A harmonic pressure differential wave energy converter

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Abstract

A pressure differential sea wave energy converter concept using the forced harmonic motion of a hydraulic water column and hydraulic piston is presented. The mechanical power from the hydraulic piston is converted to electrical power by a linear induction generator. The equation of motion, and electrical power generated are derived in terms of design dimensions and mechanical properties of the wave energy converter. The frequency response of the system is modelled numerically for a variety of wave frequencies, and its time response is modelled using a simple Euler method numerical model. It was found that the wave energy converter yields a maximum conversion efficiency of 27 % and a prompt transient response to actuation with waves around its natural frequency. High survivability and reduced visual and water-surface impact are likely advantages of this design concept, since all mechanical components may be incorporated into the sea-floor and shoreline.

Keywords: Ocean wave energy converter, Harmonic oscillator, Differential pressure

1. Introduction

Wave energy is a promising source of renewable energy which is available near the surface of seas and oceans. It is estimated to have a global potential around 3.7 TW [1], and has been the subject of research for several decades [2, 3]. Its wider deployment to sea has so far been elusive though, since it is hindered by the high cost of installation and maintenance in the harsh marine environment [4].

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The conversion of sea waves through submerged pressure differential wave energy converters provides a concept that could combine high conversion efficiency with lower maintenance costs, due to its inherently higher survivability [5, 6]. For such devices, the response of the sea floor is crucial in achieving high energy conversion efficiency [7, 8]. A concept providing a high level of control and adaptability of the response is a submerged differential pressure wave energy converter using a hydraulic water column for transmission of the pressure differential onto a hydraulic piston that is attenuated by a linear induction generator. This a concept is particularly suited to areas in which impact of the energy conversion on marine life and landscape is to be avoided since they can be integrated into the existing shape of the sea shore. These devices would be placed in arrays to obtain a scalable wave energy extraction area. Figure 1 shows a section schematic of this concept.

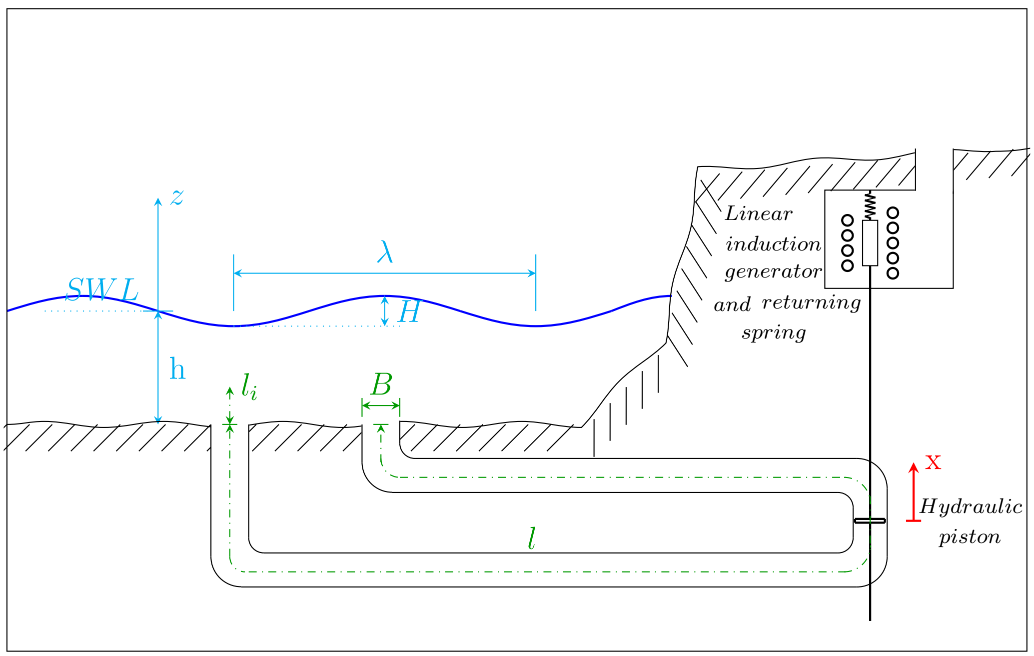

Figure 1: Schematic of the harmonic pressure wave energy converter.

The aim of this work is to describe the fundamental mechanical behaviour of this concept, and to provide basic models for its frequency response, time response, and conversion efficiency.

2. Theory

The harmonic pressure differential wave energy converter is used to mechanically convert the dynamic pressure of a sea wave into useful electrical power. The pressure difference between to locations on the sea floor is used to drive the harmonic oscillation of a water column and piston. The water column is divided into two parts by a hydraulic piston, as shown in figure 1. The hydraulic piston is actuated by the differential pressure in the two parts of the water column, and drives a linear induction generator located in an onshore nacelle. The piston may be located below the still water line (SWL) to provide protection against cavitation, while the linear induction generator may be located in a dry nacelle above the SWL.
Figure 1 shows the width B of the pressure collector inlet, and the length l of the water column consisting of both its parts. The pressure collector inlet is assumed to have a square cross-section of width B, which is transformed into a tube of circular cross-section and diameter $D = \sqrt{\frac{4}{\pi}}$ at the location of the hydraulic piston. In the vicinity of the inlets of the pressure collector, some water is entrained into the oscillating motion of the water column. A length l_i is used to represent the water close the inlets affected by the motion of the wave energy converter.

2.1. Wave energy converter motion

The displacement x, velocity $\dot{x}$, and acceleration $\ddot{x}$ of the hydraulic piston and magnet of the linear induction generator are a function of the total mass, the damping force, the restoring force and the actuation force [9]. The equation of motion of this linear system has been presented in terms of its individual components in equation 1. The water column has a cross-sectional area A, length $l' = l + 2l_i$. This comprises the length of the underground tube l and the two inlet lengths l_i. The inlet lengths reflect the mass of water accelerated in the vicinity of the pressure collector inlets. The water density is assumed to be constant and designated as ρ. The hydraulic piston and magnet of the linear induction generator have a combined mass of m_{pist}, a linear damping coefficient due to friction f and a linear damping coefficient due to electrical induction c. The piston is held around its neutral position by a spring with stiffness k_{pist}. The amplitude of the dynamic wave force is defined as F_0, and the gravitational acceleration as g.

Equation of motion:

$$\ddot{x} + \frac{(c+f)}{A\rho l' + m_{pist}}\dot{x} + 2\frac{\rho g A + k_{pist}}{A\rho l' + m_{pist}}x = \frac{F_0}{A\rho l' + m_{pist}}sin(\omega t) \tag{1}$$

The steady state solution takes the form shown in 2 [9].

$$x = X_0 \cdot sin(\omega t + \phi) \tag{2}$$

Differentiating equation 2 with respect to time yields the velocity $\dot{x}$ as equation 3, and its second time derivative yields the acceleration as equation 4 .

$$\dot{x} = X_0 \cdot \omega \cdot \cos(\omega t + \phi) \tag{3}$$

$$\ddot{x} = -X_0 \cdot \omega^2 \cdot \sin(\omega t + \phi) \tag{4}$$

Solving the differential equation yields the undamped natural frequency ω_n of the wave energy converter, which is described by equation 5.

$$\omega_n = \sqrt{\frac{2\rho g A + k_{pist}}{A\rho l' + m_{pist}}} \tag{5}$$

When the system is damped by the linear damping coefficient due to friction f and linear damping coefficient due to electrical induction c, the maximum amplitude of displacement becomes a function of the actuation frequency ω and the natural frequency ω_n. The maximum amplitude of motion of the pressure wave energy converter under these damped conditions may be derived by substituting equations 2, 3 and 4 into equation 1 [9].

$$X_0 = \frac{\frac{F_0}{A\cdot\rho\cdot l' + m_{pist}}}{\sqrt{(\omega_n^2 - \omega^2)^2 + (\frac{(c+f)}{A\cdot\rho\cdot l' + m_{pist}} \cdot \omega)^2}} \tag{6}$$

Similarly, the phase angle between the actuating force, and the piston response is derived for damped conditions as equation 7 [9].

$$\phi = \arctan\left(\frac{\frac{(c+f)}{A\rho\cdot l' + m_{pist}} \cdot \omega}{\omega_n^2 - \omega^2}\right) \tag{7}$$

The instantaneous power absorbed by the wave energy converter is the product of the instantaneous opposing force from the linear induction generator and the velocity of the piston. The instantaneous power is thus described by equation 8.

$$P(t) = \frac{dE}{dt} = F_{L.I.} \cdot \frac{dx}{dt} = (c \cdot \frac{dx}{dt}) \cdot \frac{dx}{dt} = c \cdot \dot{x}^2(t) \tag{8}$$

Substituting equation 3 into equation 8 and integrating yields the energy absorbed by the linear induction generator during one wave cycle, represented by equation 9.

$$\Delta E = c \cdot X_0^2 \cdot \omega^2 \int_0^{\frac{2\pi}{\omega}} \cos(\omega t + \phi)dt = c \cdot X_0^2 \cdot \omega \cdot \pi \tag{9}$$

The time-averaged power of the wave energy converter is found by multiplying equation 9 by the actuation frequency.

$$\bar{P} = \frac{1}{2} \cdot c \cdot \omega^2 \cdot X_0^2 \tag{10}$$

2.2. Linear inductance power generator

The energy of the system is converted into electric power by means of a linear inductance generator, whose electric inductance provides a force opposing the direction of the velocity of the hydraulic piston. The power generated by within the linear inductance generator has been originally described by Omholt [10, 2]. The amplitude of the opposing force depends on the number of turns of the coil N_e, the magnetic field strength B_e, the wire length of the coil l_e, the total resistance of the electric load and the coil R_e, as well as the velocity of the hydraulic piston $\dot{x}$.

$$P_e = \frac{N_e^2 \cdot B_e^2 \cdot l_e^2 \cdot \dot{x}^2}{R_e} \tag{11}$$

For a stationary coil, the relative velocity between the magnet of the linear inductance generator, and the coil is equal to $\dot{x}$. The force generated by the induction coil that is resisting the motion of the hydraulic piston, is described using the linear damping coefficient due to electrical induction c as shown in equation 12.

$$c = \frac{N_e^2 \cdot B_e^2 \cdot l_e^2}{R_e} \tag{12}$$

The electrical induction force constant c as described above was used to evaluate equations 1, and 6-10.

2.3. Sea wave properties

The properties of the sea wave supplying energy to the wave energy converter can take various forms depending on the types and vicinity of the winds that generate them, the water depths and detailed topography of the area. If the wave is assumed to have the low height to length ratio typical of *the swell*, it may be approximated by a linear wave. At intermediate water depths for which $\frac{\lambda}{2} < h < \frac{\lambda}{20}$, the pressure on the seabed below the wave may be approximated by equation 13, as described by McCormick [2] or Sorensen [11].

$$p \simeq -\frac{\rho \cdot g \cdot H}{2} \cdot \frac{\cosh k(h+z)}{\cosh(kh)} \cdot \cos(kx - \omega t) - \rho \cdot g \cdot z \tag{13}$$

The first term of equation 13 describes the dynamic pressure component and the second term the static pressure. The maximum force acting between two infinitesimally small inlets of the wave pressure collector is equal to twice the dynamic component of the wave pressure times the pressure collector area, as described by equation 14. This occurs when the two inlets are situated at a distance $\frac{\lambda}{2}$ from one another in the direction of wave propagation, i.e. when one inlet is exposed to a peak of the wave and the other to a trough. Equation 14 applies to situations where $\lambda >> B$.

$$F \simeq -A \cdot \rho \cdot g \cdot H \cdot \frac{\cosh k(h+z)}{\cosh(kh)} \tag{14}$$

For values of B that are not insignificant with respect to the wavelength The variation in pressure due to the wave height variation over the inlets of the wave pressure collectors should be taken into account, since it reduces the effective pressure difference available to the wave energy converter. Integrating the dynamic component of equation 13 over a square inlet area of width B at time $t = 0$ yields half of the net force acting between the inlets, and should be multiplied by two. The resulting total force is shown in equation 15.

$$F_0 = \int \text{p } dA = B \int_{\frac{-B}{2}}^{\frac{B}{2}} \text{p } dx = \int_{\frac{-B}{2}}^{\frac{B}{2}} -\rho \cdot g \cdot H \cdot B \cdot \frac{\cosh k(h+z)}{\cosh(kh)} \cdot \cos(kx) dx \ F_0$$
$$= -\frac{\rho \cdot g \cdot H \cdot B \cdot \lambda}{\pi} \cdot \frac{\cosh k(h+z)}{\cosh(kh)} \cdot \sin(\frac{\pi \cdot B}{\lambda}) \quad (15)$$

The force between the two pressure collector inlets F_0 is equal to the force driving the hydraulic piston of the wave energy converter and was used to evaluate equations 1, 6, 9, and 10.

The energy available in one wave cycle per unit crest width b has been described by McCormick [2] or Sorensen [11] and is given by equation 16.

$$E_w \simeq \frac{\rho \cdot g \cdot H^2 \cdot \lambda \cdot b}{8} \quad (16)$$

where b is the width of the crest, which was assumed to be equal to the width of the wave collector inlet B.

2.4. Interaction between sea wave and energy converter and efficiency

The sea wave provides the driving force for the motion of the wave energy converter, which dissipates its energy via the linear induction generator and a minor amount through mechanical friction. The motion of the wave energy converter and the forcing function, are coupled using a simple interference factor a describing the proportion by which the amplitude of the wave motion is reduced by the motion induced in the wave energy converter. The interference factor is defined by equation 17.

$$a = \frac{X_0}{\frac{H}{2}} \quad (17)$$

The efficiency of the wave energy converter in extracting the energy from the wave is calculated using the interference factor, the energy dissipated in the linear induction generator ΔE and the energy of the wave E_w as in equation 18.

$$\eta = (1-a) \cdot \frac{\Delta E}{E_w} \quad (18)$$

3. Results

3.1. Natural frequency

The natural frequency is a function of the length of the water column, the cross-sectional area of the pressure collector tube, the mass of the hydraulic piston, and of the spring stiffness returning the hydraulic position to its neutral position. A reference case for the dimensions of the wave energy converter was defined as a water column of 75 m, cross sectional area of 100 m^2, a mass of the hydraulic piston of 1000 kg, and a spring-stiffness of 100 kN/m. Parametric studies showing variations of these four dimensions from the reference case are presented in figure 2

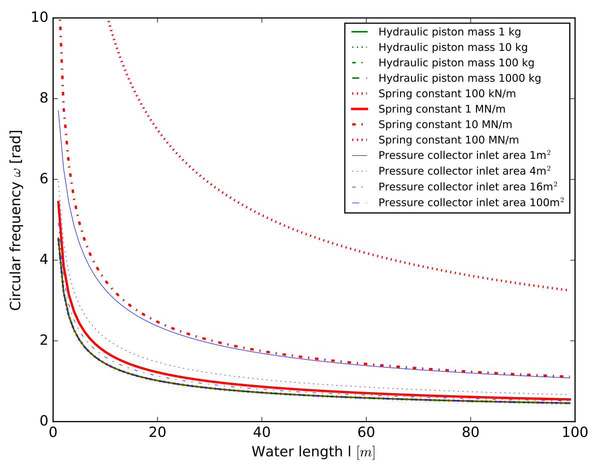

Figure 2: Natural circular frequency of the wave energy converter as function of water column length l', hydraulic piston mass m_{pist}, restoring spring stiffness k_{pist} and pressure collector inlet area A.

Figure 2 shows that the natural frequency of the wave energy converter is a strong function of the water column length l'. The water column typically represents the largest proportion of the total moving mass in the sea wave energy converter, being far larger than the mass of the hydraulic piston. The mass of the water column is linearly proportional to the water column length. If the mass of the hydraulic piston were negligible, then the natural frequency of the wave energy converter would scale with the water column length at a factor of $l^{-\frac{1}{2}}$.

Figure 2 further shows that the cross sectional area of the water column exerted a strong influence on the natural frequency. The mass of the water column scaled linearly with its cross-sectional area. If it were assumed that the mass of the hydraulic piston was negligible, then the natural frequency of the wave energy converter would scale with a factor of $A^{-\frac{1}{2}}$ of the pressure collector cross-sectional area. Increasing the pressure collector inlet tube diameter from 1 m^2 to 10 m^2 visibly reduced the natural frequency of the wave energy converter from almost 2 rad/s to below 1 rad/s for the reference case of 75 m water column length.
The influence of increasing the mass of the hydraulic piston within a realistic range was considerably smaller. Only a marginal decrease in natural frequency was observed when increasing the mass of the piston from 1 kg to 1000 kg. This is because the mass of the piston represents only a small proportion of the overall mass since it is small in comparison to the mass of the water column.

Increasing the spring-constant of the spring returning the hydraulic piston to its neutral position from 100 kN to 100 MN had a significant effect on the natural frequency of the wave energy converter, increasing it from less than 1 rad/s to over 4 rad/s. In order to keep the structural design of the wave energy converter simple, large spring-forces associated with a high spring-constant, should be avoided. Yet, the spring should have a restoring force that is significant with respect to the wave force driving it away from its equilibrium position. A spring stiffness providing a similar restoring force as the wave itself is described by equation 19.

$$k_{pist} \approx \frac{F_0}{X_0\Big|_{\omega=\omega_n}} \tag{19}$$

The maximum displacement range of the wave energy converter was calculated for different excitation frequencies using the reference dimensions described above. The friction in the hydraulic piston and the water column was assumed to have a value of 1000 Ns/m. The linear induction generator was assumed to consist of a coil with 250 windings having a total wire length of 1.45 m. The magnetic field strength was assumed to be 10 [Wb/m2]. The resulting displacement is shown in figure 3.

The phase angle between the displacement of the wave energy converter

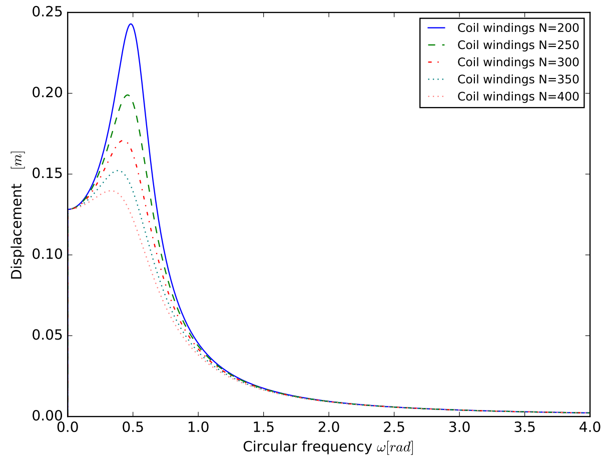

Figure 3: Maximum displacement range of the hydraulic piston.

and the instantaneous wave height is shown in figure 4. Five curves are shown representing five different number of windings in the linear induction generator. A low number of windings provided lower electrical induction and a lower resistive force the piston motion. At low sea wave frequencies the piston followed the motion of the wave with only a small phase lag. As the excitation frequency approached the natural frequency of the system, the phase lag increased up to $\frac{\pi}{2}$. At this point the maximum displacement amplitude of the piston was calculated. When the wave frequency increases further phase angle increased and approached a value of π for high wave frequencies.

The efficiency of the wave energy converter defined in equation 18 was calculated for five numbers of induction windings in the generator. The force resisting the motion of the piston increased with the number of electrical windings in the induction generator. Figure 5 shows the conversion efficiencies of the system over the excitation frequency spectrum. A maximum efficiency of 27 % was calculated for a number of 250 windings.

The results show that in order to maximise its energy conversion efficiency, the inductive load on the linear induction generator should be variable, so as to optimise the response of the hydraulic piston for the particular operating condition.

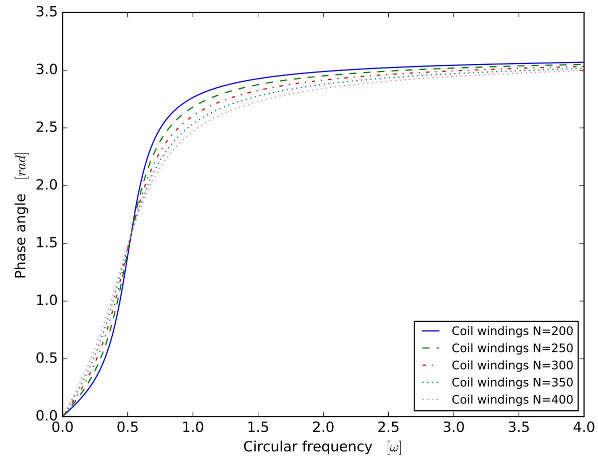

Figure 4: Phase angle between piston displacement and wave amplitude.

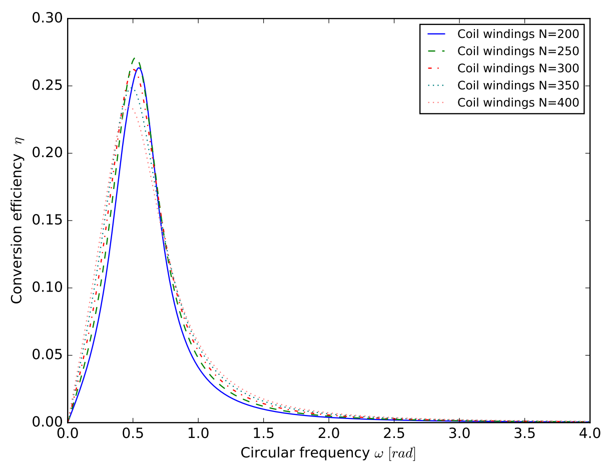

Figure 5: Calculated energy conversion efficiency at different inductive loads.

3.2. Numerical simulation of piston motion

The transient movement of the hydraulic cylinder was simulated numerically using the equation of motion 1 and the force F_0 acting on the hydraulic piston described by 15 using the Euler method. During each time step, the acceleration, velocity and displacement of the current time step N were calculated using values from the previous time-step $N-1$ as shown in equation

20, equation 21, and equation 22 below.

$$\ddot{x}_N = \frac{F_0}{A\rho l' + m_{pist}} sin(\omega t) - \frac{(c+f)}{A\rho l' + m_{pist}} \dot{x}_{N-1} - 2\frac{\rho g A + k_{pist}}{A\rho l' + m_{pist}} x_{N-1} \quad (20)$$

$$\dot{x}_N = \dot{x}_{N-1} - \ddot{x}_N \cdot dt \quad (21)$$

$$x_N = x_{N-1} - \dot{x}_N \cdot dt \quad (22)$$

Velocity and displacement were integrated in steps of 0.1 ms from the forces and masses of the system yielding the hydraulic piston displaement, velocity and acceleration in the time domain.

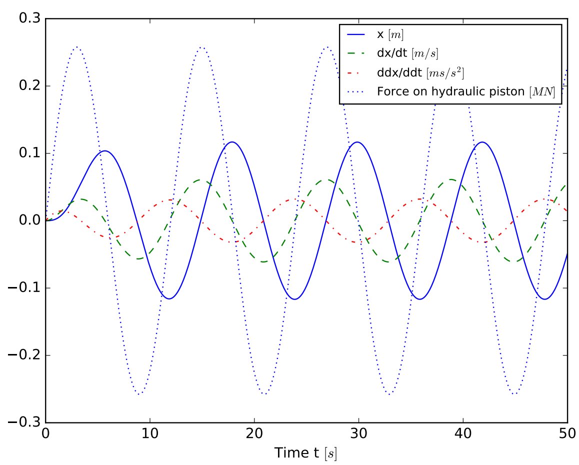

Figure 6: Numerical calculation of piston motion using an Euler model.

The hydraulic cylinder was accelerated from rest towards steady state motion when exposed to the wave force described by equation 14. Figure 6 describes the motion of the wave energy converter for a wave of the natural frequency of the wave energy converter for a linear induction generator coil having 250 windings. This condition corresponds to the peak resonance at for second condition (green dashed curve) shown in figure 3. Figure 6 shows how the wave energy converter accelerated from rest at around 0.035 m/s^2

and rapidly reached near steady state operation. The phase angle of $\pi/2$ between the force and the displacement of the hydraulic piston are visible for the its response at the natural frequency.

4. Cost

The levelised cost of electricity production using the pressure differential wave energy converter was estimated based on the capital cost and the production cost of electricity. The capital costs comprised the linear induction generator, the hydraulic piston mechanism, the pressure collection tunnel, estimates for the onshore power conditioning and transmission system and the onshore nacelle. An overview of the construction costs is given in Table 1.

Table 1: Construction costs for pressure differential wave energy converter

Construction component	Cost [EUR / W]	Fraction [%]
Linear induction generator	2.19	5.36 %
Hydraulic piston mechanism	0.47	1.15 %
Pressure collection tunnel	37.40	91.42 %
Power conditioning and transmission	0.14	0.35 %
Nacelle	0.71	1.71 %
Total	40.92	100 %

The costs were estimated based on Allan et al. [4] regarding the cost of the linear induction generator, and Rostami et al. [12] for the cost of the pressure collection tunnels. This results in a total construction cost of 40.92 EUR per installed Watt. The majority of this cost occurs due to the construction of the pressure collection tunnel. It was estimated that the operations and maintenance costs would be relatively low, at 25 EUR/W year, due to the high survivability and simplified access to the land-based power conversion system. Despite the relatively low maintenance costs, the levelised cost of energy production was estimated to be as high as 2.31 EUR/kWh.

For comparison, Allan et al. [4] have estimated the levelised cost of existing wave power technology to 0.225 EUR/kWh (189.66 GBP/MWh) [15]. The dominating portion of the costs for existing technology (81 %) was attributed to the construction costs. The construction costs for existing technology were estimated at 4.291 EUR/W(3622 GBP/kW) of installed capacity of wave power [15]. Operations and maintenance costs for existing technology tidal power were estimated at 117 EUR/kW (98.4 GBP/kW) for wave power [15].

The pressure differential sea wave energy converter concept is thus more expensive than existing technology in terms of levelised cost of electricity, but it has as its main advantage that it can be deployed in coastal areas without affecting the sea surface or requiring surface-piercing or floating structures. This is important for example near cities or ports, where shipping is important. Its ability to extract wave power without impacting the water surface by obstacles may justify its higher cost in certain applications. Innovative materials and construction methods may also provide significant cost reductions [13]. In areas where coastlines are being redeveloped or fortified for other reasons, the added cost of including the tunnels during such construction work may also be lower, since their construction may be incorporated into planned construction work.

5. Summary and conclusions

The concept of a pressure differential wave energy converter for coastal areas was described in terms of its mechanical concept, frequency response, time response and conversion efficiency. Due to its mechanical components being the sea floor, this concept of a pressure differential wave energy converter is particularly suited for high survivability and low visual and water-surface impact of coastal areas. The natural frequency of the system was described in terms of the dimensions of the pressure collector tube, the mass of the hydraulic piston, the stiffness of the spring of the hydraulic piston. The damping of the system was described by the mechanical friction and the electrical design parameters of the linear induction generator, such as number of windings, magnetic field strength, length and resistance of the coil. The sea wave was modelled as low-frequency swell.

The results showed that the dimensions of the pressure collection tube are important for the natural frequency since they contribute to the mass in motion. The mass of the hydraulic piston was comparatively small, and had a significantly smaller influence on the natural frequency. The stiffness of the restoring spring acting on the hydraulic piston also had a strong effect in increasing the natural frequency when increased from 100 kN/m to 1 MN/m.

The wave attenuation of this device was found to be adjustable by varying the inductive electrical load on the linear induction generator. The maximum efficiency of the wave energy converter was reached for an intermediated number of electrical windings, yielding an intermediate maximum displacement. This was because as the displacement amplitude becomes larger than the wave amplitude, the energy extracted from the wave is reduced. In order to maximise the energy conversion efficiency, the response of the hydraulic piston should be optimised by adjusting the inductive load on the linear induction generator. The wave energy converter showed promise of achieving a conversion efficiency of around 27 %.

A transient numerical simulation of the hydraulic piston in the wave energy converter was shown to yield a swift response to near steady state harmonic oscillation, after being accelerated from rest by the sea wave.

This initial study prompts further work and more detailed analyses of the conversion efficiency, in particular with regards to more complex wave patterns in shallow waters close to the shore, transient wave patterns, as well as varying actuation and control conditions. Wave patterns of shallow waters are described in more detail by McCormick [2] or Sorensen [11], and further work could aim to study the impact of these wave motions numerically

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