Hydrothermal scheduling in the continuous-time framework

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Abstract

Continuous-time optimization models have successfully been used to capture the impact of ramping limitations in power systems. In this paper, the continuous-time framework is adapted to model flexible hydropower resources interacting with slow-ramping thermal generators to minimize the hydrothermal system cost of operation. To accurately represent the non-linear hydropower production function with forbidden production zones, binary variables must be used when linearizing the discharge variables and the continuity constraints on individual hydropower units must be relaxed. To demonstrate the performance of the proposed continuous-time hydrothermal model, a small-scale case study of a hydropower area connected to a thermal area through a controllable high-voltage direct current (HVDC) cable is presented. Results show how the flexibility of the hydropower can reduce the need for ramping by thermal units triggered by intermittent renewable power generation. A reduction of 34% of the structural imbalances in the system is achieved by using the continuous-time model.

1. Introduction

The Norwegian power system is in an interesting state of transition towards tighter integration to the rest of Europe. New high-voltage direct current (HVDC) cable interconnections to Germany and Great Britain are under construction, which increases the potential of cross-zonal trading of both energy and balancing services. Hydropower dominates the Norwegian generation mix and is well suited to provide system balancing services due to its flexibility. A larger share of intermittent renewable generation means that hydropower will play an increasingly important role in providing flexibility to the interconnected North European system in the future. However, propagating the flexibility across HVDC cables is challenging with current practices related to the hourly day-ahead market structure. According to the Norwegian transmission system operator Statnett, changing the HVDC cable flow between areas on an hourly basis has the potential of increasing the structural (or deterministic) imbalances caused by the mismatch in the scheduled hourly production and real-time load [1]. In this paper, a modified version of the continuous-time optimization framework is proposed to impose a smooth and continuous flow of power between a hydropower area and a thermal area connected by an HVDC cable.

Continuous-time optimization was originally used to accurately describe the cost of ramping scarcity in thermal systems with large amounts of renewable power generation, such as the power system in California [2]. Ramping restrictions can be directly applied to the derivatives of the decision variables when they are allowed to be continuous and smooth functions of time instead of the usual piece-wise constant formulation. The continuous-time formulation relies on limiting the decision variables to be polynomials of degree \( r \), which allows the variables to be expressed by the Bernstein polynomials of the same degree. The optimization problem can then be defined in terms of the coefficients of the Bernstein polynomials, which is a mixed-integer linear program (MILP) in the case of the unit commitment problem. The continuous-time framework has lately been expanded in several directions. The existence of a continuous-time marginal price for the economic dispatch problem was proven and calculated in [3] for a thermal system. This work was later extended to include energy storage devices in [4], which has applications in optimal control of charging electric vehicles according to queue theory [5,6] and the scheduling of batteries in balancing markets [7]. A stochastic continuous-time model was formulated for unit commitment and reserve scheduling problem in [8], with the inclusion of energy storage in [9] and a method for load

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1. Introduction

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estimation and scenario generation in [10]. Applications to other areas within the power system operations field are also emerging, such as the active distribution network model in [11].

Hydrothermal scheduling has been an active field of research for decades, which in turn has contributed to the advanced mathematical models used for system and operational planning in hydropower-dominated systems. Good examples of this are the models used in Norway [12–14] and Brazil [15,16]. Previous hydrothermal scheduling dominated systems. Good examples of this are the models used in models used for system and operational planning in hydropower.

The core idea of the continuous-time framework is to represent time-dependent input and decision variables as polynomials of time instead of piece-wise constant functions. This increases the complexity of the model formulation, but sub-hourly effects and constraints related to derivatives with respect to time are easily captured. The motivation behind the original continuous-time unit commitment model in [2] was precisely to incorporate the impact of ramping scarcity into the market clearing. The time-dependent decision variables in the typical continuous-time optimization framework are defined through the Bernstein polynomials in each time interval, which form a basis for any polynomials of at most degree \(P\). To split the time horizon of the model into \(N\) intervals \(h \in \mathcal{T}\) of length \(\delta_h\), the time-dependent decision variables can be expressed as polynomials of the form

\[
x(t) = \sum_{h \in \mathcal{T}} x_h B_r(t_h) \Pi(h),
\]

where \(t_h\) and \(\Pi(h)\) are defined as follows:

\[
th = \frac{1}{\delta_h} \left( t - \sum_{i<h} \delta_i \right) \quad \forall h \in \mathcal{T};
\]

\[
\Pi(h) = \begin{cases} 1, & 0 \leq \delta_h \leq 1 \\ 0, & \text{otherwise} \end{cases} \quad \forall h \in \mathcal{T}.
\]

The vectors \(x_h\) contain the \(r + 1\) coefficients of the Bernstein polynomials in each time interval, which become the decision variables of the continuous-time model. It is necessary to use the scaled time \(t_h\) and the operator \(\Pi\) to project the Bernstein polynomials into the correct

2. Model

2.1. Fundamentals of a continuous-time model

The core idea of the continuous-time framework is to represent time-dependent input and decision variables as polynomials of time instead of piece-wise constant functions. This increases the complexity of the model formulation, but sub-hourly effects and constraints related to derivatives with respect to time are easily captured. The motivation behind the original continuous-time unit commitment model in [2] was precisely to incorporate the impact of ramping scarcity into the market clearing. The time-dependent decision variables in the typical continuous-time optimization framework are defined through the Bernstein polynomials of degree \(r\), \(B_r(t_h)\), which form a basis for any polynomials of at most degree \(r\) on the time interval \([0,1]\). By splitting the time horizon of the model into \(N\) intervals \(h \in \mathcal{T}\) of length \(\delta_h\), the time-dependent decision variables can be expressed as polynomials of the form

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\]
time interval while maintaining their property as basis functions. One of the main reason for using Bernstein polynomials is the convex hull property, which makes it possible to impose inequality constraints on \( x(t) \) for all times \( t \) by directly bounding the coefficients \( x_k \) [2]. This paper uses Bernstein polynomials of degree 3 as the basis:

\[
B_3(t) = [1 - (1 - t)^3, 3t(1 - t)^2, 3t^2(1 - t), t^3].
\] (4)

This is a popular choice in the literature, as it keeps the size of the model reasonable without sacrificing the ability to model complex time dependencies. Another advantage is the linear relationship to the cubic Hermite splines \( H(t) \), which can be used as an equivalent basis:

\[
H(t) = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -\frac{1}{3} & 1 \end{bmatrix} B_3(t) \equiv W \cdot B_3(t).
\] (5)

The coefficients of the Hermite splines have a physical interpretation as the value of \( x(t) \) and its derivative at the start and end of the time interval \( h \):

\[
x_{xH}^H = (W^{-1} x)^T, x_h = [x_{\text{start}}, x_{\text{start}}, x_{\text{end}}, x_{\text{end}}]^T.
\] (6)

This interpretation is useful for expressing the continuity of \( x(t) \) across the time intervals \( h \). The reader is referred to [2] for a more detailed introduction to the continuous-time formulation with further references to the properties of the Bernstein polynomials mentioned in this section.

### 2.2. Objective function

The objective of the proposed hydrothermal model is to minimize the future expected cost of the system, the penalties for bypassing and spilling water, and the operational, startup and shutdown costs of the thermal generators:

\[
Z = \alpha + \sum_{m \in M} \int_0^{t_{\text{end}}} (C_m^q q_m^i(t) + C_m^u q_m^o(t)) dt + \sum_{j \in J} \int_0^{t_{\text{end}}} C_j g_j(t) dt + \sum_{j \in J} \sum_{k \in T} (C_j^l s_{jk} + C_j^r s_{jk}).
\] (7)

Note that startup and shutdown cost are assumed to be negligible for the hydropower plants. The definite integral of the Bernstein polynomials of the third degree is \( \int_0^1 B_3(t) dt = \frac{1}{4} t^3 \), which simplifies the integrals in (7) to the sums

\[
Z = \alpha + \frac{1}{4} \sum_{m \in M} \sum_{k \in T} \delta_k F_k (C_m^q q_m^i + C_m^u q_m^o) + \sum_{j \in J} \sum_{k \in T} \left\{ \frac{1}{4} \delta_k C_j^l g_j + C_j^r s_{jk} + C_j^r s_{jk} \right\}.
\] (8)

As this paper focuses on modelling hydropower generation in the continuous-time framework, a simplified linear formulation of the thermal generation cost function is used in (8). More advanced modeling of quadratic and piece-wise linear cost functions in continuous-time unit commitment is available in the literature [2,9], and their integration in the model proposed in this paper is straightforward.

### 2.3. Hydropower topology constraints

The cascaded topology constraints dictate how water moves between the reservoirs. These constraints are equality constraints, see for instance [14], which means that equating the polynomial coefficients are sufficient to satisfy them in the continuous-time framework. The convex hull property of the Bernstein polynomials and the fact that \( t^3 \cdot B_3(t) = 1 \) is used to enforce the physical bounds on the variables:

\[
q_{\text{in}}^m = I_{\text{in}} + q_{\text{in}}^m - q_{\text{out}}^m \quad \forall \ m, h \in M, T
\] (9)

\[
q_{\text{out}}^m = q_{\text{in}}^m + q_{\text{out}}^m \quad \forall \ m, h \in M, T
\] (10)

\[
q_{\text{in}}^m = \sum_{i \in \mathcal{I}} q_{\text{in}}^i + \sum_{i \in \mathcal{I}} q_{\text{in}}^i + \sum_{i \in \mathcal{I}} q_{\text{in}}^i \quad \forall \ m, h \in M, T
\] (11)

\[
q_{\text{out}}^m = q_{\text{out}}^m + q_{\text{out}}^m - I_{\text{out}}^m \quad \forall \ m, h \in M, T
\] (12)

\[
0 \leq q_{\text{in}}^m \leq Q_{\text{in}}^m \quad \forall \ m, h \in M, T
\] (13)

\[
0 \leq q_{\text{out}}^m \leq Q_{\text{out}}^m \quad \forall \ m, h \in M, T
\] (14)

\[
0 \leq q_{\text{in}}^m \quad \forall \ m, h \in M, T
\] (15)
There are three waterways that connect reservoirs; discharge through the turbine, the bypass gate and the spill gate. Fig. 1 shows the relationship between the different waterways in addition to where natural inflow enters the system.

2.4. Volume constraints

The rate of change in the reservoir content is described by the differential equation:
\[
\frac{dv_m(t)}{dt} = q_{mw}^\text{net}(t) \quad \forall \ m \in M.
\]
(17)

The integral of Bernstein polynomials of degree 3 can be expressed using Bernstein polynomials of degree 4 using a linear mapping matrix [4,9]:
\[
\int B_3(t) dt = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} B_4(t) \equiv N \cdot B_4(t)
\]
(18)
which is further utilized to show the volume increase within a time interval as follows:
\[
v_m(t) - v_m^0 = \int_{t=0}^{t} \int_{t=0}^{t} \frac{dv_m(t)}{dt} dt' = (q_{mw}^\text{net} \cdot \cdot) \cdot B_4(t) dt' = \delta_0 (q_{mw}^\text{net} \cdot \cdot) \cdot N \cdot B_4(t)
\]
(19)
Here, \( t(h) = \sum_{i=1}^{3} \delta_i \) and the fact that \( N \cdot B_4(0) = 0 \) was used. Note that the volume variables \( v_m \) denotes the volume at the start of interval \( h \). Based on equation (19), the following volume balance constraints can be added to the optimization problem:
\[
v_{m+1} = v_m + q_{mw}^\text{net} \quad \forall \ m \in M
\]
(20)

\[
v_{m+h+1} - v_{m+h} = \frac{1}{4} \delta_0 (q_{mw}^\text{net} \cdot \cdot) \cdot N \cdot B_4(t) \quad \forall \ m, h \in M, \mathcal{T}
\]
(21)

\[0 \leq v_{m+h} + \delta_0 N \cdot q_{mw}^\text{net} \leq v_m \quad \forall \ m, h \in M, \mathcal{T}
\]
(22)

Constraint (20) sets the initial volume of each reservoir and (21) calculates the volume change from one time interval to the next by inserting \( N \cdot B_4(1) = \frac{1}{4} L \). Constraint (22) uses the convex hull property to bound the volume within the limits of the reservoir for all times \( t \).

2.5. Future cost bounds

The future expected cost of the system is represented by a set of Bohr's cuts created by a hydrothermal long-term model such as [14]. The expected future cost depends on the state of all hydropower reservoirs in the system at the end of the last time interval \( N' \):
\[
\alpha \geq \sum_{m \in M} W v_{m+N'=1} + D_k \quad \forall k \in \mathcal{K}.
\]
(23)

2.6. Hydropower production

The conversion from discharge through the turbine to generated power is a non-linear function which depends on the effective plant head and the efficiency curves of the turbine and generator [17]. By assuming a constant head for the planning horizon, the hydropower production function can be approximated as a single piece-wise linear curve, where the discharge variable is split into \( n \in \mathcal{N}_n \) segments with constant gradient \( \eta_n \). In an discrete-time model, the discharge segments will usually be uploaded in the correct order as long as the gradient is decreasing for increasing segment number. The exception is extreme situations where it is beneficial to dump as much water as possible while limiting the power produced, which can be the case in high inflow and low load scenarios. A similar effect of incorrect uploading of discharge segments has been observed in this work when the continuous-time framework was implemented. Segments with high efficiency are still favoured but there is no guarantee that segment \( n \) is at its maximal capacity for all times that segment \( n+1 \) is being used. The model will often start using the next segment too early to be able to fulfill the continuous-time power balance described in Section 2.7. To remedy this problem, binary variables \( w_{nw} \) \( (t) = \sum_{n \in \mathcal{N}_n} B_4(t) \cdot \cdot \cdot(I(I(n))) \) are used in this work to force the segments to be fully utilized before the next segment can be used:
\[
q_{nw}^d = \sum_{m \in \mathcal{M}} q_{nw}^\text{net} \quad \forall \ m, h \in M, \mathcal{T}
\]
(24)

\[
\mathcal{P}_{nw} = \sum_{m \in \mathcal{M}} P_{nw} \cdot q_{nw}^d \quad \forall \ m, h \in M, \mathcal{T}
\]
(25)

\[
Q_{nw}^l w_{nw} \cdot q_{nw}^d \leq q_{nw}^d \leq Q_{nw}^l w_{nw} \quad \forall \ m, n, h \in M, \mathcal{T}, \mathcal{N}_n
\]
(26)

\[
q_{nw}^d \leq Q_{nw}^l w_{nw} \cdot q_{nw}^d \quad \forall \ m, h \in M, \mathcal{T}, \mathcal{N}_n \setminus \{0\}
\]
(27)

This modelling choice of the hydropower production function has the unfortunate effect of introducing additional binary variables into the model but also enables the use of non-concave linearizations of the hydropower production function. It is also possible to incorporate forbidden production regions within the operating range of the turbine by modifying (26) to \( q_{nw}^d = Q_{nw}^l w_{nw} \cdot q_{nw}^d \) for the segment representing the forbidden region.

2.7. Power balance and HVDC power flow

The power balance constraints must be satisfied in each node of the system. In this work, each node represents a larger market area assuming no internal power flow limits. The areas are connected with HVDC cables where the flow can be controlled by the system operator. The power balance constraints are formulated as
\[
\sum_{m \in \mathcal{M}} P_{nw} + \sum_{a \in \mathcal{A}} g_{pa} - \sum_{a \in \mathcal{A}} G_{pa} f_{pa} = L_{pa} \quad \forall \ a, h \in \mathcal{A}, \mathcal{T}.
\]
(28)

The coefficient \( G_{pa} \) dictates the positive and negative direction of flow on each cable \( l \in \mathcal{L} \) by taking the values \( \pm 1 \), or zero if cable \( l \) is not connected to area \( a \). \( M_e \) and \( \mathcal{J}_a \) are the sets of hydropower and thermal units located in area \( a \), respectively. The flow on the HVDC cables is constrained by maximal flow limits
\[
F_{\text{max}} \leq f_{pa} \leq F_{\text{min}} \quad \forall \ a, h \in \mathcal{L}, \mathcal{T}.
\]
(29)
and additional limitations on the change of flow is imposed on the derivative \( f_{pa} \) to stay within the specified HVDC cable ramping limits used in the Nordic system [18]. By using the following property of the Bernstein polynomials,
2.8. Thermal generation constraints

The thermal generators are subject to unit commitment decisions which signify if a generator is online or producing between the minimal and maximal production limits. The thermal unit commitment constraints are modelled by the use of the binary decision variables $u_j(t)$:

$$G_{\min}^j u_j \leq G_j(t) \leq G_{\max}^j u_j\quad\forall \, j, \, h \in J, \, T$$

(32)

$$u_j = \left[ u_{j0}, u_{j,h+1}, u_{j,h+1} \right]^T \forall \, j, \, h \in J, \, T\setminus[N]$$

(33)

$$u_N = u_{Nh,1} \mathbf{1} \forall \, j \in J$$

(34)

$$s_{j,h}^+ - s_{j,h+1} = u_{j,h+1} - u_{j,h} \forall \, j, \, h \in J, \, T\setminus[N]$$

(35)

$$s_{j,h}^+ + s_{j,h+1}^i = 1 \forall \, j, \, h \in J, \, T$$

(36)

$$u_{Nh} \in [0, 1] \forall \, j \in J, \, T.$$  

(37)

The constraints closely follow the implementation used in [2] and [8], which are in turn adapted from the standard discrete-time unit commitment formulation found in for instance [19]. The choice of the commitment decision vector in (33) and (34) allows the thermal generator to use time interval $h$ to ramp up from zero to above $G_{\min}$ or conversely ramp down production to zero. The smooth transition is necessary for the continuity constraints that will be applied to the thermal production variables in Section 2.10. Constraint (35) captures the startups and shutdowns of the generators, which are accounted for in the objective function (8). The up and down ramping constraints of thermal generators, taking into account the startup and shutdown ramp limitations, are modeled as follows:

$$\frac{1}{\delta_h} \left[ R_j^+ + R_j s_{j,h}^+ \right] g_j \leq G_j(t) \leq \frac{1}{\delta_h} \left[ R_j^- + R_j s_{j,h}^- \right] g_j \quad \forall \, j, \, h \in J, \, T$$

(38)

$$\frac{1}{\delta_h} \left[ R_j^+ + R_j s_{j,h}^+ \right] g_j \leq -R_j^- \left[ s_{j,h}^- \right] g_j \leq \frac{1}{\delta_h} \left[ R_j^- + R_j s_{j,h}^- \right] g_j \quad \forall \, j, \, h \in J, \, T.$$  

(39)

The minimum up and down time constraints of thermal generation is not considered in this paper, and the readers are referred to our previous works for details on modeling these constraints in the continuous-time unit commitment model [2].

2.9. Hydropower unit commitment

Due to operating characteristics such as mechanical vibration or loss of efficiency, hydropower turbines usually have one or several forbidden production regions depending on the turbine type. It is important to model these regions when looking at short-term scheduling of a hydropower system to have an accurate representation of the operating range of the hydropower plants. The unit commitment constraints of the hydropower plants in the continuous-time optimization model must account for the forbidden production region so that the flexibility of the plant is not overestimated. The hydropower unit commitment decisions $z_{min}(t)$ are used to model this in the following way:

$$P_{m}^{min} z_{min} \leq P_{m} \leq P_{m}^{max} z_{min} \forall \, m, \, h \in M, \, T$$

(40)

$$z_{min} = z_{min} \mathbf{1} \forall \, m, \, h \in M, \, T$$

(41)

$$s_{min}^+ - s_{min}^i = z_{min} \forall \, m, \, h \in M, \, T\setminus[N]$$

(42)

$$s_{min}^+ + s_{min}^i \leq 1 \forall \, m, \, h \in M, \, T$$

(43)

$$z_{min}, s_{min}^i \in [0, 1] \forall \, m, \, h \in M, \, T.$$  

(44)

In contrast to the choice of the thermal unit commitment vector in (33), the formulation in (41) forces the hydropower unit commitment decisions to be constant for the whole time interval so that the production is never between 0 and $P_{m}^{min}$. However, this formulation is in opposition to the normal continuous-time formulation, as discontinuous jumps in power production must be allowed. If not, the hydropower plants will be unable to start and stop at all. These issues are addressed in Section 2.10.

2.10. Continuity constraints

The standard continuous-time optimization framework builds on the $C^0$ continuity of all decision variables $x(t)$. This requires both the value $x(t)$ and the value of the derivative $x(t)$ to be continuous over the change of time intervals $h \in T$. Such constraints are enforced by using the relationship between the Bernstein polynomials and the cubic spline functions, shown in (5). The interpretation of the coefficients of $H(t)$ described in (6) simplifies the implementation of the $C^0$ continuity constraints. By labelling the components of the vector $x$ as $x[i]$ for $i \in \{0, 1, 2, 3\}$, the continuity constraints become:

$$x_h^H[2] = x_{h+1}^H[0] \quad \forall \, h \in T\setminus[N]$$

(45)

$$x_h^H[3] = x_{h+1}^H[1] \quad \forall \, h \in T\setminus[N].$$

(46)

These constraints are applied to the thermal generation and HVDC flow variables:

$$g_{j,h}^H[2] = g_{j,h+1}^H[0] \quad \forall \, j, \, h \in J, \, T\setminus[N]$$

(47)

$$g_{j,h}^H[3] = g_{j,h+1}^H[1] \quad \forall \, j, \, h \in J, \, T\setminus[N]$$

(48)

$$f_{b,h}^H[2] = f_{b,h+1}^H[0] \quad \forall \, l, \, h \in L, \, T\setminus[N]$$

(49)

$$f_{b,h}^H[3] = f_{b,h+1}^H[1] \quad \forall \, l, \, h \in L, \, T\setminus[N].$$

(50)

As mentioned in Section 2.9, discontinuous jumps in power production are required to model the forbidden production region of hydropower plants. Therefore, enforcing the $C^0$ continuity constraints on the variables related to the hydropower production is not possible. In addition, requiring continuous derivatives for water flow and hydropower production is strict when $\delta_h$ is longer than a few minutes. To avoid conservative solutions underestimating the ramping capabilities of hydropower, (46) is not implemented for any variable related to hydropower. The bypass and overflow variables are $C^0$ continuous:

$$q_{inh}^B[2] = q_{inh+h+1}^H[0] \quad \forall \, m, \, h \in M, \, T\setminus[N]$$

(51)
and the reservoir volume continuity is already secured by (21). The hydropower production is forced to be \( C^1 \) continuous unless a startup or shutdown happens in the time interval. This is modelled by replacing (45) by the following two inequalities:

\[
\begin{align*}
p_{h, k}^{\text{H}}[2] - p_{h, k-1}^{\text{H}}[0] & \leq p_{\text{max}}^{\text{H}} \forall m, h \in M, \mathcal{T} \cap \mathbb{N} \\
p_{h, k}^{\text{H}}[0] - p_{h, k-1}^{\text{H}}[2] & \leq p_{\text{max}}^{\text{H}} \forall m, h \in M, \mathcal{T} \cap \mathbb{N}
\end{align*}
\]

which is consistent with the unit commitment constraints imposed in (40) to (44). Note that this relaxation produces a more constrained problem, as production in the forbidden region is impossible. Due to the connection between production and discharge in (25), the discharge variables \( q \) must also be allowed to have discontinuous jumps. However, the binary definitions of the discharge bounds in (26) and (27) take care of continuity when the hydropower plant is producing, so there is no need to apply any further constraints to the discharge variables. The continuity properties of the derived flow variables \( q^m, q^q, q^s, q^a \) and \( q^d \) are also implicitly accounted for through (9) to (12).

It is important to note that even though the individual hydropower plants may have discontinuous jumps and discontinuous derivatives in the power production curve between time intervals, their sum is still forced to be \( C^1 \) continuous through the power balance constraint (28) since all other quantities in the equation are \( C^1 \) continuous. The \( C^1 \) continuity constraints of the flexible hydropower have effectively been lifted from the individual plant to the sum on an area level. The hydropower topology is based on a real Norwegian water course only hydropower, while the other only contains thermal generation. Most of the ramping is carried out by the hydropower system, which can be seen in Fig. 4. The figure shows that the hourly model overestimates the ramping capabilities of the thermal system during the extreme ramping events. Thermal production is shut down in the morning and turned back on in the afternoon, while the hydropower producers increase their production to cover the load in both areas in the meantime. This is not the case in the continuous-time model, as shutting down all thermal generators is either infeasible or very costly when following the net load during the ramping events. The cheapest and slowest thermal generator stays on for the whole 24 hours in the continuous-time model, contributing to the ramping in a modest way.

The resulting sum production of hydropower and thermal generators are shown in Fig. 3. The figure shows that the hourly model overestimates the ramping capabilities of the thermal system during the extreme ramping events. Thermal production is shut down in the morning and turned back on in the afternoon, while the hydropower producers increase their production to cover the load in both areas in the meantime. This is not the case in the continuous-time model, as shutting down all thermal generators is either infeasible or very costly when following the net load during the ramping events. The cheapest and slowest thermal generator stays on for the whole 24 hours in the continuous-time model, contributing to the ramping in a modest way.

Most of the ramping is carried out by the hydropower system through the HVDC cable, which can be seen in Fig. 4. The figure shows how the hydropower system is able to mitigate the ramping in net load in both directions while keeping the thermal generator online. The power flow is kept close to 50 MW throughout the day in the hourly model since the hydropower is generally cheaper than the thermal generators. However, two major changes in flow occur when the thermal generators are shut down and then started back up in the thermal system. This behaviour is undesirable, as it can increase the structural imbalances in the system [1].

4. Conclusion

Hydropower is considered an important balancing resource due to its flexibility. A continuous-time hydrothermal unit commitment model with HVDC cables was formulated in this paper to show how excessive ramping in the thermal system can be avoided by hydropower and active use of the HVDC cables. The structural imbalances in the system are reduced by 34% in the continuous-time model compared to the
hourly discrete-time model since sub-hourly effects are captured by the polynomial expansion. Several modelling issues related to incorporating hydropower into the continuous-time framework have been uncovered in the process. The linearization of the hydropower production curve requires binary variables to avoid unphysical uploading, and modelling the forbidden production zone requires the relaxation of the continuity constraints of the individual hydropower plants. The overall continuity of the model is still preserved on a system level, as the power balance forces the sum of hydropower production to be continuous. Investigating other potential modelling choices of the hydropower production curve, calculating system prices, and expanding the model to cover cross-zonal reserve capacity procurement are interesting avenues of further research.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References