Estimation of tank-to-wheel efficiency functions based on type approval data

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HIGHLIGHTS

- Type approval data are useful for determining tank-to-wheel efficiency functions.
- Estimated efficiency functions outperform tank-to-wheel efficiency constants.
- The efficiency for diesel fueled vehicles is the most challenging to estimate.

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ABSTRACT

The representation of tank-to-wheel efficiency is for many modelling purposes simplified to a constant value. However, the relationship between tank-to-wheel efficiency and operational properties is not necessarily constant. As a result, situations with both favorable and unfavorable driving conditions will not be adequately represented in instruments for decision making. In this paper, we address this issue by estimating tank-to-wheel efficiency functions customized for use in transport and traffic modelling applications. The functions are estimated by sum of squares on aggregated data, with two different seed functions from a theoretical basis. The level of detail in the estimated functions provide more complexity for evaluations of transport systems in terms of energy and fuel usage, while avoiding too complex modelling of internal engine processes or vehicle specific type parameters. Data sets from vehicle type approval tests are used for the estimation process, and validation tests suggest that a tank-to-wheel efficiency function outperforms a constant value.

1. Introduction

The operational properties of specific car engines are not readily available. Engine processes are complex, where for instance the fuel consumption depends on variables such as load, speed, size and temperature of the engine, as well as ambient temperatures. For the purpose of vehicle modelling, these processes need to be simplified and expressed in theoretical terms. Yet, some very detailed models exist, e.g. the VECTO model [1] and the ADVISOR model [2]. The level of detail in these models leads to a high cost in the form of input data requirements, which might be perfectly acceptable when modelling a single vehicle. However, a higher number of different vehicle types will increase the input requirements. In this case, a simpler approach may be appropriate, e.g. a constant value representing an average tank-to-wheel (TTW) efficiency. TTW efficiency is, in this context, defined as the ratio between energy output from the wheels and the energy content of the fuel in the tank.

The TTW efficiency is determined by the total amount of losses, which in a combustion engine comprise thermal losses, pump losses and mechanical losses [3]. The thermal losses occur as not all of the fuel energy is transformed to mechanical energy, and most of these losses are dissipated through the exhaust. The combustion process is unable to utilize all the thermal energy, and the exhaust ends up at a higher temperature and pressure than the ambient air. All the rotating parts of the engine creates friction when moving and results in mechanical losses, which increase with the speed of the engine. For electric vehicles, there are losses in the battery, converter and electric engine. These components have ohmic losses due to resistance, which will typically give a loss proportional to the power squared. In addition to this, the converters have switching losses which are proportional to the switching frequency, and magnetic losses in the electric engine. In the drive train, the losses are mainly mechanical, and these friction forces

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are increasing with both speed and torque.

The TTW efficiency for combustion engines is about 16%–20% when all driving situations are included, as reported by [4]. However, this number varies depending on the driving situation. According to [5], typical values are 10%–13% on urban cycles and about 28% for highway cycles. [6] include engine speed as a variable, and state that diesel engines have an efficiency rate between 20% and 25% for engine speeds below 2000 rpm. For engine speeds higher than 2000 rpm, the efficiency was reported to be significantly lower for conventional diesel engines. In the same study, substitution to biodiesel showed an improved efficiency for engine speeds higher than 2000 rpm with a peak effect of 35%, but drop rapidly to about 5% when it reaches about 2900 rpm. [7] use a constant value for TTW efficiency in their longitudinal dynamics model, which is set to 80% for battery electric vehicles (BEVs), both use and recharge, and 30% for internal combustion engine vehicles (ICEVs). [8] developed an energy consumption model for electric freight vehicles, assuming the efficiency as a set of parameters for each vehicle. Their estimated values were in the range of 90% and 98% for the driveline, electric motor and battery efficiencies.

This shows that the TTW efficiency depends on the operating conditions. For overarching calculations and modelling, a representation of this process through a constant value will be sufficient. Problems arise when a more detailed approach is necessary. This is especially the case when the engine operates in atypical conditions, e.g. high or low speeds, steep grades, stop and go speed profiles and unfavorable temperatures. [9] reviewed fuel consumption models and classified them according to transparency, where white-box models are completely based on mathematical descriptions of physical processes, and black-box models are designed as input-output models with the physical processes hidden in a black box defined from experiments and measurements. A combination of the two models is called a grey box model, which contains both experimental data and mechanical insight. In white- and grey-box models, a typical predictor for the engine output is the vehicle specific power (VSP), which depends on the resistances acting on the vehicle. For example, [10] developed hybrid regression models to estimate the energy consumption of BEVs based on different levels of VSP but used constant values for the energy efficiency. [11] also used a constant value to represent energy efficiencies when using VSP to study the impacts of highway electrification strategies. [12] summarize the use of longitudinal dynamic models (LDM) and studies the impact of all relevant parameters through a sensitivity analysis. They find that TTW efficiency is the parameter with the greatest impact on the modelled energy use.

Although constant TTW efficiencies are widely used in the literature, there have been several attempts to estimate efficiency functions. [13] present a method for estimating the specific fuel consumption as a function of engine speed and engine load for a specific diesel engine. However, the method requires detailed experimental data about the engine as input. [14] use vehicle speed as the explanatory variable, for simplicity, although remarking that the vehicle power would be the first choice. In their concluding remarks, they stress that future work should involve a more detailed description of efficiency functions. In line with this, [15] present the TTW efficiency as a function of engine load, but only for electric motors. Their functions are based on guidelines from the [16]. The approach by [17] includes a representation of TTW efficiency functions based on Willans lines, which are linear correlations between input and output power. Similar to the study by [13], the estimation of efficiency functions require data from dynamometer experiments.

It is clear from the presented literature that it is possible to estimate accurate efficiency functions, but the methods require detailed experimental data. Thus, there is a gap for knowledge in research on the estimation of efficiency functions which can be applied without the need for advanced data collection and estimation. This is especially the case for fleet-based calculations, where the data demand renders detailed models impractical, and efficiency constants too simplified. This knowledge gap is addressed in this study, where we set out to estimate a relationship between the TTW efficiency and VSP. The estimation is based on the large and readily available dataset from the European Environment Agency (EEA) containing vehicle approval tests, which supply CO₂-emission or electric power consumption values, vehicle characteristics and test procedures for 62 000 vehicles between the years 2012 and 2017. Since the VSP can be estimated for each vehicle at each point of the test cycle, and measured CO₂-emissions can be used to derive fuel consumption for fossil based vehicles, we can assume that the only unknown variable is a TTW efficiency function. By considering these tests as single experiments, we estimate TTW efficiency functions by minimizing the error between measured and calculated CO₂-emissions for ICEVs or between measured and calculated electric power consumption for BEVs. The main contribution from this study is a set of representative TTW efficiency functions for ICEVs, both diesel and petrol, and BEVs.

2. Data collection and preparation

The United Nations Economic Commission for Europe (UNECE) provides technical requirements for vehicles through the 1958 Agreement on technical harmonization of vehicles. The 1958 Agreement regulates “performance-oriented test provisions; administrative procedures for granting type approvals; conformity of production; mutual recognition of the type approvals granted by contracting parties” [18]. The European Union provides an additional framework for technical harmonization, for which safety and environmental requirements in Directive 2007/46 sets the basis. The directive enforces the application of the Whole Vehicle Type-Approval System (WVTA) to all motor vehicles and their trailers. It also formalizes and reinforces the UNECE regulations of 1958. Type approval is defined as “the process applied by national authorities to certify that a model of a vehicle meets all EU safety, environmental and conformity of production requirements before authorising it to be placed on the EU market” [19].

The Worldwide Harmonised Light Vehicle Test Procedure (WLTP) is a laboratory test used to measure car emissions and fuel consumption and it ensures that new vehicles comply with the EU emission regulations. The WLTP test is designed to be standardized and repeatable and hence allow for comparison between various models of vehicles. From 2017/18, the WLTP replaced the previous lab test, the New European Driving Cycle (NEDC), which was introduced in the 1980s.

The WLTP test lasts for 30 min and is divided into four phases with different speed: low, middle, high and extra high. Each phase involves stops, accelerations and breaking [20]. During the test, the vehicle runs for 23 km at an average speed of 46.5 km/h and a maximum speed of 131 km/h. The test cycle consists of four dynamic phases, of which 52% features urban driving and 48% non-urban driving. Accelerations and decelerations are present during 44% and 40% respectively of the test, while constant driving accounts for 4% and stop duration of 13% [21].

Similar to WLTP, the NEDC test requires a cold start and during the test the daytime running lights must be turned on and the vehicle should comply with predefined specifications for engine oil, tires and fuel. In comparison, the NEDC test is shorter than the WLTP test, both in time and distance, running for 20 min and 11 km. Instead of four phases, the NEDC test consists of two phases, where the majority (66%) simulates urban driving. The percentage of constant driving during the test is also much higher than for the WLTP, amounting to 40%, while acceleration is 21% of the test and deceleration 15%. Stop duration is in total 24% of the test. Both the average speed, at 34 km/h, and the maximum speed, at 120 km/h, are lower than for the current WLTP test.

For this paper we rely on measurements from the NEDC test, as this test has been performed for many years and thus have a much higher data volume available. Since we are aiming at estimating energy efficiency functions through a simple form of reverse engineering, we are dependent on high volumes of data, even though WLTP is considered
Table 1
Number of type approval tests available for passenger cars between 2012–2017, including the mean of engine power and weight for each year.

<table>
<thead>
<tr>
<th>Year</th>
<th>N (NEDC)</th>
<th>N (WLTP)</th>
<th>Weight (kg)</th>
<th>Power (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>375 415</td>
<td>442 475</td>
<td>1 566</td>
<td>115</td>
</tr>
<tr>
<td>2013</td>
<td>417 709</td>
<td>440 645</td>
<td>1 527</td>
<td>111</td>
</tr>
<tr>
<td>2014</td>
<td>493 818</td>
<td>4 955 599</td>
<td>1 535</td>
<td>118</td>
</tr>
<tr>
<td>2015</td>
<td>37 5</td>
<td>645 493</td>
<td>1 533</td>
<td>118</td>
</tr>
<tr>
<td>2016</td>
<td>417 709</td>
<td>4 955 599</td>
<td>1 540</td>
<td>115</td>
</tr>
<tr>
<td>2017</td>
<td>417 709</td>
<td>4 955 599</td>
<td>1 528</td>
<td>116</td>
</tr>
</tbody>
</table>

Table 2
Number of type approval tests available for vans between 2012–2017, including the mean of engine power and weight for each year.

<table>
<thead>
<tr>
<th>Year</th>
<th>N (NEDC)</th>
<th>N (WLTP)</th>
<th>Weight (kg)</th>
<th>Power (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>148 612</td>
<td>255 707</td>
<td>2 045</td>
<td>142</td>
</tr>
<tr>
<td>2013</td>
<td>1 426</td>
<td>1 495</td>
<td>1 897</td>
<td>93</td>
</tr>
<tr>
<td>2014</td>
<td>1 495</td>
<td>1 636</td>
<td>1 760</td>
<td>84</td>
</tr>
<tr>
<td>2015</td>
<td>1 636</td>
<td>1 650</td>
<td>1 786</td>
<td>86</td>
</tr>
<tr>
<td>2016</td>
<td>1 650</td>
<td>740</td>
<td>1 815</td>
<td>88</td>
</tr>
<tr>
<td>2017</td>
<td>740</td>
<td>249</td>
<td>1 825</td>
<td>93</td>
</tr>
</tbody>
</table>

3. Method overview

3.1. Estimation of CO₂-emission in type approval tests

To estimate the CO₂-emissions for ICEVs driving through a test procedure, here the NEDC, we first calculate the theoretical work needed for each specific vehicle to complete the driving cycle. For each uniform time slot in the test cycle, we use the entity

\[ \Delta W_{i-1, i} = t_i \text{max} [v_i (mgC_f + 0.5 \rho v_i^2 AC_d + ma_i), 0], \]

where \( \Delta W_{i-1, i} \), \( t_i \), \( v_i \), and \( a_i \) is the calculated work required, time, average speed and acceleration between point \( i-1 \) and point \( i \), respectively, \( m \) is the weight of the vehicle, \( g \) is the gravitational acceleration, \( C_f \) is rolling resistance coefficient, \( \rho \) is the air density, \( C_d \) is the drag coefficient and \( A \) is the vehicle projected frontal area.

Second, the vector of utilized engine power between each point \( i-1 \) and \( i \), denoted \( p_i \) for \( i = 2, ..., N \), in the test procedure is constructed by

\[ p = (p_2, ..., p_N) = (\Delta W_{i-1, i}/W_{\text{max}})_{i=2}^N, \]

where \( W_{\text{max}} \) is the work capacity of the engine, available in the EEA dataset for each vehicle. Note that we will refer to \( p \) as utility values this paper.

Third, and last, we assume that the total work performed by the engine can be calculated by dividing the required theoretical work with an efficiency coefficient that depends on the utilized engine power, denoted \( \eta(p_i) \). Moreover, we assume that the fuel consumption, and thereby emissions, can be calculated by converting the total work in the engine to emissions through known energy content of the different fuel types, i.e.

\[ CO_2 = \frac{E_{\text{HV}}}{HV} \sum_{i=2}^{N} \Delta W_{i-1, i}/\eta(p_i). \]

where \( HV \) is the higher heating value of the fuel, and \( E_{\text{HV}} \) is the emission rate of CO₂ (g) when burning a given amount of fuel (kg). In reality, \( \eta(p_i) \) will be a function of many more variables, such as gear ratio, differences between engine manufacturers, outside temperature etc. However, for applications such as transport modelling and evaluation of environmental effects due to changes in road infrastructure or fuel type usage, we argue that the estimated energy efficiency, as a function of fuel type and utilized engine power, provide a large-scale approximation of important value.

Given that all constant values in the above equations are known (see Section 3.4 for assumed and calculated values) the remaining challenge is to estimate the energy efficiency function \( \eta(\cdot) \). Our approaches for this is described in Section 3.3. Next, we present the theoretical framework needed to estimate the power-usage of BEVs driving the test cycle.

3.2. Estimation of power-usage for electric vehicles in type approval tests

When estimating the theoretical need for power-usage of BEVs, regeneration has to be taken into account, i.e.

\[ \Delta W_{i-1, i} = t_i [v_i (mgC_f + 0.5 \rho v_i^2 AC_d + ma_i)], \]

which we note to be slightly different from Eq. (1), which was established for ICEVs.

Since the type approval test measures electric power usages directly for BEVs, we assume the following equations for calculating the total net energy needed

\[ \bar{E}_{\text{WH}} = \frac{1}{3600} \sum_{i=2}^{N} \left( I \left( \Delta W_{i-1, i} > 0 \right) \frac{\Delta W_{i-1, i}}{\eta(p_i)} + I \left( \Delta W_{i-1, i} \leq 0 \right) \Delta W_{i-1, i} \eta_{\text{reg}} \right), \]

where \( \eta_{\text{reg}} \) is an assumed constant energy regeneration efficiency coefficient. Ideally, this coefficient should also be a function of relevant parameters, however the appropriate datasets to estimate this properly are not known or available. Therefore, we simply assume \( \eta_{\text{reg}} = 0.8 \) in our calculations, based on the work by [22], but note that estimating also this function would be a natural case for further work, for instance by retrieving relevant data through controlled experiments.

3.3. Estimation of TTW efficiency function

In order to estimate the TTW efficiency function, we assume the following properties:

1. The efficiency is zero at zero load, i.e. \( \eta(0) = 0 \).
2. We assume some energy is lost due to other factors than loss from the engine, such as heating, lights etc. According to results in [23], a reasonable value to include in the estimation is about 1 kW.
3. We investigate two functional forms of \( \eta(p_i) \). The first model is based on Willans approximation (see Section 3.3.1 for details)

\[ \eta(p_i) = \frac{\alpha_1 + \alpha_2 p_i + \alpha_3 p_i^2}{\alpha_1 + \alpha_2 p_i + \alpha_3 p_i^2}, \]

where \( \alpha_1, \alpha_2, \alpha_3 \) are parameters to be estimated from data. The
second model is based on trial and error to maximize the fit to training data
\[
\eta(p) = \beta_1 p^{\beta_2} e^{\beta_3 p},
\]  
(7)
where \(\beta_1, \beta_2, \beta_3\) are parameters to be estimated from data. We denote this latter model \textit{potExp} in the remainder of this paper. Both these models ensure point 1 above, and are otherwise flexible in terms of possible continuous curve appearances.

4. To calculate the theoretical energy usage, we assume the parameters given in Table 3 for the different fuel types considered here. In this table, the front area \(A\) is calculated using the national vehicle registration in Norway, Autosys, \(C_c\) and \(C_d\) are assumed constants from the literature (e.g. [24]), and the rest are known physical constants.

The type approval test data unfortunately does not provide information on emission and/or electric power consumption on high granularity grid points along with the test procedure, but only the sum of these two parameters for the whole test. This makes the estimation of \(\eta(\cdot)\) a more complex reverse engineering problem. In particular, we estimate the energy efficiency function of two fossil fuel types by minimizing the expression (equivalent for \(\alpha, \beta_2, \alpha \) when Willans approximation is used)
\[
\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3 = \arg \min_{\beta_1, \beta_2, \beta_3} \sum_{k=1}^{K} (\hat{C}^{f}_{2} - \hat{C}^{f}_{3}),
\]  
(8)
where \(K\) is the number of vehicles run through the NEDC-test, \(\hat{C}^{f}_{2}\) is the amount of measured emissions for each vehicle, and \(\hat{C}^{f}_{3}\) is the estimated \(\hat{C}^{f}_{3}\)-emissions in Eq. (3) using assumed parameters and the estimated energy efficiency functions in Eq. (6) or (7). For the electric vehicles we minimize the following expression
\[
\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3 = \arg \min_{\beta_1, \beta_2, \beta_3} \sum_{k=1}^{K} (\hat{E}_{Wh}^{k} - \hat{E}_{Wh}^{k}),
\]  
(9)
where \(\hat{E}_{Wh}^{k}\) is the measured energy consumption for vehicle \(k\) running the NEDC-test and \(\hat{E}_{Wh}^{k} = \hat{E}_{Wh}(\hat{p}_1, \hat{p}_2, \hat{p}_3, A, m_\rho, C_c, C_d, H V, L, E_v, W_{max})\) as in Eq. (5). The estimation procedure is implemented in R [25] utilizing the \textit{optim} package.

3.3.1. Willans approximation

Willans approximation is a simplification of an engine’s energy demand [26]. The model assumes the input energy to be linearly dependent on the output energy
\[
p_{in} = \alpha_1 + \alpha_2 p_{out},
\]  
(10)
where \(\alpha_1\) and \(\alpha_2\) are constants, \(p_{in}\) and \(p_{out}\) are the input power (fuel or electricity) and output (mechanical) power of the engine, respectively. Here \(\alpha_1\) can be interpreted as the power needed at zero load, i.e. simply to spin the engine, and \(\alpha_2\) is the efficiency of the combustion process.

Internal combustion engines have nonlinear losses, which make the engine less efficient at higher loads. Therefore we have extended the model to include a quadratic loss term
\[
p_{in} = \alpha_1 + \alpha_2 p_{out} + \alpha_3 p_{out}^2,
\]  
(11)
Finally, we rearrange to the form shown in Eq. (5)
\[
\eta(p_{out}) = \frac{p_{out}}{p_{in}} = \frac{p_{out}}{\alpha_1 + \alpha_2 p_{out} + \alpha_3 p_{out}^2},
\]  
(12)

3.4. Data preparation

For ICEVs, we use type approval data from vans between 2012 and 2017, and estimate the energy efficiency for diesel and petrol vehicles that are selected according to the following criteria.

- Duplicated rows in the datasets are removed.
- The result of emissions from the NEDC test is between 75 and 400 g/km.
- The vehicle power is between 40 and 425 kW.
- The weight is between 800 kg and 3 000 kg.
- If the maximum of theoretically calculated utility values, see Eq. (2), is above 0.95, the vehicle is not included.

These restrictions are set as an approach to remove both errors and outliers from the dataset that might affect the result too much. Vans are chosen as dataset because they often are heavier and less powerful than passenger cars, resulting in more observations of higher utility values, as the NEDC test is already heavily skewed towards running the vehicles in the lower spectrum. This is especially true for petrol vehicles, so in addition to only considering vans, we perform a pruning of the dataset for petrol vehicles. This is done by random sampling, including only 10 000 vehicles with a maximum utility value in the NEDC test cycle below 0.3, and an additional 10 000 randomly chosen vehicles with a maximum utility between 0.3 and 0.4. All vehicles where the maximum utility value in the NEDC test cycle is above 0.4 are also included. Again, this is done to avoid a too strong dependency of vehicles that only run at lower values of \(p\) throughout the test procedure. All available diesel vehicles are used, since this dataset was shown to be more balanced in this regard.

For BEVs, the available dataset is much smaller than for ICEVs. Here as well, we use data from 2012 to 2017, but include data from both vans and passenger cars. Here we establish the following criteria.

- Duplicated rows in the dataset are removed.
- The result of the NEDC test is between 90 and 400 Wh/km
- The vehicle power is above 40 kW.
- The weight must be between 800 kg and 3 000 kg
- If the maximum of theoretically calculated utility values, see Eq. (2), is above 0.95, the vehicle is not included.

For these three datasets, the petrol vehicles dataset is split into a training set of 24 000 vehicles and a test set of 3 580 vehicles, while the diesel vehicles are split into a training set of 35 000 vehicles and a test set of 2 146 vehicles, and lastly, the training set of the BEVs consists of 4 500 vehicles and 696 vehicles in the test set. This is a large reduction of the dataset sizes shown in Table 1 and Table 2. First of all, vehicles not in the categories ICEV and BEV, or where relevant values are missing, are removed. This leads to a removal of about 15% of the total data amount. Next, duplicate rows are removed such that only one instance of each record remains, resulting in almost 90% of the data to filter out. With respect to vans, only 5% are petrol vehicles, while the remaining are diesel, except for a few electric vans. The rest of the aforementioned filters results in the removal of about 10% of the remaining dataset.

4. Results

When running our estimation procedure, we get the estimated functions as shown in Fig. 1. In this figure, the utilized engine power is

Table 3
Values of dimensions and coefficients for ICEVs and BEVs and fuel specifications used in the estimation.

<table>
<thead>
<tr>
<th></th>
<th>Petrol</th>
<th>Diesel</th>
<th>Battery</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) (m²)</td>
<td>3.4</td>
<td>3.4</td>
<td>2.8</td>
</tr>
<tr>
<td>(C_c) (°)</td>
<td>0.015</td>
<td>0.015</td>
<td>0.015</td>
</tr>
<tr>
<td>(C_d) (°)</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>(\rho) (kg/m³)</td>
<td>1.24</td>
<td>1.24</td>
<td>1.24</td>
</tr>
<tr>
<td>(HV) (kJ/kg)</td>
<td>43.9 × 10⁶</td>
<td>43.1 × 10⁶</td>
<td>NA</td>
</tr>
<tr>
<td>(E_v) (g/kg)</td>
<td>3 180</td>
<td>3 140</td>
<td>NA</td>
</tr>
</tbody>
</table>
shown on the x-axis along with the resulting energy efficiency coefficient on the y-axis for each fuel type and assumed functional form. First, the overall efficiency of electricity is much higher than for fossil fuels, peaking at approximately 85% efficiency, at 13% of available engine power. This result is, in general, a well-known fact and makes sense considering, for instance, the absence of heat waste in electric engines. The diesel curve shows a higher energy efficiency than petrol at low utility values, which is also a well known property in the literature [27]. The peak for diesel vehicles occurs at about 40% efficiency, while for petrol vehicles the peak is found to be just above 30% efficiency. As these are maximum values, they are typically not the same as reported by the OEMs (Original Equipment Manufacturers) and the literature (see Section 1), who instead report average values obtained through continuous driving (estimation using constant function is given below). As these results indicate, the energy efficiency varies quite a bit, e.g. from 40% in peak efficiency for diesel vehicles at \( p = 0.17 \) to an efficiency of 10% – 20% at \( p = 0.80 \) depending on whether the potExp-function or the Willans approximation is used.

As we can see from Fig. 1, there are close correlations between the two estimated model forms for all fuel types, especially for lower values of \( p \). As most of the driving cycles are running at lower utility values, and hence including most of the observations in the datasets, it is natural that the lowest estimation uncertainty is located within this area. If provided a higher amount of data with higher values of \( p \), we would expect the estimation of the two functions to become more similar to each other also in this area. For higher values of \( p \), the functions based on Willans approximation are somewhat higher for fossil fuels, while the opposite is true for the functions estimated for electricity. The parameter estimates for the functions in Fig. 1 are given in Table 4.

### 4.1. Comparison to training and test sets

Using these functions to estimate emissions and electric consumption, we report the sum of squares for the difference between calculated and measured values in Table 5. In this table we have also included the result from estimating the energy efficiency as a constant function, i.e.

\[
\eta(p) = \eta_0
\]  

(13)

where the resulting parameter estimates for \( \eta_0 \) are 0.2329, 0.3198 and 0.8194 for petrol, diesel and electricity, respectively. As we can see, both efficiency functions based on Willans approximation and our chosen potExp-form are more accurate when predicting on both the test and the training sets for all fuel types compared to the constant value estimates. Comparing the use of Willans approximation and the potExp-function, there seems to be no or very low differences. Diesel is slightly more accurately predicted with the potExp-function, while the opposite is true for electricity. In all, the two proposed function forms seem to give comparable results, and for simplicity we will therefore in the continuation of our analyses focus on the estimated models based on the potExp-function.

To investigate the predictive performance of our estimated modeling approach, we have in Fig. 2 made box plots of the difference between the predicted and the measured \( \text{CO}_2 \)-emissions or electric power consumptions for all the vehicles in the training (left) and test (right) datasets.

As we can see, these plots are all centered around 0, which indicates low bias in our estimations. However, for some of the vehicles, the difference is quite high, indicating that there are some important features not captured by our simple modelling approach. Given the very simple assumed dependency structure, where only the utility of \( p \) and fuel type is in practice taken into account, this is to be expected. We argue, that for the accuracy of relevant applications, e.g. transport modeling and other larger scale evaluations, unbiased estimates are more important than complex modeling of engine internal processes. In this discussion it is important to note that more complex modeling than average numbers are important, since for instance new road projects should be evaluated in terms of \( \text{CO}_2 \)-emissions not only as a function of road length, but also other parameters such as geometry and speed limits, both made possible by this estimated energy efficiency as functions of VSP. For Fig. 2, we also note that the difference between prediction on in- or out-of-sample data is low, meaning that our models are not overfitting to the training data.

As a last way of presenting the predictive results on the test dataset, we plot the estimated \( \text{CO}_2 \)-emissions and electric power consumptions as a function of the corresponding measurements from the test cycle, see Fig. 3. In these plots we see that petrol and electricity is more nicely distributed along the diagonal than diesel, reflecting the result in Fig. 2 for the diesel test set, which clearly is the fuel type with most

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**Table 4**  
Parameter estimates for the energy efficiency functions.

<table>
<thead>
<tr>
<th>Petrol</th>
<th>Diesel</th>
<th>Electric</th>
</tr>
</thead>
<tbody>
<tr>
<td>potExp</td>
<td>Willans</td>
<td>potExp</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>1.2856</td>
<td>NA</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>0.5795</td>
<td>NA</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>2.6255</td>
<td>NA</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>NA</td>
<td>0.1181</td>
</tr>
<tr>
<td>( \alpha_3 )</td>
<td>NA</td>
<td>2.1153</td>
</tr>
<tr>
<td></td>
<td>NA</td>
<td>3.9871</td>
</tr>
</tbody>
</table>

**Table 5**  
Comparison of prediction using estimated efficiency functions and constant efficiency. The reported values are sum of squares for the difference between estimation and observation, and the lowest value in each column is highlighted.

<table>
<thead>
<tr>
<th>Petrol</th>
<th>Diesel</th>
<th>Electric</th>
</tr>
</thead>
<tbody>
<tr>
<td>potExp</td>
<td>Willans</td>
<td>potExp</td>
</tr>
<tr>
<td>N</td>
<td>24000</td>
<td>3580</td>
</tr>
<tr>
<td>Constant</td>
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<td>3626154</td>
</tr>
<tr>
<td>Willans</td>
<td>24665965</td>
<td>3431238</td>
</tr>
<tr>
<td>potExp</td>
<td>2428548</td>
<td>3444883</td>
</tr>
</tbody>
</table>
observations outside the whiskers of the box plot. This indicates that another functional form potentially could be better for modelling diesel or that some other features could be added to the modelling approach.

4.2. Comparison to simulation results

As an additional validation of our estimation and modeling approach, we include results from a detailed simulation model. In particular, we perform simulations of the driving cycle to study the form of

![Box plots of differences between estimated and observed values for the training data set (left) and the test data set (right), for all fuel types. NB: Outliers above or below 100% difference are omitted from the figure. In Fig. 3, all observations are included.](image)

**Fig. 2.** Box plots of differences between estimated and observed values for the training data set (left) and the test data set (right), for all fuel types. NB: Outliers above or below 100% difference are omitted from the figure. In Fig. 3, all observations are included.

![Comparison of estimated CO₂-emissions and electric power consumption with corresponding measurements for the test set. With a perfect fit all observations would be aligned along the diagonal.](image)

**Fig. 3.** Comparison of estimated CO₂-emissions and electric power consumption with corresponding measurements for the test set. With a perfect fit all observations would be aligned along the diagonal.
the drivetrain efficiency curve. These simulations are done using the advanced vehicle simulation tool ADVISOR [2]. This tool offers a large variety of vehicles and different parameters to tune and outputs to study. To make comparable results, a modified NEDC driving cycle is completed for each of the vehicles given in Table 6. The BEV is equipped with 28 modules of ESS_PB65_FocusEV batteries. We modify the test slightly by adding a constant road grade of 10% to increase the power utilization as a pure NEDC implementation resulted in very few observations of high utility values. For each vehicle category, the power demand and the efficiency is logged for each time instance. Models and control strategies from ADVISOR are used without further modifications. In Fig. 4, the simulated efficiency is plotted against the estimated efficiency functions.

It is important to note that these simulations are based on a simulation tool providing calculations for one specific vehicle, while our estimated functions are based on a large dataset including many different vehicles aggregated to three representative vehicles. Hence, differences are to be expected. Firstly, we observe, in line with the results from Section 4.1, that the diesel function has the largest difference compared to the result from the simulation of one vehicle. Petrol seems to have the best fit for the simulations, and we observe that the simulations from both petrol and electric vehicles show a decrease in calculated energy efficiency with increasing $p$. In sum, given the large difference between these two modeling approaches, we find it reassuring that the level and, at least to some degree, the forms of the functions are similar. Comparing petrol and electricity, we see that electricity has the largest difference between the simulation and the estimated functions. In this regard, it is important to recall that the dataset used to estimate the efficiency curve for this fuel type is much smaller than for the ICEVs. As a continuation of this work, one should investigate the modeling approach using a much larger dataset for BEVs, which probably will be available in the future.

5. Concluding remarks

In this paper, we have estimated tank-to-wheel efficiency functions for internal combustion engine vehicles and battery electric vehicles as a function of utilized engine power. This new approach finds a middle ground of complexity for transport modelling, situating itself between the simplified assumption of constant tank-to-wheel efficiency and the complex engine modelling approach.

Our estimated functions have increasing uncertainty for higher
values of the utilized engine power \( p \). This is due to the lack of data available in this area. In this regard, it is important to note that higher certainty in lower values of \( p \) is preferred from a transport modeling perspective, as most of the driving in general is performed in this area. Therefore, an accurate fit for high values of \( p \) is primarily of theoretical interest in this case.

As discussed in the introduction, all records included in this paper are results from the NEDC test, which by many is criticized to not reflect realistic driving very well. Currently, all new vehicles are tested using the WLTP driving cycle, which is more realistic and will provide more data for higher values of \( p \) as well. As soon as higher volumes from these tests are available, the estimation procedure outlined in this paper should be applied to such a dataset for comparison.

The vehicles included in this paper are passenger cars and vans. In 2019, the EEA will start collecting data on new heavy-duty vehicles in accordance with Regulation (EU) No 2018/956. As the engine and drivetrains of these vehicles are quite different from vans and passenger cars, these data should also be investigated with the outlined approach once the data is available.

Both data from a dedicated test datasets and results from an advanced simulation tool is used to validate the proposed modeling approach and both validation procedures show good correspondence between model predictions and measurements. This supports the accuracy, relevance and usefulness of the estimated energy efficiency functions. As an additional test and future work, these results should be compared to measurements of \( CO_2 \)-emission and energy consumption collected from driving on the real road network. GPS-tracking and relevant parameters from such test vehicles should be collected and compared to theoretical estimates using the proposed efficiency functions.

Data availability


CRediT authorship contribution statement

**Odd Andre Hjelkrem:** Investigation, Methodology, Data curation, Formal analysis, Writing - original draft, Writing - review & editing, Supervision. **Petter Arnesen:** Methodology, Data curation, Formal analysis, Software, Validation, Writing - original draft, Writing - review & editing. **Torstein Aarseth Be:** Validation, Writing - original draft, Writing - review & editing. **Rebecka Snejugli Sundell:** Investigation, Writing - original draft.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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