# Economic Impact of Forecasting Errors in Residual Reserve Curves in the Day-ahead Scheduling of Pumped Storage Plants

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Abstract—The economic impact of forecasting errors in the residual demand curves of the secondary regulation reserve market is analysed in the context of the operation of a closed-loop and daily-cycle pumped-storage hydropower plant. The plant participates in the day-ahead energy market as a price-taker and in the secondary regulation reserve market as a price-maker. The secondary regulation energy due to the real-time use of the committed reserves is also considered in the optimization model. The results show that profit is significantly more sensitive to forecast errors in the day-ahead energy market prices than in the residual demand curves of the secondary regulation reserve market.

*Index Terms*—Pumped-Storage Plant, Secondary Regulation Service, Residual Reserve Curve Forecasting, Value of Perfect Information.

## NOMENCLATURE

## Acronyms

AP	Actual total profit									
DM Prices	Day-ahead energy market prices									
DM Profit	Profit in the day-ahead energy market									
ER2 Prices	Upward and downward secondary regulation									
	energy prices									
ER2UP	Income due to the real-time use of the upward									
	reserves									
ER2DW	Cost due to the real-time use of the downward									
	reserves									
MTI	Maximum theoretical income									
PSHP	Pumped-storage hydropower plant									
RRC	Residual reserve curve, i.e. the residual demand									
	curve of the secondary regulation reserve market									
RTRUs	Percentage of the real-time use of the upward and									
	downward reserves									
SM Income	Actual income in the secondary regulation									
	reserve market									
VPI	Value of perfect information									
VPI-DM	Value of perfect information of the day-ahead									
	energy prices									
VPI-RRC	Value of perfect information of the residual									
	reserve curve									

## I. INTRODUCTION

Traditionally, pumped-storage hydropower plants (PSHPs) have been operated following a price-arbitrage strategy in the day-ahead energy market (selling energy during peak hours and buying energy during off-peak hours) [1]. Recently, several papers have dealt with the joint operation of PSHPs in the day-ahead energy market and in ancillary services such as the tertiary regulation service [2] and the secondary regulation service modelling the plant as a price-taker [3], [4] or as a price-maker [5], [6].

In the context of the Spanish power system, the income from the secondary regulation reserve market is often higher than from the day-ahead energy market or from the real-time use of reserves [3], [5]. For this reason, we believe that studying the economic impact of the forecasting errors in the secondary regulation reserve market market can make a significant contribution to the technical literature.

The value of perfect information of the day-ahead energy prices has been studied in the technical literature in the context of: i) load-shifting industrial plants [7], ii) demand-side market customers [8], iii) a thermal and hydro-based generation company [9], iv) PSHPs participating in the day-ahead energy market [10] and v) PSHPs participating in the day-ahead energy market and in the secondary regulation service [5]. However, to the author's knowledge, there is no paper in the literature where the value of perfect information of the residual demand curves of the secondary regulation reserve market (hereafter referred to as the residual reserve curves, RRCs) is studied.

This paper can be considered as a continuation of [5]. Firstly, the presented case study considers the same PSHP with the same technical data and in the same time period than the one used in [5] in order to establish comparisons. And secondly, the presented paper covers some remaining questions regarding the uncertain data of the problem. According to the results presented in [5], the value of perfect information (also called the economic impact [8] or the profit loss [10]) of the day-ahead energy prices is between 26-40% of the maximum theoretical income, representing an important loss of profit

due to the errors in forecasting the day-ahead energy market prices.

Therefore, the main goal of this paper is to calculate and analyse the value of perfect information (VPI) of the residual reserve curves (hereafter referred to as the VPI-RRCs) and to compare it to the VPI of the day-ahead energy prices, in the context of the operation of conventional<sup>1</sup> PSHPs, participating in the day-ahead energy market and the secondary regulation service (power and energy) of the Spanish power system. For this purpose, a deterministic mixed integer quadratic programming model is used. The objective function of the model aims at maximizing the income of a PSHP participating in the day-ahead energy market and in the secondary regulation service, in the framework of the Iberian electricity market [11], [12].

The secondary regulation service in the Spanish electricity market comprises: 1) a day-ahead reserve market, which takes place after having cleared the day-ahead energy market and where the secondary regulation reserve requirements of the system are assigned. The assigned power reserve is remunerated by the marginal market price, and 2) power reserve delivery in real-time according to the assigned reserve in the day-ahead reserve market and the system requirements in real-time. The upward and downward secondary regulation energy is remunerated by the marginal price of the upward and downward tertiary regulation market, respectively [12]. The procedure for the procurement of secondary regulation is not exclusive of the Iberian system. For instance, a similar procedure is implemented in the Swiss system in the short-term (less than a week) and in the Dutch, Belgian, German or Danish electricity systems in a longer-term (more than a week) [13].

The model, used to estimate the VPI-RRCs of a realistic closed-loop and daily-cycle PSHP, is described in [5]. The PSHP is supposed to be a price-taker in the day-ahead energy market and a price-maker in the secondary regulation reserve market. For the purpose to estimate the VPI-RRCs, the model was run, day by day, for a period of one year (2014), in order to obtain representative enough results.

The rest of the paper is organised as follows: the data and the methodology to obtain the VPI-RRCs are described in Section II. Section III presents the results and discussion and finally, conclusions are described in Section IV.

#### II. IMPACT OF RRC FORECASTING ACCURACY ON INCOME

## A. PSHP technical Data

The PSHP considered in this paper is the same as in [5]. It is composed by a single reversible Francis pump-turbine unit, whose technical data are presented in Table I: g refers to power, q refers to flow,  $\eta$  refers to efficiency and cSU refers to start-up cost. Superscript d refers to generating mode whereas p refers to pumping mode. The gross head is 400 m with

hydraulic losses of 3% of the gross head. A constant head is assumed. In generating mode, the minimum water discharges and efficiencies at maximum and minimum water discharges are calculated following the guidelines of [14]. A linear relationship between water discharge and power generation is assumed. The efficiency in pumping mode is 90% [15]. The round-trip efficiency is 76.3%. The start-up costs in generating and pumping modes are obtained following the guidelines of [16]. The upper reservoir has a maximum and minimum storage capacity of  $5 Mm^3$  and  $0 Mm^3$ , respectively. In each day, the initial and final water volumes are the same and equal 2.5  $Mm^3$ , in order to meet the daily cycle of the PSHP.

TABLE I TECHNICAL DATA OF THE PSHP. FLOWS ARE EXPRESSED IN  $m^3/s$ , power in MW and start-up cost in  $\mathfrak{C}$ .

	$\overline{q}^d$		$\overline{q}^d$ $\overline{g}^d$ $\overline{\eta}^d$		$\overline{\eta}^d$	$\underline{q}^d$		$\underline{g}^d$		$\underline{\eta}^d$	
2	231.5		93	9	90%		9.5 28		7.9	769	%
	$\overline{q}^p$		$\overline{g}^{p}$	$\gamma \eta^p$			$cSU^d$		$cSU^p$		
	176.6		79	3	90%	%	26	67	21	18	

#### B. Electric power system historical data

The electric power system data used in this paper correspond to hourly values of: 1) the day-ahead energy price, 2) the linear approximation of the RRCs, 3) the upward and downward regulation energy prices, 4) the percentages of the real-time use of reserves and 5) the ratio between the upward and total reserves. The actual percentages of the real-time use of reserves provided by each power plant in Spain is not publicly available. It is assumed that the percentages are given by the historical hourly ratio of the aggregate power delivery and the aggregate assigned reserves in the entire Spanish electric power system.

#### C. Forecasting models

Perfect knowledge is assumed in all data except in the RRCs. Four Cases are analysed, each corresponding to a different approach to forecast the RRCs, see Table II. In this paper, the RRCs are modelled as a linear function of the upward reserve, Fig. 1b. Forecasting an RRC entails therefore the forecast of two random variables: the intercept and the slope of the linear approximation. According to the results obtained in [17], the error due to the use of a linear RRC is expected to be lower than 2% of the estimated income using the real historical RRC.

Case A assumes perfect knowledge also in the RRCs, and its results are the maximum theoretical income against which the results of the other Cases are checked. The forecasting model of Case B assumes that the intercept and the slope of the RRC in hour t of a certain day of 2014 is the average intercept and slope in the said hour across all days in 2013, respectively, Fig. 2.

<sup>&</sup>lt;sup>1</sup>A conventional PSHP is operated with fixed speed in pumping mode and, therefore, it cannot participate in the secondary regulation service in the said mode.

Case	Forecasting Model	MAPE Intercept	MAPE Slope	
Α	Perfect knowledge	No error	No error	
В	Mean hourly price of 2013	35.9%	34%	
С	Historical RRC of the previous week	27.1%	25.9%	
D	Historical RRC of the previous day	22.6%	22.2%	
E	SARIMA $(0, 1, 1)(0, 1, 1)^{24}$ model	21.2%	25.8%	

 TABLE II

 Cases to forecast the residual reserve curves, and the Mean Absolute Percentage Error (MAPE) throughout 2014

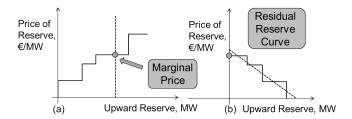


Fig. 1. (a) Supply and demand functions (solid and dashed lines, respectively), and the marginal price of the market. (b) RRC of the upward reserve and its linear approximation (solid and dashed lines, respectively)

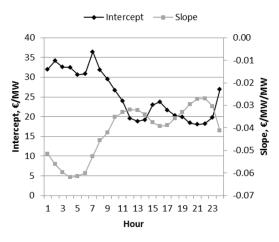


Fig. 2. Intercept and slope of each RRC from Case B

The forecasting models of Case C and D assume that the intercept and the slope of the RRCs in a day d is equal to the historical intercepts and slopes of the RRC of the day d-7 (previous week) and the day d-1 (previous day), respectively. According to Section 5.9.2 in [18], both the system supply and demand functions of the secondary regulation reserve market in Spain are publicly available daily after having cleared the day-ahead reserve market at 5:45 pm in d-1.

The forecasting model of Case E, which is a seasonal autoregressive integrated moving average model (SARIMA) with period 24, is adjusted as follows. The time series of the intercept and slope are transformed into a regular and seasonal stationary process 1) by means of a logarithmic transformation to obtain a more stable variance and 2) by means of a first order regular and seasonal differentiations to obtain a more stable mean. The regular and seasonal autoregressive and moving average parameters of the forecasting model are obtained by an iterative process in which the autocorrelation

and partial autocorrelation plots of the transformed time series and of the residuals of the fitted models are inspected at each iteration step. The iterative process is stopped selecting as few parameters as required to properly explain the data and when the residuals are a white noise process<sup>2</sup>.

## D. Methodology

The VPI-RRCs in each Case is calculated in a given day throughout 2014 by following the next three steps, Fig. 3:

- 1) the optimal generation and consumption schedules for the day-ahead energy market and the optimal upward and downward secondary regulation reserve schedules for the reserve market are obtained by solving the mixed integer quadratic programming model proposed in [5]. Imperfect information is considered with the forecast RRCs whereas perfect information is assumed in the rest of the data. The rest of the data correspond to the historical values of the day-ahead energy prices (DM Prices), the upward and downward secondary regulation energy prices (ER2 Prices) and the percentage of the real-time use of the upward and downward reserves (RTURs). The model is solved by the branch and cut algorithm in CPLEX 12.2 in a computer with a 2.4 GHz Intel Core i5-450M CPU and 4 GB of RAM memory. Each Case, composed by 365 daily problems, is solved in around 30 min.
- 2) the actual profit in the secondary regulation reserve market is calculated in a post-optimal simulation process from the optimal hourly reserve schedule obtained in the previous step and the actual hourly price of reserve. Note that the latter is the result of evaluating the optimal hourly reserve schedule with the linear approximation of the historical RRC.
- 3) the VPI-RRC in each Case is calculated as the difference between the maximum theoretical income (MTI) and the actual total profit (AP) in each Case. The MTI is obtained assuming perfect information in all data (Case A) whereas the AP considers the SM Income from the previous step. The methodology presented in Fig. 3 is based on the one proposed in [10] and [7]. Note that the AP is the result of the profit in the day-ahead energy market, the income of the secondary regulation service (capacity and energy) and the start-up costs in generating

 $^2\mathrm{A}$  white noise process has zero mean, constant variance, uncorrelated process and normal distribution.

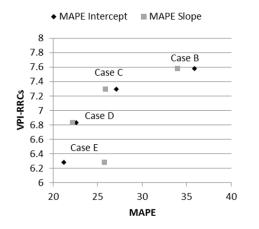


Fig. 4. Relationship between the indicator MAPE and the VPI-RRCs for all the analysed Cases.

and pumping modes (AP = DM Profit + net income in the secondary regulation service - start-up costs).

## **III. RESULTS AND DISCUSSION**

The economic results are shown in Table III. They comprise the profit in the day-ahead energy market (DM Profit) as the difference between the income due to the sold energy minus the cost due to the purchased energy, the actual income in the secondary regulation reserve market (SM Income), the income due to the real-time use of the upward reserves (ER2UP), the cost due to the real-time use of the downward reserves (ER2DW), the start-up costs in generating ( $cSU^d$ ) and pumping ( $cSU^p$ ) modes, the AP, and the VPI-RRCs in  $\in$ and in % of the maximum theoretical income.

The obtained VPI-RRCs ranges between 6.28-7.58% of the MTI. The VPI-RRCs is significantly lower in all Cases than the value of perfect information of the day-ahead energy prices (VPI-DM) published in the technical literature [5]<sup>3</sup>. The Case with the lowest VPI-RRCs is Case E, which forecasts the RRCs with a SARIMA model. The Case with the highest VPI-RRCs corresponds to Case B, which predicts the RRCs with a naive forecasting model (the hourly average intercept and slope of the RRCs across all days of the previous year).

There seems to be a certain positive correlation between the MAPE of the proposed forecasting models and the VPI-RRCs, Fig. 4, especially for the forecasting models of the RRCs intercept.

Several reasons can be discussed to understand why the VPI-RRCs is not significant in comparison to the VPI-DM.

 As the PSHP is operated with fixed speed in pumping mode, it cannot participate in the secondary regulation service in the said mode. Therefore, the forecast errors of the day-ahead energy prices at peak and off-peak hours have an impact on the PSHP income. However, only the forecast errors of the RRCs at peak hours of

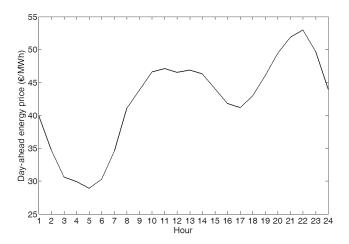


Fig. 5. Hourly average price of the day-ahead energy market across all days in 2014

the day-ahead energy prices (when the plant is typically scheduled to operate in generating mode) have an impact on the PSHP income.

2) A PSHP with fixed speed is usually operated in generating mode at peak hours (when the day-ahead energy price is high). At these hours, the intercept of the RRCs is roughly half of the day-ahead energy price (see Figs. 2 and 5). Considering that, in the Spanish power system, the hourly ratio between the upward and total (upward + downward) reserve is requested to be close to 0.5, an error in the RRC intercept has inevitably a lower impact on the PSHP income than an error of the same magnitude (in relative terms) in the day-ahead energy price.

In order to better understand this discussion, we invite the reader to imagine three different situations in a specific hour, see Table IV. In Situation 1, the PSHP sells 500 MW in the day-ahead energy market and 200 MW of upward and downward reserve in the reserve market. In Situation 2, the PSHP sells also 500 MW in the day-ahead market and only 30 MW of upward and downward reserve in the reserve market. And in Situation 3, the PSHP sells 350 MW in the day-ahead energy market and also 30 MW of upward and downward reserve in the reserve market. Considering an average peak energy price of 50 €/MWh (Fig. 5), and an average RRC intercept of 20 €/MW at peak hours of the day-ahead energy market (Fig. 2), an error of 20% means a forecast day-ahead energy price and RRC intercept of 40 €/MWh and 16 €/MW, respectively. Assuming a conservative value of the RRC slope (-0.03 €/MW/MW), the actual and forecast price of reserve would be, respectively, 14 and 10 €/MW, in Situation 1, and 19.1 and 15.1 €/MW, in Situation 2 and 3. The error in day-ahead energy price means 5000  $\in$  in Situation 1 and 2 and 3500 € in Situation 3, whereas the error in the intercept means 1600 € in Situation 1 and

 $<sup>^{3}</sup>$ The VPI-DM published in [5] ranges from 26.9 to 40.4% of the maximum theoretical income. Note that the case study in [5] uses the same PSHP with the same technical data and in the same time period (2014) than the one used in the presented paper.

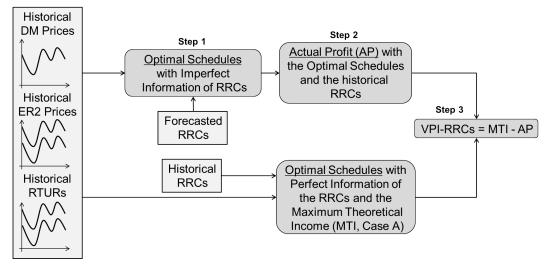


Fig. 3. Methodology to obtain the VPI-RRCs in each Case (except for Case A)

TABLE III ECONOMIC RESULTS AND THE VPI-RRCS THROUGHOUT 2014. INCOME, PROFIT AND VPI-RRCS IN €

Case	DM Profit	SM Income	ER2UP	ER2DW	$cSU^d$	$cSU^p$	AP	VPI-RR	.Cs
A	11 408 606.9	15 650 021	6 212 864.8	-4 631 307.6	-2 462 010.2	-1 303 148.1	24 875 026.9	-	-
В	10 487 924.6	15 495 998.7	6 944 690.4	-5 795 952.6	-2 686 071.8	-1 457 830.7	22 988 758.7	1 886 268.2	7.58 %
C	10 737 505	14 610 301.7	6 468 411.8	-4 844 905.1	-2 528 695.2	-1 381 548.9	23 061 069.3	1 813 957.6	7.29 %
D	11 171 481.4	14 261 552.3	6 312 814.5	-4 648 837.5	-2 544 699.6	-1 377 311	23 175 000.2	1 700 026.7	6.83 %
E	13 101 013.3	12 329 141.9	5 670 286.9	-4 259 814.8	-2 275 292.2	-1 252 293.5	23 313 041.5	1 561 985.3	6.28 %

240 € in Situation 2 and 3. Although the forecast error is the same (20%), the error in the RRC intercept has a much lower impact on the PSHP income, regardless of the scheduled reserve (high in Situation 1 or low in Situation 2 and 3).

3) Analogously, an error in the RRC slope results in an error in the price of the secondary regulation reserve that has a lower impact on the PSHP income than an error of the same magnitude (in relative terms) in the day-ahead energy price.

Using the same three Situations described above (see Table IV) and considering an average RRC slope of  $-0.03 \notin MW/MW$  at peak hours of the day-ahead energy market (Fig. 2), an error of 20% means a RRC slope of  $-0.036 \notin MW/MW$ . Assuming a RRC intercept of 20  $\notin MW$ , the actual and forecast price of reserve would be, respectively, 14 and 12.8  $\notin MW$  in Situation 1, and 19.1 and 18.92  $\notin MW$  in Situation 2 and 3. The error in day-ahead energy price means 5000  $\notin$  in Situation 1 and 2 and 3500  $\notin$  in Situation 3, whereas the error in the RRC slope means 480  $\notin$  in Situation 1 and 10.8  $\notin$  in Situation 2 and 3. Although the forecast error is the same (20%), the error in the RRC slope has a much lower impact on the PSHP income, regardless of the scheduled reserve.

TABLE IV SITUATIONS TO UNDERSTAND WHY THE VPI-RRCS IS NOT SIGNIFICANT IN COMPARISON TO THE VPL-DM

IN COMPARISON TO THE VPI-DM						
	Situation 1	Situation 2	Situation 3			
Produced Power, $g^d$ , MW	500	500	350			
Upward Reserve, MW	200	30	30			
Downward Reserve, MW	200	30	30			
Energy Price, €/MWh	50	50	50			
Energy Price with Error, €/MWh	40	40	40			
Price of Reserve, €/MW	14	19.1	19.1			
Price of Reserve with Error in the Intercept, €/MW	10	15.1	15.1			
Price of Reserve with Error in the Slope, €/MW	12.8	18.92	18.92			
VPI-DM, €	5000	5000	3500			
VPI-RRC with Error in the Intercept, €	1600	240	240			
VPI-RRC with Error in the Slope, €	480	10.8	10.8			

Consequences of the obtained results in the presented paper are the following:

 Harder effort is to be carried out in forecasting the day-ahead energy prices in comparison to forecasting the RRCs as the profit loss due to forecast errors in the former is larger than in the latter, in the context of the day-ahead scheduling of conventional PSHPs participating in the day-ahead energy market and in the secondary regulation service of the Spanish power system.

2) Among all the uncertain parameters that are presented in the day-ahead energy and reserve scheduling (the day-ahead energy prices, the RRCs, the upward and downward secondary regulation energy prices and the real-time use of the upward and downward reserves), the uncertainty of the RRCs might be reasonably modelled with an expected value in order to reduce the computational burden and the size of the scenario tree in a stochastic optimization scheduling model.

Despite the foregoing, the observed profit loss due to the RRCs forecast errors should be complemented by independently analysing the VPI of the rest of uncertain variables that are involved in the secondary regulation service, namely: the secondary regulation energy prices and the real-time use of the reserves. This is proposed as a future work. In addition to this, it is also expected an increase in the VPI-RRCs in advanced PSHPs such as those operating in hydraulic short-circuit mode or with variable speed as they are able to participate in the secondary regulation service also when they are pumping. To study the extent to which the VPI-RRCs increases in advanced PSHPs is also proposed as a future work.

## **IV. CONCLUSIONS**

The economic impact of forecasting errors in the residual demand curves of the secondary regulation reserve market has been analysed in the hourly scheduling of a closed-loop and daily-cycle PSHP. The PSHP has been allowed to participate in the day-ahead energy market as a price-taker and in the secondary regulation reserve market as a price-maker. The economic impact was found to be 6-8% of the maximum theoretical income, which is significantly lower than the impact of the errors in forecasting the day-ahead energy prices (between 26-40% of the maximum theoretical income according to the technical literature for the same PSHP in the same case study). Therefore, it seems reasonable to conclude that among all the uncertain variables involved in the day-ahead scheduling, the residual reserve curves can be modelled with an expected value in order to reduce the computational burden in a stochastic optimization scheduling model.

As a future work, it is proposed to analyse the value of perfect information of the residual reserve curves for more flexible PSHPs such as those operating in hydraulic short-circuit mode or with variable speed, and to calculate independently the value of perfect information of the rest of the uncertain variables of the secondary regulation service: the upward and downward regulation energy prices and the percentage of the real-time use of the upward and downward reserves.

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