaks in the equivalence ratio marked as B_1 and B_2 . The difference
282 between the peak equivalence ratios (i.e. $B_2 - B_1$) in the two different fuel streams increases when
283 constant velocity scaling is employed and is more than twice as big for the 250 kW case compared
284 to the base case. Furthermore, both values, B_1 and B_2 , are at richer equivalence ratios. This trend
285 is opposite for constant residence time scaling where both peaks are at lower equivalence ratios and
286 slightly closer in value to each other, when comparing the 250 kW and 10 kW simulations.

287 Another difference between constant velocity and constant residence time scaling can, however,
288 be seen in between primary and secondary fuel ports marked as point A and C respectively. Constant
289 velocity scaling leads here (see A), with increasing thermal input, to an increase of the local progress
290 variable source term. This can also be seen in the form of a second peak in the marginal plot marked
291 as A', which does not appear when constant residence time scaling is employed. The probability
292 density distribution of the progress variable source term reaches for all cases its maximum around

293 stoichiometry. However, this peak is more significant for larger thermal input and constant residence
294 time scaling, which can be seen in location D. This indicates, in combination with the “discontinuity”
295 point C, that the fuel streams on a macro scale are less premixed compared to the constant velocity
296 scaling. Indeed, figure 5, which shows the stoichiometric iso-surface for the five different CFD
297 simulations, reveals that each of the fuel stream is recognizable as a single “jet” at 250 kW when
298 constant residence time scaling is applied. All other simulations show a continuous, though wrinkled,
299 iso-surface and hence a fuel rich inner recirculation zone. Note that all subfigures are scaled to the
300 same bluff body diameter. It can also be seen that the iso-surface is characterized by four tips that
301 move closer together for constant velocity scaling and finally collapse to a single tip at 250 kW, while
302 they get further separated when constant residence time scaling is applied.

303 3.2. NO_x formation

304 The stoichiometric iso-surface in figure 5 is coloured by the volumetric NO_x formation rate. The
305 peak rate is in all cases reached in between primary and secondary fuel ports in the concave region
306 formed by the iso-surface. This is in general the location where a stoichiometric mixture of fuel
307 and combustion air meets hot products that are recirculated in the inner recirculation zone. At
308 laboratory scale of 10 kW the peak values appear as a single region spanning from the primary to
309 the secondary fuel stream. Scaling the burner up by means of constant velocity scaling leads to the
310 formation of two separate elongated regions with elevated NO_x formation rates that merge again after
311 a certain distance downstream of the bluff body trailing edge. The local peak formation rate on the
312 stoichiometric iso-surface for constant residence time scaling is, however, generally lower compared
313 to the constant velocity scaling approach. The two scaling approaches lead to a significant different
314 appearance of the iso-surface at 250 kW. Even though clear differences can be seen in figure 5 it is
315 not possible to conclude on the overall NO_x performance based on a local volumetric source term as
316 the volumes for the different scaling approaches are significantly different. Figure 6 shows the global
317 NO_x emissions at different scales for constant velocity and constant residence time scaling. Both
318 methodologies lead to increasing NO_x emissions at increasing scales. However, constant velocity
319 scaling reaches higher NO_x levels. None of the approaches reaches a plateau within the investigated
320 range. Note, that the largest reported simulation for constant velocity scaling in this section is
321 450 kW; the 500 kW case is a special case and will be discussed in section 3.3.

322 The fluid domain was subdivided into six sub domains during post processing, similar to the
323 approach presented by Hsieh et al. [36], in order to identify regions that contribute to the trend
324 seen in figure 6. The regions are the wall jet (WJ), flame (FL), inner recirculation zone (IRZ), outer
325 recirculation zone (ORZ), immediate post flame (IPF) and post flame (PF) as illustrated in figure 7.

326 Figure 8 presents the total NO_x formation rate based on the described domain subdivision. For
327 clarity, only the three main contributing regions, immediate post flame, flame and inner recirculation
328 zone are shown. The formation rate for all other regions is in general more than one order of

329 magnitude smaller than the formation rate in the inner recirculation zone, confirming the findings
330 by Hsieh et al. [36]. The constant velocity approach leads, with increasing scales, to a more rapid
331 increase of the formation rate in the immediate post flame and flame region compared to constant
332 residence time scaling. The inner recirculation zone on the other hand shows an opposite trend. The
333 rate increases here for constant residence time scaling, while it stays relatively constant for constant
334 velocity scaling.

335 The inner recirculation zone and the flame region are dominating in terms of a mean volumetric
336 rate as shown in figure 9. Constant residence time scaling leads in all three regions to larger mean
337 volumetric NO_x formation rate than constant velocity scaling, which is the opposite trend than
338 what was seen in figure 8 for the flame and the immediate post flame regions. This difference
339 can be attributed to the different volumes and different residence times accordingly. The larger
340 volume/longer residence time of the inner recirculation zone for constant velocity scaling is, however,
341 compensated by a significant decrease of the mean volumetric rate compared to constant residence
342 time scaling.

343 Figure 10 compares the NO_x formation rate at the 10 kW and the 100 kW scale based on
344 the constant velocity scaling approach, as this represents a most severe increase in global NO_x .
345 The contours are scaled to the same bluff body diameter, D . The inner recirculation zone can be
346 recognized by the iso-lines corresponding to zero axial velocity. The normalized dimensions of the
347 inner recirculation zone are comparable in both cases. In general, all conducted simulations predict
348 a recirculation zone length of approximately $1.7 D$. It can be seen that, at the laboratory scale of
349 10 kW, mainly the upper part of the inner recirculation zone contributes to the NO_x formation. The
350 formation rate is in this region significantly lower at 100 kW. The contour plot shows furthermore
351 a decrease of the formation rate in the flame and immediate post flame region, similar to what has
352 been seen in figure 9. However, the local volumetric NO_x formation rate close to the flame anchor
353 point is larger for 100 kW compared to 10 kW.

354 Thermal NO_x is the dominating NO_x route for the combustion of pure hydrogen at low pressures

355 and the temperature distribution is, therefore, an important factor for the overall NO_x formation
356 rate. Figure 11 shows the mean temperature in the three dominating regions. The mean temperature
357 in the inner recirculation zone is relatively constant and decreasing slightly at larger scales, for both
358 scaling methodologies. Hence, the temperature cannot be the leading cause of the different trend
359 for constant velocity and constant residence time scaling seen in figure 9. The spacial temperature
360 distribution within the recirculation zone is, furthermore, relatively constant while the NO_x formation
361 rate varies as has been seen in figure 10. The mean temperatures in the flame and immediate post
362 flame region change slightly more with an approximately 8% decrease in the flame region and an
363 similar large increase in the immediate post flame, when scaling with constant residence time from
364 10 kW to 500 kW. The mean temperature stays, on the other hand, nearly constant when constant
365 velocity scaling is employed.

366 Since the temperatures in the flame and inner recirculation zone are generally sufficiently high
367 for the formation of thermal NO_x , local species concentrations become the governing factor affecting
368 NO_x formation. Thermal NO_x formation is described by the extended Zeldovich mechanism [52].
369 Hence, the driving radicals are O and OH, where the latter is important particularly at near stoi-
370 chiometric conditions and fuel rich mixtures. Figure 12 shows the OH mass fraction distribution for
371 the laboratory scale burner compared to the burner scaled to 100 kW following the two investigated
372 scaling methodologies. The iso-lines show zero axial velocity to indicate the extend of the inner
373 recirculation zone. It can be seen that the OH contours display a similar distribution as seen in
374 figure 10 for the volumetric NO_x formation rate, with constant velocity scaling leading to lower and
375 constant residence time scaling leading to higher OH mass fractions in the inner recirculation zone
376 compared to the base case. A similar distribution in all three simulations was found for the O mass
377 fraction, which is not shown here. The production of OH and O radicals is almost entirely attributed
378 to the flame-sheet. Note that this is not referring to the flame region defined for post processing.
379 Hence, the concentration of these two radicals in the inner recirculation zone is dependent on the flow
380 conditions and the entrainment into inner recirculation zone. The entrainment is strongly affected

381 by the ratio between the penetration depth of the secondary fuel stream and the annular wall jet
382 height, which is differently affected by the two applied scaling laws.

383 *3.3. Constant velocity scaling up to 500 kW*

384 The main flow features of the PPBB burner, namely an inner recirculation zone and a larger outer
385 recirculation, were preserved in all simulations presented in the previous sections. The dimensions
386 of these zones were, furthermore, relatively constant with a length of approximately 1.7 bluff body
387 diameters for the inner and approximately 11 bluff body diameters for the outer recirculation zone.
388 However, scaling the burner up to a scale of 500 kW based on constant velocity led to a fundamentally
389 different flow field, causing the flame to impinge onto the combustion chamber wall, as can be seen
390 in figure 13. The temperature contours, overlaid by the velocity vector field for the 10 kW and the
391 500 kW constant velocity simulations are compared in this figure. Sudden changes in the overall flame
392 pattern, during scaling, have also been observed experimentally for other burners [35]. Applying
393 constant residence time scaling on the other hand allowed to scale the burner successfully up to
394 500 kW, although, with the disadvantage of reaching high velocities. The simulation of the 250 kW
395 and 500 kW scale with constant residence time scaling reached a local Mach number of 0.35 and 0.44
396 respectively, which exceed the upper limit of 0.3 generally applied as best practice for incompressible
397 solvers. Both cases are still considered as subsonic flows, however, the model uncertainties introduced
398 by neglecting compressibility effects become larger with increasing velocities.

399 Figure 13 shows the breakup of the inner recirculation zone at the 500 kW scale. Instead of the
400 inner recirculation zone a set of two equally sized vortices is formed in the outer chamber region,
401 which was previously characterized by a single large recirculation zone and smaller secondary vortices
402 in regions of flow separation from the chamber wall. Note that only one of these secondary vortices
403 is visible due to the coarsened resolution in the vector plot. These vortices cause the flame to be
404 bend outwards leading to an impingement of high temperature flow onto the chamber wall. The
405 breakup of the inner recirculation zone at this scale was only observed under reacting conditions.

406 An additional non-reacting simulation was conducted in which the original flow field, similar to the
407 smaller scales, was preserved.

408 Based on the employed steady state RANS simulations it is not possible to determine if the
409 breakup under reacting conditions is a transient flow instability or if the flow will remain permanently
410 attach to the chamber wall. This could possibly be assessed by conducting unsteady RANS or scale
411 resolving simulations. However, the more profound question is what causes the breakup of the
412 inner recirculation zone. An apparent difference between the investigated scaling principles is the
413 fuel concentration in the recirculation zone, which is an important factor for the flame stability
414 [53]. Constant velocity scaling leads to an increased recirculation zone equivalence ratio, reaching
415 1.13 at the 450 kW scale, while constant residence time scaling leads to a decrease, reaching a lean
416 equivalence ratio of 0.9 at the 500 kW scale. However, this effect is expected to be of less importance,
417 since neither of the scaling laws leads to equivalence ratios far from stoichiometry.

418 Other important factors for the recirculation zone characteristics are the blockage ratio [53], the
419 bluff body position [54], and the ratio between bluff body diameter and chamber diameter [35],
420 which is 3.75 in the present study. However, these factors are not affected by the employed scaling
421 methodologies. Hence, the breakup of the inner recirculation zone is most likely associated with
422 a fundamental difference between the applied scaling methodologies. An inherent difference of the
423 investigated scaling laws is the ratio between velocity and length scales, U_0/D_0 , which is per definition
424 constant for constant residence time scaling and decreases for constant velocity scaling. Hence, the
425 annular jet flow for constant velocity scaling has a lower momentum compared to the flow for constant
426 residence time scaling. The recirculation zone needs, furthermore, to span a significant larger distance
427 when constant velocity scaling is applied as the bluff body diameter is larger compared to constant
428 residence time scaling. This may indicate that the bluff body diameter and the annular throat cross
429 section require independent scaling similar to what has been suggested by Cheng et al. [37].

430 It is not yet known if the breakup of the inner recirculation zone would also occur in an uncon-
431 fined burner configuration. Hence, adjusting the chamber to bluff body diameter ratio may lead to

432 improved stability. Furthermore, the effect of different lance heights (i.e., the elevation of the bluff
433 body with respect to the burner throat) on the recirculation zone could be explored. Tong et al. [54]
434 suggested that the flame stability can be improved by modifying the bluff body position. However,
435 altering the position of the conical bluff body changes inevitable also the cross-sectional throat area.
436 This changes consequently the velocity of the annular jet flow, which will impact the flame stability
437 as well and needs therefore to be investigated further.

438 **4. Conclusion**

439 Eleven CFD simulations of a complex burner configuration, at various scales ranging from 10 kW
440 to 500 kW, were conducted. The characteristic combustion regimes and the NO_x emissions at the
441 different scales were analysed, employing a novel approach of combining hexagonal binning and
442 combustion regime diagrams. The scaling of the burner was conducted following two different scaling
443 principles; the constant velocity and the constant residence time scaling.

444 Employing constant velocity scaling shifts the flame towards lower Karlovitz numbers and leads
445 to combustion that spans multiple combustion regimes. The variation in the combustion regimes
446 was shown to be less dependent on the local equivalence ratio compared to flames that were scaled
447 by constant residence time scaling. Constant residence time scaling leads, on the other hand, to a
448 constant global Damköhler number. However, a detailed analysis by means of CFD based regime
449 diagrams showed that a considerable part of the flame follows a constant Karlovitz number, which
450 is unexpected in relation to the theory of constant residence time scaling and evidenced the need for
451 individual scaling laws for different burner regions.

452 Increasing the burner scale led, independent of the applied scaling law, to an increase of the
453 total NO_x emissions. The fact that no plateau was reached within the investigated range and the
454 observed Karlovitz number dependencies for constant residence time scaling suggest that the critical
455 thermal load for the investigated burner was not reached. The immediate post flame, flame and
456 inner recirculation zone regions dominate the NO_x formation while the influence of the post flame

457 and outer recirculation zone regions are negligible, which may explain the failure of global furnace
458 residence time models to predict NO_x emissions at different burner scales.

459 Constant velocity scaling led to a lower total NO_x formation rate in the inner recirculation zone
460 compared to constant residence time scaling, despite its larger volume and longer residence times.
461 This was linked to the different effect that the scaling approaches have on the flow conditions and
462 entrainment into the inner recirculation zone, such as the ratio between the penetration depth of the
463 secondary fuel stream and the annular wall jet height. The entrainment affects consequently the O
464 and OH concentrations, the dominating parameter for the NO_x formation in the inner recirculation
465 zone.

466 A fundamental change of the flow field was observed in the narrow band between 450 kW and
467 500 kW scale when constant velocity scaling was applied. At this scale the inner recirculation zone
468 breaks up and a vortex pair is formed in the outer region of the combustion chamber that causes
469 the flame to be bend outwards and consequently impinge onto the chamber wall. Constant residence
470 time scaling, on the other hand, preserved the inner recirculation zone at all investigated scales. The
471 sudden breakup was attributed to the different effect of the scaling laws on the velocity to length
472 scale ratio and momentum of the annular jet flow, and needs to be investigated further.

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480 **Appendix A Scaling**

481 In what follows is the examination of the effect of constant velocity and constant residence time
 482 scaling on macro- and micro-mixing in terms of Damköhler and Karlovitz number, following the
 483 considerations made by Farcy et al. [55]. Given geometric similarity, all burner length scales, l , are
 484 increased by a factor, $\beta > 1$, when the burner is scaled up from laboratory scale to larger thermal
 485 loads:

$$l^* = \beta l, \quad (4)$$

486 where the star superscript represents scaled properties. For the following considerations it is further-
 487 more assumed that the turbulent intensity stays unchanged for both scaling approaches.

488 *A.1 Constant velocity scaling*

489 Based on equation 2 the scaling factor for constant velocity scaling is:

$$\beta_v = \left(\frac{Q_v^*}{Q} \right)^{1/2}. \quad (5)$$

490 Since the velocity scale is unchanged, $u_v^* = u$, and all length scales are scaled up, $l_v^* = \beta_v l$, we obtain:

$$k_v^{*1/2} \approx l_v^* \left| \frac{\partial u_v^*}{\partial y_v^*} \right| = \beta_v l' \left| \frac{1}{\beta_v} \frac{\partial u}{\partial y} \right| = k^{1/2}, \quad (6)$$

491 where k is the turbulent kinetic energy. Furthermore we see that the turbulent Reynolds number,
 492 Re_T , scales with the factor β_v :

$$Re_{T,v}^* = \frac{k_v^{*1/2} l_v^*}{\nu} = \frac{k^{1/2} \beta_v l'}{\nu} = \beta_v Re_T, \quad (7)$$

493 the Kolmogorov scale, η_k , scales with the factor $\beta_v^{1/4}$:

$$\eta_{k,v}^* \approx \frac{l_v^*}{Re_{T,v}^{*3/4}} = \frac{\beta_v l'}{(\beta_v Re_T)^{3/4}} = \beta_v^{1/4} \eta_k, \quad (8)$$

494 and the dissipation rate, ϵ , scales with the factor β_v^{-1} :

$$\epsilon_v^* \approx \frac{\nu^3}{\eta_{k,v}^{*4}} = \frac{\nu^3}{(\beta_v^{1/4} \eta_k)^4} = \frac{1}{\beta_v} \epsilon. \quad (9)$$

495 As a result, the turbulent time scale, τ_T , scales with the factor β_v :

$$\tau_{T,v}^* = \frac{k_v^*}{\epsilon_v^*} = \frac{k}{\beta_v^{-1}\epsilon} = \beta_v \tau_T, \quad (10)$$

496 and the Kolmogorov time scale, τ_k , with the factor $\beta_v^{1/2}$:

$$\tau_{k,v}^* = \left(\frac{\nu}{\epsilon_v^*}\right)^{1/2} = \left(\frac{\nu}{\beta_v^{-1}\epsilon}\right)^{1/2} = \beta_v^{1/2} \tau_k. \quad (11)$$

497 The scaling relation for the Damköhler number is thus:

$$Da_v^* = \frac{\tau_{T,v}^*}{\tau_{c,v}^*} = \frac{\beta_v \tau_T}{\tau_c} = \beta_v Da, \quad (12)$$

498 and for the Karlovitz number:

$$Ka_v^* = \frac{\tau_{c,v}^*}{\tau_{k,v}^*} = \frac{\tau_c}{\beta_v^{1/2} \tau_k} = \frac{1}{\beta_v^{1/2}} Ka. \quad (13)$$

499 Substituting β_v with equation (5) leads to:

$$Da_v^* = \left(\frac{Q_v^*}{Q}\right)^{1/2} Da, \quad (14)$$

500 and

$$Ka_v^* = \left(\frac{Q_v^*}{Q}\right)^{-1/4} Ka. \quad (15)$$

501 A.2 Constant residence time scaling

502 Based on equation 3 the scaling factor for constant residence time scaling is:

$$\beta_{rt} = \left(\frac{Q_{rt}^*}{Q}\right)^{1/3}. \quad (16)$$

503 Additionally to the length scales also the velocity needs to be scaled up, $u_{rt}^* = \beta_{rt} u$, in order to
 504 preserve the residence time, which leads to:

$$k_{rt}^{*1/2} \approx l_{rt}^* \left| \frac{\partial u_{rt}^*}{\partial y_{rt}^*} \right| = \beta_{rt} l' \left| \frac{\beta_{rt} \partial u}{\beta_{rt} \partial y} \right| = \beta_{rt} k^{1/2}, \quad (17)$$

505 or equivalently

$$k_{rt}^* = \beta_{rt}^2 k. \quad (18)$$

506 Here the turbulent Reynolds number, Re_T , scales with the factor β_{rt}^2 :

$$Re_{T,rt}^* = \frac{k_{rt}^*{}^{1/2} l_{rt}^*}{\nu} = \frac{\beta_{rt} k^{1/2} \beta_{rt} l'}{\nu} = \beta_{rt}^2 Re_T, \quad (19)$$

507 the Kolmogorov scale, η_k , scales with the factor $\beta_{rt}^{-1/2}$:

$$\eta_{k,rt}^* \approx \frac{l_{rt}^*}{Re_{T,rt}^*{}^{3/4}} = \frac{\beta_{rt} l'}{(\beta_{rt}^2 Re_T)^{3/4}} = \frac{1}{\beta_{rt}^{1/2}} \eta_k, \quad (20)$$

508 and the dissipation rate, ϵ , scales with the factor β_{rt}^2 :

$$\epsilon_{rt}^* \approx \frac{\nu^3}{\eta_{k,rt}^*{}^4} = \frac{\nu^3}{(\beta_{rt}^{-1/2} \eta_k)^4} = \beta_{rt}^2 \epsilon. \quad (21)$$

509 This leads to a constant turbulent time scale, τ_T :

$$\tau_{T,rt}^* = \frac{k_{rt}^*}{\epsilon_{rt}^*} = \frac{\beta_{rt}^2 k}{\beta_{rt}^2 \epsilon} = \tau_T, \quad (22)$$

510 and a scaled Kolmogorov time, τ_k , by the factor β_{rt}^{-1} :

$$\tau_{k,rt}^* = \left(\frac{\nu}{\epsilon_{rt}^*} \right)^{1/2} = \left(\frac{\nu}{\beta_{rt}^2 \epsilon} \right)^{1/2} = \frac{1}{\beta_{rt}} \tau_k. \quad (23)$$

511 The Damköhler number, Da , representing macro-mixing is thus constant for a fixed chemical time
512 scale, $\tau_{c,rt}^* = \tau_c$:

$$Da_{rt}^* = \frac{\tau_{T,rt}^*}{\tau_{c,rt}^*} = \frac{\tau_T}{\tau_c} = Da. \quad (24)$$

513 The Karlovitz number, Ka , that represents micro-mixing on the other hand scales with factor β_{rt} :

$$Ka_{rt}^* = \frac{\tau_{c,rt}^*}{\tau_{k,rt}^*} = \frac{\tau_c}{\beta_{rt}^{-1} \tau_k} = \beta_{rt} Ka, \quad (25)$$

514 which together with equation (16) leads to:

$$Ka_{rt}^* = \left(\frac{Q_{rt}^*}{Q} \right)^{1/3} Ka. \quad (26)$$

515 Equation (14) and (15) together with equation (24) and (26) show the inherent problem of
516 scaling turbulent flames, namely that the non-linear character of turbulence makes it impossible to
517 preserve both macro- and micro-mixing, even when the majority of other non-dimensional groups
518 are neglected. Hence, one needs to decide which mixing mechanism is given the higher priority by
519 choosing an appropriate scaling law. The present work discusses the effect of this on the combustion
520 characteristics of of complex low emission burners, here represented by the PPBB burner.

521 **References**

- 522 [1] H. Kobayashi, A. Hayakawa, K. Somarathne, E. Okafor, Science and technology of
523 ammonia combustion, *Proceedings of the Combustion Institute* 37 (1) (2019) 109–133.
524 doi:10.1016/j.proci.2018.09.029.
- 525 [2] D. Dunn-Rankin, P. Therkelsen (Eds.), *Lean Combustion*, 2nd Edition, Academic Press, 2016.
- 526 [3] A. Uemichi, K. Kouzaki, K. Warabi, K. Shimamura, M. Nishioka, Formation of ultra-lean
527 comet-like flame in swirling hydrogen–air flow, *Combustion and Flame* 196 (2018) 314–324.
528 doi:10.1016/j.combustflame.2018.06.019.
- 529 [4] P. Stuttaford, H. Rizkalla, Y. Chen, B. Copley, T. Faucett, Extended Turndown, Fuel Flexible
530 Gas Turbine Combustion System, in: *Volume 2: Combustion, Fuels and Emissions, Parts A and*
531 *B*, ASME, 2010, pp. 483–492. doi:10.1115/GT2010-22585.
- 532 [5] A. Haj Ayed, K. Kusterer, H.-W. Funke, J. Keinz, C. Striegan, D. Bohn, Improvement study for
533 the dry-low-NOx hydrogen micromix combustion technology, *Propulsion and Power Research*
534 4 (3) (2015) 132–140. doi:10.1016/j.jprr.2015.07.003.
- 535 [6] M. Dutka, M. Ditaranto, T. Løvås, NOx emissions and turbulent flow field in a partially pre-
536 mixed bluff body burner with CH4 and H2 fuels, *International Journal of Hydrogen Energy*
537 41 (28) (2016) 12397–12410. doi:10.1016/j.ijhydene.2016.05.154.

- 538 [7] K. Souflas, P. Koutmos, On the non-reacting flow and mixing fields of an axisymmetric disk sta-
539 bilizer, under inlet mixture stratification and preheat, *Experimental Thermal and Fluid Science*
540 99 (2018) 357–366. doi:10.1016/j.expthermflusci.2018.08.008.
- 541 [8] D. Spalding, The art of partial modeling, *Symposium (International) on Combustion* 9 (1) (1963)
542 833–843. doi:10.1016/S0082-0784(63)80090-9.
- 543 [9] J. Beér, The Significance of Modelling, *Journal of Insitute of Fuel* 39 (1966) 466–473.
- 544 [10] S. R. Turns, Understanding NO_x formation in nonpremixed flames: experiments and modeling,
545 *Prog. Energy Combust. Sci.* 21 (1995) 361–385.
- 546 [11] M. Sadakata, Y. Hirose, Scaling law for pollutant emission from a combustion furnace, *Fuel*
547 73 (8) (1994) 1338–1342. doi:10.1016/0016-2361(94)90310-7.
- 548 [12] R. Weber, Scaling characteristics of aerodynamics, heat transfer, and pollutant emissions
549 in industrial flames, *Symposium (International) on Combustion* 26 (2) (1996) 3343–3354.
550 doi:10.1016/S0082-0784(96)80182-2.
- 551 [13] G. A. Lavoie, A. F. Schlander, A Scaling Study of NO Formation in Turbulent Diffusion Flames
552 of Hydrogen Burning in Air, *Combustion Science and Technology* 8 (5-6) (1973) 215–224.
553 doi:10.1080/00102207308946645.
- 554 [14] N. Peters, An Asymptotic Analysis of Nitric Oxide Formation in Turbulent Diffusion Flames,
555 *Combustion Science and Technology* 19 (1-2) (1978) 39–49. doi:10.1080/00102207808946862.
- 556 [15] M. C. Drake, R. J. Blint, Thermal NO_x in stretched laminar opposed-flow diffusion flames
557 with CO/H₂/N₂ fuel, *Combustion and Flame* 76 (2) (1989) 151–167. doi:10.1016/0010-
558 2180(89)90064-3.

- 559 [16] R. Barlow, C. Carter, Raman/Rayleigh/LIF measurements of nitric oxide formation in turbu-
560 lent hydrogen jet flames, *Combustion and Flame* 97 (3-4) (1994) 261–280. doi:10.1016/0010-
561 2180(94)90020-5.
- 562 [17] C. D. Carter, R. S. Barlow, Simultaneous measurements of NO, OH, and the major species in
563 turbulent flames, *Optics Letters* 19 (4) (1994) 299. doi:10.1364/OL.19.000299.
- 564 [18] J. Kent, R. Bilger, The prediction of turbulent diffusion flame fields and nitric oxide forma-
565 tion, *Symposium (International) on Combustion* 16 (1) (1977) 1643–1656. doi:10.1016/S0082-
566 0784(77)80443-8.
- 567 [19] R.-H. Chen, J. F. Driscoll, Nitric oxide levels of jet diffusion flames: Effects of coaxial air and
568 other mixing parameters, *Symposium (International) on Combustion* 23 (1) (1991) 281–288.
569 doi:10.1016/S0082-0784(06)80271-7.
- 570 [20] J.-Y. Chen, W. Kollmann, PDF modeling and analysis of thermal NO formation in turbu-
571 lent nonpremixed hydrogen-air jet flames, *Combustion and Flame* 88 (3-4) (1992) 397–412.
572 doi:10.1016/0010-2180(92)90042-N.
- 573 [21] N. Smith, R. Bilger, J.-Y. Chen, Modelling of nonpremixed hydrogen jet flames using a condi-
574 tional moment closure method, *Symposium (International) on Combustion* 24 (1) (1992) 263–
575 269. doi:10.1016/S0082-0784(06)80035-4.
- 576 [22] G. Szego, B. Dally, G. Nathan, Scaling of NO_x emissions from a laboratory-
577 scale mild combustion furnace, *Combustion and Flame* 154 (1-2) (2008) 281–295.
578 doi:10.1016/j.combustflame.2008.02.001.
- 579 [23] S. R. Turns, F. H. Myhr, Oxides of nitrogen emissions from turbulent jet flames: Part I—Fuel
580 effects and flame radiation, *Combustion and Flame* 87 (3-4) (1991) 319–335. doi:10.1016/0010-
581 2180(91)90116-S.

- 582 [24] S. R. Turns, F. H. Myhr, R. V. Bandaru, E. R. Maund, Oxides of nitrogen emissions from
583 turbulent jet flames: Part II—Fuel dilution and partial premixing effects, *Combustion and*
584 *Flame* 93 (3) (1993) 255–269. doi:10.1016/0010-2180(93)90107-E.
- 585 [25] A. Lutz, R. Kee, R. Dibble, J. Broadwell, A model for detailed chemical kinetics in turbulent
586 nonpremixed jet flames, in: 29th Aerospace Sciences Meeting, American Institute of Aeronautics
587 and Astronautics, Reston, Virginia, 1991. doi:10.2514/6.1991-478.
- 588 [26] N. A. Røkke, J. E. Hustad, O. K. Sønju, A study of partially premixed unconfined propane
589 flames, *Combustion and Flame* 97 (1) (1994) 88–106. doi:10.1016/0010-2180(94)90118-X.
- 590 [27] A. Santos, M. Costa, Reexamination of the scaling laws for NO_x emissions from hydro-
591 carbon turbulent jet diffusion flames, *Combustion and Flame* 142 (1-2) (2005) 160–169.
592 doi:10.1016/j.combustflame.2005.03.004.
- 593 [28] R. Weber, J. F. Driscoll, W. J. Dahm, R. T. Waibel, Scaling Characteristics of Aerodynamics
594 and Low-NO_x Properties of Industrial Natural Gas Burners. The SCALING 400 Study. Part I:
595 Test Plan, Tech. rep., GRI Topoical Report 93/0227, Gas Research Institute, Chicago (1993).
- 596 [29] U. Bollettini, F. Breussin, R. Weber, A study on scaling of natural gas burners, *IFRF Combustion*
597 *Journal*.
- 598 [30] R. Weber, F. Breussin, Scaling properties of swirling pulverized coal flames: From 180 kW to
599 50 MW thermal input, *Symposium (International) on Combustion* 27 (2) (1998) 2957–2964.
600 doi:10.1016/S0082-0784(98)80155-0.
- 601 [31] J. P. Smart, W. L. D. Van Kamp, The Impact of Scaling Criteria on the Characteristics of
602 Pulverised Coal Flames, *Developments in Chemical Engineering and Mineral Processing* 7 (3-4)
603 (1999) 301–331. doi:10.1002/apj.5500070305.

- 604 [32] N. Megalos, N. Smith, D. Zhang, The potential for low NO_x from a precessing jet burner of
605 coal, *Combustion and Flame* 124 (1-2) (2001) 50–64. doi:10.1016/S0010-2180(00)00173-5.
- 606 [33] H. Levinsky, C. van der Meij, LIF and CARS measurements performed on the 50 kW bluff-
607 body burner for the British Gas, Tech. rep., Gaz de France, Gasunie, GRI collaboration, N.V.
608 Nederlandse Gasunie (1994).
- 609 [34] M. Perrin, J. Imbach, S. Albert, J. Mariasine, British Gas/Gasunie/Gaz de France collaboration
610 on turbulent diffusion flames scaling. Velocity, Temperature and Concentration Measurements
611 on a 500 kW Bluff-Body Burner, Tech. rep., Gas de France (1994).
- 612 [35] R. Weber, M. Mancini, On scaling and mathematical modelling of large scale industrial flames,
613 *Journal of the Energy Institute* doi:10.1016/j.joei.2019.04.010.
- 614 [36] T.-C. Hsieh, W. J. Dahm, J. F. Driscoll, Scaling Laws for NO_x Emission Performance of Burn-
615 ers and Furnaces from 30 kW to 12 MW, *Combustion and Flame* 114 (1-2) (1998) 54–80.
616 doi:10.1016/S0010-2180(97)00289-7.
- 617 [37] R. Cheng, D. Yegian, M. Miyasato, G. Samuelsen, C. Benson, R. Pellizzari, P. Loftus, Scaling
618 and development of low-swirl burners for low-emission furnaces and boilers, *Proceedings of the*
619 *Combustion Institute* 28 (1) (2000) 1305–1313. doi:10.1016/S0082-0784(00)80344-6.
- 620 [38] Ø. Spangelo, O. Sonju, T. Slungaard, M. Ditaranto, US 20090220899 A1 (2009).
- 621 [39] M. Dutka, Emissions of nitrogen oxides from partially premixed flames stabilized on a conical
622 bluff body, Ph.D. thesis, NTNU (2015).
- 623 [40] M. Dutka, M. Ditaranto, T. Løvås, Application of a Central Composite Design for the Study
624 of NO_x Emission Performance of a Low NO_x Burner, *Energies* 8 (5) (2015) 3606–3627.
625 doi:10.3390/en8053606.

- 626 [41] M. Dutka, M. Ditaranto, T. Løvås, Emission characteristics of a novel low NO_x burner fueled
627 by hydrogen-rich mixtures with methane, *Journal of Power Technologies* 95 (2) (2015) 105–111.
- 628 [42] C. Meraner, T. Li, M. Ditaranto, T. Løvås, Cold flow characteristics of a novel bluff
629 body hydrogen burner, *International Journal of Hydrogen Energy* 43 (14) (2018) 7155–7168.
630 doi:10.1016/j.ijhydene.2018.02.062.
- 631 [43] C. Meraner, T. Li, M. Ditaranto, T. Løvås, Combustion and NO_x Emission Charac-
632 teristics of a Bluff Body Hydrogen Burner, *Energy & Fuels* 33 (5) (2019) 4598–4610.
633 doi:10.1021/acs.energyfuels.9b00313.
- 634 [44] B. Magnussen, On the structure of turbulence and a generalized eddy dissipation concept for
635 chemical reaction in turbulent flow, in: 19th Aerospace Sciences Meeting, American Institute of
636 Aeronautics and Astronautics, Reston, Virginia, 1981, pp. 1–6. doi:10.2514/6.1981-42.
- 637 [45] B. Magnussen, B. Hjertager, On mathematical modeling of turbulent combustion with special
638 emphasis on soot formation and combustion, *Symposium (International) on Combustion* 16 (1)
639 (1977) 719–729. doi:10.1016/S0082-0784(77)80366-4.
- 640 [46] J. Li, Z. Zhao, A. Kazakov, F. L. Dryer, An updated comprehensive kinetic model of
641 hydrogen combustion, *International Journal of Chemical Kinetics* 36 (10) (2004) 566–575.
642 doi:10.1002/kin.20026.
- 643 [47] J. R. Nanduri, D. R. Parsons, S. L. Yilmaz, I. B. Celik, P. A. Strakey, Assessment of
644 RANS-Based Turbulent Combustion Models for Prediction of Emissions from Lean Pre-
645 mixed Combustion of Methane, *Combustion Science and Technology* 182 (7) (2010) 794–821.
646 doi:10.1080/00102200903341546.
- 647 [48] D. A. Lysenko, I. S. Ertesvåg, Reynolds-Averaged, Scale-Adaptive and Large-Eddy Simulations

- 648 of Premixed Bluff-Body Combustion Using the Eddy Dissipation Concept, *Flow, Turbulence*
649 *and Combustion* 100 (3) (2018) 721–768. doi:10.1007/s10494-017-9880-4.
- 650 [49] N. Lewin-Koh, Hexagon Binning: an Overview, Tech. rep., TU Dortmund (2009).
- 651 [50] N. Peters, The turbulent burning velocity for large-scale and small-scale turbulence, *Journal of*
652 *Fluid Mechanics* 384 (1999) S0022112098004212. doi:10.1017/S0022112098004212.
- 653 [51] D. G. Goodwin, R. L. Speth, H. K. Moffat, B. W. Weber, Cantera: An object-oriented
654 software toolkit for chemical kinetics, thermodynamics, and transport processes (2018).
655 doi:10.5281/zenodo.1174508.
- 656 [52] Y. B. Zeldovich, The oxidation of nitrogen in combustion and explosions, *Acta Physicochimica*
657 *U.R.S.S.* 21 (1946) 577–628.
- 658 [53] R. W. Schefer, M. Namazian, J. Kelly, M. Perrin, Effect of Confinement on Bluff-Body Burner
659 Recirculation Zone Characteristics and Flame Stability, *Combustion Science and Technology*
660 120 (1-6) (1996) 185–211. doi:10.1080/00102209608935573.
- 661 [54] Y. Tong, M. Li, J. Klingmann, S. Chen, Z. Li, Experimental Investigation on the Influences of
662 Bluff-Body’s Position on Diffusion Flame Structures, in: *Volume 1: Boilers and Heat Recov-*
663 *ery Steam Generator; Combustion Turbines; Energy Water Sustainability; Fuels, Combustion*
664 *and Material Handling; Heat Exchangers, Condensers, Cooling Systems, and Balance-of-Plant,*
665 *ASME, 2017, p. V001T04A009. doi:10.1115/POWER-ICOPE2017-3090.*
- 666 [55] B. Farcy, L. Vervisch, P. Domingo, Large eddy simulation of selective non-catalytic reduction
667 (SNCR): A downsizing procedure for simulating nitric-oxide reduction units, *Chemical Engi-*
668 *neering Science* 139 (2015) 285–303. doi:10.1016/j.ces.2015.10.002.

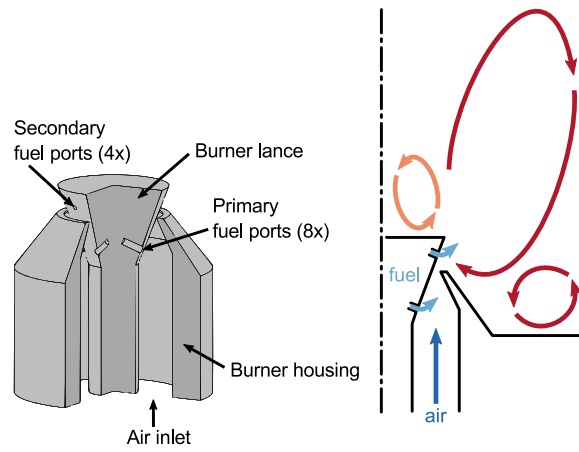


Figure 1: 3D rendering of the PPBB burner and illustration of the flow pattern. Primary and secondary fuel ports are drawn in the same plane for illustration purpose.

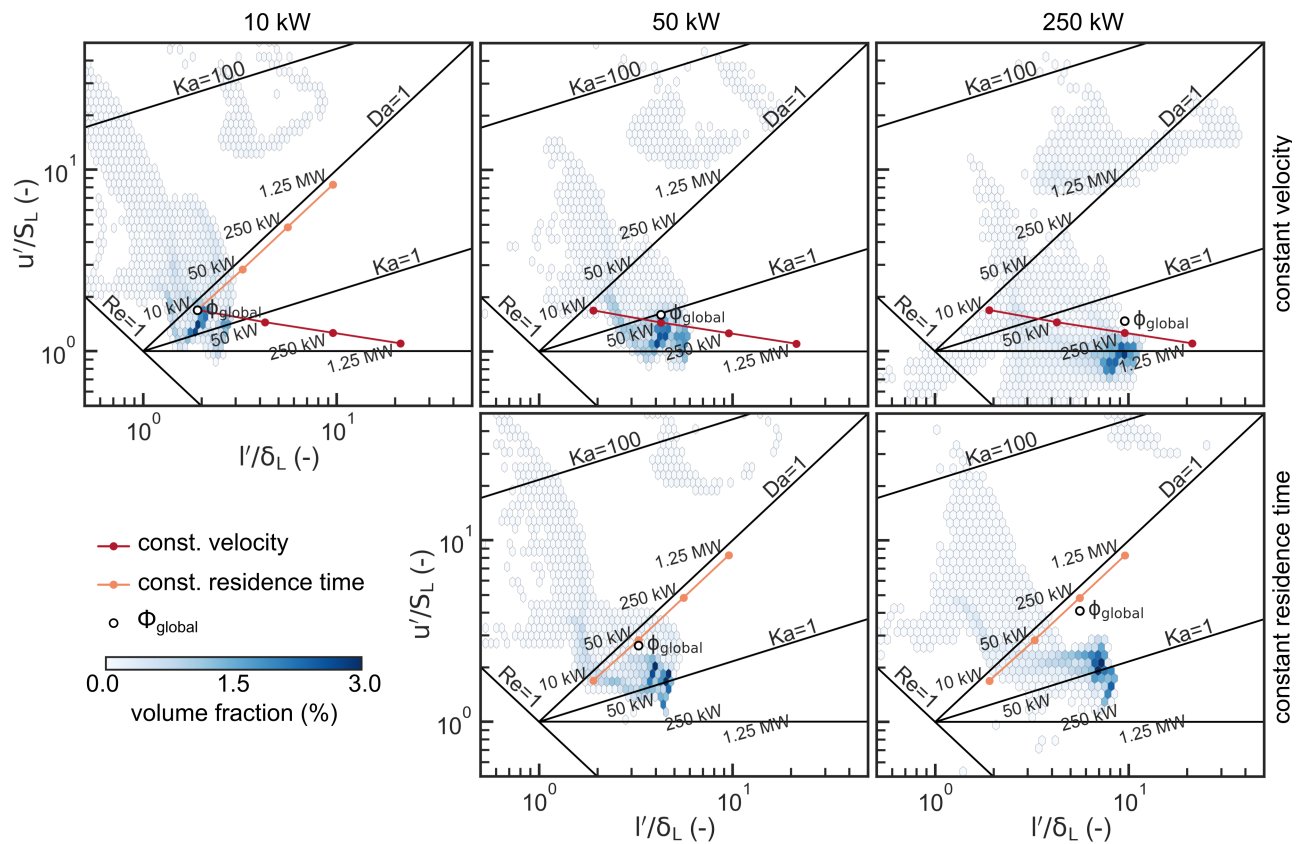


Figure 2: Modified turbulent combustion diagram based on Peters [50]. The hexbin distribution is obtained from CFD simulations and coloured by the volume fraction that is represented by each bin (i.e., the volume associated to one single bin normalized by the total volume represented in the hexbin plot). The red and orange lines show the theoretical scaling for the global burner representation. The single circular marker shows the global representation of the burner within the regime diagram.

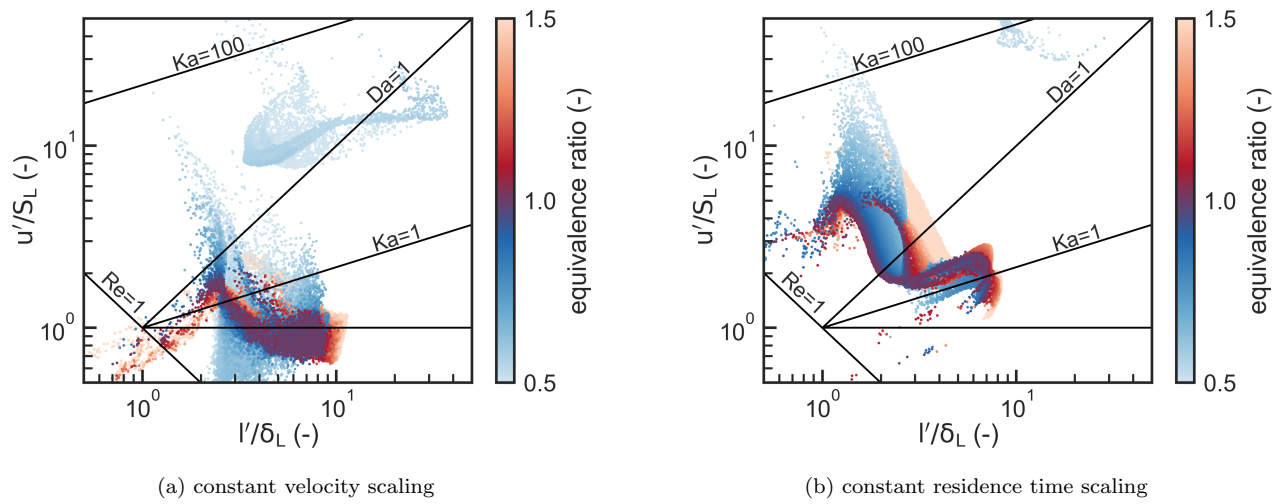


Figure 3: Modified turbulent combustion diagram based on Peters [50] containing the scatter plot for the burner at the 250 kW scale based in the two different scaling laws. The scatter plot is coloured by the local equivalence ratio. Data points closed to stoichiometric conditions overlap data points that are further away.

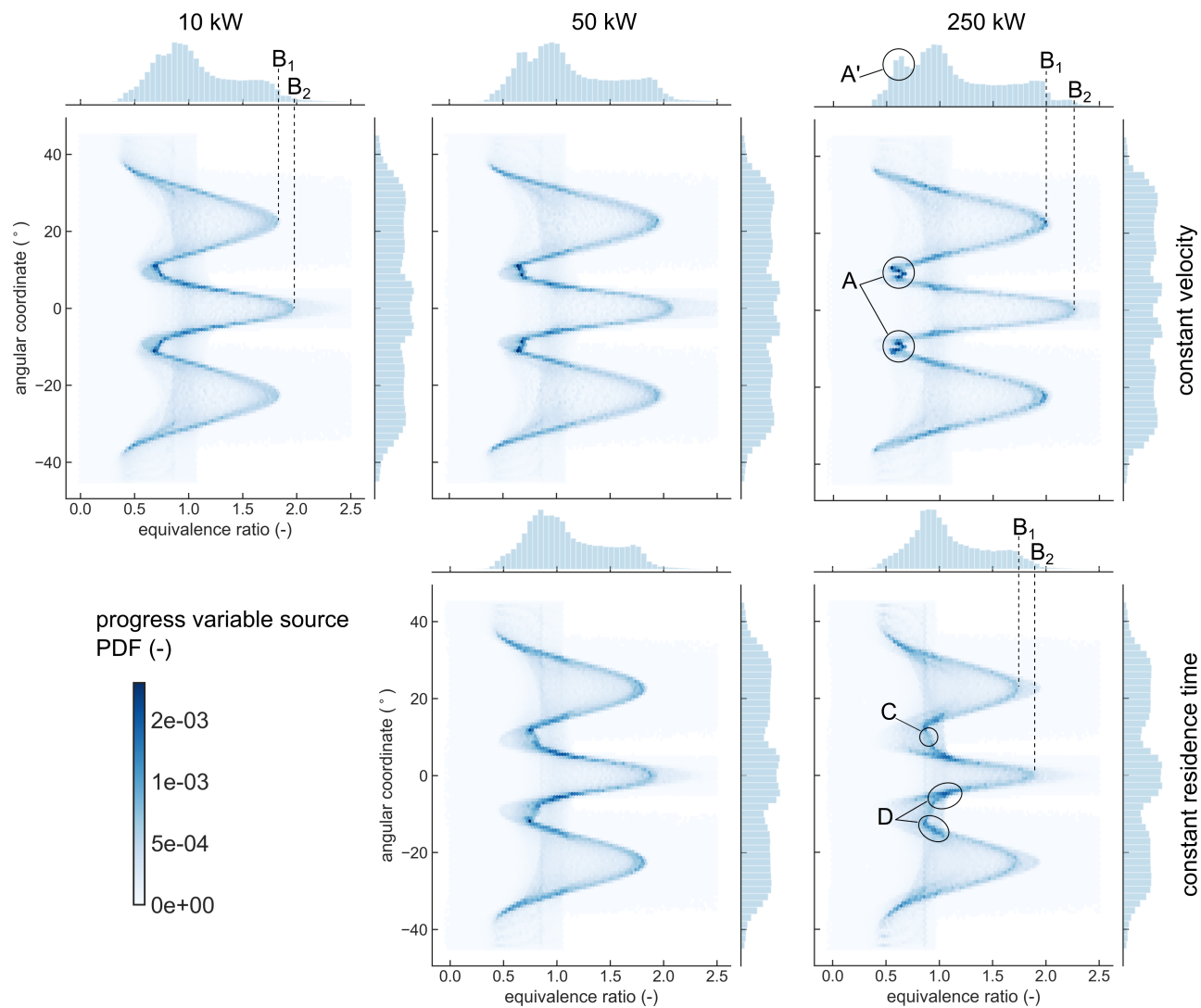


Figure 4: Hexbin plot of the the progress variable source term probability density distribution integrated along axial and radial direction, plotted in a two dimensional space formed by the equivalence ratio and the angular coordinate. The marginal plots show the probability density function of the progress variable source term along the corresponding axes.

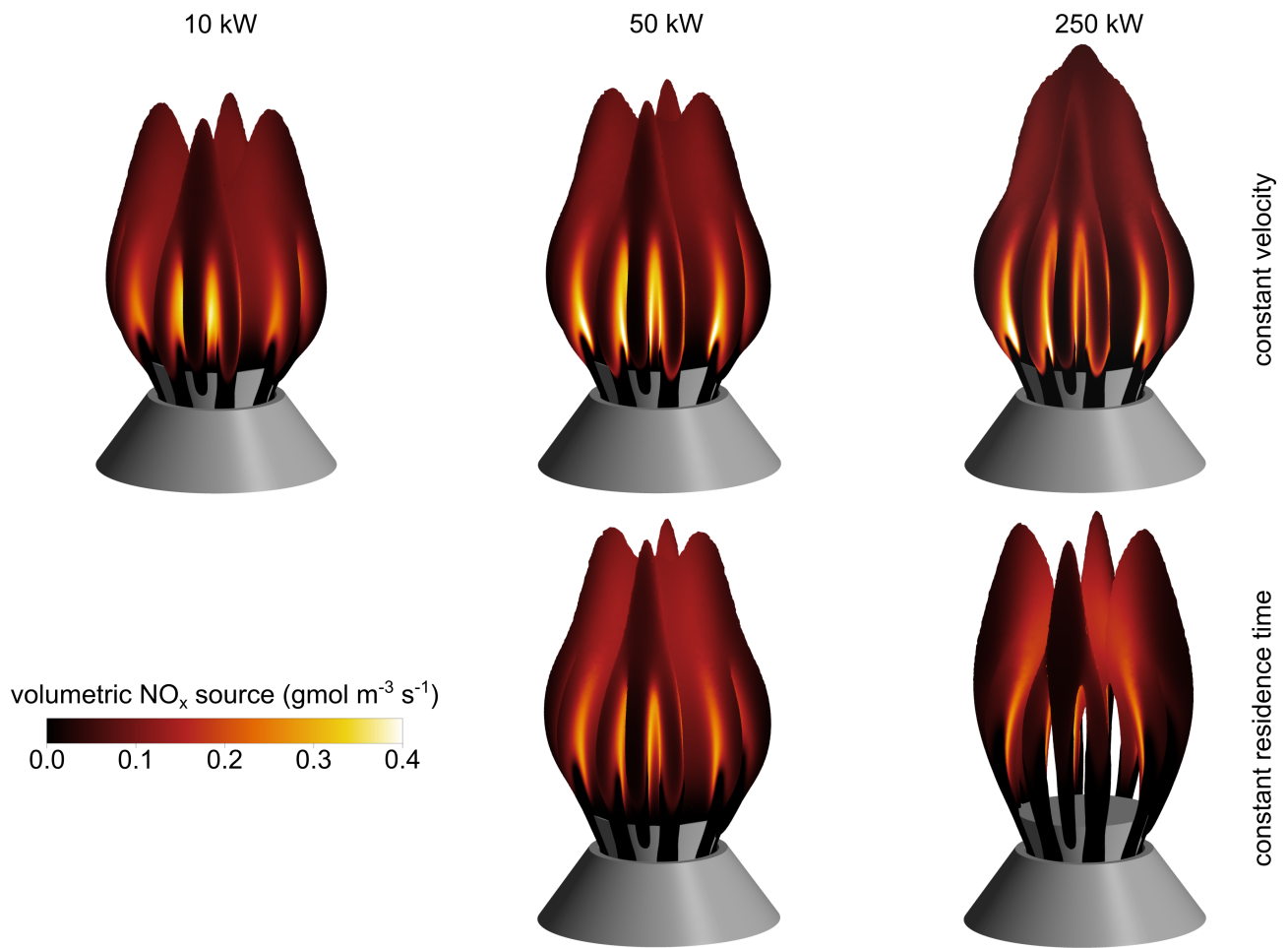


Figure 5: Iso-surface of the stoichiometric equivalence ratio coloured by the NO_x formation rate.

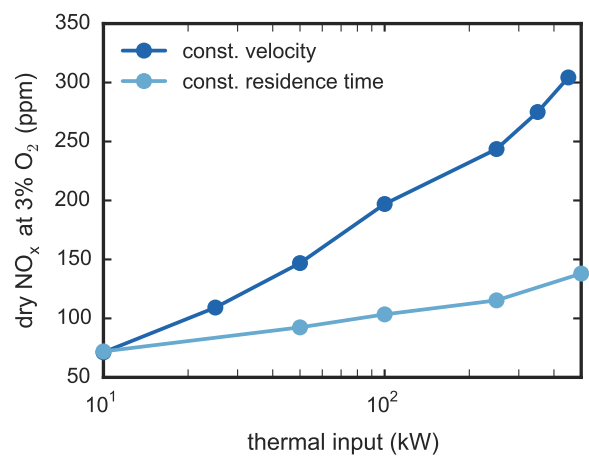


Figure 6: Global NO_x emissions monitored at the chamber outlet.

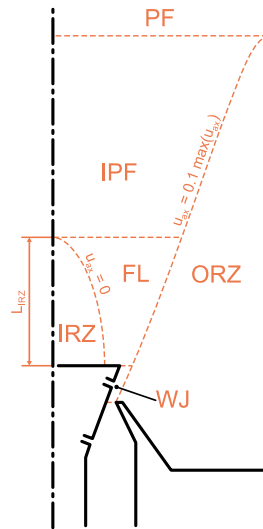


Figure 7: Illustration of the post processing regions. Note the zones are not to scale and the fuel ports are drawn in the same plane for illustration purpose.

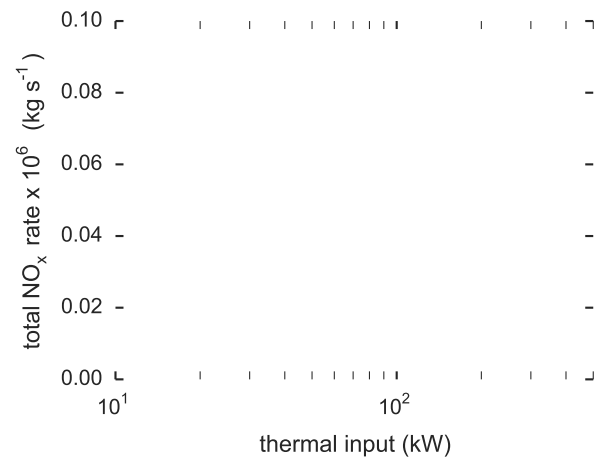


Figure 8: Volume integral of the volumetric NO_x formation rate. Solid lines and filled marker show constant velocity scaling, while dashed lines and open marker show constant residence time scaling.

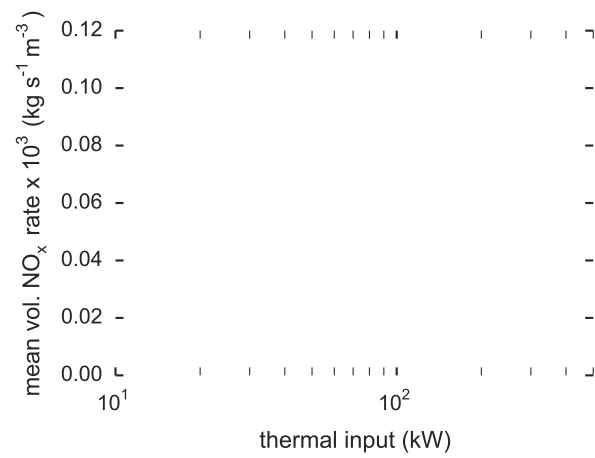


Figure 9: Volume weighted average volumetric NO_x formation rate. Solid lines and filled marker show constant velocity scaling, while dashed lines and open marker show constant residence time scaling.

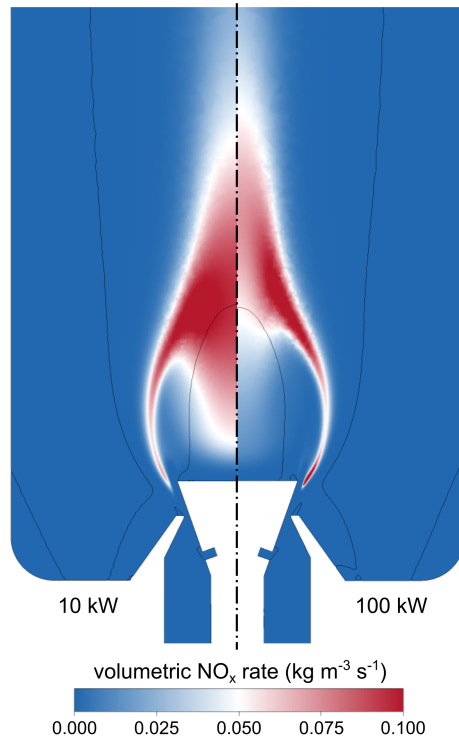


Figure 10: Volumetric NO_x formation rate contours for 10 kW (left) and 100 kW (right) thermal input, where the latter was scaled based on constant velocity scaling. The black iso-lines indicate a zero axial velocity component. Note that the dimensions in the figure are normalized by the bluff body diameter.

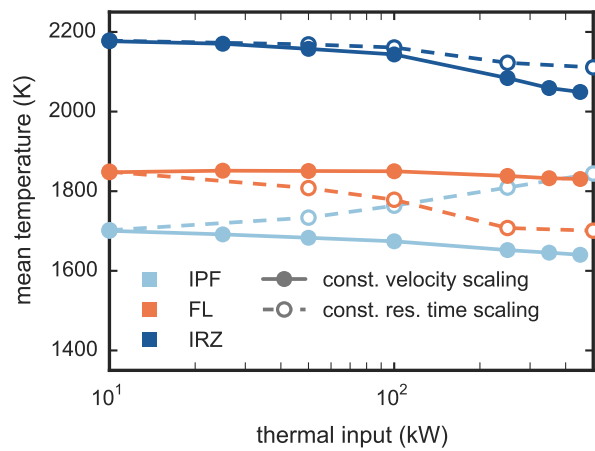


Figure 11: Volume weighted average temperature. Solid lines and filled marker show constant velocity scaling, while dashed lines and open marker show constant residence time scaling.

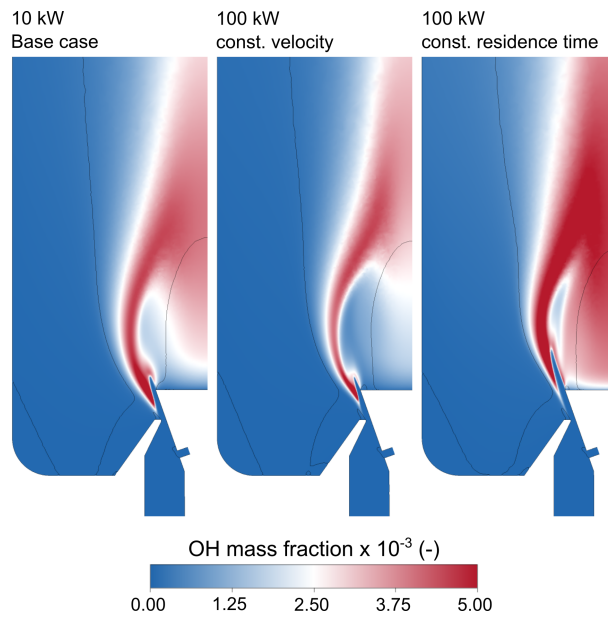


Figure 12: Contours of the OH mass fraction distribution. The black iso-lines indicate a zero axial velocity component. The size of all figures is normalized by the bluff body diameter.

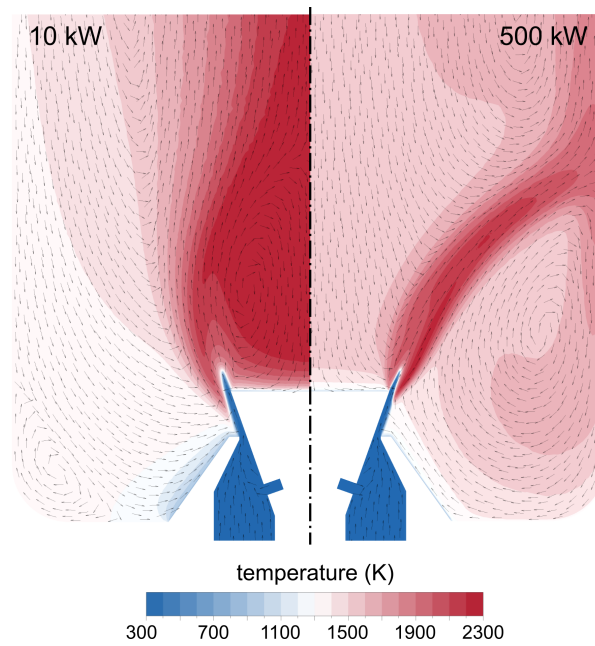


Figure 13: Temperature contours overlaid by the velocity vector field for 10 kW (left) and 500 kW (right) thermal input, where the latter was scaled based on constant velocity scaling. The size of both contours is normalized by the bluff body diameter.