

# Including power management strategies and load profiles in the mathematical optimization of energy storage sizing for fuel consumption reduction in maritime vessels



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## ARTICLE INFO

**Keywords:**  
Vessels  
Optimisation  
Battery  
Design

## ABSTRACT

Choosing the optimal type and size of energy storage for a given hybrid maritime vessels is challenging. Investment cost, fuel saving and energy storage expected life time will be affected by the choices. Furthermore, the optimum choices depend on the operation profile of the vessel as well as safety related constraints in different vessel mode of operations. In addition, the optimum power management strategy will be mode dependent as well as dependent on the type and size of onboard energy storage. Finally, the total system has to fulfill certain safety related rules and regulations that typically both favour the use of storage and set some constraints to the size and the utilization of the storage. In this paper we propose a mathematical optimisation model called OBLIVION that stands for “Optimised Battery Lifetime In Vessels Internal Operations and Networks”. OBLIVION is created to support battery investment decisions. Beyond including battery degradation and desired battery lifetime in the choice, the model facilitates analysis of how the investment decisions change for different combinations of vessel operation modes. The key contribution of this paper is the proposed methodology to formulate technical and safety constraints, represent different vessel modes of operation and battery storage degradation in a way suitable for inclusion within mathematical optimisation models. Moreover, analyses that demonstrate how these features affect the storage investment decisions are presented. Mathematical formulations of constraints such as closed and open bus-tie breaker operation, true spinning reserve requirements as well as spinning reserve provided by batteries are included as well.

## 1. Introduction

### 1.1. Motivation

All electric ships where all onboard systems are powered by electricity have over the last decades become more and more common [1]. Direct driven propulsion systems are still the preferred solution for some types of vessels, but the increased design flexibility, and the potential for fuel saving offered by all electric ship concepts have reduced the number of vessels build with direct driven propulsion systems. AC based power systems have been dominating, but DC based power systems are now becoming more widespread due to development of more efficient power electronic converters and power electronic based breakers capable of breaking high prospective DC currents. Advantages and disadvantages of DC and AC power systems for ship electrification are illustrated in [2]. For DC based systems it is common to utilize the possibility to allow diesel engines to adapt speed of the engine to the current loading in order to reduce fuel consumption. This flexibility

cannot be utilized in pure AC based systems. An upcoming alternative to reduce fuel consumption, especially for AC based systems, is to introduce on-board energy storages. On-board electrical energy storage for the purpose of reducing fuel consumption in vehicles such as cars, buses and trucks has become common practice. As discussed in [3], the same trend is now emerging in the maritime sector. New builds and retrofits are now being equipped with battery energy storages as supplement to diesel engine generator sets. There are also examples of plug-in vessels that charge in harbour, similar to the ground-based Plug-in Hybrids. Finally, there are an increasing number of vessels sailing with batteries as their only on-board source of energy, mostly short distance ferries [4].

This paper focuses on hybrid electrical power and propulsion systems for maritime vessels, that is, vessels powered by diesel generator sets supported by energy storage. The introduction of energy storage in marine power systems might be beneficial for several reasons: strategic loading of diesel/gas engines by operating the storage such that engines are running at a more optimal working point; spinning reserve by

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<https://doi.org/10.1016/j.est.2019.03.021>

Received 19 October 2018; Received in revised form 18 January 2019; Accepted 24 March 2019

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reducing the need for having more engines running as a reserve in case of sudden increase in load or sudden reduction or loss of power producing unit; power source in harbour by reducing the need of running engines at very low load or by replacing the need for installation of a dedicated harbour generator; dynamic support and peak shaving by reducing frequency transients through instant support from storage. Moreover, storage can help reducing the need for starting extra engines due to short time load peaks and reducing the rate of change in power outtake from engines that are sensitive to fast load changes (e.g. LNG engines).

However, the inclusion of energy storage presents ship designers with several challenges because there are multiple types of energy storage that can be used and estimating the lifetime of energy storage systems is a complex task. Further challenges arise since vessels modes of operation may affect the choices in the investment decisions. In fact, investment decisions in battery installation are highly dependent on the actual operations performed in the system and therefore should be addressed together in a holistic way. Furthermore, the system design must ensure that the desired lifetime of storage is guaranteed within the predicted operating profile.

The main motivation behind this study is related to the fact that optimal sizing with a properly designed energy management strategy is required to maximize the benefits of introducing the energy storage. Non-optimal use of storage can easily increase, rather than decrease fuel consumption. It can also be shown that for certain load levels it will be better to use engines only and let storage be inactive.

The knowledge and experience of how to select the best combination and size of energy storage within the required constraints is not in the public domain, and practical system designs depend on recommendations from specialized consultancy services or battery manufacturers. Thus, the present paper aims to make knowledge on design and optimization of marine energy systems available to the wider industry.

The main objective of this study is to develop a methodology for optimized sizing of energy storage in hybrid ship power and propulsion systems where energy storage is operated together with diesel or gas generators.

The potential for reduction in fuel consumption is largest for AC based power systems and less for DC based systems. The DC based systems can adapt engine speed in order to maintain higher efficiency also at low loading, reducing the potential benefit of the storage. This paper will therefore focus on optimization of battery systems for vessels with AC based power systems.

The rest of the paper is organised as follows: Section 1.2 will propose a brief literature review in the field of optimisation applied to vessels systems, followed by Section 1.3 that will illustrate the key contributions of the study proposed in this paper. Section 2 will summarise the main properties to take into account when studying vessels operations while a brief introduction to batteries will be proposed in Section 3 to outline the most important aspects that has to be taken in consideration when including batteries in mathematical optimisation models. The mathematical model will be presented in Section 4 followed by sensitivity analyses and computational experiments thoroughly illustrated in Section 5. Finally Section 6 will discuss the conclusions.

## 1.2. Literature review

Electrification, storage integration, optimal design and control strategy techniques in Vessels have received attention in literature.

The electrification and digitalisation of the marine industry has been underway for sometime already [5]. Due to an increase in electric equipment and systems used for different operational profiles, a transition towards all electric ships took place as vessels needed reliable power generation which could supply the rapidly varying load profiles [6,7].

A 2011 study by DNV GL (the world's largest maritime class society) already demonstrated that energy storage technologies represent a substantial potential for improving both fuel economy and reducing emissions in the maritime industry [8]. The importance of energy storage technologies and smart management for DC microgrid-based maritime onboard power systems has been discussed in [9,10]. The most common energy storage usage strategies for safer, smarter and greener ships have been illustrated in [11], among which it is worthy to mention in particular enhanced dynamic performance, spinning reserve, strategic loading for fuel saving, zero emission operations and peak shaving. Moreover, the potential of energy storage to reduce the fuel consumption onboard of marine vessels equipped with multiple diesel engine generators has been illustrated also in [12]. Energy management system algorithms based on mixed integer linear programming are proposed in [13] as a suitable strategy for optimal unit commitment in the power generation. The results indicate that optimal energy management algorithms can increase the operational efficiency in terms of fuel savings and reduction in genset running hours. An energy management strategy for hybrid electric dynamic positioning vessels is also presented in [14] with the objective of optimally distributing energy flows between the power sources onboard, including generators and battery energy storages. Similarly, power management optimisation strategies for hybrid power systems in electric ferries are discussed in [15]. The simulation results show the ability of such optimisation strategies to achieve fuel consumption reductions as well as emissions reduction. In addition, in [16] an optimization algorithm is proposed to minimize fuel consumption under various loading conditions and a detailed efficiency analysis of a shipboard dc hybrid power system is carried out.

A review on intelligent design and control strategies for smart ships is proposed in [17], while a comprehensive review on emerging storage solutions for transportation is available in [18] where sea transportation applications are illustrated together with road, rail and air applications. The economical value of integrating energy storage technologies within maritime vessels is discussed in [19], while in [20] the feasibility of installing renewable energy generation technologies in combination with Li-ion battery storage is investigated.

Simulation, optimisation, heuristic and meta-heuristics approaches for optimal investments, design and operations in vessels are available in literature. Optimal investment and design of vessels have been extensively studied, using both deterministic and meta-heuristics methodologies. A simulation application to the ageing estimation of a supercapacitor-based ferry is studied in [21]. In this study, a cycle-based formulation is applied to the ageing behaviour simulation of the energy storage unit of an all-electric ferry which only uses supercapacitors as energy storage. A deterministic dynamic optimization problem to find the optimal loading strategy for the ship generators in the presence of a particular energy storage size, is presented in [22]. The model is then run assuming different battery size in order to identify a good design. The optimal design of ship power systems with included photovoltaic, diesel and batteries is studied in [23] where a particle swarm algorithm together with a genetic algorithm are adopted. A genetic algorithm to optimise the design of a hybrid propulsion system for marine vessels is proposed also in [24] while particle swarm techniques are used also in [25] to define the optimal location and size of energy storage within electric ship power systems.

In terms of operational optimisation of vessels, both mixed integer linear/non-linear programming (MILP, MINLP) and meta-heuristics techniques are available in literature. A MILP model is presented in [13] as a strategy for the optimal unit commitment in the power generation of diesel-electric marine vessels. While a MINLP model is presented in [26] where the principles of optimal planning and economic dispatch problems are extended to shipboard systems with flywheel as storage devices. Moreover, an optimisation model together with spline approximation, Karush–Kuhn–Tucker method and linear interpolation is present in [27] to maximise the fuel savings on marine vessels

through optimal charge/discharge operations of onboard batteries. In [28] an optimal power management method is proposed so that ship operation cost is minimized, greenhouse gas emissions are limited, and ship power system technical and operational constraints are fulfilled. Particle swarm techniques are used in [29] where a stochastic energy management system for optimised operations in vessels is developed. While a genetic algorithm for the optimal energy production in offshore vessels is presented in [30]. A simulation approach focusing on diesel engines operations to minimise the fuel consumption is proposed in [31]. Simulation is also adopted in [32] where a detailed battery modeling is proposed in terms of technical battery properties and analyses are performed to show how long it will take for the ship owner to have saved enough to be able to buy another battery.

### 1.3. Key contributions

The main contribution of this paper is to propose a methodology to perform optimal selection and sizing of energy storage for a given vessel topology, formulate technical and safety constraints, represent different vessels mode of operation and define storage lifetime requirements in a way suitable for inclusion within mathematical optimisation models. The secondary contribution is to propose analyses that demonstrate how different combinations of load profiles, engine type and size, operational constraints (such as spinning reserve requirements) and mode of operations performed in the vessel, will affect the storage investment decisions and which conditions will penalise or not battery installation. Finally, the third contribution is to propose a methodology to understand the value of storage within vessels and, when batteries are beneficial, maximize the fuel and cost saving by choosing the right type and size of battery for a given vessel power plant.

As outlined in Section 1 there can be different motivations for installing the batteries onboard of maritime vessels. The study proposed in this paper aims at addressing optimal choice and sizing of the battery system for the purpose of reducing fuel consumption. The objective is to present a method to assess if it gives a cost saving to install batteries and, in case it is beneficial, what is the optimal battery type, size and rating to install. There is no doubt that batteries can give significant fuel saving, although not necessarily a cost saving. In fact the installation of batteries does not always automatically reduce the cost enough to pay for the installation. The analyses performed and illustrated in Section 5 clearly show that the potential will be very dependent on load profiles, engine type and size and operational constraints such as spinning reserve requirements in certain operations. Therefore the proposed model aims at providing a methodology to better investigate the actual potential of battery installations within ferries under different scenarios of load profiles, engine types and size, operational constraints and costs.

For the purposes mentioned above, a mathematical optimisation model called OBLIVION has been developed to facilitate rapid vessel design development. OBLIVION stands for “Optimised Battery Lifetime In Vessels Internal Operations and Networks”: beyond including battery degradation and desired battery lifetime in the choice, we analyse how the investment decisions change for different operational modes. Four different modes of operations are defined to cover typical ways of operating the power system. One or several of these may be relevant for a specific vessels. The operation mode on a specific vessel varies over time and is typically based on the criticality of the current vessel operation. Hence the key contribution of this study is proposing a methodology to represent particular modes performed on vessels in a way suitable for inclusion within mathematical optimisation models, and analysing how this affects the storage investment decisions taking into account battery degradation. The objective of the model is to find the optimal design and operations for a vessel system in terms of battery choice, sizing and energy flows management among the different energy units by fulfilling the technical constraints that are peculiar of the vessel system. The proposed mathematical model aims at optimising both investment decisions and operational decisions in a holistic way:

these need to be optimized together since in general, the optimal storage rating will be different for different power and energy management strategies, given a power system topology and a certain load.

In sum, the contribution of this paper lies in the methodology and the value of a novel tool that can give precious insights in the investment decision making process for maritime vessels electrification. In addition, extensive case studies are proposed to show the versatility of the model and the wide range of analyses that can be performed. The case studies show that the investment decisions in batteries are strongly affected by the different modes of operation of the vessels. Technical operational constraints that have to be fulfilled onboard, have a strong impact on the overall investment decisions. Hence the proposed tool represents a first advanced prototype that is able to provide an optimal solution by taking into account many different techno economic aspects that a traditional manual approach would not be able to address.

To our knowledge, this is the first time that the problem of vessels design and operations is addressed this way, by taking into account energy management strategies, specific operational modes performed on the ships and battery lifetime and degradation issues holistically. None of the works available in literature address the effect of power management strategies and operational modes within mathematical optimisation models. Moreover, to our knowledge, none of the available works analyse the effect that such modes can have on the investment choices in batteries and the way through which the degradation and consequent battery lifetime expectations can affect the final results.

## 2. Notes on vessels peculiarities

Maritime vessels come in different sizes and shapes, they may be built for short or long-distance voyages and they can be used for fundamental different purposes with changing criticality. The consequence is that there is a variety of power systems topologies found onboard maritime vessels. Some vessels carry large electric power plants with multiple diesel engine generators supplying onboard activities and multiple propellers and thrusters, while others are equipped with propulsion systems directly driven by diesel engines, that at the same time powers shaft generators to provide the necessary onboard electrical power. The variation in electric power demand is consequently large. A vessel on a long-distance voyage will usually have a much more constant and predictable load demand than a dynamically positions offshore supply vessel trying to stay at a fixed position fighting against varying environmental forces from wind, current and waves. The large variety in vessel design and usage also implies that many vessels are built as one of a kind. They are tailored for their specific intended use according to the owner's request. This is quite different from what is the case in for instance the car industry. In this paper we have selected one specific power system topology to illustrate the methodology and to show how variation in load profile and operation mode affects the optimal choice of energy storage. It is acknowledged that optimization of storage type and size is just an inner loop of the full optimization of the vessel power plant, since the power system topology, including number and size of diesel engines, will also have to be optimized.

### 2.1. The studied system

The system studied is represented in Fig. 1. The same scheme will be used as a reference for the computational experiments presented in Section 5. In particular, there are two buses (named sections in the document) connected by a bus tie breaker. Every section has a load and two generators. The objective is to choose the optimal type and size of batteries to be installed, either in both sections or just in one of the two sections, by fulfilling the system technical requirements and safety conditions.

The bus tie breaker can be open or close depending on the particular operation performed in a certain time interval (hence open or close bus tie breaker is a time varying input parameter). When the bus tie breaker

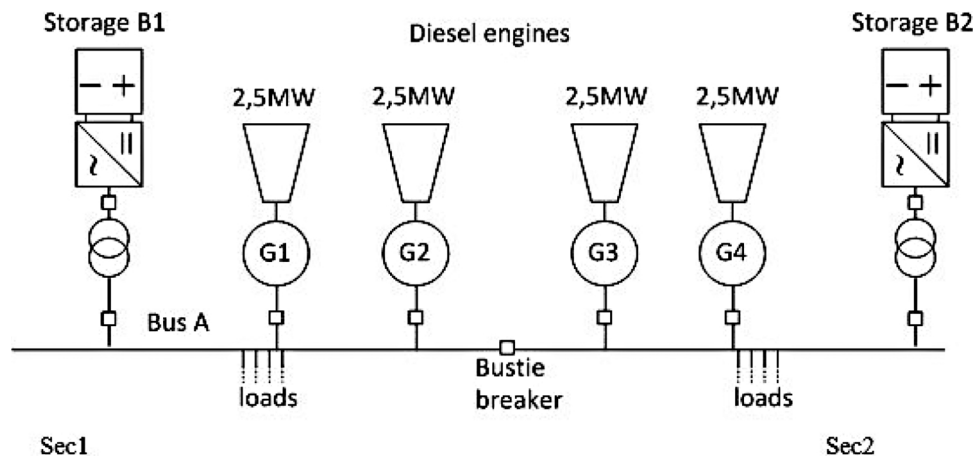


Fig. 1. Scheme of the considered system.

is close, both storage units and generator units can supply energy to both loads in both sections. When the bus tie breaker is open, every section can rely only on those units that are belonging to the section itself: namely, generators 1 and 2 supply load for section 1, generators 3 and 4 supply load for section 2; load can rely of course also on the battery bank that is eventually installed in each section.

It is assumed that the storage is only charged from onboard engines (not offshore).

Moreover, in certain time intervals some operational modes are active and this affect the system requirements in terms of allowable way to use generators and batteries to supply the load. This will be clarified in the model description.

## 2.2. Typical operating modes

Six different modes of operation are defined to cover typical ways of operating the power system. One or several of these may be relevant for a specific vessel. It is acknowledging that other modes of operation can be relevant as well. The mode of operation on a specific vessel will vary over time and will typically be chosen based on the type of activity and criticality of operations.

The modes included in this work are:

- **Mode 00** – This is the simplest mode. In this mode it is accepted that a single failure can lead to a total blackout on the vessel (included loss of all power for steering and propulsion). The only requirement is that the power plant supplies the power required by the loads. Mode 00 will for instance be applicable for an anchored vessel as well as for a vessel in harbor. It may as well be applicable for a vessel in transit in open sea as well as for operations where a blackout has no significant consequences. Operation with battery only is allowed in Mode 00. It is accepted to shut down all diesel engines if the batteries are able to supply the load on their own.
- **Mode 01** – In this mode, it is required to have minimum one diesel generator running and connected. Operation with battery only is not allowed. It is however not required to have a generator connected on each section (each side of the bus-tie). This mode will only be relevant if for some reason one does not fully thrust the batteries as the single source of power or if rules and regulations or operational procedures prescribes minimum one running generator for safety reasons.
- **Mode 02** – This mode is an extension of mode 00 with reduced risk of blackout in periods where vessel is operating on batteries only. As for Mode 00, it is allowed to operate with battery only. However, in Mode 02, it is required that a minimum (configurable) amount of total stored energy (sum of stored energy on both sections) is

available to ensure for instance that at any time there will be enough stored energy to bring the vessel an operation into a safe state in case one get problems with start of one of the diesel engine when batteries approaches a fully discharged state.

At least one diesel engine must be started and connected if the total stored energy is less than what is required as minimum.

- **Mode 03** – In this mode it is required to have at least one diesel engine running and connected to each of the sections. Operation with battery only is not allowed. In addition it is required that there is a (configurable) amount of free power (spinning reserve) on each section that is instantly available for covering of sudden load increase. This correspond to the classical dynamic positioning operation as well as other critical operations where one needs to ensure that sufficient power is available to handle a worst case single failure. In its simplest form the requirement will be that each section has sufficient spinning reserve to take over all the load of the other section in case that section fails totally.
- **Mode 04** – This mode is a variant of Mode 03 for use in critical operations where sufficient power is to be instantly available to handle a worst case single failure. For Mode 04, also the batteries are allowed to serve as a spinning reserve. This reduces the need for additional diesel engines to be connected to maintain the required spinning reserve. The requirement that applies to each section is: a (configurable) amount of free power (spinning reserve) has to be instantly available for covering of sudden load increase. The spinning reserve will be the sum of non-used capacity on running and connected diesel engines and in addition the maximum power that the storage can supply for a minimum configurable time duration.

## 3. Notes on battery peculiarities and lifetime

The main battery properties considered in this study are related to energy capacity, efficiency, state of charge, C-rate, lifetime energy throughput and investment costs. The battery capacity is given in kWh.

The nominal capacity is often measured by Ah (number of Amperes that can be taken from battery multiplied by the duration this current can be supplied). In order to work in kWh, the battery capacity will be calculated as battery voltage multiplied by Ah. It is assumed that the voltage is constant and equal to the nominal voltage, which is the reference voltage provided by manufacturers. This is a common assumption when dealing with mathematical optimisation models that involve batteries, like for instance the ones proposed in [33].

The roundtrip efficiency indicates the percentage of the energy going into the battery that can be drawn back out. We assume that the efficiency in both directions is the same (see [34,35]).

Moreover, a constant battery efficiency is assumed for modelling



purposes. Assuming a constant battery efficiency is a normal practise when building mathematical optimisation models, especially when it comes to Mixed Integer Linear Programming (MILP) models like the one proposed in this paper. Many other studies available in literature adopt the same approximation when dealing with a wide variety of energy related optimisation problems where batteries are involved. Examples of such works can be found in [36] where the value of batteries for peer to peer trading is investigated; [37] where a MILP model is developed for the design of hybrid wind-photovoltaic systems with batteries; [38] where batteries are used in the context of vehicle to grid applications; [39] where a mathematical model is developed for the sizing and analysis of renewable energy and battery systems in residential microgrids; [40] where a MILP approach for multi-microgrid planning with batteries is presented; [33] where a linear programming approach is developed for battery degradation analyses in off-grid power systems; [41] where optimal sizing of energy storage devices is addressed under uncertainty.

The state of charge is the percentage of the available battery capacity, relative to the capacity when it is fully charged. The minimum state of charge defines a limit below which a battery must not be discharged to avoid permanent damage. The so-called C-rate defines the rate at which a battery is being discharged. It is defined as the ratio between the discharge current and the theoretical current drawn under which the battery would deliver its nominal rated capacity in one hour. A 1 C discharge rate means the battery is able to deliver the entire capacity in 1 hour. While a 2 C discharge rate means the battery is able to discharge twice as fast (hence it will deliver the entire capacity in 30 min). In this paper it is assumed that C-rate is the same for charge and discharge.

As illustrated in [42], in order to track the condition of a battery, different state-of-health methodologies can be used. These methods mainly consist of electrochemical models [43], equivalent-circuit models [44] and throughput models [45]. An overview of different approaches for battery lifetime prediction can be found in [46], while a comparison of different lifetime prediction models is available in [47]. Throughput models in particular are widely used and appreciated in literature, see for instance [48–50]. Moreover, they are also particularly suitable to be integrated within mixed integer linear programming models [33]. Therefore in this paper it has been chosen to measure the lifetime of the battery by the so called *lifetime energy throughput*  $B_j^{hr}$  that defines the total amount of energy in kWh that can be discharged before the battery is expected to have degraded to such level that it is no longer suited for the purpose due to lost capacity and increase in losses and internal resistance. The lifetime throughput is derived by the lifetime curve. Such dataset has to be provided by manufacturers and

shows how different depth of discharge are associated with the number of residual cycles to end of life (the deeper the discharge, the lower the remaining cycles to end of life). As illustrated in [33], for every depth of discharge it is possible to calculate a single value of lifetime throughput (multiplying the battery capacity by the depth of discharge and the number of cycles to failure). Then the lifetime throughput of the battery is obtained by averaging all the values of lifetime throughput calculated previously, in the allowable range of depth of discharge. Further information about the lifetime throughput calculation can be found in [51].

Beyond the lifetime throughput, the model proposed in this paper makes also use of another parameter  $B_j^{life}$  that defines the desired lifetime of a battery of type  $j$  and that it is expressed in years. Through this parameter it is possible to set a target minimum battery lifetime, which defines how many years the model-user wants the chosen battery to last. Therefore, the model will make an optimal combination of investment and operational decisions in such a way that the chosen battery will last for a minimum number of years as desired by the investor who is performing analyses with the model. The set of model equations aimed at fulfilling the battery lifetime expectations will be illustrated in Section 4.3.3. This way of modelling allows performing sensitivity analyses through which the model user can investigate how the decisions change when different target battery lifetime are imposed. An example of such sensitivity analyses will be presented in the case studies of Section 5.5 that will show how prolonging the desired battery lifetime is affecting the way through which the battery charge/discharge operations are performed by the model.

Further information about battery properties can be found in [52] and [53].

Moreover a broader introduction about the battery technologies can be found in [54] while a detailed reading on a wide variety of mathematical modelling approaches for batteries can be found in [55].

#### 4. Mathematical model description

This section will discuss the proposed mixed integer linear programming approach for the design and operation of batteries in maritime vessels. Given the variables and parameters listed in Tables 1 and 2, the mathematical model for a maritime vessel design and operation follows. We convert energy flows in kW by dividing the kWh variables flows by the factor  $\Delta$ . Operational costs have to be spread throughout a time horizon  $T$  of one year in order to be consistent with the capital recovery factor definition. This means that we assume that the same operations of one typical year will repeat throughout the desired lifetime.

**Table 1**  
Nomenclature – variables.

$f_{t,i,j,s,p}^{GB}$	Energy flow on time $t$ for profile $p$ from the generator of type $i$ to the battery of type $j$ installed in section $s$ (kWh)
$f_{t,i,l,p}^{GD}$	Energy flow on time $t$ for profile $p$ from the generator of type $i$ to the load $l$ (kWh)
$f_{t,i,p}^G$	Total energy flow on time $t$ for profile $p$ from the generator of type $i$ (kWh)
$f_{t,j,s,l,p}^{BD}$	Energy flow on time $t$ for profile $p$ from the battery of type $j$ installed in section $s$ to the load $l$ (kWh)
$b_{j,s}$	Binary variable equal to 1 if a battery of type $j$ is installed in section $s$
$b_{j,s}^N$	Integer variable indicating the number of batteries of type $j$ installed in section $s$
$g_{t,i,p}$	Binary variable equal to 1 if the generator of type $i$ is running on time $t$ for profile $p$
$g_{t,i,p}^{on}$	Binary variable equal to 1 if the generator of type $i$ is turned on at time $t$ for profile $p$
$g_{t,i,p}^{off}$	Binary variable equal to 1 if the generator of type $i$ is turned off at time $t$ for profile $p$
$b_{t,j,p}^+$	Binary variable equal to 1 if the battery of type $j$ is charging on time $t$ for profile $p$
$k_{t,p}^l$	Binary variable used for critical operations constraints definition for every profile $p$
$b_{t,j,s,p}^{SOC}$	State of charge in every time $t$ for profile $p$ for the battery of type $j$ installed in section $s$ (kWh)
$TC$	Total investment costs (\$/year)
$TO$	Total operational costs (\$/year)
$z_{i,p}^1, z_{i,p}^2, z_{i,p}^3, z_{i,p}^4$	Binary variables used for the definition of constraints for equally loaded generators

**Table 2**  
Nomenclature – sets, indexes and parameters.

Sets	
$\mathcal{T}$	Set of operational time periods
$\mathcal{B}$	Set of batteries
$\mathcal{G}$	Set of generators
$\mathcal{S}$	Set of sections
$\mathcal{L}$	Set of loads
$\mathcal{P}$	Set of profiles
Indexes	
t	Time interval
j	Battery
i	Generator
s	Section
l	Load
p	Profile
Battery parameters	
$B_j^{\text{cost}}$	Investment cost of battery of type $j$ (\$)
$B_j^\eta$	Efficiency of battery of type $j$ (%)
$B_j^{\text{thr}}$	Lifetime throughput of battery of type $j$ (kWh)
$B_j^{\text{life}}$	Desired lifetime of battery of type $j$ (years)
$\bar{B}_j$	Capacity of battery of type $j$ (kWh)
$B_j^{\text{rate}}$	Power rating of battery of type $j$ (kW)
$\underline{B}_j$	Minimum state of charge of battery of type $j$ to avoid permanent damage (%)
$\bar{B}^{\text{qty}}$	Maximum number of batteries to be installed in the battery bank
$\underline{B}^{\text{qty}}$	Minimum number of batteries to be installed in the battery bank
$\bar{B}^{\text{soc}}$	Desired initial state of charge of the battery (%)
$\underline{B}^{\text{soc}}$	Desired final state of charge of the battery (%)
$B_j^{\text{CRF}}$	Capital recovery factor of battery of type $j$
$r$	Interest rate
Generator parameters	
$\bar{G}_i$	Capacity of generator of type $i$ (kW)
$G_i^\eta$	Efficiency of generator of type $i$ (%)
$G_i^K$	Initial value of the consumption curve for the generator of type $i$ (kg)
$G_i^{\text{start}}$	Start up cost of the generator of type $i$ (\$)
$G_i^{\text{fuel}}$	Fuel price (\$/kg)
$G_i^{\text{cons}}$	Marginal fuel consumption (slope of the fuel consumption curve) (kg/kWh)
Load parameters	
$D_{t,l,p}$	Total load in time $t$ for profile $p$ in section $l$ (kW)
Operations parameters	
$O_{t,p}$	Binary parameter equal to 1 if the bus tie breaker is closed on time $t$ for profile $p$
$M_{t,p}^1$	Binary parameter equal to 1 if on time $t$ for profile $p$ mode of type 1 is activated
$M_{t,p}^2$	Binary parameter equal to 1 if on time $t$ for profile $p$ mode of type 2 is activated
$M_{t,p}^3$	Binary parameter equal to 1 if on time $t$ for profile $p$ mode of type 3 is activated
$M_{t,p}^4$	Binary parameter equal to 1 if on time $t$ for profile $p$ mode of type 4 is activated
$G_{t,p}^{\text{free}}$	Desired free power to maintain for safety reasons in every time $t$ , profile $p$ (kW)
$\delta$	Minimum time duration for the storage to be able to supply power when mode of type 4 is activated
$R$	Minimum desired amount of energy stored in the batteries installed in both sections (kWh)
$\Delta$	Length of time interval, share of one hour the time interval refers to (% of 1 h)
$\text{BigM}$	A very big number
$\omega_p$	Weight defining the number of times that a typical profile of type $p$ occurs throughout a year

The year can be split into typical profiles  $p$  represented by a particular type of load. Every typical profile will repeat throughout the year and will therefore be assigned a weight  $\omega_p$ . For instance, if we assume that one typical day will repeat throughout the year, then we can run the model with a time horizon of 24 h and multiply by  $\omega = 365$  that is the number of days in a year. We can of course assume to have various

typical days in a year and therefore multiply by a different weight  $\omega_p$  that defines the number of times a certain typical day occurs in a year. Moreover, different typical periods can be taken into account by properly defining the time horizon (daily, weekly, monthly, etc.) and the related weights.

#### 4.1. Objective function

The objective function minimises the total investment costs  $TC$  and operational costs  $TO$ .

$$\min(TC + TO) \tag{1}$$

The total investment costs are given by Eq. (2). The capital cost of each battery units  $B_j^{\text{cost}}$  is multiplied by the decisional variable  $b_{j,s}^N$  that defines how many units of a battery of type  $j$  are going to be installed. Such costs are then multiplied by the capital recovery factor  $B_j^{\text{CRF}}$  that is defined in Eq. (3) and that takes into account the lifetime of the battery  $B_j^{\text{life}}$  and the interest rate  $r$ . A capital recovery factor is the ratio of a constant annuity to the present value of receiving that annuity for a given length of time. Using an interest rate  $r$ , the capital recovery factor is expressed by Eq. (3) where  $B_j^{\text{life}}$  is the number of annuities received, meaning the forecast lifetime of the battery of type  $j$ . The capital recovery factor converts a present value into a stream of equal annual payments over a specified time, at a specified discount rate (interest). It can be interpreted as the amount of equal (or uniform) payments to be received for  $n$  years such that the total present value of all these equal payments is equivalent to a payment of one dollar at present, if interest rate is  $r$ . The forecast lifetime of the battery is guaranteed by constraint (24) where we limit the battery annual throughput in such a way that the total battery throughput will last for the desired lifetime  $B_j^{\text{life}}$ .

$$TC = \sum_{j,s} B_j^{\text{CRF}} * B_j^{\text{cost}} * b_{j,s}^N \tag{2}$$

$$B_j^{\text{CRF}} = \frac{r * (1 + r)^{B_j^{\text{life}}}}{(1 + r)^{B_j^{\text{life}}} - 1} \tag{3}$$

Operational costs  $TO$  are given by Eq. (4) and they relate to the costs of operating the conventional diesel generators. The start up cost occurs only when the generator is turned on and it is obtained multiplying the startup cost  $G_i^{\text{start}}$  by the binary variable  $g_{t,i,p}^{\text{on}}$  that indicates if in time  $t$  the generator if type  $i$  installed in section  $s$  is turned on. The cost of operating the generator is illustrated in Fig. 2, through the blue plot. In particular, as soon as the generator production has a positive value, a fixed cost is incurred. This cost function is not linear and is not continuous. There is a jump at  $x$ -axis = 0, as illustrated in the diagram of Fig. 2 through the blue plot. Eq. (4) describes this function as a line equation where  $G_i^{\text{cons}}$  represents the slope of the fuel consumption curve in kg/kWh, multiplied by the generator production  $f_{t,i,p}^G$  in kWh. Finally

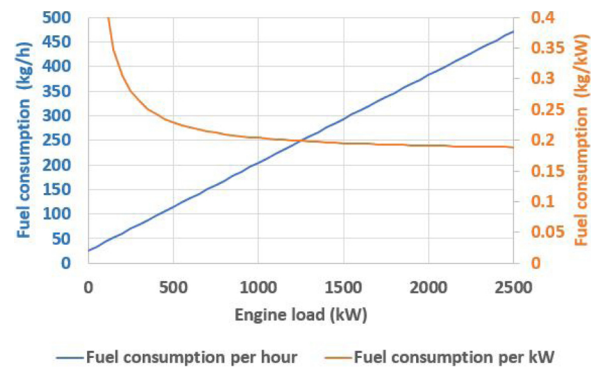


Fig. 2. Example of the discontinuous diesel cost function (kg fuel per hour) in blue. Shown in the same plot is the equivalent specific fuel consumption (kg/kWh) in red. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

$G_i^K$  in kg represents the y-intercept, that determines the point at which the line crosses the y-axis. This term occurs only when the generator is working, and therefore it is multiplied by the related binary variable  $g_{t,i,p}$ . Moreover,  $G_i^K$  has to be multiplied also by the factor  $\Delta$  in order to ensure a proper dimensional equation. Both terms are multiplied by the fuel price  $G^{fuel}$  in \$/kg in order to obtain the total cost in \$.

Note that Eq. (4) works in combination with the constraint (12) on the maximum capacity for the diesel generator in which the variable  $g_{t,i,p}$  is linked to the total flow  $f_{t,i,p}^G$ . Such constraint will be further illustrated later in the model.

$$TO = \sum_p \omega_p \left( \sum_{t,i} G_i^{start} * g_{t,i,p}^{on} + \sum_{t,i} (G_i^{cons} * f_{t,i,p}^G + G_i^K * g_{t,i,p} * \Delta) * G^{fuel} \right) \quad (4)$$

Eq. (5) explicitly defines the generator production  $f_{t,i,p}^G$  in kWh, as the summation of the energy flows from the generators to the batteries  $f_{t,i,j,s,p}^{GB}$  and the energy flows from the generators to the load  $f_{t,i,l,p}^{GD}$

$$f_{t,i,p}^G = \sum_{j,s} f_{t,i,j,s,p}^{GB} + \sum_l f_{t,i,l,p}^{GD} \quad \forall t, i, p \quad (5)$$

#### 4.2. Investment constraints

The proposed model is capable to perform both the selection of a certain type of battery among different choices, and the sizing of the battery bank. The selection of a battery of a certain type  $j$  is made by using the binary variable  $b_{j,s}$  that is equal to 1 if a battery of type  $j$  is installed, 0 otherwise. The type of battery is described by certain battery properties that appears in the parameters. The sizing of the battery bank is addressed by using the integer variable  $b_{j,s}^N$  that indicates the optimal decisions in terms of number of batteries of type  $j$  to be installed in section  $s$ . Given a topology of the vessel system, the model makes a cost optimal choice of the type and number of storage units. This is in line with the real world situation in which the storage units come with a given discretised size. In fact storage units are available in the market in different given sizes with certain given properties. Therefore, given a database containing different battery units available in different sizes and with different properties, the proposed model is able to choose the best type and how many units of that particular type should be installed to optimise the overall investment and operational costs.

Only one type of battery can be chosen for each section (6) meaning that it is not possible to combine different types of batteries in the same battery bank.

$$\sum_j b_{j,s} \leq 1 \quad \forall s \quad (6)$$

If a battery of type  $j$  is chosen, then at least one or more units of this type has to be installed (7)

$$b_{j,s}^N \geq b_{j,s} \quad \forall j, s \quad (7)$$

If a battery of type  $j$  is not chosen, then no units of such battery can be installed (8). For this purpose, the binary variable  $b_{j,s}$  is multiplied by a very big number  $BigM$ . If a battery of type  $j$  is chosen, then constraint (8) will be always verified. If a battery of type  $j$  is not chosen, then the left hand side of constraint (8) will be zero and the integer variable  $b_{j,s}^N$  will be forced to be zero as well.

$$b_{j,s} * BigM \geq b_{j,s}^N \quad \forall j, s \quad (8)$$

The number of battery units installed should be within a maximum  $\bar{B}^{qty}$  desired amount and a minimum desired amount  $\underline{B}^{qty}$  ((9) and (10))

$$\sum_j b_{j,s}^N \geq \underline{B}^{qty} \quad \forall s \quad (9)$$

$$\sum_j b_{j,s}^N \leq \bar{B}^{qty} \quad \forall s \quad (10)$$

In fact, space constraints and/or weight constraints might limit the maximum units to be installed. There may also be special reasons for imposing a lower bound on the minimum storage installation, for instance, a certain number of batteries are required anyway in harbour due to pollution or noise issues etc.

Note that the  $BigM$  used in Eq. (8) can be replaced by a smaller number such as the  $\bar{B}^{qty}$  parameter in order to have a better formulation that might improve the computational solution time.

#### 4.3. Operational constraints

##### 4.3.1. Meet the load

$$\sum_{j,s} f_{t,j,s,l,p}^{BD} * B_j^\eta + \sum_i f_{t,i,l,p}^{GD} * G_i^\eta = D_{t,l,p} * \Delta \quad \forall t, l, p \quad (11)$$

Each load  $D_{t,l,p}$  can be satisfied by the energy flows out of the battery  $f_{t,j,s,l,p}^{BD}$  and/or the flows out of the generators  $f_{t,i,l,p}^{GD}$ . Later constraints will define which generators and batteries can be connected to each load according to bus tie breaker conditions and particular safety requirements. If the load is given in kW, then it has to be multiplied by the factor  $\Delta$  in order to convert it in kWh and make it consistent with the energy flows on the left side of the equation.

Both flows out of the battery and flows out of the generators are multiplied by the related percentage efficiency, namely  $B_j^\eta$  for the battery and  $G_i^\eta$  for the generators. As outlined through the references illustrated in Section 3, a constant battery efficiency is assumed for modelling purposes and such assumption is a normal practise when building mixed integer linear optimisation models like the one proposed in this paper.

As for the generators efficiency, it is important to highlight that the constant percentage efficiency value  $G_i^\eta$  is for the generator only, not for the complete diesel engine generator set. As described in Section (1) and illustrated in Fig. 2, the model does not consider a constant efficiency for the diesel engine itself. In fact, the starting point is the specific fuel consumption curve for the diesel engine. The red plot in Fig. 2 represents the fuel consumption in kg/kWh at different kW engine loading at the fixed speed. This clearly shows that the engine is less efficient at low loads (which means non constant efficiency for the diesel engine generator set). The specific fuel consumption curve can be then used to directly create a curve showing the fuel consumption per hour (kg/h) at different kW engine loading. This curve will be much more linear than the specific fuel consumption. The linear approximation used in this paper is shown in Fig. 2 (blue plot). This is not equivalent to a constant efficiency since the curve does not cross the origin. It is however assumed a constant percentage efficiency  $G_i^\eta$  for the generator only. The load dependency of the generator efficiency has however a very minor effect on the results compared to the load dependency of the diesel engine efficiency. In addition to the observations above, it is also important to highlight that, assuming a constant generator efficiency, is a normal practise when building mathematical optimisation models, especially when it comes to mixed integer linear programming models like the one proposed in this paper. There are other studies available in literature that adopt the same approximation when dealing with a wide variety of energy related optimisation problems where conventional diesel generators are involved, for instance [13,56–59].

##### 4.3.2. Generators operations

Constraint (12) defines the generator capacity. The total energy flows from the generator to the battery  $f_{t,i,j,s,p}^{GB}$  plus the total flows from the generator to the demand  $f_{t,i,l,p}^{GD}$  should be less than or equal to the generator capacity  $\bar{G}_i$  multiplied by the binary variable  $g_{t,i,p}$  that is equal to 1 if the generator is running, 0 otherwise. This way, when the generator is on, the right hand side of constraint (12) is equal to the generator capacity, while when the generator is off, the right hand side of

constraint (12) is equal to zero and flows are forced to be zero as well.

$$\sum_{j,s} f_{t,i,j,s,p}^{GB} + \sum_l f_{t,i,l,p}^{GD} \leq \bar{G}_i^* \Delta^* g_{t,i,p} \quad \forall t, i, p \quad (12)$$

Constraint (13) is used to link binary variables in such a way that time intervals in which the generator is turned on are properly identified and start up costs can be assigned in the objective function. In particular,  $g_{t,i,p}$  is a binary variable equal to 1 if the generator is running at a certain time  $t$ , 0 otherwise. While  $g_{t,i,p}^{\text{on}}$  and  $g_{t,i,p}^{\text{off}}$  are binary variables equal to 1 if a generator is turned on or off in a certain time step respectively. If a generator is running on time  $t$  ( $g_{t,i,p} = 1$ ) and it was running also on time  $t - 1$  ( $g_{t-1,i,p} = 1$ ), then it means that no operations of starting or stopping have been performed on time  $t$  ( $g_{t,i,p}^{\text{on}} = 0$  and  $g_{t,i,p}^{\text{off}} = 0$ ). If a generator is running on time  $t$  ( $g_{t,i,p} = 1$ ) but was not running on time  $t - 1$  ( $g_{t-1,i,p} = 0$ ), then it means that a starting operation has been performed on time  $t$  ( $g_{t,i,p}^{\text{on}} = 1$  and  $g_{t,i,p}^{\text{off}} = 0$ ). If a generator is not running on time  $t$  ( $g_{t,i,p} = 0$ ) but was running on time  $t - 1$  ( $g_{t-1,i,p} = 1$ ) then it means that a stopping operation has been performed on time  $t$  ( $g_{t,i,p}^{\text{on}} = 0$  and  $g_{t,i,p}^{\text{off}} = 1$ ). Constraint (13) fulfills the above statements and controls the generators operations properly.

$$g_{t,i,p} - g_{t-1,i,p} = g_{t,i,p}^{\text{on}} - g_{t,i,p}^{\text{off}} \quad \forall t, i, p \quad (13)$$

Constraints (14), guarantees that the operation of starting and stopping can't happen simultaneously. If the generator is turned on, the binary variable  $g_{t,i,p}^{\text{on}}$  is equal to 1, but then the binary variable  $g_{t,i,p}^{\text{off}}$  is forced to be equal to zero due to the multiplication by a very big number  $\text{BigM}$ .

$$g_{t,i,p}^{\text{on}} \leq (1 - g_{t,i,p}^{\text{off}}) * \text{BigM} \quad \forall t, i, p \quad (14)$$

#### 4.3.3. Battery operations

The selection and sizing of the battery has to be done by considering also the forecast battery operations within the considered vessel system. Therefore operational variables are introduced in the model to control the battery operations given a forecast load.

Battery capacity and battery minimum state of charge to avoid permanent damage are defined in constraints (15) and (16).

The battery state of charge in each time step  $b_{t,j,s,p}^{\text{SOC}}$  is imposed to be less than the battery unitary capacity  $\bar{B}_j$  multiplied by the number of battery units to be installed  $b_{j,s}^N$ . The state of charge is also imposed to be greater than a minimum value  $\underline{B}_j$  multiplied by the units to be installed.

$$b_{t,j,s,p}^{\text{SOC}} \leq \bar{B}_j * b_{j,s}^N \quad \forall t, j, s, p: t > t_{\text{first}} \quad (15)$$

$$b_{t,j,s,p}^{\text{SOC}} \geq \underline{B}_j * b_{j,s}^N \quad \forall t, j, s, p \quad (16)$$

Desired initial state of charge  $\bar{B}^{\text{SOC}}$  and final state of charge  $\underline{B}^{\text{SOC}}$  are defined in (17) and (18). In this paper, for testing purposes, the desired initial state of charge will be assumed equal to the desired final state of charge.

$$b_{t,j,s,p}^{\text{SOC}} = \bar{B}^{\text{SOC}} * \bar{B}_j * b_{j,s}^N \quad \forall t, j, s, p: t = t_{\text{first}} \quad (17)$$

$$b_{t,j,s,p}^{\text{SOC}} = \underline{B}^{\text{SOC}} * \bar{B}_j * b_{j,s}^N \quad \forall t, j, s, p: t = t_{\text{last}} \quad (18)$$

Constraint (19) defines the state of charge of the battery in every time interval. The state of charge in each time step  $t$  is equal to the state of charge in the previous time step  $b_{t-1,j,s,p}^{\text{SOC}}$ , minus the energy flows out the battery  $f_{t,j,s,l,p}^{\text{BD}}$  plus the energy flows into the battery  $f_{t,i,j,s,p}^{\text{GB}}$ . The battery efficiency  $B_j^\eta$  and the generator efficiency  $G_i^\eta$  are taken into account as well and multiplied by the energy flows.

$$b_{t,j,s,p}^{\text{SOC}} = b_{t-1,j,s,p}^{\text{SOC}} - \sum_l f_{t,j,s,l,p}^{\text{BD}} * \frac{1}{B_j^\eta} + \sum_i f_{t,i,j,s,p}^{\text{GB}} * G_i^\eta \quad \forall t, j, s, p: t > t_{\text{first}} \quad (19)$$

Constraints (20) and (21) define upper bounds on the energy that can be charged and discharged according to the rating  $B_j^{\text{rate}}$  of the

particular battery installed.

$$\sum_l f_{t,j,s,l,p}^{\text{BD}} * \frac{1}{B_j^\eta} \leq B_j^{\text{rate}} * \Delta^* b_{j,s}^N \quad \forall t, j, s, p \quad (20)$$

$$\sum_i f_{t,i,j,s,p}^{\text{GB}} * G_i^\eta \leq B_j^{\text{rate}} * \Delta^* b_{j,s}^N \quad \forall t, j, s, p \quad (21)$$

Constraints (22) and (23) are inserted to impose mutually exclusive flows in and out the battery by using the binary variable  $b_{t,j,p}^+$  that is equal to 1 if the battery is charging on time  $t$  and 0 otherwise. If the battery is charging on time  $t$ , it can't be discharging on the same time interval.

$$\sum_i f_{t,i,j,s,p}^{\text{GB}} \leq \text{BigM} * b_{t,j,p}^+ \quad \forall t, j, s, p \quad (22)$$

$$\sum_l f_{t,j,s,l,p}^{\text{BD}} \leq \text{BigM} * (1 - b_{t,j,p}^+) \quad \forall t, j, s, p \quad (23)$$

The last constraint (24) aims at fulfilling the battery lifetime expectations by limiting the total battery throughput along a whole year.

$$\sum_p \omega_p * \sum_{t,l} f_{t,j,s,l,p}^{\text{BD}} \leq \frac{B_j^{\text{thr}}}{B_j^{\text{life}}} * b_{j,s}^N \quad \forall j, s \quad (24)$$

As outlined in Section 3, the desired lifetime of a battery  $B_j^{\text{life}}$  is an input parameter that is expressed in years. Through this parameter it is possible to set a target battery lifetime, which defines how many years the model-user wants a battery of type  $j$  to last. Constraint (24), is aimed at fulfilling the battery lifetime expectations. The left hand side of the constraint calculates the total amount of energy that is being drawn from the battery throughout a whole typical year; the right hand side of the constraint imposes an upper bound on the total amount of energy that can be drawn from the battery. By dividing the battery throughput  $B_j^{\text{thr}}$  expressed in kWh by the desired battery lifetime  $B_j^{\text{life}}$  expressed in years, it is possible to define the battery throughput per year that can be exploited for each battery unit. This value is then multiplied by the decision variable  $b_{j,s}^N$  that defines the number of battery units of type  $j$  installed in section  $s$ . Therefore the available throughput for a year increases linearly with the number of batteries installed. This way it is possible to define the total annual throughput that is available in each year. The model will therefore optimise the battery choice, size and operations in a way suitable to fulfill the desired lifetime imposed by the decision makers. Of course constraint (24) will work in combination with the other set of constraints (15)–(23) that are aimed at managing the battery investment and operations decisions. As all the constraints have to be fulfilled holistically, the investment and operations decisions made by the model will be optimal, given the battery lifetime expectations imposed. This will guarantee an optimal combination of investment and operational decisions in such a way that the chosen batteries will last for a minimum number of years as desired by the model-user who is performing analyses with the model.

Note that the BigM used in Eqs. (22) and (23) can be replaced by a smaller number such as the  $B_j^{\text{rate}}$  parameter in order to have a better formulation that might improve the computational solution time.

#### 4.3.4. Bus tie breaker

When the bus tie breaker is open, the binary variable  $O_{t,p}$  is equal to 0 and therefore certain energy flows (namely, flows from units installed in section 1 to units and load in section 2 and vice versa) are not allowed. If the bus tie breaker is open, flows from the batteries in section 1 to the load in section 2 are not allowed (25).

$$O_{t,p} * \text{BigM} \geq f_{t,j,s,l,p}^{\text{BD}} \quad \forall t, j, s, l, p: s = 1, l = 2 \quad (25)$$

If the bus tie breaker is open, flows from the battery in section 2 to the load in section 1 (26) are not allowed, as well as those from the generators in section 1 to the load in section 2 (27).



$$O_{t,p} * \text{BigM} \geq f_{t,j,s,l,p}^{\text{BD}} \quad \forall t, j, s, l, p: s = 2, l = 1 \quad (26)$$

$$O_{t,p} * \text{BigM} \geq f_{t,i,l,p}^{\text{GD}} \quad \forall t, i, l, p: i = (1, 2), l = 2 \quad (27)$$

If the bus tie breaker is open, flows from the generators in section 2 to the load in section 1 are not allowed (28) as well as those from the generators in section 1 to the batteries installed in section 2 (29).

$$O_{t,p} * \text{BigM} \geq f_{t,i,l,p}^{\text{GD}} \quad \forall t, i, l, p: i = (3, 4), l = 1 \quad (28)$$

$$O_{t,p} * \text{BigM} \geq f_{t,i,j,s,p}^{\text{GB}} \quad \forall t, i, j, s, p: i = (1, 2), s = 2 \quad (29)$$

If the bus tie breaker is open, flows from the generators in section 2 to the batteries installed in section 1 are not allowed (30).

$$O_{t,p} * \text{BigM} \geq f_{t,i,j,s,p}^{\text{GB}} \quad \forall t, i, j, s, p: i = (3, 4), s = 1 \quad (30)$$

When looking at the constraints above, remember that according to Fig. 1, generators 1 and 2 are installed in section 1, while generators 3 and 4 are installed in section 2.

When the bus tie breaker is closed, the binary variable  $O_{t,p}$  is equal to 1 and all the above constraints are verified, meaning that all energy flows among all units are allowed because the two sections of the system are connected through the bus tie breaker.

Note that the *BigM* used in the previous equations can be replaced by a smaller number such as the  $B_j^{\text{rate}}$  parameter for those equations that involve batteries and the  $\bar{G}_i$  parameter for those equations that involve generators. This may lead to a better formulation and improved computational solution time.

#### 4.3.5. Operational modes constraints

The six different modes of operation introduced in Section 2.2 are modeled as follows.

Mode 00 is just the basic vessels management without any particular requirements, therefore it is performed by the whole model as is without any additional constraint.

Mode 01 is implemented through constraint (31). When the binary parameter  $M_{t,p}^1$  will be 1, then at least one variable  $g_{t,i,p}$  will be equal to 1, hence at least one generator will be connected.

$$\sum_i g_{t,i,p} \geq M_{t,p}^1 \quad \forall t, p \quad (31)$$

Mode 02 is implemented through constraints (32) and (33). If the total amount of energy stored in the two sections  $\sum_{j,s} b_{t,j,s,p}^{\text{SOC}}$  will be less than the minimum required amount  $R$ , and the binary parameter  $M_{t,p}^2$  is activated, then the binary variable  $k1^t$  will be forced to be 1 in order to satisfy the constraint (32). This will activate the following constraint (33) and force at least one generator to be connected. On the other hand, if the total amount of energy stored in the two sections  $\sum_{j,s} b_{t,j,s,p}^{\text{SOC}}$  will be greater than the minimum required  $R$ , then the binary variable  $k1^t$  will be free and constraints (32) and (33) will always be verified.

$$\sum_{j,s} b_{t,j,s,p}^{\text{SOC}} > = R * M_{t,p}^2 * (1 - k1^t) \quad \forall t, p \quad (32)$$

$$\sum_i g_{t,i,p} > = M_{t,p}^2 * k1^t \quad \forall t, p \quad (33)$$

Mode 03 is implemented through constraints (34–37). Constraints (34, 35) impose that, if  $M_{t,p}^3$  is equal to 1 (meaning mode 03 is active), then at least one generator has to be activated in each section.

$$g_{t,i=1,p} + g_{t,i=2,p} > = M_{t,p}^3 \quad \forall t, p \quad (34)$$

$$g_{t,i=3,p} + g_{t,i=4,p} > = M_{t,p}^3 \quad \forall t, p \quad (35)$$

In addition, constraints (36), and (37) impose a free power  $G_{t,p}^{\text{free}}$  that has to be available in each section if the binary parameter  $M_{t,p}^3$  is equal to 1. Therefore, the generator capacity  $G_i * g_{t,i,p}$  has to satisfy the load  $D_{t,l,p}$  by keeping some free power available as required by the mode

properties.

$$\sum_i \bar{G}_i * g_{t,i,p} * \Delta \geq (D_{t,l,p} + G_{t,p}^{\text{free}}) * \Delta * M_{t,p}^3 \quad \forall t, p, (i = 1, 2), (s = 1) \quad (36)$$

$$\sum_i \bar{G}_i * g_{t,i,p} * \Delta \geq (D_{t,l,p} + G_{t,p}^{\text{free}}) * \Delta * M_{t,p}^3 \quad \forall t, p, (i = 3, 4), (s = 2) \quad (37)$$

Mode 04 is implemented through constraints (38) and (39) for section 1, and through constraints (40 and 41) for section 2. Compared to constraints of the previous mode, there are additional terms related to the power that the storage can supply for a minimum time duration  $\delta$  by considering that this has to be limited not only by the storage state of charge  $b_{t,j,s,p}^{\text{SOC}}$  (constraints (38) and (40)) but also by the storage rating  $B_j^{\text{rate}}$  (constraints (39) and (41)).

$$\begin{aligned} \sum_i \bar{G}_i * g_{t,i,p} * \Delta + \sum_j \frac{b_{t,j,s,p}^{\text{SOC}}}{\delta} * \Delta &\geq (D_{t,l,p} + G_{t,p}^{\text{free}}) * \Delta * M_{t,p}^4 \quad \forall t, p, (i \\ &= 1, 2), (s = 1) \end{aligned} \quad (38)$$

$$\begin{aligned} \sum_i \bar{G}_i * g_{t,i,p} * \Delta + \sum_j b_{j,s}^N * B_j^{\text{rate}} * \Delta &\geq (D_{t,l,p} + G_{t,p}^{\text{free}}) * \Delta * M_{t,p}^4 \quad \forall t, p, (i \\ &= 1, 2), (s = 1) \end{aligned} \quad (39)$$

$$\begin{aligned} \sum_i \bar{G}_i * g_{t,i,p} * \Delta + \sum_j \frac{b_{t,j,s,p}^{\text{SOC}}}{\delta} * \Delta &\geq (D_{t,l,p} + G_{t,p}^{\text{free}}) * \Delta * M_{t,p}^4 \quad \forall t, p, (i \\ &= 3, 4), (s = 2) \end{aligned} \quad (40)$$

$$\begin{aligned} \sum_i \bar{G}_i * g_{t,i,p} * \Delta + \sum_j b_{j,s}^N * B_j^{\text{rate}} * \Delta &\geq (D_{t,l,p} + G_{t,p}^{\text{free}}) * \Delta * M_{t,p}^4 \quad \forall t, p, (i \\ &= 3, 4), (s = 2) \end{aligned} \quad (41)$$

#### 4.4. Equally loaded generators

Generators can be let free to run at whatever load is required by the optimisation, or they can be forced to be equally loaded when they are connected in parallel. From a real world perspective there are reasons why it is advisable to make sure that generators connected in parallel always share power load equally (in percentage), hence additional constraints that fulfill such requirement can be added. We will need new binary variables for all the possible combinations of connected generators. In particular, when the bus tie breaker is open ( $O_{t,p} = 0$ ) that means that only the couple of generators 1, 2 belonging to section 1 and the couple of generators 3,4 belonging to section 2 will have to be equally loaded if connected in parallel. On the other hand, when the bus tie breaker is closed ( $O_{t,p} = 1$ ) a wider variety of connected generators can happen and all the different combinations have to be considered when building such constraints. Hence binary variables  $z_{t,p}$  that are equal to 1 only when a certain combination of connected generator happens, have to be added to the model and constraints have to be inserted as follows. Note constraints (45, 49), (54, 60) are “indicator constraints”, used to turn on or turn off the enforcement of the previous related group of constraints.

*Open bus tie breaker. Generators 1 and 2*

$$z_{t,p}^1 \leq g_{t,p,i=1} \quad (42)$$

$$z_{t,p}^1 \leq g_{t,p,i=2} \quad (43)$$

$$z_{t,p}^1 \geq g_{t,p,i=1} + g_{t,p,i=2} - 1 \quad (44)$$

$$\frac{f_{t,p,i=1}^G}{\bar{G}_{i=1}} = \frac{f_{t,p,i=2}^G}{\bar{G}_{i=2}} \quad \forall z_{t,p}^1 = 1 \quad (45)$$

where  $z_{t,p}^1$  is equal to 1 if both generator 1 and 2 are connected, 0 otherwise

A similar constraint has to be created for the couple of generators 3

and 4 positioned in section 2.

*Closed bus tie breaker. Generators 1 and 3*

$$z_{t,p}^2 * O_{t,p} \leq g_{t,p,i=1} * O_{t,p} \quad (46)$$

$$z_{t,p}^2 * O_{t,p} \leq g_{t,p,i=3} * O_{t,p} \quad (47)$$

$$z_{t,p}^2 * O_{t,p} \geq (g_{t,p,i=1} + g_{t,p,i=3} - 1) * O_{t,p} \quad (48)$$

$$\frac{f_{t,p,i=1}^G * O_{t,p}}{\bar{G}_{i=1}} = \frac{f_{t,p,i=3}^G * O_{t,p}}{\bar{G}_{i=3}} \quad \forall z_{t,p}^2 = 1 \quad (49)$$

where  $z_{t,p}^2$  is equal to 1 if both generator 1 and 3 are connected, 0 otherwise

A similar constraint has to be created for all the possible couple of generators namely, 3-4, 1-3, 1-4, 2-3, 2-4.

*Closed bus tie breaker. Generators 1, 2 and 3*

$$z_{t,p}^3 * O_{t,p} \leq g_{t,p,i=1} * O_{t,p} \quad (50)$$

$$z_{t,p}^3 * O_{t,p} \leq g_{t,p,i=2} * O_{t,p} \quad (51)$$

$$z_{t,p}^3 * O_{t,p} \leq g_{t,p,i=3} * O_{t,p} \quad (52)$$

$$z_{t,p}^3 * O_{t,p} \geq (g_{t,p,i=1} + g_{t,p,i=2} + g_{t,p,i=3} - 2) * O_{t,p} \quad (53)$$

$$\frac{f_{t,p,i=1}^G * O_{t,p}}{\bar{G}_{i=1}} = \frac{f_{t,p,i=2}^G * O_{t,p}}{\bar{G}_{i=2}} = \frac{f_{t,p,i=3}^G * O_{t,p}}{\bar{G}_{i=3}} \quad \forall z_{t,p}^3 = 1 \quad (54)$$

where  $z_{t,p}^3$  is equal to 1 if generator 1, 2 and 3 are connected, 0 otherwise

A similar constraint has to be created for all the possible combinations of three generators available, namely 1-2-4, 2-3-4, 1-3-4.

*Closed bus tie breaker. Generators 1, 2, 3 and 4*

$$z_{t,p}^4 * O_{t,p} \leq g_{t,p,i=1} * O_{t,p} \quad (55)$$

$$z_{t,p}^4 * O_{t,p} \leq g_{t,p,i=2} * O_{t,p} \quad (56)$$

$$z_{t,p}^4 * O_{t,p} \leq g_{t,p,i=3} * O_{t,p} \quad (57)$$

$$z_{t,p}^4 * O_{t,p} \leq g_{t,p,i=4} * O_{t,p} \quad (58)$$

$$z_{t,p}^4 * O_{t,p} \geq (g_{t,p,i=1} + g_{t,p,i=2} + g_{t,p,i=3} + g_{t,p,i=4} - 3) * O_{t,p} \quad (59)$$

$$\frac{f_{t,p,i=1}^G * O_{t,p}}{\bar{G}_{i=1}} = \frac{f_{t,p,i=2}^G * O_{t,p}}{\bar{G}_{i=2}} = \frac{f_{t,p,i=3}^G * O_{t,p}}{\bar{G}_{i=3}} = \frac{f_{t,p,i=4}^G * O_{t,p}}{\bar{G}_{i=4}} \quad \forall z_{t,p}^4 = 1 \quad (60)$$

where  $z_{t,p}^4$  is equal to 1 if generator 1, 2, 3 and 4 are connected, 0 otherwise

### 5. Computational experiments

The model has been developed and tested using Python programming language and the Python-based open-source software package PYOMO.

Computational experiments have been performed considering the system described in Section 2.1 and represented in Fig. 1. The main objective is to investigate the sensitivity of the model results to different parameters. In particular, the main focus is understanding under which conditions battery installation become valuable for such systems, given the costs and degradation issues included in the decision process.

For that purpose we run the model by assuming different typical daily load profiles  $p$  that repeat throughout a year. Figs. 3–5 show the load variations for quay load, low load and high load profiles respectively. The time interval has been set as 30 minutes long, even though lower time resolution would be possible to use as well by just changing the model input settings. Table 3 shows the average and cumulative

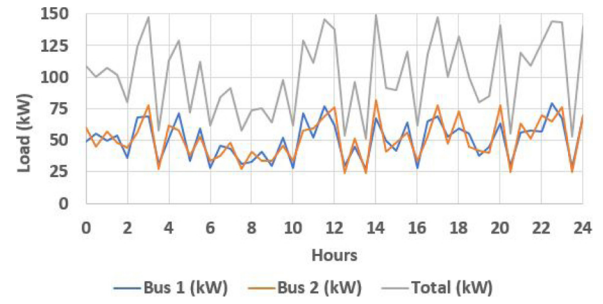


Fig. 3. Trend for a quay load profile on a vessel over a typical day of 24 h used in the computational experiments.

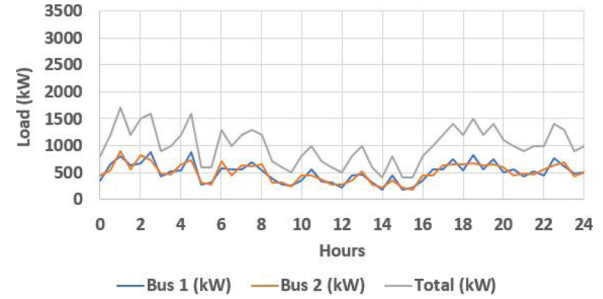


Fig. 4. Trend for a low load profile on a vessel over a typical day of 24 h used in the computational experiments.

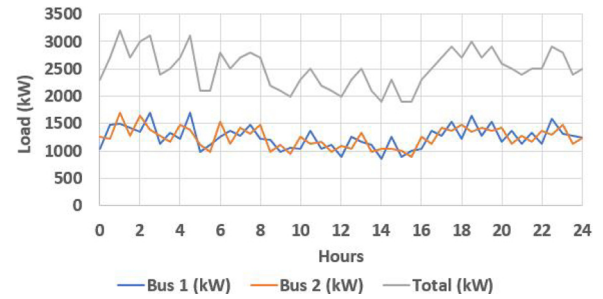


Fig. 5. Trend for a high load profile on a vessel over a typical day of 24 h used in the computational experiments.

Table 3

Average values and cumulative values for the different typical load profiles.

Profile	Bus (n)	Average (kW)
Quay	1	50
	2	51
Low	1	981
	2	976
High	1	2438
	2	2433

values of load for the two sections and the different typical profiles.

For such systems it can be assumed a certain number of days per year for quay load profile, low load profile and high load profile. This is expressed by the weight factor  $\omega_p$  in the mathematical model. However, for testing purposes and illustration purposes, we will also show case studies assuming only one profile per year to better identify the effect of the type of load on the model decisions.

Data for battery can be found in Table 4 where  $\bar{B}_j$  is the capacity of battery of type  $j$  (kWh);  $B_j^{\text{rate}}$  is the power rating of battery of type  $j$  (kW);  $B_j^{\eta}$  is the efficiency of battery of type  $j$  (%);  $B_j^{\text{thr}}$  is the lifetime throughput of battery of type  $j$  (kWh);  $B_j^{\text{cost}}$  is the investment cost of battery of type  $j$  (\$);  $B_j^{\text{lifc}}$  is the desired lifetime of battery of type  $j$  (years).

**Table 4**  
Battery data.

Type	$B_j$ (kWh)	$B_j^{rate}$ (kWh)	$B_j^{\eta}$ (%)	$B_j^{thr}$ (kW)	$B_j^{post}$ (\$)	$B_j^{life}$ (years)
A	100	100	95	800000	50000	10
B	100	200	95	800000	75000	10

**Table 5**  
Data for the diesel engines and generators.

Type	$G_i^{\eta}$ (%)	$\bar{G}_i$ (kW)	$G_i^{start}$ (\$)	$G_i^K$ (kg)	$G_i^{cons}$ (kg/kWh)	$G^{fuel}$ (\$/kg)
Wartsila 8L26	95	2500	0.6	25.35	0.17845	0.35

While data for the diesel engines and the generators can be found in Table 5 where  $\bar{G}_i$  is the capacity of generator of type  $i$  (kW);  $G_i^{\eta}$  is the efficiency of generator of type  $i$  (%);  $G_i^K$  is the initial value of the consumption curve for the diesel engine of type  $i$  (kg);  $G_i^{start}$  is the start up cost of the diesel engine of type  $i$  (\$);  $G^{fuel}$  is the fuel price (\$/kg);  $G_i^{cons}$  is the marginal fuel consumption that is represented by the slope of the fuel consumption curve (kg/kWh). The discontinuous diesel engine cost function (kg fuel per hour) is shown in Fig. 2 in blue. Shown in the same plot is the equivalent specific fuel consumption (kg/kWh) in red.

Following sections will discuss the more relevant results and summarise them in tables. Every table is made of a left side that contains the dataset used and a right side that shows the results in terms of battery installation and costs. The data shown in the left side of the tables are related to type of load, number of days covered by that type of load, type of mode that is activated, bustie condition, number of generators available, free power requirements in each section and minimum amount of total stored energy requirements. By default we considered 4 generators available, 2 for each bus of the system. Note that the free power requirements are expressed in percentage (namely, percentage of the load of the other section). The results shown in the right side of the tables are related to battery installed in each section, operational cost of generators and total costs given by the summation of operational costs and investment costs in batteries. The last column of each table with header “Cost Saving With Bat” will show the difference between the total costs in a scenario in which no batteries are available for installation, and the total cost in an optimised scenario in which batteries are installed: this shows the savings that can be obtained through battery installation, compared to scenarios in which no batteries are installed and only generators have to fulfill the load.

The battery installed will be given by a number that indicates the number of units and a letter that indicates the type of battery chosen, A or B, according to the battery properties listed in Table 4.

A yellow colour in the tables will highlight those dataset that is

varying to perform sensitivity analyses. While an orange colour in the tables will highlight those columns that differentiate two or more tables presented in the same section. This will help the reader in quickly identifying the most important data useful for the understanding of the computational experiments.

In the proposed case studies, the choice of battery types is limited to two (see Table 4), hence the model can choose if it is better to install a battery with a low power rating and a lower price (namely type A in the table) or a battery with a higher power rating and a higher price (namely type B in the table). Of course the model can handle a wider dataset and more types of batteries can be available to choose from, just like it happens when choosing batteries from a company catalogue. Within the proposed case study, the authors decided to limit the choice to two batteries, because this was allowing a smoother interpretation and explanation of the results. It is important to note that the case studies are proposed mainly to illustrate the model potential. In particular the case study is aimed at validating the model, illustrating its capabilities, showing the type of analyses that can be performed and showing that the model makes logical choices based on the technical requirements.

5.1. Effect of type of load

Some preliminary tests have been made to investigate how the different load profiles affect the model decisions in terms of battery installation. For this purpose, preliminary tests have been made assuming that for every run, one of the three load profiles will cover the whole year. Mode 00 has been chosen in order to focus the test on the load effect without having any mode affecting the decisions. Table 6 shows a summary of relevant results. It is possible to note that battery installation happens mainly when it is necessary to fulfill a quay load profile, while for higher load profiles the model will always prefer to run just the generators. This is happening because the investment in battery is proportional to the battery size. The used battery costs are too high to justify investments in big capacity. Therefore, scenarios that require small battery installation (like quay load profiles) are worthier than those that require big battery installation (like low and high load profiles). With a so small load like the quay load, running a generator all the time is not worthy. In this case it is possible to save costs from the generators through a small battery installation that is economically beneficial. With highest load profiles then the investment in battery capacity has to be bigger, which is making the use of the generators more beneficial. It is worthy to note that, for the quay load profile, when batteries are not installed the total costs are 46% higher compared to a situation in which an optimal size of battery is available.

5.2. Effect of Mode

From the previous tests it was clear that battery installation was mainly suggested with quay load profiles, hence quay load have been

**Table 6**  
Effect of the type of load.

Load	Days	Mode	Bustie	Gen	Bat Installed		Oper Cost	Tot Cost	Cost Saving with Bat
					sec1	sec2			
	(n)			(n)	(n)	(n)	(\$/year)	(\$/year)	(\$/year)
quay	365	00	open	4	5A	5A	80625	145376	67062
low	365	00	open	4	0	0	731808	731808	na
high	365	00	open	4	0	0	1595693	1595693	na

**Table 7**  
Effect of mode. Tests run with a quay load profile and open bustie.

Load	Days (n)	Mode	Bustie	Gen (n)	Free Pow (%)	Res Ener (kWh)	Bat Installed		Oper Cost (\$/year)	Tot Cost (\$/year)	Cost Saving with Bat (\$/year)
							sec1 (n)	sec2 (n)			
quay	365	00	open	4	na	na	5A	5A	80625	145376	67062
quay	365	01	open	4	na	na	3A	2A	146022	178397	34041
quay	365	02	open	4	na	100	5A	5A	80625	145376	67072
quay	365	02	open	4	na	500	5A	5A	81056	145808	66630
quay	365	03	open	4	100	na	0	0	212438	212438	na
quay	365	04	open	4	100	na	5A	5A	80625	145376	67062

used to investigate how different mode activated in the vessels affect the decision making process. Table 7 shows a summary of relevant results. It is useful to note that in real world scenarios mode 03 and mode 04 are usually not active for a quay load profile, but test cases will be shown anyway in the table for illustration and validation purposes.

The first line of Table 7 shows the basic results in terms of battery installation when no mode is active. The following lines show how this investment decision is likely to change when different modes are activated on the vessel.

Mode 01 requires to have minimum one diesel generator running and connected, and will therefore penalise battery installation. This is visible on the second line where a lower battery capacity is installed compared to mode 00. The reader might note that in this case more battery capacity is installed in section 1 compared to section 2. This is due to the fact that loads on the two sections are slightly different. If we calculate the cumulative total load of section 1 and we compare it with the cumulative total load of section 2, we notice that the total cumulative load is lower in section 1. We already observed and explained in the first test that the tendency of the model is to prioritise battery installation for smaller load profiles: therefore it is not surprising that the model prioritises battery installation in section 1 where the total cumulative load is smaller. While for a bigger load like the one in section 2, the model will penalise the battery installation and will give way to the use of generators.

Moving towards the third line of Table 7, we notice that mode 02 will prioritise battery installation due to the requirement of a minimum amount of total stored energy, hence the battery capacity installed with mode 02 is bigger than the one installed with mode 01. Moreover, in mode 02 it is also allowed to shut down generators, while this is not allowed in mode 01. Allowing shutting down all generators increases the value of the storage.

A straightforward question arises regarding how the amount of required storage can affect the model decisions. The fourth line of Table 7 is aimed at giving a first answer: the model was tested with different amount of total stored energy required in mode 02, but it was found that no more than 5 batteries were installed. For higher amount of total stored energy, it was more economical to keep the generator running instead of installing more battery capacity: this can be observed also looking at the total operational costs that are higher in line 4 where

more power as reserve is required compared to line 3 (bear in mind that there are costs associated to keep the generators running and connected even without actually serving the load). This is due to two main reasons: the first one is related to the fact that battery costs are proportional to the size, and we already showed previously that above a certain size, the battery installation becomes too expensive compared to the use of generators. The second reason is that in this mode it is possible to operate with generator only, every time that the total stored energy is less than what is required as minimum. Hence above a certain total battery capacity, it is more economical to connect a generator rather than add a further storage unit.

The fifth line of Table 7 shows results when mode 03 is active. As in this mode at least one diesel engine has to be connected in each section, then battery installation is highly penalized. In fact, no batteries are installed in this scenario. In this mode free power is required only from the generators, hence batteries would have been installed only if the generators total capacity was limited (i.e. lower than the peak load for most of the time) and if such capacity had to be used to fulfill the free power requirements. Anyway, the kind of vessels we are looking at here (dynamic positioning) will typically never be loaded to full generator capacity. The reason is simply that they need to operate in such way that if they lose half the power plant, they still are supposed to be able to produce the same amount of propulsion force as before the failure. The consequence is that there will never be a load that is close to maximum installed generator capacity.

Finally, the last line of Table 7 shows results when mode 04 is active. Compared to mode 03, the mode 04 requires an amount of free power that can come also from the batteries. Therefore battery installation is not penalised as in the previous mode.

Let us now spend few words on costs comparisons when batteries are available and when they are not. The highest costs saving is obtained when modes 00, 02 and 04 are activated: for such modes there is 46% increment of costs when batteries are not installed. A lower saving is gained when mode 01 is active, with a 19% increment of costs when batteries are not installed.

We now move to Table 8 to show how the results change when the bus tie breaker is closed.

Note that when the bus tie breaker is open, generators and batteries installed in each sections are allowed to serve only the load of their own

**Table 8**  
Effect of mode. Tests run with a quay load profile and closed bustie.

Load	Days (n)	Mode	Bustie	Gen (n)	Free Pow (%)	Res Ener (kWh)	Bat Installed		Oper Cost (\$/year)	Tot Cost (\$/year)	Cost Saving with Bat (\$/year)
							sec1 (n)	sec2 (n)			
quay	365	00	closed	4	na	na	4A		103380	129281	5642
quay	365	01	closed	4	na	na	0		134923	134923	na
quay	365	02	closed	4	na	100	4A		103599	129500	5423
quay	365	02	closed	4	na	500	8A		82703	134504	na
quay	365	03	closed	4	100	na	0		212438	212438	na
quay	365	04	closed	4	100	na	4A		103380	129281	5642



**Table 9**  
Effect of battery costs. Tests run with a low load profile and open bustie.

Load	Days (n)	Mode	Bustie	Gen (n)	Bat Cost (%)	Bat Installed		Oper Cost (\$/year)	Tot Cost (\$/year)	Cost Saving with Bat (\$/year)
						sec1 (n)	sec2 (n)			
low	365	00	open	4	100	0	0	731808	731808	na
low	365	00	open	4	30	0	4A	723686	731456	352
low	365	00	open	4	28	4A	5A	714216	730534	874

section. While a closed bustie will allow all generators and batteries to serve both loads in each section. This means that closed bustie will penalise battery installation: in fact with a closed bustie, the same generator can be used to serve both loads, meaning that lower generator operational costs will be incurred. Hence the generator use with a closed bus tie breaker will be cheaper due to lower start up operations. The same situations can happen with batteries, considering that lower capacity can be installed as the same battery unit can be used to serve both loads. This explains why in Table 8 the battery installation is lower compared to Table 7, but operational costs of generators are higher (simply because more load is served directly with generators).

It is also important to note that when the bustie is closed 100% of the time, then there is no longer an actual difference in installing battery units in a section or in another: this is because with a closed bustie then it is like having one whole section where generators and batteries are allowed to serve any load. Hence it is the total battery installation that counts in this case. This explains why in Table 8 there is only one column with total battery installation instead of two columns showing the number of batteries installed in each section. Of course this is valid only in representative tests like the one proposed here. In many real life scenarios, there will be a variation of open and closed bustie throughout the year, and therefore identify the amount of batteries installed in each section will always be necessary.

In general, the higher the number of days with a closed bustie is, the more penalised the battery installation will be.

It is therefore not surprising to note that with a closed bustie also the costs saving obtained through battery installation will be much lower. In particular, we can observe that there is just a 4% cost reduction due to battery installation.

5.3. Effect of battery costs

We now investigate how the battery price should drop to make the investment worthy in those scenarios in which the current battery costs

make them economically not feasible. For this purpose, we run sensitivity analyses with different battery prices and 100% time covered by a low load profile. This is because we noticed in previous tests that batteries were not installed at all when low load profile and high load profile occurred most of the time. Hence a low load is suitable to investigate how much should the price drop to see a first battery installation. Table 9 summarises the relevant results. Different tests are run by considering a different battery cost as a percentage of the current cost. In particular, it was noted that the battery cost had to drop at least 30% of the current cost to make the investment feasible. This is visible in the second line of the table where batteries are installed only in section 1. In order to see further installation in both sections, the cost had to drop at least 28% of the current cost.

Table 10 shows how results radically change if the bustie is closed. We already showed in previous tests how bustie can penalise battery installation: hence it is hardly surprising to note that even when the battery cost drops to 28% of the current cost, there is no battery installation for a low load profile with a closed bustie for mode 00.

5.4. Effect of fuel price and environmental charges

We now investigate how the fuel price increment can affect the worthiness of batteries in those scenarios in which the current battery costs make them economically not feasible. For this purpose, we run sensitivity analyses with different fuel prices and 100% time covered by a low load profile with mode 00 active. We choose a low load profile for the same reasons mentioned in the previous tests of Section 5.3. Table 11 summarises the relevant results. In particular, we noted that a fuel price increment towards 1.1 \$/kg is necessary to make battery worthy. This higher fuel price can be motivated by considering that for instance in Norway there are charges related to the environmental impact of the diesel generators. The charge might be added on top of the fuel cost to build up an environmental cost function. However, tests showed that higher fuel price had an impact on the battery worthiness

**Table 10**  
Effect of battery costs. Tests run with a low load profile and closed bustie.

Load	Days (n)	Mode	Bustie	Gen (n)	Bat Cost (%)	Bat Installed		Oper Cost (\$/year)	Tot Cost (\$/year)	Cost Saving with Bat (\$/year)
						sec1	sec2			
low	365	00	closed	4	100	0		653866	653866	na
low	365	00	closed	4	30	0		653866	653866	na
low	365	00	closed	4	28	0		653866	653866	na

**Table 11**  
Effect of fuel price. Tests run with a low load profile and open bustie.

Load	Days (n)	Mode	Bustie	Gen (n)	Fuel Price (\$/kg)	Bat Installed		Oper Cost (\$/year)	Tot Cost (\$/year)	Cost Saving with Bat (\$/year)
						sec1 (n)	sec2 (n)			
low	365	00	open	4	0,35	0	0	731808	731808	na
low	365	00	open	4	0,5	0	0	1045252	1045252	na
low	365	00	open	4	1,5	4A	8A	3032338	3110040	24841

**Table 12**  
Effect of fuel price. Tests run with a low load profile and closed bustie.

Load	Days (n)	Mode	Bustie	Gen (n)	Fuel Price (\$/kg)	Bat Installed	Oper Cost (\$/year)	Tot Cost (\$/year)	Cost Saving with Bat (\$/year)
low	365	00	closed	4	0,35	0	653866	653866	na
low	365	00	closed	4	0.5	0	934000	934000	na
low	365	00	closed	4	2,7	7A	4996565	5041892	746

**Table 13**  
Effect of desired battery lifetime. Tests run with a quay load profile and open bustie.

Load	Days (n)	Mode	Bustie	Gen (n)	Bat Desired Life (years)	Bat Installed		Oper Cost (\$/year)	Tot Cost (\$/year)	Cost Saving with Bat (\$/year)
						sec1 (n)	sec2 (n)			
quay	365	00	open	4	10	5A	5A	80625	145376	67062
quay	365	00	open	4	5	2A	2A	101993	148188	64250
quay	365	00	open	4	1	0	0	212438	212438	na

only for low load profiles. While for high load profiles, battery was never installed regardless of the fuel price increment.

Table 12 shows how results change with a closed bustie. As we already discussed in previous sections, closed bustie will penalise battery installation, hence it is not surprising to note that the fuel price should increase more to make the battery worthy to be installed.

5.5. Trade off between throughput and capital recovery factor

We now investigate how the desired battery lifetime parameter  $B_j^{life}$  affects the model investment decisions.

As outlined in Sections 3 and 4.3.3, the desired lifetime of a battery is an input parameter, expressed in years, which defines how many years the model-user wants each type of battery to last. The modelling approach proposed in Section 4.3.3 allows performing sensitivity analyses through which the model user can investigate how the decisions change when different target battery lifetime are imposed. An example of such sensitivity analyses is presented Table 13 that summarises the relevant results. Increasing the desired battery lifetime will have three main effects.

The first effect is a lower throughput available to exploit in every year. In fact, the longer the desired lifetime of a battery is, the lower the amount of total throughput that can be used in every year will be. On the contrary, the shorter the desired battery lifetime is, the higher the amount of throughput that can be used within every year will be.

Increasing the battery lifetime, will also affect the investment worthiness through the capital recovery factor. In fact, assuming that a battery will last longer, will make the actualised investment worthier, while assuming that a battery will last shorter, will penalise the worthiness of the investment. Hence, a shorter desired battery lifetime will bring a higher throughput to exploit (which may motivate the battery installation because more energy can be available), but for a shorter amount of time (which may penalise installation because the actualised investment costs will be too high).

Increasing the desired battery lifetime will also have a third effect on the battery operational curve. In particular, the battery curve will flatten more and more as the desired battery lifetime increases because less cycles will be possible due to a lower amount of available yearly throughput.

This will carry on until a point in which batteries will no longer be cycled because a too low throughput would be available to be exploited in every year.

Figs. 6–8 show how prolonging the desired battery lifetime is affecting the way through which the battery charge/discharge operations are performed by the model. For each figure, a certain value for the desired battery lifetime has been fixed when running the test. Namely, the target battery lifetime is 5 years in Fig. 6, 10 years in Fig. 7, 15 years in Fig. 8. Then the battery energy content throughout the time horizon has been plotted. By comparing the three figures, it is possible to observe that the number of battery charge/discharge cycles will decrease

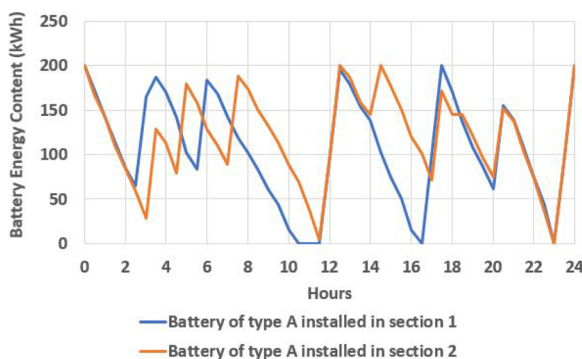


Fig. 6. Example of operational battery curve obtained when imposing a desired battery lifetime of 5 years.

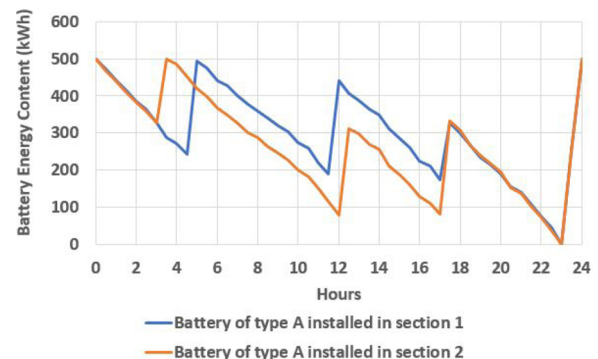


Fig. 7. Example of operational battery curve obtained when imposing a desired battery lifetime of 10 years.

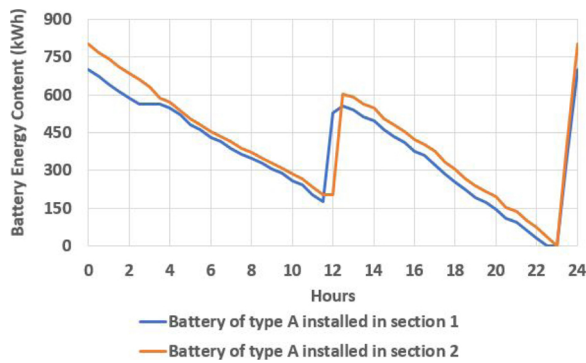


Fig. 8. Example of operational battery curve obtained when imposing a desired battery lifetime of 15 years.

as the desired battery lifetime increases because less cycles will be possible due to a lower amount of available yearly throughput.

As already discussed in previous sections, a closed bus tie breaker penalises the battery installation. Table 14 shows how the results change with a closed bustie for completeness. It is possible to note that the total battery installation is now lower.

5.6. Effect of combined types of load

After the premises of previous tests, we are now ready to go through more comprehensive tests with combined types of load and different modes. The year is now split into different typical profiles with different type of loads and different modes active. We are interested in investigating how the weight of different profiles affect the model decisions and what is the effect of different modes active in different profiles.

Table 15 summarises the relevant results when a quay load profile with mode 00 activated is combined with a low load profile and a high load profile that have mode 03 activated. It is possible to note that the number of days with quay load affects investment decisions in batteries. In particular, it was noted that 60 was the minimum number of quay

days required to make the battery installation worthy. But even with a very high number of quay days, then no more than one battery unit was installed. This is because we already discussed in previous sections that battery installation mainly happens to fulfill just a quay load profile: a case study where the quay load profile is combined with low load profiles and high load profiles, will penalise battery installation because generators are always preferred to fulfill high and low load profiles.

Table 16 shows how the results change when mode 04 is activated in low load profile and high load profile.

Compared to Table 15 where mode 03 was activated, two main observations can be discussed: the first thing to observe is that the number of quay days required to make the battery installation worthy is lower when mode 04 is active. In fact, in Table 16 battery installation happens with just 30 days of quay load compared to Table 15 where 60 days were necessary. The second observation is that the operational and total costs in Table 16 where mode 04 is active, are lower than those in Table 15 where mode 03 is active. This is due to the fact that mode 03 requires at least one generator to be always connected for safety reasons, while there is not such a requirement in mode 04. Therefore mode 03 will penalise battery installation (in fact battery installation happens later in Table 15 compared to Table 16) and will increase the operational costs due to the cost of the fuel needed to have one or more generators running than it is actually needed to produce the load power. It is acknowledged that many other combinations of open/closed bus tie breaker with different load profiles are possible, but for the purpose of this paper one is shown for illustration purposes and to give the reader an overview of the importance of analysing the impact of combined types of load on the final decisions.

5.7. Combined effect of battery price and profiles weight

In this section we evaluate the combined effect of number of quay days and battery price variations. Table 17 shows the relevant results. In particular, we show how much should the battery price drop to make the battery installation worthy, in different scenarios when different amount of quay days occur throughout the year. As we already discussed that higher number of quay days will motivate battery installation, it is not surprising to note that as the number of quay days

Table 14 Effect of desired battery lifetime. Tests run with a quay load profile and closed bustie.

Load	Days (n)	Mode	Bustie	Gen (n)	Bat Desired Life (years)	Bat Installed	Oper Cost (\$/year)	Tot Cost (\$/year)	Cost Saving with Bat (\$/year)
quay	365	00	closed	4	10	4A	103380	129281	5642
quay	365	00	closed	4	5	2A	104033	127131	7792
quay	365	00	closed	4	1	0	134923	134923	na

Table 15 Effect of combined types of load. Increasing the number of quay days. Mode 03 active on low and high load profiles.

Load	Days (n)	Mode	Bustie	Gen (n)	Free Pow (%)	Bat Installed		Oper Cost (\$/year)	Tot Cost (\$/year)	Cost Saving with Bat (\$/year)
						sec1 (n)	sec2 (n)			
quay	30	00	open	4	na	0	0	819390	819390	na
low	280	03	closed		100					
high	55	03	closed		100					
quay	60	00	open	4	na	1A	1A	761569	774520	2279
low	250	03	closed		100					
high	55	03	closed		100					
quay	110	00	open	4	na	1A	1A	678277	691157	14656
low	200	03	closed		100					
high	55	03	closed		100					

**Table 16**  
Effect of combined types of load. Increasing the number of quay days. Mode 04 active on low and high load profiles.

Load	Days (n)	Mode	Bustie	Gen (n)	Free Pow (%)	Bat Installed		Oper Cost (\$/year)	Tot Cost (\$/year)	Cost Saving with Bat (\$/year)
						sec1 (n)	sec2 (n)			
quay	30	00	open	4	na	2A	2A	760970	786871	42675
low	280	04	closed		100					
high	55	04	closed		100					
quay	60	00	open	4	na	2A	2A	719838	745740	41215
low	250	04	closed		100					
high	55	04	closed		100					
quay	110	00	open	4	na	3A	2A	636206	668582	47387
low	200	04	closed		100					
high	55	04	closed		100					

**Table 17**  
Combined effect of battery price and profiles weight. Increasing the number of quay days and observe how this affect the battery price variations to make battery installation worthy.

Load	Days (n)	Mode	Bustie	Gen (n)	Free Pow (%)	Bat Price (%)	Bat Installed		Oper Cost (\$/year)	Tot Cost (\$/year)	Cost Saving with Bat (\$/year)
							sec1 (n)	sec2 (n)			
quay	20	00	open	4	na	30	1A	1A	838740	842625	1118
low	290	03	closed		100						
high	55	03	closed		100						
quay	30	00	open	4	na	50	1A	1A	821912	828388	1158
low	280	03	closed		100						
high	55	03	closed		100						
quay	40	00	open	4	na	70	1A	1A	805475	814540	809
low	270	03	closed		100						
high	55	03	closed		100						

increase, then the battery price will have to drop less to make the investment worthy.

**6. Conclusions and future developments**

A mathematical model for the optimal investment decisions and operations of battery units within maritime vessels has been presented. The paper shows a methodology to include relevant safety and operational constraints and storage degrading effects in the investment decisions. This encompasses the inclusion of combinations of vessel operational modes, bus-tie breaker operation philosophy and spinning reserve requirements.

An implementation of the proposed model has been used to run storage investment analysis for an example vessel with 4 diesel engines. The case studies proposed in the paper aimed at showing the wide variety of sensitivity analyses that can be performed with the proposed model and therefore the strong potential of such a tool for those in charge of investment decision making. It was shown how the investment analysis is influenced by e.g. different safety constraints and different combinations of operational modes.

The case studies showed that many combinations of dataset can interact with each other and affect the final decisions. The decision making process is therefore very delicate and challenging and requires adequate tools that are able to tackle the different needs and constraints in a holistic way that would not be possible with other manual approaches. Hence a holistic approach given by optimisation techniques is key to find a suitable solution that is able to fulfill all the operational requirements and take into account all the critical properties of the system. The model decisions are seen to be very sensitive to the vessels modes of operation, therefore studying ways to mathematically

describe such operations is a key contribution for the scientific community aiming at analysing such systems. It is important to highlight the choice of including a wide variety of case studies to show the type of analyses that can be performed with the proposed model in a way that can be as neutral as possible. The case studies are neither aimed at demonstrating that batteries are strongly needed in marine vessels, nor aimed at demonstrating that batteries do not have any value in marine vessels. They are rather aimed at showing that the combination of different conditions (load, operational modes, bus tie, etc.) will affect the value of storage in different ways. Therefore a proper optimisation tool like the one proposed in the paper, can give precious indications in the decision making process when it comes to understand the value of storage for marine vessels that are supposed to operate under certain scenarios.

Further development towards stochastic techniques that can tackle the uncertainty in the load profile are going to be developed and will be proposed in a following paper where the value of a stochastic solution will be analysed.

The proposed methodology for energy storage sizing can either be utilized as stand-alone or it can also be incorporated in vessel design software tools for optimization of larger parts of the vessel design.

**Acknowledgments**

The paper presents result from a Maritime Research program primarily funded by the Research Council of Norway (RCN), project number 254766 and Singapore Maritime Institute (SMI) project number SMI-2015-MA-15.



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