Contraction Analysis for a MMC Converter

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***Abstract*—One of the present barriers to the widespread use of Modular Multilevel Converters (MMC) is the complexity of its control. We use a suitable degree of simplifications of the MMC converters, under given conditions, to allow system level studies over potentially large networks. The analysis is based on a recently proposed simplified model of the MMC. Based on this model, we propose an analysis using the theory of Contraction to define some stability conditions. Some simulation results allow to see the advantages of the proposed method.**

***Index Terms*—Contraction Analysis, Modular Multilevel Converter, stability.**

I. Introduction

THE increasing penetration of renewable energy in the traditional power system and particularly the massive integration of offshore wind farms has created the need to generate new control and analysis strategies in the Power systems. The use of the Voltage Source Converter (VSC) technology, with independent control of active and reactive power at the AC terminals is considered and advantage to support weak power systems [1].

Classic stability analysis in power systems requires three main steps: modelling, load flow calculation and dynamic analysis. For this purpose, in this paper, we used a simplified model of the MMC converter and for the dynamic analysis we applied the contraction theory instead of Lyapunov theory which is the most used technique [2]. Contraction theory is introduced as a powerful concept to treat the stability properties of nonlinear dynamical systems. Besides, the contraction theory provides an elegant way to analyze the behaviors of certain nonlinear dynamical systems [2], [3]. Contraction analysis is motivated by the elementary idea that talking about stability does not require to known what the nominal motion is: intuitively, a system is stable in some region if the final behavior of the system is independent of the initial conditions [4], [5]. Many questions of nonlinear systems theory call for an incremental version of the Lyapunov stability concept, in which the convergence to a specific target solution is replaced by the convergence or contraction between any pairs of solutions [6]. Essentially, this stronger property means that solutions forget about their initial condition. The difference between the Lyapunov and contraction stability analysis is that stabilization in the sense of Lyapunov occurs at the minima of a generally defined a given function (particular) of any solution, while contraction theory proceeds to a differential approach and the stability of trajectories of a dynamical system with respect to one another [7]. In addition, the convergence of solutions is determined to be independent of initial conditions [8].

This paper is organized as follows: Section II gives the Dynamic model of Modular Multilevel Converter. Section III presents basic concepts about Contraction Analysis. In Section IV we show simulation results using contraction analysis in one MMC. Finally in section V, the discussion and conclusions are presented respectively.

II. Dynamic model of Modular Multilevel Converter

The modular multilevel converter (MMC) is the preferred topology for voltage source converter (VSC)-based HVDC transmission schemes. Different approaches to the model of the MMC have been made, for the purpose of this paper, we will take as reference the development in [9] and [10]. It is a common procedure in power systems analysis to approximate the fast transient behaviour of the power electronics converter to an active power injection model as presented in [1]. This approximation is based on the reasonable assumption that the converters are tightly-regulated, the harmonic distortion is negligible, DC faults are outside the scope of the analysis, the transient response is typically in the range of few milliseconds, losses can be neglected and a generic model can be used to represent the dynamics. The MMC depicted in Figure 1 is the most suitable converter topology due to its improved harmonic ac-voltage output, avoiding the need of installing harmonic filters.

The system can be described with the set of equations (1)-(3).

$$L_a \frac{d\mathbf{i}}{dt} = \mathbf{E} - \mathbf{v} - R_a \mathbf{i} - \mathcal{J} \cdot \omega L_a \mathbf{i} \quad (1)$$

$$L_a \frac{di_{\Sigma z}}{dt} = -R_a i_{\Sigma z} + \frac{v_{dc}}{2} - u_{\Sigma z} \quad (2)$$

$$\frac{dW_z}{dt} = 2u_{cz} i_{\sigma z} - \frac{1}{2}(e_d i_d + e_q i_q) - \alpha_p W_z \quad (3)$$

where, the AC side variables are $\mathbf{i} = (i_d, i_q)^T$, $\mathbf{E} = (e_d, e_q)^T$, $\mathbf{v} = (v_d, v_q)^T$ $\mathcal{J} \in R^{(2\times 2)}$ is a skew-symmetric matrix. The DC side variables are v_{dc}, $u_{\Sigma z}$ and $i_{\Sigma z}$. The inductance is L_a

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and the resistance is R_a. The parameter α_p considers the power losses of the capacitors for the sub-modules of the MMC.

With the case of internal regulation of the quadrature current i_q to zero and the tight regulation of $i_{\Sigma z}$ to the reference $i^\star_{\Sigma z}$. We can write a reduced model as

$$L_a \dot{i_d} = -R_a i_d + e_d - v_d \quad (4)$$
$$\dot{W} = \xi - \frac{1}{2} e_d i_d - \alpha_p W_z \quad (5)$$

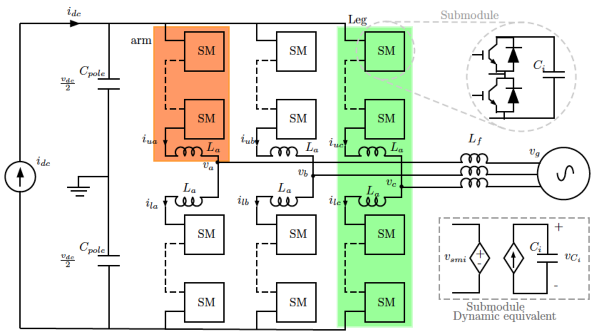

Figure 1. MMC dynamic model

III. Contraction Analysis

Contraction analysis emerges with the works of Slotine on his seminal paper [11] as an alternative to analyze the stability of a nonlinear system, without requiring the well-known Lyapunov theory. The mathematical framework applied on this theory uses differential forms and variational principles [12] to establish an incremental stability set up, then it is possible to relates the results found on contraction theory with the Lyapunov stability notion. We consider the general and deterministic dynamical systems of the form

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t) \quad (6)$$

where $\mathbf{x} \in \mathbb{R}^n$ is the system states, $\mathbf{f} : \mathbb{R}^n \to \mathbb{R}$ the vector field, $t \in \mathbb{R}^+$, is the continuous time.

Consider the local flow $s(x,t)$ at a point x where it is possible to analyze the convergence among two trajectories. If both trajectories converges to each other global exponential convergence to a single trajectory could be established. Assuming that $\mathbf{f}$ is sufficiently smooth, we have

$$\delta\dot{\mathbf{x}} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}(\mathbf{x}, t)\delta\mathbf{x}$$

where $\delta\mathbf{x}$ is a virtual displacement, formally it defines linear tangent differential form with a quadratic tangent $\delta\mathbf{x}^T\delta\mathbf{x}$, this equivalently means that the squared distance between trajectories is defined through these differential forms, so using the ideas before the rate of change is written as,

$$\frac{d}{dt}\left(\delta\mathbf{x}^T\delta\mathbf{x}\right) = 2\delta\mathbf{x}^T\delta\dot{\mathbf{x}} = 2\delta\mathbf{x}^T\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\delta\mathbf{x}$$

Defining an upper bound for the solution

$$\|\delta\mathbf{x}\| \le \|\delta\mathbf{x}_o\| \, e^{\int_o^t \lambda_{\max}(\mathbf{x},t)dt}$$

Assuming that $\lambda_{max}(\mathbf{x}, t)$ is uniformly strictly negative. Then, any infinitesimal length $\|\delta\mathbf{x}\|$ converges exponentially to zero.

Definition 1. *[11] Given a system of equations $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t)$, a region of the state space is called a contraction region if the Jacobian $\frac{\partial \mathbf{f}}{\partial \mathbf{x}}$ is uniformly negative (UND) in that region.*

The UND Jacobian condition is defined as

Definition 2. *The Jacobian uniformly negative accomplish with $\exists\beta > 0, \forall t \ge 0, \forall\mathbf{x}$*

$$\frac{1}{2}\left(\frac{\partial \mathbf{f}}{\partial \mathbf{x}} + \frac{\partial \mathbf{f}^T}{\partial \mathbf{x}}\right) \le -\beta\mathbf{I} < 0$$

in any region considered as an open connected set. As a consequence a semi-contraction region is said to $\frac{\partial \mathbf{f}}{\partial \mathbf{x}}$ being negative semi-definite, and a indifferent region to $\frac{\partial \mathbf{f}}{\partial \mathbf{x}}$ being skew-symmetric.

Consider the figure 2, any open set defined under a normed ball encircles some trajectory inside this ball and it remains there $\forall t \ge 0$. So, any trajectories distance decreases exponentially and converges exponentially to the given trajectory (ball center). So, this leads to the following theorem:

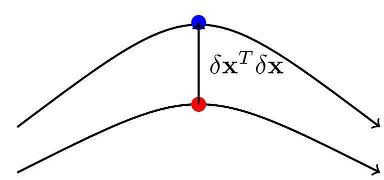

Figure 2. Squared Distance between two neighboring trajectories

Theorem 1. *[11] Given the system dynamics*

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t)$$

any trajectory, which starts in a ball of constant radius centered about a given trajectory and contains at all times a contraction region, remains in that ball and converges exponentially to this trajectory. Furthermore, global exponential convergence to the given trajectory is guaranteed if the whole state space is a contraction region

With this mathematical foundations about contraction theory, the following analysis is performed on the MMC model defined in the sections above.

Let the dynamics separation of the MMC simplyfied model be,

$$L_a \dot{i_d} = -R_a i_d + e_d - v_d$$
$$\dot{W} = \xi - \tfrac{1}{2} e_d i_d - \alpha_p W_z$$

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clearly this model has a nonlinear structure, considering the convention above, and rewriting this model we have,

$$\begin{aligned}\dot{x}_1 &= -\tfrac{R_a}{L_a}x_1 + \tfrac{u}{L_a} - \tfrac{d_1}{L_a}\\ \dot{x}_2 &= d_2 - \tfrac{1}{2}ux_1 - \alpha_p x_2\end{aligned}$$

For the ease of the analysis and without loss of generality, we propose a change of variable over the control input $u(t)$, so,

$$u = \int_0^t v(\lambda)d\lambda$$

This establishes the model alike that has used for an integral backstepping design [13], then,

$$\begin{aligned}\dot{x}_1 &= -\tfrac{R_a}{L_a}x_1 + \tfrac{x_3}{L_a} - \tfrac{d_1}{L_a}\\ \dot{x}_2 &= d_2 - \tfrac{1}{2}x_3x_1 - \alpha_p x_2\\ \dot{x}_3 &= v\end{aligned}$$

being $v(t)$ the new control input, the objective through this analysis is consider the potential use of the contraction theory to design a control law stabilizing this nonlinear system, to consider this methodology let assume that $v \in \mathbf{V}$ is defined over a set of admissible controls and $v : \mathbf{R^n} \to \mathbf{R}$ is a nonlinear function of the states. In this representation is possible to define a Jacobian which depends on $v(\mathbf{x})$, so the UND condition could be achieved.

$$\frac{\partial \mathbf{f}}{\partial \mathbf{x}} = \begin{bmatrix} -\frac{R_a}{L_a} & 0 & \frac{1}{L_a}\\ -\frac{1}{2}x_3 & -\alpha_p & -\frac{1}{2}x_1\\ \frac{\partial v}{\partial x_1} & \frac{\partial v}{\partial x_2} & \frac{\partial v}{\partial x_3}\end{bmatrix} \tag{7}$$

Using the symmetric decomposition we obtain,

$$\frac{1}{2}\left(\frac{\partial \mathbf{f}}{\partial \mathbf{x}} + \frac{\partial \mathbf{f}}{\partial \mathbf{x}}^T\right) =$$
$$\frac{1}{2}\begin{bmatrix} -2\frac{R_a}{L_a} & -\frac{1}{2}x_3 & \frac{\partial v}{\partial x_1}+\frac{1}{L_a}\\ -\frac{1}{2}x_3 & -2\alpha_p & \frac{\partial v}{\partial x_2}-\frac{1}{2}x_1\\ \frac{\partial v}{\partial x_1}+\frac{1}{L_a} & \frac{\partial v}{\partial x_2}-\frac{1}{2}x_1 & 2\frac{\partial v}{\partial x_3}\end{bmatrix} \tag{8}$$

Consequently UND Jacobian implies that, all leading principal minors are negative, so is important to

$$\left(\frac{1}{2}\right)\Big[(-\frac{1}{2}x_3)(\frac{\partial v}{\partial x_2})(\frac{\partial v}{\partial x_1}+\frac{1}{L_a}) + (\frac{\partial v}{\partial x_1}+\frac{1}{L_a})(-\frac{1}{2}x_3)(\frac{\partial v}{\partial x_2}-\frac{1}{2}x_1) - (-2\frac{R_a}{L_a})(\frac{\partial v}{\partial x_2}-\frac{1}{2}x_1)(\frac{\partial v}{\partial x_2}-\frac{1}{2}x_1) - (-\frac{1}{2}x_3)(-\frac{1}{2}x_3)(2\frac{\partial v}{\partial x_3})\Big] \leq 0 \tag{9}$$

Consider the case where the equality remains, it is possible to define an analytic form to derive $v(\mathbf{x})$ as a potential function such that its derivatives form a gradient vector. Let

$$v(\mathbf{x}) = \int_0^{x_1} g_1(y_1)dy_1 + \int_0^{x_2} g_2(x_1,y_2)dy_2 + \cdots + \int_0^{x_n} g_n(x_1\ldots,y_n)dy_n \tag{10}$$

One possible solution for $v(\mathbf{x})$ is presented in (11).

$$v(\mathbf{x}) = \frac{-x_1}{L_a}\int_0^{x_3}\frac{y_3}{{y_3}^2}dy_3 - \frac{R_a}{L_a}{x_1}^2\int_0^{x_3}\frac{dy_3}{{y_3}^2} - 16{x_1}^2\int_0^{x_3}\frac{1}{{y_3}^2}dy_3 \tag{11}$$

$$v(\mathbf{x}) = 16L_a|x_1| + \frac{R_a{x_1}^2}{L_a x_3} - \frac{x_1}{L_a}\log(x_3) \quad \square$$

This last inequality is a particular case of the expression (9), there, we can use the partial derivatives on $v(\mathbf{x})$ to establish its degree of freedom selecting a suitable function which leads the Jacobian to achieve a desired contracting behavior, obtained by (10). This implies partial differentiation in the very general case, however assuming some structure on these derivatives, and suitable boundary conditions there exists a simpler expression which can be solved. This problem in particular is alike to that used for synthesizing a control law starting with a Control Lyapunov Function (CLF) achieving asymptotically stability behavior. Although the results presented so far, defines a methodology for stabilizing a nonlinear system, the idea behind contraction theory is to analyze the stability problem of a nonlinear system in a differential framework with enriches the classical methodology like Lyapunov theory, in fact the results presented in [11] shows a generalization of exponential stability which is a strong property comparing with asymptotic stability. The conclusions about the system using contraction theory are strong and extends the results applying classic nonlinear stability studies.

Remark 1. *In the equation 7 the UND Jacobian is considered, it requires that principal minors of the matrix to be strictly negative according to the Sylvester theorem, therefore, the control law designed must hold this property for all time* □

IV. Simulation Results

For simulation we have used the following parameters about the simplified MMC model, the results are simulated using MATLAB™ 2018b The figures 3, 4, 5 show the

L_a	R_a	d_1	d_2
0.093500	0.94465	400000	13542400

state trajectories achieving a contracting behavior when $v(\mathbf{x})$ is applied, as the theory dictates the initial conditions are "forgotten", so the steady state value corresponds to the fixed point of the nonlinear system. The phase space shown in 6, demonstrates the fixed point stability, locally, as a stable spiral, since the oscillatory behavior of the nonlinear system. The behavior observed on each of the states allows us to conclude

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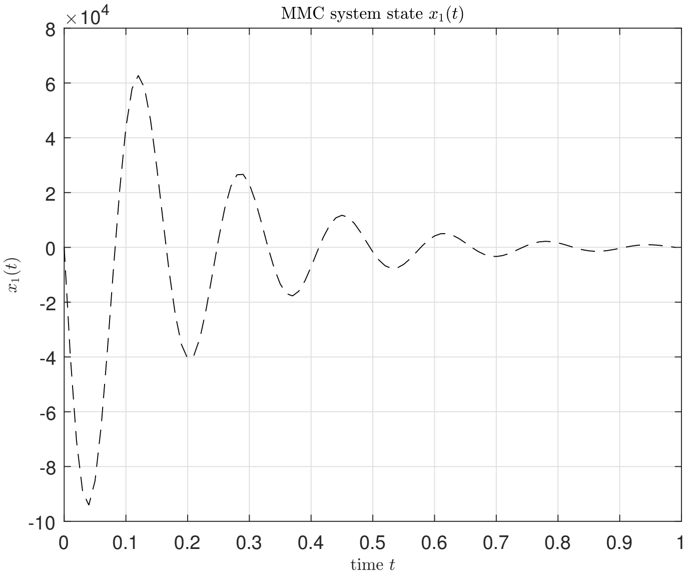

Figure 3. x_1 state evolution

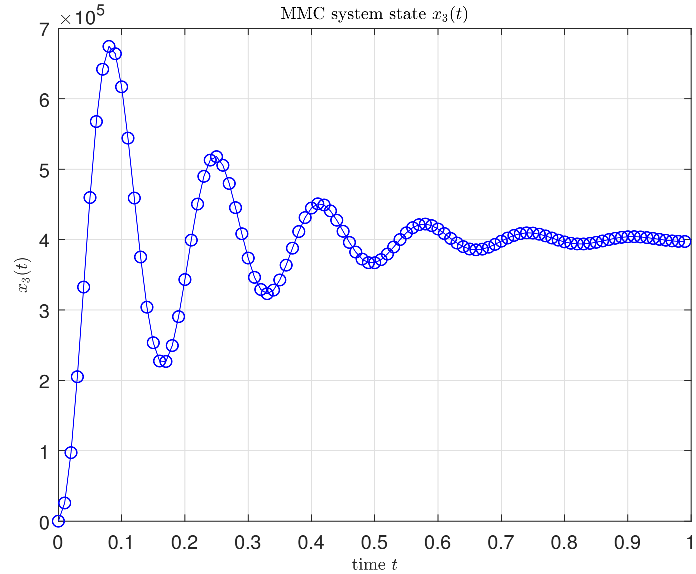

Figure 5. x_3 state evolution

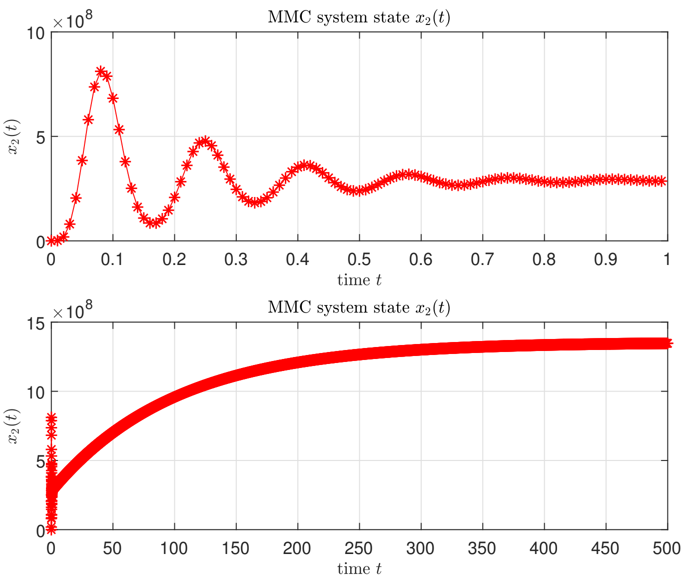

Figure 4. x_2 state evolution

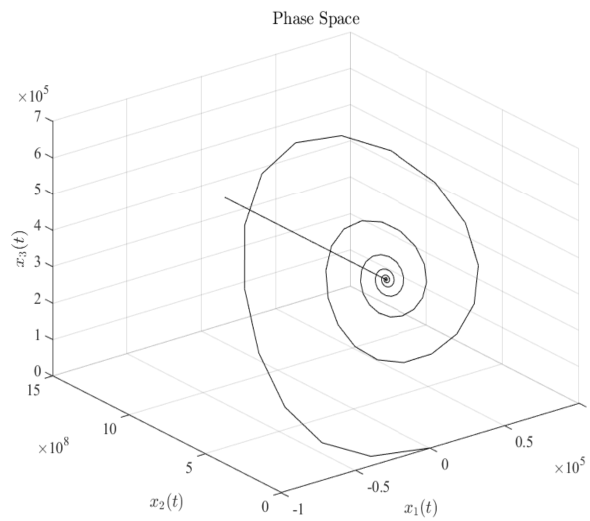

Figure 6. Phase Space

that the global stability is achieved considering the uniqueness, and isolation of the equilibrium point.

V. Discussion and Conclusions

The use of contraction theory to study the stability of a MMC with the Jacobian in function of the states eases the identification of the region of attraction. Therefore, this tool can be used by the engineers to find physical limitations on the application and design of the converter. The convergence analysis and limit behavior are in a sense treated separately. In a control context, once contraction is guaranteed through feedback, specifying the final behavior reduces to the problem of shaping one particular solution.

One of the most important problems arising in the control and the analysis of dynamical systems is to determine if the system is stable. Typically, stability is defined in the sense of Lyapunov but it is clear with the information in this paper that the contraction analysis could be used in conjunction with the Lyapunov theory to guarantee stability conditions.

Current research includes systematically guaranteeing global exponential convergence for general nonlinear systems, stable adaptation to unknown parameters, and further applications to mechanical and chemical systems. We reduced the model of the MMC based on its normal application on HVDC grids. First, a pure active power converter is used. Followed, the DC power of the converter is tightly regulated.

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Hence, the initial model can be reduced to the nonlinear structure with two states.

In the future, our recommendation is to use the complete model of the MMC instead of the reduced because with the latter case it is possible lose valuable information. Also the complete model considers all the states and this allows us to understand in a better way the overall behavior of the system.

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