A bi-objective mixed capacitated general routing problem

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Abstract

The mixed capacitated general routing problem consists of determining routes for vehicles that are to service segments of a mixed network consisting of vertices, edges and arcs. In this study we present a bi-objective version of this problem. In addition to the traditional minimize total route cost objective we introduce several ways of considering route balance as a second objective. Insights on route balance and which effects variants of this objective have on the solutions are given. An exact solution method is used to generate exact Pareto fronts for a number of small benchmark instances. We illustrate the effects on the solutions for four suggestions for route balance objectives.

Keywords: vehicle routing, mixed graph, bi-objective, route balance

1. Introduction

Routing problems arise in many practical applications and have been widely studied. Defined a network, one or several routes need to be created that visits all, or a subset of, the network's vertices, arcs and/or edges. An overview and early references can be found in Gilbert and Osman (1995). Two groups of popular problems are the capacitated vehicle routing problem (CVRP) and the capacitated arc routing problem (CARP). For the CVRP, the service occurs at the vertices in the network, while for the CARP service occurs in arcs and/or edges.

In this paper we consider the mixed capacitated general routing problem (MCGRP), a generalization of the CVRP and CARP. It is defined on a mixed graph $G = (V, E, A)$, where V is the set of vertices, E the set of undirected edges, and A the set of directed arcs. Some of the vertices, edges and arcs are required, i.e. they have a non-negative demand that needs to be serviced. Each edge and arc have a non-negative traversal cost, and required vertices, edges and arcs may have a non-negative service cost. A homogeneous fleet of vehicles with a given service capacity is available for servicing the demand. Typical applications are waste collection and newspaper distributions where arcs/edges represent housing streets and vertices larger facilities like apartment buildings, schools and commercial buildings.

The MCGRP was first introduced in Pandi and Muralidharan (1995), who propose a heuristic solution procedure based on creating one giant route first that is split into individual vehicle routes. The literature on MCGRP is scarce, but includes both heuristic procedures and exact solution approaches: Gutiérrez et al. (2002) propose a heuristic close to a partition-first-route-second procedure. Their heuristic is compared with the one by Pandi and Muralidharan (1995) and shows promising results. Prins and Bouchenoua (2005) use a memetic algorithm, and construct a set

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of benchmark instances. Other methods are simulated annealing (Kokubugata et al., 2007) and adaptive iterated local search (Dell'Amico et al., 2012). The first integer programming formulation was defined in Bosco et al. (2013) who use a branch-and-cut algorithm to find optimal solutions for small problem instances. Lower bounds for the MCGRP was identified in Bach et al. (2013). The MCGRP with stochastic demands is introduced in Beraldi et al. (2015). A chance-constrained integer programming formulation is presented, and solved exactly by a branch-and-cut algorithm and heuristically by a neighbourhood search algorithm.

Common for all previous studies of the MCGRP is that one objective is considered: minimize total route cost. We study a bi-objective version of the problem where we in addition to minimizing total travel cost also want the routes travelled by each vehicle to be balanced. In this paper we consider balanced routes to be of similar travel distance, i.e. they should take around the same time for a vehicle to finish. This is an extension to the MCGRP motivated by the relevance for many real-life problems where each vehicle driver should have similar workloads. How to define an objective for balanced routes is not trivial, and we will introduce and test the effects of several route balance objectives.

The contributions of this paper include providing insights into the bi-objective MCGRP with route balance, and how the problem solutions depend on how route balance is defined. A new solution method for bi-objective problems is used to quickly generate a good representation of the Pareto front.

The remainder of this paper is organized as follows. Section 2 contains some insight on the effects of considering both total cost and route balance for routing problems. Then Section 3 present an overview of related literature. In Section 4 we present an overview of the solution approach to the bi-objective MCGRP with route balance. Section 5 provides the numerical results of a computational study. Finally, Section 6 concludes the paper and suggests alternatives for further research.

2. Bi-objective vehicle routing problems - some insights

We are studying routing problems that have two objectives: Minimizing total travel cost and balancing the routes. These two objectives can be conflicting, and we will illustrate this in the following. In this section we use the VRP in the examples for illustrative purposes. The insights are, however, general and also apply for the MCGRP and the CARP.

Bi-objective routing problems belong to the class of multi-objective optimization problems, see e.g. Ehrgott (2005). These problems can be described as follows:

$$\min \ (f(x_1), f(x_2), ..., f(x_n)),$$
$$\text{subject to} \quad x \in X.$$

For the bi-objective case two objectives are to be minimized, i.e. $n = 2$.

When there are more than one objective, there will in most cases exist several solutions that can all be considered optimal, or non-dominated. Such solutions are often referred to as Pareto optimal. The definitions of weak and strict Pareto optimum are as follows (Caramia and Dell'Olmo, 2008):

Definition A solution x^* is a weak Pareto optimum for the multi-objective problem if and only if there is no $x \in X$ such that $f_i(x) < f_i(x^*)$ for all $i \in \{1, ..., n\}$.

Definition A solution x^* is a strict Pareto optimum for the multi-objective problem if and only if there is no $x \in X$ such that $f_i(x) \leq f_i(x^*)$ for all $i \in \{1, ..., n\}$, with at least one strict inequality.

The image of all Pareto optimal solutions is called a Pareto front.

Routing problems are most often threated as single-objective problems where the objective is some sort of minimizing total cost of the solution, i.e.:

$$Min \quad \sum_{r \in \Omega^*} t_r,$$

where Ω^* is the set of routes in the solution and t_r the cost of route r. Costs will often be associated with time or distance. In this paper, we let travel distance define the cost of a route. I.e. when we discuss total travel cost this will be equivalent to total travel distance. Route balance will then be defined with respect to the travel distance of each individual route in the solution.

In this section we operate with the following three assumptions:

1. We have a fleet of Q homogeneous vehicles with capacity C.

2. In a solution each of the Q vehicles travels one route that starts and ends at the depot and service at least one customer.

3. No symmetry, i.e. there does not exist any set V^* of vertices, arcs or edges that can be visited on a route that has the exact same travel distance as a different route visiting a different set of vertices, arcs or edges where at least one of these is not element in V^*.

It is not trivial to define route balance. When we consider travel distance, we can define a perfectly route balanced solution as one for which each vehicle will be travelling the exact same distance. To obtain route balance in this setting, the two following objectives can be considered:

$$Min \quad \sum_{r \in \Omega^*} (t_r - \mu)^2 \,,$$

$$Min \quad t^L - t^S,$$

where μ is the average travel distance of each individual route in the solution, and t^L and t^S the routes with the longest and shortest travel distance in the solution, respectively. The first of these objectives is then the same as minimizing the variance of the travel distance of each individual route in the solution.

If there for a routing problem exist at least one perfectly route balanced solution, the two balance objectives presented above will have the same optimal value, 0. But the Pareto front for these objectives when the other objective is to minimize total travel distance can be very different. The first minimization problem will require more computational effort than the second, but takes into account the travel distance of all individual routes in the solution. The second minimization problem will ensure that the difference in travel distance between t^L and t^S is small, but does not consider the travel distances of the other routes in the solution. Hence there can be many routes that have incrementally longer travel distance than t^S or many that have incrementally shorter travel distance than t^L. If a decision maker is interested in creating a fair workload, i.e. a fair distribution for the lengths of the routes, a variance objective may be more suitable. On the other hand, a variance objective may result in solutions with extremes, i.e. one substantially longer route or shorter route. This can be undesirable. To avoid such solutions lower and upper bounds on the individual routes' travel distance can be added.

A different objective that, as the variance objective, also takes into account the length of all routes is:

$$Min \quad \sum_{r \in \Omega^*} \left(t_r - t^S \right).$$

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(a) The nine node VRP. The square numbered 0 represents the depot.

(b) Solution when minimizing longest route distance.

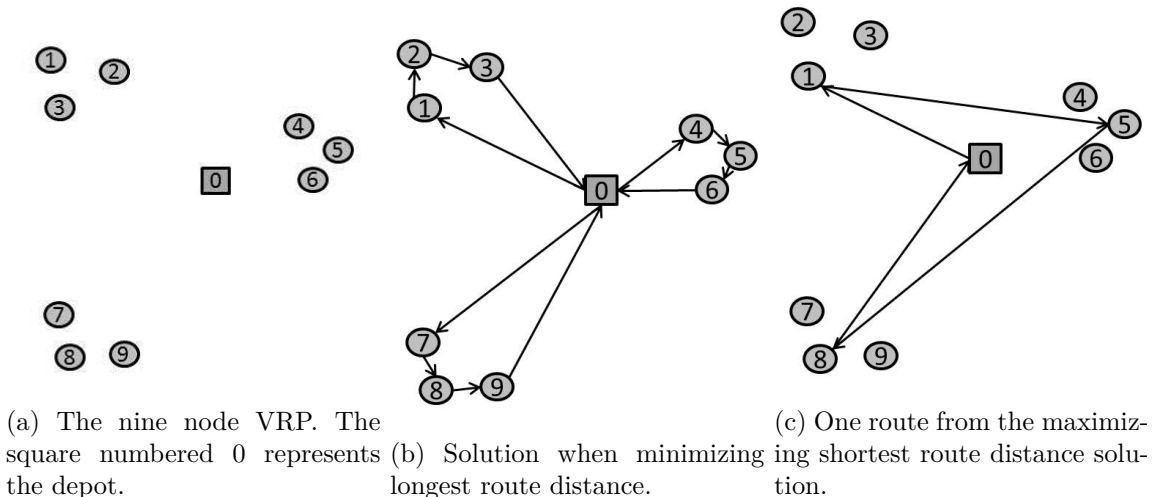(c) One route from the maximizing shortest route distance solution.

Figure 1: Consequences of minimizing longest route distance and maximizing shortest route distance for a nine node VRP.

Here Ω^* is the set of routes in the solution, t_r is the travel distance of route r and t^S the travel distance of the shortest route. This objective has the advantage of considering all route lengths without needing to consider a quadratic objective.

For some real-life routing problems there may exist a target route distance, i.e. a travel distance that all routes should be close to. This can be the case for e.g. the newspaper delivery problem where all the individual routes should be completed within a certain time and all routes for individual newspaper deliverers need to be of similar length. If we define this target distance as T^G this objective can be modelled as follows:

$$Min \quad \sum_{r \in \Omega^*} |t_r - T^G|.$$

In some situations it may not be of importance that the routes are overall balanced, but there may be a preference of either having not too long routes or too short routes. The former can for example be the case for waste collecting problems where the drivers have to sort the waste after returning to the depot (see e.g. Lacomme et al. (2003)). Then one of the following objectives can be used:

$$Max \quad t^S,$$
$$Min \quad t^L,$$

where t^S and t^L are the travel distance for the shortest and longest routes in the solution, respectively. These two objectives can lead to very diverse solutions. An example is illustrated in Figure 1. This is a nine node VRP in the Euclidian space where nodes $1-3$, $4-6$ and $7-9$ are located in clusters with varying travel distances from the depot, 0 (Figure 1a). Figure 1b shows the solution to the minimize longest route problem: Each vehicle route visit one cluster. Figure 1c shows a route in the solution to the maximize shortest route problem: Each vehicle route will visit one node in each cluster.

Maximizing the travel distance for the shortest route in the solution may sound quite conflicting for a decision maker whose primary objective is to minimize cost. However, it may for some routing problems be an efficient means to create solutions that are more route balanced when both objectives are considered. Maximizing t^S is one part of the objective of minimizing the distance between the
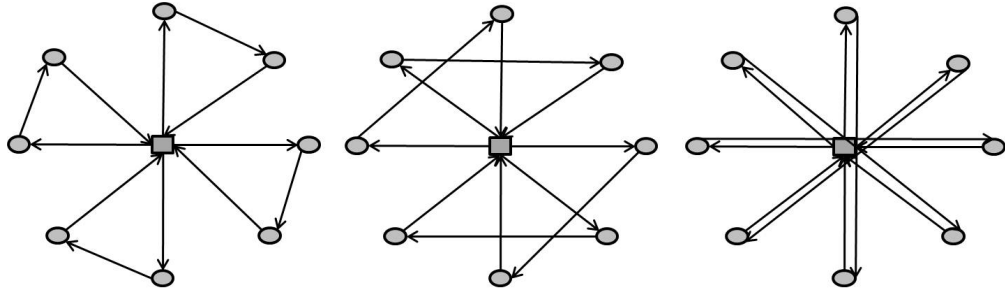
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Figure 2: Examples of solutions that are equally route balanced for a VRP with four vehicles and eight vertices. The square box represents the depot. All required vertices are equally spaced on a circle with radius R from the depot.

longest and shortest route, while minimizing t^L is the other. An interesting observation from the example in Figure 1 is that the solution to the minimizing t^L problem is close to the solution of minimizing total travel distance, while the solution to the maximizing t^S problem is close to the solution of minimizing $t^L - t^S$.

Insights from Bertazzi et al. (2015) show that also the objective of minimizing t^L may have significantly higher total travel distance than the minimizing total travel distance solution. In fact, the authors prove that in the worst case the ratio of the two solutions total travel distance tends to infinity. However, this relies on assumption 2 being relaxed. In general, for most real-life problems, there will be a limited number of capacitated vehicles where most feasible solutions require that each one of them travels one route. Then it may often be possible to find a solution to the minimizing t^L problem which total travel distance is not far from the optimal minimizing total travel distance solution.

For many routing problems there exist several solutions that have the same, or very similar route balance, but where the total travel distance will vary. One example is shown in Figure 2. Here there are eight vertices and four vehicles that visit two vertices each. The vertices are all equally spaced on a circle with distance R from the depot. All three solutions have perfect route balance, i.e. all four routes have the exact same travel distance, but the total travel distance of all routes varies. For the far left solution, each route have travel distance $(2 + \sqrt{2})R$, making the total travel distance $4(2 + \sqrt{2})R$. The next solution has total travel distance of $2(1 + cos(3\pi/8))R$, and the last has total travel distance of $16R$.

In Figure 3, we illustrate the opposite effect of Figure 2: Two solutions to a VRP where both have the same total travel distance, but different route balance. There are four vertices and two vehicles and the travel distances between the vertices are as shown in the far left illustration. Both solutions have a total travel distance of $2(a + b + 2c)$. The solution shown in the middle is perfectly route balanced with two routes each of travel distance $a + b + 2c$, while the other solution has two different routes with a travel distance difference of $2b + 2c - 2a$. In the worst case, a and c are small values (ϵ), and b is a high value, making the difference in travelled distance for the two routes close to $2b$.

For problems like the ones illustrated in Figures 2 and 3, the interesting solutions are the ones that have the lowest total travel distance (Figure 2), and the best route balance (Figure 3).

We have seen how solutions considering different objectives may vary, and one interesting question is how large difference we can have in solutions in terms of total travel distance when considering the two objectives of minimizing total travel distance and maximizing route balance (minimizing route imbalance). We introduce the following definitions:

Definition Let $\mathcal{S}$ be the set of solutions to a routing problem. Then let $T_s\,(s \in \mathcal{S})$ be the to-
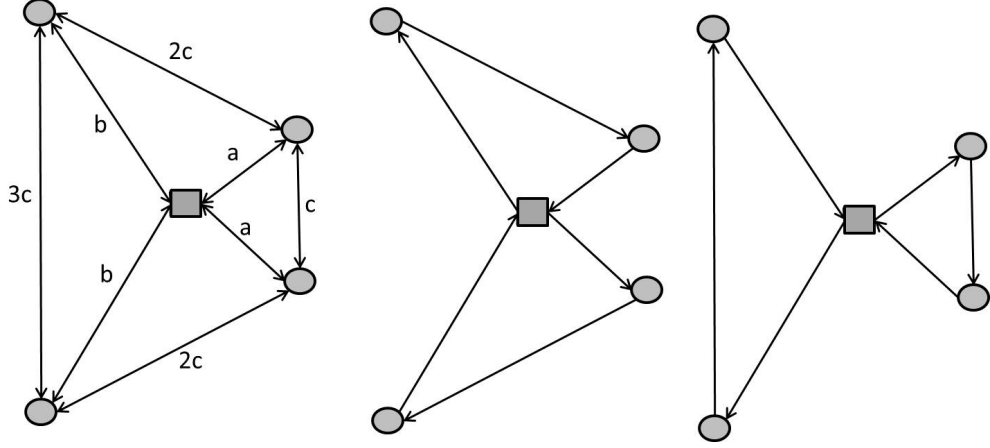
Figure 3: Two solutions with the same total travel distance for a VRP with two vehicles and two required vertices.
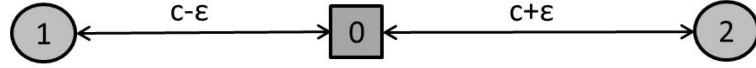

Figure 4: VRP with two vehicles and two required vertices.

tal travel distance of solution s. Further, let $\Delta_s (s \in \mathcal{S})$ be the imbalance of solution s, and $\left(V_s^1, V_s^2, ..., V_s^Q\right) (s \in \mathcal{S})$ be the partition of the required vertices, edges and arcs V visited by the vehicles associated with solution s. Let $l\left(\overline{V}\right)$ be the travel distance of the optimal traveling salesman problem (TSP) visiting the set of all required entities $\overline{V}$ so that $T_s = \sum_{k=1}^{Q} l\left(V_s^k\right)$. Finally, let s^T be the optimal solution to the single-objective minimize total travel distance problem and let s^Δ be the optimal solution to the single-objective minimize imbalance routing problem.

There will for many routing problem exist several optimal solutions to the maximize route balance problem that have different values for T_{s^Δ}. In those cases we are only interested in the one with the shortest travel distance, i.e. s^Δ will always be the solution with shortest value for T_{s^Δ}.

We start by considering a VRP with two vehicles and two required vertices, see Figure 4. We have two required vertices located in directly opposite directions from the depot. Each of them have a demand of D, and we assume that the demand can be split, i.e. a vehicle may visit one vertex and service only a portion of the demand. The travel distance to vertex 1 is $c - \epsilon$ and the travel distance to vertex 2 is $c + \epsilon$ where ϵ is a small number. Then we observe the following travel distance for the optimal solutions to the example illustrated in Figure 4:

$$T_{s^T} = 2 * (c - \epsilon) + 2 * (c + \epsilon) = 2c,$$

$$T_{s^\Delta} = 2 * (c - \epsilon) + 2 * (c + \epsilon) + 2 * (c - \epsilon) + 2 * (c + \epsilon) = 4c = 2 * T_{s^T}.$$

We observe that we get the optimal route balance solution by creating two equal routes that service both vertices. The travel distance for this solution is then twice as long as the solution to the minimize travel distance problem.

Claim 1. $l\left(V_{s^\Delta}^k\right) \leq l\left(V\right) \leq T_{s^T}$.

The first inequality in Claim 1 follows from $V_{s^\Delta}^k \subseteq V$ and the second inequality is a consequence of the length of an optimal TSP tour visiting all vertices is less than or equal to the length of an optimal VRP solution visiting all vertices. This then implies:

Theorem 1. $T_{s^\triangle} = \sum_{k=1}^{Q} l\left(V_{s^\triangle}^k\right) \leq \sum_{k=1}^{Q} T_{s^T} = Q T_{s^T}$.

By Theorem 1 we know that the best route balanced solution we can get has a total travel distance equal to, or less than, the number of routes multiplied by the best total travel distance. In the case where vertices are located in clusters where each of the vertices in a cluster can be serviced by individual routes, it is possible to create a perfectly route balanced solution where each vehicle visits each cluster exactly once. We will illustrate this by an example.

Figure 5 shows a VRP with three clusters that each contain three vertices, there are three vehicles, assumptions 1-3 holds, and each vertex has a demand of $C/3$. That is, a vehicle visiting a cluster can service one, two or three vertices, representing a demand of $C/3$, $2C/3$ or C, respectively. The travel distances from the depot 0 to the clusters and between the clusters are as shown in Figure 5a. The travel distance between each vertex in a cluster is ϵ. As balance objective, we consider minimizing the difference in travel distance between the longest and shortest routes. The following solutions exist:
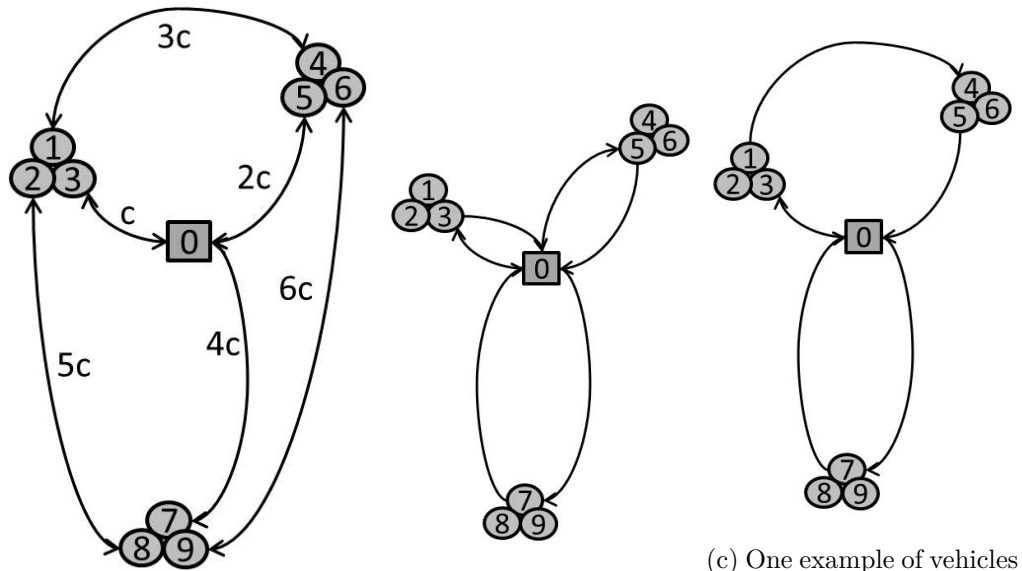
- All vehicles visit one cluster each and service all three vertices, a total demand of C, Figure 5b.

- Combinations of two vehicles visiting two clusters, servicing two vertices and a demand of $2C/3$ in one cluster and one vertex and a demand of $C/3$ in the other, and one vehicle visiting one cluster servicing all vertices and a demand of C, Figure 5c.

- All vehicles visit two clusters each, servicing two vertices in one cluster and one in the other, Figure 5d.

- Combinations of two vehicles visiting two clusters servicing two vertices in one and one vertex in the other cluster, and one vehicle servicing one vertex in all three clusters, Figure 5e.

- All vehicles visit each cluster once servicing one vertex and a demand of $C/3$ in each cluster, Figure 5f.

Figure 6 shows the objective values and Pareto front for the three cluster example when $\epsilon \to 0$. There are three non-dominated solutions: The optimal minimum travel distance solution: each vehicle visits exactly one cluster, Figure 5b, the optimal route balance solution: each vehicle visits all three clusters, Figure 5f, and the shortest travel distance alternative of the combination of routes visiting one or two clusters: Figure 5c. We observe that the total travel distance of the best route balance solution is three times the one for the best travel distance solution.

We observe from Figure 6 that we can create a solution that scores high on the route balance objective at a total travel distance that is close to the minimum total travel distance solution. To get better route balance than this solution the only option is the perfectly balanced solution that has a much higher total travel distance.

We can create a general case of the example illustrated in Figure 5: We have Q vehicles with capacity C that are to service a total of Q^2 customers, each with demand C/Q. Clusters of Q customers are located at distances $c_i, i = 1, ..., Q$ from the depot, where each c_i is unique. To get from a cluster to a different cluster, it is necessary to travel through the depot. The internal travel distances between each customer in a cluster is ϵ. When assumption 3 holds, and $\epsilon \to 0$, the best total travel distance solution will be that each vehicle service all customers in one cluster. The best route balance solution will be that each vehicle service one customer in each cluster. Total travel distances for each of the best solutions are then:

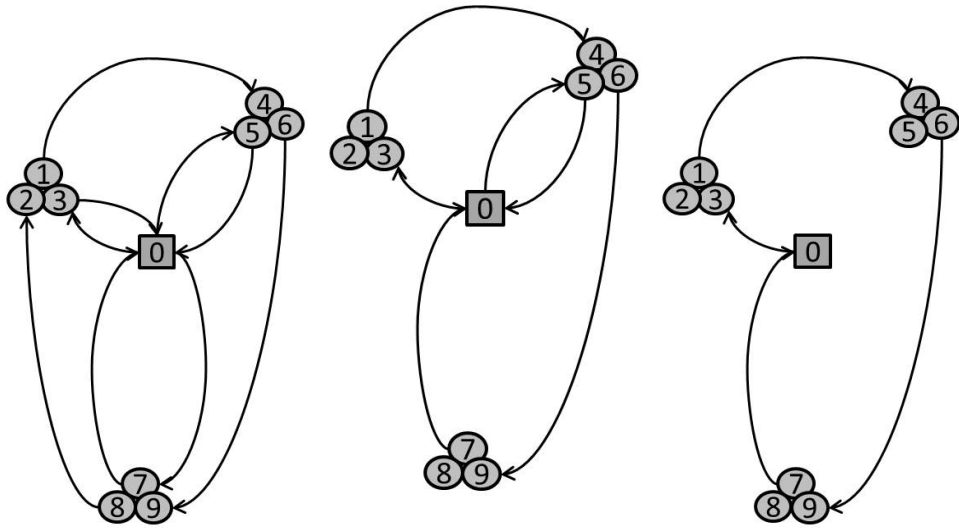$$L_{s^L} = \sum_{i=1}^{Q} 2c_i,$$

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(a) The three cluster VRP.

(b) Vehicles visit one cluster each.

(c) One example of vehicles visiting one or two clusters each.

(d) Vehicles visit two clusters each.

(e) One example of vehicles visiting two or three clusters each.

(f) All vehicles visit all three clusters.

Figure 5: Solutions to the three clusters, nine vertices, three vehicles example.
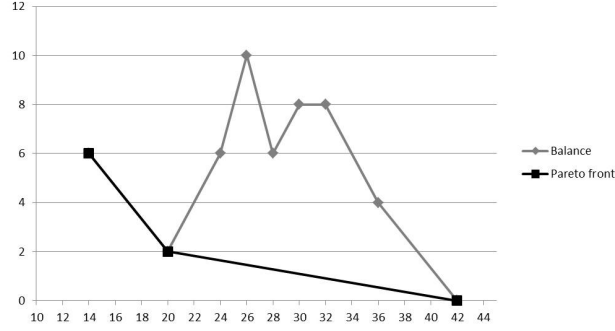
Figure 6: Objective values and Pareto front for the three clusters, nine vertices, three vehicles example. X-axis shows total travel distance and y-axis the balance measured as difference between longest and shortest route.

$$L_{s^\Delta} = Q \sum_{i=1}^{Q} 2c_i = Q L_{s^L}.$$

Bertazzi et al. (2015) show that also when the balance objective is to minimize t^L the worst case is $T_{s^\Delta}/T_{s^T} \to Q$. However, unless $Q = 1$, this ratio will never reach Q. We introduce the following definitions:

Definition Let t^L_Δ be the longest route in the solution to the single-objective $\min t^L$ problem, an let t^L_T be the longest route in the solution to the single-objective minimize total travel distance problem. As before, s^T is the solution to the single-objective minimize total travel distance problem. s^{Δ_L} is the solution to the single-objective $\min t^L$ problem.

Claim 2. If $Q \geq 2$, then $t^L_\Delta \leq t^L_T < T_{s^T}$.

The first inequality in Claim 2 is obvious as the longest route in the solution to the $\min t^L$ problem is at least no longer than the longest route to any other solution to the problem. The strict inequality holds under assumption 2 when there is more than one vehicle. This then implies:

Theorem 2. $T_{s^{\Delta_L}} = \sum_{k=1}^{Q} l\left(V_{s^{\Delta_L}}^k\right) \leq \sum_{k=1}^{Q} t^L_\Delta < QT_{s^T}$.

Theorem 2 holds whenever $Q \geq 2$, and shows that the worst total travel distance we can get is strictly less than the number of vehicles multiplied with the best possible total travel distance when the objective is to minimize t^L. It also holds when we relax our assumption 2, i.e. when all vehicles do not need to travel a route. This implies from the insights illustrated in Figure 2 and 3: We may have several solutions with different total travel distance that still is optimal relative to the $\min t^L$ problem. In these cases, we are only interested in the solution with the lowest total travel distance. Hence, we have:

Theorem 3. If $t^L_\Delta = T_{s^T}$, then $t^L_T = t^L_\Delta$ and $T_{s^{\Delta_L}} = T_{s^T}$.

The first implication in Theorem 3 follows directly from Claim 2 and the second implication follows the first as $t^L_T = t^L_\Delta$ directly implies that there exists an optimal solution to the $\min t^L$ problem that is also optimal for the single-objective minimize total travel distance problem.

For real routing problems a perfectly route balanced solution like the one in Figure 5f will never be of interest. The perfectly route balanced solution will be very expensive in terms of total travel distance, and any driver will question why there should be made several visits to the same location
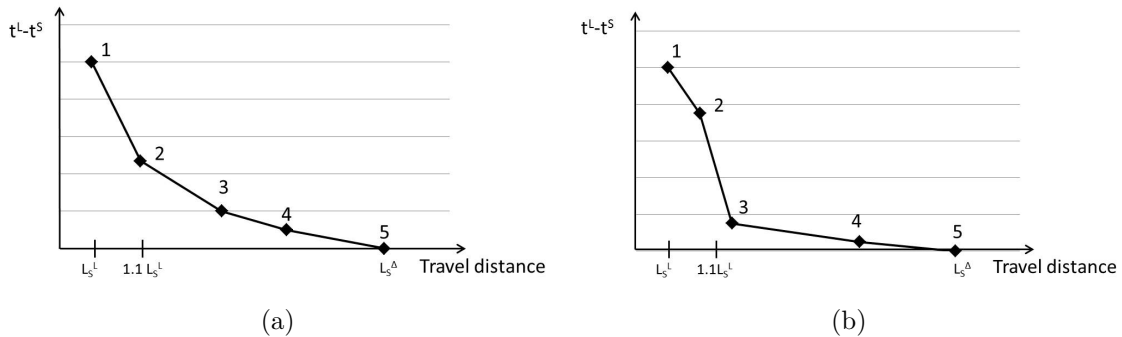
Figure 7: Examples of Pareto fronts.

if one single vehicle can service the demand. The observation from Figure 6 is interesting as there is a Pareto optimal solution with good route balance at a small increase in total travel distance from the best known solution. In this solution there is though, a duplication of one of the routes, and hence it would not be an interesting solution from a practical point of view. But for many other real routing problems that includes many more required vertices and vehicles than our little example, the Pareto front may look similar to the one in Figure 6 and hence, it will be possible to achieve a much better route balance at a low increase in cost.

It is far from trivial to decide which solutions, or what part of the Pareto front a decision maker will be interested in. Figure 7 illustrates the Pareto front from five solutions found for two different routing problems where one objective is to minimize total travel distance and the other is to minimize the difference in travel distance between the longest and shortest route. In many cases, and in particular if it is possible to split demand, the optimal route balance solution can be very expensive in terms of total travel distance as illustrated by Theorem 1 and our example in Figure 5. For most real routing problems, a decision maker will not aim for perfect route balance, but for some sort of more fair distribution of workload between the vehicles, and will be willing to add to the total travel distance to obtain this. The decision maker may have a given willingness towards how much increase in total travel distance that would be accepted for the gain of balanced routes, e.g. that a 10 % increase in total travel distance compared with the minimum total travel distance is acceptable. In that case only solutions 1 and 2 in Figures 7a and 7b will be of interest for the decision maker. But as we observe, solution 3 in Figure 7b looks quite promising, having a total travel distance just over $1.1L_{s^L}$, but a much better route balance than solutions 1 and 2. If the Pareto front has a structure similar to the one in Figure 7b it will be interesting to study the solutions where there is a large improvement in the route balance. However, if the Pareto front has a structure similar to the one in Figure 7a it is more difficult to define which areas that are of interest for the decision maker, and maybe an approach where the Pareto optimal solutions considered are within given criteria specified by the decision maker will be of good quality.

For solutions to real routing problems, we assume that demand cannot be split, i.e. each vertex needs to be serviced by exactly one vehicle. In practice most routing problems will be of this type meaning that the demand at each required vertex is sufficiently small so that routing decisions where more than one vehicle service the demand will be cost wise dominated by decisions where only one vehicle service this demand. In particular this will be valid for the MCGRP problem we consider in this paper. Figure 8 shows a VRP with ten nodes and two vehicles. In Figure 8a the optimal solution to the minimum travel distance objective is shown. This solution is not very well route balanced as we have one shorter route visiting four vertices and one longer visiting six vertices. It is possible to consider these two routes and make the solution artificially more

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(a) Minimum total travel distance. (b) Artificial route balance. (c) Two examples of solutions with better route balance.
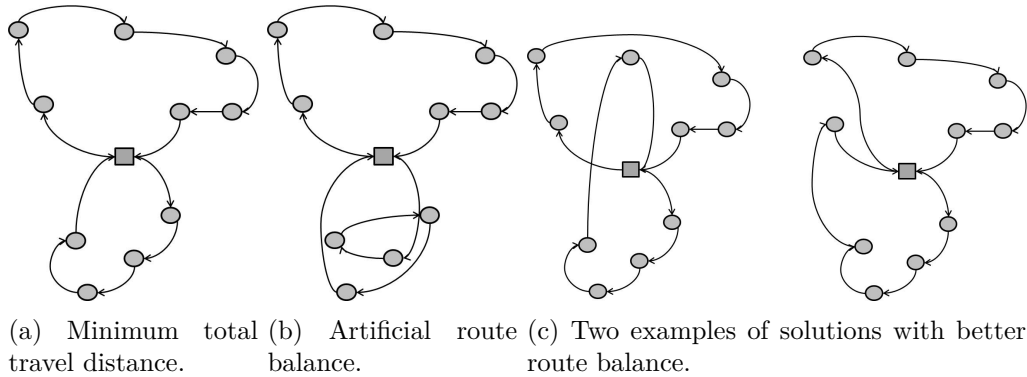
Figure 8: Minimum total travel distance, artificial route balance and route balanced solutions.

route balanced by making the shorter route longer as illustrated Figure 8b. Such routes are called artificially balanced and will not be of practical interest. Artificially balanced routes can be avoided by making each separate route optimal according to the minimize total travel distance objective, see e.g. the discussion by Jozefowiez et al. (2002). To create a solution that is better route balanced than the minimum total travel distance solution, we will have to move one node visit from the longest route to the shortest route. Examples of such solutions are shown in Figure 8c.

3. Relevant literature

The bi-objective MCGRP is a multi-objective routing problem. There exists some studies on this set of problems, a review is given by Jozefowiez et al. (2008b). Their findings are that there exist only a relative moderate number of publications. However, many authors have started to consider several objectives in routing problems, e.g. Demir et al. (2014). For most problems one of the objectives will be the standard one for general routing problems: Minimize total route cost. Cost can in that setting be defined as total travel distance, total travel time, or other means of cost relevant for the problem studied. Other popular objectives studied in the literature is minimizing the number of routes (vehicles), maximizing customer satisfaction and balancing of routes. Jozefowiez et al. (2008a) present an overview of what multi-objective optimization brings to VRPs, and illustrate this by two examples. They propose a general two-stage strategy to deal with the challenges of multi-objective optimization: Diversification and intensification.

In the following some of the relevant literature on multi-objective routing problems is presented. We focus on literature that consider balancing of routes as one of the objectives. We start with presenting references considering the CARP, then the VRP. No literature on the multi-objective MCGRP has been found.

Lacomme et al. (2003, 2006) were the first to study a multi-objective CARP where they seek to minimize both the total route length and the length of the longest trip: the makespan. The problem is motivated by waste management companies. They use one of the most widely used multi-objective genetic algorithms: Non-dominated Sorting Genetic Algorithm II (NSGA-II). Initial solutions are generated randomly, but also three good starting solutions are added by using three good heuristics for the CARP. Local search operators are added to improve solutions with respect to both objectives. Such hybridization approaches where evolutionary algorithms are combined with local search are also called memetic algorithms.

The multi-objective CARP was also studied by Mei et al. (2011) who propose a decomposition-based memetic algorithm with extended neighborhood search. In the decomposition framework,

the multi-objective CARP is split into a number of single-objective CARPs where the objectives are given different weights. An experimental study compares the authors' solution method with the one proposed by Lacomme et al. (2006) and a pure NSGA-II approach.

Grandinetti et al. (2010, 2012) propose an optimization-based heuristic procedure for the multi-objective CARP with three objectives: minimize total transportation cost, makespan and the total number of routes. The last objective is handled by solving the bi-objective CARP for all possible number of vehicle routes. The problem is modelled as a path-flow integer programming model where each path represents one possible route. The bi-objective CARP is then solved by the ϵ-constraint method where the Pareto set is approximated by solving a sequence of constrained single-objective problems: One objective is transformed into a constraint bounded by the value of ϵ. A computational study compares the results with the solutions presented by Lacomme et al. (2006), and finds that their method is competitive with the algorithm proposed by Lacomme et al. (2006).

A stochastic bi-objective CARP was studied by Fleury et al. (2005). The authors seek to minimize total costs and makespan for a CARP with stochastic demands. A stochastic memetic algorithm is proposed using a NSGA-II framework.

An early study by Sutcliffe and Board (1990) considers the problem of transporting mentally handicapped adults to a training center. They identify several objectives; among them minimizing the total travel distance, minimizing total time, equalization of the trip times for each vehicle and equalization of the number of unused seats in each vehicle. They solve the problem as a single-objective VRP with the objective of minimizing total time and add constraints on the equalization of trip times and unused seats.

Lee and Ueng (1999) study a VRP with load-balancing, motivated by the aspect of fairness among the drivers in a company. They formulate the problem as an integer program where the objective function is a weighted sum of total travel distance and the total difference between each vehicles working time and the vehicle with shortest working time. A heuristic solution algorithm is proposed for solving the integer program.

Three objectives and multiple periods are considered in the VRP studied by Ribeiro and Lourenço (2001). Objectives are to minimize total costs, minimize standard deviation of the work of each vehicle at the end of the period, and maximize the number of times a driver visits the same customer. In their solution procedure, each objective is given a weight and the sum of the weighted objectives is minimized.

Corberán et al. (2002) propose a scatter search to the problem of routing school buses when the objectives are both to minimize the total number of buses and to minimize the maximum time any student spends on a bus. Two constructive heuristics are used to generate initial routes, then the solutions are improved by local search and by generating new solutions by combining two existing ones.

In a sequence of papers Jozefowiez et al. (2002, 2006, 2007, 2009) study the VRP with route balance. Here route balance is measured in terms of difference between the longest and shortest route. In Jozefowiez et al. (2002) the problem is solved in a NSGA-II framework where a technique is proposed to maintain the population in the evolutionary algorithm diversified: Elitist diversification. For intensification of the Pareto set found by the evolutionary algorithm the authors propose the use of tabu search. Parallelization is also added to their algorithm to improve the performance. The authors do further investigations on the NSGA-II framework in Jozefowiez et al. (2006). A computational study compares the results of using only NSGA-II, NSGA-II with elitist diversification and adding parallelization to the algorithms. In Jozefowiez et al. (2007) a target aiming Pareto search using tabu search is proposed. The local search procedure is embedded with NSGA-II forming a hybrid meta-heuristic. The elitist diversification technique and parallelization procedure are

revisited in Jozefowiez et al. (2009). The two mechanisms are embedded in a multi-objective evolutionary algorithm, and computational results show a strict improvement of the generated Pareto sets.

Pasia et al. (2007a) also study the VRP with route balance. The proposed solution method combines population search with local search. In a computational study the results show that their proposed solution method performs well compared with the NSGA-II algorithm. In a second paper, Pasia et al. (2007b), the authors propose a Pareto ant colony optimization scheme for the same problem. Computational results show that the new solution method improves the performance of the solution method from Pasia et al. (2007a).

A multi-objective evolutionary algorithm with an explicit collective memory method for the VRP with route balance was proposed by Borgulya (2008). The algorithm is compared with the ones by Jozefowiez et al. (2002, 2006) and the comparison shows that the algorithm has good quality and provides similar results as the methods it is compared with.

Reiter and Gutjahr (2012) propose an exact method for the VRP with the objectives of minimizing total travel cost and the length of the longest route. The problem is formulated as an integer programming model and the solution approach is based on the adaptive ϵ-constraint method. The resulting single-objective subproblems are solved by a branch-and-cut technique. The metaheuristic algorithm NSGA-II and a single-objective genetic algorithm is combined with the ϵ-constraint method to improve the performance. A computational study shows that the method can solve small to medium sized instances to optimality.

The commercial waste collection VRP with time windows (VRPTW) was studied by Kim et al. (2006). Objectives are to minimize the number of vehicles and total travel time, maximize route compactness and balance workload among the vehicles. The solution approach contains a clustering-based waste collection VRPTW algorithm.

A column generation procedure for a bi-objective VRP is proposed by Sarpong et al. (2013). The objectives are minimize total travel cost and length of the longest route. A variant of an ϵ-constraint method is proposed, where instead of defining an ϵ-constraint for the objective of minimizing the length of the longest route, this objective defines a column's feasibility.

Melián-Batista et al. (2014) study a bi-objective VRPTW where the objectives are minimizing total travelled distance and minimizing distance between longest and shortest route length (in time units). A mixed integer linear model for the bi-objective problem is provided, and two versions of a scatter search-based algorithm are designed and implemented. Their performance is compared with an NSGA-II algorithm, and experiments show that the two proposed algorithms outperform the NSGA-II algorithm.

A bi-objective transportation location routing problem was studied by Martínez-Salazar et al. (2014). This problem is an extension to the VRP where also the locations of facilities need to be determined. The objectives are to minimize total cost and balancing workload by minimizing the distance between longest and shortest route. Both a local search based method (scatter tabu search procedure) and an evolutionary method (NSGA-II) are proposed.

Some not directly related literature references are Apte and Mason (2006) who study the delivery operations at the San Francisco public library. Two alternatives for delivery design are proposed: Balanced routes with pre-sorting and cross docking with pre-sorting and balanced routes. Groër et al. (2009) study the balanced billing cycle VRP where a utility company needs to create new meter reading routes based on the already existing routes. Objectives are to reduce total length of the routes and balance the meter readers' route in terms of total route length and number of meters read each day. He et al. (2009) propose a balanced K-means algorithm to design balanced groups of geographically close customers for a tobacco company, but do not consider the actual vehicle routes. Tsouros et al. (2006) propose a routing-loading balance heuristic algorithm for a

VRP where the objectives are to maximize uniform vehicle utilization taking into account a smooth vehicle loading throughout the fleet.

4. Solution approach

To illustrate the effects of including route balance to the MCGRP, we use a solution method for the bi-objective problem that quickly generates a good approximation to the Pareto front. The solution method is described in detail by Halvorsen-Weare (2015). In this section we present a mathematical model formulation for the MCGRP, and provide a brief description of the solution method.

4.1. Mathematical formulation for the MCGRP

We choose to formulate the MCGRP as a set partitioning model. In this model, each variable correspond to one route, and each required vertex, edge and arc needs to be serviced by exactly one route. Depending on the problem instance, the number of routes can be fixed, or it can be a part of the optimization problem, i.e. the number of routes will be determined by what gives the best objective function value.

Let Ω be the set of routes a vehicle with capacity C can travel. The sets V_R, A_R and E_R are the sets of required nodes, arcs and edges, respectively.

The parameters A_{rv}, A_{ra} and A_{re} are one if on route $r \in \Omega$ required vertex $v \in V_R$, arc $a \in A_R$ or edge $e \in E_R$ is serviced, and zero otherwise. T_r is the length in time units for each route, and Q is the number of available vehicles.

The variables are the binary variable λ_r, $r \in \Omega$, equal to 1 if route r is used and 0 otherwise, and the continuous variables t^L and t^S, equal to the length of the longest and shortest route, respectively.

We define a total of five objectives, that all should be minimized:

$$T : \sum_{r \in \Omega} T_r \lambda_r \tag{1}$$

$$\Delta_1 : t^L - t^S \tag{2}$$

$$\Delta_2 : t^L \tag{3}$$

$$\Delta_3 : \sum_{r \in \Omega} \left| T_r \lambda_r - T^G \right| \tag{4}$$

$$\Delta_4 : \sum_{r \in \Omega} \left| T_r \lambda_r - \mu \right| \tag{5}$$

T is the objective of minimizing the total distance travelled by all routes. Then Δ_1-Δ_4 are four different options for the route balance objective. Δ_1 minimizes the difference between the longest and the shortest route and Δ_2 minimizes the length of the longest route. T^G is some target route distance defined by the user, i.e. for Δ_3 the decision maker's objective is to minimize the total difference in travelled distance of all individual routes compared with some desired route distance. μ is the mean travel distance of all routes in the optimal solution, hence Δ_4 minimizes the difference in travelled distance between all individual routes and the mean travel distance. This objective resembles the variance objective. To ensure that we have a linear problem, objective Δ_4 can only be applied to problem instances where there is a fixed number of vehicles.

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For objectives Δ_3 and Δ_4 we use the absolute value. To formulate these objectives as linear objectives, we reformulate them as follows:

$$\Delta_3 : \sum_{r \in \Omega} t_r^G \tag{6}$$

$$\Delta_4 : \sum_{r \in \Omega} \sigma_r \tag{7}$$

The values of the new variables, t_r^G and σ_r are determined by the following constraints:

$$t_r^G \geq \left(T_r - T^G\right) \lambda_r, \quad r \in \Omega, \tag{8}$$

$$t_r^G \geq \left(T^G - T_r\right) \lambda_r, \quad r \in \Omega, \tag{9}$$

$$\mu = \frac{1}{Q} \sum_{r \in \Omega} T_r \lambda_r, \tag{10}$$

$$\sigma_r \geq T_r \lambda_r - \mu, \quad r \in \Omega, \tag{11}$$

$$\sigma_r \geq \mu - \left(T_r - M\right) \lambda_r - M, \quad r \in \Omega. \tag{12}$$

Constraints (8) and (9) set the value of t_r^G to be the positive difference between the travel distance of route r and the target travel distance T^G. If a route is not part of the optimal solution, λ_r will be zero, hence so will t_r^G. Constraint (10) determines the mean travel distance of all routes, and (11) and (12) set the value of σ_r to be the positive difference between the travel distance of route r and the mean travel distance. M is here set to a large value that needs to be at least as high as μ to ensure that σ_r does not take a positive value if route r is not travelled.

The constraints in the set partitioning model are the following:

$$t^L \geq T_r \lambda_r, \quad r \in \Omega, \tag{13}$$

$$t^S \leq T_r \lambda_r + M \left(1 - \lambda_r\right), r \in \Omega, \tag{14}$$

$$\sum_{r \in \Omega} A_{rv} \lambda_r = 1, \quad v \in V_R, \tag{15}$$

$$\sum_{r \in \Omega} A_{re} \lambda_r = 1, \quad e \in E_R, \tag{16}$$

$$\sum_{r \in \Omega} A_{ra} \lambda_r = 1, \quad a \in A_R, \tag{17}$$

$$\sum_{r \in \Omega} \lambda_r \leq Q, \tag{18}$$

$$\lambda_r \in \{0, 1\}, \quad r \in \Omega. \tag{19}$$

Constraints (13) sets the length of the longest route. These constraints are only necessary when objectives Δ_1 and Δ_2 are considered. Constraints (14) sets the length of the shortest route, and are only necessary when objective Δ_1 is considered. M is set to a sufficiently high number so that there will be at least one route in a solution of minimum this length. Constraints (15)-(17) are the partitioning constraints that forces all required vertices, edges and arcs to be serviced exactly once. Constraint (18) is limiting the number of routes in a solution to the number of available vehicles. This constraint is only valid for problem instances where there is a maximum number of available vehicles. If objective Δ_4 is to be used, we need a fixed number of vehicles and the constraint will be replaced by an equality constraint. Finally, (19) set the binary requirements for the route variables.

To solve the model described by constraints (13)-(19), we need to determine the routes $r \in \Omega$. For routing problems, there exist a vast amount of work where similar problem formulations have been solved to optimality. Methods consist of pre-generating all routes, and by dynamically adding promising route candidates in a branch-and-price framework. By introducing route balance as a second objective, the existing methods for dynamically adding routes can no longer be used. Also, when generating routes, we have to ensure that each individual route is cost-optimal, so we avoid solutions with routes that are artificially route balanced.

We therefore choose to pre-generate all candidate routes, where a candidate route is defined as follows:

Definition For each subset of vertices, edges and arcs that can be visited on a single route a candidate route is the cost-optimal route that visits all vertices, edges and arcs in the subset exactly once.

To generate all candidate routes, the routepool, we first enumerate all possible subsets of vertices, edges and arcs, before the optimal solution for the TSP for each of the subsets is calculated by a dynamic programming algorithm.

4.2. Solution method for the bi-objective problem

We are interested in finding Pareto optimal solutions that are well spread along the Pareto front. To generate strict Pareto optimal solutions, we use a hierarchical approach that combines two techniques from the multi-objective literature: the ϵ-constraint method and the lexicographic method, see e.g. Ehrgott (2005). For the bi-objective problem, the ϵ-constraint method can be formulated as follows:

$$\min f_1 \left(x \right),$$
$$\text{subject to} \quad f_2 \left(x \right) \leq \epsilon_2.$$

Here, f_1 and f_2 are the two objectives, and ϵ_2 is a threshold value for the second objective. All solutions in the Pareto front can be found by the ϵ-constraint method with appropriate choices for ϵ, see e.g. (Ehrgott, 2005, p. 100). The lexicographic method is a technique that can be used for multi-objective optimization problems when the objective functions can be or have to be considered in a hierarchical manner. The following optimization problems are then solved (Marler and Arora, 2004):

$$\min f_i \left(x \right),$$
$$\text{subject to} \quad f_j \left(x \right) \leq f_j \left(x_j^* \right), \ j = 1, 2, ..., i - 1, i > 1,$$
$$i = 1, 2, ..., k,$$

where i represents an objective's position in the hierarchy.

We combine ideas from the ϵ-constraint and lexicographic methods, and minimize one of the objectives, and then the other objective with a constraint on the first objective. Then the objectives swap roles. By using this method, we will find the outermost solutions in the Pareto front: the solutions to the two single-objective problems with the best possible value for the other objective. When these solutions are found, we want to find some solution that is located at the Pareto front at some point between these two. This can be obtained by taking the two objective function values for one of the solutions and define the middle point of these in an ϵ-constraint.

Let f_1 and f_2 be the objective function for the first and second objective, respectively. Further let x^i be the solution to the i'th optimization problem. The six first solutions to the bi-objective problem will be found in the following manner:
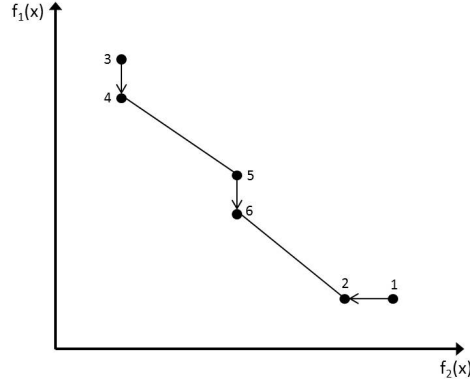
Figure 9: Illustration of how solutions 1-6 may appear for the solution procedure.

1. Minimize $f_1(x)$.

2. Minimize $f_2(x)$ subject to $f_1(x) \leq f_1(x^1)$.

3. Minimize $f_2(x)$.

4. Minimize $f_1(x)$ subject to $f_2(x) \leq f_2(x^3)$.

5. Minimize $f_2(x)$ subject to $f_1(x) \leq 1/2\left(f_1(x^1) + f_1(x^4)\right)$.

6. Minimize $f_1(x)$ subject to $f_2(x) \leq f_1(x^5)$.

Figure 9 illustrates how the solutions may be spread for the solution procedure.

Using this procedure, we may have found up to 6 Pareto optimal solutions. Solutions x_1, x_3 and x_5 will be weak Pareto optimum, while solutions x_2, x_4 and x_6 will be strict Pareto optimum. For some problem instances, the solutions may coincide.

Given that we have found three individual strict Pareto optimal solutions, we need to determine which part of the Pareto front to investigate. For this determination, we measure distance between two solutions as the area defined by them. The area, A_{12} between two Pareto optimal solutions, x_1 and x_2 is calculated as follows:

$$A_{12} = \frac{f_1(x^1) - f_1(x^2)}{f_2(x^2) - f_2(x^1)}. \tag{20}$$

We then introduce the terms explored area and unexplored area. We call the strict Pareto optimal solutions that have already been found during the search along the Pareto front existing Pareto optimal solution, and two such solutions that are next to each other, i.e. there are no other existing Pareto optimal solutions in the Pareto front between them, neighbouring solutions. At each iteration, we need to find a new ϵ-value. The new ϵ-value and its neighbouring solution defines the new area to explore. Let the set A contain all pair of neighbouring points, (p_i, p_j), where points represent both existing Pareto optimal solutions and ϵ-values defining unexplored areas. Then we have the following definitions:

Definition We call the two end points in the Pareto front, i.e. the two points representing the strict Pareto optimal solutions to the min $f_1(x)$ and min $f_2(x)$, p_1 and p_2. When these two points are found, the unexplored area is the rectangle defined by $R(p_1, p_2)$. At each iteration, each pair of neighbouring points $(p_i, p_j) \in A$, on the active Pareto front that has not yet been explored, defines such an unexplored area: $R(p_i, p_j)$. The explored area is then $R(p_1, p_2) \setminus \bigcup_{(i,j) \in A} R(p_i, p_j)$.
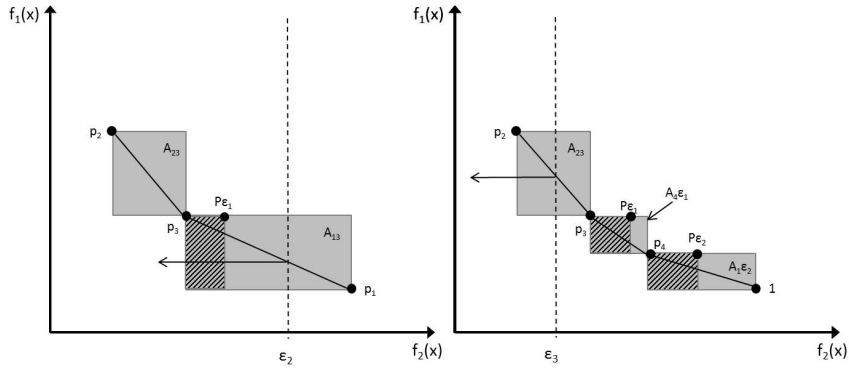
Figure 10: Determining where to search for a new Pareto optimal solution and setting right-hand-side value for epsilon constraint when there are three and four existing Pareto optimal solutions. $f_2(x)$ is the objective treated in the epsilon constraint in Step 3, and ϵ_2 and ϵ_3 are the new right-hand-side values. p_i are the existing Pareto optimal solutions, and p_{ϵ_k} are the ϵ-points that together with neighbouring solutions define unexplored area. Shaded area is explored area.

The procedure to find a new Pareto optimal solution is then as follows:

1. Choose the largest unexplored area defined by two neighbouring points on the Pareto front, A^*.

2. Determine the right-hand-side of the epsilon constraint to be the mid-point of the unexplored area between the two neighbouring points.

3. Solve the resulting MCGRP.

4. Swap objectives and set right-hand-side of epsilon constraint to the objective function value found in Step 3.

5. Solve the resulting MCGRP.

The procedure for determining the new epsilon constraints when there are three and four existing strict Pareto optimal solutions is illustrated in Figure 10. It continues until a predetermined number of Pareto optimal solutions are found, or when all the area has been explored.

This solution procedure for bi-objective problems perform binary search over the values of one of the objective functions to ensure that we quickly get a good representation of the Pareto front. Even when the procedure stops early, we will have a good approximation of the Pareto front. For a detailed description of the procedure, including pseudocodes, we refer to Halvorsen-Weare (2015).

5. Computational study

The solution method presented in Section 4 can be used to solve small problem instances of the MCGRP. We have used it to generate optimal Pareto fronts for some of the smallest benchmark instances from Bosco et al. (2013). In the following we will first present the problem instances before results are provided along with discussion.

5.1. Problem instances

The problem instances we have solved have a varying number of tasks consisting of required vertices, edges and arcs. The number of available vehicles is 3, 4 or 5. For these small problem

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Table 1: Problem instances. Q is the number of available vehicles, C is each vehicles capacity, D is total demand. V, E and A are the total number of vertices, edges and arcs in the problem, and V_R, E_R and A_R are the number of required vertices, edges and arcs. $|R|$ is the number of generated candidate routes.

Instance number	Name	Q	C	D	V	E	A	V_R	E_R	A_R	$|R|$
1	mggdb_0.25_19	3	27	66	8	2	18	3	1	6	314
2	mggdb_0.30_14	5	21	89	7	5	32	3	3	11	3423
3	mggdb_0.30_19	3	27	66	8	2	18	3	1	6	339
4	mggdb_0.35_4	4	5	19	11	4	30	6	2	9	6111
5	mggdb_0.35_19	3	27	66	8	2	18	3	1	5	188
6	mggdb_0.40_4	4	5	19	11	4	30	6	2	9	6111
7	mggdb_0.40_19	3	27	66	8	2	18	4	1	5	322
8	mggdb_0.45_1	5	5	22	12	5	34	6	2	9	4442
9	mggdb_0.45_4	4	5	19	11	4	30	7	2	8	6111
10	mggdb_0.45_6	5	5	22	12	5	34	7	2	9	5442
11	mggdb_0.45_14	5	21	89	7	5	32	6	2	8	3487
12	mggdb_0.45_15	4	37	112	7	5	32	6	2	8	7347
13	mggdb_0.45_19	3	27	66	8	2	18	3	1	4	92
14	mggdb_0.45_20	4	27	107	11	5	34	5	2	9	5502
15	mggdb_0.50_1	5	5	22	12	5	34	8	2	8	5442
16	mggdb_0.50_4	4	5	19	11	4	30	6	2	7	2277
17	mggdb_0.50_6	5	5	22	12	5	34	7	2	8	3650
18	mggdb_0.50_14	5	21	89	7	5	32	6	2	8	2521
19	mggdb_0.50_15	4	37	112	7	5	32	5	2	8	4807
20	mggdb_0.50_19	3	27	66	8	2	18	3	1	4	106
21	mggdb_0.50_20	4	27	107	11	5	34	5	2	8	1867

instances, all vehicles will always be used, hence we do not need to treat constraint (18) as an equality constraint for balance objective Δ_4. A summary of the instances are provided in Table 1. These problem instances, along with a number of other benchmark instances for the MCGRP can be found and downloaded from SINTEF (2014).

As we do not have a decision maker to set a target value for the travelled distance of each individual route, T^G, we need to set this to some default value for each problem instance when we use balance objective Δ_3. This default value is calculated as the mean route distance for the optimal solution to objective T rounded up to nearest integer. Setting T^G to this default value can result in fewer points in the optimal Pareto front since the two objectives are correlated.

The problem instances also have service costs, which are not necessary to consider for the minimum total route cost objective. However, when considering route balance, we will get different solutions depending on whether or not the service of tasks is considered as a part of what we want to balance. For this computational study, we have considered balancing travelled distance only, hence service cost is neglected.

5.2. Results and discussion

The solution method from Section 4 has been implemented in C# using Microsoft Visual Studio 2010. The mathematical programming model was modelled with Xpress-BCL 4.4.0 and solved by calling Xpress-Optimizer 22.01.04. All results presented in this paper were obtained on a 2.70 GHz Intel Core i7-2620 M PC with 8 GB RAM. All problem instances from Table 1 was solved

to optimality, i.e. all individual MCGRP instances were solved to optimality and the bi-objective solution procedure found all Pareto optimal solutions. The CPU times varied depending on the problem instance and the balance objective that was considered. For objective Δ_1 most instances was solved within a few minutes, and a few where above 1 hour, the longest CPU time being 1 hour and 50 minutes. For Δ_2 all problem instances were solved within less than a minute, for Δ_3 all were solved in less than two minutes, and for Δ_4 all were solved within a few minutes.

Table 2 shows an overview of the results from running the problem instances. Reported in the table are the values for all five objectives when each of the five objectives are minimized. For objective T the best values for each of the four balance objectives, Δ_1 - Δ_4, given the optimal value for T are reported. These are hence the worst values for the balance objectives in the Pareto fronts. For the balance objectives, it is only objective T that is minimized subsequently and the value for T is then the worst values for this objective in the Pareto fronts. For all other balance objectives than the one being minimized, there may exist other solutions that yield better values for the objectives without deteriorating the objective function values for T and the balance objective being minimized.

Table 3 reports the number of Pareto optimal solutions for each of the problem instances for each of the four options for balance objective. We observe that the Pareto front size varies for the four balance objectives. It is Δ_1 and Δ_4 that have the largest Pareto fronts. Both these balance objectives consider the range of travelled distance for each individual vehicle routes. Objective Δ_2 only considers the route with the longest travel distance and a relatively small Pareto front is obtained for all problem instances. We observe that the Pareto fronts for objective Δ_3 also contains relatively few solutions, although this balance objective considers the duration of all individual routes. But as the target route distance for this objective is set to be the average route distance for the minimum total route distance problem, the balance objective and the total route duration objective are correlated and hence, a natural consequence is few solutions in the Pareto front.

Table 4 shows the Pareto fronts for problem instance 10, mggdb_0.45_6. These Pareto fronts are in line with the observation that there are more Pareto optimal solutions for balance objectives Δ_1 and Δ_4, and that these are spread over a much wider total route distance span than balance objectives Δ_2 and Δ_3. For this particular instance, the value for objective T is increased with 65% for the best solutions for objectives Δ_1 and Δ_4, while this increase is only 6% for Δ_2 and 7% for Δ_3.

Figure 12 shows the best vehicle routes for each of the five objectives that are considered for problem instance 3, mggdb_30_19. The problem instance is illustrated in Figure 11.

Problem instance 3 is a small problem instance providing a typical example on how minimizing the balance objectives Δ_1 and Δ_4 can result in a high increase in total travelled distance. The best vehicle routes for Δ_1 and Δ_4 are the same, and the total travel distance is 31% higher than the solution with lowest travel distance, Figure 12c. We see for the solution that required arcs 1-6 and 6-8 are serviced on two different vehicle routes, a choice that yields better route balance for these objectives at the expense of a higher total travel distance.

The best solutions for balance objectives Δ_2 and Δ_3 do not require much increase in total travel distance, 8% and 2% respectively. The best vehicle routes for these objectives have a slightly better route balance at a small increase in total travel distance from the best total travel distance solution.

In Section 2 we discussed how a decision maker may be interested in only part of the Pareto front. Particularly, solutions that yield much better route balance at a low increase in total travel distance can be interesting. Figure 13 shows the Pareto fronts for balance objectives Δ_1, Δ_3 and Δ_4 for problem instance 2, and Table 5 provides the values of the points in the Pareto fronts for all objectives. For this problem instance we see that it is possible to achieve a high increase in route balance at a low increase in total travel distance. This is typically a feature for many real life route

Table 2: Results overview for all problem instances. First columns are the results when objective T is minimized, next when objective Δ_1 is minimized and so on. Note that it is only when objective T is minimized that all the other four objectives are minimized, hence all results for min T and all results for T are the best ones, but for the balance objectives not being minimized there may exist other solutions that can yield better objective function values. These results are shown in italic.

	min T					min Δ_1					min Δ_2					min Δ_3					min Δ_4				
---	T	Δ_1	Δ_2	Δ_3	Δ_4	T	Δ_1	Δ_2	Δ_3	Δ_4	T	Δ_1	Δ_2	Δ_3	Δ_4	T	Δ_1	Δ_2	Δ_3	Δ_4	T	Δ_1	Δ_2	Δ_3	Δ_4
1	53	4	20	5	1.56	65	1	22	11	0.44	53	4	20	5	1.56	57	2	20	3	0.67	65	1	22	11	0.44
2	101	9	23	12	2.56	115	0	23	10	0.00	104	1	21	1	0.32	104	1	21	1	0.32	115	0	23	10	0.00
3	51	7	21	8	2.67	67	1	23	16	0.44	55	4	20	6	1.56	53	5	21	6	2.22	67	1	23	16	0.44
4	242	26	73	46	11.50	288	0	72	44	0.00	255	1	64	11	0.38	255	1	64	11	0.38	288	0	72	44	0.00
5	51	15	25	16	5.33	65	1	22	14	0.44	57	5	21	8	2.00	57	5	21	8	2.00	65	1	22	14	0.44
6	238	33	74	54	13.50	340	0	85	100	0.00	254	20	71	32	6.50	260	10	71	20	3.00	340	0	85	100	0.00
7	38	16	20	17	5.78	54	0	18	15	0.00	46	2	16	7	0.89	46	2	16	7	0.89	54	0	18	15	0.00
8	259	48	76	69	13.84	289	5	61	29	1.44	283	14	61	33	4.08	282	14	63	28	3.28	289	5	61	29	1.44
9	228	18	67	30	7.50	380	0	95	152	0.00	250	10	66	26	3.50	246	11	67	20	3.00	380	0	95	152	0.00
10	218	26	56	42	8.48	360	0	72	140	0.00	231	9	50	17	3.04	234	6	50	14	2.56	360	0	72	140	0.00
11	66	17	19	20	4.48	67	15	19	19	3.92	66	17	19	22	4.56	68	15	19	18	3.84	68	15	19	18	3.84
12	34	3	10	4	1.00	40	0	10	4	0.00	34	5	10	6	1.75	36	2	10	2	0.50	40	0	10	4	0.00
13	48	4	18	4	1.33	64	2	22	16	0.89	48	4	18	4	1.33	48	4	18	4	1.33	64	2	22	16	0.89
14	78	21	30	40	10.00	94	1	24	14	0.50	81	14	24	10	5.13	87	4	24	7	1.75	92	2	24	12	0.50
15	214	49	63	75	14.96	315	0	63	100	0.00	219	27	60	42	8.56	223	22	60	26	6.16	315	0	63	100	0.00
16	219	50	80	55	13.75	370	1	93	150	0.50	234	12	64	20	4.00	234	13	65	20	4.00	364	2	92	144	0.50
17	276	62	76	92	18.56	382	1	77	102	0.48	290	16	66	34	6.40	301	16	72	21	4.72	385	2	78	105	0.40
18	75	7	19	8	1.60	92	2	20	17	0.64	75	7	19	8	1.60	78	5	19	5	1.36	95	2	20	20	0.40
19	37	3	11	5	0.88	41	1	11	1	0.38	37	7	11	9	2.63	41	1	11	1	0.38	41	1	11	1	0.38
20	44	12	20	13	4.44	44	12	20	13	4.44	44	12	20	13	4.44	44	12	20	13	4.44	44	12	20	13	4.44
21	81	15	28	34	5.75	101	1	26	17	0.38	83	12	25	15	3.88	91	7	26	11	2.75	101	1	26	17	0.38
Avg:	126	21	39	31	7.12	173	2	42	47	0.71	133	10	36	16	3.24	135	8	36	12	2.36	173	2	42	47	0.69

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Table 3: Number of Pareto optimal solutions for each of the balance objectives.

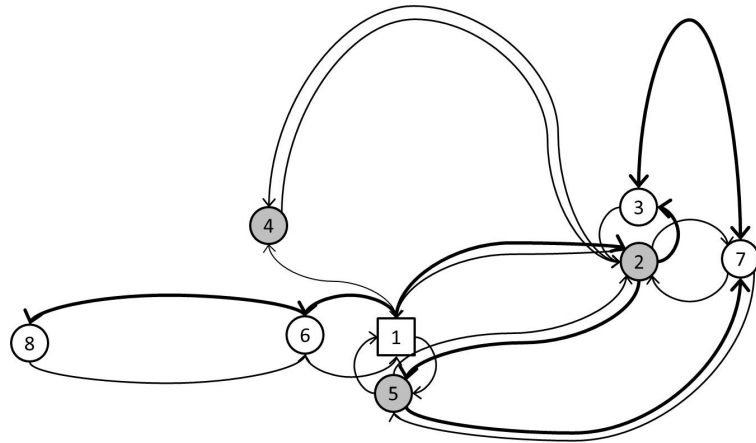| Instance number	Δ_1	Δ_2	Δ_3	Δ_4
1	3	1	2	3
2	4	2	4	5
3	5	2	2	4
4	5	3	3	4
5	5	2	3	4
6	10	3	6	10
7	5	3	4	5
8	9	4	8	9
9	10	2	5	11
10	12	3	6	13
11	2	1	3	3
12	4	1	2	3
13	2	1	1	2
14	9	3	6	9
15	18	2	4	22
16	12	3	5	11
17	13	3	8	18
18	6	1	4	7
19	3	1	3	2
20	1	1	1	1
21	7	2	4	5

Figure 11: Overview of problem instance 3 (mggdb_30_19): 1 is the depot, shaded circles represent required vertices, thick edges and arcs represent required edges and arcs.

Table 4: Pareto fronts for problem instance 10, mggdb_0.45_6. The values for the balance objectives that are not being minimized are not necessarily the best objective function values as there may exist other solutions that yield better values for these objectives. These results are shown in italic.

| | min Δ_1 | | | | | min Δ_2 | | | | | min Δ_3 | | | | | min Δ_4 | | | | |
	T	Δ_1	Δ_2	Δ_3	Δ_4	T	Δ_1	Δ_2	Δ_3	Δ_4	T	Δ_1	Δ_2	Δ_3	Δ_4	T	Δ_1	Δ_2	Δ_3	Δ_4
Point 1	218	26	56	42	8.48	218	28	56	42	8.48	218	34	56	42	8.64	218	28	56	42	8.48
Point 2	225	22	52	33	6.00	225	22	52	33	6.00	225	25	55	33	6.40	225	22	52	33	6.00
Point 3	226	20	56	34	6.56	231	9	50	17	3.04	228	18	56	32	6.08	229	16	54	23	4.96
Point 4	228	18	56	32	6.08						229	16	54	23	4.96	231	9	50	17	3.04
Point 5	229	16	54	23	4.96						231	9	50	17	3.04	234	6	50	14	2.56
Point 6	231	9	50	17	3.04						234	6	50	14	2.56	241	4	50	21	1.76
Point 7	234	6	50	14	2.56											247	6	52	27	1.52
Point 8	241	4	50	21	1.76											251	5	52	31	1.36
Point 9	257	3	53	37	1.12											257	3	53	37	1.12
Point 10	270	2	55	50	0.40											268	3	55	48	0.88
Point 11	271	1	55	51	0.32											270	2	55	50	0.40
Point 12	360	0	72	140	0.00											271	1	55	51	0.32
Point 13																360	0	72	140	0.00

(a) Best vehicle routes for objective T. (b) Best vehicle routes for objective Δ_1 and Δ_4.

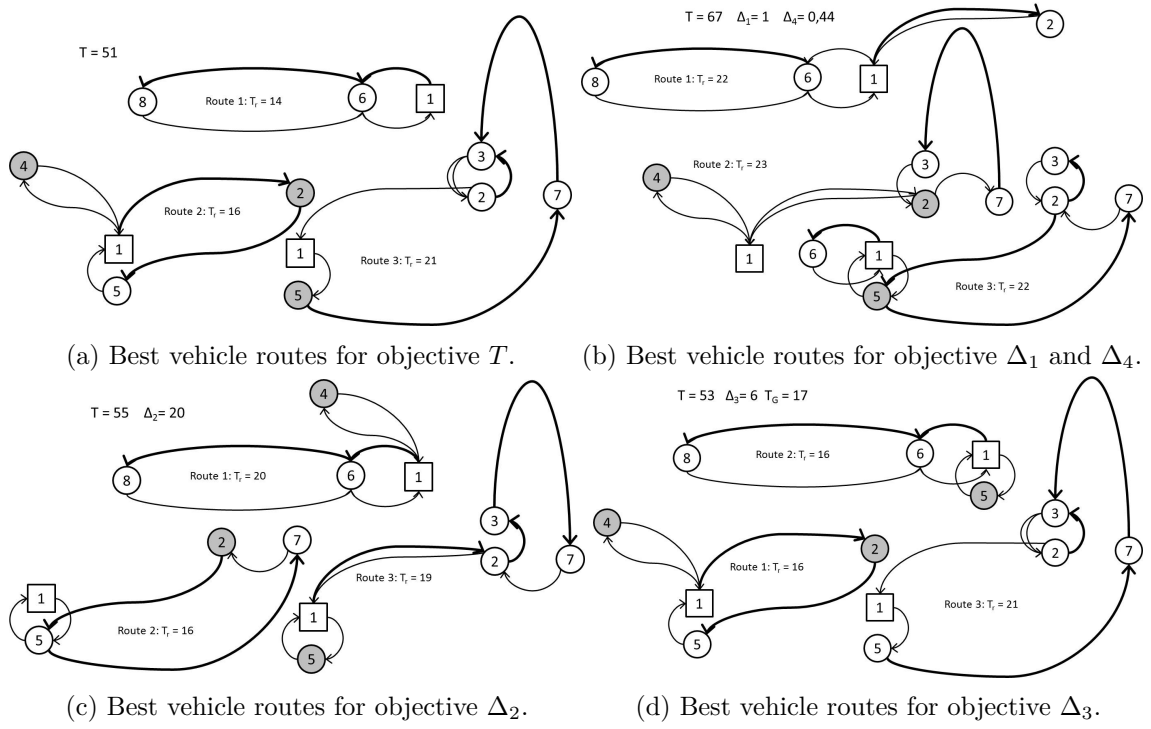(c) Best vehicle routes for objective Δ_2. (d) Best vehicle routes for objective Δ_3.

Figure 12: Best vehicle routes for all objectives for problem instance 3 (mggdb_0.30_19). Best vehicle routes for objectives Δ_1 and Δ_4 are the same and are illustrated in (b).

problems, and illustrates how it can be very beneficial to consider more than one objective and provide a decision maker with several solutions that are located at the part of the Pareto front the decision maker will be most interested in.

6. Conclusions

In this paper we have introduced the bi-objective mixed capacitated general routing problem. The MCGRP, along with other routing problems, are in previous studies mainly considered as single-objective problems where the objective is to minimize total cost which can include only routing cost, or routing cost and cost of vehicles.

However, for most practical routing problems, there will be other objectives than minimizing the total cost that are relevant. One of them, that has attracted some interest, is route balance.
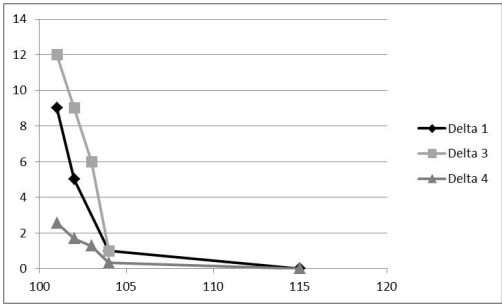

Figure 13: Pareto fronts for problem instance 2 (mggdb_0.30_14) for balance objectives Δ_1, Δ_3 and Δ_4.

Table 5: Pareto fronts for problem instance 2, mggdb_0.30_14. The values for the balance objectives that are not being minimized are not necessarily the best objective function values as there may exist other solutions that yield better values for these objectives. These results are shown in italic.

| | min Δ_1 | | | | | min Δ_2 | | | | | min Δ_3 | | | | | min Δ_4 | | | | |
	T	Δ_1	Δ_2	Δ_3	Δ_4	T	Δ_1	Δ_2	Δ_3	Δ_4	T	Δ_1	Δ_2	Δ_3	Δ_4	T	Δ_1	Δ_2	Δ_3	Δ_4
Point 1	101	9	23	12	2.56	101	9	23	12	2.56	101	9	23	12	2.56	101	9	23	12	2.56
Point 2	102	5	23	11	2.08	104	1	21	1	0.32	102	6	24	9	1.68	102	6	24	9	1.68
Point 3	104	1	21	1	0.32						103	5	23	6	1.28	103	5	23	6	1.28
Point 4	115	0	23	10	0.00						104	1	21	1	0.32	104	1	21	1	0.32
Point 5																115	0	23	10	0.00

How route balance is best defined is not trivial, and will vary depending on the actual routing problem studied, and the preferences of the decision maker.

We have in this paper introduced several versions of route balance objectives, and discussed how these affect the optimal solutions. Which one to use should be decided based on the problem at hand and which of the objectives that will provide most value for the decision maker. The chosen balance objective will have consequences for both the Pareto optimal solutions, and which solution methods that can be used.

An exact method that can be used to compute the Pareto front, or parts of it, has been applied to some small benchmark instances for the MCGRP. The results illustrate how both the problem instance, and the objectives that are considered, affect the size of the Pareto front. We have considered a total of four options for the balance objective, and have illustrated how some of these for some problem instances may be conflicting with the objective of minimizing total travel distance. This relates specially to balance objectives that consider the span of all individual route distances, i.e. minimize the distance between the longest and shortest route and minimize the variance of the individual route distances. Other options for balance objectives may have smaller impact on the total travel distance. We have seen that e.g. minimizing the distance of the longest route results in small Pareto fronts for the problem instances we have considered.

Despite the relevance, there is not much research on multi- or bi-objective routing problems. And from what exist, there is often only one version of a second objective that is presented. Future research should thus focus on introducing several objectives, and methods for solving the problems that creates a set of good and, for a decision maker, relevant Pareto optimal solutions, or a good approximation of the Pareto front within reasonable time.

Acknowledgements

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