

**INTERNAL FLUID EFFECT INSIDE A FLOATING STRUCTURE: FROM FREQUENCY  
DOMAIN SOLUTION TO TIME DOMAIN SOLUTION**

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**ABSTRACT**

Liquid inside a floating structure influences both hydrostatic and hydrodynamic response of the floating structure. Some examples of floating structure with internal liquid are: vessels with roll damping tanks, floating hydrocarbon storage facility, LNG tankers and etc. A floating oil storage tank is considered in this study, which has a cuboid – shaped external wall, cylinder – shaped internal wall and simple structure configurations for its roof and bottom. Influence of internal liquid on the hydrostatic response of floating structures is well established and must be taken into consideration. The internal liquid reduces the stability of floating structures. The focus of this study is the influence of internal liquid on the hydrodynamic response of floating tank. Frequency domain analysis is performed with WAMIT, for partially filled tank with both solid mass and liquid mass. By comparing the different cases, the force induced by the internal liquid on the floating tank is illustrated. Based on the WAMIT calculated radiation damping force for the external flow, Impulse Response Function (IRF) connecting frequency domain and time domain solution is constructed and the force and moment induced by internal liquid is considered as an excitation force. By assuming linear tank motion, the internal liquid induced force is related to the incoming wave by a set of force transfer functions and it is moved to the right hand side of the tank motion equation. In practice, this set of force transfer

function due to internal liquid has to be combined with the wave excitation force transfer function and it is the total force transfer function (internal liquid plus wave excitation) to be imported to SIMO. SIMO time domain simulation is performed in regular waves. Motion transfer functions from WAMIT frequency domain and SIMO time domain calculations are compared and reasonable agreement is achieved.

**INTRODUCTION**

The internal liquid contained in a floating structure has its own natural frequencies. When there are excitations close to one of its natural frequencies, the motion of internal liquid is amplified and the phenomenon is known as resonance. The main effects of the resonance motion of internal liquid on the floating structure are two folded. The first is the induced impact pressure onto the tank walls/roof (local effect) and the second is its influence on the motion of the floating structure (global effect). The focus of this paper is on the second issue, which has been studied both experimentally and numerically in recent years.

One of the early experimental studies is performed by Mikelis et al. [1]. In this paper, measured loads on a partially filled prismatic tank and three cargo tanks on a ship model were reported. It also reported calculated internal liquid motion and

its induced pressure, by two-dimensional finite difference approach where the Navier-Stokes equations were solved for each cell in conjunction with appropriate boundary conditions and ancillary equations. Coupling between ship motion and sloshing was considered, where only roll motion was allowed. Francescutto and Contento [2] conducted an experiment to investigate the coupling effect between the roll motion response of a ship in regular beam sea and the sloshing flow of the liquid in a compartment. The roll amplitude of the ship was obtained by a nonlinear roll motion equation accounting for the induced sloshing moment at the right hand side. The internal liquid flow was solved within potential theory, where inertial frame of reference and fully moving panel method are applied. Molin et al. [3] carried out experiments of a rectangular barge with two tanks subject to irregular beam seas. A numerical model was proposed to solve the coupled problem between internal liquid motion and barge motion. The model is based on linearized potential flow theory and decomposes the sloshing motion of internal liquid over a finite number of modes. The tanks did not have roof in these tests. Later Molin et al. [4] performed model tests where tanks were fitted with roofs, based on those tests as in [3]. Rognebakke and Faltinsen [5] performed series of model tests where the tank model at different filling conditions was excited in sway by regular waves and a comparison of numerical simulation and experimental data was also presented. The paper concluded, "even if violent sloshing occurs in the tanks, the steady-state motion is almost linear and sinusoidal with the frequency of the linear incident waves" for the considered case. The sloshing was modelled by use of multimodal approach for rectangular tank and 2-dimensional flow. Gaillardet et al. [6] published experimental results of the SALT JIP [7] on the coupling between liquid motion in tanks and vessel motion. Clauss et al. [8] conducted experiments for a 138,000m<sup>3</sup> LNG with four membrane tanks and a numerical model was also established by WAMIT [9] in the frequency domain. The focus of the study is the influence of sloshing on the surge and roll motions of the LNG. Zhao et al. [10] carried out a series of two-dimensional model tests to study the hydrodynamic performance of a FLNG section with internal sloshing. The reference FLNG section is ballasted with fresh water and equivalent solid weights respectively, to clarify the coupling effects. A review on the tank sloshing effect on vessel motion is given in Zhao et al. [11]. In [12], Zhao proposed a numerical method to solve the coupling between internal liquid motion and vessel motion. The method is based on potential flow theory with numerical solution directly performed in time domain. A section of an FLNG is considered in the calculation with only sway motion is allowed. Other experimental work on this topic can be found in Nam and Kim [13], Bunnik and Huijsmans [14] and Nasar et al. [15, 16], for examples.

A floating structure with partially filled tank has two fluid domains: one is external to the floating structure and the other is inside the tank. The external flow is treated in the same way as for standard vessels/offshore platforms. Different ways of

treating the internal liquid motion are discussed here. The first category of methods is based on potential flow theory.

Molin et al. [4], and Rognebakke and Faltinsen [5] proposed methods on expressing the internal liquid sloshing by a superposition of different sloshing modes. In these two studies, only 2-dimensional cases are considered. Malenica et al. [17] and Newman [18] developed solutions to solve velocity potential directly, under linear potential theory and boundary element approach. In [17], a barge is considered and in [18] a spheroidal hull with three tanks is considered. Mitra et al. [19] investigated the coupling between 6DOF vessel motion and fluid sloshing inside a rectangular container in the time domain. The motion of internal liquid is simulated using potential flow equation and the finite element technique.

The second category of methods is based on the Navier-Stokes equation solver for viscous flow. Lee et al. [20] investigated effects of tank sloshing on LNG vessel response. The liquid sloshing in a tank is simulated in time domain by a Navier Stokes solver. A finite difference method is applied. Single – valued free surface profile is assumed. The computed sloshing forces and moments are then applied as external excitation to the LNG motion equation. Yu et al. [21] developed a numerical method for simulation of violent sloshing flow inside a 3D LNG tank where wave breaking and liquid-gas interaction is considered. The method has advantage of giving accurate prediction of impact pressure on the LNG tank including violent free surface motion, three-dimensional instability and air trapping effect, while other methods focus on presenting an accurate estimation of the total force/moment due to sloshing onto the tank. The focus of the paper is sloshing induced pressure and its effect on vessel motion is not discussed. Arai et al. [22] presented a study based on solving Euler equation with finite difference method for the liquid inside tanks of a LNG carrier. The impact pressure on the tank ceiling is treated properly. Impact pressure at a few different locations is predicted under regular and irregular wave excitation. In addition, model experiments were carried out to verify the results of the numerical method. The focus is sloshing induced pressure and its effect on vessel motion is not discussed in the paper. Twillert [23] applied both frequency and time domain approaches to study the effect of sloshing on ship motion. Experimental study was also performed in his work.

The two commercial 3D panel codes considering internal liquid effect are: WAMIT and HYDROSTAR [24]. In WAMIT, the interior wetted surfaces of the tanks are included as an extension of the conventional computational domain defined by the exterior wetted surface. The separate fluid domains are independent and the same free surface conditions are applied for both domains. In HYDROSTAR, a fictitious force depending on the fluid velocity is introduced to represent damping effect without modifying the inviscid and irrotational properties of fluid [24]. By applying perturbation procedure, the

boundary value problems of the first order and second order are then developed.

This paper presents an engineering approach to assess the hydrodynamic response of a partially filled floating tank, by applying WAMIT and SIMO [25]. A linear frequency domain analysis is firstly performed by WAMIT, with hydrodynamic coefficients and motion transfer functions of the floating tank obtained and presented. Based on the WAMIT calculated radiation damping force for the external flow, Impulse Response Function (IRF) connecting frequency domain and time domain solution is constructed and the force and moment induced by internal liquid is considered as an excitation force. SIMO time domain simulation is performed in regular waves. Motion transfer functions from WAMIT frequency domain and SIMO time domain calculations are compared.

### PROBLEM DESCRIPTION

The floating tank considered in this paper is illustrated in Figure 1. The tank is a double – wall design and has simple roof and bottom structures. The tank has external breadth 35m, internal diameter 32m and overall height 20m. Air is between the double walls. One loading condition is considered: 50% partially filled tank. And two cases are considered: solid mass filled and liquid mass filled. It is assumed that the solid and liquid have the same density so that the mass properties of the complete floating tank are the same under these two cases. The applied density is 800kg/m<sup>3</sup>, close to the density of light crude oil. Density of water is 1025kg/m<sup>3</sup>. Water depth 18m is applied. Tank particulars applied in the hydrodynamic analysis are presented in Table 1.

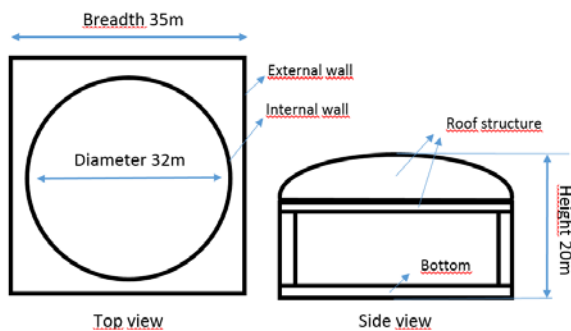


Figure 1 Illustration of floating tank

Table 1 Main parameters of floating tank

Case no.	Load. cond.	Draft (m)	Mass. (kg)	VCG* (m)	r44*/r66* (m)
1	50% (solid)	11.2	1.41E+07	6.8	14/20
2	50% (liquid)	11.2	9.67E+06*	7.8*	16*/22*

In Table 1, VCG is with reference to external tank bottom, positive up. r44 / r55 / r66 is radius of gyration for roll, pitch

and yaw and r55 = r44 due to double symmetry. r44 and r55 are referred to the calm water surface. For fluid filled case: mass properties are given for the empty tank without filling as required by WAMIT input specification.

The tank is designed to be self – stable. Stability issue is not discussed in this paper. In the following sections, the hydrodynamic response of the free floating tank under 50% loading condition is analyzed. Regular wave is considered with no wind or current.

### FREQUENCY DOMAIN SOLUTION

Frequency domain hydrodynamic analysis is performed by WAMIT, for the two cases in Table 1. Part of the WAMIT calculation results are compared with similar HYDROSTAR calculation results. Motion transfer functions are presented in the first place. The calculated first resonance frequency of internal liquid is then compared with analytical solution of the resonance frequency. Radiation added mass and damping of the floating tank are presented. The effect of adding linear damping coefficient in WAMIT calculation is illustrated in subsection "Linear damping effect".

The coordinate system applied in WAMIT calculation together with the wave direction definition is shown in Figure 2. Z-axis is positive upward and the origin O of the coordinate system is located on the calm water surface and at tank center. The WAMIT calculation is referred to the origin of this coordinate system.

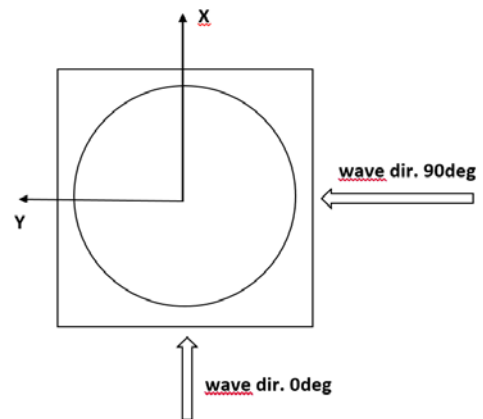


Figure 2 WAMIT coordinate system and wave direction definition

### Tank motion response

The motion equation of the floating tank in frequency domain is given in Equation (1).

$$\{-\omega^2[M + A(\omega)] + i\omega B(\omega) + C\}X(\omega) = F(\omega),$$

Equation (1)

where  $M$  is the mass matrix of the floating tank,  $A(\omega)$  and  $B(\omega)$  are the total radiation added mass and damping matrices due to external flow and internal liquid,  $C$  is the restoring matrix,  $F(\omega)$  is the amplitude vector of wave excitation force and  $X(\omega)$  is the amplitude vector of 6DOF motion.

By dividing the radiation added mass into two components, Equation (1) is re-arranged.

$$\begin{aligned} \{-\omega^2[M + A_{ext}^\infty] + i\omega[i\omega a_{ext}(\omega) + B(\omega)] + C\}X(\omega) \\ = F(\omega) + \omega^2 A_{int}X(\omega), \end{aligned}$$

$$A(\omega) = A_{ext}^\infty + a_{ext}(\omega) + A_{int}(\omega),$$

Equation (2)

where  $A_{ext}^\infty$  is the added mass value at infinite wave frequency due to external flow,  $a_{ext}$  is the remaining added mass due to external flow and  $A_{int}$  is the total added mass due to internal liquid. Damping due to internal liquid is taken as zero so  $B(\omega)$  is radiation damping due to external flow.

WAMIT calculates the radiation added mass and damping forces, hydrostatic restoring force and wave excitation forces. With the mass matrix of the floating tank and user specified extra damping and stiffness matrices, the motion transfer functions are also calculated by WAMIT. For roll and pitch motion, 15% of critical damping is added as extra damping. No extra stiffness is added in this study. Mean drift force is not considered in this study, though it can be calculated by WAMIT.

For case no. 2 in Table 1, the internal liquid effect is taken into consideration by WAMIT. The hydrostatic effect of the internal liquid is often considered in standard analysis. The existence of internal liquid reduces the transverse and longitudinal metacentric height so it decreases the stability of the floating tank. This hydrostatic effect is considered by WAMIT when internal liquid / internal free surface is specified.

In addition to this, WAMIT includes options to analyze the linear hydrodynamic parameters for liquid inside an oscillatory tank, or to analyze the coupled problem where one or more tanks are placed within the interior of one or more bodies, including their dynamic coupling. Usually the fluid in each tank is bounded above by a free surface. The free surface boundary condition in each tank is linearized in the same manner as for the exterior free surface. Special attention is required near the natural frequencies of the internal liquid, where nonlinear effects are significant [9].

Within linear potential theory, the hydrodynamic forces onto a floating body in waves are categorized into wave excitation force and radiation force. Wave excitation force is the force onto a fixed body in waves while radiation force is the force onto an oscillating body in calm water. For the case with

internal liquid, WAMIT takes the tank without internal filled liquid as the body to be analyzed. The effect of internal liquid on to the tank (without internal liquid) is included as additional radiation force.

Assuming linear tank motion, the internal liquid contributes only to the added mass force [18]. The motion of the internal liquid is constrained in the tank and it does not propagate so there is no potential damping in theory. In WAMIT calculation, the damping force induced by internal liquid onto the tank (without internal fluid) is practically zero except the vicinity of the resonance frequency of internal liquid [9]. The added mass and damping force due to internal liquid are to be illustrated in the following Sections.

The linear motion transfer functions of the floating tank are calculated for the two cases in Table 1. In the present study, wave direction 0deg is considered and surge, heave and pitch motion are evaluated. The RAOs (Response Amplitude Operator) of the motion transfer functions are presented in Figures 3 – 5, respectively. Green curve is from a similar HYDROSTAR calculation with liquid in tank; red curve is WAMIT calculation with liquid in tank; blue curve is WAMIT calculation with solid mass in tank.

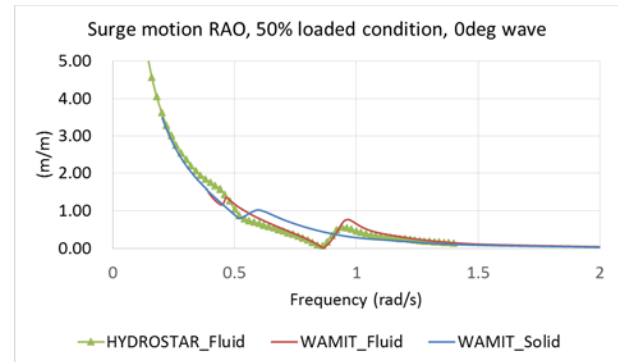


Figure 3 Surge motion RAO

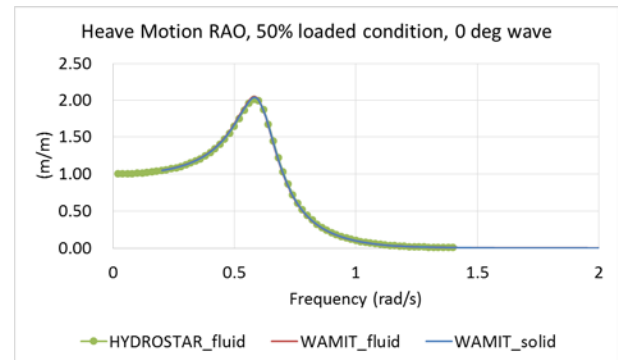


Figure 4 Heave motion RAO

WAMIT calculation indicates that the first internal liquid resonance induces large surge and pitch response at frequency 0.95rad/s or period 6.6s. Secondly, it is observed the internal

liquid does not influence the heave motion RAO much for the considered tank with double-symmetry. Due to free surface effect, the natural frequency of the floating tank itself in pitch is reduced from 5.7rad/s to 4.5 rad/s (both are from WAMIT calculation). For surge and pitch motion, HYDROSTAR and WAMIT calculation do not give identical RAO due to the slightly different mass properties.

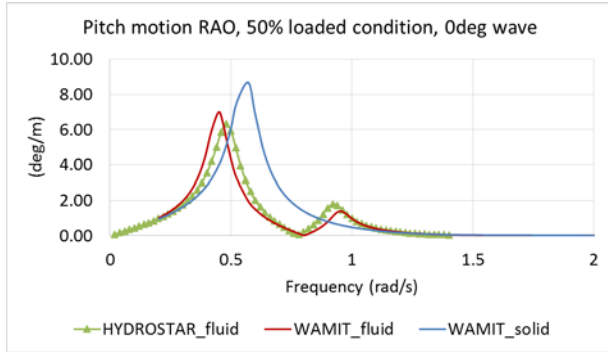


Figure 5 Pitch motion RAO

### Resonance frequency of internal fluid

The floating tank under study has a cylindrical internal tank, for which analytical solution of the internal liquid resonance frequency exists. The resonance frequencies are obtained from the following equation [26]:

$$\sigma_{mn}^2 = \frac{g}{R} \xi_{mn} \tanh\left(\xi_{mn} \frac{h}{R}\right), \quad m, n = 0, 1, \dots \quad \text{Equation (3)}$$

where  $g$  is gravitational acceleration,  $R$  is radius of the internal tank,  $h$  is the height of filled liquid and  $\xi_{mn}$  is the roots of the first derivative of Bessel function of first kind. In the case  $m=0$ , the motion is symmetrical about the longitudinal axis of the cylindrical tank and the most interesting modes of the anti-symmetrical class are those corresponding to  $m=1$ . The lowest roots of the first derivative of Bessel function of first kind in both symmetrical and anti-symmetrical cases are summarized in Table 2.

Table 2 Roots of Bessel function

m/n	$\xi_{mn}$	m/n	$\xi_{mn}$
0/1	3.8317	1/1	1.841
0/2	7.0160	1/2	5.332
0/3	10.1735	1/3	8.536

For the floating tank under consideration, it is most interesting to find the first resonance frequency (smallest resonance frequency/largest resonance period) of the internal liquid in surge/sway motion. Based on Equation (3) and the values of  $\xi_{mn}$  presented in Table 2, the first resonance frequency of the internal liquid is calculated, which is compared with the resonance frequency from WAMIT calculation (corresponding frequency at spikes of added mass and damping curves) in

Table 3. As shown, the WAMIT calculated resonance frequency agrees well with the analytical solution.

Table 3 Comparison of first resonance frequency of internal liquid analytical estimation vs. WAMIT estimation

Tank loading condition	Analytical Est.	WAMIT Cal.
50%	0.86 (rad/s)	0.86(rad/s)

### Linear damping effect

A linear damping coefficient can be specified in WAMIT calculation to represent viscous damping effect. The application of an additional linear damping term in WAMIT calculation is most relevant for the roll motion for ships and for the heave motion for semi-submersibles due to viscous effect. One way to tune the linear damping coefficient is to compare calculated motion with measured motion at a certain sea state and require a good agreement of the standard deviation of motion. It is possible that different damping coefficient needs to be applied for different sea state. The normal range of the linear damping coefficient is within 10% of the critical damping coefficient.

To illustrate the linear damping effect on the hydrodynamic response of the floating tank, the 50% loaded case with liquid inside is considered. Pitch motion is evaluated. Three different linear damping coefficients are applied and they are equivalent to 5%, 10% and 15% of the critical damping, respectively. Value of critical damping is calculated as 1.92E+09Nsm. The RAOs of pitch motion with different linear damping coefficients are presented in Figure 6.

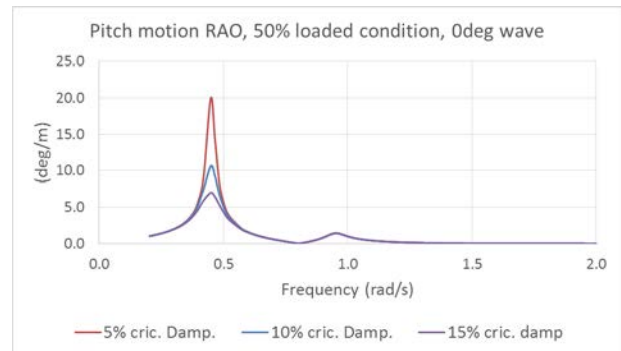


Figure 6 Pitch motion RAO with different linear damping applied

As shown in Figure 6, the linear damping is effective in controlling the motion response amplitude at the tank's own natural frequency, but not the response amplitude due to internal fluid resonance.

### Radiation force for liquid and solid mass filled tank

As explained in earlier sections, the difference of hydrodynamic coefficients for a solid mass filled and a liquid mass filled tank is in the radiation force. To illustrate the difference in radiation force, a few added mass and damping curves for the two cases

in Table 1 are presented. The selected curves include: A11/B11 and A55/B55. The added mass and damping curves are presented in Figures 7-8 and Figures 9-10, respectively.

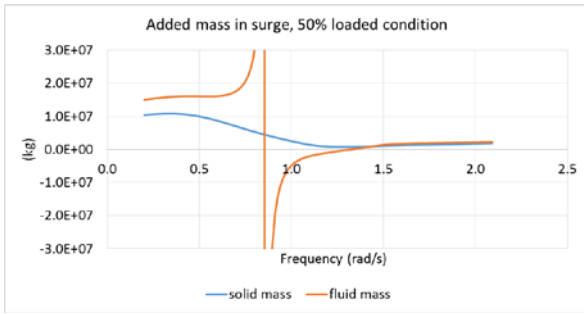


Figure 7 Added mass in surge

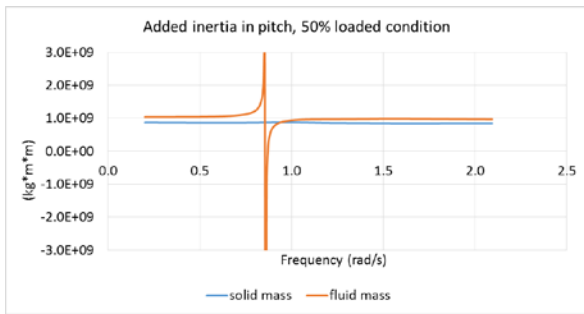


Figure 8 Added inertia in pitch

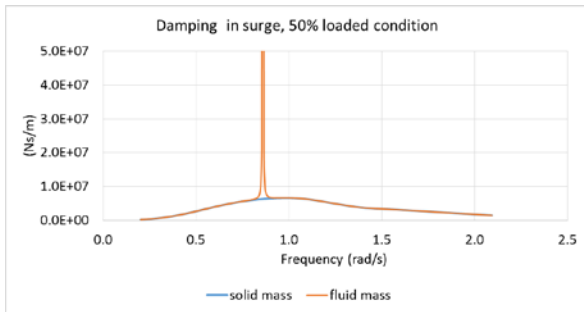


Figure 9 Damping in surge

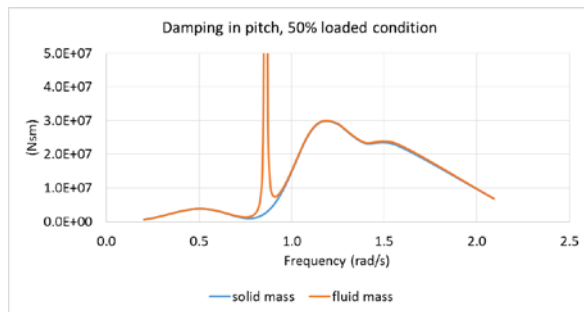


Figure 10 Damping in pitch

As shown in Figures 7-10, the internal liquid contributes to the radiation forces. For damping, the influence is only reflected around the resonance frequency of internal liquid. For added mass, the influence is observed over the complete frequency

range. For consideration of internal liquid related issue, care need to be taken to treat the spikes in the added mass and damping curves properly.

## TIME DOMAIN SOLUTION

For the case without internal liquid, the transformation from frequency to time domain is well established where radiation damping is often used to construct IRF and a convolution integral of IRF and velocity gives the hydrodynamic force in the time domain corresponding to radiation force in the frequency domain. This approach suits a linear problem. For a tank with internal liquid, there is strong nonlinearity at internal liquid resonance though the problem may still have a linear character at other frequencies. If the interest is to know the actual surface elevation inside the tank and/or impact pressure at resonance frequency, for example, one has to treat the resonance problem carefully with a Navier Stokes solver, for example. On the other hand, if the interest is to estimate tank motion when filled with liquid, one can choose a solution which gives a pragmatic approach which gives realistic results and overcomes the limitations of potential theory. In this study, the influence of internal liquid on tank motion is concerned. Therefore, IRF / convolution integral approach is applied for the external flow, and the force/moment from internal liquid onto tank is treated as an excitation force. A set of transfer functions between internal liquid induced force and incoming wave are established, which is based on the linear tank motion transfer functions and the WAMIT calculated added mass coefficients due to internal liquid.

### From frequency domain to time domain

By applying Inverse Fourier Transform and Parseval's theorem, Equation (2) becomes [27]:

$$[M + A_{ext}^{\infty}] \ddot{\mathbf{x}}(t) + \int_0^t \mathbf{k}(t-\tau) \dot{\mathbf{x}}(\tau) d\tau + \mathbf{C}\mathbf{x}(t) = \mathbf{f}(t) + \mathbf{f}_{int}(t),$$

$$\mathbf{k}(\tau) = \frac{1}{\pi} \int_0^{+\infty} [B(\omega) \cos \omega\tau - \omega a_{ext}(\omega) \sin \omega\tau] d\omega$$

Equation (4)

where  $\mathbf{k}(\tau)$  is the IRF also known as retardation function,  $\mathbf{x}(t)$ ,  $\dot{\mathbf{x}}(t)$  and  $\ddot{\mathbf{x}}(t)$  are the vector of motion, velocity and acceleration,  $\mathbf{f}(t)$  is the vector of wave excitation force and  $\mathbf{f}_{int}(t)$  is the force due to internal liquid. The time dependent forces on the right hand side of the equation are obtained by superposing the force at different incoming wave frequencies.

Equation (4) formulates time domain solution of the hydrodynamic response of the floating tank. The most complex part of the solution is related to the calculation of convolution



integral. For the external flow, causality of the radiation effect is true so  $k(\tau) = 0$ , for  $\tau < 0$  is valid. In this case, the IRF can also be calculated by only added mass or only damping as shown in Equation (5).

$$k(\tau) = \frac{2}{\pi} \int_0^{+\infty} B(\omega) \cos \omega \tau d\omega$$

$$k(\tau) = -\frac{2}{\pi} \int_0^{+\infty} \omega a_{ext}(\omega) \sin \omega \tau d\omega.$$

Equation (5)

The radiation damping is often chosen to calculate retardation function as its asymptotic value at infinite frequency is known to be zero. It is then easy to truncate the integral. SIMO [25] uses radiation damping to construct IRF.

The force from internal liquid to tank is considered as an excitation force in the present approach, as shown in Equation (2). In the frequency domain, the added mass force is related to the incoming wave directly through WAMIT calculated motion transfer function and the added mass coefficients due to internal liquid. The relation between this force and incoming wave is presented as a transfer function, similar as for the wave excitation force. The force transfer function due to internal liquid and incoming wave are combined for each wave frequency and each wave direction. This new set of force transfer function is then input to SIMO and only the added mass and damping coefficients due to external flow are imported from WAMIT to SIMO.

The added mass and damping curves have spikes at the resonance frequency of the internal liquid as shown by WAMIT calculation. A small frequency interval has to be applied when performing WAMIT calculation in order to identify these spikes. The spikes in added mass indicates that there is large added mass force around the resonance frequency combined with sign change when crossing the resonance frequency. The spikes of the damping curve are related to the spikes of the added mass curve in the numerical calculation. However, they do not mean that there is huge potential damping at the resonance frequencies.

At resonance of internal liquid, nonlinear effects are significant and WAMIT captures the resonance frequency well. However estimation of damping and further the tank response at the resonance frequency is beyond the capability of potential flow theory. To obtain a good estimation of tank motion, linear/quadratic damping need to be added in SIMO and tuned based on available measured or calculated tank motion where viscous effect is considered properly.

The calculated IRFs of external flow are presented for surge and pitch for the 50% loaded cases in Figures 11 and 12, respectively.

The applied frequency interval for IRF calculation is taken as 0.0005rad/s in the performed calculations.

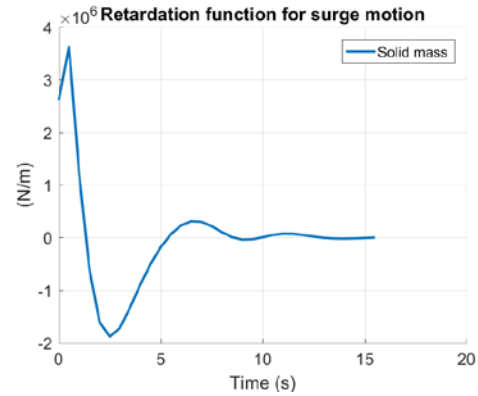


Figure 11 Retardation function of surge, solid filled

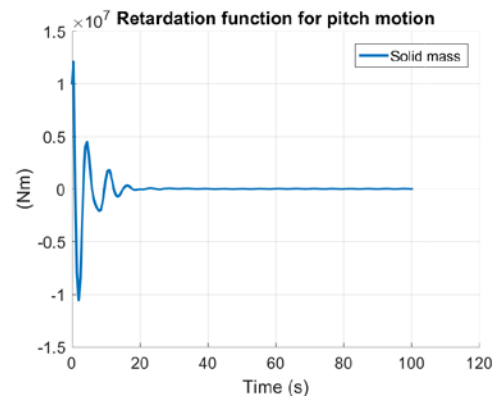


Figure 12 Retardation function of pitch, solid filled

### SIMO time domain simulation with the calculated IRF

SIMO time domain simulations are performed in regular waves, with the constructed IRF and excitation force due to incoming wave and internal liquid effect. By default, SIMO uses the radiation damping to calculate IRF. It is possible to import user specified IRF to SIMO in the .sys file as well.

When importing a WAMIT calculation model into SIMO, SIMO automatically calculates the IRF based on the radiation damping and SIMO recalculates the added mass and damping based on the calculated IRF. The difference between the recalculated added mass and the originally calculated added mass by WAMIT is reported by SIMO as the added mass at infinite frequency. If negative damping is observed in the recalculated damping, a corresponding linear damping term will be added in the SIMO model automatically to make sure the total damping is still positive.

The floating tank is kept in position in the horizontal plane by horizontal soft springs in the SIMO model. The springs are so

soft that they do not interfere with the wave frequency tank motions.

The frequencies of the regular waves applied in SIMO calculation cover the same range as WAMIT calculation. About 20 periods are simulated at each wave period. A high pass filter is applied to the directly calculated tank motion by SIMO to remove any response due to the horizontal soft spring system. Motion transfer functions of the floating tank are then constructed based on the filtered tank motion. The motion transfer functions are referred to the incoming wave at the tank center position, which is the same as WAMIT definition. Comparison of WAMIT and SIMO calculated motion transfer functions are presented and discussed in the next Section.

**Comparison of motion transfer functions from WAMIT frequency domain and SIMO time domain solutions**

Comparison of WAMIT and SIMO calculated motion transfer functions is illustrated in Figure 13 - 18. The case is 50% loading condition with liquid inside and surge, heave and pitch motions are considered.

The first resonance frequency of internal fluid is identified by WAMIT calculation. This resonance frequency is maintained in SIMO time domain simulation. WAMIT is a potential code and it can not consider the viscous effect which is important to quantify the magnitude of the actual tank response at internal liquid resonance. It is possible to add quadratic damping in SIMO time domain simulations to adjust the tank response at internal liquid resonance frequency, when measured tank motion is available, for example.

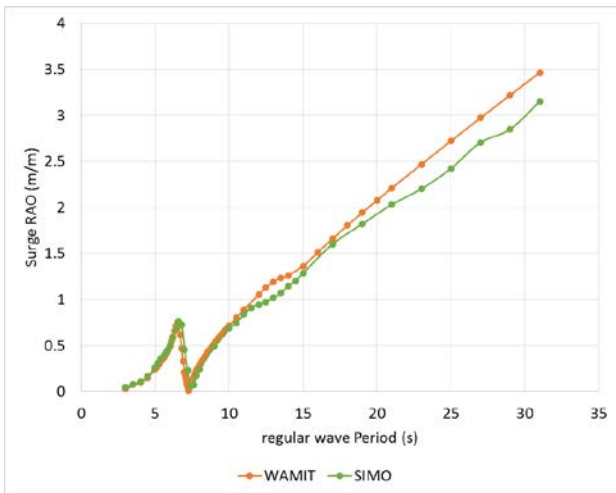


Figure 13 Motion RAO of surge, WAMIT vs. SIMO

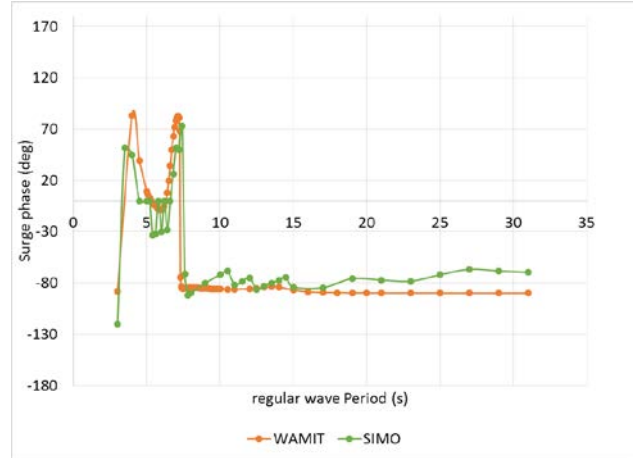


Figure 14 Phase angle of surge, WAMIT vs. SIMO

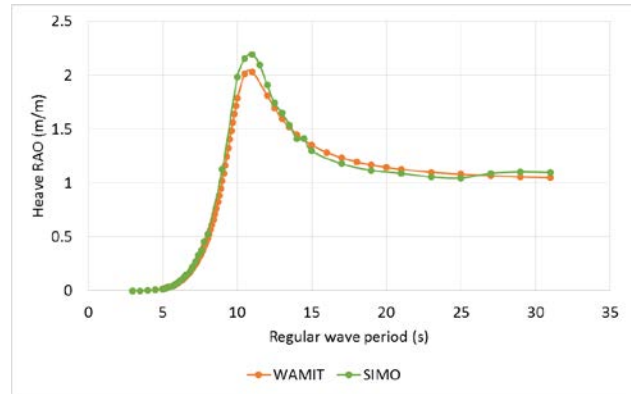


Figure 15 Motion RAO of heave, WAMIT vs. SIMO

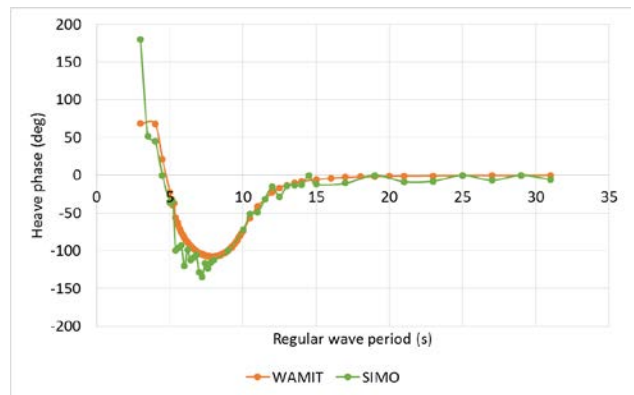


Figure 16 Phase angle of heave, WAMIT vs. SIMO



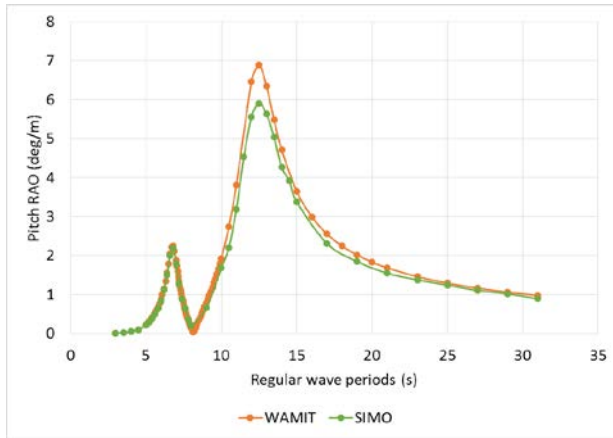


Figure 17 Motion RAO of pitch, WAMIT vs. SIMO

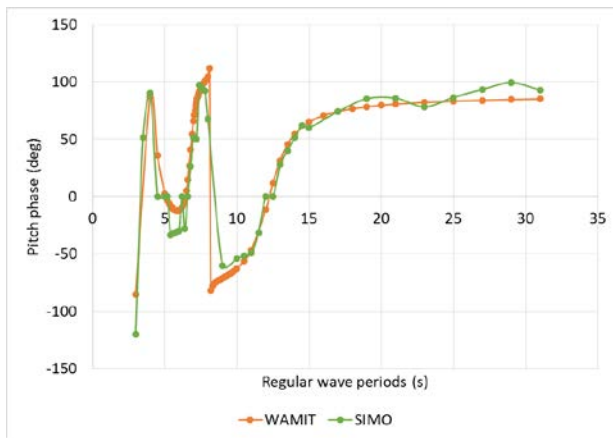


Figure 18 Phase angle of pitch, WAMIT vs. SIMO

## CONCLUSIONS

The hydrodynamic response of a partially filled floating oil storage tank in regular waves is analyzed in this paper. No wind or current condition is assumed. Both solid and liquid filled cases are evaluated to properly illustrate and understand the effect of liquid filling. The analysis is performed in frequency domain by WAMIT initially. The WAMIT calculation results are then imported to SIMO to perform time domain simulations. The purpose of the paper is to implement time domain simulation for an internal liquid related issue based on WAMIT and SIMO.

The internal liquid and external liquid flow are treated separately. To do so, one has to perform two sets of WAMIT calculations: one with liquid filling and one with solid filling. The added mass force from internal liquid onto tank is obtained by taking the difference of the two sets of WAMIT calculations. By assuming linear tank motion, the added mass force from internal liquid onto tank is related to the incoming wave by a set of force transfer functions and it is moved to the right hand side

of the tank motion equation. In practice, this set of force transfer function due to internal liquid has to be combined with the wave excitation force transfer function and it is the total force transfer function (internal liquid plus wave excitation) to be imported to SIMO. The added mass and damping due to external flow shall be imported to SIMO accordingly. This approach is valid on two conditions. First, tank motion is linearly dependent on incoming wave in the calculation of internal liquid induced force; secondly, internal liquid induced force is linearly dependent on tank motion. If one or both conditions are violated, one has to consider an iteration approach to solve the motion equation.

An example of the above mentioned frequency domain and time domain analysis is given in details in this paper. The predicted motion transfer functions of the tank show reasonable agreement between WAMIT frequency domain and SIMO time domain calculations.

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