Maintenance of existing structures requires a solid understanding of their structural behavior with their varying geometry, boundary conditions, and load cases. For the case of the masonry arches, this understanding is still lacking in the field, yet, masonry arches are crucial parts of the railway and highway network systems in many countries. In this paper, two dimensional numerical models were prepared to simulate the nonlinear response of masonry arches under static loading without soil-structure interaction effects. A custom-made Discrete Element Method (DEM) software was employed for this research, such that the models represent a discontinuous medium of rigid blocks. Different scenarios were generated on a hypothetical masonry arch model to observe the influence of different parameters on the structural behavior of masonry arches. Investigated parameters include: effect of soil infill and spandrel walls, bond pattern at the arch barrel for double layer arches, and boundary conditions. In addition, the discrete element approach and the software were validated by an experimental work from literature. The results of the analyses show that discrete element modeling is a powerful technique, which demonstrates the development of collapse mechanisms of masonry arch structures. Parametric analyses also indicated that soil infill and spandrel walls, if intact, can have beneficial effects on the load carrying capacity of arches. Based on the results of this study, the bond pattern between arch layers does not make a significant difference in the overall behavior. As expected, boundary conditions matter, and should be taken into careful consideration for each masonry arch bridge through detailed observations on site.

Bora Pulatsu, MSc
University of Nebraska-Lincoln
Omaha, UNITED STATES
bora.pulatsu@gmail.com

Bora Pulatsu, MSc
Ece Erdogmus, Professor
Eduardo Martins Bretas, PhD

Carlo Citto
ccitto@ana-usa.com
Mr. Citto is a professional engineer in Atkinson-Noland and Associates. One of the motivation of this paper to provide a link between advanced numerical models and engineering practice. In this context, Mr. Citto would make a contribution to this article to improve the link between both sides.

Jose Lemos
vlemos@lnec.pt
Dr. Lemos has a great contribution to the literature on discrete element modeling of masonry structures. In this context, numerical modeling part of this paper would be strengthened with his comments and opinions.

Authors are required to attain permission to re-use content, figures, tables, charts, No
maps, and photographs for which the authors do not hold copyright. Figures created by the authors but previously published under copyright elsewhere may require permission. For more information see http://ascelibrary.org/doi/abs/10.1061/9780784479018.ch03. All permissions must be uploaded as a permission file in PDF format. Are there any required permissions that have not yet been secured? If yes, please explain in the comment box.

| ASCE does not review manuscripts that are being considered elsewhere to include other ASCE Journals and all conference proceedings. Is the article or parts of it being considered for any other publication? If your answer is yes, please explain in the comments box below. | No |
| Is this article or parts of it already published in print or online in any language? ASCE does not review content already published (see next questions for conference papers and posted theses/dissertations). If your answer is yes, please explain in the comments box below. | No |
| Has this paper or parts of it been published as a conference proceeding? A conference proceeding may be reviewed for publication only if it has been significantly revised and contains 50% new content. Any content overlap should be reworded and/or properly referenced. If your answer is yes, please explain in the comments box below and be prepared to provide the conference paper. | No |
| ASCE allows submissions of papers that are based on theses and dissertations so long as the paper has been modified to fit the journal page limits, format, and tailored for the audience. ASCE will consider such papers even if the thesis or dissertation has been posted online provided that the degree-granting institution requires that the thesis or dissertation be posted. Is this paper a derivative of a thesis or dissertation posted or about to be posted on the Internet? If yes, please provide the URL or DOI permalink in the comment box below. | No |
| Each submission to ASCE must stand on its own and represent significant new information, which may include disproving the work of others. While it is acceptable to build upon one’s own work or replicate... | No |
other's work, it is not appropriate to fragment the research to maximize the number of manuscripts or to submit papers that represent very small incremental changes. ASCE may use tools such as CrossCheck, Duplicate Submission Checks, and Google Scholar to verify that submissions are novel. Does the manuscript constitute incremental work (i.e. restating raw data, models, or conclusions from a previously published study)?

Authors are expected to present their papers within the page limitations described in <u><i><a href="http://dx.doi.org/10.1061/9780784479018" target="_blank">Publishing in ASCE Journals: A Guide for Authors</a></i></u>. Technical papers and Case Studies must not exceed 30 double-spaced manuscript pages, including all figures and tables. Technical notes must not exceed 7 double-spaced manuscript pages. Papers that exceed the limits must be justified. Grossly over-length papers may be returned without review. Does this paper exceed the ASCE length limitations? If yes, please provide justification in the comments box below.

All authors listed on the manuscript must have contributed to the study and must approve the current version of the manuscript. Are there any authors on the paper that do not meet these criteria? If the answer is yes, please explain in the comments.

Was this paper previously declined or withdrawn from this or another ASCE journal? If so, please provide the previous manuscript number and explain what you have changed in this current version in the comments box below. You may upload a separate response to reviewers if your comments are extensive.

Companion manuscripts are discouraged as all papers published must be able to stand on their own. Justification must be provided to the editor if an author feels as though the work must be presented in two parts and published simultaneously. There is no guarantee that companions will be reviewed by the same reviewers, which complicates the review process, increases the risk for rejection and potentially lengthens the review time. If this is a companion paper, please indicate the part number and provide the title, authors and manuscript number (if available) for the companion papers along with your detailed justification for the editor in the comments box below. If there is no justification provided, or if there is
insufficient justification, the papers will be returned without review.

<table>
<thead>
<tr>
<th>If this manuscript is intended as part of a Special Issue or Collection, please provide the Special Collection title and name of the guest editor in the comments box below.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Special Issue ARCHES: Editor Franco Bontempi</td>
</tr>
</tbody>
</table>

Recognizing that science and engineering are best served when data are made available during the review and discussion of manuscripts and journal articles, and to allow others to replicate and build on work published in ASCE journals, all reasonable requests by reviewers for materials, data, and associated protocols must be fulfilled. If you are restricted from sharing your data and materials, please explain below.

<table>
<thead>
<tr>
<th>Papers published in ASCE Journals must make a contribution to the core body of knowledge and to the advancement of the field. Authors must consider how their new knowledge and/or innovations add value to the state of the art and/or state of the practice. Please outline the specific contributions of this research in the comments box.</th>
</tr>
</thead>
<tbody>
<tr>
<td>This study provides discrete element modeling (DEM) strategy on blocky structures such as masonry arches. Along with this research, understanding of the structural behavior of masonry arches and its components would be strengthened and discussed.</td>
</tr>
</tbody>
</table>

The flat fee for including color figures in print is $800, regardless of the number of color figures. There is no fee for online only color figures. If you decide to not print figures in color, please ensure that the color figures will also make sense when printed in black-and-white, and remove any reference to color in the text. Only one file is accepted for each figure. Do you intend to pay to include color figures in print? If yes, please indicate which figures in the comments box.

| No |

If there is anything else you wish to communicate to the editor of the journal, please do so in this box.
Parametric Study on Masonry Arches Using 2D Discrete Element Modeling

Bora Pulatsu¹, Ece Erdogmus² and Eduardo M. Bretas³

¹ Graduate Research Assistant, Architectural Engineering, University of Nebraska-Lincoln, 242 Peter Kiewit Institute, 1110 South 67th St., Omaha, NE 68182. (Corresponding author) E-mail: bpulatsu2@unl.edu

² Professor, Architectural Engineering, University of Nebraska-Lincoln, 203C Peter Kiewit Institute, 1110 South 67th St., Omaha, NE 68182.

³ Senior Research Scientist, Department of Infrastructure, Materials and Structures, Northern Research Institute, Narvik, Norway.

ABSTRACT: Maintenance of existing structures requires a solid understanding of their structural behavior with their varying geometry, boundary conditions, and load cases. For the case of the masonry arches, this understanding is still lacking in the field, yet, masonry arches are crucial parts of the railway and highway network systems in many countries. In this paper, two dimensional numerical models were prepared to simulate the nonlinear response of masonry arches under static loading without soil-structure interaction effects. A custom-made Discrete Element Method (DEM) software was employed for this research, such that the models represent a discontinuous medium of rigid blocks. Different scenarios were generated on a hypothetical masonry arch model to observe the influence of different parameters on the structural behavior of masonry arches. Investigated parameters include: effect of backfill and spandrel walls, bond pattern at the arch
barrel for double layer arches, and boundary conditions. In addition, the discrete element approach and the software were validated by an experimental work from literature. The results of the analyses show that discrete element modeling is a powerful technique, which demonstrates the development of collapse mechanisms of masonry arch structures. Parametric analyses also indicated that backfill and spandrel walls, if intact, can have beneficial effects on the load carrying capacity of arches. Based on the results of this study, the bond pattern between arch layers does not make a significant difference in the overall behavior. As expected, boundary conditions matter, and should be taken into careful consideration for each masonry arch bridge through detailed observations on site.

KEYWORDS: Masonry, Discrete element model, Masonry Arch, DEM, Collapse Mechanism

Introduction

Throughout history, masonry arches were used to span relatively large distances as a common structural form dating back to Roman Empire. Many of them are still in use in road and railway networks in Europe and northeast United States. Until the first half of the nineteenth century, design and analysis of these structures had been performed by using empirical methods and common construction techniques (Brencich and Morbiducci 2007). With recent developments in modern mechanics and the increased computational capacity, more detailed structural analysis of masonry arches became possible.

In the last three decades, discrete element based modeling, including DEM, discontinuous deformation analysis (DDA) and combined discrete/finite element analysis, has become a widely used solution to model masonry structures. It has an advantage over continuum-based methods, due to its inherently discontinuous medium and compatibility with the nature of masonry structures.
In this study, DEM, which can be classified as a simplified micro modeling approach, was used to investigate the response of a typical masonry arch under static loading (Lourenço 2009).

The goal of the paper is to simulate the realistic behavior of masonry arches and demonstrate the impact of different parameters: boundary conditions, bond patterns for double ring arches and existence of spandrel walls or backfill. A detailed discrete element model was generated to validate the custom-made software with previous experimental work from literature. In addition, the obtained results were compared with limit analysis for two case studies including arch with and without backfill.

**Background**

The modern mechanics of masonry arches begins with the introduction of plastic analysis by Heyman (1966) in order to assess the load carrying capacity. The collapse of the masonry arches is considered as a geometrical problem and series of assumptions are employed: (i) stone has no tensile strength; (ii) friction between voussoirs is high enough to prevent sliding failure; (iii) the masonry has infinite compressive strength. Heyman indicated that plastic hinges appear where the line of thrust touches either extrados or intrados of the arch and turns out a collapse mechanism. This is often referred to as the mechanism method. Therefore, plastic design method, previously used as a technique to analyze rigid-plastic structural frames, was applied to unreinforced masonry structures by Heyman and limit analysis became a widely employed tool to analyze masonry structures. Later, important contributions were made on the Heyman’s theorem by limiting the infinite compressive strength to material crushing strength and accounting for the possibility of sliding (Gilbert 2007).
Linear programming (LP) technique, first applied by Livesley (1978) on masonry arches formed by rigid blocks, has been exhaustively used in the literature to solve equilibrium and mechanism formulations of limit analysis. Recently, using dual LP framework, rigid elements and homogenized interfaces, where deformations were lumped at the joints, were used to analyze masonry arches and double curvature shell structures (Milani et al. 2008, Milani 2015). Still, limit analysis is one the most common methods. It is easy to use and requires fewer number of input parameters. It can provide the maximum load carrying capacity and related failure mechanisms for an arch structure, despite spending negligible computational time compared to advanced numerical techniques, such as nonlinear finite element analysis (FEA). Hence, it appears that it is an efficient and appropriate technique to analyze masonry arches and vaults (Tralli et al. 2014).

In the last three decades, more detailed and comprehensive numerical approaches were developed, which are used to analyze both modern and historical masonry structures (Lourenço 2002). These are often referred to as continuum and discontinuum (or discrete) models. Analysis may involve “micro” or “macro” modeling, depending on the level of accuracy required. The micro modeling focuses on each part of the masonry by taking into account the unit, mortar and unit/mortar interfaces. In literature, there exits 2D and 3D strategies utilized for micro modeling on different type of masonry structures (Lourenço and Rots 1997; Milani and Lourenço 2012). It is important to note that there are also hybrid and meso-scale models, falling between micro and macro modeling strategies (Zhang et al. 2016). On the other hand, in macro modeling, masonry is described as an equivalent continuum model and nonlinear models capturing the overall structural behavior are used. Unit and mortar are implicitly represented by following a continuity condition at the nodes, as in the case of standard finite element method (FEM) procedure.
Methodology: Numerical modeling

In this study, a custom-made software utilizing discrete element method (DEM) was employed for all the numerical simulations to demonstrate failure mechanism of masonry arches. The software was first employed to analyze masonry dams and then it was used to simulate out-of-plane behavior of masonry walls. For further description about software, the reader is referred to (Bretas et al. 2013, 2014, 2016). DEM falls within the classification of discontinuum analysis. This approach, originally proposed by Cundall (1971), provides an opportunity to model structures as composed of 2D and 3D polygonal blocks that may be rigid or deformable. This method is successfully applied by many researchers on different masonry structures (Bui et al. 2017; Isfeld and Shrive 2015; Lemos 2007; Pulatsu et al. 2017; Simon and Bagi 2014; Tóth et al. 2009).

For the numerical models, the masonry units were modeled as rigid blocks and mortar joints were represented as zero thickness interfaces between each block. The main reason to employ rigid blocks was to take advantage of high compressive strength of stone and brick masonry units (in comparison to mortar) and low computational cost in the analysis. Thus, nonlinear response of masonry arch models was only controlled by the joints where normal and shear springs were assigned in two orthogonal directions (Fig. 1). In the custom-made software, the governing differential equations for translational and rotational motions were integrated through each time step, using an explicit finite-difference method. The static solutions are obtained by dynamic relaxation, using scaled masses. Furthermore, out-of-balance forces are checked in each calculation step and additional load is applied after the stability of the structure is ensured.
A force-displacement law was assigned to each spring and used to calculate stress increments for normal stress ($\Delta\sigma$) and shear stresses ($\Delta\tau$), depending on the considered constitutive model, as given in equations (1) and (2).

\[ \Delta\sigma_i = k_n (\Delta u_n)_i \]  
\[ \Delta\tau_i = k_s (\Delta u_s)_i \]  

Where $k_n$ and $k_s$ are normal and shear stiffness at the joints, respectively; and $u_n$ and $u_s$ are relative displacements in the normal and tangential directions, respectively.

To illustrate a simple case, the external force, $F$, acting on a rigid block (Fig. 2a) and the corresponding stress distribution are given in Figure 2b where $\sigma_t$ indicates the tensile stress and $\sigma_c$ shows compressive stress at the joint.

Stresses, calculated at each time step, are corrected according to the given failure criteria as presented in Equations (3) and (4), where ‘$c$’ stands for cohesion and ‘$\theta$’ indicates the friction angle. In this study, it is assumed that masonry has zero tensile strength and the Coulomb model is used to determine the shear stress ($\sigma_s$).

\[ \sigma_n < \text{Tensile strength} \]
\[ |\sigma_s| < c + \sigma_n \tan \theta \] (4)

New contact forces, \( F_n \) for the normal direction and \( F_s \) for the tangential direction, are calculated, using the contact lengths, \( l_{\text{contact}} \) depending on the tension or the compression part of the contact, as shown in Figure 2b by \( l_t \) and \( l_c \), respectively. Finally, new position and displacement of the blocks are found in an explicit way. The discontinuous representation of blocks allows to model joint sliding and openings that determine the ultimate load carrying capacity of the structure. An external load is increased until the movements increase without bound, which demonstrates a non-equilibrium state for the structure.

It is important to note that implemented contact type (“face to face” or “edge to edge” in 2D), allows for the use of different stress integration schemes in order to find the resultant contact forces, \( F_t \) and \( F_c \), as shown in Figure 2b. Therefore, the contact type, to model mechanical interaction between blocks, provides linear stress distribution along the contact length. These are statically consistent with the stress diagrams and bending stiffness in the linear elastic range. Therefore, different from standard point contact model, commonly used in DEM codes, accurate results are obtained with less number of blocks. However, it is efficient to use simple contact models (e.g. point contact), in the parts where failure is not expected and more rigorous contact assumptions may be employed among the blocks actively participates in failure mechanism.

Material Properties

The strength of masonry units and mortar vary remarkably, especially in case of historical masonry structures. Heterogeneous and composite characteristics of masonry make it further difficult to
select a representative number for each of the mechanical characteristics for the constitutive laws employed in the numerical model. Here, zero tensile strength and cohesion at the joints were assigned to replicate dry-joint masonry, where there is no mortar to bind masonry units. Different contact stiffness for both orthogonal directions and related joint properties were given in Table 1. Representative unit weights for the stone units and backfill material were taken from the literature, as 24 kN/m³ and 20 kN/m³, respectively (Oliveira et al. 2010).

Validation of the Methodology

In this section, the custom-made software and the numerical approach, used in this study are benchmarked against an experimental study conducted in the Technical University of Catalonia (UPC). Two short span true-scale brick masonry arch bridges were tested under quasi-static loads, applied at quarter span, and the ultimate load carrying capacities were predicted by limit analysis (Roca and Molins 2004). Here, the semi-circular arch, named BA2, spanning 3.2 meter was selected from that study to validate our custom-made software and modeling strategy. The geometrical and material properties were taken from the related article, given in Table 2 and Table 3, respectively. In order to perform static analysis in DEM, contact stiffnesses ($k_n$ and $k_s$) were calculated according to Lourenço and Rots (1997) using the expressions below.

\[ k_n = \frac{E_u E_m}{E_u - E_m} \]  \hspace{1cm} (5)

\[ k_s = \frac{G_u G_m}{G_u - G_m} \]  \hspace{1cm} (6)
Where $E_u$ and $E_m$ are Young's modulus; $G_u$ and $G_m$ are shear modulus for unit and mortar, respectively and $t_m$ indicates the thickness of joint. Shear modulus was calculated using linear elastic relationship; $E / 2(1+v)$.

The calculated mechanical properties of contacts, tensile and cohesion strength at the joints were shown in Table 4. Specific weight of the backfill (sand) and masonry units (bricks) were considered as 18 kN/m$^3$ and 20 kN/m$^3$. Backfill load was taken as dead weight acting on extrados of the arch-barrel and external load dispersion through the backfill was applied at the quarter span according to the Boussinesq distribution model with an angle of $\pi/6$. Hence, the spandrel walls were not used actively during the load application. Both ends of spandrel wall and bottom part of the abutments were restrained during loading.

Through incremental loading, the damage procession on the masonry arch bridge was observed and a point at the intrados of the arch ring, located at $1/4$ of the span, was monitored. First, separation of the spandrel wall and arch ring was noticed and failure occurred because of the formation of 4-hinge mechanism. The capacity curve obtained by DEM and limit analysis results from Roca & Molins (2004) are given in Figure 3 together with the experimental peak load. It is worth noting that the observed failure mechanism matched experimental observations very closely as shown in Figure 4a. Then, successive plastic hinges developed starting from extrados of the arch barrel where the load was applied. Discrete element model did not only capture the experimental peak load, but also demonstrated the damage progression up to failure, as shown in Figure 4b.

According to the numerical simulations and experimental work, collapse mechanism may change significantly depending on load path. In other words, separation between arch-ring and spandrel wall may occur where the loads are not applied on the spandrel wall.
Parametric Study

In this parametric study, the validated numerical modeling method is used on a base model. The model is varied to test the effect of the following parameters: backfill, spandrel wall, boundary conditions, and the morphology of the arch barrel. Morphology will be varied in the form of layers of stones in the arch (single versus double layer) and the bond pattern (running versus stack bond).

Geometry of the Base Model

Historic masonry construction was mostly a manual trade, therefore the geometry, bond patterns, materials and other considerations vary greatly with local traditions and the architectural styles (roman era semi-circular arches to more recent shallower arches, for instance). As a result of this, it is not straightforward to find a “typical” masonry arch configuration to build a parametric study upon. However, one can hypothesize that within the constraints of a specific style, the effect of varying the common elements of a masonry arch will have similar response on the overall structural behavior. This study utilizes this hypothesis to examine the effect of spandrel walls, boundary conditions, arch thickness, number of arch rings, and bond pattern on one hypothetical masonry arch model. Authors strongly emphasize that the readers will benefit from the understanding developed from this study greatly, but for more accurate results, each masonry arch bridge should be examined considering its own geometrical characteristics. This is because, the structural response is strongly controlled by the geometrical properties (Block et al. 2006).

The base DEM model for the hypothetical arch was generated (Fig. 5), adapting some of the typical geometrical parameters presented in the literature (de Arteaga and Morer 2012; Conde et al. 2016;
Then, each structural component was studied individually, and different boundary conditions were simulated.

**Effect of the Backfill and Spandrel Wall**

For this analysis, first, an isolated semi-shallow arch (without any backfill material and spandrel wall), having a 0.4 rise to span ratio with 0.6 m thickness, was modeled. A vertical point load of $F$ was applied incrementally at quarter-span. The load carrying capacity, found as 65 kN, was compared by limit analysis approach, using an open-source software ArchNURBS, as shown in Figure. 6a (Chiozzi et al. 2016). Less than 2% difference was obtained for ultimate load and identical collapse mechanisms were observed (Fig. 6 b-c).

This is informative of masonry arch behavior under such point loading and the collapse mechanism is well known from the literature. However, while estimating the actual load carrying capacity of masonry arches, one needs to take into account other parameters than only the arch itself, as masonry arches are rarely, if ever, in this vulnerable condition. For instance, for a masonry arch bridge, the backfill material is one of the important parts of the system, which provides more strength to the arch barrel by applying compression forces around the extrados of the arch that counteract the flexural tension forming on the arch due to any point load. To present this phenomenon, the dead load from the backfill material was applied on each rigid block as an external load. Then, a point load was incrementally applied on the arch extrados without considering the load dispersion angle. At this stage of the study, soil-structure interaction was not taken into account which allows for the study of parameters one at a time using DEM. While this simplified approach has merit, authors are aware that, backfill and arch barrel has a complex relationship. The
soil can be mobilized and play an active role in the load carrying capacity and failure mechanism rather than applying a static weight (Callaway et al. 2012). Future work will incorporate this complex relationship to the models. In addition to that, there are several approaches in discrete element framework, such as modeling the backfill material as deformable blocks to represent backfill material (Bićanić et al. 2003). There are also different approaches, mentioned earlier that considers finite element limit analysis approach based on the kinematic theorem to take into account the arch-fill interaction (Cavicchi and Gambarotta 2007). Here, for the sake of simplicity, backfill loads were considered as dead loads applied as external forces on each block.

In Figure 7, the contribution coming from the backfill material to the load carrying capacity of masonry arch is presented in terms of the load-displacement curve. Limit analysis is also performed to benchmark the results (Fig. 7). According to results, more than three times higher capacity was obtained under given geometrical form when the backfill compressive forces are considered. It can be deduced that, if backfill material is not taken into consideration the capacity of masonry arches is underestimated. Therefore, it is crucial to consider individual parts of arch bridges to assess their overall behavior (Brookes 2010). This is in fact an important problem in the present day analysis and rehabilitation of masonry arch bridges.

The above analysis considers a cross-section at the middle of the arch bridge. If there is backfill, there has to be masonry spandrel walls on either end of the arch barrel to contain the backfill. To simulate the behavior one of these outer sections of the arch bridge, the same masonry arch is modeled with varying thickness of spandrel walls. Contributions of the backfill and the spandrel
walls depend on their geometrical characteristics. Spandrel wall thicknesses are varied as 0.15, 0.25 and 0.375 meters on each side were modeled together with a 1 meter width of backfill material in each case. If the backfill material was not included, this demonstration would represent the case of a 3-D masonry arch bridge model that has a spandrel wall without backfill material (Lemos 1995). Concentrated load was applied at quarter span on spandrel wall. Each displacement was monitored after getting an equilibrium state under every incremental loading. From the results of the analyses, it is concluded that, similar to the backfill, the spandrel wall also provides a significant contribution to the capacity depending on its thickness (Fig. 8). However, it should be noted that these in-plane discrete element models assume no out-of-plane action, which needs 3-D analysis, and may not be necessary in all cases.

Furthermore, both sides of the spandrel wall were left free in these examples. Boundary conditions could provide extra capacity to the structure, which will be discussed in the next sections.

**Effect of Boundary Conditions on Load Carrying Capacity**

In engineering practice, soil characteristics and exact boundary conditions are among the most difficult parameters to determine for an existing masonry arch bridge. In this case, to see the influence of boundary conditions on the load carrying capacity of the masonry arch including backfill material and spandrel walls, two case studies were prepared. Both backfill and spandrel wall have a significant contribution to the capacity of the arch, but, which one has the higher impact on the arch bridges is an ongoing research topic (Sarhosis et al. 2016). In this context, it should be underlined that, the results are obtained for a unit width of backfill and 0.15 m spandrel wall.
Therefore, readers should keep in mind that, the obtained results may increase or decrease depending on the thickness of the spandrel wall and backfill properties.

In the first case, there are no assigned constraints at the spandrel wall boundaries. In the second model, a passive earth pressure is employed at both sides of the masonry spandrel walls, as shown in Figure 9. In both models, the bottom of the models has fixed.

As expected, there is a significant change on the load carrying capacity of the models depending on the boundary condition assigned to rigid blocks (Fig. 10a). Collapse mechanisms are identical for both cases; however, the capacities at which this mechanism is achieved, are different. The formation of the plastic hinges first follows the line of action of the force, and other plastic hinges appear at both extrados and intrados on the arch. After the arch develops these hinges cracking in the spandrel wall follows, as demonstrated in Figure 10b. The contribution of the boundary conditions may have even further impact when the arch geometry is varied from shallow to deep arches, since the horizontal thrust distribution would be vastly different. However, for a comparison with the same arch geometry, it is clear that the model with free ends would underestimate the capacity of the structure and generate conservative results.
Effect of the Morphology of the Arch Barrel (Number of layers and Bond Pattern)

Construction practice of masonry structures varies from one country to another, or even within one country by region or time of construction. Characteristics of masonry structures may affect their structural behavior depending on the arch configuration (Pulatsu et al. 2016).

To account for such variability, this simulation experiment is expanded to include the analysis of arches with double layer arches and bond patterns (stack versus running bond). For comparison, the previously examined model with a single layer arch (0.6 m thick) was modified to create a double layer arch (0.3 m thickness each layer) with two different bond patterns (Fig. 11).

Several comparisons were carried out with these models. First, double layer masonry arch was compared with the single layer arch. Both of these models included masonry spandrel wall and backfill material. DEM analysis results show that single layer of arch with a thickness of 0.6 m has almost double strength than one that has two stack bonded arch layers of 0.3 m thickness (Fig. 12a), despite the fact that the total arch thickness is the same in these models. Since, the model does not have any bonding material (i.e. mortar), no tensile and cohesion strengths were presented at the joints. As such, this analysis demonstrates the worst case situation for the double layer arch structure. The collapse mechanism starts when the thrust line touches the intrados of the arch barrel and tensile forces appear at the contact that yields to opening between two adjacent blocks. To understand the influence of the tensile strength considered at the arch barrel, a sensitivity analysis was performed by adding tensile strength to the joints. Since, mortar joints are the weak planes for masonry construction, low tension capacity ($f_t$) and a cohesion value of ($1.5f_t$) were employed.
Although this affects the stiffness and the strength of the structure, shown in Figure 13, it seems difficult to reach single layer arch capacity (0.6 m thickness). The failure mechanism generally triggers by the lack of tensile strength at the joints due to aged and damaged mortar. Due to such poor mechanical properties at the joints, in existing historical masonry structures, tensile strength, if any, would be very small. This is why as a part of sensitivity analysis, the effect of a small range of tensile strengths was studied. Figure 13 shows the overall capacity is only marginally affected. It should be noted, however, that a higher tensile strength for contact in DEM for the mortar joints may result in over estimation, especially for historical masonry structures.

As a second step, different bond patterns were considered to see the possible inter-locking effects that may have an influence on the overall structural behavior and the capacity. However, there was no considerable difference observed neither at the load carrying capacity (Fig. 12b) nor at the failure mechanism of the structure (Fig. 12c). Therefore, without the binding effects of mortar in the joints, the bond pattern does not have a significant influence on the ultimate load capacity of the structure. Table 5 summarizes the maximum load carrying capacities for single and double layer arches along with the effect of the different structural components. The results clearly indicate the positive influence of backfill and spandrel walls.

Conclusions

Using two-dimensional rigid block DEM models, the load carrying capacity and the nonlinear response of a family of masonry arch structures were studied. The following conclusions are drawn from this parametric study:
The discrete element method, which was the numerical strategy employed in this project, is a powerful technique that can capture the nonlinear behavior of masonry arches and the complex relationship between its structural components and assigned boundary conditions. It allows to visualize the progression of damage and formation of hinges.

Among the parameters studied, the boundary conditions have the most significant contribution to the load carrying capacity of masonry arch structures.

A single layer of thicker units presented a higher capacity than an arch of same thickness but formed of two layers. This is partially because the analyses assumed no mortar in the joints and presented the most vulnerable case. However, in general, mortar joints are the weak planes for masonry structures and does not provide remarkable strength.

For the cases investigated, there was no significant impact due to the bond pattern between the two arch layers.

Masonry arch with a thicker spandrel wall emerged as the stiffer case scenario. This makes sense, especially for the case of a well-preserved structure with an intact masonry wall. The backfill above the arch barrel also had a significant effect on the arch’s capacity, however, modeling backfill accurately is more difficult than spandrel walls since most of the time there is a lack of knowledge about the status of the backfill material. Nevertheless, these analyses show that analyzing existing masonry arch bridges without any consideration of the backfill or the spandrel wall vastly underestimates their inherent strength.

The custom-made application was validated via an experimental study and results found demonstrated realistic collapse mechanism that matched published experimental results. It was noted that the application of load through the spandrel wall may yield different conclusions and results. The load transfer influences the damage progression of the
structure. For example, when loads are applied through the backfill, separation between arch ring and spandrel wall would be observed. Thus, the strength contribution coming from spandrel wall should be evaluated carefully via parametric studies on the numerical models of real structures.

In future work, each of the parameters will be further studied using 3D models and considering the soil-structure interactions. With a 3D analysis, the effect of the spandrel walls as a boundary condition in the orthogonal direction, transverse cracking in the arch barrel and other 3D effects will be captured.

References


Cundall, P. A. (1971). “A computer model for simulating progressive, large-scale movements in


### Table 1. Joint properties

<table>
<thead>
<tr>
<th>Normal Stiffness (kn)</th>
<th>Shear Stiffness (ks)</th>
<th>Friction Angle (degrees)</th>
<th>Cohesion (GPa/m)</th>
<th>Tensile strength (GPa/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>20</td>
<td>40</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

### Table 2. Geometrical Properties (in meters)

<table>
<thead>
<tr>
<th>Span</th>
<th>Spandrel Wall Thickness</th>
<th>Total Length</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.2</td>
<td>0.15</td>
<td>5.2</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rise</th>
<th>Arch Thickness</th>
<th>Backfill Depth on</th>
<th>Loaded Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6</td>
<td>0.14</td>
<td>0.1</td>
<td>1/4 of span</td>
</tr>
</tbody>
</table>

### Table 3. Material Properties (BA2)

<table>
<thead>
<tr>
<th>$E_{\text{unit}}$ (GPa)</th>
<th>$E_{\text{mortar}}$ (GPa)</th>
<th>$G_{\text{unit}}$ (GPa)</th>
<th>$G_{\text{mortar}}$ (GPa)</th>
<th>$t_m$ (m)</th>
<th>Poisson's ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.45</td>
<td>0.81</td>
<td>4.35</td>
<td>0.34</td>
<td>0.02</td>
<td>0.2</td>
</tr>
</tbody>
</table>

### Table 4. Contact Properties

<table>
<thead>
<tr>
<th>$k_n$ (GPa/m)</th>
<th>$k_s$ (GPa/m)</th>
<th>$f_t$, Tensile Strength (kPa/m)</th>
<th>$c$, Cohesion (kPa/m)</th>
<th>Friction Angle (deg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>43.9</td>
<td>18.29</td>
<td>40</td>
<td>1.5$f_t$</td>
<td>35</td>
</tr>
</tbody>
</table>

### Table 5. Load carrying capacities (kN) for single layer and double layer masonry arch structures

<table>
<thead>
<tr>
<th>Arch-Type (thickness)</th>
<th>Arch</th>
<th>Arch+Infill</th>
<th>Arch+Infill+Spandrel Wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single Layer (0.6 m)</td>
<td>65</td>
<td>217</td>
<td>600</td>
</tr>
<tr>
<td>Double Layer (0.3x2 m)</td>
<td>26</td>
<td>100</td>
<td>300</td>
</tr>
</tbody>
</table>
Figure
Figure

Contact point - 1

Contact point - 2

BLOCK - 1

BLOCK - 2

F
Figure

Displacement (mm)

Force (kN)

Experiment – BA2
Limit Analysis
DEM
Ring separation

Horizontal bed-joint opening

Plastic hinge
Formation of hinges
Fixed support
Passive soil pressure
Figure

Displacement (mm)

Force (kN)

Stack Bond (0.3x2)

1/2 Running Bond (0.3x2)
List of Figure Captions

**Fig. 1.** Discrete blocks and assigned springs in two orthogonal directions.

**Fig. 2.** Contact points and stress distribution.

**Fig. 2a** Contact points and external force, $F$

**Fig. 2b** Stress distribution at the contact.

**Fig. 3.** Capacity curve, obtained via discrete element code and comparison with limit analysis and experimental peak load.

**Fig. 4.** Experimental and numerical modeling of true scale brickwork masonry arch bridge.

**Fig. 4a** Damaged state of masonry arch bridge (BA2) loaded at $\frac{1}{4}$ of the clear span.

**Fig. 4b** Damaged state of masonry arch bridge using DEM incrementally loaded at $\frac{1}{4}$ of the clear span.

**Fig. 5.** Dimensions of the base model and the arrangement of rigid blocks in the numerical model.

**Fig. 6.** Collapse mechanism of masonry arch under vertical eccentric loading.

**Fig. 6a** Comparison between DEM and Limit Analysis.

**Fig. 6b** Discrete Element Model.

**Fig. 6c** Limit Analysis (ArchNURBS).

**Fig. 7.** Force-Displacement curve for masonry arch with in-fill material using DEM and Limit Analysis (ArchNURBS).
Fig. 8. Force-Displacement curves for different thickness of spandrel wall (SW) thicknesses.

Fig. 9. Boundary conditions.

Fig. 9a Fixed at the supports and both sides of the structure are free to move.

Fig. 9b Fixed at the supports and passive earth pressure applied at both sides.

Fig. 10. Capacity curves and corresponding failure mechanism.

Fig. 10a Load vs. displacement for different boundary conditions.

Fig. 10b Collapse mechanism.

Fig. 11. Double layer masonry arch models with different bond pattern.

Fig. 11a Masonry arch (double layer stack bond) with spandrel wall.

Fig. 11b Masonry arch (1/2 running bond) with spandrel wall.

Fig. 12. Force vs. displacement curves for two different arch layer and corresponding collapse mechanism.

Fig. 12a Force vs. displacement of single (SL) and double layer (DL) arch with spandrel and back-fill material.

Fig. 12b Force vs. displacement curve of two different bond pattern used for arch barrel

Fig. 12c Collapse state of the structure with double layer and ½ running bond.

Fig 13. Parametric study for tensile strength at the contacts located at the arch barrel.
Dear Eng. Pulatsu,

At your request, I am pleased to confirm you my permission to use for your publications the photographs of the experimental masonry bridges tested in the Laboratory of Structural Technology of Technical University of Catalonia and published in the proceedings of the International Arch Bridge Conference ARCH'04. Please in your publications include the corresponding credits or references of the mentioned photographs.

Yours sincerely,

Pere Roca
Universitat Politècnica de Catalunya
Departament d’Enginyeria Civil i Ambiental
Jordi Girona 1-3, mòdul C1, 08034 Barcelona
ASCE Authorship, Originality, and Copyright Transfer Agreement

Publication Title: Parametric Study on Masonry Arches Using 2D Discrete Element Modelling

Manuscript Title: Parametric Study on Masonry Arches Using 2D Discrete Element Modelling

Author(s) – Names, postal addresses, and e-mail addresses of all authors

1. Bora Pulasçu, 24 Peter Kiewit Institute 1110 S. 69th St., Omaha, NE 68182. E-mail: bopulascu2@unl.edu (corresponding author)

2. Ece Erdogmus, 203 Peter Kiewit Institute 1110 S. 69th St., Omaha, NE 68182. eerdogmus2@unl.edu

3. Eduardo M. Breña, Northern Research Institute, Narvik, Norway eduardo@tek.no

I. Authorship Responsibility

To protect the integrity of authorship, only people who have significantly contributed to the research or project and manuscript preparation shall be listed as coauthors. The corresponding author attests to the fact that anyone named as a coauthor has seen the final version of the manuscript and has agreed to its submission for publication. Deceased persons who meet the criteria for coauthorship shall be included, with a footnote reporting date of death. No fictitious name shall be given as an author or coauthor. An author who submits a manuscript for publication accepts responsibility for having properly included all, and only, qualified coauthors.

I, the corresponding author, confirm that the authors listed on the manuscript are aware of their authorship status and qualify to be authors on the manuscript according to the guidelines above.

Bora Pulasçu 9/5/2017

Print Name Signature Date

II. Originality of Content

ASCE respects the copyright ownership of other publishers. ASCE requires authors to obtain permission from the copyright holder to reproduce any material that (1) they did not create themselves and/or (2) has been previously published, to include the authors’ own work for which copyright was transferred to an entity other than ASCE. Each author has a responsibility to identify materials that require permission by including a citation in the figure or table caption or in extracted text. Materials re-used from an open access repository or in the public domain must still include a citation and URL, if applicable. At the time of submission, authors must provide verification that the copyright owner will permit re-use by a commercial publisher in print and electronic forms with worldwide distribution. For Conference Proceeding manuscripts submitted through the ASCE online submission system, authors are asked to verify that they have permission to re-use content where applicable. Written permissions are not required at submission but must be provided to ASCE if requested. Regardless of acceptance, no manuscript or part of a manuscript will be published by ASCE without proper verification of all necessary permissions to re-use. ASCE accepts no responsibility for verifying permissions provided by the author. Any breach of copyright will result in retraction of the published manuscript.

I, the corresponding author, confirm that all of the content, figures (drawings, charts, photographs, etc.), and tables in the submitted work are either original work created by the authors listed on the manuscript or work for which permission to re-use has been obtained from the creator. For any figures, tables, or text blocks exceeding 100 words from a journal article or 500 words from a book, written permission from the copyright holder has been obtained and supplied with the submission.

Bora Pulasçu 9/5/2017

Print name Signature Date

III. Copyright Transfer

ASCE requires that authors or their agents assign copyright to ASCE for all original content published by ASCE. The author(s) warrant(s) that the above-cited manuscript is the original work of the author(s) and has never been published in its present form.
The undersigned, with the consent of all authors, hereby transfers, to the extent that there is copyright to be transferred, the exclusive copyright interest in the above-cited manuscript (subsequently called the "work") in this and all subsequent editions of the work (to include closures and errata), and in derivatives, translations, or ancillaries, in English and in foreign translations, in all formats and media of expression now known or later developed, including electronic, to the American Society of Civil Engineers subject to the following:

- The undersigned author and all coauthors retain the right to revise, adapt, prepare derivative works, present orally, or distribute the work, provided that all such use is for the personal noncommercial benefit of the author(s) and is consistent with any prior contractual agreement between the undersigned and/or coauthors and their employer(s).
- No proprietary right other than copyright is claimed by ASCE.
- If the manuscript is not accepted for publication by ASCE or is withdrawn by the author prior to publication (online or in print), this transfer will be null and void.
- Authors may post a PDF of the ASCE-published version of their work on their employers' Intranet with password protection. The following statement must appear with the work: "This material may be downloaded for personal use only. Any other use requires prior permission of the American Society of Civil Engineers."
- Authors may post the final draft of their work on open, unrestricted Internet sites or deposit it in an institutional repository when the draft contains a link to the published version at www.ascelibrary.org. "Final draft" means the version submitted to ASCE after peer review and prior to copyediting or other ASCE production activities; it does not include the copyedited version, the page proof, a PDF, or full-text HTML of the published version.

Exceptions to the Copyright Transfer policy exist in the following circumstances. Check the appropriate box below to indicate whether you are claiming an exception:

☐ U.S. GOVERNMENT EMPLOYEES: Work prepared by U.S. Government employees in their official capacities is not subject to copyright in the United States. Such authors must place their work in the public domain, meaning that it can be freely copied, republished, or redistributed. In order for the work to be placed in the public domain, ALL AUTHORS must be official U.S. Government employees. If at least one author is not a U.S. Government employee, copyright must be transferred to ASCE by that author.

☐ CROWN GOVERNMENT COPYRIGHT: Whereby a work is prepared by officers of the Crown Government in their official capacities, the Crown Government reserves its own copyright under national law. If ALL AUTHORS on the manuscript are Crown Government employees, copyright cannot be transferred to ASCE; however, ASCE is given the following nonexclusive rights: (1) to use, print, and/or publish in any language and any format, print and electronic, the above-mentioned work or any part thereof, provided that the name of the author and the Crown Government affiliation is clearly indicated; (2) to grant the same rights to others to print or publish the work; and (3) to collect royalty fees. ALL AUTHORS must be official Crown Government employees in order to claim this exemption in its entirety. If at least one author is not a Crown Government employee, copyright must be transferred to ASCE by that author.

☐ WORK-FOR-HIRE: Privately employed authors who have prepared works in their official capacity as employees must also transfer copyright to ASCE; however, their employer retains the rights to revise, adapt, prepare derivative works, publish, reprint, reproduce, and distribute the work provided that such use is for the promotion of its business enterprise and does not imply the endorsement of ASCE. In this instance, an authorized agent from the authors' employer must sign the form below.

☐ U.S. GOVERNMENT CONTRACTORS: Work prepared by authors under a contract for the U.S. Government (e.g., U.S. Government labs) may or may not be subject to copyright transfer. Authors must refer to their contractor agreement. For works that qualify as U.S. Government works by a contractor, ASCE acknowledges that the U.S. Government retains a nonexclusive, paid-up, irrevocable, worldwide license to publish or reproduce this work for U.S. Government purposes only. This policy DOES NOT apply to work created with U.S. Government grants.

I, the corresponding author, acting with consent of all authors listed on the manuscript, hereby transfer copyright or claim exemption to transfer copyright of the work as indicated above to the American Society of Civil Engineers.

Bora Pulatsy
Print Name of Author or Agent

Signature of Author of Agent

Date

More information regarding the policies of ASCE can be found at http://www.asce.org/authorsandeditors
Answer to reviewer comments:

Authors would like to acknowledge the reviewers for their valuable comments and opinions. We are also grateful anonymous reviewers whose comments provided important improvements to this manuscript.

Reviewer #1:

1) DEM is not the most appropriate tool to analyze arches, even interacting with the backfill. The most appropriate way (also for vaults and 2D curvature structures) is limit analysis because it provides immediately collapse loads and failure mechanisms. In this regard, this should be acknowledged at least in introduction with a proper discussion. Good references to corroborate such idea are the following:

the following general review paper would be also very beneficial to improve the discussion:

Related valuable references were added and further explanation of limit analysis in the introduction part was given as suggested (line 67 – line 76)

2) In the examples treated it would be interesting to plot the thrust-line at collapse, to see what really changes in the load carrying capacity, especially for the spandrel model.

The proposed discrete element models were analyzed using limit analysis and thrust line was obtained at the stage of collapse. All the limit analysis was done by ArchNURBS, cited in the text.

3) I do not understand why authors use such a simplified way for the backfill. There is a classic paper by Bicanic (that should be cited) where DEM is used taking into account backfill in a proper way. Again limit analysis is more straightforward and there are at least two classic papers by Cavicchi and Gambarotta discussing this issue that should be cited.

The motivation was to model infill material in a simplified way computationally without including the details of the infill into discrete element method (e.g. deformable blocks for back-fill material).

Suggested references were included to acknowledged, there are other methods for more detailed analysis.

4) The numerical approach used should be discussed in detail. This is a static approach but traditionally DEM works in dynamics. Is it a "slow" dynamic solver? There is a special issue in International Journal of Masonry Research and Innovation (Vol 1 Issue 4) in Memory of Prof. Bicanic where some details are provided. Also, in the same Journal there is a paper by Drei, Sincuraian and Milani that study two masonry aqueducts with UDEC. Maybe a new Section could be beneficial.

Authors of the paper greatly appreciate from your suggestions. Since, discrete element method has a dynamic procedure to solve differential equations of motion, we used dynamic relaxation method to obtain static solutions. Therefore, we can model damaging process of the masonry arches. Improvements in the text was done to clarify the employed numerical approach.

5) Is there any possibility to compare DEM prediction with that provided by Limit Analysis? In all the force-displacement curves provided there are horizontal lines that I imagine indicate the collapse load. However in some cases the capacity curve at collapse still does not have a horizontal first derivative. This is theoretically
expected in any elasto-plastic model. Is there any reason about this issue or simply it indicates the first point of lack of convergence? Maybe using Ring software by Gilbert or ArchNurbs software by Chiozzi authors could have an idea of the expected collapse load, also to corroborate results.

**A new section (Validation of the Methodology) was added to validate the custom-made software.** Experimental results were from Roca & Molins (2004) compared with the limit analysis and DEM solutions. All of the capacity curves were revised and the last point where we have the successful convergence was selected as maximum load carrying capacity.

Reviewer #2:

1) line 72. It states that "custom-made software" was used. There should be a reference to some paper (or thesis) with more information about the software.

The references were added.

2) line 104 and Fig. 2b. The figure shows a linear stress variation. Is this diagram used to calculate the contact forces? Most DEM codes are based on simple vertex-face contacts. The linear diagram is an interesting approach that should be better explained.

*The stress diagram is used for contact forces. This issue is clarified further in the text.*

3) line 121. Instead of "numerical stability is lost", perhaps it would be more accurate to say "equilibrium is lost", or "movements increase without bound".

*The sentence is changed to “movements increase without bound”.*

4) The results obtained for the various conditions are quite credible. The relative values of the carrying capacity appear realistic. The weak point of the paper is that there is no quantitative comparison of any failure load to an analytical solution (for a simple arch), or to other numerical models. The arch shape is similar to the one analyzed by Lemos (1995) with UDEC/3DEC. Is it possible to compare any loads? Or at least to confirm the conclusions drawn about the arch behavior?

**A whole new section was added to validate the custom-made software as ‘Validation of the Methodology’. Experimental results from Roca & Molins (2004) were compared with the limit analysis and DEM solutions. Similar collapse mechanism and load carrying capacities were found with experimental study.**

5) line 225. The study of the model with a 2 layer arch is very interesting. There is not much in the literature about this issue, so these results are valuable. They show clearly the much lower capacity of the double layer arch. The authors comment that in this model the joint between the layers is only frictional. But, would the consideration of a bond between the 2 layers (assuming mortar cohesion and tensile strength) really increase significantly the capacity? An extra analysis to check this would be a good enhancement of the paper.

**A sensitivity analysis was performed to check how much the capacity will be influenced by the tensile strength at the joints (for the arch-ring). Although, we found that there is a contribution coming from limited tensile strength, it is still difficult to achieve single layer arch-ring capacity. This is because, strength is mostly governed by the geometrical properties of the arch form and distinct composition of the masonry units. (line 318-325)**
Associate Editor's comments:

1. Line 34: Consider changing "Throughout the history" to "Throughout history";
   
   **It is corrected.**

2. Line 41: Consider changing "Last three decades" to "In the last three decades,";
   
   **It is corrected.**

3. Line 52: Consider changing "including arch and arch with infill" to "including arch with and without infill";
   
   **It is corrected.**

4. Line 56: Consider changing "The collapse of the masonry structures" to "The collapse of the arch" as the reference for is pertinent to arches;
   
   **It is corrected.**

5. Line 58: This assumption is improperly states: "(ii) friction between voussoirs is high enough to get sliding failure". Heyman's assumption it that sliding failure cannot occur. Consider changing to "friction between voussoirs is high enough to prevent sliding failure";
   
   **The statement is revised and put into the form as suggested.**

6. Line 64: Consider changing "by extending the infinite compressive strength to material crushing strength and the possibility of sliding" to "by limiting the infinite compressive strength to material crushing strength and accounting for the possibility of sliding";
   
   **The related correction is made.**

7. Line 203: "Masonry construction is mostly a manual trade,.." This may be true for historic masonry construction, but not for modern construction which follows current standard codes for design and construction. Consider changing to "Historic masonry construction was mostly a manual trade,...";
   
   **Authors agree with your comment and changed as suggested.**

8. Line 215: Consider changing "masonry" to "arch";
   
   **It is changed.**

9. Line 231: Use either "in-fill" or "infill" throughout the paper, but not both. Also, backfill would be more appropriate than infill for masonry arches;
   
   **In the revised version of the text, the term ‘backfill’ is used.**

10. Line 233: Consider deleting "asymmetric" to make this statement more general. One would expect that even a symmetrical point load, i.e. point load at the arch crown, will produce tensile flexural stresses in the arch barrel.
   
   **The word, ‘asymmetric’ is taken out from the related sentence.**

11. Line 248, Figure 7: It would be helpful to combine Fig 6a and Fig 7 into one figure in order to directly compare the effect of infill on the arch capacity;
   
   **Due to the organization of the other figures and the text, we decided to keep as it is with all due respect to your comment.**
12. Line 258: Consider changing "To simulate behavior one of these outer planes of the arch bridge,..." to "To simulate the behavior of one of these outer sections,...".

   **It is changed as suggested.**

13. Line 311: Use either "back-fill" or "backfill" throughout the paper, but not both.

   **In the revised version of the text, it is changed to 'backfill'.**

14. Line 318: Consider changing "..., sensitivity analysis...." to "..., a sensitivity analysis...."

   **It is changed as suggested.**

15. Line 324: "The failure mechanism generally triggers by the lack of tensile strength at the joints due to aged and damaged mortar." Please revise or delete this sentence as it seems that it is not supported by the numerical results obtained in this study. Results presented in Fig 13 suggest the opposite: that the failure load and, consequently, the failure mechanism are marginal affected by the masonry tensile strength.

   **The related part is revised, and new explanations are added to clarify that the range of assigned tensile strength is quite small which does not lead to any significant difference on the overall load carrying capacity of the structure.**