A RATIONAL APPROACH TO SEEPAGE FLOW EFFECTS ON BOTTOM FRICTION BENEATH RANDOM WAVES

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ABSTRACT

The bottom friction beneath random waves is predicted taking into account the effect of seepage flow. This is achieved by using wave friction factors for rough turbulent, smooth turbulent and laminar flow valid for regular waves together with a modified Shields parameter which includes the effect of seepage flow. Examples using data typical to field conditions are included to illustrate the approach. The analytical results can be used to make assessment of seepage effects on the bottom friction based on available wave statistics. Generally, it is recommended that a stochastic approach should be used rather than using the *rms* values in an otherwise deterministic approach.

Keywords: Bottom friction; Shields parameter; Seepage flow; Random waves; Stochastic method.

1. INTRODUCTION

At intermediate and shallow water depths the bottom wave boundary layer is a thin flow region dominated by friction arising from the combined action of the wave-induced near-bottom flow and the bottom roughness. The wave boundary layer flow determines the bottom shear stress, which affects many phenomena in coastal engineering, e.g. sediment transport and assessment of the stability of scour protection in the wave environment. A review is e.g. given in Holmedal et al. (2003).

In coastal areas the seabed is often sandy and permeable, and seepage flow in the seabed may occur naturally due to the horizontal pressure gradient caused by the difference between the pressure at the seabed under the wave crest and the wave trough, respectively. This will vary in time and space following the wave motion inducing flow into the seabed as the wave crest passes and out of the seabed as the wave trough passes (see e.g. Lohmann et al. (2006, Fig. 12)). This seepage flow has two opposing effects. First, the seepage flow into and out of the bed modifies the wave boundary layer by increasing and decreasing the velocity gradient at the bed, respectively, leading to that the bed shear stress increases and decreases, respectively (see e.g. Lohmann et al. (2006, Figs. 12, 18)). Second, the seepage flow exerts a vertical force on the sediments within the bed; the seepage flow into and out of the bed stabilizes and destabilizes the sediment, respectively.

Effects of seepage flow due to regular waves have e.g. been discussed by Sleath (1984), Soulsby (1997), Nielsen (1992, 2009). Moreover, Conley and Inman (1992) observed a wave crest – wave trough asymmetry in the fluid – sediment boundary layer development due to seepage flow beneath near-breaking waves in field measurements. These observations were supported by Conley and Inman (1994) in laboratory experiments for regular waves. Lohmann et al. (2006) performed Large Eddy Simulation of a fully developed turbulent wave boundary layer subject to seepage flow for regular waves, and obtained results in accordance with the experimental results of Conley and Inman (1994). Nielsen (1997) was the first to quantify the two opposing effects of seepage flow by defining a modified Shields parameter for regular waves. He used the shear stress experiments of Conley (1993) for regular waves and the slope stability experiments of Martin and Aral (1971) for steady flow to derive the coefficients to use in this modified Shields parameter. Nielsen et al. (2001) used this modified Shields parameter together with their own experiments for regular waves to investigate the seepage effects on the mobility of sediments on a flat bed under waves. Obhrai et al. (2002) extended this work to investigate the seepage effects on suspended sediments over a flat and a rippled bed for regular waves. Myrhaug et al. (2014) provided a practical approach by which the stochastic properties of the net bedload sediment transport rate due to the seepage flow can be derived from the irregular wave motion outside the seabed wave boundary layer.

For the prediction of the bottom friction under random waves, a commonly used procedure is to use the *rms* (root-mean-square) value of the wave height (H_{rms}) or the near-bed orbital velocity amplitude (U_{rms}) in an otherwise deterministic approach. However, this approach does not account for the stochastic feature of the processes included.

The purpose of the present paper is to provide a practical approach by which the stochastic properties of the bottom friction due to seepage flow can be derived from the irregular wave motion outside the seabed wave boundary layer. For regular waves there is a variety of bottom friction coefficient formulas available (see e.g. Soulsby (1997)). However, the purpose here is not to examine the details of them, but to demonstrate how such formulas can be used to find the bottom friction due to seepage flow under linear random waves over a flat seabed. The approach is based on assuming the waves to be a stationary Gaussian narrow-band random process, using wave friction factors for regular waves including the effect of seepage flow by adopting the Nielsen (1997) modified Shields parameter. Thus the only wave asymmetry effect considered in this study is that contained in the modified Shields parameter. Examples are also included to demonstrate the applicability of the results for practical purposes using data typical for field conditions.

2. BOTTOM FRICTION BY REGULAR WAVES

The effect of seepage flow is taken into account by adopting a modified Shields parameter originally suggested by Nielsen (1997) (and re-presented by Nielsen et al. (2001)):

$$\theta_w = \frac{u_{*0}^2\left(1-\kappa\frac{w}{u_{*0}}\right)}{g\left(\gamma-1-\beta\frac{w}{K}\right)d_{50}} \tag{1}$$

where $u_{*0} = (\tau_{w0}/\rho)^{1/2}$ is the friction velocity with no seepage, τ_{w0} is the maximum bottom shear stress with no seepage, ρ is the density of the fluid, g is the acceleration due to gravity, γ is the sediment density to fluid density ratio, d_{50} is the median grain size diameter, w is the vertical seepage velocity taken as positive upwards, κ and β are dimensionless coefficients recommended as $(\kappa,\beta)=(16,0.4)$, and K is the hydraulic conductivity of the sand (to be specified later). Eq. (1) is based on obtaining $u_*^2/u_{*0}^2 = 1-16w/u_{*0}$ as the best fit to the Conley (1993) data for $-0.05 < w/u_{*0} < 0.025$, where $u_* = (\tau_w/\rho)^{1/2}$ is the friction velocity with seepage, and τ_w is the maximum bottom shear stress with seepage. Moreover, $\beta = 0.4$ was determined using the slope stability experiments of Martin and Aral (1971). This modified Shields parameter includes two opposing effects. First, the flow into and out of the bed will make the boundary layer thinner and thicker and thereby the bed shear stress increases and decreases, respectively. Second, the flow into and out of the bed stabilizes and destabilizes the sediments, respectively. The numerator in Eq. (1) includes the change in the bed shear stress, i.e. to increase the shear stress for downward seepage $(w<0)$ and to reduce it for upward seepage $(w>0)$. The denominator includes the change in the effective weight due to the seepage, i.e. to stabilize the particles for downward seepage and to destabilize the particles for upward seepage. It should also be noticed that Eq. (1) is valid for non-breaking waves over a horizontal bed; see Nielsen et al. (2001) for more details.

The maximum bottom shear stress within a wave-cycle without seepage is taken as

$$\frac{\tau_{w0}}{\rho} = \frac{1}{2} f_w U^2 \tag{2}$$

where U is the orbital velocity amplitude at the seabed, and f_w is the wave friction factor given for laminar (Eq. (3)), smooth turbulent (Eq. (5)) and rough turbulent flow (Eqs. (7) – (10)).

For laminar flow, the wave friction factor is given as that for Stokes' second problem (Schlichting, 1979)

$$f_w = 2\,\mathrm{Re}^{-0.5} \qquad \text{for} \qquad \mathrm{Re} \le 3 \times 10^5 \tag{3}$$

where

$$\mathrm{Re} = \frac{UA}{\nu} \tag{4}$$

is the Reynolds number associated with the wave motion, $A = U/\omega$ is the maximum near-bed orbital displacement, ω is the wave frequency, and ν is the kinematic viscosity of the fluid.

For smooth turbulent flow, the Myrhaug (1995) smooth bed wave friction factor is adopted

$$f_w = r\,\mathrm{Re}^{-s} \quad \text{for} \quad \mathrm{Re} > 3 \times 10^5 \tag{5}$$

with the coefficients

$$(r,s) = (0.0450, 0.175) \tag{6}$$

Alternative coefficients (r,s) for smooth turbulent flow are given in Soulsby (1997).

For rough turbulent flow, the friction factor proposed by Myrhaug et al. (2001) is used

$$f_w = c\left(\frac{A}{z_0}\right)^{-d} \tag{7}$$

$$(c,d) = (18, 1) \quad \text{for} \quad 20 \lesssim A/z_0 \lesssim 200 \tag{8}$$

$$(c,d) = (1.39, 0.52) \quad \text{for} \quad 200 \lesssim A/z_0 \lesssim 11000 \tag{9}$$

$$(c,d) = (0.112, 0.25) \quad \text{for} \quad 11000 \lesssim A/z_0 \tag{10}$$

where $z_0 = 2.5d_{50}/30$ is the bed roughness based on the median grain size diameter d_{50}. Note that for rough turbulent flow Eq. (8) is obtained as best fit to irregular wave data, and that Eqs. (9) and (10) are obtained as best fit to regular wave data (see Myrhaug et al. (2001) for more details). Also note that Eq. (9) corresponds to the coefficients given by Soulsby (1997) obtained as best fit to regular wave data for $10 \lesssim A/z_0 \lesssim 10^5$. The advantage of using these friction factors is that it is possible to derive the stochastic approach analytically. One should note that all the wave-related quantities in Eqs. (1) to (7), i.e., τ_{w0}, U and A are the quantities associated with the harmonic motion. Thus a stochastic approach based on the harmonic wave motion is feasible, as will be outlined in the forthcoming.

3. BOTTOM FRICTION BY RANDOM WAVES

The present approach is based on the following assumptions: (1) the free surface elevation $\zeta(t)$ associated with the harmonic motion is a stationary Gaussian narrow-band random process with zero expectation described by the single-sided spectral density $S_{\zeta\zeta}(\omega)$, and (2) the bottom friction for regular waves given in the previous section, are valid for irregular waves as well.

The second assumption implies that each wave is treated individually, and consequently that the bottom friction is taken to be constant for a given wave situation and that memory effects are neglected. The accuracy of this assumption has been justified by Samad (2000) for laminar and smooth turbulent flow, for which the bottom friction is given by Eq. (2); f_w is given in Eq. (3) for laminar flow, and by Eq. (5) using $(r,s) = (0.041, 0.16)$ for smooth turbulent flow. Samad (2000) found good agreement between his measured bed shear stresses (laminar and smooth turbulent) under irregular waves and simulations and bed shear stresses based on individual wave formulas. For rough turbulent flow the validity of the second assumption was confirmed for seabed shear stresses by Holmedal et al. (2003) for high values of A/z_0 (at about 30000). Characteristic statistical values of the resulting seabed shear stress amplitude deviated less than 20% from those obtained by the Monte Carlo simulation method by Holmedal et al. (2000); that essentially is based on the same two assumptions upon which the present approach is based. Regarding the second assumption that each wave is treated individually, Holmedal et al. (2003) concluded for large values of A/z_0 that the main reason for the fair agreement obtained between the Monte Carlo simulations and the (k - ε) model predictions is the good description of the wave friction factor for individual waves. This appears to be much more important than violating the assumption of independent individual waves. Thus, since the Shields parameter in Eq. (1) is essentially based on using the friction factors for laminar, smooth turbulent and rough turbulent flow in Eqs. (2) to (10), the assumption of treating each wave individually seems reasonable.

Based on the present assumptions, the (instantaneous) time-dependent bed orbital displacement and velocity $a(t)$ and $u(t)$, respectively, associated with the harmonic motion, are both stationary Gaussian narrow-band processes with zero expectations and with single-sided spectral densities

$$S_{aa}(\omega) = \frac{S_{\zeta\zeta}(\omega)}{\sinh^2 kh} \tag{11}$$

$$S_{uu}(\omega) = \omega^2 S_{aa}(\omega) = \frac{\omega^2 S_{\zeta\zeta}(\omega)}{\sinh^2 kh} \tag{12}$$

Here k is the wave number determined from $\omega^2 = gk \tanh kh$.

For a narrow-band process the waves are specified as a `harmonic´ wave with cyclic frequency ω and with slowly varying amplitude and phase. Then, for the first order, the near-bed orbital velocity amplitude U is related to the near-bed orbital displacement amplitude A by $U = \omega A$, where U is slowly varying with time as well (see e.g. Sveshnikov (1966)).

Now the orbital displacement amplitude at the seabed, A, the orbital velocity amplitude at the seabed, U, and the wave height, H, are Rayleigh-distributed with the cumulative distribution function (*cdf*) given by

$$P(\hat{x}) = 1 - \exp(-\hat{x}^2) \ ; \quad \hat{x} = x / x_{rms} \geq 0 \tag{13}$$

where x represents, A, U or H, and x_{rms} is the *rms* value of x representing A_{rms}, U_{rms} or H_{rms}. Now A_{rms}, U_{rms} and H_{rms} are related to the zeroth moments m_{0aa}, m_{0uu} and $m_{0\zeta\zeta}$ of the amplitude, velocity and free surface elevation spectral densities, respectively, (corresponding to the variances of the amplitudes (σ_{aa}^2), the velocity (σ_{uu}^2) and the free surface elevation $(\sigma_{\zeta\zeta}^2)$), given by

$$A_{rms}^2 = 2m_{0aa} = 2\sigma_{aa}^2 = 2\int_0^\infty S_{aa}(\omega)d\omega \tag{14}$$

$$U_{rms}^2 = 2m_{0uu} = 2\sigma_{uu}^2 = 2\int_0^\infty S_{uu}(\omega)d\omega \tag{15}$$

$$H_{rms}^2 = 8m_{0\zeta\zeta} = 8\sigma_{\zeta\zeta}^2 = 8\int_0^\infty S_{\zeta\zeta}(\omega)d\omega \tag{16}$$

From Eqs. (15) and (12) it also appears that $m_{0uu} = m_{2aa}$ where m_{2aa} is the second moment of the amplitude spectral density. Thus, the mean zero-crossing frequency for the bed orbital displacement, ω_z, is obtained from the spectral moments of $a(t)$ as

$$\omega_z = \left(\frac{m_{2aa}}{m_{0aa}}\right)^{1/2} = \left(\frac{m_{0uu}}{m_{0aa}}\right)^{1/2} = \frac{U_{rms}}{A_{rms}} \tag{17}$$

where Eqs. (14) and (15) have been used. This result is valid for a stationary Gaussian random process. Note that this zero-crossing frequency will generally be smaller than for surface elevation due to greater attenuation of high frequencies. However, for a narrow-band process these two zero-crossing frequencies will be equal. It should be noted that U_{rms} used by Soulsby (1997) corresponds to the standard deviation σ_{uu} used here.

For a narrow-band process, $A = U/\omega$ where ω is replaced with ω_z from Eq. (17). Then, by substitution in Eqs. (2), (3), (4), (5) and (7) (i.e. according to assuming that these regular wave friction factors are valid for irregular waves), Eq. (1) can be re-arranged to give the Shields parameter for individual narrow-band random waves as

$$\theta_w = \frac{u_{*rms}^2 \hat{U}^{\varepsilon}}{g(\gamma - 1 - \beta \frac{w}{K})d_{50}} \left(1 - \kappa \frac{w}{u_{*rms}} \hat{U}^{-\varepsilon/2}\right) \; ; \begin{cases} \varepsilon = 1, \text{ laminar} \\ \varepsilon = 2 - 2s, \text{ smooth turbulent} \\ \varepsilon = 2 - d, \text{ rough turbulent} \end{cases} \tag{18}$$

where $\hat{U} = U/U_{rms}$ and

$$u_{*rms}^2 = \mathrm{Re}_{rms}^{-0.5} U_{rms}^2, \quad \text{laminar} \tag{19}$$

$$u_{*rms}^2 = \frac{1}{2} r \,\mathrm{Re}_{rms}^{-s} U_{rms}^2, \quad \text{smooth turbulent} \tag{20}$$

$$\mathrm{Re}_{rms} = \frac{U_{rms} A_{rms}}{\nu} \tag{21}$$

$$u_{*rms}^2 = \frac{1}{2} c \left(\frac{A_{rms}}{z_0}\right)^{-d} U_{rms}^2, \quad \text{rough turbulent} \tag{22}$$

Note that (*r*,*s*) and (*c*,*d*) are the wave friction factor coefficients for smooth turbulent and rough turbulent flow, respectively. Moreover, a Shields parameter can be defined where the wave-related quantities are replaced by their *rms*-values, i.e.,

$$\theta_{wrms} = \frac{u_{*rms}^2}{g(\gamma - 1)d_{50}} \tag{23}$$

Furthermore, by taking $\omega = \omega_z$ and $k = \bar{k}$ (i.e. according to the narrow-band assumption), where $\bar{k}$ is the mean wave number determined from $\omega_z^2 = g\bar{k} \tanh \bar{k}h$, then $U_{rms} = \omega_z H_{rms} / (2\sinh \bar{k}h)$.

The bottom friction due to the effect of seepage flow that will be considered here is (1) the maximum value during the wave half-cycle beneath the wave crest $(w < 0)$; and (2) the maximum (absolute) value during the wave half-cycle beneath the wave trough $(w > 0)$. For the individual narrow-band random waves this is expressed in terms of the Shields parameters which are obtained by re-arranging Eq. (18) to (notice that w is taken as positive in the following)

$$\theta_{w\pm} = \frac{u_{*rms}^2}{C_\pm} \hat{U}^\varepsilon (1 \pm b\hat{U}^{-\varepsilon/2}) \tag{24}$$

where

$$b = \kappa \frac{w}{u_{*rms}} \tag{25}$$

$$C_\pm = g(\gamma - 1 \pm \beta \frac{w}{K}) d_{50} \left[m^2 / s^2 \right] \tag{26}$$

A statistical quantity of interest is the expected (mean) value of the bottom friction caused by the ($1/n$)th highest waves, which in terms of the Shields parameter is given as

$$E\left[\theta_{w\pm}(\hat{U}) \mid \hat{U} > \hat{U}_{1/n} \right] = n \int_{\hat{U}_{1/n}}^{\infty} \theta_{w\pm}(\hat{U}) p(\hat{U}) d\hat{U} \tag{27}$$

where $p(\hat{U})$ is the probability density function (*pdf*) of $\hat{U}$ given by $p(\hat{U}) = dP(\hat{U})/d\hat{U}$ and $P(\hat{U})$ is given in Eq. (13) and $\hat{U}_{1/n}$ is the value which is exceeded by the probability $1/n$, i.e., $1 - P(\hat{U}_{1/n}) = 1/n$, giving $\hat{U}_{1/n} = (\ln n)^{1/2}$. By substituting Eq. (24) in Eq. (27) the integral in Eq. (27) can be evaluated giving

$$E\left[\theta_{w\pm} \mid \hat{U} > \hat{U}_{1/n} \right] = \frac{u_{*rms}^2}{C_\pm} n \left[\Gamma(1 + \frac{\varepsilon}{2}, \ln n) \pm b\Gamma(1 + \frac{\varepsilon}{4}, \ln n) \right] \tag{28}$$

Here $\Gamma(.,.)$ is the incomplete gamma function, u_{*rms} is given in Eqs. (19) to (22); b in Eq. (25) where w needs to be specified according to values given in the range $-0.05 < w/u_{*rms} < 0.025$. Here the integral in Eq. (27) is evaluated analytically by using Eq. (A4) in the Appendix. Thus, the plus (+) and minus (-) signs represent the values beneath the wave crest and the wave trough, respectively.

The results without the effect of seepage flow are given by $w = 0$, i.e. the values beneath the wave crest and wave through are the same given by

$$E\left[\theta_w \,|\, \hat{U} > \hat{U}_{1/n}\right] = \frac{u_{*rms}^2}{g(\gamma-1)d_{50}} n\Gamma(1+\frac{\varepsilon}{2}, \ln n) \tag{29}$$

For prediction of the bottom friction beneath irregular waves, a commonly used procedure is to use the *rms* values of the wave-related quantities in an otherwise deterministic approach. In the present case this corresponds to taking $U = U_{rms}$, i.e. $\hat{U} = U / U_{rms} = 1$, which substituted in Eq. (24) gives

$$\theta_{w\pm,\det rms} = \frac{u_{*rms}^2}{C_\pm}(1 \pm b) \tag{30}$$

Without seepage flow $(w = 0)$ the result reduces to Eq. (23), i. e., $\theta_{w,\det} = \theta_{wrms}$.

Alternatively, the expected value of the $(1/n)$th largest values of the wave-related quantities can be used in an otherwise deterministic approach, i.e. to substitute $\hat{U} = E\left[\hat{U}_{1/n}\right] = n\Gamma(1.5, \ln n)$ in Eq. (24) (by using the results in the Appendix). For $n = 3$ this corresponds to the significant value U_s (which will be exemplified in Section 4), corresponding to taking $U_s = \sqrt{2}U_{rms}$, i.e. $\hat{U} = U_s / U_{rms} = \sqrt{2}$, which substituted in Eq. (24) gives

$$\theta_{w\pm,\det s} = \frac{u_{*rms}^2}{C_\pm} 2^{\varepsilon/2}(1 \pm b2^{-\varepsilon/4}) \tag{31}$$

Without seepage flow it follows from Eq. (18) that the result in this case reduces to

$$\theta_{w,\det s} = \frac{u_{*rms}^2 \, 2^{\varepsilon/2}}{g(\gamma-1)d_{50}} \tag{32}$$

4. EXAMPLES OF RESULTS

To the authors' knowledge no data exist in the open literature for random wave-induced bottom friction including the effects of seepage flow. Hence three examples of calculating the bottom friction for rough turbulent, smooth turbulent and laminar flow due the effect of seepage flow based on the results in Section 3 are provided. Here the purpose is to illustrate the application of the approach using data typical to field conditions, exemplified by considering the expected value of the Shields parameter for the one third highest waves, i.e. for $n=3$ in Eqs. (27), (28) and (29); also referred as the significant value. Similar results can be obtained for other statistical values of the Shields parameter by choosing other values of n. However, this will not be elaborated further here.

4.1 Rough turbulent

For rough turbulent flow the following flow conditions are given

- Horizontal bed with water depth, $h = 15$ m
- Significant wave height, $H_s = 5$ m
- Mean wave period, $T_z = 8.9$ s
- Median grain diameter (fine sand according to Soulsby (1997, Fig. 4)) $d_{50} = 0.201\,\text{mm}$
- $\gamma = 2.65$ (as for quartz sand)
- Kinematic viscosity of water at temperature 10°C and salinity 35 o/oo, $\nu = 1.36 \cdot 10^{-6}\,\text{m}^2/\text{s}$
- Median grain settling velocity (according to Soulsby (1997), Eq. SC(102)), $w_s = 0.02$ m/s

The calculated quantities are given in Table 1, where $H_{rms} = H_s / \sqrt{2}$ when H is Rayleigh-distributed. It appears that the flow corresponds to sheet flow conditions, i.e. $\theta_{wrms} = 1.121 > 0.8$.

In addition to the Shields parameter, the mode of sand transport is governed by the fall velocity parameter w_s / u_{*rms}, i.e. the sand transport will be in the suspension mode if $\theta_{wrms} > \theta_{srms}$ and $w_s / u_{*rms} < 1$; otherwise the sand transport will be in the no-suspension mode. Here θ_{srms} is the critical value for initiation of suspension from the bed, depending on the grain Reynolds number $u_{*rms}d_{50}/\nu$. According to Sumer and Fredsøe (2002, Eq. (7.7)) θ_{srms} is given by a relationship which is strictly only valid for initiation of suspension from the bed in the case of steady currents. However, to a first approximation this relationship can be used for waves (see Sumer and Fredsøe (2002) for more details).

By using this it appears for rough turbulent flow that $u_{*rms}d_{50}/\nu = 8.9$ and $\theta_{srms} = 0.49$ (Sumer and Fredsøe (2002, Eq. (7.7)); thus $\theta_{wrms} > \theta_{srms}$ and $w_s / u_{*rms} = 0.35 < 1$, and consequently the sediment transport takes place as suspended load.

The vertical seepage velocity (taken as positive upwards) is taken as $w = \pm 0.025 u_{*rms}$, which is in the validity range of Eq. (1). Moreover, the hydraulic conductivity of the sand is taken as $K = w/0.15$ based on Horn et al. (1998), who found that $w < 0.15K$ corresponds to modest seepage rates generated by uprush of swash during a rising tide (see more discussion in Nielsen et al. (2001)). It should be noted that the values of w and K are taken as characteristic values for all the individual random waves, which is consistent with the narrow-band assumption.

From Table 1 $A_{rms} / z_0 = 90000$ and accordingly the friction factor for rough turbulent flow in Eqs. (7) and (10) has been used. However, to ensure that the flow is rough turbulent the roughness Reynolds number has to be evaluated; which is defined as

$$\mathrm{Re}_* = \frac{u_* z_0}{\nu} \tag{33}$$

Then the turbulent flow regimes are defined as (Schlichting, 1979)

Smooth turbulent

$$0 < \mathrm{Re}_* < 0.17 \tag{34}$$

Intermediate smooth to rough turbulent

$$0.17 < \mathrm{Re}_* < 2.3 \tag{35}$$

Rough turbulent

$$2.3 < \mathrm{Re}_* \quad (36)$$

From Table 1 it follows that $\mathrm{Re}_* = u_{*rms} z_0 / \nu = 0.74$, i.e. the flow is intermediate smooth to rough turbulent. However, due to sheet flow there will be an additional roughness caused by sediment transport, which according to Nielsen (1992) is given as

$$z_{0transport} = \frac{1}{30} 190(\theta_w - 0.05)^{1/2} d_{50} \quad (37)$$

This is valid for irregular waves in field conditions. For $\theta_w = \theta_{wrms}$ this gives $z_{0transport} = 6.554 d_{50}$. By taking $z_{0total} = z_{0grain} + z_{0transport} = d_{50}/12 + 6.554 d_{50} = 6.64 d_{50}$, then $\mathrm{Re}_* = u_{*rms} z_{0total} / \nu = 4.9$, i.e. the flow is rough turbulent.

From Table 1 it appears that the significant values of the Shields parameter including the effect of seepage flow are 2.64 under the wave crest, 1.59 under the wave trough; without seepage the result under both the wave crest and the wave trough is 2.13. Thus it appears that the Shields parameter exceeds 0.8 (i.e. the threshold for sheet flow) under both the wave crest and the wave through. This is also the case without seepage effects.

4.2 Smooth turbulent

For smooth turbulent flow the given flow conditions are the same as for rough turbulent flow in Section 4.1 except for

- $H_s = 4\,\mathrm{m}$
- $d_{50} = 0.06\,\mathrm{mm}$ (very fine sand according to Soulsby (1997, Fig. 4))
- $w_s = 0.0022\,\mathrm{m/s}$ (according to Soulsby (1997), Eq. SC(102))

The calculated quantities are given in Table 2. It appears that the flow corresponds to sheet flow conditions, i.e. $\theta_{wrms} = 1.57 > 0.8$. Moreover, for smooth turbulent flow $u_{*rms} d_{50} / \nu$ = 1.7 and θ_{srms} = 0.65 (Sumer and Fredsøe (2002, Eq. (7.7)); thus $\theta_{wrms} > \theta_{srms}$ and $w_s / u_{*rms} = 0.056 < 1$, and consequently the sediment transport takes place as suspended load.

It also appears that $\mathrm{Re}_{rms} = 7.5 \cdot 10^5 > 5 \cdot 10^5$, i.e. smooth turbulent flow. However, to ensure that this is the case it is also required that the roughness Reynolds number $\mathrm{Re}_* = u_{*rms} z_0 / \nu < 0.17$ according to Eq. (34). It appears that $\mathrm{Re}_* = 0.14$. If u_{*rms} is calculated assuming rough turbulent flow (i.e from Eq. (22) for $A_{rms} / z_0 = 2.4 \cdot 10^5$) then $\mathrm{Re}_* = 0.16$, i.e. the flow is smooth turbulent. However, if an additional roughness due to sediment transport is taken into account by using Eq. (37) for $\theta_w = \theta_{wrms}$ this gives $z_{0total} = d_{50} / 12 + 7.808 d_{50} = 7.89 d_{50}$. Thus $\mathrm{Re}_* = u_{*rms} z_{0total} / \nu = 1.1$, i.e. the flow is transitional smooth to rough turbulent.

From Table 2 it appears that the significant values of the Shields parameter including the effect of seepage are 3.54 under the wave crest, 2.07 under the wave trough; without seepage it is 2.85 under both the wave crest and the wave trough. It appears that the flows are in the sheet flow regime both with and without seepage effects.

4.3 Laminar

For laminar flow the given flow conditions are the same as for smooth turbulent flow in Section 4.2 except for

- $H_s = 1\,\mathrm{m}$

The calculated quantities are given in Table 2. It appears that the threshold of sediment motion at the seabed is exceeded, i.e. $\theta_{wrms} = 0.21 > 0.05$. Moreover, for laminar flow $u_{*rms} d_{50} / \nu = 0.63$ and $\theta_{srms} = 0.70$ (Sumer and Fredsøe (2002, Eq. (7.7)); thus $\theta_{wrms} < \theta_{srms}$ and $w_s / u_{*rms} = 0.15 < 1$, and consequently the sediment transport is in the no-suspension mode; however, bedload transport takes place since $\theta_{wrms} > 0.05$. It also appears that $\mathrm{Re}_{rms} = 4.6 \cdot 10^4 < 5 \cdot 10^5$, i.e. laminar flow.

From Table 2 (based on a flat bed approach) it appears that the significant values of the Shields parameter including the effect of seepage are 0.39 under the wave crest, 0.20

under the wave trough; without seepage it is 0.30. All these values exceed the threshold of inception of motion suggesting that ripples may exist.

4.4 Example of finite bandwidth effects

The Rayleigh cdf in Eq. (13) is valid for a stationary Gaussian narrow-band process. Longuet-Higgins (1980) has shown that the wave heights are Rayleigh-distributed if H_{rms} is replaced with $0.925H_{rms}$, which includes finite bandwidth effects of the wave process. This modified Rayleigh distribution is used to exemplify the effects of finite bandwidth on the bottom friction (Shields parameter). For the significant Shields parameter the results are:

1. For rough turbulent flow the stochastic method gives 2.29 for downward seepage (+); 1.38 for upward seepage (-); 1.86 without seepage.
2. For smooth turbulent flow the stochastic method gives 3.12 for downward seepage; 1.82 for upward seepage; 2.51 without seepage.
3. For laminar flow the stochastic method gives 0.36 for downward seepage; 0.19 for upward seepage; 0.28 without seepage.

Thus the effect of finite bandwidth is to reduce the Shields parameters compared with the values obtained by using the narrow-band approximation. In this example the reduction is 13% for rough turbulent flow; 12% for smooth turbulent flow; 5-8 % for laminar flow.

4.5 Comments to examples

In these examples it is the effect of bed shear stress which dominates, i.e. the numerator in Eq. (1) contains the parenthesis (1 ± 0.4), while the denominator contains the parenthesis (1.65 ± 0.06) after the substitution of the values of the physical parameters in Eq. (1). Thus the effect of increased and reduced bed shear stress for flow into and out of the bed, respectively, dominates the stabilizing and destabilizing effect of the sediments, respectively.

From Tables 1 and 2 it appears for all three flow regimes that the bottom friction based on the deterministic method by substituting the significant values of the wave-induced quantities in the otherwise regular wave formulas gives values which agree very well with those obtained by the stochastic method based on the narrow-band assumption. This is the case for the Shields parameters under the wave crest and under the wave trough both with and without the effects of seepage. It appears that the stochastic to deterministic method ratios are in the range 1.00 to 1.05. For laminar flow without seepage effects it should be noted that the stochastic and deterministic methods coincide.

As demonstrated in Section 4.4 there is an uncertainty associated with the narrow-band approximation. By using the Longuet-Higgins (1980) modified Rayleigh distribution of wave heights, the effects of finite bandwidth are to reduce the bottom shear stress (Shields parameter) compared with the values obtained based on the narrow-band assumption. The reduction is in the range 5-13% percent depending on the flow regime.

4 GENERAL COMMENTS

It should be noted that the present theory is valid for linear waves only, suggesting that it should be applied outside the surf zone where the non-linearities become less important. The implications of including non-linearities (e.g. Stokes wave asymmetry with higher wave crests and shallower wave troughs) are expected to enhance the seepage effect under the wave crest leading to larger bottom friction under the wave crest than for linear waves, and to reduce the seepage effects under the wave trough leading to smaller bottom friction under the wave trough than for linear waves.

The importance of the seepage effect on the bottom friction depends on the flow conditions in a non-trivial way. Here it is inherent in the modified Shields parameter in Eq. (1), i.e. how the seepage effect changes the bed shear stress in the numerator and the effective weight in the denominator. More discussion of the seepage effect on e.g. sediment transport is given in Nielsen (1997).

Generally, it is recommended to use a stochastic approach rather than using the rms-values or other characteristic quantities in an otherwise deterministic approach since this will provide global stochastic features associated with the random waves. Although it appears in these examples that the deterministic method agrees very well with the stochastic method if the significant value is used, this is not necessarily the case if other characteristic statistical values are used in the regular wave formulas. The most appropriate statistical value to use will depend on the problem considered and the flow conditions, and can only be determined by performing an analysis spanning out a wide parameter range. All this information is contained in a stochastic method, and hence this should be used to make assessment of seepage effects on bottom friction under random waves based on available wave statistics. However, comparison with data are required before a conclusion regarding the validity of this approach can be given.

5 SUMMARY

The bottom friction beneath random waves is predicted taking into account the effect of seepage flow. The method is based on assuming the waves to be a stationary Gaussian narrow-band random process, using wave friction factors for rough turbulent, smooth turbulent and laminar flow for regular waves including seepage flow effects by adopting the Nielsen (1997) modified Shields parameter. Examples are also included to demonstrate the applicability of the results for practical purposes using data typical to field conditions. In this example the effect of increased and reduced bed shear stress for flow into (under the wave crest) and out of (under the wave trough) the bed, respectively, dominates the stabilizing and destabilizing effect of the sediments, respectively. The present analytical results should be used to provide more details of the inherent stochastic features, and to make assessment of seepage effects on the bottom friction under random waves based on available wave statistics. Generally, it is recommended to use a stochastic approach rather than using characteristic statistical values of the wave-related quantities in an otherwise deterministic approach.

Although simple, the present method should be useful as a first approximation to represent the stochastic properties of the bottom friction due to seepage flow under random waves. However, comparisons with data are required before a conclusion regarding the validity of this approach can be given. In the meantime, the method should be useful as an engineering tool for the assessment of seepage effects on the bottom friction by random waves.

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APPENDIX

Let x be Rayleigh distributed with the *pdf*

$$p(x) = 2x\exp(-x^2) \; ; \; x \geq 0 \tag{A1}$$

The expected value of the $(1/n)$ the largest values of x is given as

$$E[x_{1/n}] = n\int_{x_{1/n}}^{\infty} xp(x)dx \tag{A2}$$

where $x_{1/n}$ is the value of x which is exceeded by the probability $1/n$.

Moreover, from Abramowitz and Stegun (1972, Ch. 6.5, Eq. (6.5.3)) it is given that

$$\Gamma(a,x) = \int_{x}^{\infty} e^{-t}t^{a-1}dt \tag{A3}$$

By utilizing this, the following result is obtained

$$\int_{x_1}^{\infty} x^m p(x)dx = \int_{x_1}^{\infty} x^m 2x\exp(-x^2)dx = \Gamma\left(1+\frac{m}{2}, x_1^2\right) \quad \text{(A4)}$$

by using Eq. (A1), and where $\Gamma(.,.)$ is the incomplete gamma function: $\Gamma(x,0) = \Gamma(x)$ where Γ is the gamma function. The result in Eq. (A4) is obtained by substituting $t = x^2$ in the second integral in Eq. (A4) and using Eq. (A3).

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Table 1. Example of results for seepage flow effects on the bottom friction for rough turbulent flow by random waves.

$H_{rms}(m)$	3.54
k_p(rad/m)	0.0667
U_{rms}(m/s)	1.06
A_{rms}(m)	1.50
A_{rms}/z_0	90 000
c,d	0.112, 0.25
u_{*rms}(m/s) (Eq. (22))	0.0603
θ_{wrms} (Eq. (23))	1.121
$w = \pm 0.025 u_{*rms}$ (m/s)	± 0.00151
$K = w/0.15$(m/s)	0.0101
b(Eq.(25))	0.4
$C_+(\mathrm{m}^2/\mathrm{s}^2)$ (Eq. (26))	0.003355
$C_-(\mathrm{m}^2/\mathrm{s}^2)$ (Eq. (26))	0.003120
Stochastic with seepage flow	
θ_{w+} (Eq. (28))	2.64
θ_{w-} (Eq. (28))	1.59
Without seepage flow	
θ_w (Eq. (29))	2.13
Deterministic with seepage flow using significant values	
θ_{w+} (Eq. (31))	2.57
θ_{w-} (Eq. (31))	1.51
Without seepage flow using significant values	
θ_w (Eq. (32))	2.05

Table 2. Example of results for seepage flow effects on the bottom friction for smooth turbulent and laminar flow by random waves.

	Smooth turbulent	**Laminar**
H_{rms} (m)	2.83	0.71
k_p (rad/m)	0.0667	0.0667
U_{rms} (m/s)	0.85	0.21
A_{rms} (m)	1.20	0.30
Re_{rms}	$7.5 \cdot 10^5$	$4.6 \cdot 10^4$
r, s	0.045, 0.175	2, 0.5
u_{*rms} (m/s) $\begin{cases} \text{Eq. (20), smooth turbulent} \\ \text{Eq. (19), laminar} \end{cases}$	0.0390	0.0143
θ_{wrms} (Eq. (23))	1.57	0.21
$w = \pm 0.025 u_{*rms}$ (m/s)	±0.000975	±0.000358
$K = w / 0.15$ (m/s)	0.0065	0.0024
b (Eq. (25))	0.4	0.4
C_+ $(\mathrm{m}^2/\mathrm{s}^2)$ (Eq. (26))	$1.01 \cdot 10^{-3}$	$1.01 \cdot 10^{-3}$
C_- $(\mathrm{m}^2/\mathrm{s}^2)$ (Eq. (26))	$0.94 \cdot 10^{-3}$	$0.94 \cdot 10^{-3}$
Stochastic with seepage flow		
θ_{w+} (Eq. (28))	3.54	0.39
θ_{w-} (Eq. (28))	2.07	0.20
Without seepage flow		
θ_w (Eq. (29))	2.85	0.30
Deterministic with seepage flow using significant values		
θ_{w+} (Eq. (31))	3.47	0.38
θ_{w-} (Eq. (31))	2.01	0.20
Without seepage flow using significant values		
θ_w (Eq. (32))	2.77	0.30

Highlights

- A simple analytical tool for estimating seepage effects on bottom friction by random waves
- The results are valid for laminar, smooth turbulent and rough turbulent flow
- Example of results corresponding to typical field conditions
- Relevant for making assessment of seepage effects on bottom friction based on available wave statistics