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Power System Dynamical Analysis

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Power System Dynamical Analysis
An intelligible and general methodology for detailed response- and eigenvalue analysis

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RESULT (summary)

A methodology is presented that allows for detailed power system modelling and analysis in a practicable and conceptually intelligible way, based on the development of a stock of standard *submodels* for modelling of power system components. Such models are developed for most of the commonly applied power system components. For illustration a not-so-common system component is also dealt with, namely the adjustable speed synchronous machine. The power network related modelling part is conducted within the *synchronous d-q axis frame of reference.*

The system state variables are partitioned into *two* subsets of state variables; *the power network state variables* which are the defined network loop currents together with the voltage across the capacitors of the network, and the *remaining* or *'local' system state variables* which e.g. are angular speed and fluxes of rotating machines, electrical angles of synchronous machine rotors, and variables associated with involved control systems.

To describe the *power network state variables,* the appropriate *electrical circuit submodels* and *capacitor voltage submodels* are fetched from stock, -and lined up to form what may be denoted 'the primitive system'. Contributing in making up such submodels in the chosen frame of reference, are 2x2 matrices **R**, $\mathbf{X}_L$ and $\mathbf{X}_C$, and 2x1 e.m.f. matrices $\Delta\mathbf{E}$. All component-specific complexity is 'hidden' within the confines of such submodel matrices. Depending on which type of network component a considered submodel matrix belongs to, it may be a zero matrix, a constant element matrix, or a matrix containing elements that are functions of one or more 'local' system state variables. The definition and lineup of the primitive system is typically done once at start of analysis. So also the definition of a loop matrix **B** that conveys information on how the circuit submodels are tied together. In modelling of power network performance, *two* plain numerical processes are involved; fill-in of content of the primitive system lineup based on current value of the state variables, and standard matrix operations related to the *primitive system* and ***B.***

To describe the *remaining* or *'local' state variables* the task becomes merely one of fetching and imple-menting the proper submodels from the model stock.

The report deals with submodel development, system modelling, and model application to main tasks of analysis, the latter being *Initial condition-*, *eigenvalue-* and *time response* analysis. Numerous illustrations of model application are included.

KEYWORDS

SELECTED BY AUTHOR(S)	Component modelling	System modelling
	Response analysis	d-q axis frame of reference

Power System Dynamical Analysis

An intelligible and general methodology for detailed time response- and eigenvalue analysis

EXECUTIVE SUMMARY

The methodology allows for detailed power system modelling and analysis in a practicable and conceptually intelligible way, based on the development of a stock of standard *submodels* for modelling of power system components. Such models are produced for most of the commonly applied power system components. For illustration a not-so-common system component is also dealt with; namely the adjustable speed synchronous machine. The power network related modelling part is conducted within the synchronous *d-q axis frame of reference.*

Power system performance is described in terms of a set of simultaneous, first order, ordinary differential equations d**z**/dt =**f**(**z**,v), where **z** is the set of all system state variables, and v comprises exogenously specified excitations. For convenient formulation of the equations, **z** is partitioned into *two* subsets of state variables; *the power network state variables* which are the defined network loop currents together with the voltage across the capacitors of the system, and the *remaining* or *'local' system state variables* which e.g. are angular speed and fluxes of rotating machines, electrical angles of synchronous machine rotors, and variables associated with involved control systems.

To describe the *power network state variables,* the appropriate *electrical circuit submodels* and *capacitor voltage submodels* are fetched from stock, -and lined up to form what may be denoted 'the primitive system'. Contributing in making up such submodels in the chosen frame of reference, are 2x2 matrices **R**, $\mathbf{X}_L$ and $\mathbf{X}_C$, and 2x1 e.m.f. matrices $\Delta\mathbf{E}$. All component-specific complexity is 'hidden' within the confines of such submodel matrices. Depending on which type of network component a considered submodel matrix belongs to, it may be a zero matrix, a constant element matrix, or a matrix containing elements that are functions of one or more 'local' system state variables. The definition and lineup of the primitive system is done once at start of analysis. So also the definition of a loop matrix **B** that conveys information on how the circuit submodels are tied together. In computing the right hand side of d**z**/dt = **f**(**z**,**v**), two plain numerical pro-cesses are involved; the fill-in of content of the primitive system lineup based on current value of the state variables, and standard matrix operations related to the *primitive system* and ***B***.

To describe the *remaining* or *'local' state variables* the task becomes merely one of fetching and implementing the proper submodels from the model stock.

With the full set of differential equations established, the platform on which to conduct specific system analyses is ready. Three tasks are dealt with: *Initial condition analysis* which is a prerequisite for most other studies within the realm of system operation, *eigenvalue analysis*, and *time respons analysis* :

Initial condition analysis implies setting d/dt = 0 in all of the simultaneous differential equations and solving for the particular steady state solution that fulfills the initial load flow requirements specified for the power system.

Eigenvalue analyses aim to reveal the power system's inherent dynamic characterisitics, when incrementally disturbed from an initial, specified state. The task implies evaluating the eigenvalues associated with matrix **A** of the linearized formulartion dΔ**z**/dt = **A** Δ**z**. Three example cases are included.

Time response analyses are conducted to evaluate the variation over time of the system's state variables and their interactions, following some given disturbance to the system.Transient power system behaviour is exemplified when caused by respectively a temporary three phase short circuit, start/loading of an asynchronous motor, start/loading/disconnection of a synchronous generator, and islanding of a local power system. Also included is an example comparison of characteristic effects of the same disturbance applied to respectively a 'weak' and a 'strong' power system, - the disturbance being start and loading up of a relatively large asynchronous motor. Finally, the dynamical performance of the double-fed asynchronous machine in the appearance of the Adjustable Speed Hydro ('ASH'-) unit, is exemplified.

Content overview

0. Summary

Power System Dynamical Analysis

An intelligible and general methodology for detailed time response- and eigenvalue analysis

0. Summary

Summary description is given under *three* headings: *Conceptual overview*, *Computational overview*, and *Methodological overview*. The text pertaining to the first *two* aim at giving a snapshot picture of merit and scope of the modelling scheme presented, without delving much into definition of parameters and variables.This is left to the third and dominant overview part, which covers all methodological aspects in some depth.

0.1 Conceptual overview

The methodology allows for detailed power system modelling and analysis in a practicable and conceptually intelligible way, based on the development of a stock of standard *submodels* for modelling of power system components. Such models are produced for most of the commonly applied power system components. For illustration a not-so-common system component is also dealt with; namely the adjustable speed synchronous machine. The power network related modelling part is conducted within the synchronous *d-q axis frame of reference.*

Network-wise, any power system component is represented in terms of one or more *electrical circuit models*, each comprising a 2x2 resistance matrix $\mathbf{R}$, a 2x2 inductive reactance matrix $\mathbf{X}_L$, and a 2x1 e.m.f. matrix $\Delta\mathbf{E}$. All component complexity is 'hidden' within the confines of the circuit terms ($\mathbf{R}$,$\mathbf{X}_L$, $\Delta\mathbf{E}$). The terms are specific to each type of power system component. Depending on which type of component the considered circuit term belongs to, it may be a zero matrix, a constant element matrix, or a matrix containing elements that are functions of one or more 'local' system state variables. For definition of the latter, see next.

Given the collection of *electrical circuit models* of the power system components, *topological information* describing how the circuit models are tied together, and 2x2 capacitive reactance matrices $\mathbf{X}_C$ characterizing respective capacitors of the network, power network modelling is readily afforded by generating an appropriate set of network equations. In the present scheme of analysis a system loop matrix $\mathbf{B}$ is defined and applied to the 'machinery' of generating the differential equations that describe the performance of the *power network state variables.* These variables are made up of appropriately chosen *loop currents* together with the *voltage across the capacitors* of the system. Further insight into formulation of the equations is offered on next page.

The *remaining system state variables* are 'local' component variables like *angular speed* and *befitting fluxes* for each asynchronous machine, - *angular speed*, *electrical angle* and *befitting fluxes* for each synchronous machine, and *appropriate variables* associated with respective control systems of the power network. The differential equations describing the *remaining state variables* are fetched from the stock of submodels.

With the system state variables $\mathbf{z}$ described in terms of a set of simultaneous, first order, ordinary differential equations $d\mathbf{z}/dt = \mathbf{f}(\mathbf{z},\mathbf{v})$ - where $\mathbf{v}$ comprises exogenously specified excitations - , the platform on which to conduct specific system analyses is ready. Three main tasks are dealt with : *Initial condition analysis* which is a prerequisite for most other studies within the realm of system operation, *Eigenvalue analysis*, and *Time respons analysis* :

> *Initial condition analysis* implies setting $d/dt = 0$ in all of the simultaneous differential equations stated above, and solving for the particular steady state solution that fulfills the initial load flow requirements specified for the power system. The initial value of all control system state variables is definitionwise zero, as these state variables conveniantly are defined in terms of *incremental* quantities. An efficient gradient technique is used iteratively to converge upon the desired initial solution.
>
> *Eigenvalue analyses* are conducted to learn about the power system's inherent dynamic characterisitics, when incre-mentally disturbed from its specified initial state. The stated task implies determining the eigenvalues associated with matrix $\mathbf{A}$ of the linearized formulation $d\Delta\mathbf{z}/dt=\mathbf{A}\,\Delta\mathbf{z}$. Self- and mutual elements of matrix $\mathbf{A}$ are developed on general algorithmic form for all main types of power system components. Example eigenvalue analyses are included.

Time response analyses are conducted to evaluate the variation over time of the system's state variables and their interactions, following some given disturbance to the system. The task implies solving the set of equations $d\mathbf{z}/dt=\mathbf{f}(\mathbf{z},\mathbf{v})$ numerically over some given time horizon. To account for the fact that *electrical circuit models* themselves may be functions of local system state variables, the task of power network modelling must be continually repeated during processes of numerical inte-gration. Transients power system behaviour is exemplified when caused by respectively a temporary three phase short circuit, start/loading of an asynchronous motor, start/ loading/ dis-connection of a synchronous generator, and islanding of a local power system. Also included is a comparison of characteristic electrical consequences of the same disturbance applied to respectively a 'weak' and a 'strong' power system, - the disturbance being start and loading up of a relatively large asynchronous motor. Finally, the dynamical performance of the double-fed asynchronous machine in the appearance of the Adjustable Speed Hydro unit ('ASH'), is exemplified

0.2 Computational overview

As implied in the previous overview, the *system model* can be considered the aggregate of two submodels ; the *system submodel* comprising the differential equations of the *power network state variables*, and the *system submodel* comprising the differential equations of the *remaining, local system state variables*. This overview part focuses on the chosen approach for establishing the *first* of the two *system submodels*.

The algorithmic basis for describing the behaviour of the *power network state variables* - which are the chosen loop currents ($\mathbf{i}_{loop}$) and the voltage across the capacitors of the system ($\mathbf{e}_{tc}$), - is out-lined via comments under *four* main headings:

'Line-up' and 'fill-in' of electrical circuit models. From the stock of component models established, the appro-priate *electrical circuit models* with terms ($\mathbf{R}$, $\mathbf{X}_L$, $\Delta\mathbf{E}$) are fetched and 'lined up' in accordance with the chosen labelling (numbering) of circuit models: The aggregate of R-terms are oganized into a diagonal resistance matrix termed $\mathbf{R}_{primitive}$. The aggregate of inductive reactances $\mathbf{X}_L$ are similalarily arranged into a diagonal reactance matrix $\mathbf{X}_{Lprimitive}$. (In (the relatively rare) cases of significant electromagnetic coupling between system components – as e.g. in situations with parallell overhead lines close to each other, - off diagonal terms may have to be filled into $\mathbf{X}_{Lprimitive}$ at proper locations). Finally, the aggregate of voltage source terms $\Delta\mathbf{E}$ are arranged into a voltage source vector $\mathbf{e}_{primitive}$, which is partitioned into three subvectors denoted $\mathbf{e}_{chord}$, $\mathbf{e}_{tc}$ and $\mathbf{e}_{t\text{-}rest}$. Based on stipulated/ current value of all state variables, the content of all the model terms are computed and filled in.

Modelling of power network loop currents at the considered state of the process, is readily afforded by utilizing the information contained in the previous component lineup, together with the binary information stored in the system loop matrix $\mathbf{B}$, describing the incidence of network loops and electrical circuit models of the network. The loop currents must fulfill the following set of equations:

$$\mathbf{E}_{loop} = \mathbf{R}_{loop}\, \mathbf{i}_{loop} + (1/\omega_b)\, \mathbf{X}_{Lloop}\, d\mathbf{i}_{loop}/dt \qquad (0\text{-}1)$$

where;

$$\left.\begin{aligned} \mathbf{E}_{loop} &= -\mathbf{B}\, \mathbf{e}_{primitive} && = \text{driving voltage of respective network loops} \\ \mathbf{R}_{loop} &= \mathbf{B}\, \mathbf{R}_{primitive}\, \mathbf{B}^t && = \text{network loop resistance matrix ('t' means 'transpose')} \\ \mathbf{X}_{Lloop} &= \mathbf{B}\, \mathbf{X}_{Lprimitive}\, \mathbf{B}^t && = \text{network loop inductor matrix} \end{aligned}\right\} \qquad (0\text{-}2)$$

Modelling of power network capacitor voltages. *Circuit*-wise, the lossy capacitor banks of the network are modelled in (0-1) via their *electrical circuit models* with generic terms ($\mathbf{R}_C$,$\mathbf{X}_L$=0,$\Delta\mathbf{E}_C$). The aggregate of individual capacitor voltages $\Delta\mathbf{E}_C$ is denoted $\mathbf{e}_{tc}$. It remains to model the 'inner life' of respective ideal capacitors themselves. I.e. the variation over time of the voltages contained in $\mathbf{e}_{tc}$: Each ideal capacitor is characterized by a constant 2x2 capacitive reactance term $\mathbf{X}_C$. The collection of all such terms pertaining to the net-work, is organized into a diagonal reactance matrix $\mathbf{X}_{Cprimitive}$. This 'lineup' together with the submatrix $\mathbf{B}_{tc}$ of $\mathbf{B}$, describing the incidence of network loops and capacitors, provide the basis for modelling of the set of capacitor voltages. $\mathbf{1}_{tc}$ is a trivial/sparse transformation matrix (with elements '0','1','-1') of the same size as $\mathbf{X}_{Cprimitive}$:

$$de_{tc}/dt = \omega_0 \left(X_{Cprimitive} \, B_{tc}^{t} \cdot i_{loop} + \bar{\mathbf{1}}_{tc} \, e_{tc} \right) \qquad (0\text{-}3)$$

Modelling of all the power network state variables together is afforded by formulating (0-1) and (0-3) as *one* simultaneous set of equations, after first solvng (0-1) with respect to $\mathbf{d}i_{loop}/dt$. The sought *system submodel* then becomes as follows, when considering the partitioning applied to **B** and $\mathbf{e}_{primitive}$:

$$\begin{bmatrix} \mathbf{d}i_{loop}/dt \\ \hline de_{tc}/dt \end{bmatrix} = \omega_0 \left[\begin{array}{c|c} -\mathbf{X}_{Lloop}^{-1} \cdot R_{loop} & -X_{Lloop}^{-1} \cdot B_{tc} \\ \hline \mathbf{X}_{Cprimitive} \, B_{tc}^{t} & \bar{\mathbf{1}}_{tc} \end{array}\right] \cdot \begin{bmatrix} i_{loop} \\ \hline e_{tc} \end{bmatrix} - \omega_0 \begin{bmatrix} \mathbf{X}_{Lloop}^{-1} \\ \hline \mathbf{0} \end{bmatrix} e_{chord} - \omega_0 \begin{bmatrix} \mathbf{X}_{Lloop}^{-1} \cdot \mathbf{B}_{t\text{-}rest} \\ \hline \mathbf{0} \end{bmatrix} e_{t\text{-}rest} \qquad (0\text{-}4)$$

Equations (0-4) describe the *power network state variables* in a structured and computationally conveniant way: The definition and 'lineup' of circuit models ($\mathbf{R}_{primitive}$, $\mathbf{X}_{Lprimitive}$, $\mathbf{X}_{Cprimitive}$, $\mathbf{e}_{primitive}$) is done once at start of analysis. So also definition of the loop matrix **B** and its submatrices $\mathbf{B}_{tc}$ and $\mathbf{B}_{t\text{-}rest}$. In recomputing the right hand side of (0-4) during initial condition analysis or integration, only *two* distinct numerical processes are involved: The fill-in of network model terms based on current value of the state variables, and matrix operations as directed by (0-2) & (0-4).

0.3 Methodological overview

From a methodological as well as computational viewpoint it appears appropriate to focus on the following main steps of analysis: *Component modelling*, *Power network modelling*, *System modelling, Initial condition analysis*, *Eigenvalue Analysis*, and *Time response analysis* :

Component modelling

A stock of five *component models* have been developed for modelling of the common power network components like overhead lines, cables, the infinite bus, capacitor banks, transformers, synchronous machines, and asynchronous machines. The five component models are 'The Lossy Inductor', 'The Lossy Capacitor Bank', 'The Synchronous Motor' in two versions, and 'The Asynchronous Motor' :

'The Lossy Inductor' models directly the three phase, inductive series impedance, the three phase inductive impedance load, and the infinite bus. Transformers, overhead lines and cables are modelled by suitably arranging together component models of the type *'Lossy Inductor'* and *'Lossy Capacitor Bank'* .

'The Lossy Capacitor Bank' models directly the three phase, lossy series capacitor, and the three phase, lossy shunt capacitor. It also contributes to the modelling of other network components as stated above.

'The Synchronous Motor' models the two main modes of operation of the synchronous machine; the voltage controlled synchronous *motor*, and the voltage- and power controlled synchronous *generator*. For conceptual clearness, *motor* mode of operation is the 'default' modelling mode.

'The Asynchronous Motor' models *motor*- as well as *generator* mode of operation of the asynchonous machine. *Motor* mode of operation is the 'default' modelling mode.

The *component model* is made up of one or more *submodels*, the configuration of which is defined by the set of *state variables* that abides with the *component model.* Table 0.1 summarizes how *submodels* add up to *component models*, and how the latter are configured to model the *main power system components.*

Table 0.1 Overview of how *submodels* add up to *component models* and *component models* add up to models of main power system components.

Main power system components ←	Component models ←	Submodel(s)
Inductive series impedance	'The Lossy Inductor'	*Electrical circuit model*
Inductive impedance load	'The Lossy Inductor'	*Electrical circuit model*
Infinite bus voltage	'The Lossy Inductor'	*Electrical circuit model*
Capacitor bank	'The Lossy Capacitor Bank'	*Electrical circuit model* Capacitor voltage model
Overhead line/Cable	'The lossy Inductor' 'The Lossy Capacitor Bank'	*Electrical circuit models* Capacitor voltage models
Transformer	'The Lossy Inductor' ('The Lossy Capacitor Bank')	*Electrical circuit models* (Capacitor voltage models)
Synchronous machine (The 'ordinary' version) (The adjustable speed version)	'The Synchronous Motor' (The '5-coil' model) (The '6-coil model)	*Electrical circuit model* Rotorflux model *) Electromechanical model *) Control system models *)
Asynchronous machine	'The Asynchronous Motor'	*Electrical circuit model* Rotorflux model *) Electromechanical model *)

*) Subsystem models that describe 'remaining' (or 'local') state variables, see previous page.

One of the *submodels* is the *electrical circuit model.* In terms of *formal* representation, the *electrical circuit model* is made common to all *component models.* The set of *electrical circuit models* that go into the network, interlink to contribute to describing integrated power network performance, - i.e. to produce the differential equations that govern the variation of the *power network state variables.*

The formal content of the *electrical circuit model* is shown in Figure 0.1. It comprises three main parts:

- *An oriented terminal graph* showing positive direction of the *circuit model variables* (**i**,**e**) that connect electrically with the external network. For a stock of 2-terminal *component models* the *oriented terminal graph* becomes an *oriented line segment.* See figure 0.1a) below. Figure b) shows the standardized d-q axis serial interconnection of circuit elements that make up the *electrical circuit model,* - and is fronted by the just stated graph.

- *Impedance terms **R** and* X_L describing the power network related 'passive' electrical properties of the *component model.* Index 'L' denotes *inductive* character of the reactance. (The effect of *capacitive* reactances appear modellingwise in terms of separate state variables). **v** is the voltage across the serial interconnection of **R** and $\mathbf{X_L}$, see figure b) below.

- *A voltage source* **e** giving the power network related source impact of the *component model. Ill.:*

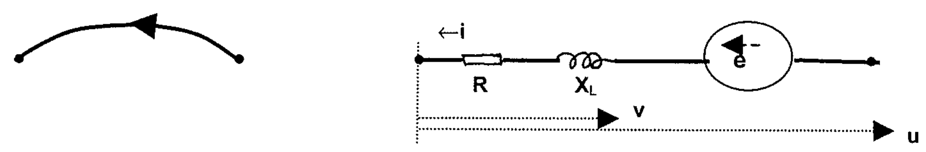
  If the *component model* is **The Lossy Inductor**, the voltage source **e** may or may not be zero : If this *component model* is applied to model e.g. an inductive series impedance, or an inductive impedance load, then **e** =0. If the task is to model a fixed voltage 'behind' some specified inductive series impedance, then **e** will be a fixed phasor. Chapter 1.6 .

  If the *component model* is **The Lossy Capacitor**, the voltage source $\mathbf{e} = \Delta\mathbf{E}_c$, where $\Delta\mathbf{E}_c$ is the voltage across the capacitor. Chapter 1.3.

  If the *component model* is **The Synchronous Motor**, the voltage source $\mathbf{e} = \Delta\mathbf{E}_{SM}$, where $\Delta\mathbf{E}_{SM}$ is a formal electromotive force (e.m.f) contributing to modelling the synchronous motor. Chapter 1.4, Chapter 1.7 and Appendix 2.

  If the *component model* is **The Aynchronous Motor**, the voltage source $\mathbf{e} = \Delta\mathbf{E}_{AM}$, where $\Delta\mathbf{E}_{AM}$ is a formal e.m.f contributing to modelling the aynchronous motor. Chapter 1.5 and 1.7.

u is the voltage across the teminals of the electrical circuit model, see figure b) below.



$$\mathbf{v} = \mathbf{R}\ \mathbf{i} + (1/\omega_b)\ \mathbf{X_L}\ di/dt \quad \& \quad \mathbf{v} - \mathbf{e} = \mathbf{u}$$

a) Oriented terminal graph

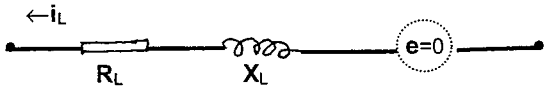
b) Serial circuit elements fronted by graph a)

Figure 0.1 *The electrical circuit model*; formal structure of submodel common to all *component models.*

Table 0.1 together with Figure 0.1 summarize the logic of power network component modelling. To illustrate the concrete content of such models, the full model of *an inductive impedance load* (as well as of *an inductive series impedance*) , *a capacitor bank*, and *a synchronous machine* are given in the following.

For details on model development, and overview of the full stock of practical models, see Chapter 1.2-1.6 and Chapter 1.7, respectively. For special treatment of the adjustable speed synchronous machine, see Appendix 2, where an 'extended' version of the synchronous machine model is developed.

The model of *the inductive impedance load* is shown in Figure 0.2. From Table 0.1 it is seen that *the electrical circuit model* is the only submodel contributing to form the sought model.



where;

$$\mathbf{R}_L = \begin{bmatrix} r_L & -x_L \\ x_L & r_L \end{bmatrix}$$

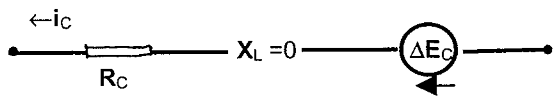
$$\mathbf{X}_L = \begin{bmatrix} \omega_b L & \\ & \omega_b L \end{bmatrix} = \begin{bmatrix} x_L & \\ & x_L \end{bmatrix}$$

$z = (r_L + jx_L)$ = given per phase description of *lossy inductor*

Electrical circuit model

Figure 0.2 Component model ***'The Lossy Inductor'***.

The model of the *capacitor bank* is shown in Figure 0.3. From Table 0.1 it is observed that *two* sub-models contribute to form the sought model; *the electrical circuit model* and *the capacitor voltage model :*



where;

$$\mathbf{R}_C = \begin{bmatrix} r_c & \\ & r_c \end{bmatrix}$$

$$\Delta \mathbf{E}_C = \begin{bmatrix} \Delta E_{Cd} \\ \Delta E_{Cq} \end{bmatrix}$$ = Capacitor voltage. See model below.

(r_c, C) = given per phase parameters of *lossy capacitor bank*

Electrical circuit model

$$d\Delta \mathbf{E}_C/dt = \omega_b \left(\mathbf{X}_C \, \mathbf{i}_C + \bar{\mathbf{1}} \, \Delta \mathbf{E}_C \right)$$

with initial condition;

$$\Delta \mathbf{E}_{C(o)} = (\bar{\mathbf{1}} \, \mathbf{X}_C) \, \mathbf{i}_{C(o)}$$

where;

$$\mathbf{X}_C = \begin{bmatrix} 1/(\omega_b C) & \\ & 1/(\omega_b \cdot C) \end{bmatrix}$$

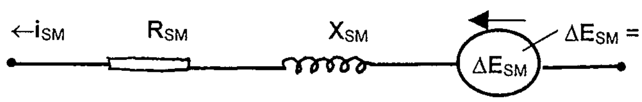
$$\bar{\mathbf{1}} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Capacitor voltage model

Figure 0.3 Component model ***'The Lossy Capacitor Bank'***.

The model of the *synchronous motor* [*)] is shown in Figure 0.4. From Table 0.1 it is inferred that *five* submodels contribute to form the sought model ; *the electrical circuit model, the rotorflux model, the electromechanical* model, and 2 *control system models* (i.e. models for *voltage-* and *power* control). Figure 0.4 covers nearly three pages. The figure intendeds to be 'self-contained' in the sense that it defines and displays on compact form all model parameters involved :

Electrical circuit model



where ;

$\Delta E_{SM} = (V_{SM} + H_{SM}\cdot\phi_{SM})$ = synchronous motor emf.

$$R_{SM} = \begin{pmatrix} (R_a+\hat{X}''_r) + (1+2\cdot\Delta\Omega_{SM})\cdot\bar{X}''\cdot\sin2\beta_{SM} + \bar{X}''_r\cdot\cos2\beta_{SM} & -\hat{X}'' + (1+2\cdot\Delta\Omega_{SM})\cdot\bar{X}''\cdot\cos2\beta_{SM} - \bar{X}''_r\cdot\sin2\beta_{SM} \\ \hat{X}'' + (1+2\cdot\Delta\Omega_{SM})\cdot\bar{X}''\cdot\cos2\beta_{SM} - \bar{X}''_r\cdot\sin2\beta_{SM} & (R_a+\hat{X}''_r) - (1+2\cdot\Delta\Omega_{SM})\cdot\bar{X}''\cdot\sin2\beta_{SM} - \bar{X}''_r\cdot\cos2\beta_{SM} \end{pmatrix}$$

$$X_{SM} = \begin{pmatrix} \hat{X}'' + \bar{X}''\cdot\cos2\beta_{SM} & -\bar{X}''\cdot\sin2\beta_{SM} \\ -\bar{X}''\cdot\sin2\beta_{SM} & \hat{X}'' - \bar{X}''\cdot\cos2\beta_{SM} \end{pmatrix} \qquad V_{SM} = \begin{pmatrix} C_f\cdot E_f\cdot\cos\beta_{SM} \\ -C_f\cdot E_f\cdot\sin\beta_{SM} \end{pmatrix}$$

$\Delta\Omega_{SM} = (\Omega_{SM}-1)$ = deviation of rotor speed from synchronous/target value. In pu.
$E_f = (E_{fo} + \Delta E_f)$ = field voltage, where E_{fo} is initial value and ΔE_f is voltage control system response. See following p 0/9.
$C_f = (\sqrt{2}/(\omega_o\cdot T'_{do}))\cdot(X''_{ad}/X'_{ad})$

$$H_{SM} = \begin{pmatrix} \Omega_{SM}\cdot f_1\cdot\sin\beta_{SM} + f_2\cdot\cos\beta_{SM} & \Omega_{SM}\cdot f_3\cdot\sin\beta_{Sm} + f_4\cdot\cos\beta_{SM} & \Omega_{SM}\cdot f_5\cdot\cos\beta_{SM} + f_6\cdot\sin\beta_{SM} \\ \Omega_{SM}\cdot f_1\cdot\cos\beta_{SM} - f_2\cdot\sin\beta_{SM} & \Omega_{SM}\cdot f_3\cdot\cos\beta_{SM} - f_4\cdot\sin\beta_{SM} & -\Omega_{SM}\cdot f_5\cdot\sin\beta_{SM} + f_6\cdot\cos\beta_{SM} \end{pmatrix}$$

$\hat{X}'' = 0.5(X''_d + X''_q)$ $\quad \hat{X}''_r = 0.5(X''_{rd} + X''_{rq})$ $\quad \leftarrow \quad X''_{rd} = (1/(\omega_o\cdot T'_{do}))\cdot(X''_{ad}/X'_{ad})^2\cdot(X_d - X'_d) + (1/(\omega_o\cdot T''_{do}))\cdot(X'_d - X''_d)$

$\bar{X}'' = 0.5(X''_d - X''_q)$ $\quad \bar{X}''_r = 0.5(X''_{rd} - X''_{rq})$ $\quad \leftarrow \quad X''_{rq} = (1/(\omega_o\cdot T''_{qo}))\cdot(X_q - X''_q)$

$f_1 = (X_d - X'_d)\cdot(X''_{ad}/(X_{ad}\cdot X'_{ad}))$ $\quad \leftarrow \quad X_{ad} = X_d - X_{a\sigma}$
$f_2 = f_1\cdot[(X'_d - X''_d)\cdot [(1/(\omega_o\cdot T''_{do}))\cdot(1/X''_{ad}) - (1/(\omega_o\cdot T'_{do}))\cdot(X_{ad}/X'^2_{ad})] - (1/(\omega_o\cdot T'_{do}))\cdot(X''_{ad}/X'_{ad})]$ $\quad \leftarrow \quad X'_{ad} = X'_d - X_{a\sigma}$
$f_3 = (X'_d - X''_d)/X'_{ad}$ $\quad \leftarrow \quad X''_{ad} = X''_d - X_{a\sigma}$
$f_4 = f_3\cdot[(1/(\omega_o\cdot T'_{do}))\cdot(X''_{ad}/X'^2_{ad})\cdot(X_d - X'_d) - (1/(\omega_o\cdot T''_{do})]$
$f_5 = - (X_q - X''_q)/X_{aq}$ $\quad \leftarrow \quad X_{aq} = X_q - X_{a\sigma}$
$f_6 = f_5\cdot(1/(\omega_o\cdot T''_{qo}))$ $\quad \leftarrow \quad X''_{aq} = X''_q - X_{a\sigma}$

Figure 0.4 (start of..) Component model ***'The Synchronous Motor'***.

[*)] Formal basis for the synchronous machine model above is the ***five-coil, salient pole generalised machine*** as defined and further elaborated in Chapter 1.4. *Synchronous Machine parameters* to be specified for this model (with example hydro-generator data in parenthesis) :

$X_{a\sigma}$ (0.12pu)	X'_d (0.30pu)	R_a (0.005pu)	T''_q (0.16s)	$\cos\varphi_N$ (0.9pu)
X_d (1.2pu)	X''_d (0.20pu)	T'_{do} (6.0s))	T_a (5.0s)	S_N (100MVA)
X_q (0.75pu)	X''_q (0.30pu)	T''_d (0.04s)	C_D (7.5pu)	E_N (16kV)

The corresponding model based on the *six-coil generalised machine*, is developed in App.2. This model may allow for simulating performance of the *double-fed asynchronous machine* as well as the *adjustable speed synchronous machine.*

$$\underset{(3x1)}{d\phi_{SM}/dt} = \omega_o\cdot(\mathbf{e}_{SMr} + \mathbf{F}_{SMi}\cdot\mathbf{i}_{SM} + \mathbf{F}_{SM\phi}\cdot\phi_{SM})$$

Rotorflux model

Here:

$\mathbf{F}_{SMi}$ =

$(1/(\omega_o\cdot T'_{do}))\cdot(X_{ad}/X'_{ad})\cdot X''_{ad}\cdot\cos\beta_{SM}$	$-(1/(\omega_o\cdot T'_{do}))\cdot(X_{ad}/X'_{ad})\cdot X''_{ad}\cdot\sin\beta_{SM}$
$(1/(\omega_o\cdot T''_{do}))\cdot X'_{ad}\cdot\cos\beta_{SM}$	$-(1/(\omega_o\cdot T''_{do}))\cdot X'_{ad}\cdot\sin\beta_{SM}$
$(1/(\omega_o\cdot T''_{qo}))\cdot X'_{aq}\cdot\sin\beta_{SM}$	$(1/(\omega_o\cdot T''_{qo}))\cdot X'_{aq}\cdot\cos\beta_{SM}$

$\mathbf{e}_{SMr}$ =

$K_f\cdot E_f$
0
0

$E_f = (E_{fo} + \Delta E_f)$ = field voltage
$K_f = (\sqrt{2}/(\omega_o\cdot T'_{do}))\cdot X_{ad}/(X_d - X'_d)$
ΔE_f = voltage control response

$\mathbf{F}_{SM\phi}$ =

$-(1/(\omega_o\cdot T'_{do}))\cdot(1/X'_{ad})\cdot[(X_{ad}/X'_{ad})\cdot(X'_d - X''_d) + X''_{ad}]$	$(1/(\omega_o\cdot T'_{do}))\cdot(X_{ad}/X'^2_{ad})\cdot(X'_d - X''_d)$	
$(1/(\omega_o\cdot T''_{do}))\cdot(1/X_{ad})\cdot(X_d - X'_d)$	$-1/(\omega_o\cdot T''_{do})$	
		$-1/(\omega_o\cdot T''_{qo})$

At any time during integration the rotor currents may be derived from the equations $\phi_{SM} = \mathbf{X}_{DQr}\cdot\mathbf{i}_{SM} + \mathbf{X}_{rr}\cdot\mathbf{i}_{SMr}$:

$$\mathbf{i}_{SMr} = (\mathbf{X}_{rr})^{-1}\cdot[\,\phi_{SM} - \mathbf{X}_{DQr}\cdot\mathbf{i}_{SM}\,]$$

where ;

$\mathbf{X}_{DQr}$ =

$X_{ad}\cdot\cos\beta_{SM}$	$-X_{ad}\cdot\sin\beta_{SM}$
$X_{ad}\cdot\cos\beta_{SM}$	$-X_{ad}\cdot\sin\beta_{SM}$
$X_{aq}\cdot\sin\beta_{SM}$	$X_{aq}\cdot\cos\beta_{SM}$

$\mathbf{X}_{rr}$ =

$X^2_{ad}/(X_d - X'_d)$	X_{ad}	
X_{ad}	$X_{ad} + X'_{ad}\cdot X''_{ad}/(X'_d - X''_d)$	
		$X^2_{aq}/(X_q - X''_q)$

Electromechanical model

$$\underset{(1x1)}{d\Omega_{SM}/dt} = (S_{Bas}/S_{SM})\cdot(1/(T_a\cdot\cos\varphi_N))\cdot(T_{SMel} - T_{SMmec})$$

Here:

T_{Smel} = $0.5\cdot\mathbf{i}^t_{SM}\cdot\mathbf{T}_{SM1}\cdot\phi_{dq}$ = electrical motor torque , - where $\phi_{dq} = \mathbf{X}''_{SM}\cdot\mathbf{T}_{SM}\cdot\mathbf{i}_{SM} + \mathbf{f}_{SM}\cdot\phi_{SM}$

T_{SMmec} = $T_{SMmec(o)}\cdot\Omega_{SM}{}^{\kappa}$ = mechanical torque in **motor** mode of operation. (Motor operation implies pos. sign of mech. torque)

If the motor is up and running at t=-0: $T_{SMmec(o)} = T_{SMel(o)}$ = electrical motor torque at t = -0. This is found from equation (1-117) applied to the initial power system load flow. κ= (say) 1.5-3.5
If the motor is to be started from stillstand (as e.g. an asynchronous motor) : $T_{SMmec(o)}$ = coefficient to model mechanical friction, air resistance, etc. during startup. Probable range: 0.02-0.05

T_{SMmec} = $(T_{SMel(o)} + \Delta T_{mec})$ = mechanical torque in **generator** mode of operation. ΔT_{mec} is the response from the *power control system*. See below for a sample hydro generator power control system.

S_{Bas}, S_{SM} = Chosen VA system power base, and rated VA motor capacity, respectively
T_a, $\cos\varphi_N$ = Dynamical system's inertia constant, and motor's rated power factor, respectively

$\mathbf{T}_{SM1}$ =

$\sin\beta_{SM}$	$-\cos\beta_{SM}$
$\cos\beta_{SM}$	$\sin\beta_{SM}$

$\mathbf{T}_{SM}$ =

$\cos\beta_{SM}$	$-\sin\beta_{SM}$
$\sin\beta_{SM}$	$\cos\beta_{SM}$

$\mathbf{X}''_{SM}$ =

X''_d	
	X''_q

$\mathbf{f}_{SM}$ =

f_1	f_3	
		$-f_5$

$f_1 = (X_d - X'_d)\cdot X''_{ad}/(X_{ad}\cdot X'_{ad})$
$f_3 = (X'_d - X''_d)/X'_{ad}$
$f_5 = -(X_q - X''_q)/X_{aq}$

The electrical angle of the rotor is defined as

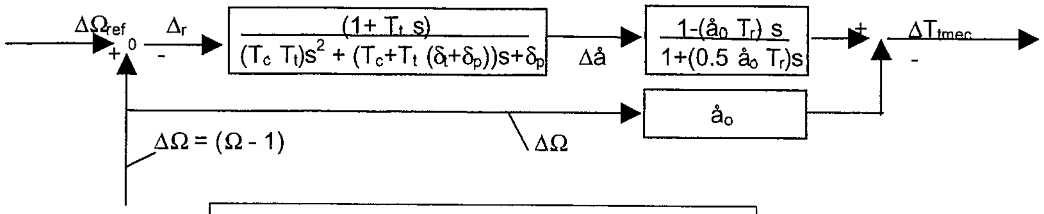
$$\beta_{SM} = (\omega_o\cdot t - \theta_{SM})$$

giving rise to the following differential equation describing the angular movement of the Synchronous Motor:

$$\underset{(1x1)}{d\beta_{SM}/dt} = \omega_o\cdot(1 - \Omega_{SM})$$

Figure 0.4 (continued..) Component model ***'The Synchronous Motor'***.

Power control system



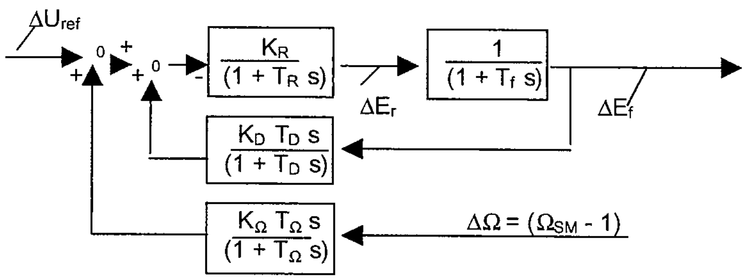
For 'normal' frequency control : $\Delta\Omega_{ref} = 0$.
For 'P-control' :
$\Delta\Omega_{ref} = c\ (P_{target} - P_m)$

$d\Delta å/dt = K_1\ (\Delta\Omega_{ref} - (1 - \Omega_{SM}) + \Delta w) - K_2\ \Delta å$	Regulator system
$d\Delta w/dt = K_3\ \Delta å - K_4\ \Delta w$	Regulator system
$d\Delta g/dt = (3K_o/\Omega_{SM})\ \Delta å - K_ó\ \Delta g$ ↓	Hydraulic system
$\Delta T_{mec} = \Delta g - å_ó\ (1 - \Omega_{SM}) - (2/\Omega_{SM})\ \Delta å$	Net change of mechanical torque

$K_o = 2/(å_o\ T_r)$
$K_1 = 1/T_c$
$K_2 = (\delta_p + \delta_t)/T_c$ ←
$K_3 = \delta_t/T_t$
$K_4 = 1/T_t$
P_m = absorbed motor power $= 0.5\ i_{SM}^t\ e_{SM}$
P_m is negative in generator mode.
P_{target} = target value of P_m
c = per unit scaling factor (eg.: c=0.1)

Control system parameters:
T_r : Time constant for hydraulic system (eg 0.3s)
T_c : Time constant for main servo (eg 0.08s)
T_t : Transient droop time constant (eg 17s)
δ_t : Transient droop (eg 0.15pu)
δ_p : Permanent droop (eg 0.0 – 0.03. The value 0.0 if frequency sustained by a single unit)
$å_o$: Initial pu turbine opening ($t_å$). (If $t_å$<0.3 then $å_o$ =0.3)

Voltage control system



$E_f = (E_{f(o)} + \Delta E_f)$
$E_{f(min)} \le E_f \le E_{f(max)}$
ΔU_{ref} = 0, unless new voltage ref. is set

$$d\Delta E_{qf}/dt = C_1\ (\Delta E_r - \Delta E_{qf})$$
$$d\Delta E_r/dt = C_2\ [\Delta U_{ref} + U_o - U + K_\Omega\ (\Omega_{SM} - 1) - \Delta h] - C_3\ \Delta E_{qf} - C_4\ \Delta E_r + C_2\ \Delta E_{ss}$$
$$d\Delta E_{ss}/dt = C_5\ \Delta E_{qf} - C_6\ E_{ss}$$
$$d\Delta h/dt = C_7\ (\Omega_{SM} - 1) - C_8\ \Delta h$$

$U = (1/\sqrt{2})\ (e_{SM}^2 + e_{SM}^2)^{0.5}$
$C_1 = 1/T_f$
$C_2 = K_R/T_R$
$C_3 = K_R\ K_o/T_R = K_D\ C_2$ ←
$C_4 = 1/T_R$
$C_5 = K_o/T_o$
$C_6 = 1/T_D$
$C_7 = K_\Omega/T_\Omega$
$C_8 = 1/T_\Omega$
$\Delta U = (U-U_o)$ = pu voltage deviation

Control system parameters:
T_f = field circuit time constant (eg 0.1s)
K_R = resulting forward amplification (eg 70pu)
T_R = regulator time constant (eg 0.1s)
K_D = transient feedback amplification (eg 0.25pu)
T_D = transient feedback time constant (eg 0.25s)
K_Ω = power stabilizer amplification (eg 1pu)
$E_{f(max)}$ = ceiling field voltage (eg 3pu)
$E_{f(min)}$ = bottom field voltage (eg –2pu)
T_Ω = power stabilizer time constant (eg 2s)

Figure 0.4 (end of) Component model **'The Synchronous Motor'**: Example hydro generator control system models.

Power network modelling

The concept and content of netwotk modelling is summarized by way of a simple illustration*) :Given the task of modelling the performance of the small power system to the left in Figure 0.5. The system com-prises *three* power network components; *the infinite bus*, *a synchronous motor*, and *an impedance type inductive load.* The *electrical circuit submodel* of respective power system components is previously given in figures 0.2-0.4. Moving from left to right in Figure 0.5b, the *oriented terminal graph* of respective circuit submodels is first given , followed by a description of the *content* of the *circuit models* fronted by the teminal graphs. For brevity of notation the oriented terminal graphs are identified by labels '1' to '3'.

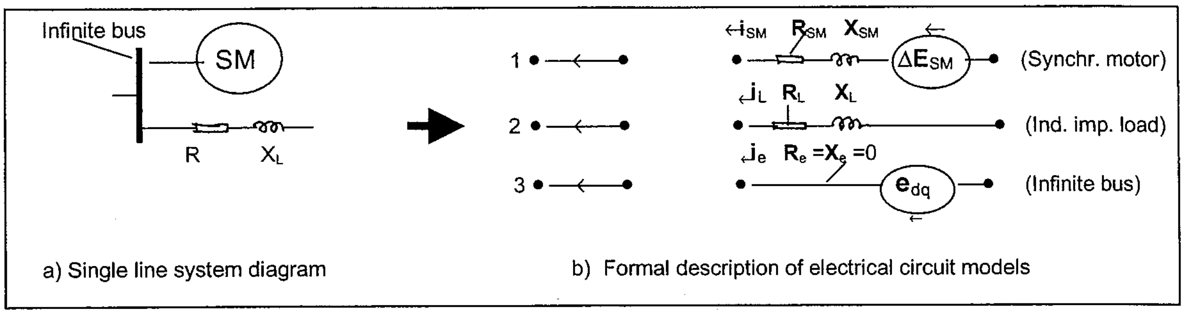


Figure 0.5 Simple three-component power system. System components and their interconnection identified in figure a). The components' electrical submodels given i figure b).

The aggregate of *separate electrical circuit models* (that constitute the *network* model if inter-connected,) may be said to form the *primitive network* of the system. Once the *primitive network* is given and it is specified how the network components are tied together, a general basis for *power network modelling* is established.

The methodology of *power network modelling* is next summarized. It is inherently a three-stage process to which the following subheadings may apply: *'The primitive network'*, *'Network topology'* and *'Network modelling'* :

The primitive network

The content of the primitive network is readily illustrated for the example system of Figure 0.5 : With the labelling '1' to '3' chosen, only a suitable arrangement of the given electrical component data is required to produce its *primitive network* shown in Figure 0.6.

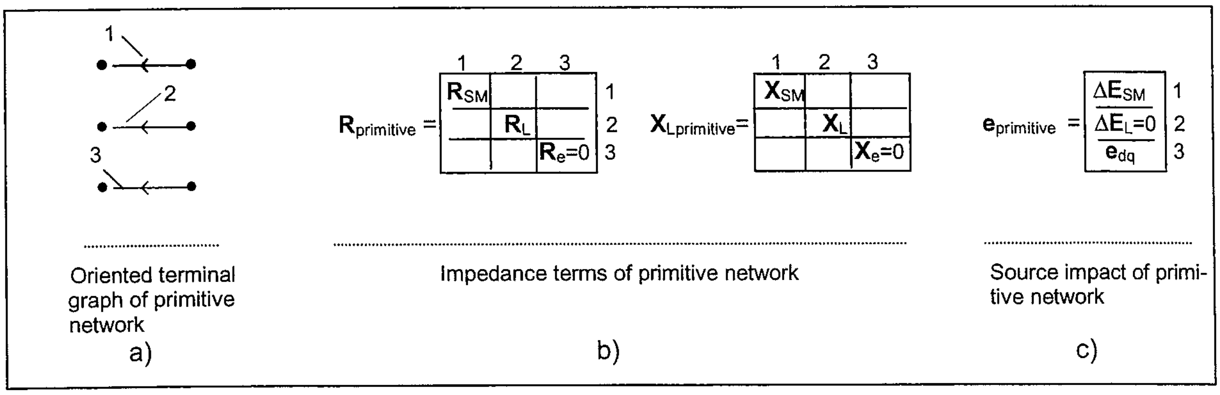


Figure 0.6 *The primitive network* of the three-component power system of Figure 0.5

*) For modelling of more complex networks where capacitors are present, reference is made to Chapter 2.

The primitive network comprises *three* main parts in the same way as each of its contributing *electrical circuit models* :

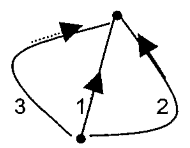
- *An oriented terminal graph* showing positive direction of the circuit model variables ($\mathbf{i}_{primitive}$, $\mathbf{e}_{primitive}$) that connect with the external network to interplay in the operation of the specified network. By convention each of the graph's line segments fronts a standardized d-q axis circuit element as defined in Figure 0.1b.

- *Impedance terms* $\boldsymbol{R}_{primitive}$ *and* $\boldsymbol{X}_{Lprimitive}$ describing the 'passive' properties of the set of circuit elements that are contained in the network. $\mathbf{R}_{primitive}$ is diagonal. Index 'L' denotes that $\mathbf{X}_{Lprimitive}$ always is of *inductive* character. See previous comment to Figure 0.1 concerning handling of *capacitive* reactances. If there is electro-magnetic coupling between system components (as e.g. may be the case for parallell overhead lines close to each other), $\mathbf{X}_{Lprimitive}$ may contain off-diagonal terms. Otherwise, $\mathbf{X}_{Lprimitive}$ will be diagonal as exemplified above in Figure 0.6.

- *A voltage source vector* $\mathbf{e}_{primitive}$ giving the source impact of the defined set of voltage sources contained in the network.

Network topology

Graphwise the topology of a network is established by connecting together the graph elements of its primitive system, as directed by the single line diagram of the power network at hand. The oriented graph of the small system in Figure 0.5a is formed by inteconnecting the primitive network graph elements of Figure 0.6a, as advised by the single line diagram implied by Figure 0.5a. The system graph is shown in Figure 0.7 :



1 : Synchronous motor
2 : Inductive impedance load
3 : Infinite bus

b = no. of network graph elements = 3
N_{node} = no. of nodes of network graph = 2
N_{tree} = no. of network tree elements = (N_{node} −1) = (2-1) = 1
N_{loop} = no. of cotree elements (chords) = (b -N_{tree}) =(3 -1) =2

Figure 0.7 Oriented graph of the three-component system in Figure 0.5a.

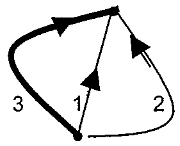
The formal modelling of interconnection of components may be afforded by different topological matrices comprising plus/minus '1', or '0' as matrix elements. In the present outline a *system loop matrix* $\boldsymbol{B}$ is used to formally describe how the power network components are tied together.

The system loop matrix $\boldsymbol{B}$ is conveniently defined on the basis of a chosen *tree* and *cotree* of the oriented network graph:

- The *tree* is a set of N_{tree} graph elements that connects all nodes of the network graph without closing any circuit. N_{tree} = (N_{node} -1), where N_{node} is the total number of nodes of the connected graph. For the network graph of Figure 0.7, N_{tree} = (2 −1) = 1. The chosen tree of this graph is shown in thick line in Figure 0.8a. Capacitor elements must belong to the *tree*, and the capacitor graph elements should conveniently be numbered first among the tree elements. It is also suitable to have any exogenously specified voltages located to *tree* elements.

- The remaining N_{loop} = (b −N_{tree}) graph elements constitute the corresponding *cotree* of the oriented network graph. b is the number of elements of the network graph. Each cotree element - or chord - identifies a unique loop of the network graph. Thus the collection of chosen cotree elements identifies a necessary and sufficient set of independent system loops for evaluation of network flow solutions. When deciding on the numbering of elements in the primitive network graph, the elements that are being defined as members of the cotree should conveniently be numbered first. For the network graph of Figure 0.7 the number of cotree elements is N_{loop} = (3 −1) = 2. The chords are identified by thin lines in Figure 0.8a.

The system loop matrix **B** describes the incidence of independent network loops as defined by the set of cotree elements, and the set of all graph elements of the connected network. See Figure 0.8b for simple illustration.



Thin lines : cotree elements (chords)
Thick line : tree element

a) Oriented network graph

Element no →

Loop no ↓

$$\mathbf{B} = \begin{array}{c|cc|c|} & 1 & 2 & 3 \\ \hline 1 & 1 & 0 & -1 \\ 2 & 0 & 1 & -1 \\ \hline \end{array} = [\mathbf{B}_{chord}, \mathbf{B}_{tree}]$$

b) Loop matrix **B**

Figure 0.8 Graph description in terms of *tree, cotree (chords)* and *B-matrix.*

The labels (read numbers) attached to the *cotree* elements can conveniently identify also the associated set of independent network loops. Furthermore, the chosen orientation of the cotree elements can suitably define positive direction of the network loop currents.

B can be partitioned into a submatrix $\mathbf{B}_{chord}$ that describes the incidence of *loops* and *cotree elements* (or *chords),* and submatrix $\mathbf{B}_{tree}$ that gives the incidence of *loops* and *tree* elements. Given the conventions above, $\mathbf{B}_{chord}$ will always be a unit matrix. Figure 0.8b illustrates the definition of submatrices in the simple three-component case.

To facilitate definition and processing of the differential equations that describe the capacitor voltage state variables ($\mathbf{e}_{tc}$), it is desirable to partition the submatrix $\mathbf{B}_{tree}$, - when relevant. With the stated numbering scheme for *tree*-elements, the following definition is made:

$$\mathbf{B}_{tree} = [\mathbf{B}_{tc}, \mathbf{B}_{t\text{-}rest}] \qquad (0\text{-}5)$$

where $\mathbf{B}_{tc}$ is a submatrix describing the incidence of *loops* and the *subset of tree elements* that relate to *capacitors.* $\mathbf{B}_{t\text{-}rest}$ is a submatrix describing the incidence of *loops* and the *'rest' of the tree elements.* The loop matrix **B** of Figure 0.8 does not allow for definition of $\mathbf{B}_{tc}$, since no capacitors are present in the simple power system of Figure 0.5. For illustration of the content of all the defined submatrices of **B**, see Chapter 2.2.

Network modelling

The *network model* is a set of simultaneous, first order, ordinary differential equations describing the behaviour of the power network state variables, - which are the *network loop currents* $\mathbf{i}_{loop}$ together with the *voltages* $\mathbf{e}_{tc}$ *across the capacitors* of the system.

The *network model* is readily generated based on 1) the lineup of the *electrical circuit models* of the power network into the standardized form of a *primitive system,* and 2) the *system loop matrix* ***B***, describing how the *electrical circuit models* are tied together into a network. The *network model* which is developed in Chapter 2.3, can be compactly expressed as follows:

$$\begin{bmatrix} d\mathbf{i}_{loop}/dt \\ \hline d\mathbf{e}_{tc}/dt \end{bmatrix} = \omega_0 \begin{bmatrix} -\mathbf{X}_{Lloop}^{-1} \cdot \mathbf{R}_{loop} & -\mathbf{X}_{Lloop}^{-1} \cdot \mathbf{B}_{tc} \\ \hline \mathbf{X}_{Cprimitive} \cdot \mathbf{B}_{tc}^{t} & \bar{\mathbf{1}}_{tc} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{i}_{loop} \\ \hline \mathbf{e}_{tc} \end{bmatrix} - \omega_0 \begin{bmatrix} \mathbf{X}_{Lloop}^{-1} \\ \hline \mathbf{0} \end{bmatrix} \cdot \mathbf{e}_{chord} - \omega_0 \begin{bmatrix} \mathbf{X}_{Lloop}^{-1} \cdot \mathbf{B}_{t\text{-}rest} \\ \hline \mathbf{0} \end{bmatrix} \cdot \mathbf{e}_{t\text{-}rest} \qquad (0\text{-}6)$$

where;

$\mathbf{i}_{loop}$ = (N_{loop}x1) loop current vector, comprising the current in respective cotree elements (chords) of the network graph. Each loop current comprises a d- and a q-component, so that numerically, $\mathbf{i}_{loop}$ is of dimension ($2N_{loop}$x1). N_{loop} is the number of independent loops of the graph. It is also the number of cotree elements - or chords - of the graph.

$\mathbf{e}_{tc}$ = (N_Cx1) capacitor voltage vector, comprising the voltage across respective ideal capacitors of the network. Each voltage comprises a d-and a q-component, so that numerically $\mathbf{e}_{tc}$ is of dimension ($2N_C$x1). N_C is the number of capacitors of the single line diagram of the power network.

$\mathbf{e}_{chord}$ = (N_{loop}x1) voltage source vector associated with respective cotree elements (or chords) of the graph. Each voltage comprises a d-and a q-component, so that numerically $\mathbf{e}_{chord}$ is of dimension ($2N_{loop}$x1). Synchronous and asynchronous machines are conveniently defined as cotree elements of the graph, with defined source impacts as follows :

The synchronous motor : $\Delta\mathbf{E}_{SM} = (\mathbf{V}_{SMf} + \mathbf{H}_{SM}\ \phi_{SM})$ see Figure 0.4 and Chapter 1.7
The asynchronous motor : $\Delta\mathbf{E}_{AM} = \mathbf{H}_{AM}\ \phi_{AM}$ see Chapter 1.7

$\mathbf{e}_{t\text{-}rest}$ = ((N_{tree} - N_C)x1) voltage source vector associated with the 'rest of the tree elements', i.e. the tree elements that are not representing capacitors. Each voltage comprises a d- and a q-component, so that $\mathbf{e}_{t\text{-}rest}$ numerically is of dimension 2(N_{tree} - N_C)x1. N_{tree} = number of tree elements of the graph. An exogenously specified voltage should suitably be associated with such a tree element. The infinite bus voltage which is dealt with in Chapter 1.6, can be conveniently expressed as

$$\mathbf{e}_{dq} = [e_d, e_q]^t = \sqrt{2}\ E_{y(eff)}\ [-\sin\gamma_{ref}, \cos\gamma_{ref}]^t$$

where $E_{y(eff)}$ is per unit *root mean square (r.m.s.)* value of the given three phase voltage, and γ_{ref} accounts for an arbitrary phase shift of the given voltage relative to zero time.

B = (N_{loop}xb) system loop matrix in the d-q axis frame of reference. Entries are **+/-1** or **0**. '**1**' means numerically a 2x2 unit matrix. '**0**' means a 2x2 emty matrix. b is the number of elements of the oriented graph. **B** is partitioned as follows:

$\mathbf{B} = [\mathbf{B}_{chord}, \mathbf{B}_{tree}]$ see Figure 0.8 for ill.
$\mathbf{B}_{tree} = [\mathbf{B}_{tc}, \mathbf{B}_{t\text{-}rest}]$ see equation (0-5) and associated text

$\mathbf{R}_{loop} = \mathbf{B}\ \mathbf{R}_{primitive}\ \mathbf{B}^t$ = (N_{loop}xN_{loop}) network loop resistance matrix in the d-q axis frame of reference. Each element $\mathbf{R}_{lopp}[i,j]$ of this matrix 'hides' a 2x2 'local' d-q description. Numerically $\mathbf{R}_{loop}$ then is of dimension ($2N_{loop}$x$2N_{loop}$)

$\mathbf{R}_{primitive}$ is - in the d-q frame of reference – a bxb matrix displaying the R-term of each and every element of the system graph. $\mathbf{R}_{primitive}$ is diagonal. Each element $\mathbf{R}_{primitive}[i,j]$ of this matrix 'hides' a 2x2 'local' d-q description. Numerically $\mathbf{R}_{primitive}$ then is of dimension (2bx2b). For simple illustration of $\mathbf{R}_{primitive}$, see Figure 0.6.

$\mathbf{X}_{Lloop} = \mathbf{B}\ \mathbf{X}_{Lprimitive}\ \mathbf{B}^t$ = (N_{loop}xN_{loop}) network loop *inductor* matrix in the d-q frame of reference. Each element $\mathbf{X}_{Lloop}[i,j]$ 'hides' a 2x2 'local' d-q description. Numerically $\mathbf{X}_{Lloop}$ then is of dimension ($2N_{loop}$x$2N_{loop}$).

$\mathbf{X}_{Lprimitive}$ is - in the d-q frame of reference – a bxb matrix displaying the X_L-term of each and every element of the system graph. If there are no mutual couplings between circuits, $\mathbf{X}_{Lprimitive}$ is diagonal. For the small example system included, $\mathbf{X}_{Lprimitive}$ is diagonal. See Figure 0.6b for ill.. Each element $\mathbf{X}_{Lprimitive}[i,j]$ 'hides' a 2x2 'local' d-q description. Numerically $\mathbf{X}_{Lprimitive}$ then is of dimension (2bx2b).

System modelling

The set of simultaneous, first order, ordinary differential equations that fully describes the performance of the power system at hand, is here denoted 'the system model'. In compact notation *the system model* can be written as;

$$d\mathbf{z}/dt = \mathbf{f}(\mathbf{z},\mathbf{v}) \qquad (0\text{-}7)$$

where **z** is the set of all system state variables, and **v** comprises exogenously specified excitations such as e.g. infinite bus voltage(s) and initial value of synchronous machine field voltages. As outlined In the 'Conceptual overview', **z** can be partitioned into *two* subsets of state variable ; *the power network state variables* which are the loop currents together with the voltage across the capacitors of the system, and the *remaining system state variables* which could also be termed 'local' component variables.

The part of the system model comprising the differential equations of the *power network state variables* ($\mathbf{i}_{loop}$,$\mathbf{e}_{Tc}$), is given by (0-6) :

$$\begin{aligned} d\mathbf{i}_{loop}/dt &= -\,\omega_0\, \mathbf{X}_{Lloop}^{-1} \cdot (\mathbf{R}_{loop}\, \mathbf{i}_{loop} + \mathbf{B}_{tc}\, \mathbf{e}_{tc} + \mathbf{e}_{chord} + \mathbf{B}_{t\text{-}rest}\, \mathbf{e}_{t\text{-}rest}) \\ d\mathbf{e}_{tc}/dt &= \omega_0\, (\mathbf{X}_{Cprimitive}\, \mathbf{B}_{tc}^{t}\, \mathbf{i}_{loop} + \mathbf{1}_{tc}\, \overline{\mathbf{e}}_{tc}) \end{aligned} \qquad (0\text{-}8)$$

The residual part of the system model comprising the differential equations of the *remaining system state variables,* is summarized next qua main types of 'local' component variables. Such variables relate first of all to rotating machines, and to the control systems governing bus voltages and power generation :

The 'local' state variables pertaining to the synchronous motor are the motor fluxes $\phi_{SM} = [\phi_f ,\phi_{kd}, \phi_{kq}]^t$, the motor's pu angular speed Ω_{SM} , and its electrical rotor angle β_{SM}. Figure 0.4 presents the model of the synchronous motor. From the subsystem models that make up this model, the differential equations of the local variables (ϕ_{SM}, Ω_{SM}, β_{SM}) are as follows:

$$\begin{aligned} d\phi_{SM}/dt &= \omega_0\, (\mathbf{V}_{SMr} + \mathbf{F}_{SMi}\, i_{SM} + \mathbf{F}_{SM\phi}\, \phi_{SM}) \\ d\Omega_{SM}/dt &= (S_{Bas}/S_{SM})\, (1/(T_å \cos\varphi_N))\, (T_{SMel} - T_{SMmec}) \\ d\beta_{SM}/dt &= \omega_0 \cdot (1 - \Omega_{SM}) \end{aligned} \qquad (0\text{-}9)$$

The 'local' state variables pertaining to the asynchronous motor are the motor fluxes $\phi_{AM} = [\phi_{r(d)},\phi_{r(q)}]^t$, and the motor's pu angular speed Ω_{AM}. From Chapter 1.7 titled Component model summary, is fetched:

$$\begin{aligned} d\phi_{AM}/dt &= \omega_0\, (\mathbf{F}_{AMi}\, i_{AM} + \mathbf{F}_{AM\phi}\, \phi_{AM}) \\ d\Omega_{AM}/dt &= (S_{Bas}/S_{AM})\, (1/(T_å \cos\varphi))\, (T_{AMel} - T_{AMmec}) \end{aligned} \qquad (0\text{-}10)$$

The 'local' state variables pertaining to the synchronous motor's voltage control system, are the incremental quantities (ΔE_{qf},ΔE_r,ΔE_{ss},Δh). See Chapter 1.7 for further definitions. From that Chapter :

$$\begin{aligned} d\Delta E_{qf}/dt &= C_1\, (\Delta E_r - \Delta E_{qf}) \\ d\Delta E_r/dt &= C_2\, [\Delta U_{ref} + U_0 - U + K_\Omega\, (\Omega - 1) - \Delta h] - C_3\, \Delta E_{qf} - C_4\, \Delta E_r + C_2\, \Delta E_{ss} \\ d\Delta E_{ss}/dt &= C_5\, \Delta E_{qf} - C_6\, E_{ss} \\ d\Delta h/dt &= C_7\, \Delta\Omega - C_8\, \Delta h \end{aligned} \qquad (0\text{-}11)$$

The 'local' state variables pertaining to the power control system of the synchronous motor in generator mode of operation, are the incremental quantities ($\Delta å$, Δw, Δg). See Chapter 1.7 for definitions and eqns:

$$\begin{aligned} d\Delta å/dt &= K_1\, (\Delta\Omega_{ref} - (1 - \Omega) + \Delta w) - K_2\, \Delta å \\ d\Delta w/dt &= K_3\, \Delta å - K_4\, \Delta w \\ d\Delta g/dt &= (3K_0/\Omega)\, \Delta å - K_0\, \Delta g \end{aligned} \qquad (0\text{-}12)$$

$$\downarrow$$

$$\Delta T_{SMmec} = \Delta g - å_0\, (1 - \Omega) - (2/\Omega)\, \Delta å$$ (Net change of mechanical torque, see (0-9))

The *system model* is the aggregate of differential equations (0-8) describing the *power network state variables*, and differential equations (0-9) – (0-12) describing the *remaining* ('local') *state variables.*

Initial condition analysis

Whether eigenvalue- or time dynamical analysis is to be conducted next, an appropriate initial state has to be defined for the system. In the following the process of arriving at the proper initial value of all state variables is summarized.

The initial state variables of machines should be set/computed in accordance with the situation at hand:

If a *synchronous motor* is to be started, its pu speed $\Omega_{SM(0)} = 0$, and so also all initial machine current and flux variables. The electrical angle $\beta_{SM(0)}$ can arbitrarely be set to zero. The synchronous motor's field voltage $E_{f(0)}$ may also be zero, if the field winding is kept short circuited during the initial part of the start-up sequence.

If a *synchronous motor* is initially in a synchronous mode of operation, $\Omega_{SM(0)} = 1$. In this case it is custom-ary to specify initial conditions in terms of absorbed power $P_{SM(0)}$ and voltage $U_{SM(0)}$ at the machine terminals. Thus $\beta_{SM(0)}$ and $E_{f(0)}$ should be specified so as to contribute to fullfilling these requirements. Computationally, this is afforded either by determining $\beta_{SM(0)}$ and $E_{f(0)}$ from an initial phasor diagram (which may be feasible only for a very small system), or by an iterative solution process in which $\beta_{SM(0)}$ and $E_{f(0)}$ are simultaneously corrected (together with all other such 'control variables') until stated initial conditions are reached to required accuracy. Absorbed (or produced) reactive motor power is in principle a byproduct from the stated solution process. The process is further described below, and exemplified in Chapter 3.11.2.

If *an asynchronous motor* i s to be started, its pu speed $\Omega_{AM(0)} = 0$, and so also all initial machine current and flux variables.

If an asynchronous motor is initially in a steady state mode of operation, it may be appropriate to specify initial conditions in terms of absorbed motor power $P_{AM(0)}$. Thus $\Omega_{AM(0)}$ should be specified so as to fulfill this power requirement. Computationally, this is afforded by including $\Omega_{AM(0)}$ as one of the simultaneously corrected 'control variables' of the above mentioned iterative solution process. Absorbed reactive motor power flows as a byproduct from the solution process. – Assume for the sake of generality of the ensuing algorithmic presentation, that an asynchronous motor in steady state mode of operation, is also present at the bus of the simple system of Figure 0.5. (This will alter the primitive system of Figure 0.6, as well as the loop matrix **B**, but these aspects are not the issues right now.)

With (finally or tentatively) specified values of variables like ($\beta_{SM(0)}$, $E_{f(0)}$, $\Omega_{SM(0)}$, $\Omega_{AM(0)}$), the premises are given for computing initial values of the rest of the pertinent power system variables. I.e: The network loop currents $\mathbf{i}_{loop(0)}$, the capacitor voltages $\mathbf{e}_{tc(0)}$, the asynchronous motor fluxes $\phi_{AM(0)}$, and the synchronous motor fluxes $\phi_{SM(0)}$.

The sought solution vector $\mathbf{z}_{(0)} = [\mathbf{i}_{loop(0)}^t, \mathbf{e}_{tc(0)}^t, \phi_{AM(0)}^t, \phi_{SM(0)}^t]^t$ is found by simultaneously solving the *network model* (0-6), the *synchronous motor rotorflux model* given on top of Figure 0.4, and the corresponding *asyn-chronous motor rotorflux model* (1-126) for steady state conditions, - i.e. after setting the derivative terms =0. At the outset we then have:

$$\begin{bmatrix} d\mathbf{i}_{loop}/dt \\ \hline d\mathbf{e}_{tc}/dt \end{bmatrix} = 0 = \omega_b \left[\begin{array}{c|c} -\mathbf{X}_{Lloop}^{-1} \cdot \mathbf{R}_{loop} & -\mathbf{X}_{Lloop}^{-1} \cdot \mathbf{B}_{tc} \\ \hline \mathbf{X}_{Cprimitive}\, \mathbf{B}_{tc}^t & \bar{\mathbf{1}}_{tc} \end{array}\right] \begin{bmatrix} \mathbf{i}_{loop(o)} \\ \hline \mathbf{e}_{tc(o)} \end{bmatrix} - \omega_b \begin{bmatrix} \mathbf{X}_{Lloop}^{-} \\ \hline \mathbf{0} \end{bmatrix} \cdot \mathbf{e}_{chord(o)} - \omega_b \begin{bmatrix} \mathbf{X}_{Lloop}^{-1} \cdot \mathbf{B}_{t\text{-}rest} \\ \hline \mathbf{0} \end{bmatrix} \cdot \mathbf{e}_{t\text{-}rest} \quad (0\text{-}13)$$

$$d\phi_{AM}/dt = 0 = \omega_b \left(\mathbf{F}_{AMi}\, \mathbf{i}_{AM(o)} + \mathbf{F}_{AM\phi(o)}\, \phi_{AM(o)} \right) \quad (0\text{-}14)$$

$$d\phi_{SM}/dt = 0 = \omega_b \left(\mathbf{V}_{SMr(o)} + \mathbf{F}_{Smi(o)}\, \mathbf{i}_{SM(o)} + \mathbf{F}_{SM\phi}\, \phi_{SM(o)} \right) \quad (0\text{-}15)$$

We notice that $\mathbf{X}_{Lloop}^{-1}$ is common factor to all terms of the upper system of equations of (0-13), and hence can be omitted in the present context. As a common factor to all equations ω_b can also be omitted. The set of equations above may then take on the following form:

$$\left[\begin{array}{c|c} \mathbf{R}_{loop} & \mathbf{B}_{tc} \\ \hline \mathbf{X}_{Cprimitive}\, \mathbf{B}_{tc}^t & \bar{\mathbf{1}}_{tc} \end{array}\right] \begin{bmatrix} \mathbf{i}_{loop(o)} \\ \hline \mathbf{e}_{tc(o)} \end{bmatrix} = \begin{bmatrix} -\mathbf{e}_{chord(o)} \\ \hline \mathbf{0} \end{bmatrix} + \begin{bmatrix} -\mathbf{B}_{t\text{-}rest} \cdot \mathbf{e}_{t\text{-}rest} \\ \hline \mathbf{0} \end{bmatrix} \quad (0\text{-}16)$$

$$\left(\mathbf{F}_{AMi}\, \mathbf{i}_{AM(o)} + \mathbf{F}_{AM\phi(o)}\, \phi_{AM(o)} \right) = 0 \quad (0\text{-}17)$$

$$\left(\mathbf{F}_{Smi(o)}\, \mathbf{i}_{SM(o)} + \mathbf{F}_{SM\phi}\, \phi_{SM(o)} \right) = -\mathbf{V}_{SMr(o)} \quad (0\text{-}18)$$

or compactly;

$$\mathbf{H}_{syst(0)}\ \mathbf{z}_{(0)} = \mathbf{E}_{syst(0)} \qquad (0\text{-}19)$$

The content of $\mathbf{H}_{syst(o)}$ and $\mathbf{E}_{syst(o)}$ flows directly from (0-16) - (0-19). The content is dealt with in Chapter 2.4. The 'remaining' inital state variables $\mathbf{z}_{(o)}$, are found from solving (0-19). The set of equations provides the desired initial load flow, when such specified variables $\beta_{SM(o)}$, $E_{f(o)}$, $\Omega_{SM(o)}$, $\Omega_{AM(o)}$ are applied, that all imposed component- as well as systems related operational constraints are met. In case of infeasibility, proper adjustment of the 'specified variables' must be made until the desired initial system status is established.

An efficient gradient technique for establishing the desired initial conditions is outlined next, by way of applying it to a small system that among its components comprises a synchronous motor and an asynchronous motor. Both machines are presumed up and running at t= -0, implying that $\Omega_{SM(o)}$ =1 and $\Omega_{AM(o)}$ is to be determined. Example initial operational constraints :

Power supplied to the synchronous motor : $P_{SMtarget(o)}$ = (say) -0.8pu (generator mode of operation)
Synchronous motor voltage : $E_{SMtarget(o)}$ = (say) 1.0pu.
Power supplied to the asynchronous motor : $P_{AMtarget(o)}$ = (say) 0.5 pu.

To contribute to fulfilling the *two* synchronous motor target values ($P_{SMtarget(o)}$, $E_{SMtarget(o)}$), *two* synchronous motor 'specified variables' ($\beta_{SM(o)}$, $E_{f(o)}$) are avalable for that purpose. To contribute to fulfilling the *one* asynchronous motor target value ($P_{AMtarget(o)}$), *one* asynchronous motor 'specified variable' ($\Omega_{AM(o)}$) is avalable for that purpose.The iterative solution process comprises in general the following three main steps :

1) **Stipulate initial value of $\beta_{SM(o)}$, $E_{f(o)}$, and $\Omega_{AM(o)}$**. Set final value $\Omega_{SM(o)}$ =1.0.

2) **Solve for the rest of the initial state variables $\mathbf{z}_{(o)}$**. Solution found from (0-19). Register as byproduct from the solution, the quantities ($P_{SM(o)}$, $E_{SM(o)}$, $P_{AM(o)}$) for which there are specified target values. Compute the deviations ($\Delta\mathbf{D}$) from target values:

$$\Delta\mathbf{D} = \begin{bmatrix} \Delta P_{SM(o)} \\ \Delta E_{SM(o)} \\ \Delta P_{AM(o)} \end{bmatrix} = \begin{bmatrix} P_{SM(o)} - P_{SMtarget(o)} \\ E_{SM(o)} - E_{SMtarget(o)} \\ P_{AM(o)} - P_{AMtarget(o)} \end{bmatrix} \qquad (0\text{-}20)$$

If the absolute value of each deviation is below some individually set treshold, the sought initial solution is found, and exit is made from the iterative process.

If the sought solution is (still) not found, a set of more appropriate values ($\beta_{SM(o)}$, $E_{f(o)}$, $\Omega_{AM(o)}$) have to be applied. To derive such a set ; go to label 3) below.

3) **Incrementally and simultaneously adjust $\beta_{SM(o)}$, $E_{f(o)}$, and $\Omega_{AM(o)}$** so that an improved initial power flow balance can be attained. The updated and improved value of respectively $\beta_{SM(o)}$, $E_{f(o)}$, and $\Omega_{AM(o)}$ to apply to this end, can conveniently be defined as currently available value plus a proper incremental correction to be determined at this stage of analysis :

To evaluate the proper corrections ($\Delta\beta_{SM(o)}$, $\Delta E_{f(o)}$, $\Delta\Omega_{AM(o)}$) on a simultaneous basis, a sensitivity analysis is conducted to find the elements of the sensitivity matrix **S** of the defined relationship (0-21) :

$$\begin{bmatrix} \Delta P_{SM} \\ \Delta E_{SM} \\ \Delta P_{AM} \end{bmatrix} = \begin{bmatrix} \mathbf{S} \end{bmatrix} \cdot \begin{bmatrix} \Delta\beta_{SM} \\ \Delta E_{f} \\ \Delta\Omega_{AM} \end{bmatrix} \qquad (0\text{-}21)$$

In the present case where **S** is a (3x3) matrix, this intermediate sensitivity analysis comprises 3 separate sensitivity computations:

increase of β_{SM} is investigated. This is afforded by setting $\beta_{SM} = \beta_{SM(o)} + \Delta\beta_{SM}$, and solving (0-19) while $E_{f(o)}$ and $\Omega_{AM(o)}$ are kept unchanged (i.e. ΔE_f and $\Delta\Omega_{AM}$ of (0-21) are both zero). $\Delta\beta_{SM}$= (say) 0.01rad. From the solution of (0-19), the new values (P_{SM} , E_{SM} , P_{AM}) are evaluated, and so also the associated incremental increases $\Delta P_{SM} = P_{SM} - P_{SM(o)}$, $\Delta E_{SM} = E_{SM} - E_{SM(o)}$ and $\Delta P_{AM} = P_{AM} - P_{AM(o)}$. The first column of **S** can now be computed, as we definitionwise have - since both ΔE_f and $\Delta\Omega_{AM} = 0$ in (0-21) -that $S_{11} = \Delta P_{SM}/\Delta\beta_{SM}$, $S_{21} = \Delta E_{SM}/\Delta\beta_{SM}$, $S_{31} = \Delta P_{AM}/\Delta\beta_{SM}$. β_{SM} is reset to $\beta_{SM(0)}$

Secondly, the sensitivity of P_{SM} ,E_{SM} and P_{AM} w.r.t. an incremental increase of E_f is investigated. This is afforded by setting $E_f = E_{f(o)} + \Delta E_f$, and solving (0-19) while $\beta_{SM(o)}$ and $\Omega_{AM(o)}$ are kept unchanged (i.e. $\Delta\beta_{SM}$ and $\Delta\Omega_{AM}$ of (0-21) are both zero). ΔE_f = (say) 0.01pu. From the new solution (0-19), values (P_{SM} , E_{SM} , P_{AM}) and associated marginal increases $\Delta P_{SM} = P_{SM} - P_{SM(0)}$, $\Delta E_{SM} = E_{SM} - E_{SM(o)}$ and $\Delta P_{AM} = P_{AM} - P_{AM(o)}$ are evaluated. The second column of **S** can now be found, as we definitionwise have that $S_{12} = \Delta P_{SM}/\Delta E_f$, $S_{22} = \Delta E_{SM}/\Delta E_f$, $S_{32} = \Delta P_{AM}/\Delta E_f$. E_f is reset to $E_{f(o)}$.

Thirdly, the sensitivity of P_{SM} , E_{SM} and P_{AM} w.r.t. an incremental increase of Ω_{AM} is investigated. This is afforded by setting $\Omega_{AM} = \Omega_{AM(o)}+\Delta\Omega_{AM}$, and solving (0-19) while $\beta_{SM(o)}$ and $E_{f(o)}$ are kept unchanged (i.e. $\Delta\beta_{SM}$ and ΔE_f of (0-21) are both zero). $\Delta\Omega_{AM}$ = (say) 0.01pu. From the new solution (0-19) we evaluate the values (P_{SM},E_{SM}, P_{AM}) and the associated marginal increases $\Delta P_{SM} = P_{SM} - P_{SM(o)}$, $\Delta E_{SM} = E_{SM} - E_{SM(o)}$ and $\Delta P_{AM} = P_{AM} - P_{AM(o)}$. The third and last column of **S** can now be found, as we definitionwise have that $S_{13} = \Delta P_{SM}/\Delta\Omega_{AM}$, $S_{23} = \Delta E_{SM}/\Delta\Omega_{AM}$, $S_{33} = \Delta P_{AM}/\Delta\Omega_{AM}$. Ω_{AM} is reset to $\Omega_{AM(o)}$.

With given sensitivity matrix **S** and prevailing deviations $\Delta\mathbf{D}$ relative to target values ($P_{SMtarget(o)}$, $E_{SMtarget(o)}$,$P_{AMtarget(o)}$), equations (0-21) are applied to estimate the set of increments ($\Delta\beta_{SM}$, ΔE_f , $\Delta\Omega_{AM}$) that will eliminate prevailing deviations- if processes were linear: Using (- $\Delta\mathbf{D}$) as 'excitation' on the left side in (0-21), and solving w.r.t. the desired simultaneous increments, we get :

$$\begin{bmatrix} \Delta\beta_{SM} \\ \Delta E_f \\ \Delta\Omega_{AM} \end{bmatrix} = -\mathbf{S}^{-1}\cdot \Delta\mathbf{D} \qquad (0\text{-}22)$$

The computed incremental values from (0-22) are then used to produce an updated and improved set of initial values ($\beta_{SM(o)}$, $E_{f(o)}$, $\Omega_{AM(o)}$) , in accordance with e.g. the dynamic update logic illustrated by (0-23), and observing boundary constraints on (in this case) E_f.

$$\begin{aligned} \beta_{SM(o)} &\leftarrow \beta_{SM(o)} + k\ \Delta\beta_{SM} \\ E_{f(o)} &\leftarrow E_{f(o)} + k\ \Delta E_f \\ \Omega_{AM(o)} &\leftarrow \Omega_{AM(o)} + k\ \Delta\Omega_{AM} \end{aligned} \qquad (0\text{-}23)$$

k is a scalar factor of default value 1.0 . An alternative value in the prospective range ($0.0<k\leq 1.0$) implies in principle safer but slower convergence. Following update of initial conditions as specified by (0-23), return is made to step 2) of the above iterative process.

Eigenvalue analysis

For the power system at hand a *system model* is presumed available for analyzing the system's dynamical response to any operations related disturbance. In compact notation the system model is expressed in this way:

$$d\mathbf{z}/dt = \mathbf{f}(\mathbf{z},\nu) \qquad (0\text{-}7)$$

where **z** is the set of all system state variables, and ν comprises exogenously specified excitations.

Given the initial state of operation of the power system. We now seek the system's *eigenvalues* which describe the *inherent dynamic characteristics of the system*, when *incrementally disturbed* from the specified initial state.The task implies determining the eigenvalues associated with matrix **A** of the linearized formulation

$$\underline{d\Delta\mathbf{z}/dt} = \Delta\mathbf{f}(\mathbf{z},\mathbf{v}) = \underline{\mathbf{A}\ \Delta\mathbf{z}} \qquad (0\text{-}24)$$

where $\Delta\mathbf{z}$ comprises the *incremental* state variables, and **A** is a square matrix produced from the indicated 'deltaoperation' performed on the right hand side $\mathbf{f}(\mathbf{z},\mathbf{v})$ of the system model. The system's eigenvalues are next computed using the QR-algorithm on **A**, after first reducing it to Hessenberg form.

The elaboration of $\Delta\mathbf{f}(\mathbf{z},\mathbf{v})$ is covered in Chapter 3, where self- and mutual elements of matrix **A** are developed on general algorithmic form for all main types of power system component models. To present the main features of the stated 'deltaoperation', the relatively simple such operation related to a system submodel that describes 'local' state variables, is pursued next :

We focus arbitrarely on the *rotorflux submodel* of a *synchronous motor* connected to the system. This submodel describes the motor's flux variables which are 'local' state variables. From Figure 0.4 we copy the rotorflux submodel of the synchronous motor:

$$d\phi_{SM}/dt = \omega_b\ (\mathbf{e}_{SMr} + \mathbf{F}_{SMi}\ \mathbf{i}_{SM} + \mathbf{F}_{SM\phi}\ \phi_{SM}) = \omega_b\ \mathbf{G}\phi_{SM} \qquad (0\text{-}25)$$

where;

$$\mathbf{G}\phi_{SM} = (\mathbf{e}_{SMr} + \mathbf{F}_{SMi}\cdot \mathbf{i}_{SM} + \mathbf{F}_{SM\phi}\ \phi_{SM}) \qquad (0\text{-}26)$$

$\boldsymbol{\phi}_{SM} = [\phi_f, \phi_{kd}, \phi_{kq}]^t$ = state variables in terms of synchronous motor flux components
$\mathbf{i}_{SM}$ = state variables in terms of synchronous motor current components

$$\mathbf{e}_{SMr} = \begin{bmatrix} K_f E_f \\ 0 \\ 0 \end{bmatrix}$$

$E_f = (E_{fo} + \Delta E_f)$ = field voltage
$K_f = (\sqrt{2}/(\omega_b\ T'_{do}))\ X_{ad}/(X_d - X'_d))$
ΔE_f = voltage control response, see equations (0-11)

$$\mathbf{F}_{Smi} = \begin{bmatrix} (1/(\omega_b T'_{do}))\ (X_{ad}/X'_{ad})\ X''_{ad}\ \cos\beta_{SM} & -(1/(\omega_b T'_{do}))\cdot (X_{ad}/X'_{ad})\ X''_{ad}\ \sin\beta_{SM} \\ (1/(\omega_b T''_{do}))\ X'_{ad}\ \cos\beta_{SM} & -(1/(\omega_b T''_{do}))\cdot X'_{ad}\ \sin\beta_{SM} \\ (1/(\omega_b T''_{qo}))\ X_{aq}\ \sin\beta_{SM} & (1/(\omega_b T''_{qo}))\ X_{aq}\ \cos\beta_{SM} \end{bmatrix}$$

$$\mathbf{F}_{SM\phi} = \begin{bmatrix} -(1/(\omega_b T'_{do}))\ (1/X'_{ad})\ [(X_{ad}/X'_{ad})\ (X'_d - X''_d) + X''_{ad}] & (1/(\omega_b T'_{do}))\ (X_{ad}/X'^2_{ad})\ (X'_d - X''_d) & \\ (1/(\omega_b T''_{d0}))\ (1/X_{ad})\ (X_d - X'_d) & -1/(\omega_b T''_{do}) & \\ & & -1/(\omega_b T''_{qo}) \end{bmatrix}$$

From the foregoing we see that $\mathbf{G}\phi_{SM}$ is a function of a subset of the state variables:

$$\mathbf{G}\phi_{SM} = f(\mathbf{i}_{SM}, \boldsymbol{\phi}_{SM}, \beta_{SM}, \Delta E_f) \qquad (0\text{-}27)$$

Equation (0-27) says that $\mathbf{G}\phi_{SM}$ will depend on i = (2+3+1+1)=7 individual state variables. Accordingly, the 'deltaoperation' on the right hand side of (0-25) can compactly be expressed in this way:

$$\underline{\Delta(\omega_b\ \mathbf{G}\phi_{SM})} = \omega_b\ \Delta\mathbf{G}\phi_{SM} = \omega_b \sum_{j=1}^{Nj=7}(\partial\mathbf{G}\phi_{SM}/\partial z_j)\ \Delta z_j = \sum_{j=1}^{Nj=7} A\phi_{SM\,j}\ \Delta z_j = \underline{\mathbf{A}\phi_{SM}\cdot \Delta\mathbf{z}\phi_{SM}} \qquad (0\text{-}28)$$

where;

$\mathbf{A}\phi_{SM} = [\mathbf{A}\phi_{SM}\mathbf{i}_{SM}, \mathbf{A}\phi_{SM}\phi_{SM}, \mathbf{A}\phi_{SM}\beta_{SM}, \mathbf{A}\phi_{SM\Delta Ef}]$
= the coefficient elements of **A** associated with the considered rotor-flux model. $\mathbf{A}\phi_{SM}$ is here on packed form, since only the 7 elements of $\Delta\mathbf{z}$ that are influencing on $\Delta(\omega_b\ \mathbf{G}\phi_{SM})$, are included in $\Delta\mathbf{z}\phi_{SM}$.

$\Delta\mathbf{z}\phi_{SM} = [\Delta\mathbf{i}^t_{SM}, \Delta\phi^t_{SM}, \Delta\beta_{SM}, \Delta(\Delta E_f)]^t$
= submatrix of $\Delta\mathbf{z}$, see comment above.

It remains to elaborate the algorithmic content of the coefficient terms of matrix $\mathbf{A}\phi_{SM}$. Based on (0-28) and the rotorflux submodel data above, the following results are readily observed :

The elements $A\phi_{SM}i_{SM}$ of coefficient matrix **A** :

$$A\phi_{SM}i_{SM} = \omega_\delta\ (\partial G\phi_{SM}/\partial i_{SM})$$

⊃:

$\underset{(3x2)}{A\phi_{SM}i_{SM}} = \omega_\delta\ F_{SMi(o)}$ = The elements of coefficient matrix **A** defining the influence on $d\Delta\phi_{SM}/dt$ of the synchronous motor's own incremental stator currents Δi_{SM}. $F_{SMi(o)}$ is initial value of F_{SMi} given on previous page. (0-29)

The elements $A\phi_{SM}\phi_{SM}$ of coefficient matrix **A** :

$$A\phi_{SM}\phi_{SM} = \omega_\delta\ (\partial G\phi_{SM}/\partial \phi_{SM})$$

⊃:

$\underset{(3x3)}{A\phi_{SM}\phi_{SM}} = \omega_\delta\ F_{SM\phi}$ = The elements of coefficient matrix **A** defining the influence on $d\Delta\phi_{SM}/dt$ of the synchronous motor's own incremental flux components $\Delta\phi_{SM}$. $F_{SM\phi}$ is given on previous page. (0-30)

The elements $A\phi_{SM}\beta_{SM}$ of coefficient matrix **A** :

$$A\phi_{SM}\beta_{SM} = \omega_\delta\ (\partial G\phi_{SM}/\partial \beta_{SM})$$

⊃:

$\underset{(3x1)}{A\phi_{SM}\beta_{SM}} = \omega_\delta\ (\partial F_{SMi}/\partial \beta_{SM})\ i_{SM(o)}$ = The elements of coefficient matrix **A** defining the influence on $d\Delta\phi_{SM}/dt$ of the synchronous motor's own incremental rotor angle $\Delta\beta_{SM}$. $(\partial F_{SMi}/\partial \beta_{SM})$ is produced from F_{SM} by taking the stated derivative of each matrix element. (0-31)

The elements $A\phi_{SM\,\Delta Eqf}$ of coefficient matrix **A** :

$$A\phi_{SM\,\Delta Ef} = \omega_\delta\ (\partial G\phi_{SM}/\partial \Delta E_f)$$

⊃:

$\underset{(3x1)}{A\phi_{SM\,\Delta Ef}} = \omega_\delta\ (\partial V_{SMr}/\partial \Delta E_f) = \begin{bmatrix} \omega_\delta K_f \\ 0 \\ 0 \end{bmatrix}$ = The elements of coefficient matrix **A** defining the influence on $d\Delta\phi_{SM}/dt$ of the synchronous motor's own incremental field voltage $\Delta(\Delta E_f)$. K_f is given on previous page. (0-32)

Time response analysis

Given the *system model* of the power system, and the *initial value* of all its *state variables.* We seek the variation over time of the system's state variables and their interactions, following some specified disturbance to the system.

The task implies solving the set of equations $dz/dt = f(z,v)$ numericlly over some desired time horizon. The solution process must observe the fact that some of the electrical circuit elements (**R**,**X**,**ΔE**) pertaining to e.g. rotating machines, are themselves functions of one or more of the system's 'local' state variables. If saturation phenómena are to be accounted for, circuit elements may become functions of currents and voltages as well.

Thus it is evident that the *system loop resistance matrix* R_{loop} and the *system loop inductor matrix* X_{Lloop} both become functions of a subset of the system state variables. To account for this functionality, R_{loop} and X_{Lloop} (and naturally also X_{Lloop}^{-1}) have to be continually updated during processes of numerical integration. In the studies accounted for in this report, a Runge-Kutta fourth order integration algorithm has been applied for solving the differential equations. 'Continous update' of the network model implies in this case the choice of generating new and updated versions of R_{loop} and X_{Lloop} (and also X_{Lloop}^{-1}) *multiple times* in the course of advancing the solution *one* integration time step.

1. Component Models

1. Component Models

1.1 The *component modelling* concept

Chapter 1 develops a stock of four *component models* to apply in modelling of the common power network components like overhead lines, cables, the infinite bus, capacitor banks, transformers, synchronous machines, and asynchronous machines. The four *component models* are *'The Lossy Inductor', 'The Lossy Capacitor Bank', 'The Synchronous Motor'*, and *'The Asynchronous Motor'* :

'The Lossy Inductor' models directly the three phase, inductive series impedance, the three phase inductive impedance load, and the infinite bus. Transformers, overhead lines and cables are modelled by suitably arranging together component models of the type *'Lossy Inductor'* and *'Lossy Capacitor Bank'* , - see next.

'The Lossy Capacitor Bank' models directly the three phase, lossy series capacitor, and the 3- phase, lossy shunt capacitor. It also contributes to the modelling of other network components as stated above.

'The Synchronous Motor' models the two main modes of operation of the synchronous machine; the voltage controlled synchronous *motor*, and the voltage- and power controlled synchronous *generator*. For conceptual clearness, *motor* mode of operation is the 'default' modelling mode. The component model of Chapter 1 is based on a '5-coil generalised model machine', and aims at describing the *ordinary synchronous machine*. For study of the performance of the *adjustable speed synchronous machine*, Appendix 2 develops an 'extended' component model based on a 6-coil generalised machine.

'The Asynchronous Motor' models *motor*- as well as *generator* mode of operation of the asynchonous machine. *Motor* mode of operation is the 'default' modelling mode.

The *component model* is made up of one or more *submodels*, the configuration of which is defined by the set of *state variables* that abides with the *component model*. Table 1.1 summarizes how *submodels* add up to *component models*, and how the latter are configured to model the *main power system components*.

Table 1.1 Overview of how *submodels* add up to *component models* and *component models* add up to models of main power system components.

Main power system components ←	Component models ←	Submodel(s)
Inductive series impedance	'The Lossy Inductor'	*Electrical circuit model*
Inductive impedance load	'The Lossy Inductor'	*Electrical circuit model*
Infinite bus voltage	'The Lossy Inductor'	*Electrical circuit model*
Capacitor bank	'The Lossy Capacitor Bank'	*Electrical circuit model* Capacitor voltage model
Overhead line/Cable	'The lossy Inductor' 'The Lossy Capacitor Bank'	*Electrical circuit models* Capacitor voltage models
Transformer	'The Lossy Inductor' ('The Lossy Capacitor Bank')	*Electrical circuit models* (Capacitor voltage models)
Synchronous machine (Ordinary synchronous machines) (Adjustable speed version)	'The Synchronous Motor' (Based on 5-coil generalised machine) (Based on 6-coil generalised machine)	*Electrical circuit model* Rotorflux model *) Electromechanical model *) Control system models *)
Asynchronous machine	'The Asynchronous Motor'	*Electrical circuit model* Rotorflux model *) Electromechanical model *) (Control system model) *)

*) Subsystem models that describe 'remaining' (or 'local') state variables, see previous page.

The *submodels* that go into respective *component models*, are developed in Chapter 1.2 – 1.6, and summarized in Chapter 1.7. The extended machine model from Appendix 2, is also added into Chapter 1.7. One of the *submodels* is the *electrical circuit model*. In terms of *formal* representation, the *electrical circuit model* is made common to all four *component models*.

A structural description of the *electrical circuit model* is given in Figure 1.1.

The electrical circuit model of respective *component models* of the network, interlink to describe integrated power network performance.

From a structural point of view the *electrical circuit model* is common to all four *component models.* Parameter interpretation however, will depend on which *component model* the *electrical circuit model* is a part of. The *electrical circuit model* comprises three main parts :

An oriented terminal graph displaying positive direction associated with the *circuit model variables* (**i,e**) that connect electrically with the external network. For the stock of 2-terminal *component models* developed in Chapter 1.2-1.6, the *oriented terminal graph* becomes an *oriented line segment.* See figure a) below. Figure b) shows the standardized d-q axis serial circuit elements that make up the *electrical circuit model*, - and is fronted by the just stated graph.

Impedance terms R and X_L describing the power network related 'passive' electrical properties of the *component model.* Index 'L' denotes *inductive* character of the reactance. The impedance terms for the lossy capacitor are $\mathbf{R_C}$ and $\mathbf{X_L}$=0. The capacitor voltage is treated as a state variable, see next on the voltage sources **e**. **v** is the voltage across the serial interconnection of **R** and $\mathbf{X_L}$, see figure b) below.

A voltage source e giving the power network related source impact of the *component model.* Illustrations:

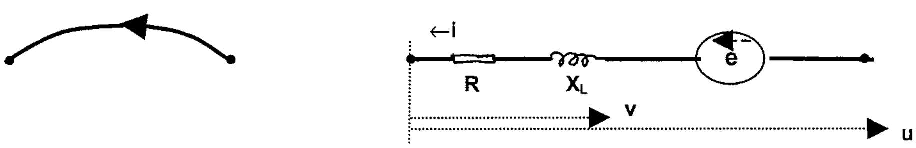
If the *component model* is **The Lossy Inductor**, the voltage source **e** may or may not be zero : If this *component model* is applied to model e.g. an inductive series impedance, or an inductive impedance load, **e** = 0. If the task is to model a fixed voltage 'behind' some specified inductive series impedance, **e** = $\mathbf{e_{dq}}$. See Chapter 1.6 for description of $\mathbf{e_{dq}}$.

If the *component model* is **The Lossy Capacitor**, the voltage source **e** = $\Delta\mathbf{E_c}$, where $\Delta\mathbf{E_c}$ is the voltage across the capacitor. See Chapter 1.3 for details.

If the *component model* is **The Synchronous Motor**, the voltage source **e** = $\Delta\mathbf{E_{SM}}$, where $\Delta\mathbf{E_{SM}}$ is a formal electro-motive force (e.m.f) contributing to modelling the synchronous motor. See Chapter 1.4 and 1.7 for details.

If the *component model* is **The Aynchronous Motor**, the voltage source **e** = $\Delta\mathbf{E_{AM}}$, where $\Delta\mathbf{E_{AM}}$ is a formal e.m.f contributing to modelling the aynchronous motor. See Chapter 1.5 and 1.7 for details.

u is the voltage across the teminals of the electrical circuit model, see figure b) below.



$$\mathbf{v = R\ i} + (1/\omega_b)\ \mathbf{X_L}\ di/dt \quad \& \quad \mathbf{v - e = u}$$

a) Oriented terminal graph

b) Serial circuit elements fronted by graph a)

Figure 1.1 *The electrical circuit model* ; formal structure of submodel common to all *component models*

1.2 The Lossy Inductor

The Lossy Inductor models directly the three phase, inductive series impedance, the three phase inductive impedance load, and the infinite bus voltage. Transformers, overhead lines and cables are modelled by suitably arranging together component models of the type *Lossy Inductor* and *Lossy Capacitor Bank.*

A brief development is given of the network terms (**R**, $\mathbf{X}_L$, $\Delta\mathbf{E}$=0) that describe the *lossy inductor* in the d-q axis frame of reference. For summary d-q axis model description, see Chapter 1.7.

Currents ($\mathbf{i}_{dqo}$), voltages ($\mathbf{v}_{dqo}$) and fluxes ($\mathbf{\Psi}_{dqo}$) within the d-q-o axis frame of reference, are definition-wise related to their corresponding phase (RST) variables in the following way:

$$\mathbf{i}_{dqo} = \mathbf{P}\ \mathbf{i}_{RST} \qquad \mathbf{v}_{dqo} = \mathbf{P}\ \mathbf{v}_{RST} \qquad \psi_{dqo} = \mathbf{P}\ \psi_{RST} \tag{1-1}$$

P is the *Park transformation*, where θ is the angular displacement of the axes of the (RST) reference frame relative to the axes of the (dq) reference frame:

$$\mathbf{P} = 2/3 \begin{array}{c} \begin{array}{ccc} R & S & T \end{array} \\ \left[\begin{array}{ccc} \cos\theta & \cos(\theta-2\pi/3) & \cos(\theta-4\pi/3) \\ -\sin\theta & -\sin(\theta-2\pi/3) & -\sin(\theta-4\pi/3) \\ ½ & ½ & ½ \end{array}\right] \end{array} \begin{array}{c} d \\ q \\ o \end{array} \tag{1-2}$$

See Appendix 3 for detailed development of **P**. For back transformation we invert (1-1):

$$\mathbf{i}_{RST} = \mathbf{P}^{-1}\ \mathbf{i}_{dqo} \qquad \mathbf{v}_{RST} = \mathbf{P}^{-1}\ \mathbf{v}_{dqo} \qquad \psi_{RST} = \mathbf{P}^{-1}\ \psi_{dqo} \tag{1-3}$$

where;

$$\mathbf{P}^{-1} = \left[\begin{array}{ccc} \cos\theta & -\sin\theta & 1 \\ \cos(\theta-2\pi/3) & -\sin(\theta-2\pi/3) & 1 \\ \cos(\theta-4\pi/3) & -\sin(\theta-4\pi/3) & 1 \end{array}\right] \tag{1-4}$$

In the physical three phase (RST) reference frame, we can for (say) phase 'R', express the voltage v_R across the considered impedance (r+jx) as:

$$v_R = i_R\ r + d\Psi_R/dt \tag{1-5}$$

where i_R and Ψ_R is - respectively - current and flux linkages of phase 'R'. The per phase variables v_R, i_R and Ψ_R are related to their respective d - q - o axis components in the following way, see (1-4) :

$$\begin{aligned} v_R &= v_d \cos\theta - v_q \sin\theta + e_o \\ i_R &= i_d \cos\theta - i_q \sin\theta + i_o \\ \Psi_R &= \Psi_d \cos\theta - \Psi_q \sin\theta + \Psi_o \end{aligned} \tag{1-6}$$

Inserting expressions from (1-6) into (1-5), and observing that

$$d\Psi_R/dt = \cos\theta\ d\Psi_d/dt - \sin\theta\ d\Psi_q/dt + d\Psi_o/dt - \omega\ \Psi_d \sin\theta - \omega\ \Psi_q \cos\theta \tag{1-7}$$

we get the following 'd-q-o version' of (1-5):

$$\begin{aligned} 0 = &\ [- v_d + r\ i_d + d\Psi_d/dt - \omega\ \Psi_q] \cos\theta \\ &+ [v_q - r\ i_q - d\Psi_q/dt - \omega\ \Psi_d] \sin\theta \\ &+ [- v_o + r\ i_o + d\Psi_o/dt] \end{aligned} \tag{1-8}$$

For general validity of (1-8), the following d-q conditions must be observed to equivalence (1-5) :

$$\begin{aligned} v_d &= r\ i_d + d\Psi_d/dt - \omega\ \Psi_q \\ v_q &= r\ i_q + d\Psi_q/dt + \omega\ \Psi_d \\ v_o &= r\ i_o + d\Psi_o/dt \end{aligned} \tag{1-9}$$

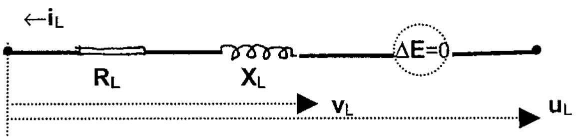
We assume in the present outline that zero sequence currents are absent. Hence the last equation of (1-9) can be disregarded. Since component symmetry is presumed at the outset, we further-more have that $\Psi_d = L_d\, i_d = L\, i_d$ and $\Psi_q = L_q\, i_q = L\, i_q$. Setting $x = \omega_b L$, (1-9) can be expressed in the following form, when referred to nominal frequency :

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = r \begin{bmatrix} i_d \\ i_q \end{bmatrix} + (1/\omega_b) \begin{bmatrix} x & \\ & x \end{bmatrix} \begin{bmatrix} di_d/dt \\ di_q/dt \end{bmatrix} + \begin{bmatrix} & -x \\ x & \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} \qquad (1\text{-}10)$$

In summary, the d-q axis reference frame model (1-10) of the symmetrical three phase impedance, is conveniently expressed as follows:

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} r & -x \\ x & r \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + (1/\omega_b) \begin{bmatrix} x & \\ & x \end{bmatrix} \begin{bmatrix} di_d/dt \\ di_q/dt \end{bmatrix} \qquad (1\text{-}11)$$

This implies the following electrical circuit model of the *lossy inductor*, hereby indexed 'L' :



↓

$\mathbf{v}_L = \mathbf{R}_L \cdot \mathbf{i}_L + (1/\omega_b)\, \mathbf{X}_L\, d\mathbf{i}_L/dt$ & $\mathbf{v}_L = \mathbf{u}_L$

where;

$$\mathbf{R}_L = \begin{bmatrix} r_L & -x_L \\ x_L & r_L \end{bmatrix} \qquad \mathbf{v}_L = \begin{bmatrix} v_{Ld} \\ v_{Lq} \end{bmatrix}$$

$$\mathbf{X}_L = \begin{bmatrix} \omega_b L & \\ & \omega_b L \end{bmatrix} = \begin{bmatrix} x_L & \\ & x_L \end{bmatrix} \qquad \mathbf{i}_L = \begin{bmatrix} i_{Ld} \\ i_{Lq} \end{bmatrix}$$

$z = (r_L + jx_L)$ = given per phase description of *lossy inductor*

Figure 1.2 Electrical circuit model of *lossy inductor 'L'*. d-q axis frame of reference

Summary model description followed by a simple/qualitative illustration of model application, is given in Chapter 1.7.

1.3 The Lossy Capacitor Bank

The Lossy *Capacitor Bank* models directly the three phase, lossy series capacitor and the three-phase, lossy shunt capacitor. It also contributes to the modelling of other network components as pointed to above.

A brief two-step development is given of the *network terms* ($\mathbf{R}_c$, $\mathbf{X}_L$=0, $\Delta\mathbf{E}_c$) describing the lossy capacitor bank in the d-q axis frame of reference. Step 1 elaborates the *electrical circuit model* com-prising the previous three network terms. Step 2 develops the *capacitor voltage model*, i.e. the differential equation governing the variation of the above formal network term $\Delta\mathbf{E}_c$. For summary d-q axis component description, see Chapter 1.7.

Electrical Circuit Model

Observing the adopted conventions of Figure 1.1, we can for the physical three phase (RST) reference frame for (say) phase 'R', express the voltage u_R across the considered lossy capacitor with parameters (r,C), as:

$$u_R = i_R\, r - \Delta E_R \qquad (1\text{-}12)$$

where

$$\Delta E_R = (1/C)\int i_R dt \qquad (1\text{-}13)$$

i_R and ΔE_R is - respectively - current of phase 'R' and voltage across the 'pure' capacitor element C of phase 'R'. The per phase variables u_R, i_R and ΔE_R are related to their respective d-q-o axis components in the following way, see (1-4) :

$$\begin{aligned} u_R &= u_d \cos\theta - u_q \sin\theta + u_o \\ i_R &= i_d \cos\theta - i_q \cdot \sin\theta + i_o \\ \Delta E_R &= \Delta E_d \cos\theta - \Delta E_q \sin\theta + \Delta E_o \end{aligned} \qquad (1\text{-}14)$$

Inserting expressions from (1-14) into (1-12), we get the following 'd-q-o version' of (1-12) :

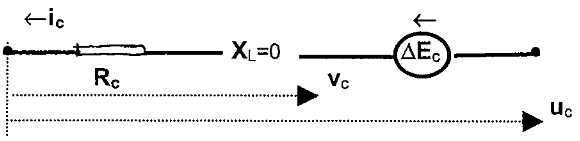
$$\begin{aligned} 0 = & [- u_d + r\, i_d - \Delta E_d] \cos\theta \\ & + [\, u_q - r\, i_q + \Delta E_q] \sin\theta \\ & + [- u_o + r\, i_o - \Delta E_o] \end{aligned} \qquad (1\text{-}15)$$

For general validity of (1-15), the following d-q conditions must be observed to equivalence (1-12) :

$$\begin{aligned} u_d &= r\, i_d - \Delta E_d \\ u_q &= r\, i_q - \Delta E_q \\ u_o &= r\, i_o - \Delta E_o \end{aligned} \qquad (1\text{-}16)$$

We assume for our present purpose that zero sequence currents and voltages are inconsequential. Hence, we can disregard the last equation of (1-16).

The two remaining equations of (1-16) implies the following *electrical circuit model* of the lossy capacitor bank, hereby indexed 'C' :



$$\downarrow$$

$$\mathbf{v_c} = \mathbf{R_c}\, \mathbf{i_c} \quad \& \quad \mathbf{v_c} - \Delta \mathbf{E_c} = \mathbf{u_c}$$

where;

$$\mathbf{R_c} = \begin{bmatrix} r_c & \\ & r_c \end{bmatrix} \qquad \mathbf{v_c} = \begin{bmatrix} v_{cd} \\ v_{cq} \end{bmatrix}$$

$$\Delta \mathbf{E_c} = \begin{bmatrix} \Delta E_{cd} \\ \Delta E_{cq} \end{bmatrix} \text{ = Capacitor voltage. See capacitor voltage model.} \qquad \mathbf{i_c} = \begin{bmatrix} i_{cd} \\ i_{cq} \end{bmatrix}$$

(r_c,C) = given per phase parameters for lossy capacitor bank

Figure 1.3 Electrical circuit model of *lossy capacitor bank 'C'*. d-q axis frame of reference

Capacitor Voltage Model

Returning to (1-13) and taking the derivative, we find that $d\Delta E_R/dt = i_R/C$. Inserting into this equation the expression for i_R from (1-14), and the derivative

$$d\Delta E_R/dt = \cos\theta \; d\Delta E_d/dt - \sin\theta \; d\Delta E_q/dt + d\Delta E_o/dt - \omega \, \Delta E_d \sin\theta - \omega \, \Delta E_q \cos\theta \qquad (1\text{-}17)$$

developed from that same set of equations, we get the following 'd-q-o version' of the derivative $d\Delta E_R/dt$ of the three phase frame of reference:

$$\begin{aligned} 0 = \; & [\, i_d/C - d\Delta E_d/dt + \omega \, \Delta E_q \,] \cos\theta \\ & + [\, -i_q/C + d\Delta E_q/dt + \omega \, \Delta E_d \,] \sin\theta \\ & + [\, i_o/C - d\Delta E_o/dt \,] \end{aligned} \qquad (1\text{-}18)$$

For general validity of (1-18), the following conditions must be fulfilled:

$$\begin{aligned} d\Delta E_d/dt &= i_d/C + \omega \, \Delta E_q \\ d\Delta E_q/dt &= i_q/C - \omega \, \Delta E_d \\ d\Delta E_o/dt &= i_o/C \end{aligned} \qquad (1\text{-}19)$$

We assume again that zero sequence currents and voltages are inconsequential. Hence, the last equation of (1-19) can be disregarded.

The two remaining equations of (1-19) implies the following *capacitor voltage model* of the lossy capacitor bank, - here indexed 'C' , - when referred to nominal frequency :

$$d\Delta \mathbf{E}_c/dt = \omega_b \, (\mathbf{X}_c \, \mathbf{i}_c + \bar{\mathbf{1}} \, \Delta \mathbf{E}_c) \qquad (1\text{-}20)$$

with initial condition (see outline of (1-21) below) :

$$\Delta \mathbf{E}_{c(o)} = (\bar{\mathbf{1}} \, \mathbf{X}_c) \, \mathbf{i}_{c(o)}$$

where;

$$\mathbf{X}_c = \begin{bmatrix} 1/(\omega_b C) & \\ & 1/(\omega_b \cdot C) \end{bmatrix} \qquad \Delta \mathbf{E}_c = \begin{bmatrix} \Delta E_{cd} \\ \Delta E_{cq} \end{bmatrix}$$

$$\bar{\mathbf{1}} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \qquad \mathbf{i}_c = \begin{bmatrix} i_{cd} \\ i_{cq} \end{bmatrix}$$

Figure 1.4 Capacitor voltage model of *lossy capacitor bank 'C'.*
d-q axis frame of reference.

Initial value of $\Delta \mathbf{E}_c$ flows from (1-20) : For t= -0 we have that $d\Delta \mathbf{E}_c/dt = 0$. Solving with respect to $\Delta \mathbf{E}_c = \Delta \mathbf{E}_{c(o)}$, we find :

$$\Delta \mathbf{E}_{c(o)} = -(\bar{\mathbf{1}})^{-1} \; \mathbf{X}_c \, \mathbf{i}_{c(o)} \; = \; (\bar{\mathbf{1}} \, \mathbf{X}_c) \, \mathbf{i}_{c(o)} \qquad (1\text{-}21)$$

Summary model description followed by a simple/qualitative illustration of model application, is given in Chapter 1.7.

1.4 The Synchronous Motor

Formal basis for the ensuing model development is the d-q *diagram of a generalised machine* as presented by B.Adkins [1]. In view of actual synchronous machine design as well as availability of practical data, it is deemed appropriate to specify a ***five-coil**, salient pole generalised machine* as main basis for analysis. See Figure 1.5. For special or more detailed analyses, modelling based on the ***six-coil** generalised machine* may be appropriate; see Appendix 2. The three phase stator winding is assumed to be the rotating part, while the d-q axes with associated windings are considered fixed. See App.3.

> The 'pseudo-stationary' d- and q coils equivalence in a suitable way the electromagnetic effects of the stator windings of the physical three phase machine. The currents, voltages and fluxes associated with these coils, are definitionwise related to their corresponding physical phase variables via the Park transformation, see App. 3. The fixed coil 'f' of the diagram represents the field circuit of the synchronous machine. The fixed coils denoted 'kd' and 'kq', aim at equivalencing the effects of all damper circuits in the machine.

The five-coil representation implies 5 state variables to describe the electrical performance of the synchronous machine. As such variables we choose to apply the stator current - represented by the two components (i_d,i_q), - and the flux linkages associated with respectively the f-, kd- and kq- coil.

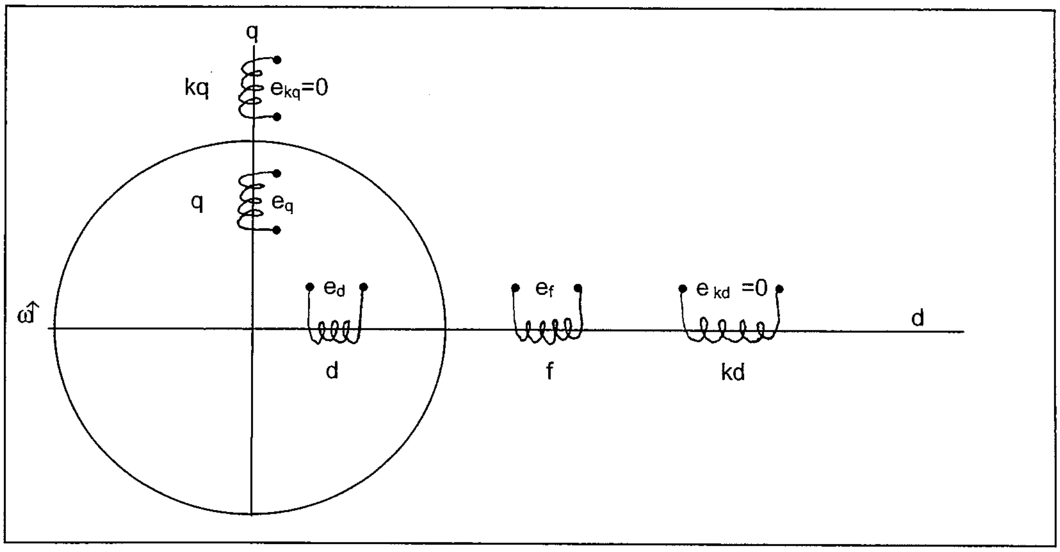


Figure 1.5 Diagram of *five-coil salient pole generalised machine.*

The elaboration of a practical synchronous motor model that – in the context of power network modelling – takes the form of a standardized d-q axis circuit element, is presented in four steps: Step 1 develops the *basic motor equations* that form the platform for the ensuing algorithmic development. Step 2 generates the *rotorflux model*, step 3 the *electrical circuit model*, and step 4 the *electro-mechanical model.*

Summary synchronous motor model description followed by a simple/qualitative illustration of model application, is given in Chapter 1.7. For extended model description and application, see Appendix.2.

Basic synchronous motor equations

The main premises for the *voltage*- and *flux* equations to be developed, are as follows:

> With reference to the *generalised machine* of Figure 1.5 : Positive direction of rotation: clockwise. The electrical torque is defined positive in *motor* mode of operation. External applied voltage is positive voltage, giving rise to positive coil currents. Positive flux is directed away from rotor centre. Positive current produce positive flux.
>
> The machine is presumed linear. Definition-wise the following basic outline then holds true :
>
> $e_\Psi = d\Psi/dt = d\Psi/di \; di/dt = (\Psi/i) \; di/dt = L \; di/dt$, where per def. : $L = \Psi/i$ (1-22)
>
> The machine is characterized in terms of per unit (pu) quantities : For the main (d and q-)circuits, base values of current and voltage are the nominal values. For a circuit magnetically coupled to a main circuit, base current is the current that produces the same magneto-motive force (m.m.f.) as the main

circuit's nominal current. By defining pu variables in this way, inductivity description of mutually coupled circuits becomes conveniant. Ill.: For two such circuits we may write: $l_1 = (l_{1\sigma} + m)$ and $l_2 = (l_{2\sigma} + m)$, where l_x is selfinductivity of coil 'x', $l_{x\sigma}$ is its leakage inductivity, and m is the mutual inductivity. In consistency with the above premises, the following inductivities are defined for the model machine:

For the d-axis: $L_d = L_{a\sigma}+L_{ad}$, $L_f = L_{f\sigma}+L_{ad}$, $L_{kd} = L_{kd\sigma}+L_{ad}$ For the q-axis: $L_q = L_{a\sigma}+L_{aq}$, $L_{kq} = L_{kq\sigma}+L_{aq}$ (1-23)

Total power supplied to the motor is the sum of power supplied to the d- and q coil. We wish to have pu supplied power =1 (or close to this value) when current and voltage of the main circuits are 1pu. Thus, in the 3-phase frame of reference we wish to define pu power as $p_{RST} = (1/3)\,(e_R\, i_R+e_S\, i_S+e_T\, i_T)$, and in the d-q axis frame of reference we wish to define pu power as $p_{dq} =(1/2)\,(e_d\, i_d+e_q\, i_q)+e_0\, i_0$. The factor 2/3 that appear in the Park transformation **P** of (1-2) (and the corresponding factor 1 of the inverse $\mathbf{P}^{-1}$), reflects the constraint that these expressions apply when transforming from the one frame of reference to the other. See Appendix 3 for details.

Based on the above premises the following defining relationships are set up between flux linkages and currents within respective axes:

$$\begin{aligned}
\Psi_d &= L_d\, i_d+L_{ad}\, i_f+L_{ad}\, i_{kd}\\
\Psi_q &= L_q\, i_q+L_{aq}\, i_{kq}\\
\Psi_f &= L_f\, i_f+L_{ad}\, i_d+L_{ad}\, i_{kd}\\
\Psi_{kd} &= L_{kd}\, i_{kd}+L_{ad}\, i_f+L_{ad}\, i_d\\
\Psi_{kq} &= L_{kq}\, i_{kq}+L_{aq}\, i_q
\end{aligned}$$

→ In matrix form :

$$\begin{bmatrix}\Psi_d\\ \Psi_q\\ \Psi_f\\ \Psi_{kd}\\ \Psi_{kq}\end{bmatrix} = \begin{bmatrix} L_d & & L_{ad} & L_{ad} & \\ & L_q & & & L_{aq}\\ L_{ad} & & L_f & L_{ad} & \\ L_{ad} & & L_{ad} & L_{kd} & \\ & L_{aq} & & & L_{kq}\end{bmatrix} \cdot \begin{bmatrix} i_d\\ i_q\\ i_f\\ i_{kd}\\ i_{kq}\end{bmatrix} \rightarrow \Psi = L\cdot i \qquad (1\text{-}24)$$

(matrix columns labelled d, q, f, kd, kq)

For the d- and q-coil the sought voltage balance is readily established by starting from the 3-phase frame of reference (as also done previously,- see Chapter 1.1 & 1.2) : In the physical three phase (RST) reference frame, we can for (say) phase 'R' of the motor, express the voltage balance as:

$$e_R = i_R\, r_a + d\Psi_R/dt \qquad (1\text{-}25)$$

where e_R, i_R, Ψ_R and r_a is - respectively – impressed voltage, current, flux linkages and resistance of motor phase 'R'. The per phase variables e_R , i_R and Ψ_R are related to their respective d -q axis components in the following way, see (1-4) :

$$\begin{aligned}
e_R &= e_d \cos\theta - e_q \sin\theta + e_o\\
i_R &= i_d \cos\theta - i_q \sin\theta + i_o\\
\Psi_R &= \Psi_d \cos\theta - \Psi_q \sin\theta + \Psi_o
\end{aligned} \qquad (1\text{-}26)$$

θ is the angular displacement of the axes of the (RST) reference frame relative to the axes of the d-q reference frame. Inserting expressions from (1-26) into (1-25), and observing that

$$d\Psi_R/dt = \cos\theta\; d\Psi_d/dt - \sin\theta\; d\Psi_q/dt + d\Psi_0/dt - \omega\, \Psi_d \sin\theta - \omega\, \Psi_q \cos\theta \qquad (1\text{-}27)$$

we get the following 'd-q-o version' of (1-25), where ω is angular rotor speed, see App. 3 :

$$\begin{aligned}
0 = \; & [- e_d + r_a\, i_d + d\Psi_d/dt - \omega\, \Psi_q] \cos\theta\\
&+ [\; e_q - r_a\, i_q - d\Psi_q/dt - \omega\, \Psi_d] \sin\theta\\
&+ [- e_o + r_a\, i_o + d\Psi_o/dt]
\end{aligned} \qquad (1\text{-}28)$$

For general validity of (1-28), the following d-q conditions must be observed to equivalence (1-25) :

$$\begin{aligned}
e_d &= r_a\, i_d + d\Psi_d/dt - \omega\, \Psi_q\\
e_q &= r_a\, i_q + d\Psi_q/dt + \omega\, \Psi_d\\
e_o &= r_a\, i_o + d\Psi_o/dt
\end{aligned} \qquad (1\text{-}29)$$

We assume in the present outline that zero sequence phenomena are inconsequential. Hence the last equation of (1-29) can be disregarded. In conclusion at this stage, we get the the following equations describing the voltage balance of the d- and q-coil of the synchronous motor:

$$\boxed{\begin{aligned}
e_d &= r_a\, i_d + d\Psi_d/dt - \omega\, \Psi_q\\
e_q &= r_a\, i_q + d\Psi_q/dt + \omega\, \Psi_d
\end{aligned}} \qquad (1\text{-}30)$$

For the remaining three fixed coils 'f', 'kd' and 'kq' of the generalised machine, the respective voltage balances can readily be defined in this way:

$$\begin{aligned} e_f &= r_f\, i_f + d\Psi_f/dt \\ e_{kd}=0 &= r_{kd}\, i_{kd} + d\Psi_{kd}/dt \\ e_{kq}=0 &= r_{kq}\, i_{kq} + d\Psi_{kq}/dt \end{aligned} \qquad (1\text{-}31)$$

Equations (1-30) and (1-31) form together the voltage equations of the model synchronous machine. The defining flux equations of the machine are given by (1-24). In a summary fashion, all these equations are shown in Figure 1.6. They form the platform for the ensuing algorithmic development.

Voltage equations of the model synchronous machine. (Equations (1-30) and (1-31))

$$\begin{bmatrix} e_d \\ e_q \\ e_f \\ e_{kd} \\ e_{kq} \end{bmatrix} = \begin{bmatrix} r_a & & & & \\ & r_a & & & \\ & & r_f & & \\ & & & r_{kd} & \\ & & & & r_{kq} \end{bmatrix} \cdot \begin{bmatrix} i_d \\ i_q \\ i_f \\ i_{kd} \\ i_{kq} \end{bmatrix} + \begin{bmatrix} d\Psi_d/dt \\ d\Psi_q/dt \\ d\Psi_f/dt \\ d\Psi_{kd}/dt \\ d\Psi_{kq}/dt \end{bmatrix} + \omega \begin{bmatrix} & -1 & & & \\ 1 & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix} \begin{bmatrix} \Psi_d \\ \Psi_q \\ \Psi_f \\ \Psi_{kd} \\ \Psi_{kq} \end{bmatrix}$$

(columns: d, q, f, kd, kq)

↓

$$\begin{bmatrix} \mathbf{e}_{dq} \\ \mathbf{e}_{fk} \end{bmatrix} = \begin{bmatrix} \mathbf{r}_a & \\ & \mathbf{r}_{fk} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{i}_{dq} \\ \mathbf{i}_{fk} \end{bmatrix} + \begin{bmatrix} d\boldsymbol{\Psi}_{dq}/dt \\ d\boldsymbol{\Psi}_{fk}/dt \end{bmatrix} + \omega \begin{bmatrix} \mathbf{H}_{dq} & \\ & \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{\Psi}_{dq} \\ \boldsymbol{\Psi}_{fk} \end{bmatrix} \qquad (1\text{-}32)$$

where;

$$\mathbf{r}_a = \begin{bmatrix} r_a & \\ & r_a \end{bmatrix} \quad \mathbf{e}_{dq} = \begin{bmatrix} e_d \\ e_q \end{bmatrix} \quad \mathbf{i}_{dq} = \begin{bmatrix} i_d \\ i_q \end{bmatrix} \quad \boldsymbol{\Psi}_{dq} = \begin{bmatrix} \Psi_d \\ \Psi_q \end{bmatrix} \quad \mathbf{H}_{dq} = \begin{bmatrix} & -1 \\ 1 & \end{bmatrix} \qquad (1\text{-}33)$$

$$\mathbf{r}_{fk} = \begin{bmatrix} r_f & & \\ & r_{kd} & \\ & & r_{kq} \end{bmatrix} \quad \mathbf{e}_{fk} = \begin{bmatrix} e_f \\ e_{kd}=0 \\ e_{kq}=0 \end{bmatrix} \quad \mathbf{i}_{fk} = \begin{bmatrix} i_f \\ i_{kd} \\ i_{kq} \end{bmatrix} \quad \boldsymbol{\Psi}_{fk} = \begin{bmatrix} \Psi_f \\ \Psi_{kd} \\ \Psi_{kq} \end{bmatrix} \qquad (1\text{-}34)$$

Equations copied from (1-24)

$$\begin{bmatrix} \Psi_d \\ \Psi_q \\ \Psi_f \\ \Psi_{kd} \\ \Psi_{kq} \end{bmatrix} = \begin{bmatrix} L_d & & L_{ad} & L_{ad} & \\ & L_q & & & L_{aq} \\ L_{ad} & & L_f & L_{ad} & \\ L_{ad} & & L_{ad} & L_{kd} & \\ & L_{aq} & & & L_{kq} \end{bmatrix} \cdot \begin{bmatrix} i_d \\ i_q \\ i_f \\ i_{kd} \\ i_{kq} \end{bmatrix}$$

(columns: d, q, f, kd, kq)

↓

$$\begin{bmatrix} \boldsymbol{\Psi}_{dq} \\ \boldsymbol{\Psi}_{fk} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_{dq} & \mathbf{L}_{(dq)(fk)} \\ \mathbf{L}_{(fk)(dq)} & \mathbf{L}_{fk} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{i}_{dq} \\ \mathbf{i}_{fk} \end{bmatrix} \qquad (1\text{-}35)$$

where;

$$\mathbf{L}_{dq} = \begin{bmatrix} L_d & \\ & L_q \end{bmatrix} \quad \mathbf{L}_{(dq)(fk)} = \begin{bmatrix} L_{ad} & L_{ad} & \\ & & L_{aq} \end{bmatrix} \qquad (1\text{-}36)$$

$$\mathbf{L}_{(fk)(dq)} = \begin{bmatrix} L_{ad} & \\ L_{ad} & \\ & L_{aq} \end{bmatrix} \quad \mathbf{L}_{fk} = \begin{bmatrix} L_f & L_{ad} & \\ L_{ad} & L_{kd} & \\ & & L_{kq} \end{bmatrix} \qquad (1\text{-}37)$$

Figure 1.6 Basic synchronous machine equations: The platform for the ensuing algorithmic development.

The rotor flux model

We seek the description of the flux variables $\mathbf{\Psi}_{fk} = [\Psi_f\ \Psi_{kd}\ \Psi_{kq}\ ]^t$ (and - if desired - also their implied currents $\mathbf{i}_{ifk}$) . For brevity of expression, we denote the algorithms that are developed in this context, 'the rotor flux model'.

Two sets of equations from Figure 1.6 provide the appropriate basis of this analysis; the lower set of equations from respectively (1-32) and (1-35) :

$$\mathbf{e}_{fk} = \mathbf{r}_{fk}\ \mathbf{i}_{fk} + d\mathbf{\Psi}_{fk}/dt \qquad (1\text{-}38)$$

$$\mathbf{\Psi}_{fk} = \mathbf{L}_{(fk)(dq)}\ \mathbf{i}_{dq} + \mathbf{L}_{fk}\ \mathbf{i}_{ifk} \qquad (1\text{-}39)$$

We wish to retain the flux variables $\mathbf{\Psi}_{fk}$ as state variables, while eliminating the currents $\mathbf{i}_{fk}$ from the 'surface' of analysis. Thus we eliminate $\mathbf{i}_{fk}$ from (1-39) and insert the expression of it into (1-38), yielding:

$$d\mathbf{\Psi}_{fk}/dt = \mathbf{e}_{fk} + (-\mathbf{r}_{fk}\ \mathbf{L}_{fk}^{-1})\ \mathbf{\Psi}_{fk} + (\mathbf{r}_{fk}\ \mathbf{L}_{fk}^{-1}\cdot\ \mathbf{L}_{(fk)(dq)})\ \mathbf{i}_{dq} \qquad (1\text{-}40)$$

The currents $\mathbf{i}_{dq}$ are referred to the model machine's local d-q axes. We want generally to refer them to the chosen global system reference phasor. The shift from global to local description is given by the following transformation:

$$\mathbf{i}_{dq} = \mathbf{T}\cdot\ \mathbf{i}_{DQ} \qquad \text{where;} \qquad \mathbf{T} = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} \qquad (1\text{-}41)$$

Here small letters (dq) signal locally referred currents, and capital letters (DQ) globally referred. β is the angular displacement of the local reference axis relative to the global axis.

We insert $\mathbf{i}_{dq}$ from (1-41) into (1-40) and define for conveniance new flux variables $\phi_{fk} = \omega_\delta\ \mathbf{\Psi}_{fk}$.
We then get the following sought form of the equations for modelling of the fluxes ϕ_{fk} :

$$d\phi_{fk}/dt = = \omega_\delta\ \mathbf{e}_{fk} + (-\mathbf{r}_{fk}\ \mathbf{L}_{fk}^{-1})\ \phi_{fk} + (\omega_\delta\ \mathbf{r}_{fk}\ \mathbf{L}_{fk}^{-1}\cdot\ \mathbf{L}_{(fk)(dq)}\ \mathbf{T})\ \mathbf{i}_{DQ} \qquad (1\text{-}42)$$

(1-42) is inconveniant to use. By setting in the appropriate matrices from Figure 1.6, and doing some further reductions and definitions, we arrive at the following practical version of (1-42), see Figure 1.7: For further on how *machine* parameters relate to basic *model* parameters, see 'Addendum' on p.1/16.

$$\underset{(3x1)}{d\phi_{fk}}/dt = \omega_\delta\ (\mathbf{e}_{fk} + \mathbf{F}_{fki}\cdot\ \mathbf{i}_{DQ} + \mathbf{F}_{fk\phi}\cdot\ \phi_{fk}) \qquad (1\text{-}43)$$

where:

$$\mathbf{e}_{fk} = \begin{bmatrix} K_f\cdot E_f \\ 0 \\ 0 \end{bmatrix} \begin{matrix} f \\ kd \\ kq \end{matrix}$$

$E_f = (E_{fo} + \Delta E_f) =$ field voltage. See Chapter 1.6
$K_f = (\sqrt{2}/(\omega_\delta\ T'_{do}))\ X_{ad}/(X_d - X'_d)$
ΔE_f = voltage control response

$X_{ad} = X_d - X_{a\sigma}$
$X'_{ad} = X'_d - X_{a\sigma}$
$X''_{ad} = X''_d - X_{a\sigma}$

$X_{aq} = X_q - X_{a\sigma}$
$X''_{aq} = X''_q - X_{a\sigma}$

$\mathbf{F}_{fki} =$

D	Q
$(1/(\omega_\delta\ T'_{do}))\ (X_{ad}/X'_{ad})\ X''_{ad}\ \cos\beta$	$-(1/(\omega_\delta\ T'_{do}))\ (X_{ad}/X'_{ad})\ X''_{ad}\ \sin\beta$
$(1/(\omega_\delta\ T''_{do}))\ X'_{ad}\ \cos\beta$	$-(1/(\omega_\delta\ T''_{do}))\ X'_{ad}\ \sin\beta$
$(1/(\omega_\delta\ T''_{qo}))\ X_{aq}\ \sin\beta$	$(1/(\omega_\delta\ T''_{qo}))\ X_{aq}\ \cos\beta$

$\mathbf{F}_{fk\phi} =$

$-(1/(\omega_\delta\ T'_{do}))\ (1/X'_{ad})\ [(X_{ad}/X'_{ad})\ (X'_d - X''_d) + X''_{ad}\ ]$	$(1/(\omega_\delta\ T'_{do}))\ (X_{ad}/X'^2_{ad})\ (X'_d - X''_d)$	
$(1/(\omega_\delta\ T''_{do}))\ (1/X_{ad})\ (X_d - X'_d)$	$-\ 1/(\omega_\delta\ T''_{do})$	
		$-\ 1/(\omega_\delta\ T''_{qo})$

X_d, X'_d, X''_d : direct-axis synchronous, transient and subtransient reactance (pu)
X_q, X''_q : quadrature-axis synchronous and subtransient reactance (pu)
$X_{a\sigma}$: stator leakage reactance (pu)
T'_{do}, T''_{do} : direct axis open stator transient and subtransient time constant (s)
T''_{qo} : quadrature axis open stator subtransient time constant (s)

Figure 1.7 Rotor flux model. Main model part describing synchronous motor state variables $\phi_{fk}=[\phi_f\ \phi_{kd}\ \phi_{kq}]^t$

At any time during integration the rotor- and damper currents $\mathbf{i}_{fk}$ may be derived from equation (1-39), after introducing $\phi_{fk} = \omega_b \Psi_{fk}$ and $\mathbf{i}_{dq} = \mathbf{T} \cdot \mathbf{i}_{DQ}$:

$$\mathbf{i}_{fk} = (\mathbf{X}_{fk})^{-1} \cdot (\phi_{fk} - \mathbf{X}_{DQr} \cdot \mathbf{i}_{DQ}) \qquad (1\text{-}44)$$

where:

$\mathbf{X}_{fk}$ =

	f	kd	kq
f	$X^2_{ad}/(X_d - X'_d)$	X_{ad}	
kd	X_{ad}	$X_{ad} + X'_{ad} \cdot X''_{ad}/(X'_d - X''_d)$	
kq			$X^2_{aq}/(X_q - X''_q)$

$\mathbf{X}_{DQr} = \mathbf{X}_{(fk)(dq)} \cdot \mathbf{T}$ =

	D	Q
f	$X_{ad}\cos\beta$	$-X_{ad}\sin\beta$
kd	$X_{ad}\cos\beta$	$-X_{ad}\sin\beta$
kq	$X_{aq}\sin\beta$	$X_{aq}\cos\beta$

Figure 1.8 Rotor flux model. Residual part of model describing (locally referred) currents $\mathbf{i}_{fk} = [i_f, i_{kd}, i_{kq}]^t$, given (locally referred) machine fluxes ϕ_{fk} and (globally referred) stator currents $\mathbf{i}_{DQ}$.

In the elaboration of equations (1-43) and (1-44), letter combinations like 'fk', 'dq' and 'DQ' have been used for indexing to (hopefully) ease understanding of the algorithmic development. From a systems analysis point of view (once the component models have been established), better notations should be applied. See Chapter 1.7 for summary model descriptions that aim at being user-oriented.

The electrical circuit model

In the context of power network analysis the task at hand is that of equivalencing the synchronous motor model of Figure 1.6, by a standardized d-q axis series element comprising an **R**-term, an inductive **X**-term, and an emf. $\Delta\mathbf{E}$. See Figure 1.1 and associated text.

Three sets of equations from Figure 1.6 form the basis for the ensuing analysis, namely the upper set from (1-32), and both sets contained in (1-35) :

$$\mathbf{e}_{dq} = \mathbf{r}_a \, \mathbf{i}_{dq} + d\Psi_{dq}/dt + \omega \, \mathbf{H}_{dq} \, \Psi_{dq} \qquad (1\text{-}45)$$

$$\Psi_{dq} = \mathbf{L}_{dq} \, \mathbf{i}_{dq} + \mathbf{L}_{(dq)(fk)} \, \mathbf{i}_{fk} \qquad (1\text{-}46)$$

$$\Psi_{fk} = \mathbf{L}_{(fk)(dq)} \, \mathbf{i}_{dq} + \mathbf{L}_{fk} \, \mathbf{i}_{ifk} \qquad (1\text{-}47)$$

$\mathbf{i}_{ifk}$ solved from (1-47) is inserted into (1-46), which then describes Ψ_{dq} as a function of $\mathbf{i}_{dq}$ and Ψ_{fk} . The expression thus found for Ψ_{dq} is inserted into (1-45), yielding finally the applied stator voltage $\mathbf{e}_{dq}$ as a function of the machine's state variables $\mathbf{i}_{dq}$ and Ψ_{fk} . Introducing also the new flux variables $\phi_{fk} = \omega_b \Psi_{fk}$ and $\phi_{dq} = \omega_b \Psi_{dq}$, we find as a result from this process:

$$\mathbf{e}_{dq} = \mathbf{r}_a \, \mathbf{i}_{dq} + (\mathbf{L}_{dq} - \mathbf{L}_{(dq)(fk)} \mathbf{L}_{fk}^{-1} \cdot \mathbf{L}_{(fk)(dq)}) \, d\mathbf{i}_{dq}/dt + (1/\omega_b) \, \mathbf{L}_{(dq)(fk)} \mathbf{L}_{fk}^{-1} \cdot d\phi_{fk}/dt$$
$$+ \, \omega \, \mathbf{H}_{dq} \, (\mathbf{L}_{dq} - \mathbf{L}_{(dq)(fk)} \mathbf{L}_{fk}^{-1} \cdot \mathbf{L}_{(fk)(dq)}) \, \mathbf{i}_{dq} + (\omega/\omega_b) \, \mathbf{H}_{dq} \, \mathbf{L}_{(dq)(fk)} \mathbf{L}_{fk}^{-1} \cdot \phi_{fk} \qquad (1\text{-}48)$$

By introducing the appropriate submatrices from Figure 1.6 into (1-48), and then elaborating some further on the equation, we find the following 'intermediate' state of it, see Figure 1.9. The state is termed intermediate since stator voltage $\mathbf{e}_{dq}$ and stator current $\mathbf{i}_{dq}$ still are referred to the machine's own d-q axes. It remains to replace these variables by their globally referred counterparts $\mathbf{e}_{DQ}$ and $\mathbf{i}_{DQ}$, respectively. The machine fluxes ϕ_{fk} are locally referred, and will conveniantly be kept so throughout all modelling processes.

$$\mathbf{e}_{dq} = r_a\, \mathbf{i}_{dq} + (1/\omega_b)\, \mathbf{X''}\, d\mathbf{i}_{dq}/dt + \Omega\, \overset{\dots}{\mathbf{X''}}\, \mathbf{i}_{dq} + \mathbf{B}_1\, d\phi_{fk}/dt + \Omega\, \mathbf{B}_2\, \phi_{fk} \qquad (1\text{-}49)$$

where;

$\Omega = (\omega/\omega_b)$ = pu rotor speed

$$\mathbf{X''} = \begin{pmatrix} X''_d & \\ & X''_q \end{pmatrix} \quad \text{and} \quad \overset{\dots}{\mathbf{X''}} = \begin{pmatrix} & -X''_q \\ X''_d & \end{pmatrix}$$

$$\mathbf{B}_1 = \begin{pmatrix} (1/\omega_b)\,(X''_{ad}/(X_{ad}\cdot X'_{ad}))\,(X_d - X'_d) & (1/\omega_b)\,(1/X'_{ad})\cdot(X'_d - X''_d) & \\ & & (1/\omega_b)\,(1/X_{aq})\,(X_q - X''_q) \end{pmatrix}$$

$$\mathbf{B}_2 = \begin{pmatrix} & & -(1/X_{aq})\,(X_q - (X''_q) \\ (X''_{ad}/(X_{ad}\cdot X'_{ad}))\,(X_d - X'_d) & (1/X'_{ad})\,(X'_d - X''_d) & \end{pmatrix}$$

From Figure 1.7 ;

$$d\phi_{fk}/dt = \omega_b\, (\mathbf{e}_{fk} + \mathbf{F}_{fki}\cdot \mathbf{i}_{DQ} + \mathbf{F}_{fk\phi}\cdot \phi_{fk}) \qquad (1\text{-}43)$$

Figure 1.9 'Intermediate state 1' of equation (1-48): It remains to replace locally referred stator voltage $\mathbf{e}_{dq}$ and stator current $\mathbf{i}_{dq}$ by their globally referred counterparts $\mathbf{u}_{DQ}$ and $\mathbf{i}_{DQ}$

As part of the basis for finalizing (1-49), we point to a few premises and rules that are crucial to the process of shifting from local to global reference (or vice versa) :

For stator voltage and current the following holds true:

$$\mathbf{e}_{dq} = \mathbf{T}\cdot \mathbf{e}_{DQ} \qquad \mathbf{i}_{dq} = \mathbf{T}\cdot \mathbf{i}_{DQ} \qquad \text{where;} \quad \mathbf{T} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \quad \text{and} \quad \mathbf{T}^{-1} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \qquad (1\text{-}50)$$

Definitionwise we have for electrical rotor angle and rotor speed:

$$\beta = (\omega_b\, t - \theta) \quad \rightarrow \quad d\beta/dt = (\omega_b - \omega) = \omega_b\cdot (1 - \Omega) \qquad (1\text{-}51)$$

From mathematics:

$$d\mathbf{e}_{dq}/dt = d(\mathbf{T}\cdot \mathbf{e}_{DQ})/dt = (d\mathbf{T}/dt)\, \mathbf{e}_{DQ} + \mathbf{T}\cdot d\mathbf{e}_{DQ}/dt \qquad (1\text{-}52)$$

From mathematics and (1-51) :

$$d\mathbf{T}/dt = (d\beta/dt)\,(d\mathbf{T}/d\beta) = \omega_b\cdot (1 - \Omega)\; d\mathbf{T}/d\beta \qquad (1\text{-}53)$$

Introducing the global variables $\mathbf{e}_{DQ}$ and $\mathbf{i}_{DQ}$ into (1-49), and processing the set of equations in accordance with premises and rules above, we arrive at 'Intermediate state 2' of equation (1-48). See Figure 1.10. It remains to develop more userfriendly expressions for the terms $\mathbf{R}_{DQ}$, $\mathbf{X}_{DQ}$ and $\Delta\mathbf{E}_{DQ}$, while abiding with the adopted definitions associated with *an electrical circuit model.* See Figure 1.1 for summary of definitions.

$$\mathbf{e}_{DQ} = \mathbf{R}_{DQ}\, \mathbf{i}_{DQ} + (1/\omega_b)\, \mathbf{X}_{DQ}\, d\mathbf{i}_{DQ}/dt + \Delta\mathbf{E}_{DQ} \qquad (1\text{-}54)$$

where;

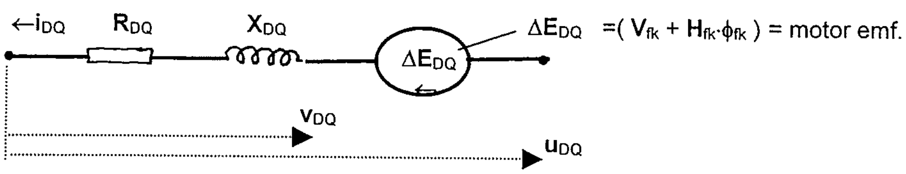
$$\mathbf{R}_{DQ} = r_a + (1\text{-}\Omega)\, \mathbf{T}^{-1}\, (\mathbf{X''}\, d\mathbf{T}/d\beta - \overset{\dots}{\mathbf{X''}}\, \mathbf{T}) + (\mathbf{T}^{-1}\, \overset{\dots}{\mathbf{X''}}\, \mathbf{T}) + (\omega_b\, \mathbf{T}^{-1}\, \mathbf{B}_1\, \mathbf{F}_{fki})$$

$$\mathbf{X}_{DQ} = \mathbf{T}^{-1}\, \mathbf{X''}\, \mathbf{T}$$

$$\Delta\mathbf{E}_{DQ} = [\omega_b\, \mathbf{T}^{-1}\, \mathbf{B}_1]\, \mathbf{e}_{fk} + [(\omega_b\, \mathbf{T}^{-1}\, \mathbf{B}_1\, \mathbf{F}_{fk\phi}) + (\Omega\, \mathbf{T}^{-1}\, \mathbf{B}_2)]\, \phi_{fk}$$

Figure 1.10 'Intermediate state 2' of equation (1-48). It remains to develop more userfriendly expressions for $\mathbf{R}_{DQ}$, $\mathbf{X}_{DQ}$ and $\Delta\mathbf{E}_{DQ}$, abiding with adopted model conventions.

After some straightforward but tedious laborations, we arrive at the sought *electrical circuit model* shown in Figure 1.11. At the end of the finalizing process sign conventions and terminology in line with that adopted for the formal *electrical circuit model* of Figure 1.1, are observed.



$$\underset{(2x1)}{\mathbf{v}_{DQ}} = \mathbf{R}_{DQ}\cdot\mathbf{i}_{DQ} + (1/\omega_o)\cdot\mathbf{X}_{DQ}\cdot d\mathbf{i}_{DQ}/dt \quad \& \quad \mathbf{v}_{DQ} - \Delta\mathbf{E}_{DQ} = \mathbf{u}_{DQ} \qquad (1\text{-}55)$$

Electrical circuit model of the synchronous motor in d-q axis frame of reference

ΔE_{DQ} = (V_{fk} + $H_{fk}\cdot\phi_{fk}$) = synchronous motor emf.

$$R_{DQ} = \begin{bmatrix} (R_a + \hat{X}''_r) + (2\Omega - 1)\cdot\bar{X}''\cdot\sin2\beta + \bar{X}''_r\cdot\cos2\beta & -\hat{X}'' + (2\Omega - 1)\cdot\bar{X}''\cdot\cos2\beta - \bar{X}''_r\cdot\sin2\beta \\ \hat{X}'' + (2\Omega - 1)\cdot\bar{X}''\cdot\cos2\beta - \bar{X}''_r\cdot\sin2\beta & (R_a + \hat{X}''_r) - (2\Omega - 1)\cdot\bar{X}''\cdot\sin2\beta - \bar{X}''_r\cdot\cos2\beta \end{bmatrix}$$

$$X_{DQ} = \begin{bmatrix} \hat{X}'' + \bar{X}''\cdot\cos2\beta & -\bar{X}''\cdot\sin2\beta \\ -\bar{X}''\cdot\sin2\beta & \hat{X}'' - \bar{X}''\cdot\cos2\beta \end{bmatrix} \qquad V_{fk} = \begin{bmatrix} C_f\cdot E_f\cdot\cos\beta \\ -C_f\cdot E_f\cdot\sin\beta \end{bmatrix}$$

E_f = (E_{fo} + ΔE_f) = field voltage where E_{fo} is initial value and ΔE_f is voltage control system response. See p. 1/29

$C_f = (\sqrt{2}/(\omega_o\cdot T'_{do}))\cdot(X''_{ad}/X'_{ad})$

$$H_{fk} = \begin{bmatrix} \Omega\cdot f_1\cdot\sin\beta + f_2\cdot\cos\beta & \Omega\cdot f_3\cdot\sin\beta + f_4\cdot\cos\beta & \Omega\cdot f_5\cdot\cos\beta + f_6\cdot\sin\beta \\ \Omega\cdot f_1\cdot\cos\beta - f_2\cdot\sin\beta & \Omega\cdot f_3\cdot\cos\beta - f_4\cdot\sin\beta & -\Omega\cdot f_5\cdot\sin\beta + f_6\cdot\cos\beta \end{bmatrix}$$

$\hat{X}'' = 0.5(X''_d + X''_q)$ $\quad\hat{X}''_r = 0.5(X''_{rd} + X''_{rq})$ $\quad\leftarrow\quad X''_{rd} = (1/(\omega_o\cdot T'_{do}))\cdot(X''_{ad}/X'_{ad})^2\cdot(X_d - X'_d) + (1/(\omega_o\cdot T''_{do}))\cdot(X'_d - X''_d)$

$\bar{X}'' = 0.5(X''_d - X''_q)$ $\quad\bar{X}''_r = 0.5(X''_{rd} - X''_{rq})$ $\quad\leftarrow\quad X''_{rq} = (1/(\omega_o\cdot T''_{qo}))\cdot(X_q - X''_q)$

$f_1 = (X_d - X'_d)\cdot(X''_{ad}/(X_{ad}\cdot X'_{ad}))$

$f_2 = f_1\cdot[(X'_d - X''_d)\cdot [\ (1/(\omega_o\cdot T''_{do}))\cdot(1/X''_{ad}) - (1/(\omega_o\cdot T'_{do}))\cdot(X_{ad}/X'^2_{ad})\ ] - (1/(\omega_o\cdot T'_{do}))\cdot(X''_{ad}/X'_{ad})\ ]$

$f_3 = (X'_d - X''_d)/X'_{ad}$

$f_4 = f_3\cdot[\ (1/(\omega_o\cdot T'_{do}))\cdot(X''_{ad}/X'^2_{ad})\cdot(X_d - X'_d) - (1/(\omega_o\cdot T''_{do})\ ]$

$f_5 = -\ (X_q - X''_q)/X_{aq}$

$f_6 = f_5\cdot(1/(\omega_o\cdot T''_{qo}))$

$\leftarrow\ X_{ad} = X_d - X_{a\sigma}$

$\leftarrow\ X'_{ad} = X'_d - X_{a\sigma}$

$\leftarrow\ X''_{ad} = X''_d - X_{a\sigma}$

$\leftarrow\ X_{aq} = X_q - X_{a\sigma}$

$\leftarrow\ X''_{aq} = X''_q - X_{a\sigma}$

Figure 1.11 Electrical circuit model of the synchronous motor

In the elaboration of equation (1-55), letter combinations like 'fk' and 'DQ' have been applied for indexing to (hopefully) enhance understanding of the algorithmic development. From an ensuing application point of view, better notations could be devised. See Chapter 1.7 for for summary model descriptions that generally aim at being user-oriented.

The electromechanical model

We seek here the description of the final two synchronous motor state variables, - namely pu rotor speed Ω, and rotor's electrical angle β relative to some chosen synchronous reference. For brevity of characterization, the algorithms developed in this context is denoted *'the electromechanical model'*.

The algorithm that governs motor speed performance is the torque equation of the machine unit. A brief elaboration of this equation on pu form referred to common system base, follows: In SI units Newton's second law states the following torque balance for the rotor system of the motor :

$$J\cdot d\omega_{mec}/dt = (T_{NM(el)} - T_{NM(mec)}) \qquad [NM] \qquad (1\text{-}56)$$

where J is *total moment of inertia* of the rotating masses in NMs^2, ω_{mec} is mechanical angular speed, $T_{NM(el)}$ is electrical motor torque and $T_{NM(mec)}$ is mechanical load torque, both in NM. $T_{NM(el)}$ is by definition positive in motor mode of operation. $T_{NM(mec)}$ is definitionwise positive for a mechanical torque that contributes to slowing down rotor speed. With S_{Bas} as rated system VoltAmpere (VA) power base, base torque becomes $S_{Bas}/\omega_{mec(o)}$. The pu form of (1-56) is found by dividing both sides of the equation by the system base torque:

$$(J\cdot\omega_{mec(o)}^2/S_{Bas})\cdot d\Omega/dt = (T_{(el)} - T_{(mec)}) \qquad [pu] \qquad (1\text{-}57)$$

Here $\Omega = (\omega_{mec}/\omega_{mec(o)}) = \omega/\omega_{(o)}$ = pu rotor speed, $T_{(el)}$ is electrical motor torque in pu, and $T_{(mec)}$ is pu mechanical torque. It remains to express the inertia constant in recognizable unit(s) : We choose here to develop the expression of J in terms of a *nominal acceleration time* T_a. T_a is the time (in s) required to accelerate the unit from stillstand to synchronous speed, given an accelerating torque that is constant and equal to the rated torque $(S_{Motor}\cdot\cos\varphi_{Motor})/\omega_{mec(o)}$ of the motor. S_{Motor} is rated (VA) motor power, and $\cos\varphi_{Motor}$ is the unit's rated power factor. Applying this torque on the right hand side of (1-56) and integrating ω_{mec} from zero to $\omega_{mec(o)}$, we find the following expression for total moment of inertia: $J = T_a\cdot S_{Motor}\cdot\cos\varphi_{Motor}/\omega_{mec(o)}^2$. This expression is finally inserted for J into (1-57), yielding this practical pu form of the motor torque equation:

$$((S_{Motor}/S_{Bas})\cdot T_a\cdot\cos\varphi_{Motor})\cdot d\Omega/dt = (T_{(el)} - T_{(mec)}) \qquad [pu] \qquad (1\text{-}57)$$

Part of *electromechanical model*: The description of synchronous motor rotor speed Ω. Acceleration time T_a used for characterizing total moment of inertia of rotating masses.

Another widely used normalized inertia figure is the H-constant. H is defined as *stored kinetic energy at synchronous speed divided by machine voltampere rating*, i.e.: $H = 0.5\cdot J\cdot\omega_{mec(0)}^2/S_{Motor}$. This implies the following relationship between T_a and H : $H = 0.5\cdot T_a\cdot\cos\varphi_{Motor}$.

The electrical motor torque $T_{(el)}$ is developed next. Per definition we have the following expression for power supplied to the synchronous motor, see Figure 1.6 and comments on basic premises in Chapter 1.4 :

$$P_{(el)} = 0.5\cdot \mathbf{e}_{dq}^t\cdot\mathbf{i}_{dq} \qquad (1\text{-}58)$$

Setting in for $\mathbf{e}_{dq}$ and $\mathbf{i}_{dq}$ from (1-32) and (1-33), and observing that $\phi_{dq} = \omega_o\cdot\Psi_{dq}$, we find that;

$$P_{(el)} = 0.5\cdot r_a\cdot(i_d^2+i_q^2) + 0.5\cdot\Omega\cdot\mathbf{i}_{dq}^t\cdot\bar{\mathbf{1}}^t\cdot\phi_{dq} + (0.5/\omega_o)\cdot\mathbf{i}_{dq}^t\cdot d\phi_{dq}/dt \qquad (1\text{-}59)$$

$\bar{\mathbf{1}}$ is defined earlier, see Figure 1.4 and also below. Replacing the locally referenced current $\mathbf{i}_{dq}$ by its globally referenced counterpart $\mathbf{i}_{DQ}$ according to the transformation $\mathbf{T}$ of (1-41), we find:

$$P_{(el)} = \underbrace{0.5\cdot r_a\cdot(i_D^2+i_Q^2)}_{\text{Losses in stator resistance}} + \underbrace{0.5\cdot\Omega\cdot\mathbf{i}_{DQ}^t\cdot\mathbf{T}_1\cdot\phi_{dq}}_{\text{Airgap power}} + \underbrace{(0.5/\omega_o)\cdot\mathbf{i}_{DQ}^t\cdot\mathbf{T}^t\cdot d\phi_{dq}/dt}_{\text{Oscillating power (zero power over time)}} \qquad (1\text{-}59)$$

The *electrical torque* is found by dividing the expression for *airgap power* by Ω. $\mathbf{T_1} = (\bar{\mathbf{1}}\,\mathbf{T})^t$. The flux vector ϕ_{dq} is determined as a function of $\mathbf{i_{DQ}}$ and ϕ_{fk} from equations (1-46), (1-47), (1-41). After some elaborations the following practical algorithm emerges for determining the electrical motor torque $T_{(el)}$:

$$T_{(el)} = 0.5\ \mathbf{i_{DQ}}^t\ \mathbf{T_1}\ \phi_{dq} \qquad (1\text{-}60)$$

where;

$$\phi_{dq} = \mathbf{X''}\ \mathbf{T}\ \mathbf{i_{DQ}} + \mathbf{f}\ \phi_{fk}$$

and

$$\mathbf{i_{DQ}} = \begin{bmatrix} i_D \\ i_Q \end{bmatrix} = \text{Synchronous motor current, global reference.}$$

$$\phi_{fk} = \begin{bmatrix} \phi_f \\ \phi_{kd} \\ \phi_{kq} \end{bmatrix} = \text{Field- and damper flux linkages, local reference.}$$

} Synchronous motor state variables

$$\mathbf{T} = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}$$ = Transformation that shifts stator current from global to local reference axis : $\mathbf{i_{dq}} = \mathbf{T}\ \mathbf{i_{DQ}}$ See (1-41).

$$\mathbf{T_1} = (\bar{\mathbf{1}}\,\mathbf{T})^t = \left[\begin{bmatrix} & 1 \\ -1 & \end{bmatrix} \cdot \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} \right]^t = \begin{bmatrix} \sin\beta & -\cos\beta \\ \cos\beta & \sin\beta \end{bmatrix}$$

$$\mathbf{X''} = \begin{bmatrix} X''_d & \\ & X''_q \end{bmatrix}$$

$$\mathbf{f} = \begin{bmatrix} f_1 & f_3 & \\ & & -f_5 \end{bmatrix}$$

where; $f_1 = (X_d - X'_d)\, X''_{ad}/(X_{ad}\, X'_{ad})$
$f_3 = (X'_d - X''_d)/(X'_{ad})$
$f_5 = -(X_q - X''_q)/(X_{aq})$

Figure 1.12 Practical algorithm for computing synchronous motor electrical torque $T_{(el)}$ in (1-57)

The *mechanical torque* $T_{(mec)}$ will take on different forms, depending on the operational regime; whether motor or generator mode of operation :

For motor operation (which per definition implies a positive sign of the mechanical torque), the following premises may in many cases prevail:

$T_{(mec)} = T_{(mec(o))}\ \Omega^{\kappa}$, where $T_{(mec(o))}$ and exponent κ depend on the 'rotational status' of the motor at $t = -0$; whether already up and running, or to be started from $\Omega = 0$: (1-61)

If the motor is up and running ;

$T_{(mec(o))} = T_{(el(o))} =$ electrical motor torque at $t = -0$. The proper value is found by applying equation (1-60) to data from the initial power system load flow.

$\kappa =$ exponent that depends on the load torque's sensitivity to rotational speed for Ω close to 1.0 . In many cases: $\kappa =$ (say) 1.5 – 3.5.

If the motor is to be started from stillstand (as e.g. an asynchronous motor) ;

$T_{(mec(o))}$ = coefficient that contributes to modelling the effect of mechanical friction, air resistance, etc, during the startup phase. Expected range: (say) 0.02 – 0.05

$\kappa =$ exponent reflecting speed dependency of $T_{(mec)}$. Prospective area of variation: $\kappa =$ (say) 1-5. κ as well as $T_{(mec(o))}$ may change over the range $\Omega = 0 \rightarrow 1$.

For generator operation (which per definition implies a neg. sign of the mechanical torque):

$T_{(mec)} = (T_{(el(o))} + \Delta T_{(mec)})$, where $T_{(el(0))}$ is initial electrical motor torque, and $\Delta T_{(mec)}$ is given by the respons of the *power control system.* For details, see 'model stock' of Chapter 1.7. (1-62)

Figure 1.13 Example algorithms for computing synchronous motor load torque $T_{(mec)}$ in (1-57)

The algorithm that governs the variation of rotor's electrical angle β relative to some chosen synchronous reference phasor, is definitionwise given by equation (1-51) :

$$\beta = \omega_\delta\, t - \theta \qquad (1\text{-}63)$$

where θ is the angular displacement of the axes of the three phase (RST) reference frame, relative to the axes of the (dq) frame of reference.The sought description flows from (1-63) :

$$\boxed{d\beta/dt = \omega_\delta\,(1 - \Omega)} \qquad (1\text{-}64)$$

Part of *electromechanical model*: The description of synchronous motor rotor angle β in radians

Summary synchronous motor model description followed by a simple/qualitative illustration of of model application, is presented in Chapter 1.7.

Addendum
To enlighten the laborations towards compact motor model descriptions like those of Figures 1.7, 1.8, 1.11 and also 1.14 and 1.15, the interrelationship between *'commercial' machine parameters* like (X_d,X'_d,X''_d,X_q,X''_q,T'_{do},T''_{do},T''_{qo}) and *basic model parameters* like ($L_{a\sigma}$,L_{ad},L_{aq},$L_{f\sigma}$,$L_{kd\sigma}$,$L_{kq\sigma}$,r_a,r_{kd},r_f,r_{kq}), are briefly summed up:

$X_d = X_{a\sigma} + X_{ad}$
$X_f = X_{f\sigma} + X_{ad}$
$X_{kd} = X_{kd\sigma} + X_{ad}$
$X_q = X_{a\sigma} + X_{aq}$
$X_{kq} = X_{kq\sigma} + X_{aq}$

$X'_d = X_{a\sigma} + X'_{ad}$ where $1/X'_{ad} = (1/X_{ad}) + (1/X_{f\sigma})$
$X''_d = X_{a\sigma} + X''_{ad}$ where $1/X''_{ad} = (1/X_{ad}) + (1/X_{f\sigma}) + (1/X_{kd\sigma}) = (1/X'_{ad}) + (1/X_{kd\sigma})$
$X''_q = X_{a\sigma} + X''_{aq}$ where $1/X''_{aq} = (1/X_{aq}) + (1/X_{kq\sigma})$

$T'_{do} = L_f/r_f = X_f/(\omega_0\, r_f)$ (Open stator. 'Seen' from the field circuit)
$T''_{do} = L/r_{kd} = X/(\omega_0\, r_{kd})$ where $X = X_{kd\sigma} + 1/(\,(1/X_{ad}) + (1/X_{f\sigma})\,)$ (Open stator. 'Seen' from the kd-circuit)
$T''_{qo} = L_{kq}/r_{kq} = X_{kq}/(\omega_0\, r_{kq})$ (Open stator. 'Seen' from the kq-circuit)

1.5 The Asynchronous Motor
Compared to the normal synchronous machine the traditional asynchronous machine lacks the field winding, and symmetry prevails regarding the electromagnetic effect of its rotor circuits.

For the synchronous machine it was presumed appropriate to base 'default' mathematical modelling on a *five coil, salient pole generalised machine.* See Figure 1.5. In view of availability of data for asynchronous machines as well as the aspect of similar level of precision in modelling of rotating machines, it would appear reasonable to specify a *four-coil, cylindrical pole generalised machine* for modelling of the asynchronous motor/generator. Thus, the machine diagram of Figure 1.5 provides the proper basis, when noticing the following interpretations/ observations :

The 'pseudostationary' d- and q coils equivalence in the same way as outlined earlier for the synchronous machine, the electromagnetic effects of the stator windings of the three phase asynchronous machine.

There is *one* superflous coil in the d-axis, since the field winding now is lacking. For ease of (later on) interpretation we choose to remove the kd-coil. Thus, the remaining 2 coils labelled respectively 'f' and 'kq', take on the function of equivalencing the rotor winding of the asyn-chronous machine.

Since the phenomenon of magnetic saliency is absent, the former f- and kq- coil now become identical in their new roles. Index 'r' for 'rotor' is in the following assigned to both of them. Inductivities and flux linkages may be written as follows, see (1-23) and (1-24) for comparison with the synchronous machine:

For the d-axis: $L_d = L_{a\sigma}+L_m$, $L_{rd} = L_{r\sigma}+L_m$ For the q-axis: $L_q = L_{a\sigma}+L_m$, $L_{rq} = L_{r\sigma}+L_m$ (1-65)

$$\begin{aligned}\Psi_d &= L_d\, i_d + L_m\, i_{rd}\\ \Psi_q &= L_q\, i_q + L_m\, i_{rq}\\ \Psi_{rd} &= L_{rd}\, i_{rd} + L_m\, i_d\\ \Psi_{rq} &= L_{rq}\, i_{rq} + L_m\, i_q\end{aligned} \quad \rightarrow \text{On matrix form: } \begin{bmatrix}\Psi_d\\ \Psi_q\\ \Psi_{rd}\\ \Psi_{rq}\end{bmatrix} = \begin{bmatrix}L_d & & L_m & \\ & L_q & & L_m\\ L_m & & L_{rd} & \\ & L_m & & L_{rq}\end{bmatrix} \cdot \begin{bmatrix}i_d\\ i_q\\ i_{rd}\\ i_{rq}\end{bmatrix} \rightarrow \mathbf{\Psi} = \mathbf{L\, i} \quad (1\text{-}66)$$

(matrix columns labelled d, q, rd, rq)

The working out of a practical asynchronous motor model that - in the context of power network modelling - take the form of a standadized d-q axis circuit element, is conveniantly afforded by reducing the current synchronous motor model in consistency with the special premises set above. The elaboration is presented in three steps: Step 1 develops the *electrical circuit model*, step 2 the *rotor flux model*, and step 3 the *electromechanical model.*

Summary asynchronous motor model description followed by a simple/qualitative illustration of model application, is given in Chapter 1.7.

The electrical circuit model

The source for our model reduction process is the *electrical circuit model* of the synchronous motor shown in Figure 1.11. To establish the appropriate basic algorithmic platform, we first find the 'asynchronous motor version' of the set of synchronous motor equations assembled in the bottom part of Figure 1.11. Going from right to left as advised by the arrows, and being aware of the following implications of the above special premises ;

$X''_d = X'_d$ (since our 'simplified' synchronous motor has no third circuit in the d-axis)
$X''_{ad} = X'_{ad}$ " " " "
$X_d = X_q = (x_{a\sigma} + X_m)$ (See (1-65) above)
$X_{ad} = X_{aq} = X_m$ " "
$X_{rd} = X_{rq} = X_r = \omega_b L_r = (x_{r\sigma} + X_m)$ " "
$X'_{ad} = 1/(\,(1/X_m) + (1/X_{r\sigma})\,)$ (For comparison: See Addendum at the end of Chapter 1.4.)
$X''_{aq} = X'_{ad} = X'_m = 1/(\,(1/X_m) + (1/X_{r\sigma})\,)$ " " "
$X'_M = X'_d = (x_{a\sigma} + X'_m)$ " " "
$T'_{do} = T'_{ro} = T''_{qo} = L_r/r_r = X_r/(\omega_b\, r_r)$ " " "

we find that

$$\hat{X}'' = X'_M = (X_{a\sigma} + X'_m) \qquad \hat{X}''_r = r_r\,(X_m/X_r)^2 \qquad \leftarrow \quad X''_{rd} = (1/(\omega_b\, T'_{ro}))\,(X_m - X'_m) = r_r\,(X_m/X_r)^2 \qquad (1\text{-}67)$$

$$\bar{X}'' = 0 \qquad \bar{X}''_r = 0 \qquad \leftarrow \quad X''_{rq} = (1/(\omega_b\, T'_{ro}))\,(X_m - X'_m) = X''_{rd} \qquad (1\text{-}68)$$

$$\left.\begin{aligned} f_1 &= (X_{ad} - X'_{ad})\,(X'_{ad}/(X_{ad}\, X'_{ad})) = (X_m - X'_m)/X_m = X_m/X_r\\ f_2 &= f_1\,[-(1/(\omega_b\, T'_{ro}))\,(X'_{ad}/X'_{ad})\,] = -r_r\,(X_m/X_r^2)\\ f_3 &= (X'_d - X'_d)/X'_{ad} = 0\\ f_4 &= 0\\ f_5 &= -(X_{aq} - X''_{aq})/X_{aq} = -(X_m - X'_m)/X_m = -f_1\\ f_6 &= f_5 \cdot (1/(\omega_b\, T'_{ro})) = -(X_m/X_r)\,(r_r/X_r) = -r_r\,(X_m/X_r^2) = f_2 \end{aligned}\right\} \qquad (1\text{-}69)$$

Figure 1.14 'Asynchronous motor version' of synchronous motor equations at bottom of Figure 1.11

For the synchronous motor the equivalent e.m.f. is expressed as $\Delta \mathbf{E}_{DQ} = \mathbf{V}_{fk} + \mathbf{H}_{fk}\,\phi_{fk}$, see (1-55). In modifying this equation to cover the asynchronous motor, three obsevations should be accounted for :

There is no separate e.m.f. associated with the rotor coils. Thus ; $\mathbf{V}_{fk} = 0$.

Three motor flux linkages $\phi_{fk} = [\phi_f, \phi_{kd}, \phi_{kq}]^t$ contribute to producing the e.m.f. $\Delta \mathbf{E}_{DQ}$ of the synchronous motor. Thus the matrix $\mathbf{H}_{fk}$ is of dimension (2x3). The asynchronous motor model lacks per definition the kd-coil, meaning that the flux linkages $[\phi_f, \phi_{kq}]^t$ are to be interpreted as the ones associated with the rotor coils of the asynchronous motor. The path to establish the H-matrix for the latter machine implies therefore deletion of column no 2 of $\mathbf{H}_{fk}$. We denote the so reduced version $\mathbf{H}_{fk(reduced)}$.

The flux linkages $[\phi_f, \phi_{kq}]^t$ for the synchronous motor are referred to the local d-q axes of the model machine, see comments after equations (1-48) . Since the concept of local axes is of little relevance to the normal asynchronous machine, we choose to refer also the motor *fluxes* to the global reference phasor. The logic of transformation together with the transformation matrix **T** itself are given in (1-41).

From the foregoing observations the e.m.f. $\Delta \mathbf{E}_{DQ}$ for the asynchronous motor can be formulated as follows, based on $\mathbf{H}_{fk}$ and **T** :

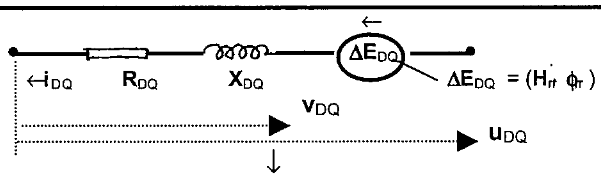
$$\Delta \mathbf{E}_{DQ} = (\mathbf{H}_{fk(reduced)}\,\mathbf{T})\,\phi_r = \mathbf{H}_{rr}\,\phi_r \qquad (1\text{-}70)$$

where ϕ_r are the globally referenced flux linkages associated with the rotor coils of the model machine of the asynchronous motor. Inserting from Figure 1.11 and (1-41), we find:

$$\mathbf{H}_{rr} = (\mathbf{H}_{fk(reduced)}\cdot\mathbf{T}) = \begin{bmatrix} \Omega f_1 \sin\beta + f_2 \cos\beta & \Omega f_5 \cos\beta + f_6 \sin\beta \\ \Omega f_1 \cos\beta - f_2 \sin\beta & -\Omega f_5 \sin\beta + f_6 \cos\beta \end{bmatrix} \cdot \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}$$

$$= \begin{bmatrix} 0.5\,\Omega (f_1+f_5)\sin 2\beta + f_2 \cos^2\beta + f_6 \sin^2\beta & 0.5\,(f_6-f_2)\sin 2\beta - \Omega f_1 \\ 0.5\,(f_6-f_2)\sin 2\beta - \Omega f_5 & -0.5\,\Omega (f_1+f_5)\sin 2\beta + f_2 \sin^2\beta + f_6 \cos^2\beta \end{bmatrix} \qquad (1\text{-}71)$$

Based on the adaptions made above, the elements $(\mathbf{R}_{DQ}, \mathbf{X}_{DQ}, \mathbf{H}_{rr})$ of the *electrical circuit model* of the asynchronous motor can now be finalized: The resistance matrix $\mathbf{R}_{DQ}$ and the inductive reactance matrix $\mathbf{X}_{DQ}$ of the asynchronous motor are readily established by applying (1-67) and (1-68) to matrix $\mathbf{R}_{DQ}$ and $\mathbf{X}_{DQ}$ of Figure 1.11. The matrix $\mathbf{H}_{rr}$ of (1-71) is finalized by implementing the results from (1-69), where it appears that $f_1 = -f_5$ and $f_2 = f_6$. The result is summarized as follows:



$$\mathbf{v}_{DQ} = \mathbf{R}_{DQ}\,\mathbf{i}_{DQ} + (1/\omega_b)\,\mathbf{X}_{DQ}\,d\mathbf{i}_{DQ}/dt \quad \& \quad \mathbf{v}_{DQ} - \Delta\mathbf{E}_{DQ} = \mathbf{u}_{DQ} \qquad (1\text{-}72)$$

Electrical circuit model of the asynchronous motor in d-q axis frame of reference

$\Delta \mathbf{E}_{DQ} = (\mathbf{H}_{rr}\,\phi_r)$ = asynchronous motor emf.

$$\mathbf{R}_{DQ} = \begin{bmatrix} R_a + R_r\cdot(X_m/X_r)^2 & -X'_M \\ X'_M & R_a + R_r\cdot(X_m/X_r)^2 \end{bmatrix}$$

$$\mathbf{X}_{DQ} = \begin{bmatrix} X'_M & \\ & X'_M \end{bmatrix}$$

$$\mathbf{H}_{rr} = (X_m/X_r)\cdot \begin{bmatrix} -(R_r/X_r) & -\Omega \\ \Omega & -(R_r/X_r) \end{bmatrix}$$

	(Example data)
$\mathbf{X}_{a\sigma}$ = **stator leakage reactance**	**(0.08)**
$\mathbf{X}_{r\sigma}$ = **rotor leakage reactance**	**(0.08)**
$\mathbf{X}_m$ = **magnetizing reactance**	**(2.50)**
$X_r = (X_{r\sigma} + X_m)$	
$X'_M = (X_{a\sigma} + X'_m) = (X_{a\sigma} + (X_m X_{r\sigma}/X_r))$	
$\mathbf{R}_a$ = **stator resistance**	**(0.03)**
$\mathbf{R}_r$ = **rotor resistance**	**(0.03)**
$T'_{ro} = L_r/R_r = X_r/(\omega_b R_r)$	
s = slip = $(1-\Omega_{AM})$	
$X'_m = (X_m X_{r\sigma}/X_r)$	

(1-73)

Figure 1.15 Electrical circuit model of the asynchronous motor

In the elaboration of equation (1-72), letter combinations like 'DQ' and 'rr' have been used for indexing to support understanding of the algorithmic development. From an application point of view better notations should be devised. See Chapter 1.7 for summary model descriptions that strive at being more user-oriented.

The rotor flux model

The source for our model reduction process is the *rotor flux model* of the synchronous motor shown in Figure 1.7. To have the appropriate basic algorithmic platform, we first evaluate the 'asynchronous motor version' of the matrices $(\mathbf{e}_{fk}, \mathbf{F}_{fki}, \mathbf{F}_{fk\phi})$ of Figure 1.7. To this end we bring to bear the implications that were ascertained via equations (1-67) to (1-69) and their surrounding text. It readily follows - when noticing that rows and columns associated with the kd-coil are to be deleted. - that ;

$$\mathbf{e}_{rr} = 0 \qquad \text{(Since there normally are no e.m.f.s in the rotor circuits)}$$

$$\bar{\mathbf{F}}_{rri} = (r_r X_m/X_r) \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} \qquad \text{(Since } T'_{do}=T''_{qo}=T'_{ro}=L_r/r_r,\ X_{ad}=X_{aq}=X_m,\ X''_{ad}=X'_{ad}) \tag{1-74}$$

$$\bar{\mathbf{F}}_{rr\phi} = (r_r/X_r) \begin{bmatrix} -1 & \\ & -1 \end{bmatrix} \qquad (\quad \text{"} \quad \text{"} \quad \text{"} \quad)$$

With the new 'asynchronous motor versions' $(\mathbf{e}_{rr}, \bar{\mathbf{F}}_{rri}, \bar{\mathbf{F}}_{rr\phi})$ to replace $(\mathbf{e}_{fk}, \mathbf{F}_{fki}, \mathbf{F}_{fk\phi})$ in (1-43), we get the following tentative expression for the *rotor flux model* of the asynchronous motor, when taking into account that the rotor flux linkages ϕ_r for the asynchronous motor are presumed globally referenced ;

$$d(\mathbf{T}\cdot \phi_r)/dt = \omega_b\, \bar{\mathbf{F}}_{rri}\, \mathbf{i}_{DQ} + \omega_b\, \bar{\mathbf{F}}_{rr\phi}\, (\mathbf{T}\cdot \phi_r) \tag{1-75}$$

Taking the derivative of the matrix product and and rearranging the equation, we find the sought after formal form of the rotor flux model:

$$d\phi_r/dt = \omega_b\, \underbrace{(\mathbf{T}^{-1}\, \bar{\mathbf{F}}_{rri})}_{\mathbf{F}_{rri}}\, \mathbf{i}_{DQ} + \omega_b\, \underbrace{[\mathbf{T}^{-1}\, \bar{\mathbf{F}}_{rr\phi}\, \mathbf{T} - (1/\omega_b)\cdot(d\beta/dt)\, \mathbf{T}^{-1}\, d\mathbf{T}/d\beta]}_{\mathbf{F}_{rr\phi}}\, \phi_r \tag{1-76}$$

Evaluating the expressions we find the following *rotor flux model* for the asynchronous motor:

$$\underset{(2\times1)}{d\phi_r/dt} = \omega_b\, (\mathbf{F}_{rri}\, \mathbf{i}_{DQ} + \mathbf{F}_{rr\phi}\, \phi_r) \tag{1-77}$$

where;

$$\mathbf{F}_{rri} = \begin{bmatrix} (R_r\cdot X_m/X_r) & \\ & (R_r\cdot X_m/X_r) \end{bmatrix}$$

$$\mathbf{F}_{rr\phi} = \begin{bmatrix} -(R_r/X_r) & (1-\Omega) \\ (\Omega-1) & -(R_r/X_r) \end{bmatrix}$$

For parameter interpretation, see Figure 1.15

Figure 1.16 Rotor flux model of the asynchronous motor

The electromechanical model

We seek the description of the remaining asynchronous motor state variable,- namely pu rotor speed Ω. The set of algorithms developed in this context are denoted *'the electromechanical model'*.

The algorithm that governs motor speed performance is the torque equation of the motor. This equation has already been developed on a practical pu form for the synchronous motor. See (1-57). We choose to apply the same pu form for the asynchronous motor :

$$((S_{Motor}/S_{Bas})\ T_a \cos\varphi_{Motor})\ d\Omega/dt = (T_{(el)} - T_{(mec)}) \qquad [pu] \qquad (1\text{-}78)$$

Main part of *electromechanical model*: The description of asynchronous motor rotor speed Ω. Acceleration time T_a used for characterizing total moment of inertia of rotating masses. H is an alternative normalized parameter: $H=0.5\ T_a \cos\varphi_{Motor}$

S_{Motor} and S_{Bas} are rated (VA) motor power and system (VA) power base, respectively. $\cos\varphi_{Motor}$ is rated power factor of the asynchronous motor. $T_{(el)}$ is electrical motor torque in pu, and $T_{(mec)}$ is pu mechanical load torque.

The electrical motor torque $T_{(el)}$ is developed next by modifying the synchronous motor algorithm of Figure 1.12: The given expression $\boldsymbol{\phi}_{dq} = \mathbf{X''}\ \mathbf{T}\ \mathbf{i}_{DQ} + \mathbf{f}\ \boldsymbol{\phi}_{fk}$ is first introduced into equation (1-60), giving:

$$T_{(el)} = 0.5\ \mathbf{i}^t_{DQ}\ \mathbf{T_1}\ \mathbf{X''}\ \mathbf{T}\ \mathbf{i}_{DQ} + 0.5\ \mathbf{i}^t_{DQ}\ \mathbf{T_1}\ \mathbf{f}\ \boldsymbol{\phi}_{fk} \qquad (1\text{-}79)$$

Here again $\boldsymbol{\phi}_{fk} = [\phi_f, \phi_{kq}]^t$ is to be interpreted as the flux linkages associated with the two symmetrical coils equivalencing the rotor winding of the asynchronous motor. $\boldsymbol{\phi}_{fk}$ is at the outset locally referenced. For the asynchronous motor it is conveniant to also have the flux state variables globally referenced. Thus we have definitionwise, see (1-41) : $\boldsymbol{\phi}_{fk} = \mathbf{T}\ \boldsymbol{\phi}_{fr}$, where $\boldsymbol{\phi}_{fr}$ is the flux vector globally referenced. Inserted into (1-79), we get the following electrical torque equation to investigate in view of all model simplifications made for the asynchronous motor relative to the synchronous one :

$$T_{(el)} = 0.5\ \mathbf{i}^t_{DQ}\ \mathbf{T_1}\ \mathbf{X''}\ \mathbf{T}\ \mathbf{i}_{DQ} + 0.5\ \mathbf{i}^t_{DQ}\ \mathbf{T_1}\ \mathbf{f}\ \mathbf{T}\boldsymbol{\phi}_{fr} \qquad (1\text{-}79)$$

X" now becomes symmetrical as $X''_d=X''_q$ ($=X'_M$). From this it follows that the first term of (1-79) becomes zero. **f** also becomes symmetrical as evidenced from (1-69). Inserting for $\mathbf{T_1}$,**f** and **T**, the following practical algorithm comes forth for determining the electrical motor torque $T_{(el)}$:

$$T_{(el)} = 0.5\ (X_m/X_r)\ (\bar{\mathbf{1}}\ \mathbf{i}_{DQ})^t\ \boldsymbol{\phi}_{fr} \qquad (1\text{-}80)$$

where;

$$\mathbf{i}_{DQ} = \begin{bmatrix} i_D \\ i_Q \end{bmatrix}$$ = Asynchronous motor current, global reference.

$$\boldsymbol{\phi}_{fr} = \begin{bmatrix} \phi_{fD} \\ \phi_{fQ} \end{bmatrix}$$ = Flux linkages associated with resp. rotor coils. Global reference

(Asynchronous motor state variables)

$$\bar{\mathbf{1}} = \begin{bmatrix} & 1 \\ -1 & \end{bmatrix}$$

X_m = magnetizing reactance
X_r = $X_{r\sigma} + X_m$
$X_{r\sigma}$ = rotor leakage reactance

See Figure 1.15 for more on data

Figure 1.17 Practical algorithm for computing asynchronous motor electrical torque $T_{(el)}$ in (1-78).

Pu mechanical torque $T_{(mec)}$ will take on different forms, depending on the operational regime; whether motor or generator mode of operation. In principle the situation is identical to that of the synchronous motor :

For motor operation (which per definition implies a positive sign of the mechanical torque), the following premises may in many cases prevail:

$T_{(mec)} = T_{(mec(o))}\ (\Omega/\Omega_{(o)})^{\kappa}$, where $T_{(mec(o))}$ and exponent κ depend on the 'rotational status' of the motor at $t = -0$; whether already up and running, or to be started from $\Omega = 0$: (1-81)

If the motor is up and running ;

$T_{(mec(o))} = T_{(el(o))} =$ *electrical* motor torque at $t = -0$. The proper value is found by applying equation (1-80) to data from the initial power system load flow.

$\kappa =$ exponent depending on the load torque's sensitivity to rotational speed for Ω not far from 1.0 . In many cases: $\kappa =$ (say) 1.5 – 4.5.

If the motor is to be started from stillstand ;

$T_{(mec(o))} =$ coefficient contributing to model the effect of mechanical friction, air resistance, etc, during the startup phase. Expected range: (say) 0.03 – 0.07

$\kappa =$ exponent that reflects speed dependency of $T_{(mec)}$. κ may change over the range $\Omega = 0 \rightarrow 1$ .Prospective area of variation: $\kappa =$ (say) 1 - 5.

For generator operation (which per definition implies a neg. sign of the mechanical torque):

$T_{(mec)} = (T_{(el(o))} + \Delta T_{(mec)})$, where $T_{(el(o))}$ is initial *electrical* motor torque, and $\Delta T_{(mec)}$ is given by the respons of the *(wind-)power turbine- and control system.* (1-82)

Figure 1.18 Example algorithms for computing asynchronous motor load torque $T_{(mec)}$ in (1-78)

Summary asynchronous motor model description followed by a simple/qualitative illustration of model application, is presented in Chapter 1.7.

1.6 Modelling of specal voltages in the d-q frame of reference

Two voltage aspects are ealt with; the transformation of the three phase voltage at some reference system bus, and the relationship between the voltage e_f of the model equations of Figure 1.6 and (1-43), and the equivalent pu voltage component of the machine's phasor diagram. The latter aspect is dealt with in a 'shortcut' manner in the following. For more detailed/general outline, see Appendix 2, p. A2/12- A2/17.

At the outset of network analysis the voltage phasor at some reference bus is often a declared quanity. In specific terms : Given the symmetrical three phase voltages

$$\mathbf{E}_{RST} = \sqrt{2}\, E_{eff} \begin{bmatrix} \cos(\alpha) \\ \cos(\alpha-2\pi/3) \\ \cos(\alpha-4\pi/3) \end{bmatrix} \qquad (1\text{-}83)$$

at some specified bus of the system. E_{eff} is the *root mean square* (*r.m.s.*) value of the three phase voltage. $\alpha = (\omega_b\, t + \gamma)$, where γ accounts for an arbitrary phase shift of the voltages relative to zero time. For convenience of final expressions – see (1-85) – we further define $\gamma = (\gamma_{ref} + \pi/2)$. This being the premise, *two* main modelling tasks are dealt with in the following: First on how the specification of $\mathbf{E}_{RST}$ is equivalenced in the d-q axis frame of reference. After that, on the related task of appropriately modelling the synchronous machine field voltage e_f contained in voltage vector $\mathbf{e}_{fk}$ of equation (1-43).

The transition of $\mathbf{E}_{RST}$ in (1-83) to $\mathbf{e}_{dqo}$ of the d-q-o axis frame of reference, is afforded by the Park transformation – see Appendix 3:

$$\mathbf{e}_{dqo} = \mathbf{P}\ \mathbf{E}_{RST} \qquad (1\text{-}84)$$

where;

$$\mathbf{P} = 2/3 \begin{array}{c} \quad R \qquad\qquad S \qquad\qquad T \\ \begin{bmatrix} \cos\theta & \cos(\theta-2\pi/3) & \cos(\theta-4\pi/3) \\ -\sin\theta & -\sin(\theta-2\pi/3) & -\sin(\theta-4\pi/3) \\ \tfrac{1}{2} & \tfrac{1}{2} & \tfrac{1}{2} \end{bmatrix} \end{array} \begin{array}{c} \\ d \\ q \\ o \end{array} \qquad (1\text{-}2)$$

In the present 'synchronous phasor' context $\theta = \omega_b\, t$. Evaluating the product in (1-84), the sought source voltage description in the d-q-o frame of reference is established:

$$e_{dqo} = \begin{bmatrix} e_d \\ e_q \\ e_o \end{bmatrix} = \sqrt{2}\, E_{eff} \cdot \begin{bmatrix} -\sin\gamma_{ref} \\ \cos\gamma_{ref} \\ 0 \end{bmatrix} \qquad (1\text{-}85)$$

d-q axis model of given voltage $\mathbf{E}_{RST}$ in the three phase frame of reference. See (1-83). E_{eff} = r.m.s. value of given symmetrical three phase voltage. γ_{ref} = arbitrary chosen phase shift. Often convenient choice: $\gamma_{ref} = 0$.

Simple examples of a specified voltage $\mathbf{e}_{dq0}$ in analysis, are given in the ensuing Chapter 1.7. The treatment of $\mathbf{e}_{dqo}$ in arbitrary complex power networks, is covered in systems modelling Chapter 2.

Next on model representation of the synchronous motor's field voltage e_f, see Figure 1.6 and (1-43): It is convenient to express this voltage as $e_f = K_f E_f = K_f (E_{f(o)} + \Delta E_f)$, where $E_{f(o)}$ is initial pu field voltage read from the machine's phasor diagram for the initial state, ΔE_f is additional voltage caused by the voltage control system, and K_f is a scaling factor to be determined next: In synchronous, idle operation with zero stator current the above equation becomes; $e_{f(o)} = K_f E_{f(o)} = K_f E_{eff(o)}$, where $E_{eff(o)}$ is initial pu machine voltage. The latter equation for the stated idle condition provides the key for determining K_f:

From (1-24): $\Psi_{d(o)} = L_d i_{d(o)} + L_{ad} i_{f(o)} + L_{ad} i_{kd(o)} = L_{ad} i_{f(o)}$ and $\Psi_{q(o)} = L_q i_{q(o)} + L_{aq} i_{kq(o)} = 0$ (1-86)
From Addendum of Chapter 1.4: $r_f = X_{ad}^2/(\omega_b T'_{do} (X_d - X'_d))$ (1-87)

From (1-32), (1-86), $e_{d(o)} = r_a \cdot i_{d(o)} + d\Psi_{d(o)}/dt - \omega_b \Psi_{q(o)} = 0$ and $e_{f(o)} = r_f i_{f(o)} + d\Psi_{f(o)}/dt = r_f \cdot i_{f(o)}$ (1-88)
(1-87) : $e_{q(o)} = r_a \cdot i_{q(o)} + d\Psi_{q(o)}/dt + \omega_b \Psi_{d(o)} = X_{ad} i_{f(o)} = (X_{ad}/r_f)\ e_{f(o)} = \omega_b T'_{do} (X_d - X'_d)\ e_{f(o)}/X_{ad}$

Observing (1-85) and (1-88) and applying the second of Kirchoff's laws – the voltage law – to our case of idle motor connected to infinite bus:

$$\mathbf{e}_{dqo} = \mathbf{e}_{dq(o)} \quad \rightarrow \quad e_{f(o)} = [\sqrt{2} \cos\gamma_{ref} X_{ad}/(\omega_b T'_{do} (X_d - X'_d))] \cdot E_{eff(o)} \qquad (1\text{-}89)$$

Here γ_{ref} =0. Then finally from (1-89):

$$K_f = \sqrt{2}\, X_{ad}/(\omega_b T'_{do} (X_d - X'_d)) \qquad (1\text{-}90)$$

Scaling factor K_f in $E_f = K_f (E_{f(o)} + \Delta E_{qf})$, see Figure 1.7

1.7 Component model summary

In the previous development of component models letters and letter combinations used to characterize variables and parameters were chosen in part to enhance understanding of the elaboration process as such. In the summary of component models that follows, new or altered notations may occur. They are all implemented to ease prospective practical use of the given stock of models. The model summary comprises:

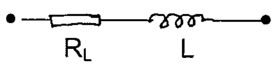
*) For development of an 'extended' machine model ; see Appendix 2. For summaru model description, see p. 1/33 – 1/35 .

The Lossy Inductor

Inductor behaviour is described in terms of 2 state variables:

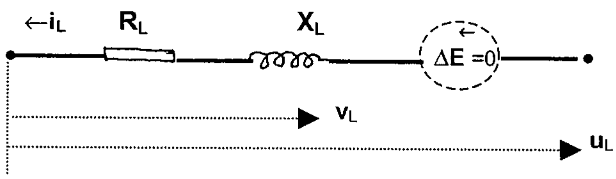
2 current variables $\mathbf{i}_L = [\ i_{L(d)}\ \ i_{L(q)}\ ]^t$ (where 't' stands for 'transpose')

Component parameters.
Per phase description :



R_L : Per phase resistance of inductor element. In Ω or pu
L : Per phase inductance of element. " " "
L defines inductive reactance $X_L = \omega_b L$

Electrical Circuit Model



↓

$$\mathbf{v}_L = \mathbf{R}_L\ \mathbf{i}_L + (1/\omega_b)\ \mathbf{X}_L\ d\mathbf{i}_L/dt \quad \& \quad \mathbf{v}_L = \mathbf{u}_L \qquad (1\text{-}91)$$

Electrical circuit model of *lossy inductor* in d-q axis frame of reference

where;

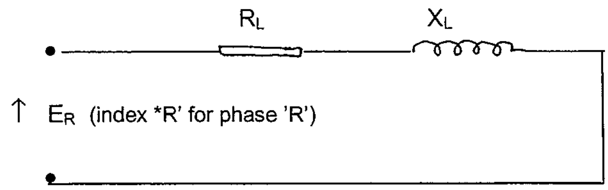
$$\mathbf{R}_L = \begin{bmatrix} R_L & -X_L \\ X_L & R_L \end{bmatrix} \qquad (1\text{-}92)$$

$$\mathbf{X}_L = \begin{bmatrix} \omega_b L & \\ & \omega_b L \end{bmatrix} = \begin{bmatrix} X_L & \\ & X_L \end{bmatrix} \qquad (1\text{-}93)$$

Example Lossy Inductor study

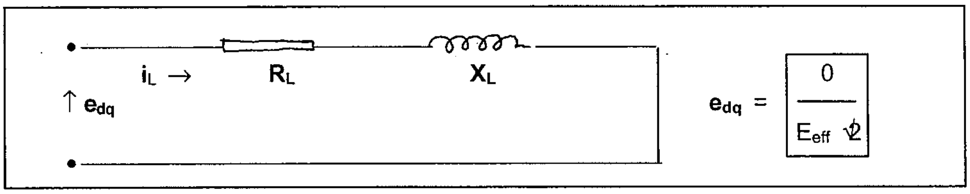
Analysis Task

Consider the simple case illustrated below : A lossy inductor is connected to an infinite bus of voltage $\mathbf{E}_{RST}$. $\mathbf{E}_{RST}$ is a symmetrical three phase voltage as defined in (1-83). Steady state conditions prevail. The current in phase 'R' is sought, following a three phase short circuit at the point of supply ($\mathbf{E}_{RST}$=0).



Lossy Inductor connected to infinite bus

Problem formulation in d-q axis frame of reference



System model in d-q axis frame of reference

$\mathbf{e}_{dq}$ is found from (1-85) with the conveniant choice $\gamma_{ref} = 0$. The above *system model* is based on the *electrical circuit model* (1-91) of the lossy inductor. Applying Kirchoff's voltage law to the system model we get by inspection:

$$\mathbf{e}_{dq} = \mathbf{R}_L\, \mathbf{i}_L + (1/\omega_b)\, \mathbf{X}_L\, d\mathbf{i}_L/dt \qquad (1\text{-}94)$$

which yields:

$$\underset{(2x1)}{d\mathbf{i}_L/dt} = \omega_b\, \mathbf{X}_L^{-1} (\mathbf{e}_{dq} - \mathbf{R}_L\, \mathbf{i}_L) \qquad (1\text{-}95)$$

where;

$\mathbf{i}_{L(o)}$ = initial current to be determined from *initial condition analysis*, see below

Problem solution is afforded in two main steps: 1) *Initial condition analysis*, and 2) *Transient performance analysis*:

1) Initial condition analysis. ($d\mathbf{i}_L/dt = 0$)
(1-95) solved w.r.t. $\mathbf{i}_L = \mathbf{i}_{L(o)}$:

$$\mathbf{i}_{L(o)} = \begin{bmatrix} i_{Ld(o)} \\ \hline i_{Lq(o)} \end{bmatrix} = \mathbf{R}^{-1}_L\, \mathbf{e}_{dq} \qquad \text{where} \qquad \mathbf{R}_L = \begin{bmatrix} R_L & -X_L \\ X_L & R_L \end{bmatrix}$$

2) Transient performance analysis
Solution found by simultaneous integration of equations (1-95) above, given initial value of the state variables $\mathbf{i}_{L(0)}$.

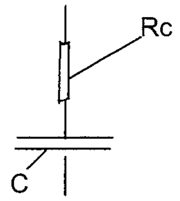
Current i_R of phase 'R' : $i_R = i_{L(d)} \cos\theta - i_{L(q)} \sin\theta$ where $\theta = (\omega_b t+\delta_o)$, and δ_o is an arbitrary chosen reference angle. Conveniant choice in many cases: $\delta_o = 0$.

The Lossy Capacitor Bank

Capacitor Bank behaviour is described in terms of 2 state variables:

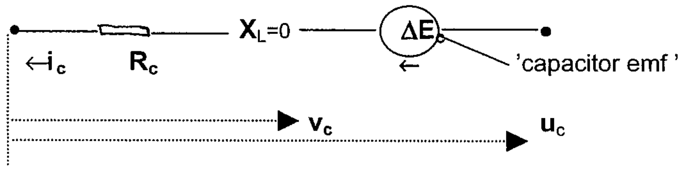
2 voltage variables $\mathbf{\Delta E_c} = [\, \Delta E_{c(d)} \;\; \Delta E_{c(q)} \,]^t$, which is the voltage across the capacitor

Component parameters.
Per phase description :



Rc : Per phase resistance of capacitor bank. In Ω or pu
C : Per phase capacitance of bank. " " "
C defines capacitive reactance $X_c = 1/(\omega_b\, C)$

Electrical Circuit Model



$$\mathbf{v_c} = \mathbf{R_c}\, \mathbf{i_c} \quad \& \quad \mathbf{v_c} - \mathbf{\Delta E_c} = \mathbf{u_c} \qquad (1\text{-}96)$$

Electrical circuit model of *lossy capacitor bank* in d-q axis frame of reference

where:

$$\mathbf{R_c} = \begin{bmatrix} R_c & \\ & R_c \end{bmatrix} \qquad (1\text{-}97)$$

Capacitor Voltage Model

$$\underset{(2x1)}{d\mathbf{\Delta E_c}}/dt = \omega_b\, (\mathbf{X_c}\, \mathbf{i_c} + \bar{\mathbf{1}}\, \mathbf{\Delta E_c}) \qquad (1\text{-}98)$$

with initial condition;

$$\mathbf{\Delta E_{c(o)}} = \bar{\mathbf{1}}\, \mathbf{X_c}\, \mathbf{i_{c(o)}} \qquad (1\text{-}99)$$

where;

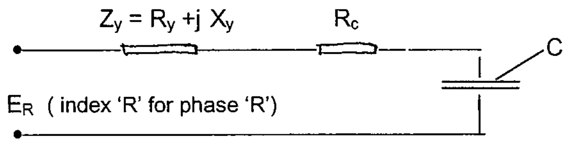
$$\mathbf{X_c} = \begin{bmatrix} 1/(\omega_b\, C) & \\ & 1/(\omega_b \cdot C) \end{bmatrix} = \begin{bmatrix} X_c & \\ & X_c \end{bmatrix} \qquad (1\text{-}100)$$

$$\bar{\mathbf{1}} = \begin{bmatrix} & 1 \\ -1 & \end{bmatrix} \qquad (1\text{-}101)$$

Example Capacitor + Inductor study

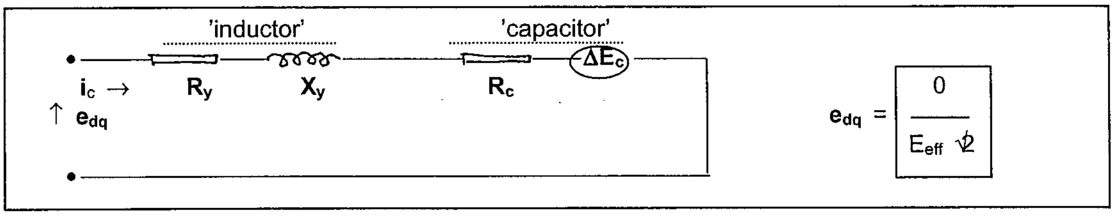
Analysis Task

Consider the simple case illustrated below: A lossy capacitor bank is connected to an infinite bus voltage $\mathbf{E}_{RST}$ via a series impedance. $\mathbf{E}_{RST}$ is a symmetrical three phase voltage as defined in (1-83). Steady state conditions prevail. The current in phase 'R' is sought, following a three phase short circuit at the point of supply ($\mathbf{E}_{RST} = 0$).



Capacitor bank connected to infinite bus via series impedance

Problem formulation in d-q axis frame of reference



System model in d-q axis frame of reference

$\mathbf{e}_{dq}$ is found from (1-85) with $\gamma_{ref} = 0$. The above system model is based on the *electrical circuit model* (1-96) of the lossy capacitor bank. Applying Kirchoff's voltage law to the model circuit,- as dealt with on a formal basis at the end of Appendix 1,- we have:

⊃:

$$\mathbf{e}_{dq} = \mathbf{R}_y \, \mathbf{i}_c + (1/\omega_b)\, \mathbf{X}_y \, d\mathbf{i}_c/dt + \mathbf{R}_c \, \mathbf{i}_c + \Delta\mathbf{E}_c \qquad (1\text{-}102)$$

(1-102) together with the developed capacitor voltage model (1-98), yield:

$$d\mathbf{i}_c/dt = \omega_b \, \mathbf{X}^{-1}_y \, [\, \mathbf{e}_{dq} - \Delta\mathbf{E}_c - (\mathbf{R}_y + \mathbf{R}_c)\, \mathbf{i}_c] \qquad (1\text{-}103)$$

$$d\Delta\mathbf{E}_c/dt = \omega_b \, [\, \mathbf{X}_c \, \mathbf{i}_c + \bar{\mathbf{1}} \, \Delta\mathbf{E}_c] \qquad (1\text{-}104)$$

where;

$\mathbf{i}_{c(o)}$ = initial current to be determined from _*initial condition analysis*, see below

$$\Delta\mathbf{E}_{c(o)} = \mathbf{1}\, \mathbf{X}_c \, \mathbf{i}_{c(0)} \qquad (1\text{-}105)$$

Problem solution is afforded in two main steps: 1) *Initial condition analysis*, and 2) *Transient performance analysis*:

1) **Initial condition analysis**. ($d\mathbf{i}_c/dt = d\Delta\mathbf{E}_c/dt = 0$)
   (1-105) inserted into (1-103) and solved w.r.t. $\mathbf{i}_{c(o)}$:

$$\mathbf{i}_{c(o)} = \mathbf{R}^{-1}_{syst} \cdot \mathbf{e}_{dq} \qquad \text{where} \qquad \mathbf{R}_{syst} = \begin{bmatrix} R_y + R_c & X_c - X_y \\ X_y - X_c & R_y + R_c \end{bmatrix}$$

2) **Transient performance analysis**
   Solution found by simultaneous integration of equations (1-103) and (1-104) above, given initial values of the state variables $\mathbf{i}_{c(o)}$ and $\Delta\mathbf{E}_{c(o)}$.

   Current i_R of phase 'R' : $i_R = i_{c(d)} \cos\theta - i_{c(q)} \sin\theta$ where $\theta = (\omega_b \, t + \delta_o)$, and δ_0 is an arbitrarely chosen reference angle. Conveniant choice in many cases : $\delta_0 = 0$.

The Synchronous Motor ('SM')

(based on the d-q diagram of a 5-coil generalised machine)

Synchronous motor/generator behaviour is 'per default' described in terms of 7 state variables :

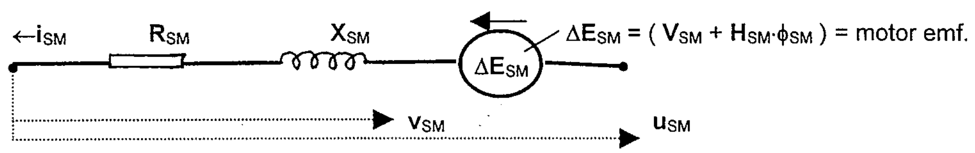
- 2 stator current components $\mathbf{i}_{SM} = [i_{SM(d)}\ i_{SM(q)}]^t$ (where 't' stands for 'transpose ')
- 3 rotor flux components $\phi_{SM} = [\phi_f\ \phi_{kd}\ \phi_{kq}]^t$ (fieldw. 'f' d-axis damperw. 'kd' q-axis damperw. 'kq')
- 1 speed variable $\Omega_{SM} = \omega_{SM}/\omega_o$
- 1 rotor angle variable β_{SM}

To handle *voltage control* 4 state variables are introduced. *Power control* in (hydro) generator mode of operation i: afforded via 3 additional state variables. Altogether (7+7) =14 state variables to model generator mode of operatior

Synchronous Motor parameters to be specified (with example hydro-generator data in parenthesis) :

$X_{a\sigma}$ (0.12pu)	X'_d (0.30pu)	R_a (0.005pu)	T''_q (0.16s)	$\cos\varphi_N$ (0.9pu)
X_d (1.2pu)	X''_d (0.20pu)	T'_{do} (6.0s))	T_a (5.0s)	S_N (100MVA)
X_q (0.75pu)	X''_q (0.30pu)	T''_d (0.04s)	C_D (7.5pu)	E_N (16kV)

Electrical Circuit Model



$$\mathbf{v}_{SM\,(2x1)} = \mathbf{R}_{SM}\cdot\mathbf{i}_{SM} + (1/\omega_o)\cdot\mathbf{X}_{SM}\cdot d\mathbf{i}_{SM}/dt \quad \& \quad \mathbf{v}_{SM} - \Delta\mathbf{E}_{SM} = \mathbf{u}_{SM} \qquad (1\text{-}106)$$

Electrical circuit model of the synchronous motor in d-q axis frame of reference

$$\Delta\mathbf{E}_{SM} = (\mathbf{V}_{SM} + \mathbf{H}_{SM}\cdot\phi_{SM}) = \text{synchronous motor emf.} \qquad (1\text{-}107)$$

$$\mathbf{R}_{SM} = \begin{bmatrix} (R_a + \hat{X}''_r) + (1+2\cdot\Delta\Omega_{SM})\cdot\bar{X}''\cdot\sin2\beta_{SM} + \bar{X}''_r\cdot\cos2\beta_{SM} & -\hat{X}'' + (1+2\cdot\Delta\Omega_{SM})\cdot\bar{X}''\cdot\cos2\beta_{SM} - \bar{X}''_r\cdot\sin2\beta_{SM} \\ \hat{X}'' + (1+2\cdot\Delta\Omega_{SM})\cdot\bar{X}''\cdot\cos2\beta_{SM} - \bar{X}''_r\cdot\sin2\beta_{SM} & (R_a + \hat{X}''_r) - (1+2\cdot\Delta\Omega_{SM})\cdot\bar{X}''\cdot\sin2\beta_{SM} - \bar{X}''_r\cdot\cos2\beta_{SM} \end{bmatrix} \qquad (1\text{-}108)$$

(columns D, Q; rows D, Q)

$$\mathbf{X}_{SM} = \begin{bmatrix} \hat{X}'' + \bar{X}''\cdot\cos2\beta_{SM} & -\bar{X}''\cdot\sin2\beta_{SM} \\ -\bar{X}''\cdot\sin2\beta_{SM} & \hat{X}'' - \bar{X}''\cdot\cos2\beta_{SM} \end{bmatrix} \qquad \mathbf{V}_{SM} = \begin{bmatrix} C_f\cdot E_f\cdot\cos\beta_{SM} \\ -C_f\cdot E_f\cdot\sin\beta_{SM} \end{bmatrix}$$

$\Delta\Omega_{SM} = (\Omega_{SM}-1)$ = rotor speed's deviation from synchronous value. In pu.

$E_f = (E_{fo} + \Delta E_f)$ =f ield voltage, where E_{fo} is initial value and ΔE_f is voltage control system response. See following p 1/29 .

$C_f = (\sqrt{2}/(\omega_o\cdot T'_{do}))\cdot(X''_{ad}/X'_{ad})$ (1-109)

$$\mathbf{H}_{SM} = \begin{bmatrix} \Omega_{SM}\cdot f_1\cdot\sin\beta_{SM} + f_2\cdot\cos\beta_{SM} & \Omega_{SM}\cdot f_3\cdot\sin\beta_{Sm} + f_4\cdot\cos\beta_{SM} & \Omega_{SM}\cdot f_5\cdot\cos\beta_{SM} + f_6\cdot\sin\beta_{SM} \\ \Omega_{SM}\cdot f_1\cdot\cos\beta_{SM} - f_2\cdot\sin\beta_{SM} & \Omega_{SM}\cdot f_3\cdot\cos\beta_{SM} - f_4\cdot\sin\beta_{SM} & -\Omega_{SM}\cdot f_5\cdot\sin\beta_{SM} + f_6\cdot\cos\beta_{SM} \end{bmatrix} \qquad (1\text{-}110)$$

(columns f, kd, kq)

$\hat{X}'' = 0.5(X''_d + X''_q)$ $\quad \hat{X}''_r = 0.5(X''_{rd} + X''_{rq})$ $\quad \leftarrow \quad X''_{rd} = (1/(\omega_o\cdot T'_{do}))\cdot(X''_{ad}/X'_{ad})^2\cdot(X_d - X'_d) + (1/(\omega_o\cdot T''_{do}))\cdot(X'_d - X''_d)$

$\bar{X}'' = 0.5(X''_d - X''_q)$ $\quad \bar{X}''_r = 0.5(X''_{rd} - X''_{rq})$ $\quad \leftarrow \quad X''_{rq} = (1/(\omega_o\cdot T''_{qo}))\cdot(X_q - X''_q)$ (1-111)

$f_1 = (X_d - X'_d)\cdot(X''_{ad}/(X_{ad}\cdot X'_{ad}))$ $\quad \leftarrow \quad X_{ad} = X_d - X_{a\sigma}$

$f_2 = f_1\cdot[(X'_d - X''_d)\cdot[(1/(\omega_o\cdot T''_{do}))\cdot(1/X''_{ad}) - (1/(\omega_o\cdot T'_{do}))\cdot(X_{ad}/X'^2_{ad})] - (1/(\omega_o\cdot T'_{do}))\cdot(X''_{ad}/X'_{ad})]$ $\quad \leftarrow \quad X'_{ad} = X'_d - X_{a\sigma}$

$f_3 = (X'_d - X''_d)/X'_{ad}$ $\quad \leftarrow \quad X''_{ad} = X''_d - X_{a\sigma}$

$f_4 = f_3\cdot[(1/(\omega_o\cdot T'_{do}))\cdot(X''_{ad}/X'^2_{ad})\cdot(X_d - X'_d) - (1/(\omega_o\cdot T''_{do})]$ (1-112)

$f_5 = -(X_q - X''_q)/X_{aq}$ $\quad \leftarrow \quad X_{aq} = X_q - X_{a\sigma}$

$f_6 = f_5\cdot(1/(\omega_o\cdot T''_{qo}))$ $\quad \leftarrow \quad X''_{aq} = X''_q - X_{a\sigma}$

Rotorflux Model

$$\underset{(3x1)}{d\phi_{SM}}/dt = \omega_o\cdot(\mathbf{e_{SMr}} + \mathbf{F_{SMi}}\cdot\mathbf{i_{SM}} + \mathbf{F_{SM\phi}}\cdot\phi_{SM}) \qquad (1\text{-}113)$$

Here:

$$\mathbf{F_{SMi}} = \begin{array}{|c|c|} \multicolumn{1}{c}{D} & \multicolumn{1}{c}{Q} \\ \hline (1/(\omega_o\cdot T'_{do}))\cdot(X_{ad}/X'_{ad})\cdot X''_{ad}\cdot\cos\beta_{SM} & -(1/(\omega_o\cdot T'_{do}))\cdot(X_{ad}/X'_{ad})\cdot X''_{ad}\cdot\sin\beta_{SM} \\ \hline (1/(\omega_o\cdot T''_{do}))\cdot X'_{ad}\cdot\cos\beta_{SM} & -(1/(\omega_o\cdot T''_{do}))\cdot X'_{ad}\cdot\sin\beta_{SM} \\ \hline (1/(\omega_o\cdot T''_{qo}))\cdot X'_{aq}\cdot\sin\beta_{SM} & (1/(\omega_o\cdot T''_{qo}))\cdot X'_{aq}\cdot\cos\beta_{SM} \\ \hline \end{array} \qquad \mathbf{e_{SMr}} = \begin{array}{|c|} \hline K_f\cdot E_f \\ \hline 0 \\ \hline 0 \\ \hline \end{array}$$

$E_f = (E_{fo} + \Delta E_f)$ = field voltage
$K_f = (\sqrt{2}/(\omega_o\cdot T'_{do}))\cdot X_{ad}/(X_d - X'_d)$
ΔE_f = voltage control response

(1-114)

$$\mathbf{F_{SM\phi}} = \begin{array}{|c|c|c|l} \multicolumn{1}{c}{f} & \multicolumn{1}{c}{kd} & \multicolumn{1}{c}{kq} & \\ \hline -(1/(\omega_o\cdot T'_{do}))\cdot(1/X'_{ad})\cdot[(X_{ad}/X'_{ad})\cdot(X'_d - X''_d) + X''_{ad}] & (1/(\omega_o\cdot T'_{do}))\cdot(X_{ad}/X'^2_{ad})\cdot(X'_d - X''_d) & & f \\ \hline (1/(\omega_o\cdot T''_{do}))\cdot(1/X_{ad})\cdot(X_d - X'_d) & -1/(\omega_o\cdot T''_{do}) & & kd \\ \hline & & -1/(\omega_o\cdot T''_{qo}) & kq \\ \hline \end{array}$$

At any time during integration the rotor currents may be derived from the equations $\phi_{SM} = \mathbf{X_{DQr}}\cdot\mathbf{i_{SM}} + \mathbf{X_{rr}}\cdot\mathbf{i_{SMr}}$:

$$\mathbf{I_{SMr}} = (\mathbf{X_{rr}})^{-1}\cdot[\,\phi_{SM} - \mathbf{X_{DQr}}\cdot\mathbf{i_{SM}}\,] \qquad (1\text{-}115)$$

where ;

$$\mathbf{X_{DQr}} = \begin{array}{|c|c|} \hline X_{ad}\cdot\cos\beta_{SM} & -X_{ad}\cdot\sin\beta_{SM} \\ \hline X_{ad}\cdot\cos\beta_{SM} & -X_{ad}\cdot\sin\beta_{SM} \\ \hline X_{aq}\cdot\sin\beta_{SM} & X_{aq}\cdot\cos\beta_{SM} \\ \hline \end{array} \qquad \mathbf{X_{rr}} = \begin{array}{|c|c|c|} \hline X^2_{ad}/(X_d - X'_d) & X_{ad} & \\ \hline X_{ad} & X_{ad} + X'_{ad}\cdot X''_{ad}/(X'_d - X''_d) & \\ \hline & & X^2_{aq}/(X_q - X''_q) \\ \hline \end{array}$$

Electromechanical Model

$$\underset{(1x1)}{d\Omega_{SM}}/dt = (S_{Bas}/S_{SM})\cdot(1/(T_a\cdot\cos\varphi_N))\cdot(T_{SMel} - T_{SMmec}) \qquad (1\text{-}116)$$

Here:

$T_{Smel} = 0.5\cdot\mathbf{i}^t_{SM}\cdot\mathbf{T_{SM1}}\cdot\phi_{dq}$ = electrical motor torque , - where $\phi_{dq} = \mathbf{X''_{SM}}\cdot\mathbf{T_{SM}}\cdot\mathbf{i_{SM}} + \mathbf{f_{SM}}\cdot\phi_{SM}$ (1-117)

$T_{SMmec} = T_{SMmec(o)}\cdot\Omega_{SM}{}^{\kappa}$ = mechanical torque in **motor** mode of operation. (Motor operation implies pos. sign of mech. torque)

If the motor is up and running at t=-0: $T_{SMmec(o)} = T_{SMel(o)}$ = electrical motor torque at t = -0. This is found from equation (1-117) applied to the initial power system load flow. κ= (say) 1.5-3.5
If the motor is to be started from stillstand (as e.g. an asynchronous motor) : $T_{SMmec(o)}$ = coefficient to model mechanical friction, air resistance, etc. during startup. Probable range: 0.02-0.05

$T_{SMmec} = (T_{SMel(o)} + \Delta T_{mec})$ = mechanical torque in **generator** mode of operation. ΔT_{mec} is the response from the *power control system.* See below for a sample hydro generator power control system.

S_{Bas}, S_{SM} = Chosen VA system power base, and rated VA motor capacity, respectively
T_a, $\cos\varphi_N$ = Dynamical system's inertia constant, and motor's rated power factor, respectively

$$\mathbf{T_{SM1}} = \begin{array}{|c|c|} \hline \sin\beta_{SM} & -\cos\beta_{SM} \\ \hline \cos\beta_{SM} & \sin\beta_{SM} \\ \hline \end{array} \quad \mathbf{T_{SM}} = \begin{array}{|c|c|} \hline \cos\beta_{SM} & -\sin\beta_{SM} \\ \hline \sin\beta_{SM} & \cos\beta_{SM} \\ \hline \end{array} \quad \mathbf{X''_{SM}} = \begin{array}{|c|c|} \hline X''_d & \\ \hline & X''_q \\ \hline \end{array} \quad \mathbf{f_{SM}} = \begin{array}{|c|c|c|} \hline f_1 & f_3 & \\ \hline & & -f_5 \\ \hline \end{array}$$

$f_1 = (X_d - X'_d)\cdot X''_{ad}/(X_{ad}\cdot X'_{ad})$
$f_3 = (X'_d - X''_d)/X'_{ad}$
$f_5 = -(X_q - X''_q)/X_{aq}$

(1-118)

The electrical angle of the rotor is defined as

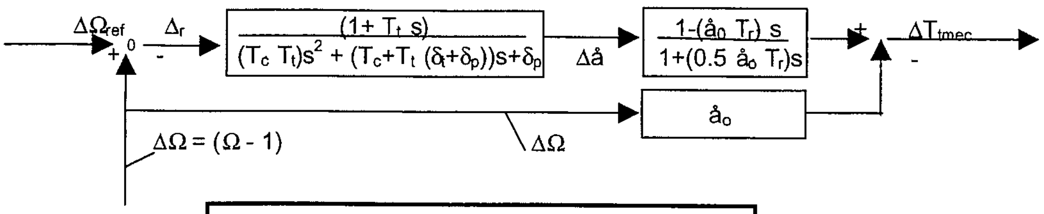
$$\beta_{SM} = (\omega_o\cdot t - \theta_{SM})$$

giving rise to the following differential equation describing the angular movement of the Synchronous Motor:

$$\underset{(1x1)}{d\beta_{SM}}/dt = \omega_o\cdot(1 - \Omega_{SM}) \qquad (1\text{-}119)$$

The Synchronous Motor, cont…

Power Control System Model for generator mode of operation (here: *hydro* generator as illustration.)



For 'normal' frequency control : $\Delta\Omega_{ref} = 0$.
For 'P-control' :
$\Delta\Omega_{ref} = c\,(P_{target} - P_m)$

$$d\Delta å/dt = K_1\,(\Delta\Omega_{ref} - (1 - \Omega_{SM}) + \Delta w) - K_2\,\Delta å$$ Regulator system

$$d\Delta w/dt = K_3\,\Delta å - K_4\,\Delta w$$ Regulator system

$$d\Delta g/dt = (3K_o/\Omega_{SM})\,\Delta å - K_ó\,\Delta g$$ Hydraulic system (1-120)

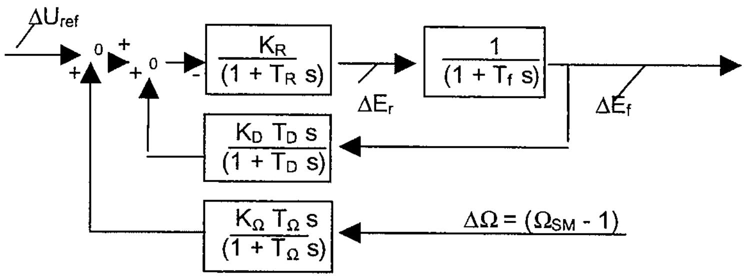
↓

$$\Delta T_{mec} = \Delta g - å_ó\,(1 - \Omega_{SM}) - (2/\Omega_{SM})\,\Delta å$$ Net change of mechanical torque

$K_o = 2/(å_o T_r)$
$K_1 = 1/T_c$
$K_2 = (\delta_p + \delta_t)/T_c$ ←
$K_3 = \delta_t/T_t$
$K_4 = 1/T_t$
P_m = absorbed motor power $= 0.5\, i_{SM}^t\, u_{SM}$
P_m is negative in generator mode.
P_{target} = target value of P_m
c = per unit scaling factor (eg.: c=0.1)

Control system parameters:
T_r : Time constant for hydraulic system (eg 0.3s)
T_c : Time constant for main servo (eg 0.08s)
T_t : Transient droop time constant (eg 17s)
δ_t : Transient droop (eg 0.15pu)
δ_p : Permanent droop (eg 0.0 – 0.03. The value 0.0 if frequency sustained by a single unit)
$å_o$: Initial pu turbine opening($t_å$). (if $t_å$<0.3 then $å_o$=0.3)

Voltage Control System Model



$E_f = (E_{qf(o)} + \Delta E_f)$
$E_{f(min)} \le E_f \le E_{f(max)}$
$\Delta U_{ref} = 0$, unless new voltage ref. is set

$$d\Delta E_f/dt = C_1\,(\Delta E_r - \Delta E_f)$$
$$d\Delta E_r/dt = C_2\,[\Delta U_{ref} + U_o - U + K_\Omega\,(\Omega_{SM} - 1) - \Delta h] - C_3\,\Delta E_f - C_4\,\Delta E_r + C_2\,\Delta E_{ss} \quad (1\text{-}121)$$
$$d\Delta E_{ss}/dt = C_5\,\Delta E_f - C_6\,E_{ss}$$
$$d\Delta h/dt = C_7\,(\Omega_{SM} - 1) - C_8\,\Delta h$$

$U = (1/\sqrt{2})\,(u_{SM}^2 + u_{SM}^2)^{0.5}$
$C_1 = 1/T_f$
$C_2 = K_R/T_R$
$C_3 = K_R K_o/T_R = K_D C_2$ ←
$C_4 = 1/T_R$
$C_5 = K_o/T_o$
$C_6 = 1/T_D$
$C_7 = K_\Omega/T_\Omega$
$C_8 = 1/T_\Omega$
$\Delta U = (U - U_o)$ = pu voltage deviation

Control system parameters:
T_f = field circuit time constant (eg 0.1s)
K_R = resulting forward amplification (eg 70pu)
T_R = regulator time constant (eg 0.1s)
K_D = transient feedback amplification (eg 0.25pu)
T_D = transient feedback time constant (eg 0.25s)
K_Ω = power stabilizer amplification (eg 1pu)
$E_{f(max)}$ = ceiling field voltage (eg 3pu)
$E_{f(min)}$ = bottom field voltage (eg –2pu)
T_Ω = power stabilizer time constant (eg 2s)

Example Synchronous Machine Study

Consider the simple case of a synchronous machine (to be) connected to an infinite bus of symmetrical voltage $\mathbf{E}_{RST}$. The given voltage transforms into $\mathbf{e}_{dq}$ in the d-q axis frame of reference, as shown by equation (1-85).

System analysis will often imply :

- Initial condition analysis
- System respons analysis in terms of *eigenvalue analysis,* and/or *time response analysis* subsequent to some additional excitation(s) to the system.

Initial condition analysis

The state variables $\Omega_{SM(o)}$ and $\beta_{SM(o)}$ together with the field voltage $E_{f(o)}$, have to be determined by special consideration of the task at hand. E.g. :

If the machine is to be started from stillstand, $\Omega_{SM(o)} = 0$ and so also all current and flux variables. $\beta_{SM(o)}$ can arbitrarely be set to zero. $E_{f(o)}$ may also be 0, if the field winding is kept short circuited during the initial part of the start-up sequence.

If the machine is initially in a steady state mode of operation, $\Omega_{SM(o)} =1$. In this case it is customary to specify initial conditions in terms of absorbed power $P_{SM(o)}$ and voltage U_o at the machine terminals. Thus $\beta_{SM(o)}$ and $E_{f(o)}$ should be specified so as to contribute to fulfilling these conditions. This is afforded either by determining $\beta_{SM(o)}$ and $E_{f(0)}$ from an initial state phasor diagram, or by an iterative solution process in which $\beta_{SM(o)}$ and $E_{f(o)}$ are simultaneously corrected until stated initial conditions are reached to required accuracy. The latter computational scheme is as follows:

Given $\Omega_{SM(o)} =1$ and tentative values of $\beta_{SM(o)}$ and $E_{f(o)}$. Initial values $i_{SM(o)}$ and $\phi_{SM(0)}$ are found by observing the constraints implied by Kirchoff's voltage law as e.g. expressed by equation (A-19) of Appendix - and the machine flux constraints of (1-113). In the present steady state context, derivative terms are set to zero. The equations become:

$$\mathbf{R}_{SM}\cdot \mathbf{i}_{SM(o)} + \mathbf{H}_{SM(o)}\cdot \boldsymbol{\phi}_{SM(o)} = (\mathbf{e}_{dq} - \mathbf{V}_{SM(o)}) \qquad \text{(from (1-106), (1-85) \& (A-19))}$$

$$\mathbf{F}_{SMi}\cdot \mathbf{i}_{SM(o)} + \mathbf{F}_{SM\phi(o)}\cdot \boldsymbol{\phi}_{SM(o)} = -\mathbf{e}_{SMr(o)} \qquad \text{(from (1-113))}$$

In matrix form:

$$\begin{bmatrix} \mathbf{R}_{SM} & \mathbf{H}_{SM(o)} \\ \mathbf{F}_{SMi} & \mathbf{F}_{SM\phi(o)} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{SM(o)} \\ \boldsymbol{\phi}_{SM(o)} \end{bmatrix} = \begin{bmatrix} (\mathbf{e}_{dq} - \mathbf{V}_{SM(o)}) \\ -\mathbf{e}_{SMr(o)} \end{bmatrix} \qquad (1\text{-}122)$$

from which the sought initial vectors $\mathbf{i}_{SM(o)}$ and $\boldsymbol{\phi}_{SM(o)}$ are found. If the initial conditions implied by present solution are ok, the desired solution has been found. If not ok, $\beta_{SM(o)}$ and $E_{f(0)}$ are adjusted appropriately , and a new and improved solution is found from (1-122). This process continues until 'ok' provides for exit. For fuller algorithmic details, see Chapter 2.4.

All power- and voltage control variables are defined in terms of incremental quantities. Their initial values are therefore zero.

Eigenvalue- and/or time response analysis

Given initial condition for the system state variables, which in this simple 'motor alone' study are the 7 synchronous motor state variables plus the 7 control system state variables. The subsequent dynamic behaviour of the system is governed by the aggregate of alltogether 14 first order, ordinary differential equations, as laborated above for the synchronous motor unit in a hydro generator setting, See below. The linearization and processing of

$$\begin{aligned}
d\mathbf{i}_{SM}/dt &= \omega_b\,(\mathbf{X}_{SM})^{-1}\,(\mathbf{e}_{dq} - \Delta\mathbf{E}_{SM} - \mathbf{R}_{SM}\,\mathbf{i}_{SM}) && \text{(from (1-106) \& Kirchhoff)}\\
d\boldsymbol{\phi}_{SM}/dt &= \omega_b\,(\mathbf{e}_{SMr} + \mathbf{F}_{SMi}\,\mathbf{i}_{SM} + \mathbf{F}_{SM\phi}\,\boldsymbol{\phi}_{SM}) && \text{(from (1-113))}\\
d\Omega_{SM}/dt &= (S_{Bas}/S_{SM})\,(1/(T_{å}\cos\varphi_N))\,(T_{SMel} - T_{SMmec}) && \text{(from (1-116))}\\
d\beta_{SM}/dt &= \omega_b\,(1-\Omega_{SM}) && \text{(from (1-119))}\\
\\
d\Delta å/dt &= K_1\,(\Delta\Omega_{ref} - (1-\Omega_{SM}) + \Delta w) - K_2\,\Delta å\\
d\Delta w/dt &= K_3\,\Delta å - K_4\,\Delta w \quad \rightarrow \quad \Delta T_{mec} = \Delta g - å_o\,(1 - \Omega_{SM}) - (2/\Omega_{SM})\,\Delta å && \text{(from (1-120))}\\
d\Delta g/dt &= (3K_o/\Omega_{SM})\,\Delta å - K_ö\,\Delta g\\
\\
d\Delta E_f/dt &= C_1\,(\Delta E_r - \Delta E_f)\\
d\Delta E_r/dt &= C_2\,[\Delta U_{ref} + U_o - U + K_\Omega\,(\Omega_{SM} - 1) - \Delta h] - C_3\,\Delta E_f - C_4\,\Delta E_r + C_2\,\Delta E_{ss} && \text{(from (1-121))}\\
d\Delta E_{ss}/dt &= C_5\,\Delta E_f - C_6\,E_{ss}\\
d\Delta h/dt &= C_7\,(\Omega_{SM} - 1) - C_8\,\Delta h
\end{aligned}$$

such equations for eigenvalue analysis, is found in Chapter 3.Time respons analyses are dealt with in Chapter 4.

The 'Extended' Synchronous Motor

(based on the d-q diagram of a 6-coil generalised machine)

Synchronous motor/generator behaviour is described in terms of 8 state variables :

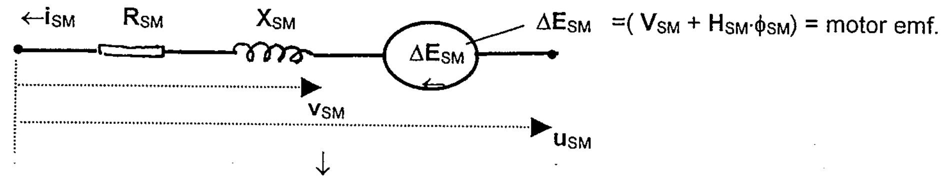
- 2 stator current components $\mathbf{i}_{SM} = [i_{SM(d)}\ i_{SM(q)}]^t$ (where 't' stands for 'transpose ')
- 4 rotor flux components $\phi_{SM} = [\phi_f\ \phi_{fq}\ \phi_{kd}\ \phi_{kq}]^t$ (fieldw.'f' fieldw.'q damperw. 'kd' damperw. 'kq')
- 1 speed variable $\Omega_{SM} = \omega_{SM}/\omega_o$
- 1 rotor angle variable β_{SM}

To handle *voltage control* 4 state variables are introduced. *Power control* in (hydro) generator mode of operation is afforded via 3 additional state variables. Altogether (8+7) =15 state variables to model generator mode of operation:

Synchronous Motor parameters to be specified (with example adjustable speed machine data in parenthesis) :

$X_{a\sigma}$ (0.1pu)	X'_d (0.35pu)	R_a (0.008pu)	T''_q (0.04s)	$\cos\varphi_N$ (0.9pu)
X_d (1.4pu)	X''_d (0.22pu)	T'_{do} (1.0s))	T'_{qo} (1.0s))	T_a (10.0s)
X_q (1.4pu)	X''_q (0.22pu)	X'_q (0.35pu)	T''_d (0.04s)	C_D (12pu)

Electrical Circuit Model



$$\mathbf{v}_{SM\,(2x1)} = \mathbf{R}_{SM}\cdot\mathbf{i}_{SM} + (1/\omega_o)\cdot\mathbf{X}_{SM}\cdot d\mathbf{i}_{SM}/dt \quad \& \quad \mathbf{v}_{SM} - \Delta\mathbf{E}_{SM} = \mathbf{u}_{SM} \qquad \text{(A2-35)}$$

Electrical circuit model of the extended synchronous motor in d-q axis frame of reference

$\Delta\mathbf{E}_{SM} = (\mathbf{V}_{SM} + \mathbf{H}_{SM}\cdot\phi_{SM})$ = synchronous motor emf.

$$\mathbf{R}_{SM} = \begin{bmatrix} (R_a+\hat{X}''_r) + (\Omega_{SMref}+2\cdot\Delta\Omega_{SM})\cdot\bar{X}''\cdot\sin2\beta_{SM}+\bar{X}''_r\cdot\cos2\beta_{SM} & -\Omega_{SMref}\cdot\hat{X}''+(\Omega_{SMref}+2\cdot\Delta\Omega_{SM})\cdot\bar{X}''\cdot\cos2\beta_{SM} - \bar{X}''_r\cdot\sin2\beta_{SM} \\ \Omega_{SMref}\cdot\hat{X}''+(\Omega_{SMref}+2\cdot\Delta\Omega_{SM})\cdot\bar{X}''\cdot\cos2\beta_{SM} - \bar{X}''_r\cdot\sin2\beta_{SM} & (R_a+\hat{X}''_r) - (\Omega_{SMref}+2\cdot\Delta\Omega_{SM})\cdot\bar{X}''\cdot\sin2\beta_{SM} - \bar{X}''_r\cdot\cos2\beta_{SM} \end{bmatrix}$$

$\Omega_{SMref} = (1-\Omega_f)$, where Ω_f = pu angular speed of rotor mmf relative to rotor. See p.A2/11

$\Delta\Omega_{SM} = (\Omega_{SM} - \Omega_{SMref})$, where Ω_{SM} =pu rotor speed

$$\mathbf{X}_{SM} = \begin{bmatrix} \hat{X}''+\bar{X}''\cdot\cos2\beta_{SM} & -\bar{X}''\cdot\sin2\beta_{SM} \\ -\bar{X}''\cdot\sin2\beta_{SM} & \hat{X}''-\bar{X}''\cdot\cos2\beta_{SM} \end{bmatrix} \qquad \mathbf{V}_{SM} = \begin{bmatrix} C_f\cdot\bar{E}_f\cdot\cos(\beta_{SM}-\beta_f) \\ -C_f\cdot\bar{E}_f\cdot\sin(\beta_{SM}-\beta_f) \end{bmatrix}$$

$\bar{E}_f = (\sqrt{2}\cdot E_{f\text{-}eff} + \Delta E_f)$ = peak field excitation.

ΔE_f = voltage regulator response. See p.1/29.

$C_f = (\sqrt{2}/(\omega_o\cdot T'_{do}\cdot\varepsilon_f))\cdot(X''_{ad}/X'_{ad})$. See p.A2/8.

β_{SM} = synchronous motor angle, see Fig.A2-3.

β_f = phase shift of magnetizing ac voltage. See Figure A2-3.

	f	fq	kd	kq	
$\mathbf{H}_{SM}$ =	$(\Omega_{SM}\cdot f_1 - f_{pu}\cdot f_7)\cdot\sin\beta_{SM}+f_2\cdot\cos\beta_{SM}$	$(\Omega_{SM}\cdot f_7 - f_{pu}\cdot f_1)\cdot\cos\beta_{SM}+f_8\cdot\sin\beta_{SM}$	$\Omega_{SM}\cdot f_3\cdot\sin\beta_{SM}+f_4\cdot\cos\beta_{SM}$	$\Omega_{SM}\cdot f_5\cdot\cos\beta_{SM}+f_6\cdot\sin\beta_{SM}$	D
	$(\Omega_{SM}\cdot f_1 - f_{pu}\cdot f_7)\cdot\cos\beta_{SM}-f_2\cdot\sin\beta_{SM}$	$-(\Omega_{SM}\cdot f_7 - f_{pu}\cdot f_1)\cdot\sin\beta_{SM}+f_8\cdot\cos\beta_{SM}$	$\Omega_{SM}\cdot f_3\cdot\cos\beta_{SM}-f_4\cdot\sin\beta_{SM}$	$-\Omega_{SM}\cdot f_5\cdot\sin\beta_{SM}+f_6\cdot\cos\beta_{SM}$	Q

$\hat{X}'' = 0.5(X''_d + X''_q)$ $\quad \hat{X}''_r = 0.5(X''_{rd} + X''_{rq})$ $\quad \leftarrow \quad X''_{rd} = (1/(\omega_o\cdot T'_{do}))\cdot(X''_{ad}/X'_{ad})^2\cdot(X_d - X'_d) + (1/(\omega_o\cdot T''_{do}))\cdot(X'_d - X''_d)$

$\bar{X}'' = 0.5(X''_d - X''_q)$ $\quad \bar{X}''_r = 0.5(X''_{rd} - X''_{rq})$ $\quad \leftarrow \quad X''_{rq} = (1/(\omega_o\cdot T'_{do}))\cdot(X''_{aq}/X'_{aq})^2\cdot(X_q - X'_q) + (1/(\omega_o\cdot T''_{qo}))\cdot(X'_q - X''_q)$

f_{pu} = shortened notation for $f_{rotor(pu)}$ = pu frequency of 3-phase voltage applied to field winding. Not subject to sign shift.

$f_1 = (X_d - X'_d)\cdot(X''_{ad}/(X_{ad}\cdot X'_{ad}))$ $\quad \leftarrow \quad X_{ad} = X_d - X_{a\sigma}$

$f_2 = f_1\cdot[(X'_d - X''_d)\cdot[(1/(\omega_o\cdot T''_{do}))\cdot(1/X''_{ad}) - (1/(\omega_o\cdot T'_{do}))\cdot(X_{ad}/X'^2_{ad})] - (1/(\omega_o\cdot T'_{do}))\cdot(X''_{ad}/X'_{ad})]$ $\quad \leftarrow \quad X'_{ad} = X'_d - X_{a\sigma}$

$f_3 = (X'_d - X''_d)/X'_{ad}$ $\quad \leftarrow \quad X''_{ad} = X''_d - X_{a\sigma}$

$f_4 = f_3\cdot[(1/(\omega_o\cdot T'_{do}))\cdot(X''_{ad}/X'^2_{ad})\cdot(X_d - X'_d) - (1/(\omega_o\cdot T''_{do})]$

$f_5 = -(X'_q - X''_q)/X'_{aq}$ $\quad \leftarrow \quad X_{aq} = X_q - X_{a\sigma}$

$f_6 = -f_5\cdot[(1/(\omega_o\cdot T'_{qo}))\cdot(X''_{aq}/X'^2_{aq})\cdot(X_q - X'_q) - (1/(\omega_o\cdot T''_{qo})]$ $\quad \leftarrow \quad X'_{aq} = X'_q - X_{a\sigma}$

$f_7 = -(X_q - X'_q)\cdot(X''_{aq}/(X_{aq}\cdot X'_{aq}))$ $\quad \leftarrow \quad X''_{aq} = X''_q - X_{a\sigma}$

$f_8 = -f_7\cdot[(X'_q - X''_q)\cdot[(1/(\omega_o\cdot T''_{qo}))\cdot(1/X''_{aq}) - (1/(\omega_o\cdot T'_{qo}))\cdot(X_{aq}/X'^2_{aq})] - (1/(\omega_o\cdot T'_{qo}))\cdot(X''_{aq}/X'_{aq})]$

Rotor Flux Model

$$d\phi_{SM}/dt = \omega_0 \cdot (\mathbf{e}_{SMr} + \mathbf{F}_{SMi} \cdot \mathbf{i}_{SM} + \mathbf{F}_{SM\phi} \cdot \phi_{SM}) \quad \text{(4x1)}$$ (A2-94)

where:

$\mathbf{e}_{SMr} =$

$K_f \cdot E_f$	f
$K_{fq} \cdot E_{fq}$	fq
0	kd
0	kq

$E_f = (\sqrt{2} \cdot E_{f\text{-}eff(o)} + \Delta E_f) \cdot \cos\beta_f$ = voltage of field coil 'f'. See page A2/14 - 15

$E_{fq} = (\sqrt{2} \cdot E_{f\text{-}eff(o)} + \Delta E_f) \cdot \sin\beta_f$ = voltage of field coil 'fq'. See page A2/14 -15

$K_f = [(\sqrt{2}/(\omega_o \cdot T'_{do} \cdot \varepsilon_f)) \cdot X_{ad}/(X_d - X'_d)]$

$K_{fq} = [(\sqrt{2}/(\omega_o \cdot T'_{qo} \cdot \varepsilon_{fq})) \cdot X_{aq}/(X_q - X'_q)]$

$(\varepsilon_f, \varepsilon_{fq})$ = factors =1, unless adjusted speed SM.

} For the adjustable speed SM (ie. the symmetrical machine): $K_f = K_{fq}$ & $\varepsilon_f = \varepsilon_{fq}$, see p. A2/15.

ΔE_f = voltage control response. (Voltage *phase* not a control variable here).

$X_{ad} = X_d - X_{a\sigma}$
$X'_{ad} = X'_d - X_{a\sigma}$
$X''_{ad} = X''_d - X_{a\sigma}$
$X_{aq} = X_q - X_{a\sigma}$
$X'_{aq} = X'_q - X_{a\sigma}$
$X''_{aq} = X''_q - X_{a\sigma}$

$\mathbf{F}_{SMi} =$

D	Q	
$(1/(\omega_o \cdot T'_{do})) \cdot (X_{ad}/X'_{ad}) \cdot X''_{ad} \cdot \cos\beta$	$-(1/(\omega_o \cdot T'_{do})) \cdot (X_{ad}/X'_{ad}) \cdot X''_{ad} \cdot \sin\beta$	f
$(1/(\omega_o \cdot T'_{qo})) \cdot (X_{aq}/X'_{aq}) \cdot X''_{aq} \cdot \sin\beta$	$(1/(\omega_o \cdot T'_{qo})) \cdot (X_{aq}/X'_{aq}) \cdot X''_{aq} \cdot \cos\beta$	fq
$(1/(\omega_o \cdot T''_{do})) \cdot X'_{ad} \cdot \cos\beta$	$-(1/(\omega_o \cdot T''_{do})) \cdot X'_{ad} \cdot \sin\beta$	kd
$(1/(\omega_o \cdot T''_{qo})) \cdot X'_{aq} \cdot \sin\beta$	$(1/(\omega_o \cdot T''_{qo})) \cdot X'_{aq} \cdot \cos\beta$	kq

$\mathbf{F}_{SM\phi} = (1/\omega_o) \cdot$

f	fq	kd	kq
$F_{fk\phi}(f,f)$	$-f_{rotor(pu)}$	$(X_{ad}/X'^{2}_{ad}) \cdot (X'_d - X''_d)/T'_{do}$	
$f_{rotor(pu)}$	$F_{fk\phi}(fq,fq)$		$(X_{aq}/X'^{2}_{aq}) \cdot (X'_q - X''_q)/T'_{qo}$
$(1/T''_{do}) \cdot (1/X_{ad}) \cdot (X_d - X'_d)$		$-1/T''_{do}$	
	$(1/T''_{qo}) \cdot (1/X_{aq}) \cdot (X_q - X'_q)$		$-1/T''_{qo}$

$F_{fk\phi}(f,f) = -(1//(T'_{do} \cdot X'_{ad})) \cdot [(X_{ad}/X'_{ad}) \cdot (X'_d - X''_d) + X''_{ad}]$

$F_{fk\phi}(fq,fq) = -(1//(T'_{qo} \cdot X'_{aq})) \cdot [(X_{aq}/X'_{aq}) \cdot (X'_q - X''_q) + X''_{aq}]$

β = angular displacement of the local machine reference axes relative to the global axes
β_f = specified phase shift (relative to local axes) of applied three phase field voltage.
$f_{rotor(pu)}$ = pu frequency of applied 3-phase rotor voltage. (Base frequency: 50Hz. Not subject to sign change)

X_d, X'_d, X''_d : direct-axis synchronous, transient and subtransient reactance (pu)
X_q, X'_q, X''_q : quadrature-axis synchronous, transient and subtransient reactance (pu)
$X_{a\sigma}$: stator leakage reactance (pu)
T'_{do}, T''_{do} : direct axis open stator transient and subtransient time constant (s)
T'_{qo}, T''_{qo} : quadrature axis open stator transient and subtransient time constant (s)

Model application alternatives:
- If adjustable speed SM: Symmetrical machine; $X_d = X_q$, $X'_d = X'_q$, $X''_d = X''_q$, $T'_{do} = T'_{qo}$, $T''_{do} = T''_{qo}$, $K_f = K_{fq}$. β_f to be set.
- If 'traditional' SM : Individual parameter setting. $\beta_f = 0$. $f_f = 0$ (i.e. dc to the field circuit)
- If 'traditional' AM : Symmetrical machine. No field voltage excitation : $E_f = E_{fq} = 0$. $f_f = 0$. No P&U-control.

At any time during integration the rotor currents may be derived from (A2-19) :

$$\mathbf{i}_{SMr} = (\mathbf{X}_{rr})^{-1} \cdot (\phi_{SM} - \mathbf{X}_{DQr} \cdot \mathbf{i}_{SM})$$ (A2-95)

where:

$\mathbf{X}_{rr} =$

f	fq	kd	kq	
$X^2_{ad}/(X_d - X'_d)$		X_{ad}		f
	$X^2_{aq}(X_q - X'_q)$		X_{aq}	fq
X_{ad}		$X_{ad} + X'_{ad} \cdot X''_{ad}/(X'_d - X''_d)$		kd
	X_{aq}		$X_{aq} + X'_{aq} \cdot X''_{aq}/(X'_q - X''_q)$	kq

$\mathbf{X}_{DQr} = \mathbf{X}_{(fk)(dq)} \cdot \mathbf{T} =$

D	Q	
$X_{ad} \cdot \cos\beta$	$-X_{ad} \cdot \sin\beta$	f
$X_{aq} \cdot \sin\beta$	$X_{aq} \cdot \cos\beta$	fq
$X_{ad} \cdot \cos\beta$	$-X_{ad} \cdot \sin\beta$	kd
$X_{aq} \cdot \sin\beta$	$X_{aq} \cdot \cos\beta$	kq

The 'Extended' Synchronous Motor, cont...

Electromechanical Model

$$\underset{(1x1)}{d\Omega_{SM}/dt} = (S_{Bas}/S_{SM})\cdot(1/(T_a\cdot\cos\varphi_N))\cdot(T_{SMel} - T_{SMmec}) \qquad \text{(A2-96)}$$

Here:

T_{Smel} = $0.5\cdot i^t_{SM}\cdot T_{SM1}\cdot\phi_{dq}$ = electrical motor torque , - where $\phi_{dq} = X''_{SM}\cdot T_{SM}\cdot i_{SM} + f_{SM}\cdot\phi_{SM}$ (A2-97)

T_{SMmec} = $T_{SMmec(o)}\cdot\Omega_{SM}^{\kappa}$ = mechanical torque in **motor** mode of operation. (Motor operation implies pos. sign of mech. torque)

If the motor is up and running at t=-0: $T_{SMmec(o)} = T_{SMel(o)}$ = electrical motor torque at t = -0. This is found from equation (1-117) applied to the initial power system load flow. κ= (say) 1.5-3.5
If the motor is to be started from stillstand (as e.g. an asynchronous motor) : $T_{SMmec(o)}$ = coefficient to model mechanical friction, air resistance, etc. during startup. Probable range: 0.02-0.05

T_{SMmec} = $(T_{SMel(o)} + \Delta T_{mec})$ = mechanical torque in **generator** mode of operation. ΔT_{mec} is the response from the *power control system*. See below for a sample hydro generator power control system.

S_{Bas}, S_{SM} = Chosen VA system power base, and rated VA motor capacity, respectively
T_a, $\cos\varphi_N$ = Dynamical system's inertia constant, and motor's rated power factor, respectively

$$T_{SM1} = \begin{bmatrix} \sin\beta_{SM} & -\cos\beta_{SM} \\ \cos\beta_{SM} & \sin\beta_{SM} \end{bmatrix} \quad T_{SM} = \begin{bmatrix} \cos\beta_{SM} & -\sin\beta_{SM} \\ \sin\beta_{SM} & \cos\beta_{SM} \end{bmatrix} \quad X''_{SM} = \begin{bmatrix} X''_d & \\ & X''_q \end{bmatrix} \quad f_{SM} = \begin{bmatrix} f_1 & & f_3 & \\ & -f_7 & & -f_5 \end{bmatrix}$$

$f_1 = (X_d - X'_d)\cdot X''_{ad}/(X_{ad}\cdot X'_{ad})$
$f_3 = (X'_d - X''_d)/X'_{ad}$
$f_5 = -(X'_q - X''_q)/X'_{aq}$
$f_7 = -(X_q - X'_q)\cdot X''_{aq}/(X_{aq}\cdot X'_{aq})$
(A2-98)

The electrical angle of the rotor is defined as, see equation (A2-43);

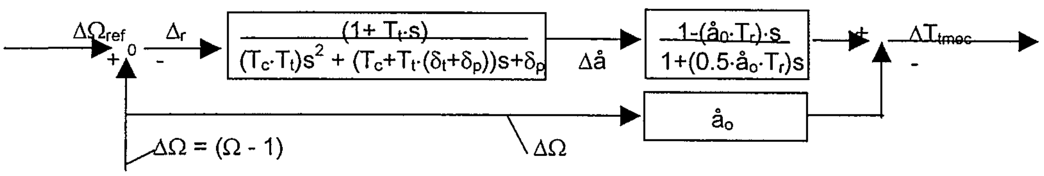
$$\beta_{SM} = (\omega_o\cdot t - (\theta_{SM} + \theta_f)$$

giving rise to the following differential equation describing the angular movement of the 'Extended' Synchronous Motor:

$$\underset{(1x1)}{d\beta_{SM}/dt} = \omega_o\cdot(1 - \Omega_f - \Omega_{SM}) \qquad \text{(A2-99)}$$

where; Ω_{SM} = pu angular speed of rotor
$\Omega_f = f_f/f_o$ =pu angular speed of rotor m.m.f. relative to rotor. See p. A2/11.

Power Control System Model for generator mode of operation (here: *hydro* generator as illustration.)



For 'normal' frequency control : $\Delta\Omega_{ref} = \Omega_f$.
For 'P-control' :
$\Delta\Omega_{ref} = c\cdot(P_{target} - P_m) + \Omega_f$

$$\begin{aligned} d\Delta å/dt &= K_1\cdot(\Delta\Omega_{ref} - (1 - \Omega_{SM}) + \Delta w) - K_2\cdot\Delta å && \text{Regulator system} \\ d\Delta w/dt &= K_3\cdot\Delta å - K_4\cdot\Delta w && \text{Regulator system} \\ d\Delta g/dt &= (3K_o/\Omega_{SM})\cdot\Delta å - K_o\cdot\Delta g && \text{Hydraulic system} \qquad \text{(A2-100)} \\ &\downarrow \\ \Delta T_{mec} &= \Delta g - å_o\cdot(1 - \Omega_f - \Omega_{SM}) - (2/\Omega_{SM})\cdot\Delta å && \text{Net change of mechanical torque} \end{aligned}$$

$K_o = 2/(å_o\cdot T_r)$
$K_1 = 1/T_c$
$K_2 = (\delta_p + \delta_t)/T_c$ ←
$K_3 = \delta_t/T_t$
$K_4 = 1/T_t$
P_m = absorbed motor power $=0.5\cdot i_{SM}^t\cdot u_{SM}$
P_m is negative in generator mode.
P_{target} = target value of P_m
c = per unit scaling factor (eg.: c=0.1)

Control system parameters:

T_r : Time constant for hydraulic system (eg 0.3s)
T_c : Time constant for main servo (eg 0.08s)
T_t : Transient droop time constant (eg 17s)
δ_t : Transient droop (eg 0.15pu)
δ_p : Permanent droop (eg 0.0 – 0.03. The value 0.0 if frequency sustained by a single unit)
$å_o$: Initial pu turbine opening ($t_å$). (if $t_å$ <0.3 then $å_o$=0.3)

Voltage Control System Model → See chosen example system on p. 1/29

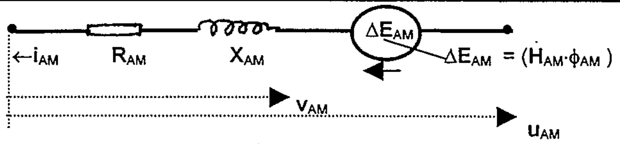
The Asynchronous Motor ('AM')

Asynchronous motor/generator behaviour is described in terms of 5 state variables:

2 stator current components $\mathbf{i}_{AM} = [i_{AM(d)}\ i_{AM(q)}]^t$ (where 't' stands for 'transpose ')
2 rotor flux components $\phi_{AM} = [\phi_{r(d)}\ \phi_{r(q)}]^t$ (1-123)
1 speed variable $\Omega_{AM} = \omega_{AM}/\omega_o$

Motor parameters: Electrical data:($X_{a\sigma}$, $X_{r\sigma}$, X_m, X_r) , see parameters in bold type below. Inertia const.T_a & friction torque, see below

Electrical Circuit Model



$$\mathbf{v}_{AM} = \mathbf{R}_{AM}\cdot\mathbf{i}_{AM} + (1/\omega_o)\cdot\mathbf{X}_{AM}\cdot d\mathbf{i}_{AM}/dt \quad \& \quad \mathbf{v}_{AM} - \Delta\mathbf{E}_{AM} = \mathbf{u}_{AM} \qquad (1\text{-}124)$$

Electrical circuit model of the asynchronous motor in d-q axis frame of reference

$\Delta\mathbf{E}_{AM} = (\mathbf{H}_{AM}\cdot\phi_{AM})$ = asynchronous motor emf.

$$\mathbf{R}_{AM} = \begin{bmatrix} R_a+R_r\cdot(X_m/X_r)^2 & -X'_M \\ X'_M & R_a+R_r\cdot(X_m/X_r)^2 \end{bmatrix}$$

$$\mathbf{X}_{AM} = \begin{bmatrix} X'_M & \\ & X'_M \end{bmatrix}$$

$$\mathbf{H}_{AM} = (X_m/X_r)\cdot\begin{bmatrix} -(R_r/X_r) & -\Omega_{AM} \\ \Omega_{AM} & -(R_r/X_r) \end{bmatrix}$$

(Example data)

$X_{a\sigma}$ = stator leakage reactance (**0.08**pu)
$X_{r\sigma}$ = rotor leakage reactance (**0.08**pu)
X_m = magnetizing reactance (**2.50**pu)
$X_r = (X_{r\sigma} + X_m)$
(1-125)
$X'_M = (X_{a\sigma} + X'_m) = (X_{a\sigma} + (X_m\cdot X_{r\sigma}/X_r))$
R_a = stator resistance (**0.03**pu)
R_r = rotor resistance (**0.03**pu)
$T'_{ro} = L_r/R_r = X_r/(\omega_o\cdot R_r)$
s = slip = $(1-\Omega_{AM})$
$X'_m = (X_m\cdot X_{r\sigma}/X_r)$

Rotorflux Model

$$\underset{(2x1)}{d\phi_{AM}}/dt = \omega_o\cdot(\mathbf{F}_{AMi}\cdot\mathbf{i}_{AM} + \mathbf{F}_{AM\phi}\cdot\phi_{AM}) \qquad (1\text{-}126)$$

Here:

$$\mathbf{F}_{AMi} = \begin{bmatrix} (R_r\cdot X_m/X_r) & \\ & (R_r\cdot X_m/X_r) \end{bmatrix}$$

(1-127)

$$\mathbf{F}_{AM\phi} = \begin{bmatrix} -(R_r/X_r) & (1-\Omega_{AM}) \\ (\Omega_{AM}-1) & -(R_r/X_r) \end{bmatrix}$$

Electromechanical Model

$$\underset{(1x1)}{d\Omega_{AM}}/dt = (S_{Bas}/S_{AM})\cdot(1/(T_a\cdot\cos\varphi))\cdot(T_{AMel} - T_{AMmec}) \qquad (1\text{-}128)$$

Here:

$T_{AMel} = 0.5\cdot(X_m/X_r)\cdot(\bar{\mathbf{1}}\cdot\mathbf{i}_{AM}\cdot)^t\cdot\phi_{AM}$ = electrical motor torque , where $\bar{\mathbf{1}} = \begin{bmatrix} & 1 \\ -1 & \end{bmatrix}$ (1-129)

$T_{AMmec} = T_{AMec(o)}\cdot(\Omega_{AM}/\Omega_{AM(o)})^{\kappa}$ = mechanical torque in **motor** mode of operation. (Motor operation implies positive sign of mechanical torque).

If the motor is up and running at t= -0 : $T_{AMec(o)} = T_{Amel(o)}$ = electrical motor torque at t= -0. Found from (1-129). κ = (say) 1.5-4.5, depending on type of load. $\Omega_{AM(o)}$ = initial rotor speed
If the motor is to be started from stillstand :$T_{AMec(o)}$= (say) 0.03-0.07= coefficient modelling mech. friction, air resistance, etc. κ = (say)1-5. $T_{AMec(o)}$ and κ may vary over the range Ω_{AM}=0→1

$T_{AMmec} = (T_{Amel(o)} + \Delta T_{mec})$ = mechanical torque in **generator** mode of operation. (Generator operation implies negative sign of mechanical torques) $T_{Amel(o)}$ is initial electrical motor torque. ΔT_{mec} is the respons from the power control system.
S_{Bas}, S_{AM} = Chosen VA system power base, and rated VA motor capacity, respectively.
T_a, $\cos\varphi$ = Dynamical system's inertia constant, and motor's rated power factor, respectively.

Example Asynchronous Motor Study

Consider the simple case of an asynchronous motor (to be) connected to an infinite bus of symmetrical voltage $\mathbf{E}_{RST}$. The given voltage transforms into $\mathbf{e}_{dq}$ in the d-q axis frame of reference, as given by (1-85).

System analysis will often imply:
- Initial condition analysis
- System respons analysis in terms of *eigenvalue analysis*, and/or *time response analysis* subsequent to some additional excitation(s) to the system

Initial condition analysis

The state variable $\Omega_{AM(o)}$ has to be determined by special consideration of the task at hand:

If the motor is to be started from stillstand, $\Omega_{AM(o)} = 0$, and so also the rest of the machine state variables. I.e. the currents $\mathbf{i}_{AM(o)}$ and fluxes $\phi_{AM(o)}$.

If the machine is initially in a steady state mode of operation, $\Omega_{AM(o)} = (1 - s_{AM(o)})$, where $s_{AM(o)}$ is initial motor slip. In such a case it is customary to specify initial condition in terms of absorbed motor power $P_{AM(o)}$ from the network. Initial pu speed $\Omega_{AM(o)}$ (or equivalentlty slip $s_{AM(o)}$) should then be set so as to contribute to fulfilling this specification. The determination of $\Omega_{AM(o)}$ is conveniantly afforded by an iterative process in which $\Omega_{AM(o)}$ is adjusted until specified initial condition is reached to required accuracy. The computational scheme may be outlined as follows:

Given a tentative value (= e.g. 1.0pu) of $\Omega_{AM(o)}$. Corresponding initial values $\mathbf{i}_{AM(o)}$ and $\phi_{AM(o)}$ are found by observing the constraints implied by Kirchoff's voltage law as (by analogy) expressed by equation (A-19) of Appendix, - and the machine flux constraints of (1-126). In the current steady state context, derivative terms are set to zero. The equations become:

$$\mathbf{R}_{AM} \cdot \mathbf{i}_{AM(o)} + \mathbf{H}_{AM(o)} \cdot \phi_{AM(o)} = \mathbf{e}_{dq} \qquad \text{(from (1-85) \& (1-124))}$$

$$\mathbf{F}_{AMi} \cdot \mathbf{i}_{AM(o)} + \mathbf{F}_{AM\phi(o)} \cdot \phi_{AM(o)} = 0 \qquad \text{(from (1-126))}$$

In matrix form:

$$\begin{bmatrix} \mathbf{R}_{AM} & \mathbf{H}_{AM(o)} \\ \mathbf{F}_{AMi} & \mathbf{F}_{AM\phi(o)} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{i}_{AM(o)} \\ \phi_{AM(o)} \end{bmatrix} = \begin{bmatrix} \mathbf{e}_{dq} \\ 0 \end{bmatrix} \qquad (1\text{-}130)$$

from which the sought initial vectors $\mathbf{i}_{AM(o)}$ and $\phi_{AM(o)}$ are found. Absorbed motor power is given as $P_{AM(o)} = 0.5\, \mathbf{i}_{AM}^{t} \cdot \mathbf{u}_{dq}$. If computed value is ok, an acceptable initial solution has been found. If $P_{AM(o)}$ is not (yet) ok, $\Omega_{AM(o)}$ is adjusted appropriately, and a new and improved solution is solved from (1-130). This process continues until 'ok' allows for exit from *initial system analysis*. For algorithmic details on the general multimachine case, see Chapter 2.4.

Eigenvalue- and/or time response analysis

Given initial value of the system state variables, which in this simple 'motor alone' study are the 5 asynchronous motor variables ($\mathbf{i}_{AM}$ ϕ_{AM} Ω_{AM}). The subsequent dynamic behaviour of the system is governed by the 5 first order, ordinary differential equations for the asynchronous motor, that are developed earlier:

$$d\mathbf{i}_{AM}/dt = \omega_0 \, (\mathbf{X}_{AM})^{-1} \, (\mathbf{e}_{dq} - \Delta\mathbf{E}_{AM} - \mathbf{R}_{AM}\, \mathbf{i}_{AM}) \qquad \text{(From (1-124) \& Kirchoff)}$$

$$d\phi_{AM}/dt = \mathbf{F}_{AMi} \cdot \mathbf{i}_{AM} + \mathbf{F}_{AM\phi}\, \phi_{AM} \qquad \text{(From (1-126))} \qquad (1\text{-}131)$$

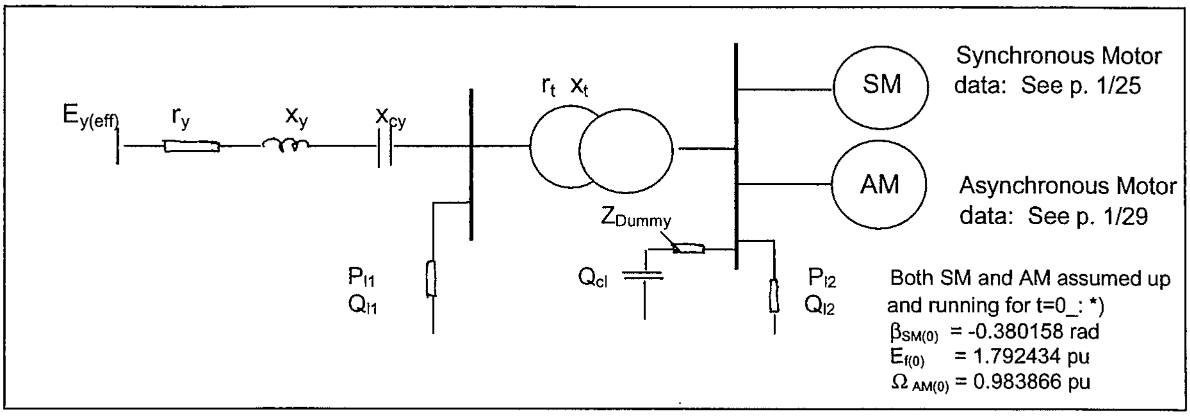
$$d\Omega_{AM}/dt = (S_{Bas}/S_{AM})\, (1/(T_{\dot{a}} \cos\varphi))\, (T_{AMel} - T_{AMmec}) \qquad \text{(From (1-128))}$$

The linearization and processing of such equations for eigenvalue analysis, is covered in Chapter 3. Time respons analyses are dealt with in Chapter 4.

2. The Power Network Model

2. The Power Network Model

The modelling of power network loop currents is summarized in Appendix1. In the following the methodology will be extended to also cover modelling of power network capacitor voltages. The full network modelling process is covered via processing and discussing of the concrete system given below. Although small in size, the example system is assumed representative qua content of main t*ypes* of power network components.



*)The following initial conditions are specified: P_{SM} =-0.8pu, E_{SM} =1.0pu, P_{AM} =0.5pu. For load flow solutuio; see Chapter 3.11.2.

Figure 2.1 : Single line diagram of specified power system at given point in time

In the per phase frame of reference, the following *per unit* (*pu*) data are given for the system at t= -0 :

External system	: Infinite bus, where r.m.s. voltage $E_{y(eff)} = 1.05$ and $\gamma_{ref} = 0$. See (1-85).
Lossless series capacitor bank	: $r_{cy} - jx_{cy}$; $r_{cy} = 0$ $x_{cy} = 0.025$
Series impedance	: $r_y + jx_y$; $r_y = 0.03$ $x_y = 0.125$
Transformer	: $r_t + jx_t$; $r_t = 0.01$ $x_t = 0.07$
Inductive Load no 1	: $P_{l1} = 0.60$ and $Q_{l1} = 0.20$ at $V_{load1} = 1$, implying $r_{l1} = 1.5$ and $x_{l1} = 0.5$
Inductive Load no 2	: $P_{l2} = 0.25$ and $Q_{l2} = 0.80$ at $V_{load2} = 1$, implying $r_{l2} = 0.356..$ and $x_{l2} = 1.139..$
Lossless shunt capacitor bank	: $P_{cl} = 0.0$ and $Q_{cl} = 0.70$ at $V_{loadc} = 1$, implying $r_{cl} = 0.0$ and $x_{cl} = 1.4285...$
Asynchronous motor ('AM')	:The motor model comprises three submodels; the *Electrical Circuit Model*, the *Rotor Flux* Model, and the *Electromechanical Model*, see (1-124) to (1-129), Chapter 1. The *Electrical Circuit Model* is the submodel of *network* relevance.
Synchronous motor ('SM')	: In *generic* terms the comment above for AM is valid also for the SM. In *specific* terms; see (1-106) - (1-119).

2.1 Electrical Circuit Models and The Primitive System

Electrical circuit models to apply in modelling of the main power network components, are summarized in Chapter 1.6 & 1.7. Applying the pertinent data from above to the 'Infinite bus voltage' (1-85) and the 'Lossy Inductor' (1-91/92), we readily get the following d-q axis circuit models of the given *voltage*, *series impedance*, *dummy impedance* *), *transformer*, and *impedance type inductive loads* :

Infinite bus voltage : $\mathbf{e_{dq}} = \begin{bmatrix} e_d \\ e_q \end{bmatrix} = \sqrt{2}\, E_{y(eff)} \begin{bmatrix} -\sin\gamma_{ref} \\ \cos\gamma_{ref} \end{bmatrix} = \begin{bmatrix} 0 \\ 1.485 \end{bmatrix}$ *Dummy imp.*: *) $\mathbf{R_D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\mathbf{X_D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Series impedance : $\mathbf{R_y} = \begin{bmatrix} 0.03 & -0.125 \\ 0.125 & 0.03 \end{bmatrix}$ $\mathbf{X_y} = \begin{bmatrix} 0.125 & 0.00 \\ 0.00 & 0.125 \end{bmatrix}$ *Transformer* : $\mathbf{R_t} = \begin{bmatrix} 0.01 & -0.07 \\ 0.07 & 0.01 \end{bmatrix}$ $\mathbf{X_t} = \begin{bmatrix} 0.07 & 0 \\ 0 & 0.07 \end{bmatrix}$

Inductive load no 1 : $\mathbf{R_{l1}} = \begin{bmatrix} 1.50 & -0.50 \\ 0.50 & 1.50 \end{bmatrix}$ $\mathbf{X_{l1}} = \begin{bmatrix} 0.50 & 0 \\ 0 & 0.50 \end{bmatrix}$ *Ind. Load no2*: $\mathbf{R_{l2}} = \begin{bmatrix} 0.356 & -1.139 \\ 1.139 & 0.356 \end{bmatrix}$ $\mathbf{X_{l2}} = \begin{bmatrix} 1.139 & 0 \\ 0 & 1.139 \end{bmatrix}$

*) The *'dummy impedance'* (here set to 0) allows for definition of a conveniant set of system loops. See later.

For the above circuit models other than the one modelling the infinite bus voltage, the e.m.f. is zero. For space-saving reasons these 'zero-vectors' are omitted from the above display.

Circuit-wise the lossy capacitor bank is represented by the terms ($\mathbf{R}_C, \mathbf{X}_L=0, \Delta\mathbf{E}_C$). *'Locally'*, the *capacitor voltage submodel* applies $\mathbf{X}_C$ for description. For the two capacitor banks the specified initial conditions imply setting of the voltage source vectors according to (1-99). The capacitor related terms become:

Series capacitor : $\mathbf{R}_{cy} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\quad \mathbf{X}_{cy} = \begin{bmatrix} 0.025 & 0 \\ 0 & 0.025 \end{bmatrix}$ $\quad \Delta\mathbf{E}_{cy(o)} = \bar{\mathbf{1}}\, \mathbf{X}_{cy}\, \mathbf{i}_{cy(o)} = \begin{bmatrix} 0.019081 \\ -0.010619 \end{bmatrix}$

Shunt capacitor : $\mathbf{R}_{cl} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\quad \mathbf{X}_{cl} = \begin{bmatrix} 1.428571 & 0 \\ 0 & 1.428571 \end{bmatrix}$ $\quad \Delta\mathbf{E}_{cl} = \bar{\mathbf{1}}\, \mathbf{X}_{cl}\, \mathbf{i}_{d(o)} = \begin{bmatrix} 0.057253 \\ 1.413120 \end{bmatrix}$

The d-q axis circuit model (at t=0_) of the *asynchronous motor*, is found by applying motor data and initial condition data to the algorithms (1-125) :

Asynchr. motor : $\mathbf{R}_{AM} = \begin{bmatrix} 0.058168 & -0.157519 \\ 0.157519 & 0.058168 \end{bmatrix}$ $\quad \mathbf{X}_{AM} = \begin{bmatrix} 0.157519 & 0 \\ 0 & 0.157519 \end{bmatrix}$ $\quad \Delta\mathbf{E}_{AM(0)} = \mathbf{H}_{AM(0)}\, \boldsymbol{\phi}_{AM(0)} = \begin{bmatrix} 0.128647 \\ 1.276071 \end{bmatrix}$

The d-q axis circuit model (at t=0_) of the *synchronous motor* is found by applying motor data and initial condition data to the algorithms (1-107) – (1-110) :

Synchr. motor : $\mathbf{R}_{SM(o)} = \begin{bmatrix} 0.045654 & -0.285186 \\ 0.214814 & -0.025459 \end{bmatrix}$ $\quad \mathbf{X}_{SM(o)} = \begin{bmatrix} 0.213769 & -0.03446 \\ -0.03446 & 0.286231 \end{bmatrix}$ $\quad \Delta\mathbf{E}_{SM(0)} = \mathbf{V}_{SM(0)} + \mathbf{H}_{SM(0)}\, \boldsymbol{\phi}_{SM(0)} = \begin{bmatrix} -0.22720 \\ 1.52919 \end{bmatrix}$

The primitive system for any considered point in time , is merely a suitable lineup of the electrical circuit models of the network components valid at that point in time. Re-arranging the component description established above, the primitive system of the power network of Figure 2.1, can take on the form shown in Figure 2.2. See Appendix1. For convenient identification the circuit models are assigned numbers from '1' to '9', as given in Figure 2.3. In principle the numbering is arbitrary. To enhance system overview as well as speed of computation, the numbering logic should observe both topological and component-oriented considerations, - see later.

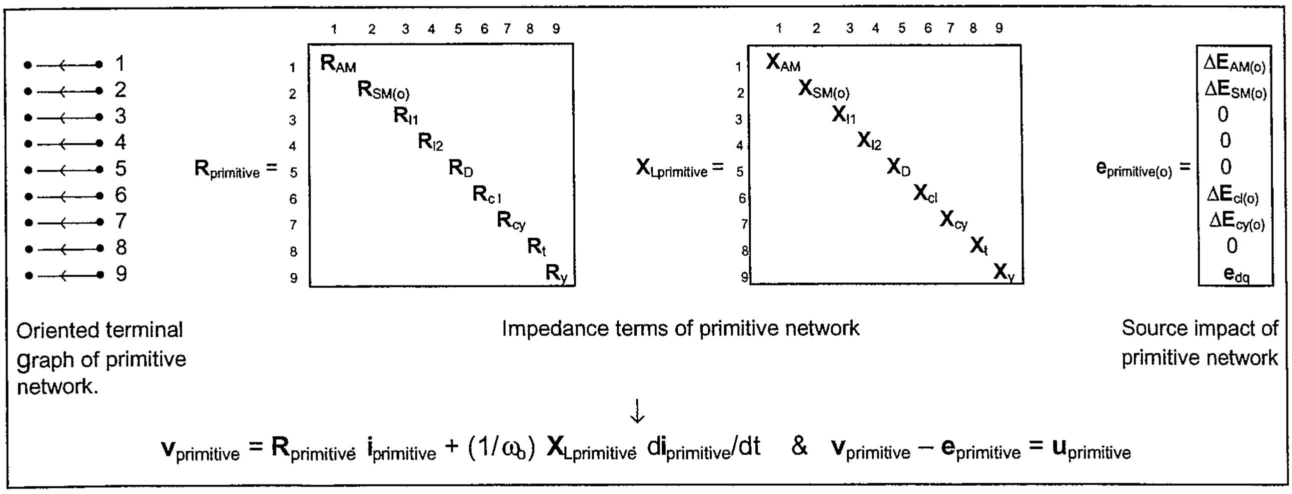
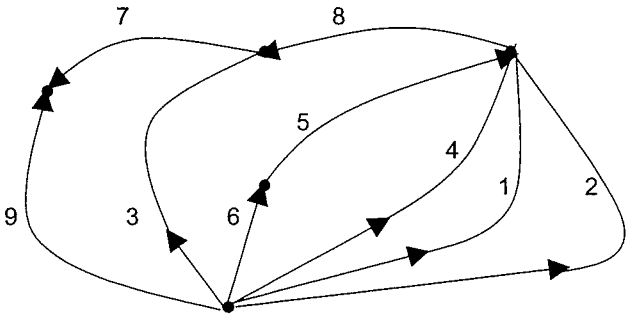


Figure 2.2 The primitive network of the power system of Figure 2.1, valid for t= -0.

2.2 Network topology

Graphwise the topology of a network is established by connecting together the graph elements of its primitive system, as directed by the single line diagram of the network at hand. The oriented graph of the system in Figure 2.1 is formed by inteconnecting the primitive network graph elements of Figure 2.2, as advised by the single line diagram of Figure 2.1. The system graph is shown in Figure 2.3 :



1 : Asynchronous machine
2 : Synchronous machine
3 : Inductive load no 1
4 : Inductive load no 2
5 : Dummy connection
6 : Shunt capacitor bank
7 : Series capacitor bank
8 : Transformer
9 : Series impedance+ infinite bus

b = no. of nework graph elements = 9
N_{node} = no. of nodes of network graph = 5

Figure 2.3: Oriented graph of the system in Figure 2.1.

The formal modelling of the interconnection of electrical circuit models may be afforded by different topological matrices comprising plus/minus '1', or '0' as matrix elements. In the present outline a *system loop matrix **B*** is used to formally describe how the circuit models are tied together.

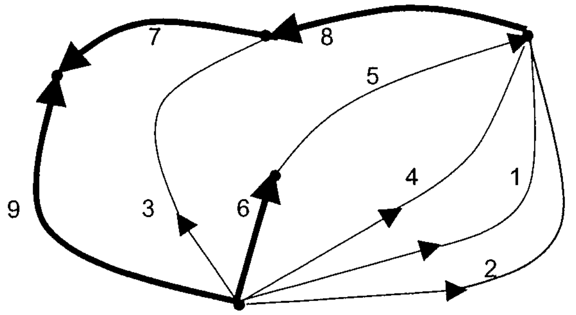
*The system loop matrix **B*** is conveniently defined on the basis of a chosen *tree* and *cotree* of the oriented graph of the network, see Figure 2.4a :

- The *tree* is a set of N_{tree} graph elements that connects all nodes of the network graph without closing any circuit. $N_{tree} = (N_{node} - 1)$, where N_{node} is the number of nodes of the connected network graph. For the graph of Figure 2.3, $N_{tree} = (5 - 1) = 4$. The chosen tree of this graph is shown with thick line in Figure 2.4a.

  Capacitor elements must belong to the *tree*, and the capacitor elements should conveniently be numbered first among the tree elements. It is also conveniant to locate any exogenously specified voltages to elements contained in the *tree.*

- The remaining $N_{loop} = (b - N_{tree})$ graph elements constitute the corresponding *cotree* of the oriented network graph. b is the number of elements of the graph. Each cotree element - or chord - identifies a unique loop of the network graph. Thus the collection of chosen cotree elements identifies a necessary and sufficient set of independent system loops for evaluation of network flow solutions.

  The *cotree* elements of the graph are suitably numbered first. The chosen cotree of the oriented graph of Figure 2.3, is shown by thin lines in Figure 2.4. With b=9 we note that $N_{loop} = (9-4) = 5$.



a) Oriented network graph

Element no →

Loop no ↓ (rows 1–5), elements 1–9 (columns):

$$\mathbf{B} = \left[\begin{array}{ccccc|cccc} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & -1 \end{array}\right] = [\mathbf{B}_{chord}, \mathbf{B}_{tree}] \qquad (2\text{-}1)$$

b) Loop matrix **B**

Figure 2.4: Graph description in terms of *tree*, *cotree* and *B-matrix*

The system loop matrix **B** gives the incidence of the independent system loops as defined by the set of cotree elements – or chords - , and the oriented graph elements of the system. The numbering of the cotree elements can conveniently identify also the set of independent system loops, and the chosen orientation of the cotree elements can similarily define positive direction of the system loop currents.

The loop matrix **B** of the oriented graph of Figure 2.4a is shown in figure 2.4b. **B** can be partitioned into a submatrix $\mathbf{B}_{chord}$ that describes the incidence of *loops* and *chords* (=cotree elements), and sub-matrix $\mathbf{B}_{tree}$ that describes the incidence of *loops* and *tree elements*. Given the conventions above, $\mathbf{B}_{chord}$ will always be a unit matrix. In the chosen compact d-q axis matrix notation '**1**' stands numerically for a 2x2 unit matrix and '**0**' for a 2x2 emty matrix. See Figure 2.4b.

To facilitate the processing of matrix equations, it is desirable to partition the submatrix $\mathbf{B}_{tree}$. With the chosen numbering convention regarding tree elements, we define;

$$\mathbf{B}_{tree} = [\,\mathbf{B}_{tc}, \mathbf{B}_{t\text{-}rest}\,] \qquad (2\text{-}2)$$

where;

$\mathbf{B}_{tc}$ =submatrix describing the incidence of loops and the subset of tree elements that comprise capacitors

$$\text{Here: } \mathbf{B}_{tc} = \begin{array}{c|cc|} & 6 & 7 \\ 1 & \mathbf{0} & \mathbf{1} \\ 2 & \mathbf{0} & \mathbf{1} \\ 3 & \mathbf{0} & \mathbf{1} \\ 4 & \mathbf{0} & \mathbf{1} \\ 5 & \mathbf{1} & \mathbf{1} \end{array} \qquad (2\text{-}3)$$

$\mathbf{B}_{t\text{-}rest}$ = submatrix describing the incidence of loops and the 'rest' of the tree elements.

$$\text{Here: } \mathbf{B}_{t\text{-}rest} = \begin{array}{|cc|} 8 & 9 \\ \mathbf{1} & \mathbf{-1} \\ \mathbf{1} & \mathbf{-1} \\ \mathbf{0} & \mathbf{-1} \\ \mathbf{1} & \mathbf{-1} \\ \mathbf{1} & \mathbf{-1} \end{array} \qquad (2\text{-}4)$$

2.3 Network modelling

With *loop currents* and *voltage across capacitors* defined as power network state variables, the network loop equations for the power system can - as shown in Appendix 1 - be compactly expressed as follows, in the d-q axis frame of reference:

$$\mathbf{E}_{loop} = \mathbf{R}_{loop}\, \mathbf{I}_{loop} + (1/\omega_b)\, \mathbf{X}_{Lloop} \cdot d\mathbf{I}_{loop}/dt \qquad (2\text{-}5)$$

where;

$\mathbf{E}_{loop}$ = - **B** $\mathbf{e}_{primitive}$ = (N_{loop}x1) d-q axis *loop voltage vector* comprising the driving voltage of respective N_{loop} loops of the network graph

$\mathbf{e}_{primitive}$ = (bx1) *voltage vector* in the d-q axis frame of reference. $\mathbf{e}_{primitive}$ comprisies the e.m.f. associated with respective b graph elements of the network

$\mathbf{I}_{loop}$ = (N_{loop}x1) d-q axis *loop current vector* comprising the current of respective N_{loop} cotree elements of the network graph. Orientation of the currents is here opposite the orientation of the cotree elements. Current from a source is thus defined negative. Current supplied to a load is correspondingly defined positive. See special discussion under heading 'Network Modelling' in Appendix 1.

$\mathbf{R}_{loop}$ = **B** $\mathbf{R}_{primitive}$ $\mathbf{B}^t$ = (N_{loop}xN_{loop}) network loop resistance matrix in the d-q axis frame of reference.

(2-6)

$\mathbf{X}_{Lloop}$ = **B** $\mathbf{X}_{Lprimitive}$ $\mathbf{B}^t$ = (N_{loop}xN_{loop}) network loop *inductive* reactance matrix in the d-q axis frame of reference.

$\mathbf{R}_{primitive}$ = (bxb) primitive network resistance matrix. $\mathbf{R}_{primitive}$ is the collection of resistances associated with resp. b elements of the network graph. $\mathbf{R}_{primitive}$ is diagonal.

$\mathbf{X}_{Lprimitive}$ = (bxb) primitive network inductive reactance matrix. $\mathbf{X}_{Lprimitive}$ is the collection of *inductive* reactances associated with respective b elements of the network graph. $\mathbf{X}_{Lprimitive}$ is normally diagonal. See text following Figure A.3 of the Appendix.

B = (N_{loop}xb) network loop matrix giving the incidence of network loops and elements of the network graph.

To bring (2-5) on a more useful form, the product $\mathbf{B}\ \mathbf{e}_{primitive}$ should be further developed. To this end $\mathbf{B}$ is already partitioned, see (2-1) - (2-4). $\mathbf{e}_{primitive}$ is partitioned as follows :

$$\mathbf{e}_{primitive} = \begin{bmatrix} \mathbf{e}_{chord} \\ \mathbf{e}_{tree} \end{bmatrix} \qquad (2\text{-}7)$$

where $\mathbf{e}_{chord}$ comprises the voltage sources associated with *chords* or *cotree elements*, and $\mathbf{e}_{tree}$ comprises the voltage sources associated with *tree* elements.

$\mathbf{e}_{tree}$ is again partitioned:

$$\mathbf{e}_{tree} = \begin{bmatrix} \mathbf{e}_{tc} \\ \mathbf{e}_{t\text{-}rest} \end{bmatrix} \qquad (2\text{-}8)$$

where $\mathbf{e}_{tc}$ is the (N_Cx1) subvector of $\mathbf{e}_{tree}$ that comprises the formal voltage sources defined by capacitors. N_C is the number of capacitors of the single line diagram of the power network. In the current example N_c=2. (Recalled from earlier: Each capacitor is structurally accounted for in terms of a *tree element*, and these capacitor graph elements are assigned numbers before (any) remaining tree elements are given numbers). $\mathbf{e}_{t\text{-}rest}$ is the subvector of $\mathbf{e}_{tree}$ comprising the voltage sources that belong to the *remaining tree elements* .

From the foregoing definitions flow :

$$\begin{aligned} \mathbf{B}\ \mathbf{e}_{primitive} &= [\mathbf{B}_{chord}, \mathbf{B}_{tree}]\ [\mathbf{e}_{chord}, \mathbf{e}_{tree}]^t \\ &= \mathbf{B}_{chord}\ \mathbf{e}_{chord} + \mathbf{B}_{tree}\ \mathbf{e}_{tree} \\ &= \mathbf{B}_{chord}\ \mathbf{e}_{chord} + [\mathbf{B}_{tc}, \mathbf{B}_{t\text{-}rest}]\ [\mathbf{e}_{tc}, \mathbf{e}_{t\text{-}rest}]^t \\ &= \mathbf{B}_{chord}\ \mathbf{e}_{chord} + \mathbf{B}_{tc}\ \mathbf{e}_{tc} + \mathbf{B}_{t\text{-}rest}\ \mathbf{e}_{t\text{-}rest} \\ &= \mathbf{e}_{chord} + \mathbf{B}_{tc}\ \mathbf{e}_{tc} + \mathbf{B}_{t\text{-}rest}\ \mathbf{e}_{t\text{-}rest} \qquad \text{(since } \mathbf{B}_{chord} = \mathbf{1}, \text{ - per definition)} \qquad (2\text{-}9) \end{aligned}$$

Equation (2-9) states : *Resulting voltage source excitation of respective network loops* can be viewed as the sum of three contributions:

$\mathbf{e}_{chord}$: The contribution from voltage sources located to cotree elements

In the present case: $\mathbf{e}_{chord} = \begin{bmatrix} \Delta E_{AM} \\ \Delta E_{SM} \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix}$ (Loop no ↓)

$\mathbf{B}_{tc}\ \mathbf{e}_{tc}$: The contribution from voltage sources formally equivalencing capacitors.

In the present case: $\mathbf{B}_{tc}\ \mathbf{e}_{tc} = \overset{\;6\quad 7}{\begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}} \cdot \begin{bmatrix} \Delta E_{cl} \\ \Delta E_{cy} \end{bmatrix} = \begin{bmatrix} \Delta E_{cl} \\ \Delta E_{cl} \\ \Delta E_{cl} \\ \Delta E_{cl} \\ \Delta E_{cl} + \Delta E_{cy} \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix}$

$\mathbf{B}_{t\text{-}rest}\ \mathbf{e}_{t\text{-}rest}$: The contribution from voltage sources that reside in the 'rest' of the tree elements.

In the present case: $\mathbf{B}_{t\text{-}rest}\ \mathbf{e}_{t\text{-}rest} = \overset{\;8\quad 9}{\begin{bmatrix} 1 & -1 \\ 1 & -1 \\ 0 & -1 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}} \cdot \begin{bmatrix} 0 \\ e_{dq} \end{bmatrix} = \begin{bmatrix} -e_{dq} \\ -e_{dq} \\ -e_{dq} \\ -e_{dq} \\ -e_{dq} \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix}$

Inserting (2-9) into (2-5) and solving w.r.t. the derivative of the network loop currents, we get the following set of first order ordinary differential equations to govern the variation of power network loop currents:

$$d\mathbf{i}_{loop}/dt = \omega_0\, \mathbf{X}_{Lloop}^{-1} \cdot [-\mathbf{R}_{loop}\, \mathbf{i}_{loop} - \mathbf{B}_{tc}\, \mathbf{e}_{tc} - \mathbf{e}_{chord} - \mathbf{B}_{t\text{-}rest}\, \mathbf{e}_{t\text{-}rest}] \tag{2-10}$$

The voltage across the capacitors of the system is given by the (N_Cx1) voltage vector $\mathbf{e}_{tc}$. These voltages are defined as state variables, and in the following the differential equations governing their time variation are developed.

In our adopted d-q frame of reference, the differential equations governing the voltage $\Delta\mathbf{E}_c$ across any given capacitor of 'throughput' current $\mathbf{i}_c$, is fetched from our stock of models in Chapter 1.7, - see equations (1-98) and (1-99) there :

$$d\Delta\mathbf{E}_C/dt = \omega_0\, (\mathbf{X}_C\, \mathbf{i}_C + \bar{\mathbf{1}} \cdot \Delta\mathbf{E}_C) \tag{2-11}$$

with initial condition;

$$\Delta\mathbf{E}_{C(o)} = \bar{\mathbf{1}} \cdot \mathbf{X}_C\, \mathbf{i}_{C(o)} \tag{2-12}$$

and definitions (1-97), (1-100) & (1-101);

$$\mathbf{R}_C = \begin{bmatrix} R_c & \\ & R_c \end{bmatrix} \tag{2-13}$$

$$\mathbf{X}_C = \begin{bmatrix} 1/(\omega_0 C) & \\ & 1/(\omega_0 C) \end{bmatrix} = \begin{bmatrix} X_c & \\ & X_c \end{bmatrix} \tag{2-14}$$

$$\bar{\mathbf{1}} = \begin{bmatrix} & 1 \\ -1 & \end{bmatrix} \tag{2-15}$$

Equations (2-11) relate to *one* capacitor voltage. Extending the description to cover the set of N_C capacitor voltages $\mathbf{e}_{tc}$, one can write ;

$$d\mathbf{e}_{tc}/dt = \omega_0\, (\mathbf{X}_{Cprimitive} \cdot \mathbf{i}_{tc} + \bar{\mathbf{1}}_{tc} \cdot \mathbf{e}_{tc}) \tag{2-16}$$

where $\mathbf{X}_{Cprimitive}$ - in d-q axis frame of reference - is defined to be a diagonal (N_CxN_C) matrix containing the sequence of reactances $\mathbf{X}_c$ along the main diagonal. $\mathbf{i}_{tc}$ is the set of capacitor currents that constitute a *subset* of the current vector $\mathbf{i}_{tree}$ that comprise the full set of *tree* element currents of the network graph.

$\bar{\mathbf{1}}_{tc}$ is - in the d-q axis frame of reference - a (N_CxN_C) diagonal matrix that repeats the matrix $\bar{\mathbf{1}}$ along its main diagonal.

The currents $\mathbf{i}_{tc}$ in (2-16) are per se not state variables. They are however, related to the loop currents which are the chosen state variables. Equation (A-7) of Appendix1 states that the currents $\mathbf{i}_{tree}$ associated with the *tree elements* of an oriented network graph, are generally expressible as linear combinations of the currents $\mathbf{i}_{loop}$ associated with the *cotree elements* –or *chords* – of the graph. In formal terms the following holds true:

$$\mathbf{i}_{tree} = \mathbf{B}_{tree}^{t} \cdot \mathbf{i}_{loop} \tag{2-17}$$

For the subset of capacitor currents $\mathbf{i}_{tc}$, (2-17) takes on this form:

$$\mathbf{i}_{tc} = \mathbf{B}_{tc}^{t} \cdot \mathbf{i}_{loop} \tag{2-18}$$

Equation (2-18) is used in (2-16), and the following useful system of equations finally appears for describing capacitor voltage performance :

$$d\mathbf{e}_{tc}/dt = \omega_0\, (\mathbf{X}_{Cprimitive} \cdot \mathbf{B}_{tc}^{t} \cdot \mathbf{i}_{loop} + \bar{\mathbf{1}}_{tc} \cdot \mathbf{e}_{tc}) \tag{2-19}$$

Equations (2-10) and (2-19) describe the electrical performance of the power network. Put together they represent the electrical network model in a d-q axis reference frame of formulation:

$$\begin{bmatrix} di_{loop}/dt \\ \hline de_{tc}/dt \end{bmatrix} = \omega_b \begin{bmatrix} -X_{Lloop}^{-1} \cdot R_{loop} & -X_{Lloop}^{-1} \cdot B_{tc} \\ \hline X_{Cprimitive} B_{tc}^{t} & \bar{1}_{tc} \end{bmatrix} \begin{bmatrix} i_{loop} \\ \hline e_{tc} \end{bmatrix} - \omega_b \begin{bmatrix} X_{Lloop}^{-1} \\ \hline 0 \end{bmatrix} e_{chord} - \omega_b \begin{bmatrix} X_{Lloop}^{-1} \cdot B_{t-rest} \\ \hline 0 \end{bmatrix} e_{t-rest} \qquad (2\text{-}20)$$

where;

i_{loop} = $(N_{loop}x1)$ vector of loop currents comprising the current in resp. chords. Each loop current comprises a d-and a q-component, so that numerically i_{loop} is of dimension $(2N_{loop}x1)$. N_{loop} is the number of independent loops of the oriented network graph. It is also the number of cotree elements -or chords- of the graph.

In present example:

$$i_{loop} = \begin{bmatrix} i_{loop1} \\ i_{loop2} \\ i_{loop3} \\ i_{loop4} \\ i_{loop5} \end{bmatrix} = \begin{bmatrix} i_{AM} \\ i_{SM} \\ i_{l1} \\ i_{l2} \\ i_{D} \end{bmatrix}$$

The orientation of the *loop* currents i_{loop} is here defined opposite the orientation of the currents of the *cotree* elements. See Appendix 1. Power supplied to loads will then be positive, when derived from i_{loop}. (2-21)

e_{tc} = (N_Cx1) vector of capacitor voltages in the d-q axis frame of reference. Each voltage comprises a d-and a q-component, so that numerically e_{tc} is of dimension $(2N_Cx1)$. N_C is the number of capacitors of the single line diagram of the power network.

Here: (Elem. no 6, 7)

$$e_{tc} = \begin{bmatrix} \Delta E_{cl} \\ \hline \Delta E_{cy} \end{bmatrix} \begin{matrix} 6 \\ 7 \end{matrix} \qquad (2\text{-}22)$$

e_{chord} = $(N_{loop}x1)$ voltage source vector associated with resp. cotree elements of the graph, d-q axis frame of reference. Each voltage comprises a d-and a q-component, so that numerically e_{chord} is of dimension $(2N_{loop}x1)$.
For the *synchronous motor* : $\Delta E_{SM} = (V_{SM} + H_{SM}\ \phi_{SM})$, see (1-106).
For the *asynchronous motor* : $\Delta E_{AM} = H_{AM}\ \phi_{AM}$, see (1-124).

Here: (Loop no 1–5)

$$e_{chord} = \begin{bmatrix} \Delta E_{AM} \\ \Delta E_{SM} \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \qquad (2\text{-}23)$$

e_{t-rest} = $(N_{tree} - N_C)x1$ voltage source vector associated with the 'rest' of the tree elements, ie the tree elements that are not representing capacitors. Each voltage comprises a d- and a q- component, so that e_{t-rest} numerically is of dimension $2(N_{tree} - N_C)x1$. N_{tree} = number of tree elements of the graph.
Infinite bus voltage $e_{dq} = [e_d, e_q]^t = \sqrt{2}\ E_{y(eff)}\ [-\sin\gamma_{ref}, \cos\gamma_{ref}]^t = [\ 0\ , 1.485..]^t$

Here: (Elem.no 8, 9)

$$e_{t-rest} = \begin{bmatrix} 0 \\ \hline e_{dq} \end{bmatrix} \begin{matrix} 8 \\ 9 \end{matrix} \qquad (2\text{-}24)$$

B = $(N_{loop}xb)$ system loop matrix in the d-q axis frame of reference. Entries are **+/-1** or **0**. '**1**' means numerically a 2x2 unit matrix. '**0**' means a 2x2 emty matrix. b is the number of elements of the oriented graph.

$B = [B_{chord}, B_{tree}]$

$B_{tree} = [B_{tc}, B_{t-rest}]$

Here (Elem.: → chords | tree-elem; Loop no 1–5):

$$B = \left[\begin{array}{ccccc|cccc} 1&0&0&0&0&0&1&1&-1 \\ 0&1&0&0&0&0&1&1&-1 \\ 0&0&1&0&0&0&1&0&-1 \\ 0&0&0&1&0&0&1&1&-1 \\ 0&0&0&0&1&1&1&1&-1 \end{array}\right] \qquad B_{tc} = \overset{6\ \ 7}{\begin{bmatrix} 0&1 \\ 0&1 \\ 0&1 \\ 0&1 \\ 1&1 \end{bmatrix}} \qquad B_{t-rest} = \overset{8\ \ 9}{\begin{bmatrix} 1&-1 \\ 1&-1 \\ 0&-1 \\ 1&-1 \\ 1&-1 \end{bmatrix}} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \qquad (2\text{-}25)$$

(B_{chord} = left block, B_{tree} = right block)

$R_{loop} = B\ R_{primitive}\ B^t$ = $(N_{loop}xN_{loop})$ network loop resistance matrix in the d-q axis frame of reference. Each element $R_{lopp}[i,j]$ of this matrix 'hides' a 2x2 'local' d-q description. Numerically R_{loop} thus is of dimension $(2N_{loop}x2N_{loop})$

$R_{primitive}$ is - in the d-q frame of reference – a bxb matrix displaying the R-term of each and every element of the system graph. $R_{primitive}$ is diagonal. Each element $R_{primitive}[i,j]$ of this matrix 'hides' a 2x2 'local' d-q description. Numerically $R_{primitive}$ then is of dimension (2bx2b). For simple illustration of $R_{primitive}$, see Figure 2.2.

$X_{Lloop} = B\ X_{Lprimitive}\ B^t$ = $(N_{loop}xN_{loop})$ network loop *inductor* matrix in the d-q frame of reference. Each element $X_{Lloop}[i,j]$ 'hides' a 2x2 'local' d-q description. Numerically X_{Lloop} then is of dimension $(2N_{loop}x2N_{loop})$.

$X_{Lprimitive}$ is - in the d-q frame of reference – a bxb matrix displaying the X_L-term of each and every element of the system graph. If there are no mutual coupling between circuits, $X_{Lprimitive}$ is diagonal. In the present example $X_{Lprimitive}$ is diagonal. See Figure 2.2 for ill. . Each element $X_{Lprimitive}[i,j]$ 'hides' a 2x2 'local' d-q description. Numerically $X_{Lprimitive}$ then is of dimension (2bx2b).

Figure 2.5 Summary description of the electrical state variables (loop currents and capacitor voltages) of a power network

2.4 Initial Condition Analysis

Whether eigenvalue- or time dynamical analysis is to be conducted next, an appropriate initial state has to be defined for the system. In the following the process of arriving at the proper initial value of all state variables is dealt with.

The initial value of state variables of rotating machines have to be determined by special consideration:

If a *synchronous motor* is to be started, its pu speed $\Omega_{SM(0)}$ = 0, and so also all initial machine current and flux variables.The electrical angle $\beta_{SM(0)}$ can arbitrarely be set to zero. The synchronous motor's field voltage $E_{f(0)}$ may also be zero, if the field winding is kept short circuited during the initial part of the start-up sequence.

If a *synchronous motor* is initially in a synchronous mode of operation, $\Omega_{SM(0)}$ = 1. In this case it is customary to specify initial conditions in terms of absorbed power $P_{SM(0)}$ and voltage $U_{SM(0)}$ at the machine terminals. Thus $\beta_{SM(0)}$ and $E_{f(0)}$ should be specified so as to contribute to fullfilling these requirements. Computationally, this is afforded either by determining $\beta_{SM(0)}$ and $E_{f(0)}$ from an initial phasor diagram (which may be feasible only for a very small system), or by an iterative solution process in which $\beta_{SM(0)}$ and $E_{f(0)}$ are simultaneously corrected (together with all other such 'control variables') until stated initial conditions are reached to required accuracy. Absorbed (or produced) reactive motor power is in principle a byproduct from the stated solution process. The process is further developed below, and exemplified in Chapter 3.11.2.

If *an asynchronous motor* is to be started, its pu speed $\Omega_{AM(0)}$ = 0, and so also all initial machine current and flux variables.

If the asynchronous motor is initially in a steady state mode of operation, it may be appropriate to specify initial conditions in terms of absorbed motor power $P_{AM(0)}$. Thus $\Omega_{AM(0)}$ should be specified so as to fulfill this power requirement. Computationally, this is afforded by including $\Omega_{AM(0)}$ as one of the simultaneously corrected 'control variables' of the above mentioned iterative solution process. Absorbed reactive motor power flows as a byproduct from the solution process.

With (finally or tentatively) specified values ($\beta_{SM(0)}$, $E_{f(0)}$, $\Omega_{SM(0)}$, $\Omega_{AM(0)}$), the premises are set for computing initial values of the rest of the pertinent power system variables. I.e: The network loop currents $\mathbf{i}_{loop(0)}$, the capacitor voltages $\mathbf{e}_{tc(0)}$, the asynchronous motor fluxes $\phi_{AM(0)}$, and the synchronous motor fluxes $\phi_{SM(0)}$.

The solution vector $\mathbf{z}_{(0)}=[\mathbf{i}_{loop(0)}^t, \mathbf{e}_{tc(0)}^t, \phi_{AM(0)}^t, \phi_{SM(0)}^t]^t$ is found by simultaneously solving (2-20), (1-126) and (1-113) for steady state conditions, - i.e. after setting the derivative terms =0. At the outset we then have:

$$\begin{bmatrix} di_{loop}/dt \\ \hline de_{tc}/dt \end{bmatrix} = 0 = \omega_b \begin{bmatrix} -X_{Lloop}^{-1}\cdot R_{loop} & -X_{Lloop}^{-1}\cdot B_{tc} \\ \hline X_{Cprimitive}\, B_{tc}^{t} & \bar{1}_{tc} \end{bmatrix} \cdot \begin{bmatrix} i_{loop(0)} \\ \hline e_{tc(0)} \end{bmatrix} - \omega_b \begin{bmatrix} X_{Lloop}^{-1} \\ \hline 0 \end{bmatrix} e_{chord(0)} - \omega_b \begin{bmatrix} X_{Lloop}^{-1}\cdot B_{t\text{-}rest} \\ \hline 0 \end{bmatrix} e_{t\text{-}rest} \qquad (2\text{-}26)$$

$$d\phi_{AM}/dt = 0 = \omega_b\,(F_{AMi}\cdot i_{AM(o)} + F_{AM\phi(o)}\,\phi_{AM(o)}) \qquad (2\text{-}27)$$

$$d\phi_{SM}/dt = 0 = \omega_b\,(e_{SMr(o)} + F_{Smi(o)}\,i_{SM(o)} + F_{SM\phi}\,\phi_{SM(o)}) \qquad (2\text{-}28)$$

We notice that X_{Lloop}^{-1} is common factor to all terms of the upper system of equations of (2-26), and hence can be omitted in the present context. As a common factor to all equations ω_b can also be omitted. The set of equations above may then take on the following form:

$$\begin{bmatrix} R_{loop} & B_{tc} \\ \hline X_{Cprimitive}\, B_{tc}^{t} & \bar{1}_{tc} \end{bmatrix} \begin{bmatrix} i_{loop(o)} \\ \hline e_{tc(o)} \end{bmatrix} = \begin{bmatrix} -e_{chord(o)} \\ \hline 0 \end{bmatrix} + \begin{bmatrix} -B_{t\text{-}rest}\cdot e_{t\text{-}rest} \\ \hline 0 \end{bmatrix} \qquad (2\text{-}29)$$

$$(F_{AMi}\,i_{AM(o)} + F_{AM\phi(o)}\,\phi_{AM(o)}) = 0 \qquad (2\text{-}30)$$

$$(F_{Smi(o)}\,i_{SM(o)} + F_{SM\phi}\,\phi_{SM(o)}) = -e_{SMr(o)} \qquad (2\text{-}31)$$

The vectors on the right hand side are avalable from previous Chapters: From Chapter 2.3 we have;

$e_{chord(o)}$ = (N_{loop}x1) voltage source vector associated with resp cotree elements of the graph. d-q axis frame of reference.

For the *synchronous motor* : $\Delta E_{SM(o)} = (V_{SM(o)} + H_{SM(o)}\ \phi_{SM(o)})$.
For the *asynchronous motor*: $\Delta E_{AM(o)} = H_{AM(o)}\ \phi_{AM(o)}$.
(See (1-106) and (1-124))

$$\supset:\ e_{chord(o)} = \begin{bmatrix} \Delta E_{AM(o)} \\ \Delta E_{SM(o)} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} H_{AM(o)}\ \phi_{AM(o)} \\ H_{SM(o)}\ \phi_{SM(o)} \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ V_{SM(o)} \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \quad \text{(Loop no)}$$

$B_{t\text{-}rest}\ e_{t\text{-}rest}$ = the loop voltage contribution from the voltage sources that reside in the 'rest' of the tree elements. It is convenient to choose the tree of the oriented graph so that external sources - if any - belong to the 'rest' of the tree elements. The infinite bus voltage:

$$\supset:\ B_{t\text{-}rest}\ e_{t\text{-}rest} = \begin{bmatrix} 1 & -1 \\ 1 & -1 \\ 0 & -1 \\ 1 & -1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ e_{dq} \end{bmatrix} = \begin{bmatrix} -e_{dq} \\ -e_{dq} \\ -e_{dq} \\ -e_{dq} \\ -e_{dq} \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix}$$

$$e_{dq} = [e_d\ ,\ e_q]^t = [-\sqrt{2}\ E_{y(eff)} \sin\gamma_{ref}\ ,\ \sqrt{2}\ E_{y(eff)} \cos\gamma_{ref}]^t$$

The above expressions for e_{chord} and $B_{t\text{-}rest}\ e_{t\text{-}rest}$ are applied to (2-29). For given initial values ($\beta_{SM(0)}, E_{f(0)}, \Omega_{SM(0)}, \Omega_{AM(0)}$) equations (2-29) -(2-31) can then be compactly expressed as follows, for determining initial conditions:

$$H_{syst(0)}\ z_{(o)} = E_{syst(o)} \qquad (2\text{-}32)$$

The sought initial values $z_{(0)}$ are solved from (2-32). The algorithmic details are summarized as follows:

$$\boxed{z_{(o)} = H^{-1}_{syst(o)}\ E_{syst(o)}} \qquad (2\text{-}33)$$

where;

$$z_{(o)} = \begin{bmatrix} i_{loop(o)} \\ e_{tc(o)} \\ \phi_{AM(o)} \\ \phi_{SM(o)} \end{bmatrix}$$

= initiial value of electrical network state variables, when values are (finally or tentatively) specified for ($\beta_{SM(o)}, E_{f(o)}, \Omega_{SM(o)}, \Omega_{AM(o)}$).

$$H_{syst(0)} = \begin{array}{c|c|c} i_{loop} & e_{tc} & \phi_{AM}\quad \phi_{SM} \\ \hline R_{loop(0)}\ (10\text{x}10) & B_{tc}\ (10\text{x}4) & \begin{matrix} H_{AM(0)} & \\ & H_{SM(0)} \end{matrix}\ (4\text{x}5) \\ & & (10\text{x}5) \\ \hline X_{Cprimitive}\ B_{tc}^t\ (4\text{x}10) & 1_{tc}\ (4\text{x}4) & \\ \hline \begin{matrix} F_{AMi} & \\ & F_{Smi(o)} \end{matrix}\ (5\text{x}4) \quad (5\text{x}10) & & \begin{matrix} F_{AM\phi(0)} & \\ & F_{SM\phi} \end{matrix}\ (5\text{x}5) \end{array} \qquad (2\text{-}34)$$

Comments:
↓

$$(2\text{-}35) \qquad E_{syst(0)} = \begin{bmatrix} e_{dq} \\ e_{dq} - V_{SM(o)} \\ e_{dq} \\ e_{dq} \\ e_{dq} \\ \hline 0 \\ \hline 0 \\ -e_{SMr(o)} \end{bmatrix} \begin{matrix} \text{10x1} \\ \\ \text{(4x1)} \\ \text{(5x1)} \end{matrix} \quad (19\text{x}1)$$

where;

$$e_{dq} = \begin{bmatrix} -\sqrt{2}\ E_{y(eff)} \sin\gamma_{ref} \\ \hline \sqrt{2}\ E_{y(eff)} \cos\gamma_{ref} \end{bmatrix}$$

In most cases: γ arbitrarely =0. $E_{y(eff)}$ =(e.g) 1.05

$$V_{SM(o)} = \begin{bmatrix} C_f\ E_{f(0)} \cos\beta_{SMo} \\ \hline -C_f\ E_{f(0)} \sin\beta_{SMo} \end{bmatrix}$$

For details: See Chapter 1.6, eqn. (1-109).

$$e_{SMr(o)} = \begin{bmatrix} K_f \cdot E_{f(o)} \\ 0 \\ 0 \end{bmatrix}$$

See (1-114).

Figure 2.6 Initial value of electrical network state variables $z_{(o)}$, when the variables $\beta_{SM(o)}, E_{f(o)}, \Omega_{SM(o)}, \Omega_{AM(o)}$ are (finally or tentatively) specified.

The solution (2-33) will provide the desired initial load flow, when such sets of specified variables $\beta_{SM(o)}$, $\mathbf{E}_{f(o)}$, $\Omega_{SM(o)}$, $\Omega_{AM(o)}$ are applied, that all imposed component- as well as systems related operational constraints are met. In case of infeasibility, proper adjustments must be made until the desired initial system status is established.

An efficient gradient technique for establishing the desired initial conditions is outlined next by way of applying it to the example system of Figure 2.1. Example initial operational constraints :

External infinite bus voltage : $E_{y(eff)}$ = 1.0pu.
Power supplied to the synchronous motor : $P_{SMtarget(o)}$ = -0.8pu (generator mode of operation)
Synchronous motor voltage : $E_{SMtarget(o)}$ = 1.0pu.
Power supplied to the asynchronous motor : $P_{AMtarget(o)}$ = 0.5 pu.

The iterative solution process comprises three main steps. Applied to the current case, the process becomes as follows:

1. *Stipulate initial value of $\beta_{SM(o)}$, $E_{f(o)}$, and $\Omega_{AM(o)}$*. Set final value $\Omega_{SM(o)}$ =1.0. Set $E_{y(eff)}$ as specified.

2. *Solve for the rest of the initial state variables* according to (2-33). Register as byproduct from the solution, the quantities ($P_{SM(o)}$, $E_{SM(o)}$, $P_{AM(o)}$) for which there are specified target values. Compute the deviations ($\Delta\mathbf{D}$) from target values:

$$\Delta\mathbf{D} = \begin{bmatrix} \Delta P_{SM(o)} \\ \Delta E_{SM(o)} \\ \Delta P_{AM(o)} \end{bmatrix} = \begin{bmatrix} P_{SM(o)} - P_{SMtarget(o)} \\ E_{SM(o)} - E_{SMtarget(o)} \\ P_{AM(o)} - P_{AMtarget(o)} \end{bmatrix} \qquad (2\text{-}36)$$

   If the absolute value of each deviation is below some individually set treshold, the sought initial solution is found, and exit is made from the iterative process.

   If the sought solution is (still) not found, a set of more appropriate values ($\beta_{SM(o)}$, $E_{f(o)}$, and $\Omega_{AM(o)}$) have to be applied. To this end ; go to label 3).

3. *Incrementally and simultaneously adjust $\beta_{SM(o)}$, $E_{f(o)}$, and $\Omega_{AM(o)}$* so that an improved initial power flow balance can be attained. The updated and improved value of respectively $\beta_{SM(o)}$, $E_{f(o)}$, and $\Omega_{AM(o)}$, can conveniantly be defined as currently available value plus a proper incremental correction to be determined at this stage of analysis :

   To evaluate the corrections ($\Delta\beta_{SM(o)}$, $\Delta E_{f(o)}$, $\Delta\Omega_{AM(o)}$) on a simultaneous basis, a sensitivity analysis is conducted to find the elements of the sensitivity matrix **S** of the defined relationship (2-37) :

$$\begin{bmatrix} \Delta P_{SM} \\ \Delta E_{SM} \\ \Delta P_{AM} \end{bmatrix} = \begin{bmatrix} & & \\ & \mathbf{S} & \\ & & \end{bmatrix} \cdot \begin{bmatrix} \Delta\beta_{SM} \\ \Delta E_{f} \\ \Delta\Omega_{AM} \end{bmatrix} \qquad (2\text{-}37)$$

   In the present case where **S** is a (3x3) matrix, this intermediate sensitivity analysis comprises 3 separate sensitivity computations:

   Firstly, the sensitivity of P_{SM}, E_{SM} and P_{AM} w.r.t. an incremental increase of β_{SM} is investigated. This is afforded by setting $\beta_{SM} = \beta_{SM(o)} + \Delta\beta_{SM}$, and solving (2-32) while $E_{f(o)}$ and $\Omega_{AM(o)}$ are kept unchanged (i.e. ΔE_f and $\Delta\Omega_{AM}$ of (2-37) are both zero). $\Delta\beta_{SM}$ = (say) 0.01rad. From solution (2-33), the new values (P_{SM}, E_{SM}, P_{AM}) are evaluated, and so also the associated incremental increases $\Delta P_{SM} = P_{SM} - P_{SM(o)}$, $\Delta E_{SM} = E_{SM} - E_{SM(o)}$ and $\Delta P_{AM} = P_{AM} - P_{AM(o)}$. The first column of **S** can now be computed, as we definitionwise have - since both ΔE_f and $\Delta\Omega_{AM} = 0$ in (2-37) - that $S_{11} = \Delta P_{SM}/\Delta\beta_{SM}$, $S_{21} = \Delta E_{SM}/\Delta\beta_{SM}$, $S_{31} = \Delta P_{AM}/\Delta\beta_{SM}$. β_{SM} is reset to $\beta_{SM(o)}$.

   Secondly, the sensitivity of P_{SM}, E_{SM} and P_{AM} w.r.t. an incremental increase of E_f is investigated.

This is afforded by setting $E_f = E_{f(o)} + \Delta E_f$, and solving (2-32) while $\beta_{SM(o)}$ and $\Omega_{AM(o)}$ are kept unchanged (i.e. $\Delta\beta_{SM}$ and $\Delta\Omega_{AM}$ of (2-37) are both zero). ΔE_f = (say) 0.01pu. From the new solution (2-33), values (P_{SM} , E_{SM} , P_{AM}) and associated marginal increases $\Delta P_{SM} = P_{SM} - P_{SM(0)}$, $\Delta E_{SM} = E_{SM} - E_{SM(o)}$ and $\Delta P_{AM} = P_{AM} - P_{AM(o)}$ are evaluated. The second column of **S** can now be found, as we definitionwise have that $S_{12} = \Delta P_{SM}/\Delta E_f$, $S_{22} = \Delta E_{SM}/\Delta E_f$, $S_{32} = \Delta P_{AM}/\Delta E_f$. E_f is reset to $E_{f(o)}$.

Thirdly, the sensitivity of P_{SM} , E_{SM} and P_{AM} w.r.t. an incremental increase of Ω_{AM} is investigated. This Is afforded by setting $\Omega_{AM} = \Omega_{AM(o)} + \Delta\Omega_{AM}$, and solving (2-32) while $\beta_{SM(o)}$ and $E_{f(o)}$ are kept unchanged (i.e. $\Delta\beta_{SM}$ and ΔE_f of (2-37) are both zero). $\Delta\Omega_{AM}$ = (say) 0.01pu. From the new solution (2-33) we evaluate the values (P_{SM} , E_{SM} , P_{AM}) and the associated marginal increases $\Delta P_{SM} = P_{SM} - P_{SM(o)}$, $\Delta E_{SM} = E_{SM} - E_{SM(o)}$ and $\Delta P_{AM} = P_{AM} - P_{AM(o)}$. The third and last column of **S** can now be found, as we definitionwise have that $S_{13} = \Delta P_{SM}/\Delta\Omega_{AM}$, $S_{23} = \Delta E_{SM}/\Delta\Omega_{AM}$, $S_{33} = \Delta P_{AM}/\Delta\Omega_{AM}$. Ω_{AM} is reset to $\Omega_{AM(o)}$.

With given sensitivity matrix **S** and prevailing deviations $\Delta\mathbf{D}$ relative to target values ($P_{SMtarget(o)}$, $E_{SMtarget(o)}$, $P_{AMtarget(o)}$), equations (2-37) is applied to estimate the set of increments ($\Delta\beta_{SM}$, ΔE_f , $\Delta\Omega_{AM}$) that will eliminate prevailing deviations: Using (- $\Delta\mathbf{D}$) as 'excitation' on the left side in (2-37), and solving w.r.t. the desired simultaneous increments become

$$\begin{bmatrix}\Delta\beta_{SM}\\ \Delta E_f\\ \Delta\Omega_{AM}\end{bmatrix} = -\mathbf{S}^{-1}\cdot\Delta\mathbf{D} \qquad (2\text{-}38)$$

The computed incremental values from (2-38) are then used to produce an updated and improved set of initial values ($\beta_{SM(o)}$, $E_{f(o)}$, $\Omega_{AM(o)}$) , in accordance with e.g. the dynamic update logic illustrated by (2-39).

$$\begin{aligned}\beta_{SM(o)} &\leftarrow \beta_{SM(o)} + k\,\Delta\beta_{SM}\\ E_{f(o)} &\leftarrow E_{f(o)} + k\,\Delta E_f\\ \Omega_{AM(o)} &\leftarrow \Omega_{AM(o)} + k\,\Delta\Omega_{AM}\end{aligned} \qquad (2\text{-}39)$$

k is a scalar factor of default value 1.0 . An alternative value in the prospective range ($0.0<k\leq 1.0$) implies in principle safer but slower convergence. Following update of initial conditions as specified by (2-39), return is made to step 2. of the iterative process.

3. Eigenvalue Analysis

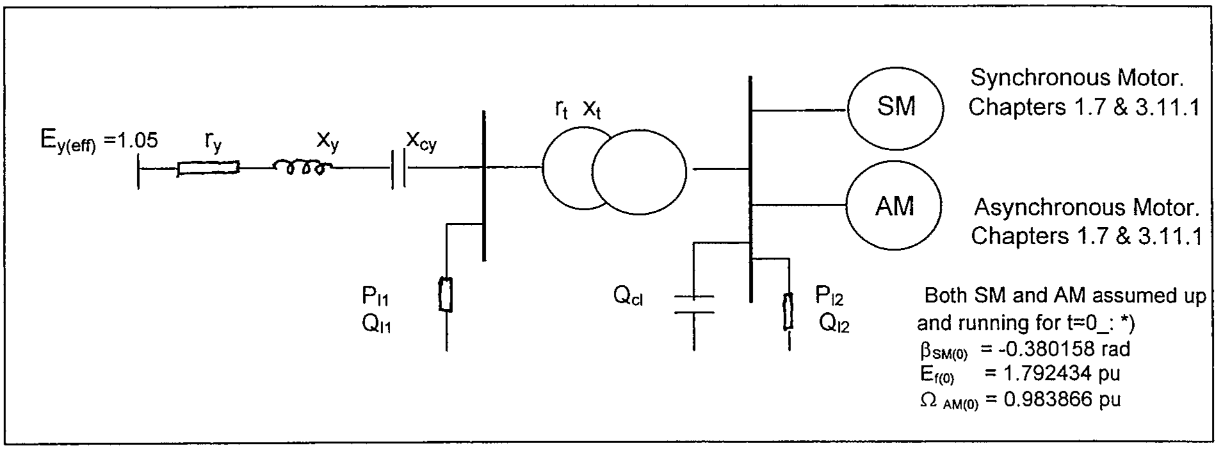
3. Eigenvalue Analysis

Given the description of an initial operational state of a power system. We seek the system's eigenvalues which describe the inherent dynamic characteristics of the system, when incrementally disturbed from the specified initial state.

If the power system's state variables are **z**, the stated task implies determining the eigenvalues associated with matrix **A** of the linearized formulation;

$$d\Delta\mathbf{z}/dt = \mathbf{A}\cdot\Delta\mathbf{z} \qquad (3\text{-}1)$$

where $\Delta\mathbf{z}$ comprises the incremental state variables. In the following the content of **A** pertaining to the system of Figure 3.1, is developed. Although specific in its approach, the presentation is generic in the sense that a general methodology is applied in modelling of incremental power system performance, and self- and mutual elements of matrix **A** are developed on general algorithmic form for state variables associated with all main types of power system components. As synchronous machine model is applied the model of Chapter 1.4 (which is based on the 5-coil generalised machine).



*)Specified initial conditions: P_{SM}=-0.8pu, E_{SM}=1.0pu, P_{AM}=0.5pu.The above implied values are found iteratively. Chapter 3.11.2

Figure 3.1: Example power system under study

The presentation is a three-stage outline; Chapter 3.1 - 3.9 develop the self- and mutual elements of **A**, Chapter 3.10 deals with placing them together to form **A**, and 3.11 discusses/checks sample results.

3.1 Performance of the derivative of incremental power system loop currents Δi_{loop}

From equation (2-20) of the systems analysis chapter 2, we have:

$$d\mathbf{i}_{loop}/dt = \omega_o\cdot\mathbf{X}_{Lloop}^{-1}\cdot(-\mathbf{R}_{loop}\cdot\mathbf{i}_{loop} - \mathbf{B}_{tc}\cdot\mathbf{e}_{tc} - \mathbf{e}_{chord} - \mathbf{B}_{t\text{-}rest}\cdot\mathbf{e}_{t\text{-}rest}) = (\omega_o\cdot\mathbf{X}_{Lloop}^{-1})\cdot\mathbf{Gi}_{loop} \qquad (3\text{-}2)$$

Here;

$\mathbf{Gi}_{loop}$ = $(-\mathbf{R}_{loop}\cdot\mathbf{i}_{loop}-\mathbf{B}_{tc}\cdot\mathbf{e}_{tc}- \mathbf{e}_{chord} - \mathbf{B}_{t\text{-}rest}\cdot\mathbf{e}_{t\text{-}rest})$ (For initial,steady state conditions: $\mathbf{Gi}_{loop}=\mathbf{Gi}_{loop(o)}=0$) (3-3)

$\mathbf{i}_{loop}$ = state variables in terms of system loop currents

$\mathbf{e}_{tc}$ = state variables in terms of capacitor voltages

$\mathbf{R}_{loop}$ = $\mathbf{B}\cdot\mathbf{R}_{primitive}\cdot\mathbf{B}^t$ = $(N_{loop}xN_{loop})$ system loop resistance matrix in the d-q axis frame of reference. $\mathbf{R}_{loop}$ is a function of the 'local' state variables β, Ω_{SM} and Ω_{AM}. (3-4)

$\mathbf{B}_{tc}$ = topological submatrix where the elements can be 0 , 1 and -1. See Chapter 2.2

$\mathbf{e}_{chords}$ = $[\Delta\mathbf{E}^t_{AM}, \Delta\mathbf{E}^t_{SM}, 0, 0, 0]^t$ = voltage sources vector associated with the *cotree* elements or *chords*. Fetched from the stock of machine models of Chapter 1.7:

$\Delta\mathbf{E}_{AM} = \mathbf{H}_{AM}\cdot\boldsymbol{\phi}_{AM}$

$\Delta\mathbf{E}_{SM} = \mathbf{V}_{SM} + \mathbf{H}_{SM}\cdot\boldsymbol{\phi}_{SM}$ (3-5) where;

$\phi_{AM} = [\phi_{rd}\ \phi_{rq}]^t$ = *asynchronous motor state variables* in terms of d- and q-axis rotor flux components

$\phi_{SM} = [\ \phi_f\ \ \phi_{kd}\ \ \phi_{kq}\ ]^t$ = *synchronous motor state variables* in terms of three rotor flux components associated with respectively the field winding ('f'), the equivalent d-axis damper winding ('kd') and the equivalent q-axis damper winding ('kq').

The state dependent machine matrices $\mathbf{H_{AM}}$, $\mathbf{H_{SM}}$ and $\mathbf{V_{SMf}}$ are fetched from Chapter 1.7 :

$$H_{AM} = (X_m/X_r)\cdot\begin{bmatrix} -(R_r/X_r) & -\Omega_{AM} \\ \Omega_{AM} & -(R_r/X_r) \end{bmatrix} \qquad (3\text{-}6)$$

$$H_{SM} = \begin{bmatrix} \Omega_{SM}\cdot f_1\cdot\sin\beta_{SM} + f_2\cdot\cos\beta_{SM} & \Omega_{SM}\cdot f_3\cdot\sin\beta_{SM} + f_4\cdot\cos\beta_{SM} & \Omega_{SM}\cdot f_5\cdot\cos\beta_{SM} + f_6\cdot\sin\beta_{SM} \\ \Omega_{SM}\cdot f_1\cdot\cos\beta_{SM} - f_2\cdot\sin\beta_{SM} & \Omega_{SM}\cdot f_3\cdot\cos\beta_{SM} - f_4\cdot\sin\beta_{SM} & -\Omega_{SM}\cdot f_5\cdot\sin\beta_{SM} + f_6\cdot\cos\beta_{SM} \end{bmatrix} \qquad (3\text{-}7)$$

$$V_{SM} = \begin{bmatrix} C_f\cdot(E_{f(o)} + \Delta E_f)\cdot\cos\beta_{SM} \\ -C_f\cdot(E_{f(o)} + \Delta E_f)\cdot\sin\beta_{SM} \end{bmatrix} \qquad (3\text{-}8)$$

From the foregoing it is observed that the derivative of the loop currents $\mathbf{i}_{loop}$ is a function of a subset of the state variables;

$$d\mathbf{i}_{loop}/dt = \mathbf{f}(\mathbf{i}_{loop}, \mathbf{e}_{tc}, \phi_{AM}, \phi_{SM}, \beta_{SM}, \Omega_{SM}, \Omega_{SM}, \Delta E_f) \qquad (3\text{-}9)$$

We seek the corresponding *incremental* network loop current behaviour, and can on the basis of (3-2) make the following formal algorithmic development for the ensuing analysis:

$$\Delta(d\mathbf{i}_{loop}/dt) = \Delta[(\omega_o\cdot\mathbf{X}_{Lloop}^{-1})\cdot\mathbf{G}i_{loop}] = \Delta[(\omega_o\cdot\mathbf{X}_{Lloop}^{-1})]\cdot\mathbf{G}i_{loop(o)} + (\omega_o\cdot\mathbf{X}_{Lloop(o)}^{-1})\cdot\Delta\mathbf{G}i_{loop} \qquad (3\text{-}10)$$

Since $\mathbf{G}i_{loop(o)} = 0$ for the initial steady state condition specified as basis for the eigenvalue analysis, we have;

$$d\Delta\mathbf{i}_{loop}/dt = (\omega_o\cdot\mathbf{X}_{Lloop(o)}^{-1})\cdot\Delta\mathbf{G}i_{loop} \qquad (3\text{-}11)$$

(3-11) provides the platform for determining the sought incremental performance $d\Delta\mathbf{i}_{loop}/dt$. To evaluate the content of $\Delta\mathbf{G}i_{loop}$ we formulate (3-12): Equation (3-9) implies that $d\mathbf{i}_{loop}/dt$ of the example, may depend on J =(5x2+2x2+(2+3)+3+1)= 23 individual state variables Δz_j . Thus we can formulate :

$$d\Delta\mathbf{i}_{loop}/dt = (\omega_o\cdot\mathbf{X}^{-1}_{Lloop(o)})\cdot\sum_{j=1}^{J=23}(\partial\mathbf{G}i_{loop}/\partial z_j)\cdot\Delta z_j = \sum_{j=1}^{J=23} Ai_{loopj}\cdot\Delta z_j = \mathbf{A}i_{loop}\cdot\Delta\mathbf{z}_{iloop} \qquad (3\text{-}12)$$

⊃:

$$d\Delta\mathbf{i}_{loop}/dt = [\mathbf{A}i_{loop}i_{loop}, \mathbf{A}i_{loop}e_{tc}, \mathbf{A}i_{loop}\phi_{AM}, \mathbf{A}i_{loop}\phi_{SM}, \mathbf{A}i_{loop}\beta_{SM}, \mathbf{A}i_{loop}\Omega_{SM}, \mathbf{A}i_{loop}\Omega_{AM}, \mathbf{A}i_{loop}E_{qf}]\cdot\begin{bmatrix} \Delta\mathbf{i}_{loop} \\ \Delta\mathbf{e}_{tc} \\ \Delta\phi_{AM} \\ \Delta\phi_{SM} \\ \Delta\beta_{SM} \\ \Delta\Omega_{SM} \\ \Delta\Omega_{SM} \\ \Delta(\Delta E_{qf}) \end{bmatrix} \qquad (3\text{-}13)$$

In the following the content of the coefficient matrix $\mathbf{A}i_{loop}$ is developed.

3.1.1 The elements $Ai_{loop}i_{loop}$ of coefficient matrix Ai_{loop}.

From (3-12) and (3-3):

$$Ai_{loop}i_{loop} = (\omega_o \cdot \mathbf{X}_{Lloop(o)}{}^{-1}) \cdot (\partial \mathbf{G}i_{loop} / \partial \mathbf{i}_{loop}) \qquad (3\text{-}14)$$

⊃:

$$\underset{(10x10)}{Ai_{loop}i_{loop}} = -(\omega_o \cdot \mathbf{X}_{Lloop(o)}{}^{-1}) \cdot \mathbf{R}_{loop} \qquad (3\text{-}15)$$

where;

$\mathbf{R}_{loop} = \mathbf{B} \cdot \mathbf{R}_{primitive} \cdot \mathbf{B}^t$ (see Appendix1)

$\mathbf{X}_{Lloop} = \mathbf{B} \cdot \mathbf{X}_{Lprimitive} \cdot \mathbf{B}^t$ ”

The elements of coefficient matrix $\mathbf{A}i_{loop}$ defining the influence on $d\Delta\mathbf{i}_{loop}/dt$ of the incremental system loop currents $\Delta\mathbf{i}_{loop}$

3.1.2 The elements $Ai_{loop}e_{tc}$ of coefficient matrix Ai_{loop}.

From (3-12) and (3-3):

$$Ai_{loop}e_{tc} = (\omega_o \cdot \mathbf{X}_{Lloop(o)}{}^{-1}) \cdot (\partial \mathbf{G}i_{loop} / \partial \mathbf{e}_{tc}) \qquad (3\text{-}16)$$

⊃:

$$\underset{(10x4)}{Ai_{loop}e_{tc}} = -(\omega_o \cdot \mathbf{X}_{Lloop(o)}{}^{-1}) \cdot \mathbf{B}_{tc} \qquad (3\text{-}17)$$

where;

$\mathbf{B}_{tc}$ = submatrix describing the incidence of loops and the tree elements that comprise capacitors. See Chapter 2.2.

The elements of coefficient matrix $\mathbf{A}i_{loop}$ defining the influence on $d\Delta\mathbf{i}_{loop}/dt$ of the incremental capacitor voltage components $\Delta\mathbf{e}_{tc}$.

3.1.3 The elements $Ai_{loop}\phi_{AM}$ of coefficient matrix Ai_{loop}.

From (3-12), (3-3) and (3-5):

$$Ai_{loop}\phi_{AM} = (\omega_o \cdot \mathbf{X}_{Lloop(o)}{}^{-1}) \cdot (\partial \mathbf{G}i_{loop} / \partial \phi_{AM}) = -(\omega_o \cdot \mathbf{X}_{Lloop(o)}{}^{-1}) \cdot (\partial \mathbf{e}_{chord} / \partial \phi_{AM}) \qquad (3\text{-}18)$$

where;

$$\partial \mathbf{e}_{chord} / \partial \phi_{AM} = \begin{bmatrix} \underset{(2x2)}{\mathbf{H}_{AM(o)}} \\ \hline \underset{(8x2)}{\mathbf{0}} \end{bmatrix} \qquad (3\text{-}19)$$

⊃:

$$\underset{(10x2)}{Ai_{loop}\phi_{AM}} = (\omega_o \cdot \mathbf{X}_{Lloop(o)}{}^{-1}) \cdot \begin{bmatrix} \underset{(2x2)}{-\mathbf{H}_{AM(0)}} \\ \hline \underset{(8x2)}{\mathbf{0}} \end{bmatrix} \qquad (3\text{-}20)$$

where;

$$\mathbf{H}_{AM(o)} = (X_m/X_r) \cdot \begin{bmatrix} -(R_r/X_r) & -\Omega_{AM(o)} \\ \Omega_{AM(o)} & -(R_r/X_r) \end{bmatrix} \qquad (3\text{-}21)$$

The elements of coefficient matrix $\mathbf{A}i_{loop}$ defining the influence on $d\Delta\mathbf{i}_{loop}/dt$ of the incremental asynchronous motor flux components $\Delta\phi_{AM}$.

3.1.4 The elements $Ai_{loop}\phi_{SM}$ of coefficient matrix Ai_{loop} .

From (3-12), (3-3), (3-5) and (3-7):

$$\mathbf{Ai}_{loop}\phi_{SM} = (\omega_o \cdot \mathbf{X}_{Lloop(o)}{}^{-1}) \cdot (\partial \mathbf{G}i_{loop}/\partial\phi_{\underline{SM}}) = -(\omega_o \cdot \mathbf{X}_{Lloop(o)}{}^{-1}) \cdot (\partial \mathbf{e}_{chord}/\partial\phi_{\underline{SM}}) \quad (3\text{-}22)$$

where;

$$\partial \mathbf{e}_{chord}/\partial\phi_{\underline{SM}} = \begin{bmatrix} \mathbf{0}\ (2x3) \\ \mathbf{H}_{SM(o)} \\ \mathbf{0}(6x3) \end{bmatrix} \quad (3\text{-}23)$$

⊃:

$$\underset{(10x3)}{\mathbf{Ai}_{loop}\phi_{SM}} = (\omega_o \cdot \mathbf{X}_{Lloop(o)}{}^{-1}) \cdot \begin{bmatrix} \mathbf{0}\ (2x3) \\ -\mathbf{H}_{SM(o)} \\ \mathbf{0}\ (6x3) \end{bmatrix} \quad (3\text{-}24)$$

where;

$$\mathbf{H}_{SM(o)} = \begin{bmatrix} f_1 \cdot \sin\beta_{SM(o)} + f_2 \cdot \cos\beta_{SM(o)} & f_3 \cdot \sin\beta_{SM(o)} + f_4 \cdot \cos\beta_{SM(o)} & f_5 \cdot \cos\beta_{SM(o)} + f_6 \cdot \sin\beta_{SM(o)} \\ f_1 \cdot \cos\beta_{SM(o)} - f_2 \cdot \sin\beta_{SM(o)} & f_3 \cdot \cos\beta_{SM(o)} - f_4 \cdot \sin\beta_{SM(o)} & -f_5 \cdot \sin\beta_{SM(o)} + f_6 \cdot \cos\beta_{SM(o)} \end{bmatrix} \quad (3\text{-}25)$$

The elements of coefficient matrix $\mathbf{Ai}_{loop}$ defining the influence on $d\Delta\mathbf{i}_{loop}/dt$ of the incremental synchronous motor flux components $\Delta\phi_{SM}$.

3.1.5 The elements $Ai_{loop}\beta_{SM}$ of coefficient matrix Ai_{loop} .

From (3-12), (3-3), (3-5), (3-7) and (3-8):

$$\mathbf{Ai}_{loop}\beta_{SM} = (\omega_o \cdot \mathbf{X}_{Lloop(o)}{}^{-1}) \cdot (\partial \mathbf{G}i_{loop}/\partial\beta_{SM})$$

$$= -(\omega_o \cdot \mathbf{X}_{Lloop(o)}{}^{-1}) \cdot (\partial(\mathbf{R}_{loop} \cdot \mathbf{i}_{loop(o)})/\partial\beta_{SM} + \partial \mathbf{e}_{chord}/\partial\beta_{SM}) \quad (3\text{-}26)$$

⊃:

$$\partial \mathbf{G}i_{loop}/\partial\beta_{SM} = \begin{bmatrix} \mathbf{0}\ (2x1) \\ -\mathbf{D}i_{SM}\beta_{SM(o)} \\ \mathbf{0}(6x1) \end{bmatrix} \quad (3\text{-}27)$$

⊃:

$$\underset{(10x1)}{\mathbf{Ai}_{loop}\beta_{SM}} = (\omega_o \cdot \mathbf{X}_{Lloop(o)}{}^{-1}) \cdot \begin{bmatrix} \mathbf{0}\ (2x1) \\ -\mathbf{D}i_{SM}\beta_{SM(o)} \\ \mathbf{0}(6x1) \end{bmatrix} \quad (3\text{-}28)$$

where ;

$$\mathbf{D}i_{SM}\beta_{SM(o)} = (\mathbf{D}i_{SM}R_{loop} + \mathbf{D}i_{SM}E_{SM} + \mathbf{D}i_{SM}h_{SM}) \quad (3\text{-}29)$$

$$\mathbf{D}i_{SM}R_{loop} = 2 \begin{bmatrix} (X'' \cdot \cos2\beta_{SM(o)} - X''_r \cdot \sin2\beta_{SM(o)}) & -(X'' \cdot \sin2\beta_{SM(o)} + X''_r \cdot \cos2\beta_{SM(o)}) \\ -(X'' \cdot \sin2\beta_{SM(o)} + X''_r \cdot \cos2\beta_{SM(o)}) & -(X'' \cdot \cos2\beta_{SM(o)} - X''_r \cdot \sin2\beta_{SM(o)}) \end{bmatrix} \cdot \mathbf{i}_{SM(o)} \quad (3\text{-}30)$$

$$\mathbf{D}i_{SM}E_{SM} = \begin{bmatrix} -C_f \cdot E_{f(0)} \cdot \sin\beta_{SM(o)} \\ -C_f \cdot E_{f(0)} \cdot \cos\beta_{SM(o)} \end{bmatrix} \quad (3\text{-}31)$$

$$\mathbf{D}i_{SM}h_{SM} = \begin{bmatrix} f_1 \cdot \cos\beta_{SM(o)} - f_2 \cdot \sin\beta_{SM(o)} & f_3 \cdot \cos\beta_{SM(o)} - f_4 \cdot \sin\beta_{SM(o)} & -f_5 \cdot \sin\beta_{SM(o)} + f_6 \cdot \cos\beta_{SM(o)} \\ -f_1 \cdot \sin\beta_{SM(o)} - f_2 \cdot \cos\beta_{SM(o)} & -f_3 \cdot \sin\beta_{SM(o)} - f_4 \cdot \cos\beta_{SM(o)} & -f_5 \cdot \cos\beta_{SM(o)} - f_6 \cdot \sin\beta_{SM(o)} \end{bmatrix} \cdot \phi_{SM(0)} \quad (3\text{-}32)$$

The elements of coefficient matrix $\mathbf{Ai}_{loop}$ defining the influence on $d\Delta\mathbf{i}_{loop}/dt$ of the synchronous motor incremental rotor angle $\Delta\beta$.

3.1.6 The elements $\mathbf{Ai}_{loop}\Omega_{SM}$ of coefficient matrix $\mathbf{Ai}_{loop}$.

From (3-12), (3-3), (3-5) and (3-7):

$$\mathbf{Ai}_{loop}\Omega_{SM} = (\omega_o \cdot \mathbf{X}_{Lloop(o)}{}^{-1}) \cdot (\partial \mathbf{Gi}_{loop}/\partial \Omega_{SM})$$

$$= -(\omega_o \cdot \mathbf{X}_{Lloop(0)}{}^{-1}) \cdot (\, \partial(\mathbf{R}_{loop} \cdot \mathbf{i}_{loop(o)})/\partial \Omega_{SM} + \partial \mathbf{e}_{chord}/\partial \Omega_{SM}\,) \qquad (3\text{-}33)$$

⊃:

$$\partial \mathbf{Gi}_{loop}/\partial \Omega_{SM} = \begin{bmatrix} \mathbf{0}\,(2x1) \\ -\mathbf{Di}_{SM}\Omega_{SM(o)} \\ \mathbf{0}(6x1) \end{bmatrix} \quad \text{where} \quad \mathbf{Di}_{SM}\Omega_{SM(o)} = \mathbf{Di}_{SM}\Omega_{SM(o)}(1) + \mathbf{Di}_{SM}\Omega_{SM(o)}(2) \qquad (3\text{-}34)$$

⊃:

$$\underset{(10x1)}{\mathbf{Ai}_{loop}\Omega_{SM}} = (\omega_o \cdot \mathbf{X}_{Lloop(o)}{}^{-1}) \cdot \begin{bmatrix} \mathbf{0}\,(2x1) \\ -\mathbf{Di}_{SM}\Omega_{SM(O)} \\ \mathbf{0}(6x1) \end{bmatrix} \qquad (3\text{-}35)$$

$$\mathbf{Di}_{SM}\Omega_{SM(o)}(1) = \begin{bmatrix} f_1 \cdot \sin\beta_{SM(o)} & f_3 \cdot \sin\beta_{SM(o)} & f_5 \cdot \cos\beta_{SM(o)} \\ f_1 \cdot \cos\beta_{SM(o)} & f_3 \cdot \cos\beta_{SM(o)} & -f_5 \cdot \sin\beta_{SM(o)} \end{bmatrix} \cdot \phi_{SM(o)} = \mathbf{T}_{SM1(o)} \cdot \mathbf{f}_{SM} \cdot \phi_{SM(o)}$$

$$\mathbf{Di}_{SM}\Omega_{SM(o)}(2) = 2 \cdot \bar{X}'' \cdot \begin{bmatrix} \sin 2\beta_{SM(o)} & \cos 2\beta_{SM(o)} \\ \cos 2\beta_{SM(o)} & -\sin 2\beta_{SM(o)} \end{bmatrix} \cdot \mathbf{i}_{SM(o)} \quad \text{where} \quad \bar{X}'' = 0.5(X''_d - X''_q) \qquad (3\text{-}36)$$

(see (1-111))

$$\mathbf{T}_{SM1(o)} = \begin{bmatrix} \sin\beta_{SM(o)} & -\cos\beta_{SM(o)} \\ \cos\beta_{SM(o)} & \sin\beta_{SM(o)} \end{bmatrix} \quad \text{and} \quad \mathbf{f}_{SM} = \begin{bmatrix} f_1 & f_3 & \\ & & -f_5 \end{bmatrix} \qquad (3\text{-}37)$$

The elements of coefficient matrix $\mathbf{Ai}_{loop}$ defining the influence on $d\Delta\mathbf{i}_{loop}/dt$ of the synchronous motor incremental rotor speed $\Delta\Omega_{SM}$.

3.1.7 The elements $\mathbf{Ai}_{loop}\Omega_{AM}$ of coefficient matrix $\mathbf{Ai}_{loop}$.

From (3-12), (3-3), (3-5) and (3-6):

$$\mathbf{Ai}_{loop}\Omega_{AM} = (\omega_o \cdot \mathbf{X}_{Lloop(o)}{}^{-1}) \cdot (\partial \mathbf{Gi}_{loop}/\partial \Omega_{AM}) = -(\omega_o \cdot \mathbf{X}_{Lloop(o)}{}^{-1}) \cdot \partial \mathbf{e}_{chord}/\partial \Omega_{AM} \qquad (3\text{-}38)$$

⊃:

$$\partial \mathbf{e}_{chord}/\partial \Omega_{AM} = \begin{bmatrix} \mathbf{Di}_{SM}\Omega_{AM(o)} \\ \mathbf{0}(8x1) \end{bmatrix} \qquad (3\text{-}39)$$

⊃:

$$\underset{(10x1)}{\mathbf{Ai}_{loop}\Omega_{AM}} = (\omega_o \cdot \mathbf{X}_{Lloop(o)}{}^{-1}) \cdot \begin{bmatrix} -\mathbf{Di}_{SM}\Omega_{AM(o)} \\ \mathbf{0}(8x1) \end{bmatrix} \qquad (3\text{-}40)$$

where;

$$\mathbf{Di}_{SM}\Omega_{AM(o)} = (X_m/X_r) \cdot \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \phi_{AM(0)} = (X_m/X_r) \cdot \bar{\mathbf{1}}^t \cdot \phi_{AM(o)} \quad \text{(see (1-129) for def. of } \bar{1}) \qquad (3\text{-}41)$$

The elements of coefficient matrix $\mathbf{Ai}_{loop}$ defining the influence on $d\Delta\mathbf{i}_{loop}/dt$ of the asynchronous motor incremental rotor speed $\Delta\Omega_{AM}$.

3.1.8 The elements $Ai_{loop}E_f$ of coefficient matrix Ai_{loop} .

From (3-12), (3-3), (3-5) and (3-8):

$$Ai_{loop}E_f = (\omega_o \cdot X_{Lloop(o)}{}^{-1}) \cdot (\partial Gi_{loop}/\partial \Delta E_f)$$
$$= -(\omega_o \cdot X_{Lloop(o)}{}^{-1}) \cdot \partial e_{chord}/\partial \Delta E_f \qquad (3\text{-}42)$$

⊃:

$$\partial e_{chord}/\partial \Delta E_{qf} = \begin{bmatrix} 0 \ (2x1) \\ Di_{SM}E_{f(o)} \\ 0 (6x1) \end{bmatrix} \qquad (3\text{-}43)$$

⊃:

$$\underset{(10x1)}{Ai_{loop}E_f} = (\omega_o \cdot X_{Lloop(o)}{}^{-1}) \cdot \begin{bmatrix} 0 \ (2x1) \\ -Di_{SM}E_{f(o)} \\ 0 (6x1) \end{bmatrix} \qquad (3\text{-}44)$$

where;

$$Di_{SM}E_{f(o)} = \begin{bmatrix} C_f \cdot \cos\beta_{SM(o)} \\ -C_f \cdot \sin\beta_{SM(o)} \end{bmatrix} \qquad (3\text{-}45)$$

The elements of coefficient matrix Ai_{loop} defining the influence on $d\Delta i_{loop}/dt$ of the synchronous motor incremental field voltage $\Delta(\Delta E_f)$.

3.2 Performance of the derivative of incremental capacitor voltages Δe_{tc}

The variation of the voltages e_{tc} is given by equations (2-20) of the systems analysis Chapter 2:

$$de_{tc}/dt = \omega_o \cdot (X_{Cprimitive} \cdot B_{tc}{}^t \cdot i_{loop} + \bar{1}_{tc} \cdot e_{tc}) = \omega_o \cdot Ge_c \qquad (3\text{-}46)$$

Here;

$$Ge_c = (X_{Cprimitive} \cdot B_{tc}{}^t \cdot i_{loop} + \bar{1}_{tc} \cdot e_{tc}) \qquad (3\text{-}47)$$

- e_{tc} = state variables in terms of capacitor voltages
- i_{loop} = state variables in terms of system loop currents
- $X_{Cprimitive}$ = diagonal (N_cxN_c) matrix (in the d-q axis frame of reference) containing the sequence of component reactances X_c along the main diagonal. N_C is the number of capacitor banks in the system. For definition of X_c, see Chapter 1.3, p.1/6.
- $\bar{1}_{tc}$ = (N_CxN_C) special 'diagonal' matrix that comprises '0'- and '1'-elements. See p.2/6 for definition.
- B_{tc} = submatrix describing the incidence of loops and the *tree* elements that comprise capacitors. See (2-3) of Chapter 2.2 for illustration.

From the foregoing we see that Ge_c is a function of a subset of the state variables:

$$Ge_c = f(i_{loop}, e_{tc}) \qquad (3\text{-}48)$$

We seek the incremental capacitor voltage behaviour and can on the basis of (3-46) make the following 'platform' for the ensuing analysis:

$$\Delta(de_{tc}/dt) = \Delta(\omega_o \cdot Ge_c)$$

⊃:

$$d\Delta e_{tc}/dt = \omega_o \cdot \Delta Ge_c \qquad (3\text{-}49)$$

(3-49) provides the basis for determining the sought incremental performance $d\Delta e_{tc}/dt$. Equation (3-48) implies that ΔGe_c may depend on J=(5x2+2x2)=14 individual state variables Δz_j . Thus we can formulate:

$$d\Delta e_{tc}/dt = \omega_o \cdot \sum_{j=1}^{J=14} (\partial Ge_c/\partial z_j) \cdot \Delta z_j = \sum_{j=1}^{J=14} Ae_{cj} \cdot \Delta z_j = Ae_c \cdot \Delta z_{ec} \qquad (3\text{-}50)$$

⊃:

$$d\Delta e_{tc}/dt = [Ae_c i_{loop}, Ae_c e_c] \cdot [\Delta i^t_{loop}, \Delta e^t_{tc}]^t \qquad (3\text{-}51)$$

In the following we will develop the (here 14) elements of the coefficient vector Ae_c .

3.2.1 The elements $Ae_c i_{loop}$ of coefficient matrix Ae_c.

From (3-47) and (3-50):

$$Ae_c i_{loop} = \omega_\delta\ (\partial \mathbf{G}e_c / \partial \mathbf{i}_{loop}) \qquad (3\text{-}52)$$

⊃:

$$\underset{(4x10)}{Ae_c i_{loop}} = \omega_\delta\ \mathbf{X}_{Cprimitive} \cdot \mathbf{B}_{tc}^{\,t} \qquad (3\text{-}53)$$

The elements of coefficient matrix $\mathbf{Ae_c}$ defining the influence on $d\Delta\mathbf{e}_{tc}/dt$ of the incremental system loop currents $\Delta\mathbf{i}_{loop}$

3.2.2 The elements $Ae_c e_c$ of coefficient matrix Ae_c.

From (3-47) and (3-50):

$$Ae_c e_c = \omega_\delta\ (\partial \mathbf{G}e_c / \partial \mathbf{e}_{tc}) \qquad (3\text{-}54)$$

⊃:

$$\underset{(4x4)}{Ae_c e_c} = \omega_\delta\ \bar{\mathbf{1}}_{tc} \qquad (3\text{-}55)$$

The elements of coefficient matrix $\mathbf{Ae_c}$ defining the influence on $d\Delta\mathbf{e}_{tc}/dt$ of the incremental capacitor voltages $\Delta\mathbf{e}_{tc}$.

3.2 Performance of the derivative of asynchronous motor incremental flux components $\Delta\phi_{AM}$

The variation of the asynchronous motor fluxes $\boldsymbol{\phi}_{AM}$ is given by equations (1-126) of systems component Chapter 1.7:

$$d\phi_{AM}/dt = \omega_\delta\ (\mathbf{F}_{AMi} \cdot \mathbf{i}_{AM} + \mathbf{F}_{AM\phi}\ \boldsymbol{\phi}_{AM}) = \omega_\delta\ \mathbf{G}\phi_{AM} \qquad (3\text{-}56)$$

Here;

$$\mathbf{G}\phi_{AM} = (\mathbf{F}_{AMi} \cdot \mathbf{i}_{AM} + \mathbf{F}_{AM\phi}\ \boldsymbol{\phi}_{AM}) \qquad (3\text{-}57)$$

$\boldsymbol{\phi}_{AM}$ = state variables in terms of asynchronous motor flux components

$\mathbf{i}_{AM}$ = state variables in terms of asynchronous motor current components (=current components of network loop no. '1')

$\mathbf{F}_{AMii}$ =

$(R_r \cdot X_m/X_r)$	
	$(R_r \cdot X_m/X_r)$

(see (1-127))

$\mathbf{F}_{AM\phi}$ =

$-(R_r/X_r)$	$(1-\Omega_{AM})$
$(\Omega_{AM}-1)$	$-(R_r/X_r)$

From the foregoing it is observed that $\mathbf{G}\phi_{AM}$ is a function of a subset of the state variables:

$$\mathbf{G}\phi_{AM} = f(\mathbf{i}_{AM}, \boldsymbol{\phi}_{AM}, \Omega_{AM}) \qquad (3\text{-}58)$$

We seek the incremental asynchronous motor flux behaviour and can on the basis of (3-56) develop the following 'platform' for the ensuing analysis:

$$\Delta(d\phi_{AM}/dt) = \Delta(\omega_\delta\ \mathbf{G}\phi_{AM})$$

⊃:

$$d\Delta\phi_{AM}/dt = \omega_\delta\ \Delta\mathbf{G}\phi_{AM} \qquad (3\text{-}59)$$

(3-59) provides the basis for determining the sought incremental performance $d\Delta\phi_{AM}/dt$. Equation (3-58) implies that $\Delta\mathbf{G}\phi_{AM}$ may depend on J=(2+2+1)=5 individual state variables Δz_j . Thus we can formulate:

$$d\Delta\phi_{AM}/dt = \omega_\delta \sum_{j=1}^{J=5} (\partial \mathbf{G}\phi_{AM}/\partial z_j)\ \Delta z_j = \sum_{j=1}^{J=5} A\phi_{AM}j\ \Delta z_j = \mathbf{A}\phi_{AM} \cdot \Delta\mathbf{z}\phi_{AM} \qquad (3\text{-}60)$$

⊃:

$$d\Delta\phi_{AM}/dt = [\mathbf{A}\phi_{AM}i_{AM}, \mathbf{A}\phi_{AM}\phi_{AM}, \mathbf{A}\phi_{AM}\Omega_{AM}]\ [\Delta\mathbf{i}^t_{AM}, \Delta\boldsymbol{\phi}^t_{AM}, \Delta\Omega_{AM}]^t \qquad (3\text{-}61)$$

In the following we will develop the (here 5) elements of the coefficient matrix $\mathbf{A}\phi_{AM}$.

3.3.1 The elements $A\phi_{AM}i_{AM}$ of coefficient matrix $A\phi_{AM}$.

From (3-57) and (3-60):

$$A\phi_{AM}i_{AM} = \omega_b\ (\partial \mathbf{G}\phi_{AM}/\partial \mathbf{i}_{AM}) \quad (3\text{-}62)$$

⊃:

$$\underset{(2x2)}{A\phi_{AM}i_{AM}} = \omega_b\ \mathbf{F}_{AMi} \quad (3\text{-}63)$$

where;

F_{AMi} =

$(R_r\ X_m/Xr)$	
	$(R_r\ X_m/Xr)$

The elements of coefficient matrix $\mathbf{A}\phi_{AM}$ defining the influence on $d\Delta\phi_{AM}/dt$ of the asynchronous motor's incremental stator currents $\Delta \mathbf{i}_{AM}$

3.3.2 The elements $A\phi_{AM}\phi_{AM}$ of coefficient matrix $A\phi_{AM}$.

From (3-57) and (3-60):

$$A\phi_{AM}\phi_{AM} = \omega_b\ (\partial \mathbf{G}\phi_{AM}/\partial \boldsymbol{\phi}_{AM}) \quad (3\text{-}64)$$

⊃:

$$\underset{(2x2)}{A\phi_{AM}\phi_{AM}} = \omega_b\ \mathbf{F}_{AM\phi(o)i} \quad (3\text{-}65)$$

where;

$F_{AM\phi(o)}$ = (3-66)

$-(R_r/X_r)$	$(1-\Omega_{AM(o)})$
$(\Omega_{AM(o)}-1)$	$-(R_r/X_r)$

The elements of coefficient matrix $\mathbf{A}\phi_{AM}$ defining the influence on $d\Delta\phi_{AM}/dt$ of the asynchronous motor's incremental flux components $\Delta\boldsymbol{\phi}_{AM}$

3.3.3 The elements $A\phi_{AM}\Omega_{AM}$ of coefficient matrix $A\phi_{AM}$.

From (3-57) and (3-60):

$$A\phi_{AM}\Omega_{AM} = \omega_b\ (\partial \mathbf{G}\phi_{AM}/\partial \Omega_{AM}) \quad (3\text{-}67)$$

⊃:

$$\underset{(2x1)}{A\phi_{AM}\Omega_{AM}} = \omega_b\ \bar{1}^t \cdot \phi_{AM(o)i} \quad (3\text{-}68)$$

where;

$\bar{1}^t$ =

	-1
1	

(For def. of $\bar{1}$, see Chapter 1.3)

The elements of coefficient matrix $\mathbf{A}\phi_{AM}$ defining the influence on $d\Delta\phi_{AM}/dt$ of the asynchronous motor's incremental speed $\Delta\Omega_{AM}$

3.4 Performance of the derivative of synchronous motor incremental flux components $\Delta\phi_{SM}$

The variation of the synchronous motor fluxes ϕ_{SM} is given by equations (1-113) of systems component Chapter 1.7 :

$$d\phi_{SM}/dt = \omega_b\ (e_{SMr} + F_{SMi}\ i_{SM} + F_{SM\phi}\ \phi_{SM}) = \omega_b\ G\phi_{SM} \qquad (3\text{-}69)$$

Here;

$$G\phi_{SM} = (e_{SMr} + F_{SMi}\cdot i_{SM} + F_{SM\phi}\ \phi_{SM}) \qquad (3\text{-}70)$$

ϕ_{SM} = state variables in terms of synchronous motor flux components

i_{SM} = state variables in terms of synchronous motor current components (=current components of network loop '2')

e_{SMr} =

$K_f\ (E_{f(o)}+\Delta E_f)$
0
0

(where $K_f = (\sqrt{2}/(\omega_b\ T'_{do}))\ X_{ad}/(X_d - X'_d)$)

F_{Smi} =

$(1/(\omega_b T'_{do}))\cdot (X_{ad}/X'_{ad})\ X''_{ad}\ \cos\beta_{SM}$	$-(1/(\omega_b T'_{do}))\ (X_{ad}/X'_{ad})\ X''_{ad}\ \sin\beta_{SM}$
$(1/(\omega_b T''_{do}))\cdot X'_{ad}\ \cos\beta_{SM}$	$-(1/(\omega_b T''_{do}))\ X'_{ad}\ \sin\beta_{SM}$
$(1/(\omega_b T''_{qo}))\cdot X_{aq}\ \sin\beta_{SM}$	$(1/(\omega_b T''_{qo}))\ X_{aq}\ \cos\beta_{SM}$

$F_{SM\phi}$ =

$-(1/(\omega_b T'_{do}))\ (1/X'_{ad})\ [\ (X_{ad}/X'_{ad})\ (X'_d - X''_d) + X''_{ad}\ ]$	$(1/(\omega_b T'_{do}))\ (X_{ad}/X'^2_{ad})\ (X'_d - X''_d)$	
$(1/(\omega_b T''_{do}))\ (1/X_{ad})\ (X_d - X'_d)$	$-1/(\omega_b T''_{do})$	
		$-1/(\omega_b T''_{qo})$

From the foregoing it is noticed that $G\phi_{SM}$ is a function of a subset of the state variables:

$$G\phi_{SM} = f(i_{SM},\ \phi_{SM},\ \beta_{SM},\ \Delta E_f) \qquad (3\text{-}71)$$

We seek the incremental synchronous motor flux behaviour and can on the basis of (3-69), (3-70) and (3-71), develop the following 'platform' for the ensuing analysis:

$$\Delta(d\phi_{SM}/dt) = \Delta(\omega_b\ G\phi_{SM})$$

⊃:

$$d\Delta\phi_{SM}/dt = \omega_b\ \Delta G\phi_{SM} \qquad (3\text{-}72)$$

(3-72) provides the basis for determining the sought incremental performance $d\Delta\phi_{SM}/dt$. Equation (3-71) implies that $\Delta G\phi_{SM}$ will depend on J = (2+3+1+1)=7 individual state variables Δz_j . Thus we can formulate:

$$d\Delta\phi_{SM}/dt = \omega_b \sum_{j=1}^{J=7} (\partial G\phi_{SM}/\partial z_j)\ \Delta z_j = \sum_{j=1}^{J=7} A\phi_{SMj}\ \Delta z_j = A\phi_{SM}\ \Delta z\phi_{SM} \qquad (3\text{-}73)$$

⊃:

$$d\Delta\phi_{SM}/dt = [A\phi_{SMi_{SM}},\ A\phi_{SM\phi_{SM}},\ A\phi_{SM\beta_{SM}},\ A\phi_{SM\Delta Ef}]\ [\Delta i^t_{SM},\ \Delta\phi^t_{SM},\ \Delta\beta_{SM},\ \Delta(\Delta E_f)]^t \qquad (3\text{-}74)$$

_In the following we will develop the (here 7) elements of the coefficient vector $A\phi_{SM}$.

3.4.1 The elements $A\phi_{SMi_{SM}}$ of coefficient matrix $A\phi_{SM}$.

From (3-69) - (3-74):

$$A\phi_{SMi_{SM}} = \omega_b\ (\partial G\phi_{SM}/\partial i_{SM}) \qquad (3\text{-}75)$$

⊃:

$$\mathbf{A}\phi_{\underset{(3\times2)}{SM}}i_{SM} = \omega_b\ \mathbf{F}_{\mathbf{SMi(o)}} \qquad (3\text{-}76)$$

where;

$F_{SMi(o)}$ =

$(1/(\omega_b T'_{do}))\ (X_{ad}/X'_{ad})\ X''_{ad}\ \cos\beta_{SM(o)}$	$-\ (1/(\omega_b T'_{do}))\ (X_{ad}/X'_{ad})\ X''_{ad}\ \sin\beta_{SM(o)}$
$(1/(\omega_b T''_{do}))\ X'_{ad}\ \cos\beta_{SM(o)}$	$-\ (1/(\omega_b T''_{do}))\ X'_{ad}\ \sin\beta_{SM(o)}$
$(1/(\omega_b T''_{qo}))\ X'_{aq}\ \sin\beta_{SM(o)}$	$(1/(\omega_b T''_{qo}))\ X'_{aq}\ \cos\beta_{SM(o)}$

The elements of coefficient matrix $\mathbf{A}\phi_{SM}$ defining the influence on $d\Delta\phi_{SM}/dt$ of the synchronous motor's incremental stator currents $\Delta\mathbf{i}_{\mathbf{SM}}$

3.4.2 The elements $\mathbf{A}\phi_{SM}\phi_{SM}$ of coefficient matrix $\mathbf{A}\phi_{SM}$.

From (3-69) - (3-74):

$$\mathbf{A}\phi_{SM}\phi_{SM} = \omega_b\ (\partial \mathbf{G}\phi_{SM}/\partial \boldsymbol{\phi}_{\mathbf{SM}}) \qquad (3\text{-}77)$$

⊃:

$$\mathbf{A}\phi_{\underset{(3\times3)}{SM}}\phi_{SM} = \omega_b\ \mathbf{F}_{\mathbf{SM}\boldsymbol{\phi}} \qquad (3\text{-}78)$$

where;

$F_{SM\phi}$ =

$-(1/(\omega_b T'_{do}))\ (1/X'_{ad})\ [\ (X_{ad}/X'_{ad})\ (X'_d - X''_d) + X''_{ad}\ ]$	$(1/(\omega_b T'_{do}))\ (X_{ad}/X'^{2}_{ad})\ (X'_d - X''_d)$	
$(1/(\omega_b T''_{do}))\ (1/X_{ad})\ (X_d - X'_d)$	$-\ 1/(\omega_b T''_{do})$	
		$-\ 1/(\omega_b T''_{qo})$

The elements of coefficient matrix $\mathbf{A}\phi_{SM}$ defining the influence on $d\Delta\phi_{SM}/dt$ of the synchronous motor's incremental flux components $\Delta\boldsymbol{\phi}_{\mathbf{SM}}$.

3.4.3 The elements $\mathbf{A}\phi_{SM}\beta_{SM}$ of coefficient matrix $\mathbf{A}\phi_{SM}$.

From (3-69) - (3-74):

$$\mathbf{A}\phi_{SM}\beta_{SM} = \omega_b\ (\partial \mathbf{G}\phi_{SM}/\partial \boldsymbol{\beta}_{\mathbf{SM}}) \qquad (3\text{-}79)$$

⊃:

$$\mathbf{A}\phi_{\underset{(3\times2)}{SM}}\beta_{SM} = \omega_b\ (\partial \mathbf{F}_{SMi}/\partial \boldsymbol{\beta}_{\mathbf{SM}}) \qquad (3\text{-}80)$$

where ;

$\partial F_{SMi}/\partial \beta_{SM}$ =

$-(1/(\omega_b T'_{do}))\ (X_{ad}/X'_{ad})\ X''_{ad}\ \sin\beta_{SM(o)}$	$-\ (1/(\omega_b T'_{do}))\ (X_{ad}/X'_{ad})\ X''_{ad}\ \cos\beta_{SM(o)}$
$-(1/(\omega_b T''_{do}))\ X'_{ad}\ \sin\beta_{SM(o)}$	$-\ (1/(\omega_b T''_{do}))\ X'_{ad}\ \cos\beta_{SM(o)}$
$(1/(\omega_b T''_{qo}))\ X'_{aq}\ \cos\beta_{SM(o)}$	$-\ (1/(\omega_b T''_{qo}))\ X'_{aq}\ \sin\beta_{SM(o)}$

$\cdot\ i_{SM(o)}$

The elements of coefficient matrix $\mathbf{A}\phi_{SM}$ defining the influence on $d\Delta\phi_{SM}/dt$ of the synchronous motor's incremental rotor angle $\Delta\beta_{\mathbf{SM}}$.

3.4.4 The elements $\mathbf{A}\phi_{SM\Delta Ef}$ of coefficient matrix $\mathbf{A}\phi_{SM}$.

From (3-69)- (3-74) :

$$\mathbf{A}\phi_{SM\Delta Ef} = \omega_b\ (\partial \mathbf{G}\phi_{SM}/\partial \Delta E_f) \qquad (3\text{-}81)$$

⊃:

$$\mathbf{A}\phi_{\underset{(3\times1)}{SM\Delta Ef}} = \omega_b\ (\partial \mathbf{e}_{SMr}/\partial \Delta E_f) = \begin{bmatrix} \omega_b K_f \\ 0 \\ 0 \end{bmatrix} \qquad (3\text{-}82)$$

The elements of coefficient matrix $\mathbf{A}\phi_{SM}$ defining the influence on $d\Delta\phi_{SM}/dt$ of the synchronous motor's incremental field voltage $\Delta(\Delta E_f)$.

3.5 Performance of the derivative of synchronous motor incremental rotor angle $\Delta\beta_{SM}$

The variation of the synchronous motor rotor angle β is given by equation (1-119) of the systems component Chapter 1.6:

$$d\beta_{SM}/dt = \omega_b\ (1 - \Omega_{SM}) = \omega_b\ G\beta_{SM} \quad (3\text{-}83)$$

Here;

$$G\beta_{SM} = (1 - \Omega_{SM}) \quad (3\text{-}84)$$

β_{SM} = state variable in terms of synchronous motor rotor angle β_{SM}. β_{SM} describes the movement of motor rotor relative to the synchronous speed of the chosen reference phasor (which here is the exogenously applied voltage).

Ω_{SM} = state variable in terms of synchronous motor per unit (pu) rotor speed.

From the foregoing we see that $G\beta_{SM}$ is a function of a very limited subset of the state variables:

$$G\beta_{SM} = f(\Omega_{SM}) \quad (3\text{-}85)$$

We seek the incremental synchronous motor rotor angle behaviour and can on the basis of (3-83) – (3-85) develop the following 'platform' for the ensuing analysis:

$$\Delta(d\beta_{SM}/dt) = \Delta(\omega_b\ G\beta_{SM})$$

$\supset$:

$$d\Delta\beta_{SM}/dt = \omega_b\ \Delta G\beta_{SM} \quad (3\text{-}86)$$

(3-86) provides the basis for determining the sought incremental performance $d\Delta\beta_{SM}/dt$. Equation (3-85) implies that $\Delta G\beta_{SM}$ will depend on J=1 state variable; $\Delta z_j = \Delta z_1 = \Omega_{SM}$. Thus we can formulate:

$$d\Delta\beta_{SM}/dt = \omega_b \sum_{j=1}^{J=1}(\partial G\beta_{SM}/\partial z_j)\ \Delta z_j = A\beta_{SM}\Omega_{SM} \cdot \Delta\Omega_{SM} = \mathbf{A}\beta_{SM}\ \mathbf{\Delta z}\beta_{SM} \quad (3\text{-}87)$$

_In the following we will develop the single element $A\beta_{SM}\Omega_{SM}$ that is the content of $\mathbf{A}\beta_{SM}$.

3.5.1 The element $A\beta_{SM}\Omega_{SM}$ of coefficient matrix $\mathbf{A}\beta_{SM}$.

From (3-83) - (3-87):

$$A\beta_{SM}\Omega_{SM} = \omega_b\ (\partial G\beta_{SM}/\partial\Omega_{SM}) \quad (3\text{-}88)$$

$\supset$:

$$\underset{(1x1)}{A\beta_{SM}\Omega_{SM}} = - \omega_b \quad (3\text{-}89)$$

The element of coefficient matrix $\mathbf{A}\beta_{SM}$ defining the influence on $d\Delta\beta_{SM}$ of the synchronous motor's incremental rotor speed $\Delta\Omega_{SM}$.

3.6 Performance of the derivative of synchronous motor incremental speed $\Delta\Omega_{SM}$

The variation of the synchronous motor pu speed $\mathbf{\Omega_{SM}}$ is given by equation (1-116) of the systems component Chapter 1.7 :

$$d\Omega_{SM}/dt = (S_{Bas}/S_{SM})\ (1/(T_{å}\ \cos\varphi_N))\ (\ T_{SMel} - T_{SMmec}\ ) = C_{SM}\cdot G\Omega_{SM} \qquad (3\text{-}90)$$

Here;

C_{SM} = $(S_{Bas}/S_{SM})\ (1/(T_{å}\cos\varphi_N))$

$G\Omega_{SM}$ = $(T_{SMel} - T_{SMmec})$ (3-91)

$\mathbf{\Omega_{SM}}$ = state variable in terms of synchronous motor pu rotor speed

T_{Smel} = $0.5\ \mathbf{i}^t_{SM}\ \mathbf{T_{SM1}}\ \phi_{dq}$ = electrical motor torque, where $\phi_{dq} = \mathbf{X}''_{SM}\ \mathbf{T_{SM}}\ \mathbf{i_{SM}} + \mathbf{f}_{SM}\ \phi_{SM}$

= $0.5\ \mathbf{i}^t_{SM}\ \mathbf{T_{SM1}}\ \mathbf{X}''_{SM}\ \mathbf{T_{SM}}\ \mathbf{i_{SM}} +\ 0.5\ \mathbf{i}^t_{SM}\ \mathbf{T_{SM1}}\ \mathbf{f}_{SM}\ \phi_{SM}$. See (1-118) for definition of $\mathbf{f}_{SM}$. (3-92)

$\mathbf{i_{SM}}$ = state variables in terms of synchronous motor current components (=current components of system loop '2')

$$\mathbf{T_{SM1}} = \begin{bmatrix} \sin\beta & -\cos\beta \\ \cos\beta & \sin\beta \end{bmatrix} \qquad \mathbf{T_{SM}} = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} \qquad \mathbf{X}''_{SM} = \begin{bmatrix} X''_d & \\ & X''_q \end{bmatrix} \qquad \mathbf{f}_{SM} = \begin{bmatrix} f_1 & f_3 & \\ & & -f_5 \end{bmatrix}$$

$T_{SMmec} = T_{SMmec(o)}\ \Omega_{SM}^{\kappa}$ = mechanical torque in **motor** mode of operation. (Motor operation implies pos. sign of mech. torque)

If the motor is up and running at t=-0: $T_{SMmec(o)} = T_{SMel(o)}$ = electrical motor torque at t = -0. This is found from equation (1-117) applied to the initial power system load flow. κ= (say) 1.5-3.5

If the motor is to be started from stillstand (as e.g. an asynchronous motor) : $T_{SMmec(o)}$ = coefficient to model mechanical friction, air resistance, etc. during startup. Probable range: 0.02-0.05

$T_{SMmec} = (T_{SMel(o)} + \Delta T_{mec})$= mechanical torque in **generator** mode of operation. ΔT_{mec} is the response from the *power control system.* See below for a sample hydro generator power control system.

S_{Bas}, S_{AM} = Chosen MVA system power base and rated MVA motor capacity, respectively

$T_a, \cos\varphi_N$ = Dynamical system's inertia constant and motor's rated power factor, respectively

From the foregoing we see that $G\Omega_{SM}$ can be expressed as a function of a subset of the state variables:

$$G\Omega_{SM} = f(\mathbf{i_{SM}}, \phi_{SM}, \beta_{SM}, \Omega_{SM}, \Delta å, \Delta g) \qquad (3\text{-}94)$$

We seek the incremental synchronous motor speed behaviour and can on the basis of (3-90) develop the following 'platform' for the ensuing analysis:

$$\Delta(d\Omega_{SM}/dt) = \Delta(C_{SM}\ G\Omega_{SM})$$

⊃:

$$d\Delta\Omega_{SM}/dt = C_{SM}\ \Delta G\Omega_{SM} \qquad (3\text{-}95)$$

(3-95) provides the basis for finding the sought incremental performance $d\Delta\Omega_{SM}/dt$ .Equation (3-94) implies that $\Delta G\Omega_{SM}$ may depend on J= (2+3+1+1+1+1) = 9 individual state variables Δz_j. Thus we can formulate:

$$d\Delta\Omega_{SM}/dt = C_{SM} \sum_{j=1}^{J=9}(\partial G\Omega_{SM}/\partial z_j)\ \Delta z_j = \sum_{j=1}^{J=9} A\Omega_{SMj}\ \Delta z_j = \mathbf{A\Omega_{SM}}\ \mathbf{\Delta z_{\Omega SM}} \qquad (3\text{-}96)$$

⊃:

$$d\Delta\Omega_{SM}/dt = [\mathbf{A\Omega_{SM}i_{SM}}, \mathbf{A\Omega_{SM}\phi_{SM}}, A\Omega_{SM}\beta_{SM}, A\Omega_{SM}\Omega_{SM}, A\Omega_{SM}\Delta_{å}, A\Omega_{SM}\Delta_g]\cdot \begin{bmatrix} \mathbf{\Delta i_{SM}} \\ \mathbf{\Delta\phi_{SM}} \\ \Delta\beta_{SM} \\ \Delta\Omega_{SM} \\ \Delta(\Delta å) \\ \Delta(\Delta g) \end{bmatrix} \qquad (3\text{-}97)$$

In the following we will develop the (here) 9 elements of the coefficient matrix $\mathbf{A\Omega_{SM}}$.

3.6.1 The elements $A\Omega_{SM}i_{SM}$ of coefficient matrix $A\Omega_{SM}$.
From (3-90) - (3-97):

$$A_{\Omega SM}i_{SM} = C_{SM}\ (\partial G_{\Omega SM}/\partial \mathbf{i}_{SM}) \qquad (3\text{-}98)$$

⊃:

$$\underset{(1\times 2)}{A\Omega_{SM}i_{SM}} = A_{\Omega SM}i_{SM}(1) + A_{\Omega SM}i_{SM}(2) \qquad (3\text{-}99)$$

where;

$$A\Omega_{SM}i_{SM}(1) = 0.5\ C_{SM}\ \mathbf{i}^t_{SM(o)}\ \left((\mathbf{T}_{SM1(o)}\ \mathbf{X}''\ \mathbf{T}_{SM(o)}) + (\mathbf{T}_{SM1(o)}\ \mathbf{X}''\ \mathbf{T}_{SM(o)})^t \right) \qquad (3\text{-}100)$$

$$= 0.5\ C_{SM}\ (X''_d - X''_q)\ \mathbf{i}^t_{SM(o)} \begin{bmatrix} \sin 2\beta_{SM(o)} & \cos 2\beta_{SM(o)} \\ \cos 2\beta_{SM(o)} & -\sin 2\beta_{SM(o)} \end{bmatrix}$$

$$A\Omega_{SM}i_{SM}(2) = 0.5\ C_{SM}\ (\mathbf{T}_{SM1(o)}\ \mathbf{f}_{SM}\ \phi_{SM(o)})^t = 0.5\ C_{SM}\ \phi^t_{SM(o)} \begin{bmatrix} f_1 \sin\beta_{SM(o)} & f_1 \cdot \cos\beta_{SM(o)} \\ f_3 \sin\beta_{SM(o)} & f_3 \cos\beta_{SM(o)} \\ f_5 \cos\beta_{SM(o)} & -f_5 \sin\beta_{SM(o)} \end{bmatrix} \qquad (3\text{-}101)$$

The elements of coefficient matrix $A\Omega_{SM}$ defining the influence on $d\Delta\Omega_{SM}/dt$ of the Synchronous Motor's incremental stator current $\Delta \mathbf{i}_{SM}$

3.6.2 The elements $A\Omega_{SM}\phi_{SM}$ of coefficient matrix $A\Omega_{SM}$.
From (3-90) - (3-97):

$$A\Omega_{SM}\phi_{SM} = C_{SM}\ (\partial G_{\Omega SM}/\partial \boldsymbol{\phi}_{SM}) \qquad (3\text{-}102)$$

⊃:

$$\underset{(1\times 3)}{A\Omega_{SM}\phi_{SM}} = 0.5\ C_{SM}\ \mathbf{i}^t_{SM(o)}\ \mathbf{T}_{SM1(o)}\ \mathbf{f}_{SM} \qquad (3\text{-}103)$$

The elements of coefficient matrix $A\Omega_{SM}$ defining the influence on $d\Delta\Omega_{SM}/dt$ of the synchronous motor's incremental flux components $\Delta\boldsymbol{\phi}_{SM}$

3.6.3 The element $A\Omega_{SM}\beta_{SM}$ of coefficient matrix $A\Omega_{SM}$.
From (3-90) - (3-97):

$$A_{\Omega SM}\beta_{SM} = C_{SM}\ (\partial G_{\Omega SM}/\partial \beta_{SM}) \qquad (3\text{-}104)$$

⊃:

$$\underset{(1\times 1)}{A\Omega_{SM}\beta_{SM}} = 0.5\ C_{SM}\ (X''_d - X''_q)\ \mathbf{i}^t_{SM(o)} \begin{bmatrix} \cos 2\beta_{SM(o)} & -\sin 2\beta_{SM(o)} \\ -\sin 2\beta_{SM(o)} & -\cos 2\beta_{SM(o)} \end{bmatrix} i_{SM(o)} + 0.5\ \mathbf{i}^t_{SM(o)} \begin{bmatrix} \cos\beta_{SM(o)} & \sin\beta_{SM(o)} \\ -\sin\beta_{SM(o)} & \cos\beta_{SM(o)} \end{bmatrix} \mathbf{f}_{SM}\ \phi_{SM(o)} \qquad (3\text{-}105)$$

The element of coefficient matrix $A\Omega_{SM}$ defining the influence on $d\Delta\Omega_{SM}/dt$ of the Synchronous Motor's incremental rotor angle $\Delta\beta_{SM}$.

3.6.4 The element $A\Omega_{SM}\Omega_{SM}$ of coefficient matrix $\mathbf{A\Omega_{SM}}$.
From (3-90) - (3-97) and (1-120):

$$A_{\Omega SM\Omega SM} = C_{SM}\ (\partial G_{\Omega SM}/\partial\Omega_{SM}) \qquad (3\text{-}106)$$

⊃:

$$\underset{(1x1)}{A\Omega_{SM}\Omega_{SM}} = -C_{SM}\ å_o \qquad (3\text{-}107)$$

The element of coefficient matrix $\mathbf{A\Omega_{SM}}$ defining the influence on $d\Delta\Omega_{SM}/dt$ of the synchronous motor's incremental rotor speed $\Delta\Omega_{SM}$, when in *generator* mode of operation.

3.6.5 The element $A\Omega_{SM}\Delta_{å}$ of coefficient matrix $\mathbf{A\Omega_{SM}}$.
From (3-90) - (3-97) and (1-120):

$$A_{\Omega SM}\Delta_{å} = C_{SM}\ (\partial G_{\Omega SM}/\partial\Delta å) \qquad (3\text{-}108)$$

⊃:

$$\underset{(1x1)}{A\Omega_{SM}\Delta_{å}} = 2\ C_{SM} \qquad (3\text{-}109)$$

The element of coefficient matrix $\mathbf{A_{\Omega SM}}$ defining the influence on $d\Delta\Omega_{SM}/dt$ of the synchronous motor's incremental gate opening $\Delta(\Delta å)$, when in *generator* mode of operation.

3.6.6 The element $A\Omega_{SM}\Delta g$ of coefficient matrix $\mathbf{A\Omega_{SM}}$.
From (3-90) - (3-97):

$$A\Omega_{SM}\Delta g = C_{SM}\ (\partial G_{\Omega SM}/\partial\Delta g) \qquad (3\text{-}110)$$

⊃:

$$\underset{(1x1)}{A\Omega_{SM}\Delta g} = -C_{SM} \qquad (3\text{-}111)$$

The element of coefficient matrix $\mathbf{A\Omega_{SM}}$ defining the influence on $d\Delta\Omega_{SM}/dt$ of the synchronous motor's incremental power control variable $\Delta(\Delta g)$, when in *generator* mode of operation.

3.7 Performance of the derivative of asynchronous motor incremental speed $\Delta\Omega_{AM}$

The variation of the asynchronous motor pu speed Ω_{AM} is given by equation (1-128) of the systems component Chapter 1.7 :

$$d\Omega_{AM}/dt = (S_{Bas}/S_{AM})\ (1/(T_a \cos\varphi))\ (T_{AMel} - T_{AMmec}) = C_{AM}\ G\Omega_{AM} \qquad (3\text{-}112)$$

Here;

C_{AM} = $(S_{Bas}/S_{AM})\ (1/(T_a \cos\varphi))$

$G\Omega_{AM}$ = $(T_{AMel} - T_{AMmec})$ (3-113)

Ω_{AM} = state variable in terms of asynchronous motor rotor speed

$T_{AMel} = 0.5\ (X_m/X_r)\ (\bar{1}\ i_{AM})^t\ \phi_{AM}$ = electrical motor torque , see (1-129), where $\bar{1} = \begin{bmatrix} & 1 \\ -1 & \end{bmatrix}$

$T_{AMmec} = T_{AMec(o)}\ (\Omega_{AM}/\Omega_{AM(o)})^{\kappa}$ = mechanical torque in **motor** mode of operation. (Motor operation implies positive sign of mechanical torque).

If the motor is up and running at t= -0 : $T_{AMec(o)} = T_{Amel(o)}$ = electrical motor torque at t= -0. Found from (1-129). κ = (say) 1.5-4.5, depending on type of load. $\Omega_{AM(o)}$ = initial rotor speed

If the motor is to be started from stillstand : $T_{AMec(o)}$= (say) 0.03-0.07= coefficient modelling mech. friction, air resistance,etc. κ = (say)1-5. $T_{AMec(o)}$ and κ may change with Ω_{AM}.

$T_{AMmec} = (T_{Amel(o)} + \Delta T_{mec})$= mechanical torque in **generator** mode of operation. (Generator operation implies negative sign of mechanical torques) $T_{Amel(o)}$ is initial electrical motor torque. ΔT_{mec} is the respons from the power control system.

S_{Bas}, S_{AM} = Chosen VA system power base, and rated VA motor capacity, respectively.

$T_a, \cos\varphi$ = Dynamical system's inertia constant, and motor's rated power factor, respectively.

From the foregoing we see that $G_{\Omega AM}$ is a function of a subset of the state variables:

$$G\Omega_{AM} = f(\mathbf{i}_{AM}, \boldsymbol{\phi}_{AM}, \Omega_{AM}) \qquad (3\text{-}114)$$

We seek the incremental asynchronous motor speed behaviour and can - on the basis of (3-112) - (3-113) - develop the following 'platform' for the ensuing analysis:

$$\Delta(d\Omega_{AM}/dt) = \Delta(C_{AM}\ G\Omega_{AM})$$

$\supset$:

$$d\Delta\Omega_{AM}/dt = C_{AM}\ \Delta G\Omega_{AM} \qquad (3\text{-}115)$$

(3-115) provides the basis for determining the sought incremental performance $d\Delta\Omega_{AM}/dt$. Equation (3-114) says that $\Delta G\Omega_{AM}$ depends on J = (2+2+1) = 5 individual state variables Δz_j .Thus we can formulate :

$$d\Delta\Omega_{AM}/dt = C_{AM} \sum_{j=1}^{J=5} (\partial G\Omega_{AM}/\partial z_j)\ \Delta z_j = \sum_{j=1}^{J=5} A\Omega_{AMj}\ \Delta z_j = \mathbf{A}\Omega_{AM} \cdot \Delta\mathbf{z}_{\Omega AM} \qquad (3\text{-}116)$$

$\supset$:

$$d\Delta\Omega_{AM}/dt = [\mathbf{A}\Omega_{AM}i_{AM}, \mathbf{A}\Omega_{AM}\phi_{AM}, A\Omega_{AM}\Omega_{AM}] \cdot [\Delta\mathbf{i}^t_{AM}, \Delta\boldsymbol{\phi}^t_{AM}, \Delta\Omega_{AM}]^t \qquad (3\text{-}117)$$

In the following we will develop the (here) 5 elements of the coefficient matrix $\mathbf{A}\Omega_{AM}$.

3.7.1 The elements $\mathbf{A}\Omega_{AM}i_{AM}$ of coefficient matrix $\mathbf{A}\Omega_{AM}$.

From (3-112) - (3-117):

$$\mathbf{A}\Omega_{AM}i_{AM} = C_{AM}\ (\partial G\Omega_{AM}/\partial \mathbf{i}_{AM}) \qquad (3\text{-}118)$$

$\supset$:

$$\underset{(1\times2)}{\mathbf{A}\Omega_{AM}i_{AM}} = 0.5\ C_{AM}\ (X_m/X_r) \cdot \boldsymbol{\phi}^t_{AM(o)}\ \bar{\mathbf{1}} \qquad (3\text{-}119)$$

The elements of coefficient matrix $\mathbf{A}\Omega_{AM}$ defining the influence on $d\Delta\Omega_{AM}/dt$ of the asynchronous motor's incremental stator current $\Delta\mathbf{i}_{AM}$

3.7.2 The elements $A\Omega_{AM}\phi_{AM}$ of coefficient matrix $A\Omega_{AM}$.
From (3-112) - (3-117):

$$A\Omega_{AM}\phi_{AM} = C_{AM}\ (\partial G\Omega_{AM}/\partial\phi_{AM}) \qquad (3\text{-}120)$$

⊃:

$$\underset{(1x2)}{A\Omega_{AM}\phi_{AM}} = 0.5\ C_{AM}\ (X_m/X_r)\ (\bar{1}\ i_{AM(o)})^t \qquad (3\text{-}121)$$

The elements of coefficient matrix $A\Omega_{AM}$ defining the influence on $d\Delta\Omega_{AM}/dt$ of the asynchronous motor's incremental flux $\Delta\phi i_{AM}$

3.7.3 The element $A\Omega_{AM}\Omega_{AM}$ of coefficient matrix $A\Omega_{AM}$.
From (3-112) - (3-117):

$$A\Omega_{AM}\Omega_{AM} = C_{AM}\ (\partial G\Omega_{AM}/\partial\Omega_{AM}) \qquad (3\text{-}122)$$

⊃:

$$\underset{(1x1)}{A\Omega_{AM}\Omega_{AM}} = -C_{AM}\ \kappa\ D_{AM}/\Omega_{AM(o)} \qquad (3\text{-}123)$$

where;

κ = (say) 1 – 3 = exponent describing pu speed sensitivity of the asynchronous motor load torque. (3-124)

D_{AM} = $T_{AMel(o)}$, if the AM is up and running at t=0_

D_{AM} = (say) 0.03-0.07 , if the AM is in a starting sequence.

The element of coefficient matrix $A\Omega_{AM}$ defining the influence on $d\Delta\Omega_{AM}/dt$ of the asynchronous motor's incremental rotor speed $\Delta\Omega_{AM}$, - in *motor* operation.

3.8 Incremental power control performance of the synchronous motor in generator mode of operation.

The variation of the power control state variables is governed by e.g. equations (1-120) of the systems component Chapter 1.7, assuming that a (high head/Francis) hydro power plant is at hand:

$d\Delta å/dt = K_1\ (\Delta\Omega_{ref} -(1-\Omega_{SM}) + \Delta w) - K_2\ \Delta å$ = $G\Delta å$ = $f(\Omega_{SM},\Delta w,\Delta å)$ Regulator system

$d\Delta w/dt = K_3\ \Delta å - K_4\ \Delta w$ = $G\Delta w$ = $f(\Delta å, \Delta w)$ Regulator system

$d\Delta g/dt = (3K_0/\Omega_{SM})\ \Delta å - K_0\ \Delta g$ = $G\Delta g$ = $f(\Omega_{SM},\Delta å,\Delta g)$ Hydraulic system (3-125)

↓

$\Delta T_{mec} = \Delta g - å_0\ (1 - \Omega_{SM}) - (2/\Omega_{SM})\ \Delta å$ Net change of mechanical torque

Control System model parameters/variables:

$K_0 = 2/(å_0\ T_r)$
$K_1 = 1/T_c$
$K_2 = (\delta_p + \delta_t)/T_c$
$K_3 = \delta_t/T_t$
$K_4 = 1/T_t$
P_m = absorbed motor power =0.5 $i^t_{SM} \cdot e_{SM}$ P_m is negative in generator mode.
P_{target} = target value of P_m
c = per unit scaling factor (eg.: c=0.1)

←

Control System data input:

T_r	: Time constant for hydraulic system (eg 0.3s)
T_c	: Time constant for main servo (eg 0.08s)
T_t	: Transient droop time constant (eg 17s)
δ_t	: Transient droop (eg 0.15pu)
δ_p	: Permanent droop (eg 0.0 – 0.03. The value 0.0 if frequency sustained by a single unit)
P_{target}	: Target value of of absorbed motor power (eg -0.8)
$å_o$	: Initial pu turbine opening ($t_å$). (if $t_å$<0.3 then $å_o$=0.3)

We seek the incremental behaviour of the state variables ($\Delta å$, Δw, Δg), and can on the basis of (3-125) develop the following 'platform' for the ensuing analysis:

$$\Delta(d\Delta å/dt) = \Delta G_{\Delta å}$$
$$\Delta(d\Delta w/dt) = \Delta G_{\Delta w} \qquad (3\text{-}126)$$
$$\Delta(d\Delta g/dt) = \Delta G_{\Delta g}$$

⊃:

$$\underline{d\Delta(\Delta å)/dt = \Delta G_{\Delta å}} \qquad a)$$
$$\underline{d\Delta(\Delta w)/dt = \Delta G_{\Delta w}} \qquad b) \qquad (3\text{-}127)$$
$$\underline{d\Delta(\Delta g)/dt = \Delta G_{\Delta g}} \qquad c)$$

(3-127) provides the basis for determining the sought incremental state variable performance. Equation (3-125) implies that $\Delta G_{\Delta å}$, $\Delta G_{\Delta w}$ and $\Delta G_{\Delta g}$ will depend on respectively J = 3, 2 and 2 individual state variables Δz_j. Thus we can formulate:

$$\Delta G_{\Delta å} = \sum_{j=1}^{J=3}(\partial G_{\Delta å}/\partial z_j)\ \Delta z_j = \sum_{j=1}^{J=3} A_{\Delta å_j}\ \Delta z_j \qquad a)$$

$$\Delta G_{\Delta w} = \sum_{j=1}^{J=2}(\partial G_{\Delta w}/\partial z_j)\ \Delta z_j = \sum_{j=1}^{J=2} A_{\Delta w_j}\ \Delta z_j \qquad b) \qquad (3\text{-}128)$$

$$\Delta G_{\Delta g} = \sum_{j=1}^{J=2}(\partial G_{\Delta g}/\partial z_j)\ \Delta z_j = \sum_{j=1}^{J=2} A_{\Delta g_j}\ \Delta z_j \qquad c)$$

_In the following we will develop the 3 , 2 , 2 individual partial derivative terms that contribute to $\Delta G_{\Delta å}$, $\Delta G_{\Delta w}$ and $\Delta G_{\Delta g}$, respectively.

3.8.1 Performance of the derivative of incremental turbine regulator opening $\Delta(\Delta å)$

From (3-125), (3-127a) and (3-128a):

$$\underline{d\Delta(\Delta å)/dt} = \sum_{j=1}^{J=3} A_{\Delta å_j}\ \Delta z_j = \mathbf{A}_{\Delta å}\ \mathbf{\Delta z}_{\Delta å}$$

$$= [\, A_{\Delta å \Omega_{SM}},\ A_{\Delta å \Delta å},\ A_{\Delta å \Delta w}\,]\ [\Delta\Omega_{SM},\ \Delta(\Delta å),\ \Delta(\Delta w)\,]^t \qquad (3\text{-}129)$$

where;

$$\underline{A_{\Delta å \Omega_{SM}} = K_1}$$
$$\underline{A_{\Delta å \Delta å} = -K_2} \qquad (3\text{-}130)$$
$$\underline{A_{\Delta å \Delta w} = K_1}$$

Power Control System. Description of *regulator opening* gradient $d\Delta(\Delta å)/dt$

3.8.2 Performance of the derivative of incremental power control variable $\Delta(\Delta w)$

From (3-125), (3-127b) and (3-128b):

$$\underline{d\Delta(\Delta w)/dt} = \sum_{j=1}^{J=2} A_{\Delta w_j}\ \Delta z_j = \mathbf{A}_{\Delta w}\ \mathbf{\Delta z}_{\Delta w}$$

$$= [\, A_{\Delta w \Delta å},\ A_{\Delta w \Delta w}\,]\ [\Delta(\Delta å),\ \Delta(\Delta w)\,]^t \qquad (3\text{-}131)$$

where;

$$\underline{A_{\Delta w \Delta å} = K_3} \qquad (3\text{-}132)$$
$$\underline{A_{\Delta w \Delta w} = -K_4}$$

Power Control System. Description of *control variable* gradient $d\Delta(\Delta w)/dt$

3.8.3 Performance of the derivative of incremental power control variable $\Delta(\Delta g)$
From (3-125), (3-127c) and (3-128c):

$$\underline{d\Delta(\Delta g)/dt} = \sum_{j=1}^{J=2} A\Delta g_j\ \Delta z_j = \mathbf{A}_{\Delta g}\ \mathbf{\Delta z}_{\Delta g}$$

$$= \underline{[\ A\Delta g\Delta \mathring{a}\ ,\ A\Delta g\Delta g\ ]\ [\Delta(\Delta \mathring{a}),\ \Delta(\Delta g)\ ]^t} \quad (3\text{-}133)$$

where;

$$\underline{A\Delta g\Delta \mathring{a}} = 3\ K_o/\Omega_{SM(o)} = \underline{3\ K_o} \quad (3\text{-}134)$$

$$\underline{A\Delta g\Delta g} = \underline{-K_o}$$

Power Control System. Description of control variable gradient $d\Delta(\Delta g)/dt$.

3.9 Incremental voltage control performance of the synchronous motor

The variation of the voltage control state variables is given by equations (1-121) of the systems component Chapter 1.7 :

$$d\Delta E_f/dt = C_1\ (\Delta E_r - \Delta E_f) \qquad = G_{\Delta Ef} = f(\Delta E_r, \Delta E_f)$$

$$d\Delta E_r/dt = C_2\ [\Delta U_{ref} + U_o - U + K_\Omega\ (\Omega_{SM} - 1) - \Delta h] - C_3\ \Delta E_f - C_4 \cdot \Delta E_r + C_2\ \Delta E_{ss} = G\Delta E_r$$

$$= f(i_{SM}, \phi_{SM}, \beta_{SM}, \Omega_{SM}, \Delta E_{qf}, \Delta E_r, \Delta E_{ss}, \Delta h)$$

$$d\Delta E_{ss}/dt = C_5\ \Delta E_f - C_6\ E_{ss} \qquad = G\Delta E_{ss} = f(\Delta E_f, \Delta E_{ss})$$

$$d\Delta h/dt = C_7\ (\Omega_{SM} - 1) - C_8 \cdot \Delta h \qquad = G\Delta h = f(\Omega_{SM}, \Delta h) \quad (3\text{-}135)$$

Control system model parameters/variables:		Control System data input:
$U = (1/\sqrt{2})\ (e_{SM}^2 + e_{SM}^2)^{0.5}$ $C_1 = 1/T_f$ $C_2 = K_R/T_R$ $C_3 = K_R\ K_D/T_R = K_D\ C_2$ $C_4 = 1/T_R$ $C_5 = K_D/T_D$ $C_6 = 1/T_D$ $C_7 = K_\Omega/T_\Omega$ $C_8 = 1/T_\Omega$ $\Delta U = (U - U_0)$ = pu voltage deviation	←	T_f ; field circuit time constant (eg 0.1s) K_R ; resulting forward amplification (eg 70pu) T_R ; regulator time constant (eg 0.1s) K_D ; transient feedback amplification (eg 0.25pu) T_D ; transient feedback time constant (eg 0.25s) K_Ω ; power stabilizer amplification (eg 1pu) $E_{f(max)}$; ceiling field voltage (eg 3pu) $E_{f(min)}$; bottom field voltage (eg –2pu) T_Ω ; power stabilizer time constant (eg 2s)

We seek the incremental behaviour of the state variables (ΔE_f, ΔE_r, ΔE_{ss}, Δh), and can on the basis of (3-135) develop the following 'platform' for the ensuing analysis:

$$\Delta(d\Delta E_f/dt) = \Delta G_{\Delta Ef}$$
$$\Delta(d\Delta E_r/dt) = \Delta G\Delta E_r \quad (3\text{-}136)$$
$$\Delta(d\Delta E_{ss}/dt) = \Delta G\Delta E_{ss}$$
$$\Delta(d\Delta h/dt) = \Delta G\Delta h$$

⊃:

$$\underline{d\Delta(\Delta E_f)/dt = \Delta G_{\Delta Ef}} \quad a)$$
$$\underline{d\Delta(\Delta E_r)/dt = \Delta G\Delta E_r} \quad b) \quad (3\text{-}137)$$
$$\underline{d\Delta(\Delta E_{ss})/dt = \Delta G\Delta E_{ss}} \quad c)$$
$$\underline{d\Delta(\Delta h)/dt = \Delta G\Delta h} \quad d)$$

(3-137) provides the basis for determining the sought incremental state variable performance. Equation (3-135) implies that $\Delta G_{\Delta Ef}$, $\Delta G\Delta E_r$, $\Delta G\Delta E_{ss}$ and $\Delta G\Delta h$ will depend on respectively J = 2, 10, 2 and 2 individual state variables Δz_j . Thus we can formulate:

$$\Delta G_{\Delta Ef} = \sum_{j=1}^{J=2}(\partial G_{\Delta Ef}/\partial z_j)\ \Delta z_j = \sum_{j=1}^{J=2} A_{\Delta Efj}\ \Delta z_j \qquad a)$$

$$\Delta G_{\Delta E_r} = \sum_{j=1}^{J=10}(\partial G_{\Delta E_r}/\partial z_j)\ \Delta z_j = \sum_{j=1}^{J=10} A_{\Delta E_{rj}}\ \Delta z_j \qquad b) \quad (3\text{-}138)$$

$$\Delta G_{\Delta E_{ss}} = \sum_{j=1}^{J=2}(\partial G_{\Delta E_{ss}}/\partial z_j)\ \Delta z_j = \sum_{j=1}^{J=2} A_{\Delta E_{ssj}}\ \Delta z_j \qquad c)$$

$$\Delta G_{\Delta h} = \sum_{j=1}^{J=2}(\partial G_{\Delta h}/\partial z_j)\ \Delta z_j = \sum_{j=1}^{J=2} A_{\Delta h_j}\ \Delta z_j \qquad d)$$

In the following we will develop the 2 , 10 , 2 , 2 individual partial derivative terms that contribute to $\Delta G_{\Delta Ef}$, $\Delta G_{\Delta E_r}$, $\Delta G_{\Delta E_{ss}}$ and $\Delta G_{\Delta h}$, respectively. For overview reasons (3-138) is processed in sequence a), c), d),

3.9.1 Performance of the derivative of incremental excitation voltage $\Delta(\Delta E_f)$

From (3-135), (3-137a) and (3-138a):

$$d\Delta(\Delta E_f)/dt = \sum_{j=1}^{J=2} A_{\Delta Ef\,j}\ \Delta z_j = \mathbf{A}_{\Delta Ef} \cdot \mathbf{\Delta z}_{\Delta Ef}$$

$$= [A_{\Delta Ef\Delta Ef}, A_{\Delta Ef\Delta Er}]\ [\Delta(\Delta E_f), \Delta\Delta(E_r)\ ]^t \qquad (3\text{-}139)$$

where;

$$A_{\Delta E_f\Delta E_f} = -C_1 \qquad (3\text{-}140)$$
$$A_{\Delta E_f\Delta E_r} = C_1$$

Voltage Control System. Description of *excitation voltage* gradient $d\Delta(\Delta E_f)/dt$.

3.9.2 Performance of the derivative of incremental voltage control variable $\Delta(\Delta E_{ss})$

From (3-135), (3-137c) and (3-138c):

$$d\Delta(\Delta E_{ss})/dt = \sum_{j=1}^{J=2} A_{\Delta Ess\,j}\ \Delta z_j = \mathbf{A}_{\Delta Ess} \cdot \mathbf{\Delta z}_{\Delta Ess}$$

$$= [\ A_{\Delta Ess\Delta Eqf}, A_{\Delta Ess\Delta Ess}]\ [\Delta(\Delta E_{qf}), \Delta(\Delta E_{ss})\ ]^t \qquad (3\text{-}141)$$

where;

$$A_{\Delta E_{ss}\Delta E_{qf}} = C_5 \qquad (3\text{-}142)$$
$$A_{\Delta E_{ss}\Delta E_{ss}} = -C_6$$

Voltage Control System. Description of *voltage control variable* gradient $d\Delta(\Delta E_{ss})/dt$.

3.9.3 Performance of the derivative of incremental speed stabilizer variable $\Delta(\Delta h)$

From (3-135), (3-137d) and (3-138d):

$$\underline{d\Delta(\Delta h)/dt} = \sum_{j=1}^{J=2} A_{\Delta h\,j}\,\Delta z_j = \mathbf{A}_{\Delta h}\cdot\,\mathbf{\Delta z}_{\Delta h}$$

$$= [\,A_{\Delta h\Delta\Omega SM}\,,\;A_{\Delta h\Delta h}]\;\underline{[\Delta\Omega_{SM},\,\Delta(\Delta h)\,]^t} \qquad (3\text{-}143)$$

where;

$$A_{\Delta h\Omega SM} = C_7 \qquad (3\text{-}144)$$

$$A_{\Delta h\Delta h} = -C_8$$

Voltage Control System. Description of *speed stabilizer variable* gradient $d\Delta(\Delta h)/dt$.

3.9.4 Performance of the derivative of incremental voltage control variable $\Delta(\Delta E_r)$

From (3-135), (3-137b) and (3-138b):

$$\underline{d\Delta(\Delta E_r)/dt} = \sum_{j=1}^{J=10} A_{\Delta Er\,j}\,\Delta z_j = \mathbf{A}_{\Delta Er}\,\mathbf{\Delta z}_{\Delta Er}$$

$$= [\,\mathbf{A}_{\Delta Eri SM},\,\mathbf{A}_{\Delta Er\phi SM},\,A_{\Delta Er\beta SM},\,A_{\Delta Er\Omega SM}\,,\,A_{\Delta Er\Delta E_{qf}}\,,\,A_{\Delta Er\Delta Er}\,,\,A_{\Delta Er\Delta Ess},\,A_{\Delta Er\Delta h}]\begin{bmatrix}\Delta \mathbf{i}_{SM}\\ \Delta\boldsymbol{\phi}_{SM}\\ \Delta\beta\\ \Delta\Omega_{SM}\\ \Delta(\Delta E_f)\\ \Delta(\Delta E_r)\\ \Delta(\Delta E_{ss})\\ \Delta(\Delta h)\end{bmatrix} \qquad (3\text{-}145)$$

The elements of coefficient matrix $\mathbf{A}_{\Delta Er}$ are outlined in the following :

$$\underset{(1\times2)}{\mathbf{A}_{\Delta Eri SM}} = \partial G\Delta E_r/\partial \mathbf{i}_{SM} = -C_2\,(\,0.5/U_{SM(o)})\;\mathbf{e}^t_{SM(o)}\,\mathbf{R}_{SM(o)} \qquad (3\text{-}146)$$

It appears from (3-135) that establishing (3-146) implies knowing pu SM voltage $U_{SM} = f(\mathbf{i}_{SM})$:
From of Chapter 1.7, and the chosen premise of shifting direction of network loop currents as given by (A-10) of Appendix, we have (in d-q axis frame of reference) the following relevant expression for the synchronous machine terminal voltage, see (A-19) of Appendix :

$$\mathbf{e}_{SM} = \mathbf{R}_{SM}\,\mathbf{i}_{SM} + (\,\mathbf{V}_{SM} + \mathbf{H}_{SM}\,\boldsymbol{\phi}_{SM}\,) \qquad (3\text{-}147)$$

In the *per phase* frame of reference the pu magnitude of this voltage is

$$U_{SM} = (1/\sqrt{2})\,(\,(e_{SMd}\,)^2 + (e_{SMq}\,)^2\,)^{\,0.5} \qquad (3\text{-}148)$$

From (3-147) – (3-148) it is seen that $U_{SM} = f(\mathbf{i}_{SM};\,\boldsymbol{\phi}_{SM},\beta_{SM},\Omega_{SM}\,)$. The partial derivative of U_{SM} w.r.t. a subset $\mathbf{z}$ of these state variables, can generally be expressed as ;

$$\partial U_{SM}/\partial\mathbf{z} = (0.5/U_{SM(0)})\;\mathbf{e}^t_{SM(0)}\cdot\,\partial\mathbf{e}_{SM}\,/\partial\mathbf{z} \qquad (3\text{-}149)$$

With $\mathbf{z} = \mathbf{i}_{SM}$, and using (3-147) in (3-149), the sought equation (3-146) is readily found.

Voltage control system. Start of description of coefficient matrix $\mathbf{A}_{\Delta Er}$, see (3-145) .

$$\underset{(1x3)}{\mathbf{A}_{\Delta Er\phi SM}} = \partial A\Delta E_r/\partial\phi_{SM} = -C_2\ (0.5/U_{SM(0)})\ \mathbf{e}^t_{SM(o)}\ \mathbf{H}_{SM(o)} \quad (3\text{-}150)$$

With $\mathbf{z} = \phi_{SM}$, and using (3-147) in (3-149), the equation (3-150) is readily found.

$$\underset{(1x1)}{A_{\Delta Er\beta SM}} = \partial A\Delta E_r/\partial\beta_{SM} = -C_2\ (0.5/U_{SM(o)})\ (\mathbf{e}^t_{SM(o)}\ \bar{\mathbf{1}}\ (\mathbf{V}_{SM(o)} + \mathbf{H}_{SM(o)}\ \phi_{SM(o)}) + \mathbf{e}^t_{SM(o)}\ \mathbf{R}_{SM\beta(o)}\ \mathbf{i}_{SM(o)}) \quad (3\text{-}151)$$

where;

$$\bar{\mathbf{1}} = \begin{bmatrix} & 1 \\ -1 & \end{bmatrix} \quad (3\text{-}152)$$

$$\mathbf{R}_{SM\beta(0)} = 2\begin{bmatrix} (\bar{X}''\cos2\beta_{SM(o)} - \bar{X}''_r\sin2\beta_{SM(o)}) & -(\bar{X}''\sin2\beta_{SM(o)} + \bar{X}''_r\cos2\beta_{SM(o)}) \\ -(\bar{X}''\sin2\beta_{SM(o)} + \bar{X}''_r\cos2\beta_{SM(o)}) & -(\bar{X}''\cos2\beta_{SM(o)} - \bar{X}''_r\sin2\beta_{SM(o)}) \end{bmatrix} \quad (3\text{-}153)$$

Since both $\mathbf{R}_{SM}$, $\mathbf{V}_{SM}$, and $\mathbf{H}_{SM}$ are functions of β_{SM}, (3-151) is the sum of three contributions.
See equations (1-108) -(1-112) for definition of synchronous motor terms R_{SM}, V_{SM}, H_{SM}, as well as $\bar{X}''$ & $\bar{X}''_r$

$$\underset{(1x1)}{A_{\Delta E_r\Omega SM}} = \partial A\Delta E_r/\partial\Omega_{SM} = C_2\ K_\Omega - C_2\ (0.5/U_{SM(o)})\ (\mathbf{e}^t_{SM(o)}\ (\bar{X}''\ \mathbf{A}_{\beta(o)}\ \mathbf{i}_{SM(o)} + \mathbf{T}_{SM1(o)}\ \mathbf{f}_{SM}\ \phi_{SM(o)})) \quad (3\text{-}154)$$

where;

$$\mathbf{A}_{\beta(o)} = 2\begin{bmatrix} \sin2\beta_{SM(o)} & \cos2\beta_{SM(o)} \\ \cos2\beta_{SM(o)} & -\sin2\beta_{SM(o)} \end{bmatrix} \quad (3\text{-}155)$$

See (3-37) for $\mathbf{T}_{SM1(o)}$ & $\mathbf{f}_{SM}$.

$$\underset{(1x1)}{A_{\Delta Er\Delta E_f}} = \partial A\Delta E_r/\partial\Delta E_f = -C_3 \quad (3\text{-}156)$$

$$\underset{(1x1)}{A_{\Delta Er\Delta Er}} = \partial A\Delta E_r/\partial\Delta Er = -C_4 \quad (3\text{-}157)$$

$$\underset{(1x1)}{A_{\Delta Er\Delta Ess}} = \partial A\Delta E_r/\partial\Delta Ess = C_2 \quad (3\text{-}158)$$

$$\underset{(1x1)}{A_{\Delta Er\Delta h}} = \partial A\Delta E_r/\partial\Delta h = -C_2 \quad (3\text{-}159)$$

Voltage Control System. End of description of coefficient matrix $\mathbf{A}_{\Delta Er}$. See (3-145).

3.10 System matrix A

The incremental performance of all state variables has been investigated in preceeding Chapters 3.1 -3.9, leading to the coefficients of the defined A-matrix of the system. Figure 3.2 summarizes the established incremental/ linearized description of power system performance around some specified operating point. The A-matrix thus defined is on compact form, and further detailed in (3-160).

$d\Delta \mathbf{i}_{loop}/dt = \mathbf{Ai}_{loop}\ \mathbf{\Delta z}_{iloop}$	(3-12)
$d\Delta \mathbf{e}_{tc}/dt = \mathbf{Ae}_{c}\ \mathbf{\Delta z}_{ec}$	(3-50)
$d\Delta \phi_{AM}/dt = \mathbf{A\phi}_{AM} \cdot \mathbf{\Delta z\phi}_{AM}$	(3-60)
$d\Delta \phi_{SM}/dt = \mathbf{A\phi}_{SM}\ \mathbf{\Delta z\phi}_{SM}$	(3-73)
$d\Delta \beta_{SM}/dt = \mathbf{A\beta}_{SM}\ \mathbf{\Delta z\beta}_{SM}$	(3-87)
$d\Delta \Omega_{SM}/dt = \mathbf{A\Omega}_{SM}\ \mathbf{\Delta z}_{\Omega SM}$	(3-96)
$d\Delta \Omega_{AM}/dt = \mathbf{A\Omega}_{AM} \cdot \mathbf{\Delta z}_{\Omega AM}$	(3-116)
$d\Delta(\Delta å)/dt = \mathbf{A}_{\Delta å}\ \mathbf{\Delta z}_{\Delta å}$	(3-129)
$d\Delta(\Delta w)/dt = \mathbf{A}_{\Delta w} \cdot \mathbf{\Delta z}_{\Delta w}$	(3-131)
$d\Delta(\Delta g)/dt = \mathbf{A}_{\Delta g} \cdot \mathbf{\Delta z}_{\Delta g}$	(3-133)
$d\Delta(\Delta E_f)/dt = {}_{\Delta Ef} \cdot \mathbf{\Delta z}_{\Delta Ef}$	(3-139)
$d\Delta(\Delta E_r)/dt = \mathbf{A}_{\Delta Ef}\ \mathbf{\Delta z}_{\Delta Er}$	(3-145)
$d\Delta(\Delta E_{ss})/dt = \mathbf{A}_{\Delta Ess} \cdot \mathbf{\Delta z}_{\Delta Ess}$	(3-141)
$d\Delta(\Delta h)/dt = \mathbf{A}_{\Delta h} \cdot \mathbf{\Delta z}_{\Delta h}$	(3-143)

Figure 3.2 : Incremental/ linearized description of power system performance around specified operating point.'Compact' performance description.

$$
\mathbf{A}_{compact} = \begin{bmatrix} \mathbf{Ai}_{loop} \\ \mathbf{Ae}_{c} \\ \mathbf{A\phi}_{AM} \\ \mathbf{A\phi}_{SM} \\ \mathbf{A\beta}_{SM} \\ \mathbf{A\Omega}_{SM} \\ \mathbf{A\Omega}_{AM} \\ \mathbf{A}_{\Delta å} \\ \mathbf{A}_{\Delta w} \\ \mathbf{A}_{\Delta g} \\ \mathbf{A}_{\Delta Ef} \\ \mathbf{A}_{\Delta Er} \\ \mathbf{A}_{\Delta Ess} \\ \mathbf{A}_{\Delta h} \end{bmatrix} = \begin{bmatrix} [\mathbf{Ai}_{loop}i_{loop},\ \mathbf{Ai}_{loop}e_{tc},\ \mathbf{Ai}_{loop}\phi_{AM},\ \mathbf{Ai}_{loop}\phi_{SM},\ \mathbf{Ai}_{loop}\beta_{SM},\ \mathbf{Ai}_{loop}\Omega_{SM},\ \mathbf{Ai}_{loop}\Omega_{AM},\ \mathbf{Ai}_{loop}E_f] \\ [\mathbf{Ae}_c i_{loop},\ \mathbf{Ae}_c e_c] \\ [\mathbf{A\phi}_{AM}i_{AM},\ \mathbf{A\phi}_{AM}\phi_{AM},\ \mathbf{A\phi}_{AM}\Omega_{AM}] \\ [\mathbf{A\phi}_{SM}i_{SM},\ \mathbf{A\phi}_{SM}\phi_{SM},\ \mathbf{A\phi}_{SM}\beta_{SM},\ \mathbf{A\phi}_{SM\Delta Ef}] \\ [A\beta_{SM}\Omega_{SM}] \\ [\mathbf{A\Omega}_{SM}i_{SM},\ \mathbf{A\Omega}_{SM}\phi_{SM},\ A\Omega_{SM}\beta_{SM},\ A\Omega_{SM\Omega SM},\ A\Omega_{SM}\Delta_{å},\ A\Omega_{SM}\Delta_g] \\ [\mathbf{A\Omega}_{AM}i_{AM},\ \mathbf{A\Omega}_{AM}\phi_{AM},\ A\Omega_{AM}\Omega_{AM}] \\ [A\Delta å\Omega_{SM},\ A\Delta å\Delta å,\ A\Delta å\Delta w] \\ [A\Delta w\Delta å,\ A\Delta w\Delta w] \\ [A\Delta g\Delta å,\ A\Delta g\Delta g] \\ [A_{\Delta Ef\Delta Ef},\ A_{\Delta Ef\Delta Er}] \\ [\mathbf{A}_{\Delta Er}i_{SM},\ \mathbf{A}_{\Delta Er}\phi_{SM},\ A_{\Delta Er}\beta_{SM},\ A_{\Delta Er\Omega SM},\ A_{\Delta Er}\Delta E_f,\ A_{\Delta Er\Delta Er},\ A_{\Delta Er\Delta Ess},\ A_{\Delta Er\Delta h}] \\ [A_{\Delta Ess\Delta Ef},\ A_{\Delta Ess\Delta Ess}] \\ [A_{\Delta h\Delta\Omega SM},\ A_{\Delta h\Delta h}] \end{bmatrix} \qquad (3\text{-}160)
$$

The Δz – vectors of Figure 3.2 are subvectors that comprise only those state variables that incrementally influence on the derivative of respective incremental state variables. This feature is what makes the A-matrix of (3-160) compact or 'non-sparse'.

We seek the sparse description (3-1) in which $\mathbf{\Delta z}$ is the set of all incremental state variables. In the present study case we have:

$$\mathbf{\Delta z} = \begin{bmatrix} \Delta \mathbf{i}_{loop} \\ \Delta \mathbf{e}_{c} \\ \Delta \phi_{AM} \\ \Delta \phi_{SM} \\ \Delta \beta_{SM} \\ \Delta \Omega_{SM} \\ \Delta \Omega_{AM} \\ \Delta(\Delta \mathring{a}) \\ \Delta(\Delta w) \\ \Delta(\Delta g) \\ \Delta(\Delta E_{f}) \\ \Delta(\Delta E_{r}) \\ \Delta(\Delta E_{ss}) \\ \Delta(\Delta h) \end{bmatrix}$$

Adapting the A-matrix description of (3-160) to the full vector $\mathbf{\Delta z}$, we at long last arrive at the sought $\mathbf{A}$ of (3-1):

A=

	$\mathbf{A}i_{loop}i_{loop}$		$\mathbf{A}i_{loop}e_{c}$	$\mathbf{A}i_{loop}\phi_{AM}$	$\mathbf{A}i_{loop}\phi_{SM}$	$\mathbf{A}i_{loop}\beta_{SM}$	$\mathbf{A}i_{loop}\Omega_{SM}$	$\mathbf{A}i_{loop}\Omega_{AM}$				$\mathbf{A}i_{loop}E_{qf}$			
	$\mathbf{A}e_{c}i_{loop}$		$\mathbf{A}e_{c}e_{c}$												
$\mathbf{A}\phi_{AM}i_{AM}$				$\mathbf{A}\phi_{AM}\phi_{AM}$				$\mathbf{A}\phi_{AM}\Omega_{AM}$							
	$\mathbf{A}\phi_{SM}i_{SM}$				$\mathbf{A}\phi_{SM}\phi_{SM}$	$\mathbf{A}\phi_{SM}\beta_{SM}$						$\mathbf{A}\phi_{SMEqf}$			
							$A\beta_{SM}\Omega_{SM}$								
	$\mathbf{A}\Omega_{SM}i_{SM}$				$\mathbf{A}\Omega_{SM}\phi_{SM}$	$A\Omega_{SM\beta SM}$	$A\Omega_{SM\Omega SM}$		$A\Omega_{SM}\Delta_{\mathring{a}}$		$A\Omega_{SM}\Delta_{g}$				
$\mathbf{A}\Omega_{AM}i_{AM}$				$\mathbf{A}\Omega_{AM}\phi_{AM}$				$A\Omega_{AM}\Omega_{AM}$							
							$A\Delta\mathring{a}\Omega_{SM}$		$A\Delta\mathring{a}\Delta\mathring{a}$ $A\Delta w\Delta\mathring{a}$ $A\Delta g\Delta\mathring{a}$	$A\Delta\mathring{a}\Delta w$ $A\Delta w\Delta w$	$A\Delta g\Delta g$				
	$\mathbf{A}_{\Delta Er iSM}$				$\mathbf{A}_{\Delta Er\phi SM}$	$A_{\Delta Er\beta SM}$	$A_{\Delta Er\Omega SM}$ $A_{\Delta h\Delta\Omega SM}$					$A_{\Delta Eqf\Delta Eqf}$ $A_{\Delta Er}\Delta E_{qf}$ $A_{\Delta Ess\Delta Eqf}$	$A_{\Delta Eqf\Delta Er}$ $A_{\Delta Er\Delta Er}$	$A_{\Delta Er\Delta Ess}$ $A_{\Delta Ess\Delta Ess}$	$A_{\Delta Er\Delta h}$ $A_{\Delta h\Delta h}$

Figure 3.3 A-matrix for sample power system of Figure 3.1

3.11 Example eigenvalue analysis

3.11.1 System data

For overview reasons all the data pertaining to system components, their interconnection and to initial system status, are summarized in the following.

The single-line diagram of the power system is shown in Figure 3.4. All per unit (pu) data given are referred to common system basis.

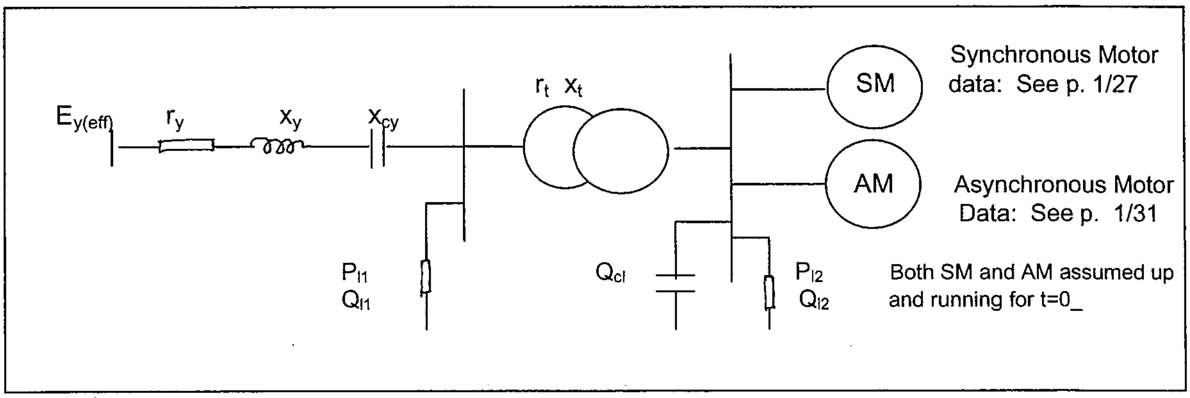


Figure 3.4: Example power system under study

External system:

The external system is assumed to be an infinite bus with per phase r.m.s. voltage $E_{y(eff)}$ =1.05 pu. Angle γ_{ref} is arbitrarely set to zero. See Chapter 1.6.

Initial conditions:

As initial operational conditions for the system we specify the following:

Power supplied to the *synchronous motor* $P_{SMtarget(0)}$ = -0.8pu. This means generator mode of operation.

Synchronous motor voltage $E_{SMtarget(0)}$ = 1.0pu.

Power supplied to the *asynchronous motor* $P_{AMtarget(0)}$ = 0.5pu. The reactive power supplied to the motor ($Q_{AMtarget(0)}$) , will flow as a byproduct from the initial condition analysis, see next .

Series impedance (r_y + j x_y) :

r_y = 0.030pu
x_y = 0.125pu

Series capacitor bank (r_{cy} – j x_{cy}):

r_{cy} = 0.000pu
x_{cy} = 0.025pu

Transformer (r_t + j x_t) :

r_t = 0.01pu
x_t = 0.07pu

Impedance type, inductive load '1' (P_{l1} +j Q_{l1}) :

P_{l1} = 0.60pu — Load defined at 1.0pu voltage
Q_{l1} = 0.20pu (inductive)

Impedance type, inductive load '2' (P_{l2} +j Q_{l2}) :

P_{l2} = 0.25pu — Load defined at 1.0pu load voltage
Q_{l2} = 0.80pu (inductive)

Shunt capacitor bank (P_{cl} - j Q_{cl}) :

P_{cl} = 0.00pu — Load defined at 1.0pu load voltage
Q_{cl} = 0.70pu (capacitive)

Dummy series impedance (r_D +jx_D)
Introduced for 'multiple purpose' reasons. For the present load flow:

r_D = 0.0pu
x_D = 0.0pu

The synchronous motor (here representing a hydroelectric generator) :

$X_{a\sigma}$ = 0.12pu	X'_d = 0.34pu	R_a = 0.005pu	T''_q = 0.16s	$\mathrm{Cos}\varphi_N$ = 0.9pu
X_d = 1.20pu	X''_d = 0.20pu	T'_{do} = 6.0s	T_a = 5.0s	
X_q = 0.75pu	X''_q = 0.30pu	T''_d = 0.04s	C_D = 7.5pu	

Synchronous motor voltage control system:

T_f	= 0.1s	; field circuit time constant
K_R	= 70pu	; resulting forward amplification
T_R	= 0.1s	; regulator time constant
K_D	= 0.25pu	; transient feedback amplification
T_D	= 0.25s	; transient feedback time constant
K_Ω	= 1.0pu	; power stabilizer amplification
$E_{f(max)}$	= 3.0pu	; ceiling field voltage
$E_{f(min)}$	= -2.0pu	; floor field voltage
T_Ω	= 2s	; power stabilizer time constant

(See equations (1-106)- (1-121) for full parameter interpretation)

Synchronous motor power control system (when in hydro generator mode of operation):

T_r	= 0.3s	; Time constant for hydraulic system
T_c	= 0.08s	; Time constant for main servo (eg 0.08s)
T_t	= 17s	; Transient droop time constant
δ_t	= 0.15pu	; Transient droop
δ_p	= 0.00pu	; Permanent droop. (0-0.04) (The value 0.0 apply if the frequency is to be sustained by this unit alone)
P_{target}	= -0.8	; Target value of of absorbed motor power. (Applicable when loading up automatically, following synchronization)

The Asynchronous Motor (in motor mode of operation)

$X_{a\sigma}$ = 0.08pu	$X_{r\sigma}$ = 0.08pu	X_m = 2.5pu
R_a = 0.03pu	R_r = 0.03pu	κ = 2.0pu
T_a = 4.0s		

(See equations (1-125)-(1-129) for parameter interpretation)

3.11.1 Initial conditions

With initial conditions specified as stated above, the iterative solution process described in section 2.4 of systems chapter 2, is called upon for targeting the given operating point to required accuracy.

As arbitrary starting values for the set of *'load flow control variables'* ($\beta_{SM(0)}$,$E_{f(0)}$, $\Omega_{AM(0)}$) discussed in section 2.4, we choose ; $\beta_{SM(o)}=0$, $E_{f(o)}=1.5$pu, $\Omega_{AM(o)}=1.0$pu. Applying the three-step logic that comprises equations (2-36) to (2-39), and using the default value 1.0 of the factor k of (2-39), - we arrive at a feasible solution after 6 iterations. Exit from the iterative process is made when

$$Res < 0.001pu \quad \text{('Res' is abbreviation for 'Residual')} \qquad (3\text{-}161)$$

where Res = ($ResV_{SM}$ + $ResP_{SM}$ + $ResP_{AM}$). The contributions in parenthesis are the deviations in absolute (pu) terms from target value, of respectively *synchronous motor voltage*, *power supplied to the synchronous motor*, and *power supplied to the asynchronous motor.*

The iteratively determined *'load flow control variables'* contributing to giving a valid initial load flow, are :

$$\begin{aligned} \beta_{SM(0)} &= -0.380158rad. \\ E_{f(0)} &= 1.792434pu \\ \Omega_{AM(0)} &= 0.983866pu \end{aligned} \qquad (3\text{-}162)$$

Main characteristics of the established initial load flow for the system in Figure 3.4 are as follows:

The infinite bus:	Voltage	: 1.0500pu	(specified)
	Active power	: 0.5667pu	(delivered to the study system)
	Reactive power	: 0.3154pu	(the study system acts reactively as an *inductuctor*)
Load bus '1' :	Voltage	: 1.0048pu	
	Active load '1'	: 0.6057pu	
	Reactive load '1'	: 0.2019pu	(inductive)
	Active power	: -0.0505pu	(delivered to the transformer)
	Reactive power	: 0.0753pu	(the transf. acts reactively an *inductor*)
Motor bus :	Voltage	: 1.0000pu	(specified)
	Active SM power	: -0.7999pu	(specified power to synchronous motor : −0.8pu)
	Reactive SM power	: -0.4427pu	(the SM acts reactively as a capacitor bank)
	Active AM power	: 0.4993pu	(specified power to asynchronous motor : 0.5pu)
	Reactive AM power	: 0.4174pu	(the AM acts reactively as an inductor)
	Capacitor load	: -0.7001pu	(capacitive)
	Active load '2'	: 0.2500pu	
	Reactive load '2'	: 0.8001pu	(inductive)

3.11.2 Systen eigenvalues

The A-matrix defined in (3-1) is etablished as advised in previous sections 3.1 to 3.10. The system's eigen-values are then computed using the QR-algorithm on **A,** after first reducing it to Hessenberg form.[1]

Three different cases based on the above defined power system, are dealt with in the following. The first is denoted *'Normal system case'* and presents the eigenvalues for the system in Figure 3.4, given initial conditions as shown above. The other two; *'Special system case 1'* and *'Special system case 2'*, are included to illustrate how the use of special (i.e. simplifying) premises to the system of Figure 3.4, can allow for simple/ manual 'spot-checks' of the correctness of the devised processes of system modelling and analysis :

'Normal system case' :
With the system as given in Figure 3.4 and further detailed above, the computed eigenvalue picture may be drawn as shown in Figure 3.5.

Re(Lambda) *)	Im(Lambda) *)	Frequency(Hz) DQ ref frame *)	Frequency(Hz) Phase ref frame **)	Timeconst. (s) ***)	
¯47.939	1869.945	297.611	247.611	0.021	
¯47.939	¯1869.945	¯297.611	¯247.611	0.021	Associated with shunt capacitor C_L
¯53.055	1236.798	196.843	146.843	0.019	
¯53.055	¯1236.798	¯196.843	¯146.843	0.019	
¯841.307	313.943	49.966	0.034	0.001	
¯841.307	¯313.943	¯49.966	¯0.034	0.001	
¯30.025	400.461	63.735	13.735	0.033	
¯30.025	¯400.461	¯63.735	¯13.735	0.033	
¯27.307	311.751	49.617	0.383	0.037	Associated with series capacitor C_y
¯27.307	¯311.751	¯49.617	¯0.383	0.037	
¯94.965	313.079	49.828	0.172	0.011	
¯94.965	¯313.079	¯49.828	¯0.172	0.011	
¯27.645	224.663	35.756	14.244	0.036	
¯27.645	¯224.663	¯35.756	¯14.244	0.036	
¯11.867	42.195	6.716	6.716	0.084	Associated with voltage regulator.
¯11.867	¯42.195	¯6.716	¯6.716	0.084	
¯33.589	9.933	1.581	1.581	0.030	Asynchronous motor
¯33.589	¯9.933	¯1.581	¯1.581	0.030	oscillation frequency
¯21.390	0.000	0.000	0.000	0.047	
¯1.249	12 .137	1.932	1.932	0.800	Synchronous motor oscillation frequency
¯1.249	¯12.137	¯1.932	¯1.932	0.800	
¯9.403	0.000	0.000	0.000	0.106	
¯8.330	0.000	0.000	0.000	0.120	
¯3.684	0.000	0.000	0.000	0.271	
¯1.829	0.000	0.000	0.000	0.547	Associated with SM field circuit & voltage regulator
¯0.359	0.818	0.130	0.130	2.782	
¯0.359	- 0.818	- 0.130	-0.130	2.782	
0.000	0.000	0.000	0.000	999.000	
¯0.500	0.000	0.000	0.000	2.000	

*) Electrical network related eigenvalues are referred to the synchronous 50Hz d-q frame of reference.

**) Electrical network related frequencies are referred to per phase frame of reference (by shifting the d-q referenced values +/- 50Hz).
Eigenvalues with |Real(Lanbda)|<50 and |Im(Lambda)<50 (i.e. f< ca 8Hz) are assumed to relate to electromechanical transients or control gear transients, and their associated frequencies are not given the stated "post-treatment of being shifted +/- 50Hz

***)The time constant associated with a real or complex eigenvalue is defined as T=1/|Real(Lambda)|. If both real and imaginary part of Lambda are zero, T is arbitrarely set to 999.

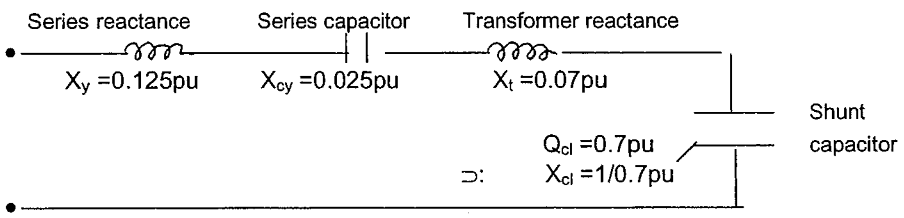
Figure 3.5 : Eigenvalue picture associated with the specified system of Figure 3.4

The two leftmost columns are the 'raw' results from the eigenvalue subroutine. Column no. 3 gives the frequencies in Herz implied by column 2. It is noted that the power circuit related frequencies of column 3 are referred to the synchronous 50Hz d-q frame of reference.

The two rightmost columns give -respectively- system frequencies when consistently referred to per phase frame of reference, and time constants associated with real and complex eigenvalues. The 'origin' of main oscillatory modes is commented on in the figure. Identification has been afforded via sensitivity analysis.

'Special system case 1'

Via special data input we wish to degenerate the system of Figure 3.4 into the series circuit of Figure 3.6. It is then possible to evaluate an oscillatory *circuit* mode in two ways; by longhand use of a simple formula, and by application of the devised computational scheme to a degenerated system model.



$X_{L(tot)} = X_y + X_t = 0.125+0.07 = 0.195\text{pu}$

$X_{C(tot)} = X_{cy} + X_{cl} = 0.025+(1/0.7) = 1.453571429\text{pu}$

Figure 3.6: System of Figure 3.3 degenerated into a simple L-C series circuit.

For the circuit of Figure 3.6 we readily have the following resonant frequency:

$$\mathbf{f_{circuit} = f_0 \cdot \sqrt{(X_{C(tot)} / X_{L(tot)})}} \quad \text{[Hz]} \qquad (3\text{-}163)$$

⊃:

$$\underline{f_{circuit}} = 50\sqrt{(1.453571429/0.195)} = \underline{\mathbf{136.512}} \quad \text{[Hz]}$$

Next approach is eigenvalue analysis : To have the system model of Figure 3.4 altered so that (in numerical terms) the series circuit of Figure 3.6 emerges as a part of it, new dummy data must be specified for some of the circuit models. The components that are influenced :

Series impedance $(r_y + j\,x_y)$:
- r_y = 0.0pu (new)
- x_y = 0.125pu (as before)

Transformer $(r_t + j\,x_t)$:
- r_t = 0.0pu (new)
- x_t = 0.07pu (as before)

Impedance type, inductive load '1' $(P_{l1} + j\,Q_{l1})$:
- P_{l1} = 0.0pu (new)
- Q_{l1} = 0.0001pu (new)

Impedance type, inductive load '2' $(P_{l2} + j\,Q_{l2})$:
- P_{l2} = 0.0pu (new)
- Q_{l2} = 0.0001pu (new)

The synchronous motor : All data unchanged, except;
- R_a = 50000pu (to approx. the effect of electrically disconnecting the motor)

The asynchronous motor : All data unchanged, except;
- R_a = 50000pu (to electrically disconnect the motor)

With system data modified as given above, the eigenvalue picture becomes as shown in Figure 3.7:

Re(Lambda) *)	Im(Lambda) *)	Frequency(Hz) DQ-ref frame *)	Frequency(Hz) Phase ref frame **)	Timeconst. (s) ***)
¯99720885.972	314.159	50.000	0.000	0.000
¯99720885.972	¯314.159	¯50.000	0.000	0.000
¯78539826.716	0.000	0.000	0.000	0.000
¯52359881.305	0.000	0.000	0.000	0.000
¯0.009	1171.901	186.514	**136.514**	113.358
¯0.009	¯1171.901	¯186.514	**¯136.514**	113.358
¯0.009	543.582	86.514	36.514	113.358
¯0.009	¯543.582	¯86.514	¯36.514	113.358
0.000	314.856	50.111	0.111	6478.013
0.000	¯314.856	¯50.111	¯0.111	6478.013
0.000	313.463	49.889	0.111	6478.013
0.000	¯313.463	¯49.889	¯0.111	6478.013
0.000	314.159	50.000	0.000	999.000
0.000	¯314.159	¯50.000	0.000	999.000
¯11.897	42.226	6.721	6.721	0.084
¯11.897	¯42.226	¯6.721	¯6.721	0.084
¯17.205	0.000	0.000	0.000	0.058
¯15.125	0.000	0.000	0.000	0.066
¯3.653	5.069	0.807	0.807	0.274
¯3.653	¯5.069	¯0.807	¯0.807	0.274
¯2.500	0.000	0.000	0.000	0.400
¯0.667	1.462	0.233	0.233	1.500
¯0.667	¯1.462	¯0.233	¯0.233	1.500
¯0.207	0.000	0.000	0.000	4.836
¯0.062	0.000	0.000	0.000	16.095
¯0.162	0.000	0.000	0.000	6.171
0.000	0.000	0.000	0.000	348097296.032
0.000	0.000	0.000	0.000	999.000
¯0.500	0.000	0.000	0.000	2.000

*) Electrical network related eigenvalues are referred to the 50HZ d-q frame of reference. To have them referred to per phase frame of reference, + or- 50Hz is added.

Eigenvalues with |Real(Lanbda)|<50 and |Im(Lambda)<50 (i.e. f< ca 8Hz) are assumed to relate to electromechanical transients, and are not given the "post-treatment"of adding + or -50 Hz.

**) The time constant associated with a real or complex eigenvalue is defined as T=1/|Real(Lambda)|. If both real and imaginary part of Lambda is zero, T is arbitrarely set to 999.'

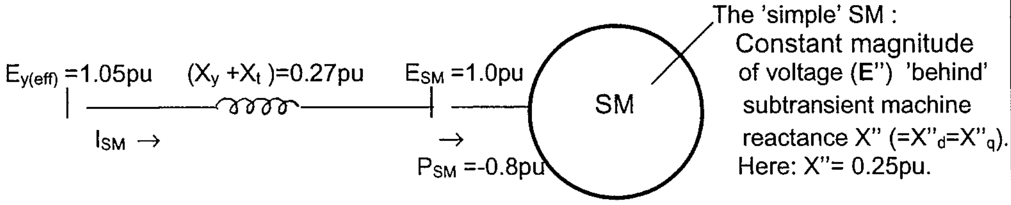
Figure 3.7 : Eigenvalue picture associated with 'Special system case 1'

As should be expected, we recognise from the above menu of modes, also the resonant frequency that previously emerged from equation (3-163). As the sought mode is an inherent *electrical circuit*-defined frequency, no need has been of establishing valid initial conditions for this particular eigenvalue analysis.

'Special system case 2'

Whereas the special case above provided for a check of a *network* related resonant frequency, the present case aims at demonstrating a similar check of a *machine* oscillatory mode. In particular, we will focus on the oscillatory mode of the synchronous motor. The motor's operational conditions (i.e. voltage and absorbed power) are arbitrarely chosen the same as before, but the network to which the motor is connected, is being modified :

Via adjusted data input we will alter the system of Figure 3.4 into the series circuit case of Figure 3.8. It is then possible to evaluate an *oscillatory eigenvalue* mode in two ways; by use of a simple formula that approximates the undamped, oscillatory frequency of a synchronous machine ('SM') connected to an infinite bus via a series reactance, and by application of the current computational tool to a suitably reduced system model.



$X_{ext} = (X_y + X_t) = 0.27$pu
$X_\Sigma = (X_y + X_t + X'') = 0.27 + 0.25 = 0.52$pu

Initial conditions:	External voltage	: $E_{y(eff)}$ = 1.05pu	(specified)
	Motor voltage	: E_{SM} = 1.0pu	(")
	Power to motor	: P_{SM} = -0.8pu	(")
	Motor current	: I_{SM} = 0.806194pu	(computed from load flow)

Figure 3.8 : System of Figure 3.4 modified into case 'Simple SM connected to infinite bus'. Model basis for evaluating oscillatory frequency from formula (3-164).

Presuming a 'simple' synchronous machine as defined in Figure 3.8, we have from basic system theory that the *undamped oscillatory machine frequency* of the configuration of Figure 3.8, can be expressed as

$$\mathbf{f_{SM} = (1/(2\pi)) [\omega_0 E_{y(eff)} E'' \cos\beta/(X_\Sigma T_a \cos\varphi_N)]^{0.5}} \quad \text{[Hz]} \qquad (3\text{-}164)$$

Definitions:

$E_{y(eff)}$ = external specified voltage. Here: $E_{y(eff)}$ =1.05pu
E_{SM} = synchronous machine voltage. Here: E_{SM} = specified = 1.0pu
I_{SM} = pu synchronous machine current from initial load flow. Here; I_{SM}=0.806194pu.
E" = pu voltage 'behind' the machine's subtransient reactance X".
E" can be computed on the basis of ($E_{y(eff)}$, E_{SM}, I_{SM}, X_{ext} , X_Σ) :
Setting $a = I_{SM} X_\Sigma$ and $b = I_{SM} X_{ext}$, we have;

$$E'' = [(E_{y(eff)} - a)^2 - (a/b) (E_{y(eff)} - b - E'') (E_{y(eff)} - b + E'')]^{0.5} \qquad (3\text{-}165)$$

= 0.994755 in the current case.

β = electrical angle between phasors $\mathbf{E}_{y(eff)}$ and **E"**. To determine cosβ we have, - according to the 'Law of cosines' ;

$$\cos\beta = (E_{y(eff)}^2 + E''^2 - a^2)/(2 E_{y(eff)} E'') \qquad (3\text{-}166)$$

= 0.917368 in the current case.

X_Σ = (X_{ext} +X") = total pu reactance between $\mathbf{E}_{y(eff)}$ and **E"**.
X_{ext} = (X_y+X_t) = total pu *'external'* reactance ; reactance between $\mathbf{E}_{y(eff)}$ and $\mathbf{E}_{SM}$.
T_a = inertia time constant for synchronous machine. Here. T_a = 5s.
$\cos\varphi_N$= rated power factor of synchronous machine. Here: $\cos\varphi_N$ = 0.9.

Inserting the proper values into (3-164), we get the following oscillatory frequency for the synchronous machine;

$$\underline{f_{SM}} = (1/(2\pi)) [100\pi\ 1.05\ 0.994755\ 0.917368/(0.52\ 5\ 0.9)]^{0.5} = \underline{\mathbf{1.805}} \text{ [Hz]} \qquad (3\text{-}167)$$

Next approach is eigenvalue analysis: To have the system model of Figure 3.4 altered so that (in numerical terms) the series circuit of Figure 3.8 becomes a part of it, new dummy data must be specified for some of the electrical circuit models. The components that are influenced :

Series impedance (r_y + j x_y) :

r_y = 0.0pu	(new)
x_y = 0.2pu	(new; changed arbitrarely from 0.125pu)

Transformer (r_t + j x_t) :

r_t = 0.0pu	(new)
x_t = 0.07pu	(as before)

Impedance type, inductive load '1' (P_{l1} +j Q_{l1}) :

P_{l1} = 0.0pu (new)
Q_{l1} = 0.0001pu (new)

Impedance type, inductive load '2' (P_{l2} +j Q_{l2}) :

P_{l2} = 0.0pu (new)
Q_{l2} = 0.0001pu (new)

Shunt capacitor bank (P_{cl} - j Q_{cl}) :

P_{cl} = 0.00pu (as before)
Q_{cl} =any value =here 50000pu (new; any effect of Q_{cl} is eliminated via setting of r_D , see next)

Dummy series impedance (r_D+ j x_D): (in series with the shunt capacitor bank; for topological reasons. See 2.2)

r_D = 50000pu (new ; to in effect disconnect the shunt capacitor bank from the system)
x_D = 0.0pu (as before)

The Synchronous Motor : All data unchanged, except;

X''_d = 0.25pu (changed from 0.2pu)
X''_q = 0.25pu (changed from 0.3pu)
T''_d = 100s (increased from initial value =0.04s to contibute to mimicking fixed magnitude of **E"**)
T''_q = 100s (" " " " = 0.16s " " " " " " " ")

Synchronous motor voltage control system : All data unchanged, except;

K_R = 0.0pu (to in effect disconnect the regulator)

Synchronous motor power control system : All data unchanged, except;

Tc = 50000s (to in effect disconnect the regulator)

The Asynchronous Motor : All data unchanged, except;

R_a = 50000pu (to in effect disconnect the motor from the system)

With system data modified as given above, the eigenvalue picture becomes as shown in Figure 3.9 :

Re(Lambda)	Im(Lambda)	Frequency(Hz) DQ-ref. frame	Frequency(Hz) Phase ref. frame	Timeconst. (s)
¯276868229.397	314.159	50.000	0.000	0.000
¯276868229.397	¯314.159	¯50.000	0.000	0.000
¯43585417.190	314.159	50.000	0.000	0.000
¯43585417.190	¯314.159	¯50.000	0.000	0.000
¯3.023	314.159	50.000	0.000	0.331
¯3.023	¯314.159	¯50.000	0.000	0.331
¯3.653	314.128	49.995	0.005	0.274
¯3.653	¯314.128	¯49.995	¯0.005	0.274
¯0.115	11.386	1.812	**1.812**	8.663
¯0.115	¯11.386	¯1.812	**¯1.812**	8.663
0.000	314.159	50.000	0.000	7957746.538
0.000	¯314.159	¯50.000	0.000	7957746.538
0.000	314.159	50.000	0.000	999.000
0.000	¯314.159	¯50.000	0.000	999.000
0.000	314.159	50.000	0.000	999.000
0.000	¯314.159	¯50.000	0.000	999.000
¯8.304	0.000	0.000	0.000	0.120
¯0.478	0.000	0.000	0.000	2.091
¯0.059	0.000	0.000	0.000	16.999
¯0.007	0.000	0.000	0.000	153.194
¯0.003	0.000	0.000	0.000	396.365
0.000	0.000	0.000	0.000	999.000
0.000	0.000	0.000	0.000	999.000
0.000	314.159	50.000	0.000	999.000
0.000	¯314.159	¯50.000	0.000	999.000
¯10.000	0.000	0.000	0.000	0.100
¯4.000	0.000	0.000	0.000	0.250
¯10.000	0.000	0.000	0.000	0.100
¯0.500	0.000	0.000	0.000	2.000

Figure 3.8 : Eigenvalue picture associated with 'Special system case 2'

As we would anticipate ; the above menu of modes comprises one resonant frequency that is fairly close to the value given by (3-167), - the difference being about 0.4%.

[1] : W.H.Press, B.P.Flannery, S.A.Teukolsky, W.T.Vetterling : Numerical Recipes (Fortran Version), Cambridge University Press, 1989

4. Time Response Analysis

4. Time Response Analysis

Given the description of an initial state of operation of a power system. We seek the variation over time of the system's state variables and their interactions, following some specified disturbance to the system.

Again we devote our attention to the system of Figure 3.4, -the detailed description of which is given in Chapter 3.11.1. In the following we will consider the main transients of interest, subsequent to respectively a *three phase short circuit*, *start/ loading of an asynchronous motor*, *start/ loading/ disconnection of a synchronous generator*, *islanding of a local power system,-* and finally ; a comparison of responses when *applying the same disturbance to respectively a 'weak' and 'strong' power system*. The transient performance of the *adjustable speed synchronous machine* is dealt with in Appendix 2.

4.1 The differential equations

The conceptual as well as specific basis for producing the differential equations needed to model the dynamical performance of the given power system, have been developed in Chapter 1 and 2.

The (5x2+2x2 =14) equations describing the performance of the power network loop currents $\mathbf{i}_{loop}$ and the power network capacitor voltages $\mathbf{e}_{tc}$ are contained in (2-20) of Network Modelling Chapter 2.3 :

$$d\mathbf{i}_{loop}/dt = -\omega_0\, \mathbf{X}_{Lloop}^{-1} \cdot (R_{loop}\, \mathbf{i}_{loop} + \mathbf{B}_{tc}\, \mathbf{e}_{tc} + \mathbf{e}_{chord} + \mathbf{B}_{t\text{-}rest}\, \mathbf{e}_{t\text{-}rest}) \qquad (4\text{-}1)$$

$$d\mathbf{e}_{tc}/dt = \omega_0\, (\mathbf{X}_{Cprimitive}\, \mathbf{B}_{tc}^{t} \cdot \mathbf{i}_{loop} + \bar{\mathbf{1}}_{tc}\, \mathbf{e}_{tc}) \qquad (4\text{-}2)$$

The (2+1=3) equations describing the performance of the asynchronous motor's fluxes and angular speed are given by (1-126) and (1-128) of Systems Component Summary Chapter 1.7 :

$$d\phi_{AM}/dt = \omega_0\, (\mathbf{F}_{AMi}\, i_{AM} + \mathbf{F}_{AM\phi}\, \phi_{AM}) \qquad (4\text{-}3)$$
$$d\Omega_{AM}/dt = (S_{Bas}/S_{AM})\, (1/(T_a \cos\varphi))\, (T_{AMel} - T_{AMmec}) \qquad (4\text{-}4)$$

The (3+1+1=5) equations describing the performance of the traditional synchronous motor's fluxes, angular speed and electrical rotor angle are given by (1-113), (1-116) and (1-119) of Chapter 1.7 :

$$d\phi_{SM}/dt = \omega_0\, (\mathbf{e}_{SMr} + \mathbf{F}_{SMi}\, \mathbf{i}_{SM} + \mathbf{F}_{SM\phi}\, \phi_{SM}) \qquad (4\text{-}5)$$
$$d\Omega_{SM}/dt = (S_{Bas}/S_{SM})\, (1/(T_a \cos\varphi_N))\, (T_{SMel} - T_{SMmec}) \qquad (4\text{-}6)$$
$$d\beta_{SM}/dt = \omega_0 \cdot (1 - \Omega_{SM}) \qquad (4\text{-}7)$$

The performance of the synchronous motor's voltage control system is in the present outline described by the four differential equations (1-121) of Chapter 1.7 :

$$d\Delta E_f/dt = C_1\, (\Delta E_r - \Delta E_f) \qquad (4\text{-}8)$$
$$d\Delta E_r/dt = C_2\, [\Delta U_{ref} + U_0 - U + K_\Omega\, (\Omega_{SM} - 1) - \Delta h] - C_3\, \Delta E_f - C_4\, \Delta E_r + C_2\, \Delta E_{ss} \qquad (4\text{-}9)$$
$$d\Delta E_{ss}/dt = C_5\, \Delta E_f - C_6\, E_{ss} \qquad (4\text{-}10)$$
$$d\Delta h/dt = C_7\, (\Omega - 1) - C_8\, \Delta h \qquad (4\text{-}11)$$

The performance of the power control system of the synchronous motor in generator mode of operation, is exemplified by the three differential equations (1-120) of Chapter 1.7 :

$$\Delta å/dt = K_1\, (\Delta\Omega_{ref} - (1 - \Omega_{SM}) + \Delta w) - K_2\, \Delta å \qquad \text{Regulator system} \qquad (4\text{-}12)$$
$$d\Delta w/dt = K_3\, \Delta å - K_4\, \Delta w \qquad \text{Regulator system} \qquad (4\text{-}13)$$
$$d\Delta g/dt = (3K_o/\Omega_{SM})\, \Delta å - K_ó\, \Delta g \qquad \text{Hydraulic system} \qquad (4\text{-}14)$$
$$\downarrow$$
$$\Delta T_{mec} = \Delta g - å_ó\, (1 - \Omega_{SM}) - (2/\Omega_{SM})\, \Delta å \qquad \text{Net change of mechanical torque}$$

The differential equations (4-1) to (4-14) describe the behaviour of alltogether 29 state variables. These variables go into the (29x1) vector of system state variables, here denoted **Respons**:

Respons (29x1) =

$\mathbf{i}_{loop}$	(5 loop currents, giving 5x2=10 d-q axis current variables)
$\mathbf{e}_{tc}$	(2 capacitor voltages, giving 2x2=4 d-q axis current variables)
ϕ_{AM}	(1 asynchronous motor,giving 1x2=2 d-q axis flux variables)
ϕ_{SM}	(1 synchronous motor, giving 1x3=3 d-q axis flux variables)
β_{SM}	(1 synchronous motor rotor angle)
Ω_{SM}	(1 synchronous motor rotor speed)
Ω_{AM}	(1 asynchronous motor rotor speed) (4-15) (4-15)
$\Delta å$	(1 turbine gate opening beyond initial setting. (I.e.: $\Delta å_{(o)}=0$))
Δw	(1 power control variable, where $\Delta w_{(o)}=0$)
Δg	(1 power control variable, where $\Delta g_{(o)}=0$)
ΔE_f	(1 excitation voltage 'beyond' initial setting. $\Delta E_{f(o)}=0$)
ΔE_r	(1 voltage control variable, where $\Delta E_{r(o)}=0$)
ΔE_{ss}	(1 voltage control variable, where $\Delta E_{ss(o)}=0$)
Δh	(1 SM speed stabilizer variable, where $\Delta h_{(o)}=0$)

The initial value of all control system related state variables is per definition zero, as stated in parentheses above. Initial value of all other state variables is provided from an initial condition analysis, see Chapter 2.4.

The circuit elements ($\mathbf{R}_{SM}$, $\mathbf{X}_{SM}$, $\Delta\mathbf{E}_{SM}$) that networkwise model an ideal synchronous motor, are functions of one or more of the machine's own 'local' state variables (β_{SM}, ω_{SM}, ΔE_{qf}) . See Chapter 1.7, equations (1-108) - (1-110), for functional details. The circuit elements ($\mathbf{R}_{AM}$, $\mathbf{X}_{AM}$, $\Delta\mathbf{E}_{AM}$) representing an ideal asynchronous motor, are in part a function of the machine's own 'local' state variable ω_{AM}. See Chapter 1.7, equations (1-125), for functional details. If e.g. saturation phenomena are to be accounted for, the above circuit elements may become functions of machine currents and voltages as well.

From the foregoing it is evident that the *system loop resistance matrix* $\mathbf{R}_{loop}$ and the *system loop inductor matrix* $\mathbf{X}_{Lloop}$ both become functions of a subset of the system's 'local' state variables. To account for this functionality, $\mathbf{R}_{loop}$ and $\mathbf{X}_{Lloop}$ (and naturally also $\mathbf{X}_{Lloop}^{-1}$) have to be continually updated during processes of numerical integration. In the example studies accounted for below, a Runge-Kutta fourth order integration algorithm has been applied for solving equations (4-1) - (4-14). 'Continous update' of the network model implies in this case by choice generating updated versions of $\mathbf{R}_{loop}$ and $\mathbf{X}_{Lloop}$ (and also $\mathbf{X}_{Lloop}^{-1}$) multiple times in the course of advancing the solution one integration time step.

4.2 On presentation of power network currents and voltages

During integration processes power network currents and voltages are conveniently dealt with in terms of d-q axis variables $\mathbf{i}_{dqo}$, and e.g. $\mathbf{u}_{dqo}$. For practical result presentation it is often suitable to transform these variables back into their per phase equivalents $\mathbf{i}_{RST}$ and $\mathbf{u}_{RST}$:

At the outset the d-q axis variable like $\mathbf{i}_{dqo}$ and $\mathbf{u}_{dqo}$ are computed from their corresponding per phase variables, by the Park transformation $\mathbf{P}$, - where $\theta=(\omega_0 t-\beta)$. See p.1/3 and Appendix 3 ;

$$\mathbf{i}_{dqo} = \mathbf{P}\ \mathbf{i}_{RST} \qquad (4\text{-}16)$$
$$\mathbf{u}_{dqo} = \mathbf{P}\ \mathbf{u}_{RST}$$

where;

$$\mathbf{P} = 2/3 \begin{array}{c} \\ d \\ q \\ o \end{array} \begin{array}{ccc} R & S & T \\ \cos\theta & \cos(\theta-2\pi/3) & \cos(\theta-4\pi/3) \\ -\sin\theta & -\sin(\theta-2\pi/3) & -\sin(\theta-4\pi/3) \\ ½ & ½ & ½ \end{array} \qquad (4\text{-}17)$$

For back transformation we invert (4-16):

$$\mathbf{i}_{RST} = \mathbf{P}^{-1}\cdot\ \mathbf{i}_{dqo} \qquad (4\text{-}18)$$
$$\mathbf{u}_{RST} = \mathbf{P}^{-1}\cdot\ \mathbf{u}_{dqo}$$

where;

$$\mathbf{P}^{-1} = \begin{array}{|c|c|c|} \cos\theta & -\sin\theta & 1 \\ \cos(\theta-2\pi/3) & -\sin(\theta-2\pi/3) & 1 \\ \cos(\theta-4\pi/3) & -\sin(\theta-4\pi/3) & 1 \end{array} \qquad (4\text{-}19)$$

For the current in (say) phase 'R', we have from (4-18) and (4-19), when observing that zero sequence currents are absent in our studies:

$$i_R = i_d \cos\theta - i_q \sin\theta$$

We define:

$$i_d = k \sin\delta \quad ; \quad i_q = k \cos\delta \quad \} \rightarrow k = (i_d^2 + i_q^2)^{0.5}$$

and get;

$$i_R = k(\sin\delta \cos\theta - \cos\delta \sin\theta) = k \sin(\delta - \theta)$$

⊃:

$$i_R = (i_d^2 + i_q^2)^{0.5} \cdot \sin(\delta - \theta)$$

Per definition:

$$i_R = \sqrt{2}\, I_{Rms} \sin(\delta - \theta)$$

where; $\delta = \arctan(i_d/i_q)$, $\theta = (\omega_0 t - \beta)$

From the two foregoing expressions for i_R we deduce the following basic algorithm for determining the *root mean square* (r.m.s) value of per phase current and voltage, based on the corresponding d-q axis variables:

$$\mathbf{i_{rms} = (1/\sqrt{2})\,(i_d^2 + i_q^2)^{0.5}}$$

and by analogy;

$$\mathbf{u_{rms} = (1/\sqrt{2})\,(u_d^2 + u_q^2)^{0.5}} \qquad (4\text{-}20)$$

r.m.s. value of power network currents and voltages produced from corresponding d-q axis variables.

Power network currents and voltages presented in the following, are r.m.s. values as defined by (4-20). r.m.s. values will per definition be equal to, or greater than zero. In the diagrams to follow, an r.m.s. variable is described in terms of its time response curve, plus three specific figures ; its value at *start of analysis*, its *maximum* value within the time range analyzed, and its correspondingly defined *mimimum* value.

Other diagram variables such as e.g absorbed motor power, electrical torque, rotor field- and damper currents, and synchronous motor rotor angle, may attain positive as well as negative values as they are instantaneous quantities. Such quantities are in this presentation characterized by 3x2 = six figures in addition to the curve depicting its time variation: The first three are the ones defined above for r.m.s. variables. The last three are included to enhance the understanding of where origo is placed along the y-axis in this special (APL-graphical) presentation. Thus, the last set of 3 figures is a 'lifted' version of the first set, so that - if minimum value of the variable is negative - this minimum becomes the 'zero reference' in the graphical presentation.

4.3 Three phase short circuit

Referring to Figure 3.4 a three phase short circuit of duration 0.25s, is implemented by transiently replacing load '2' (P_{l2}=0.25 , Q_{l2}=0.80ind.) by a new one (P_{l2new} = 1000.0 , Q_{l2new} = 0). This is equivalent to introducing a short circuit impedance $z \approx 0.001+j0$ at the motor bus of the existing system . All figures being pu data.

The same sequence of events is repeated for two different analysis durations in order to illustrate the detailed 'inner life' of a short circuit, as well as the more longterm/overall consequences on system performance of this type of disturbance. Time increment Δt during integration: 0.0005s.

Figure 4.1 – 4.9 give sample detailed results for a period of analysis of 0.5s. The short circuit is applied at t = 0.05s and removed 0.25s later at t = 0.3s. A few comments to the results:

As the fault is implemented via setting of new parameters for load '2', the short circuit current appears as the current supplied to load '2'. See Figure 4.8. We notice that the short circuit current in the d-q frame of reference, comprises a distinct oscillating frequency. From the figure we estimate the duration of 9 cycles to 0.5x2.2/6.1 = 0.1803s, giving an estimated frequency of = 9/0.1803 =49.92Hz. For a decay of 63% we estimate from the figure a duration of 0.5x1.5/6.1≈ 0.123s, and thus a decay time constant of that size for the oscillating frequency. This fits well with formal analysis: Computing eigenvalues for the short-circuited state (i.e. after freezing initial value of β_{SM}, E_f and asynchronous motor slip, and setting load '2' to its new value), we find an oscillatory eigenvalue of frequency 49.969Hz and time constant T=0.128s.

The asynchronous motor contributes to the peak of the short circuit current of Figure 4.8. This can be seen from Figure 4.6. Short-circuiting the shunt battery connected to the motor bus implies a current pulse that will also contribute to increasing the short circuit current peak. This is evidenced from Figure 4.9. To limit this current pulse, the dummy series impedance is here set to $(r_D + jx_D) = 0.01+j0.005$pu. The pulse peak of the diagram is most likely not a 'hit' of the true peak; to be sure of having a computed value close to the this peak, the integration time step would have to be considerably smaller.

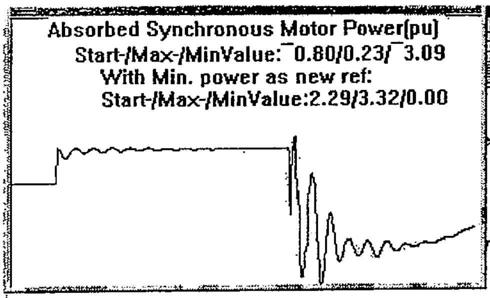


Figure 4.1 Absorbed Synchronous Motor ('SM') power

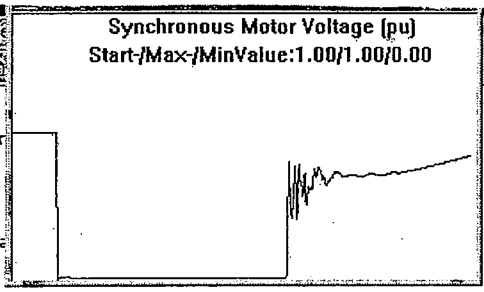


Figure 4.2 SM terminal voltage

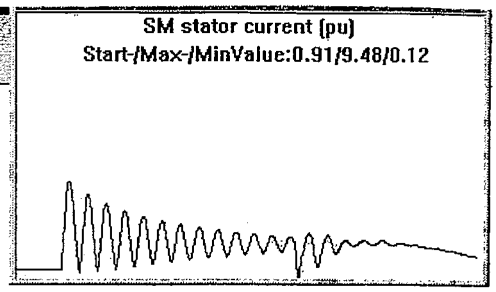


Figure 4.3 SM stator current

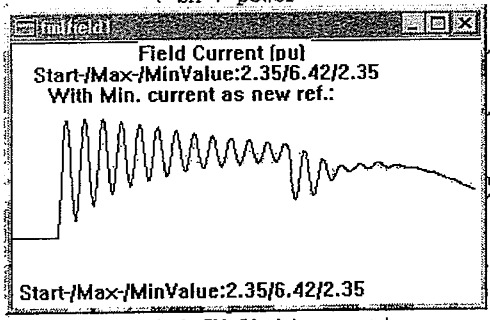


Figure 4.4 SM field current

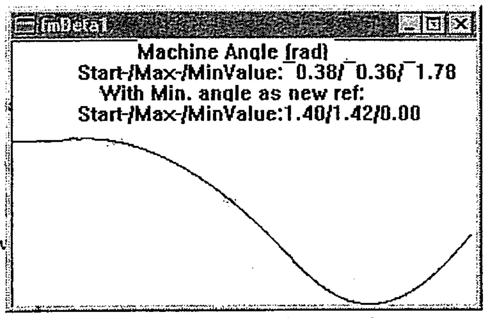


Figure 4.5 SM rotor angle

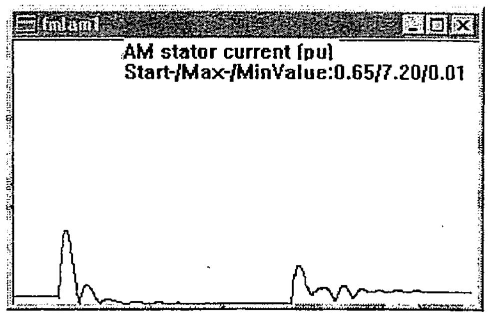


Figure 4.6 Asynchronous Motor ('AM') stator current

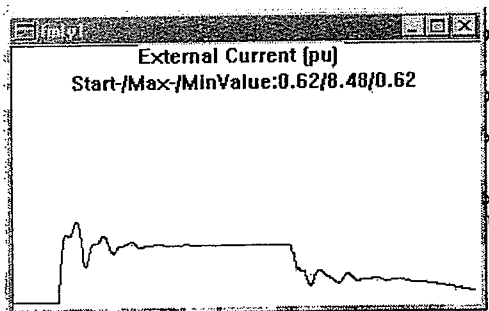


Figure 4.7 Current at external bus

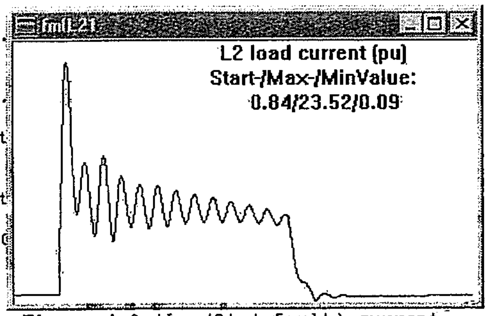


Figure 4.8 'Load2' (=fault) current

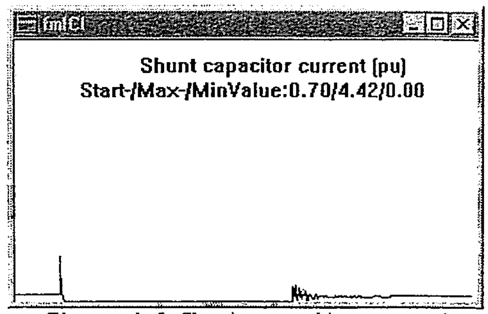


Figure 4.9 Shunt capacitor current

Figure 4.1 – 4.9 **Three phase short circuit** at the motor bus of the system of Figure 3.4. Sample results for an analysis period of **Tmax = 0.5s**. The short circuit is applied at t=0.05s and removed 0.25s later at t=0.3s.

Figure 4.10 – 4.18 give sample results for a period of analysis of 3s *) . The short circuit is applied at t=0.1s and removed 0.25s later at t = 0.35s. We observe that the system fully recovers from the disturbance in the course of the chosen period of analysis.

*) Within the time interval (up to ca 0.5s) that is common to Figures 4.1-4.9 and 4.10- 4.18, characteristic figures (like max. and min.) of any given variable may or may not be registered the same for both durations of analysis. This is due to the chosen logic of result presentation: Regardless of value of T_{max}, 1000 discrete values of each variable is retained for drawing and characterization purposes. Thus, time resolution for presentation becomes a function of T_{max}, causing an increasing number of 'intermediate' variable values to be omitted with increasing T_{max}.

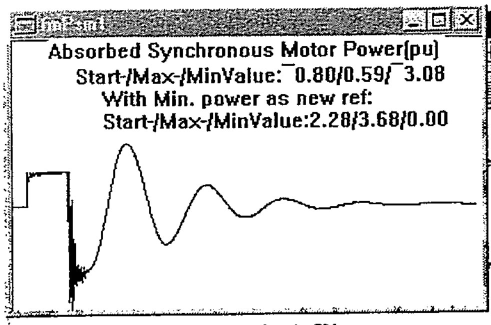


Figure 4.10 Absorbed SM power

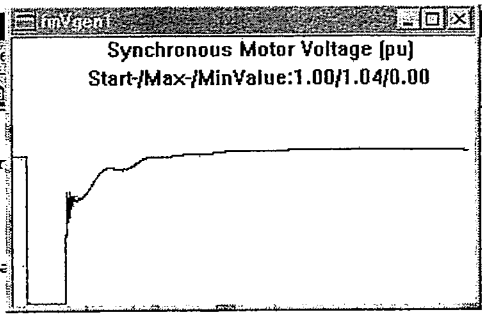


Figure 4.11 SM terminal voltage

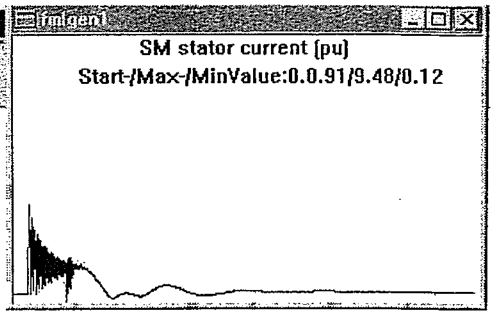


Figure 4.12 SM stator current

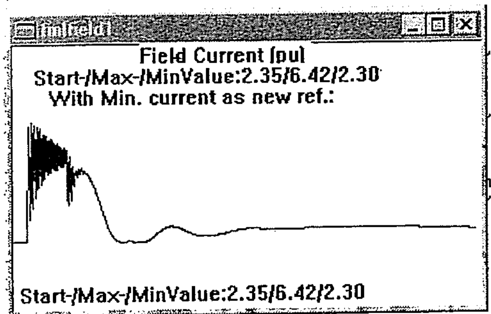


Figure 4.13 SM field current

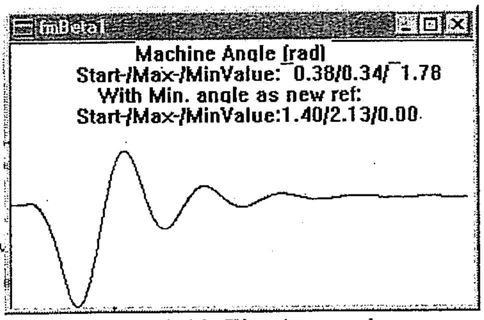


Figure 4.14 SM rotor angle

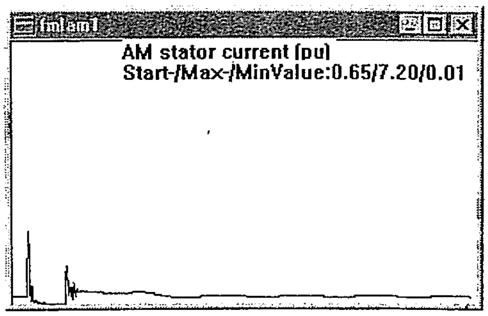


Figure 4.15 AM stator current

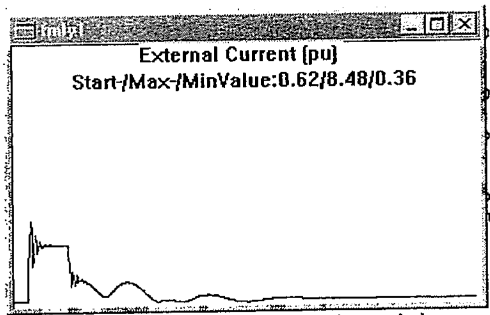


Figure 4.16 Current at external bus

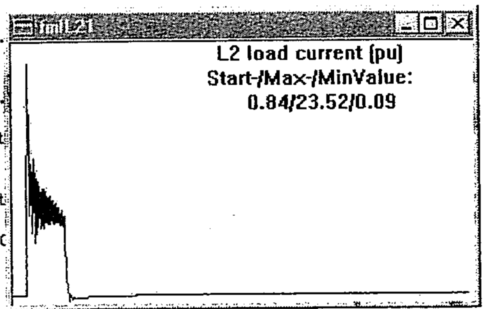


Figure 4.17 'Load2' (=fault) current

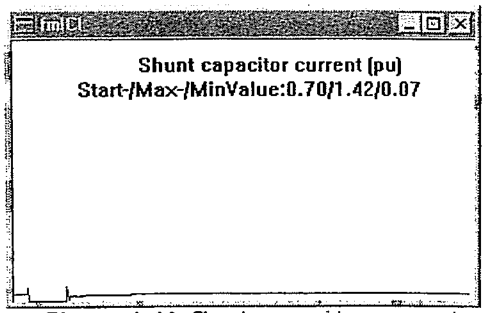


Figure 4.18 Shunt capacitor current

Figure 4.10 – 4.18 **Three phase short circuit** at the motor bus of the system of Figure 3.4. Sample results for an analysis period of $\mathbf{T_{max}}$ **=3s**. The short circuit is applied at t=0.1s and removed 0.25s later at t=0.35s.

4.4 Start of an asynchronous motor

Given an initial steady state operating point of the system in Figure 3.4, with the asynchronous motor disconnected from the network. The motor bus voltage is 1.0pu, and this value is also the target voltage value set for the voltage regulator of the synchronous motor. Power supplied to the synchronous motor is -0.8pu, implying generator mode of operation of the unit. The power control system of the unit in generator mode, is set to respond to deviations from nominal value of system frequency. Since the external infinite bus voltage sustains system frequency at all times, the power output from the synchronous generator should eventually (if remaining in synchronism) resume its initial value, following the disturbance caused by starting of the asynchronous motor.

The dynamical study is run for two different settings of T_{max} to demonstrate 1) the detailed initial impact of an AM-start, and 2) the overall system consequences of this type of disturbance to the system. Time increment during integration: $\Delta t = 0.0005s$.

Figure 4.19 – 4.27 give sample detailed results for a period of analysis of 0.5s. The asynchronous motor – initially at standstill – is connected to the system at t = 0.01s.

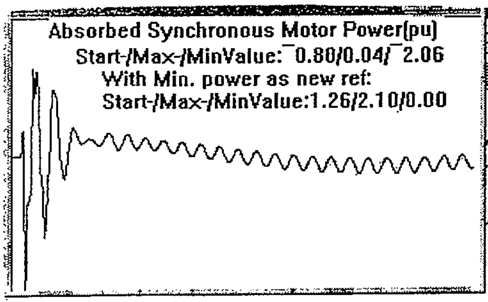


Figure 4.19 Absorbed SM power

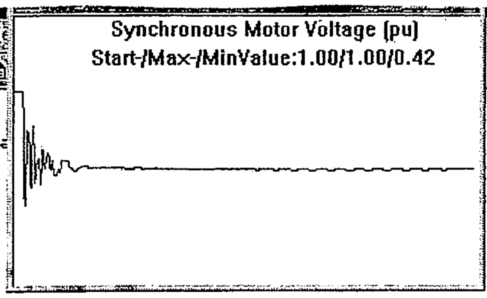


Figure 4.20 SM terminal voltage

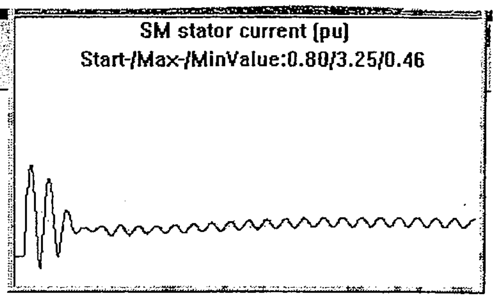


Figure 4.21 SM stator current

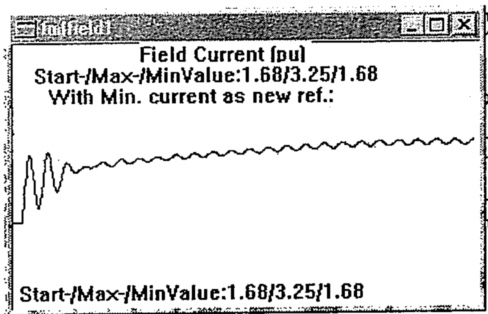


Figure 4.22 SM field current

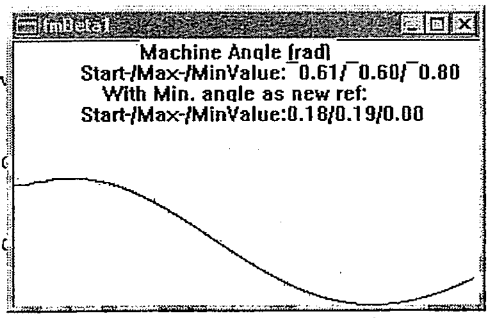


Figure 4.23 SM rotor angle

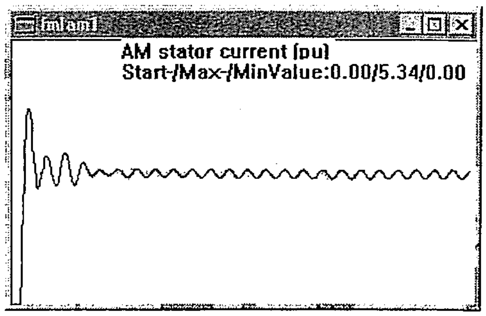


Figure 4.24 AM stator current

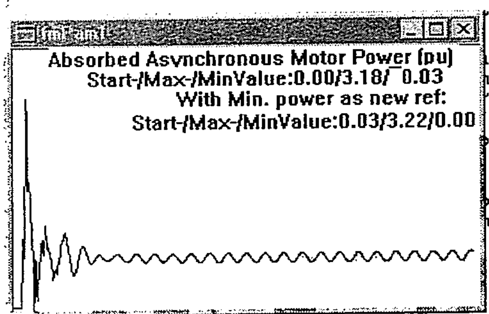


Figure 4.25 Absorbed AM power

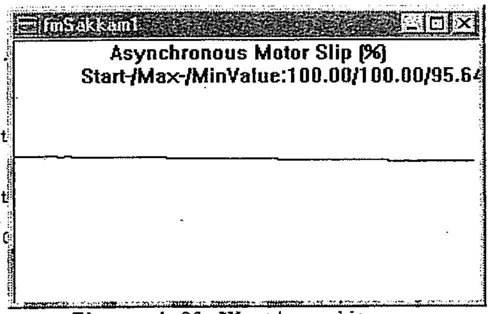


Figure 4.26 AM rotor slip

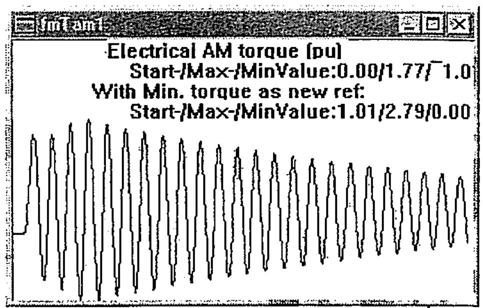


Figure 4.27 Electrical AM torque

Figure 4.19 – 4.27 **Start of an asynchronous motor** at the motor bus of the system of Figure 3.4. Sample results for an analysis period $\mathbf{T_{max} = 0.5s}$. The motor - initially at standstill - is connected to the system at t=0.01s.

Figure 4.28 – 4.36 give sample results for a period of analysis of 7s. The asynchronous motor – initially at standstill – is connected to the system at t = 0.3s. During start-up when the rotor speed of the asynchronous motor is in the range Ω_{AM} = 0 – 0.97pu, the mechanical torque -then reflecting the net effect of friction,- is set to 0.05 $(\Omega_{AM})^2$. When the motor speed (for the first time) exceeds the here chosen 0.97pu 'limit', the mechanical rotor torque is suddenly increased to the fixed value 0.5pu, to model the desired loading up of the asynchronous motor.

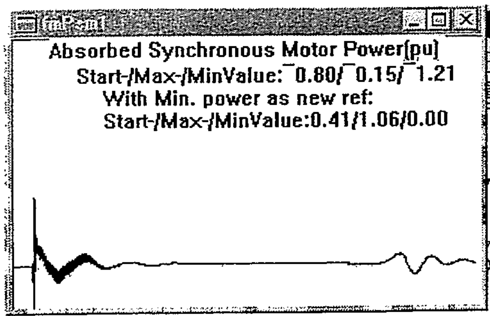


Figure 4.28 Absorbed SM power

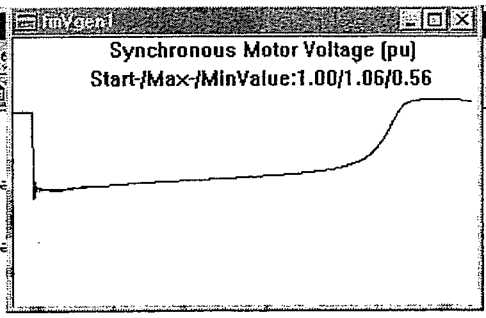


Figure 4.29 SM terminal voltage

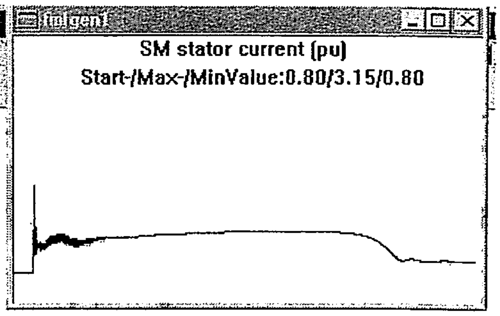


Figure 4.30 SM stator current

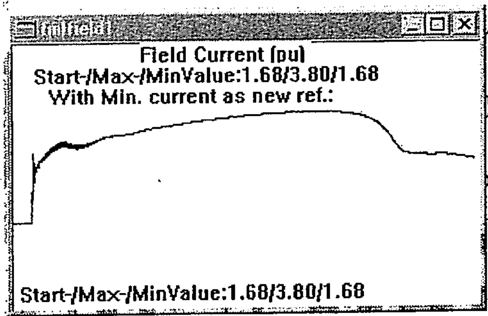


Figure 4.31 SM field current

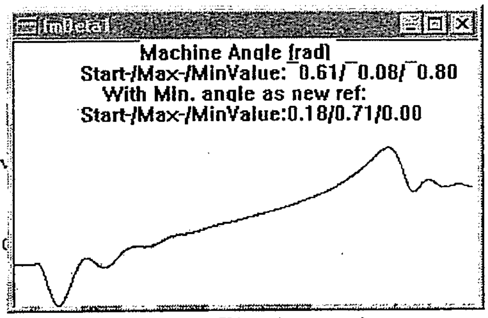


Figure 4.32 SM rotor angle

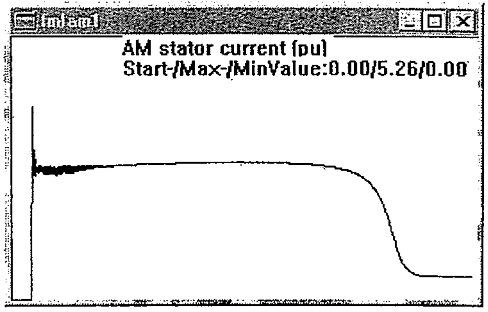


Figure 4:33 AM stator current

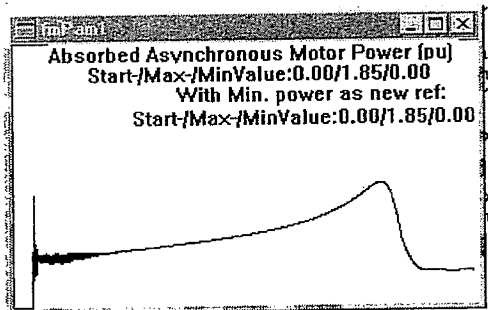


Figure 4.34 Absorbed AM power

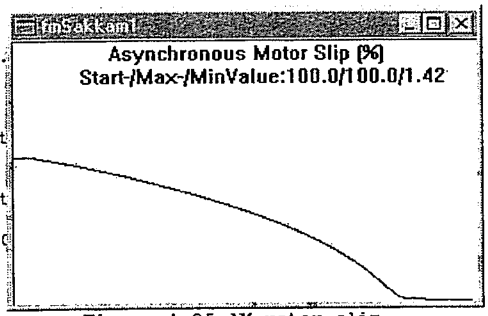


Figure 4.35 AM rotor slip

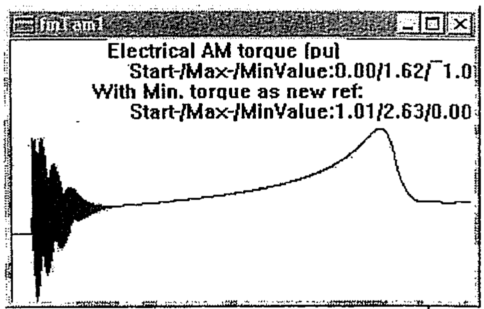


Figure 4.36 Electrical AM torque

Figure 4.28 – 4.36 **Start of an asynchronous motor** at the motor bus of the system of Figure 3.4. Sample results for an analysis period $\mathbf{T_{max} = 7s}$. The motor - initially at stand-still - is connected to the system at t=0.3s. See text for the logic of handling of the mechanical (i.e. load-) torque as function of motor speed.

4.5 Start of a synchronous motor

Given an initial steady state operating point of the system of Figure 3.4, with the synchronous motor disconnected from the network. See Chapter 3.11.2 for other load flow specifications.The initial motor bus voltage is computed to 0.891pu. After startup and synchronization of the motor, a target motor bus voltage = 1.0pu is set for its voltage regulator system. The synchronous machine is started from stillstand as an asynchronous motor; the rotor circuit being initially short-circuited over an additional series resistance, to enhance the development of an accelerating torque. During start-up when the rotor speed of the synchronous motor is in the range Ω_{SM} = 0 - 0.95pu, the mechanical torque -then reflecting the net effect of friction, is set to 0.02 $(\Omega_{SM})^2$. When the motor speed (for the first time) exceeds the here chosen 0.95pu 'trigger' value, the additional field circuit series resistance is short-circuited and an 'initial' field voltage of 1.0pu is applied. Immediately after this, the voltage control system of the machine is activated.

In the present analyses we have chosen to demonstrate (hydro-)*generator* mode of operation of the synchronous motor. When the synchronous motor's rotor speed (for the first time) during the starting process exceeds the here chosen second 'trigger' value 0.99pu, the power control system of the unit is activated. Depending on the setting of a 'task identifier', one of two control modes is then implemented:

Frequency control mode with excitation signal $\Delta r=(\Omega_{SM}-1)$ applied to the synchronous machine's power control system. I.e.: $\Delta\Omega_{ref}=0$, see equations (1-120) of Chapter 1.7. This control mode will in the present case lead to idle/no-load synchronous operation of the newly started machine.

Power control mode with 'reference setting' $\Delta\Omega_{ref} = c\,(P_{target} - P_{SM})$ applied to the synchronous machine's power control system. See again equations (1-120). c is a scaling factor (here set to 0.1). Choosing target motor load $P_{target} = -0.8$, this control mode implies *generator* mode of operation, and opening of the turbine gate to eventually sustain 0.8pu power production. The current example study applies the *power contol mode* with the setting just described.

The dynamical study is made for three different time horizons in order to illustrate the detailed initial impact of a SM start, as well as the ensuing overall system consequences of this type of disturbance to the system.

Figure 4.37- 4.45 give sample detailed results for a period of analysis of 0.5s. The motor is connected at t = 0.05s. Time increment $\Delta t = 0.0005$s.

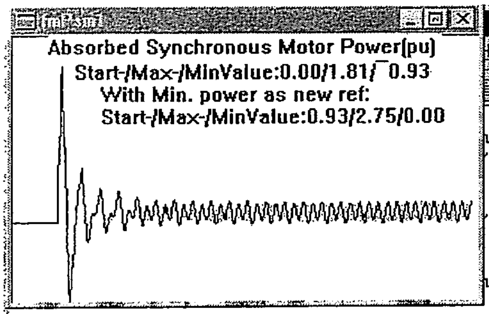


Figure 4.37 Absorbed SM power

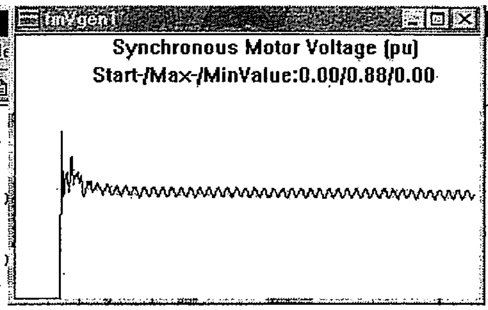


Figure 4.38 SM terminal voltage

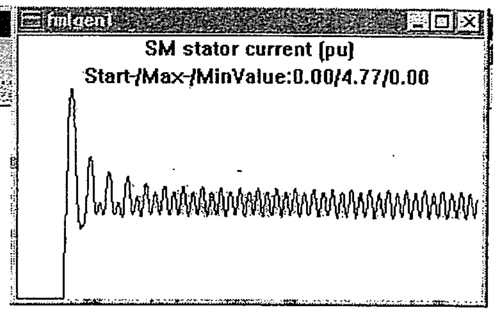


Figure 4.39 SM stator current

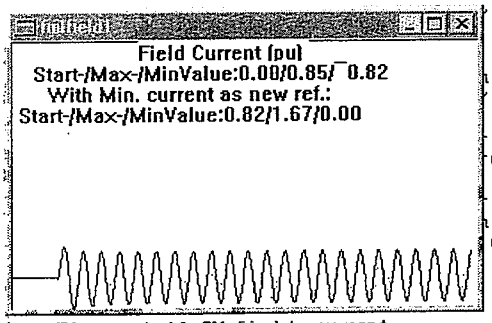


Figure 4.40 SM field current

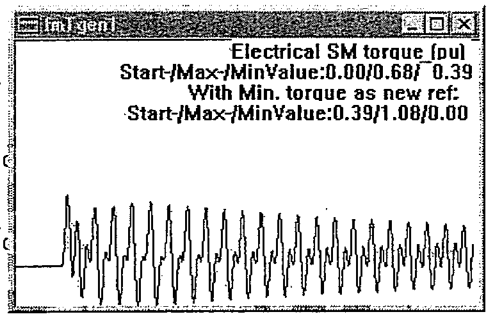


Figure 4.41 Electrical SM torque

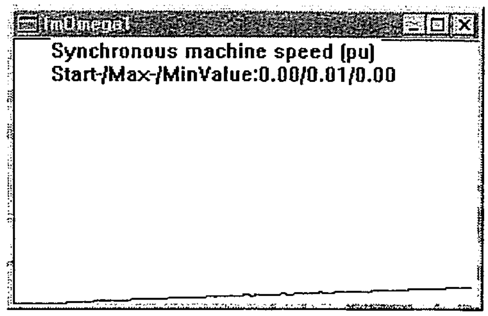


Figure 4.42 SM rotor speed

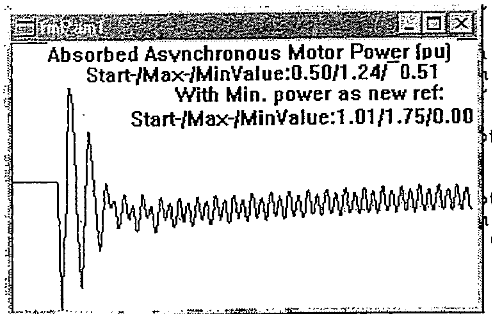


Figure 4.43 Absorbed AM power

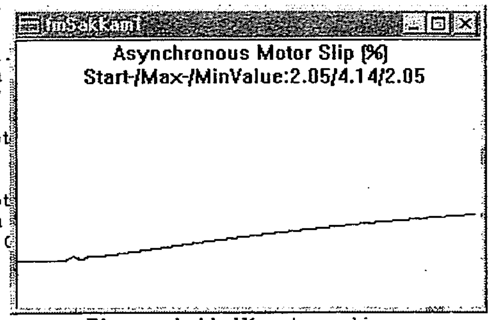


Figure 4.44 AM rotor slip

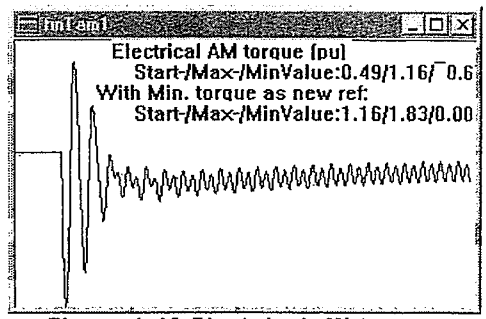


Figure 4.45 Electrical AM torque

Figure 4.37 – 4.45 **Start of a synchronous machine** at the motor bus of the system of Figure 3.4. Sample results for an analysis period $\mathbf{T_{max} = 0.5s}$. The motor – initially at stand-still – is connected to the system at t = 0.05s.

Figure 4.46 - 4.54 investigate start of the synchronous machine over a period of analysis of 5s. The motor is connected at t = 0.5s. At $t=T_{max}=5s$, the speed of the rotor has just passed the value 0.08pu. Time increment Δt = 0.001s.

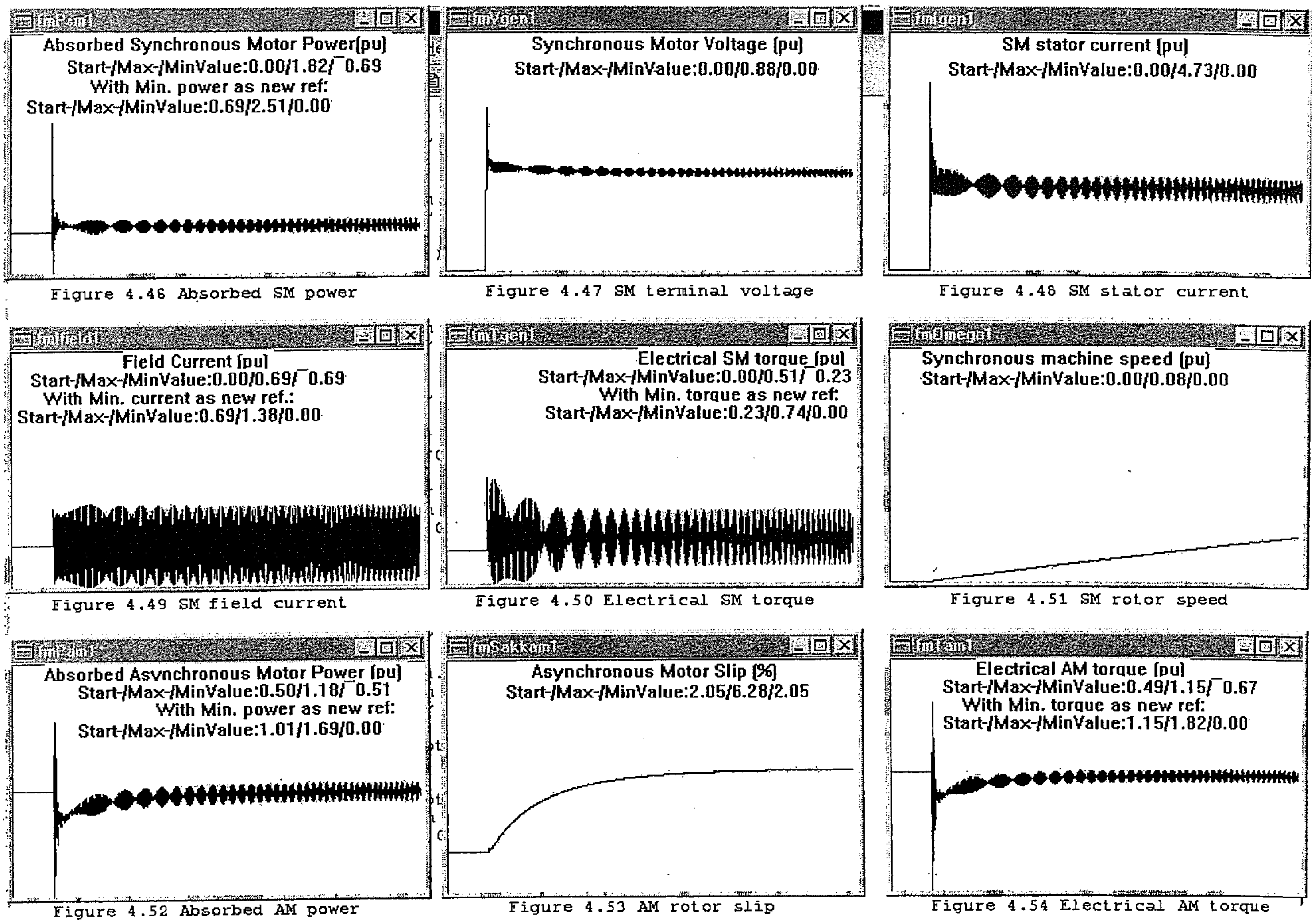


Figure 4.46 Absorbed SM power
Figure 4.47 SM terminal voltage
Figure 4.48 SM stator current
Figure 4.49 SM field current
Figure 4.50 Electrical SM torque
Figure 4.51 SM rotor speed
Figure 4.52 Absorbed AM power
Figure 4.53 AM rotor slip
Figure 4.54 Electrical AM torque

Figure 4.46 – 4.54 **Start of a synchronous machine** at the motor bus of the system of Figure 3.4. Sample results for an analysis period T_{max} **= 5s**. The motor – initially at standstill – is connected to the system at t= 0.5s.

Figure 4.55 – 4.63 investigate start, synchronization and subsequent loading in generator mode of operation, of the synchronous machine over a period of analysis of 60s. The motor - initially at standstill - is connected to the system at t =1s. At t = 57s the loaded generator is suddenly dis-connected from the network. Time increment during integration: Δt = 0.001s.

A brief comment to main results in view of initial conditions and subsequent 'events' over the period of analysis:

Before start of the synchronous motor the voltage at the motor bus is 0.89pu. The asyn-chronous motor is running and absorbes 0.5pu power from the motor bus at slip 2.05%.

After connection of the synchronous motor (at t=1s), acceleration takes place for about 40 seconds before rotor speed reaches 0.95pu. During this period the motor bus voltage appears confined to the reduced level of about 0.55pu. To sustain its nearly constant mechanical load torque at reduced voltage, the asynchronous motor quickly attains its new and higher slip level of about 6.29% during this acceleration period of time.

When the synchronous motor's rotor speed (for the first time) passes 0.95pu, an 'initial' field voltage =1.0pu is applied, immediately followed by activation of the voltage control system. It is observed from the diagrams that synchronization as well as successful voltage control then is implemented in the course of very few seconds. When 99% of synchronous speed is attained (for the first time), the *power control mode* of the power control system is activated, and close to desired generator output of 0.8pu, is then reached within the ensuing (say) 8s.

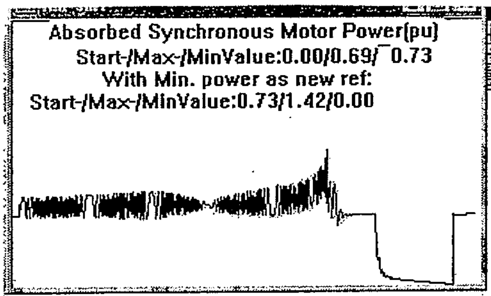


Figure 4.55 Absorbed SM power

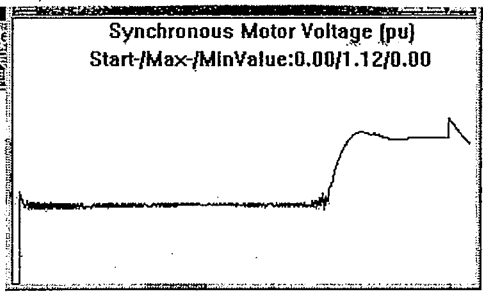


Figure 4.56 SM terminal voltage

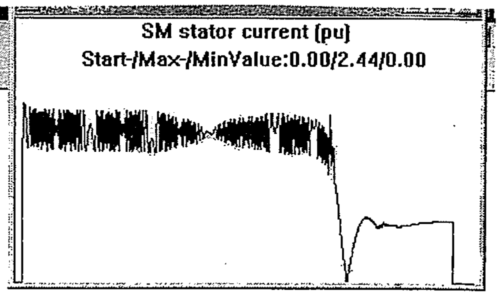


Figure 4.57 SM stator current

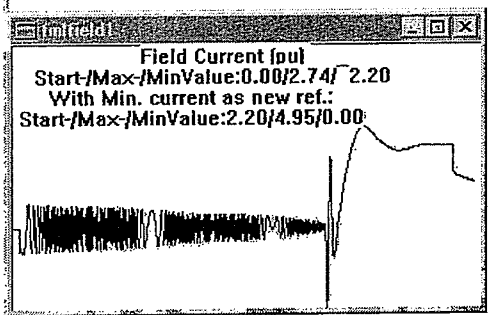


Figure 4.58 SM field current

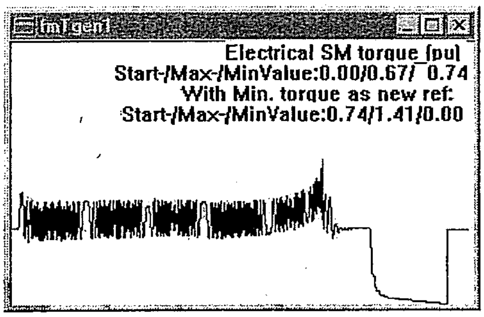


Figure 4.59 Electrical SM torque

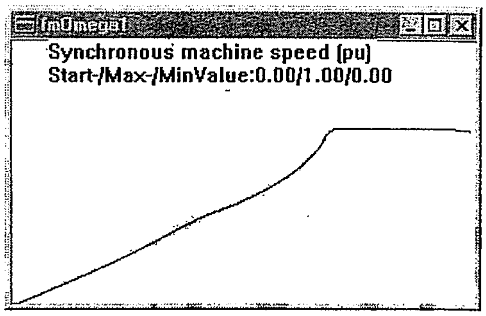


Figure 4.60 SM rotor speed

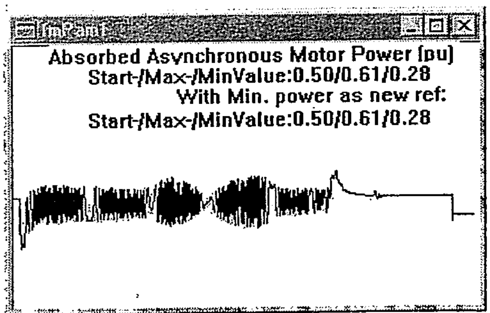


Figure 4.61 Absorbed AM power

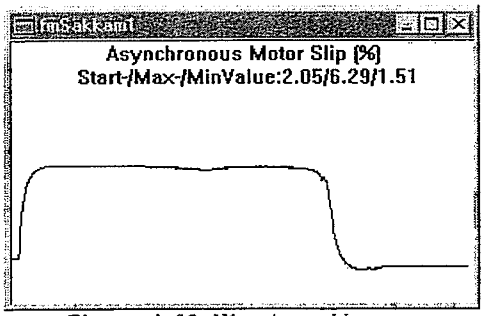


Figure 4.62 AM rotor slip

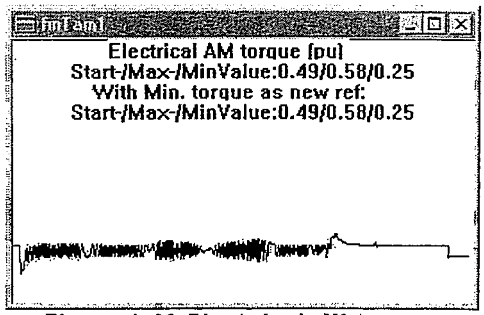


Figure 4.63 Electrical AM torque

Figure 4.55 – 4.63 **Start/loading up/disconnection of a synchronous generator** at the motor bus of the system of Figure 3.4. Sample results for an analysis period T_{max} **= 60s.** The motor -initially at standstill- is connected to the system at t=1s. The generator is abruptly disconnected from the system at t = 57s. See text.

During the just commented period of synchronous operation of the machine, the motor bus voltage is by and large noticed to be close to its target value 1.0pu, thanks to the generator's voltage control system. To sustain its nearly constant mechanical load torque at the elevated and now ≈ nominal voltage, the asynchronous motor quickly attains the new, reduced slip of about 1.51% - a figure that naturally is lower than the initial slip value (2.05%) associated with the early and low motor bus voltage of .89pu.

At $t = 57s$ the synchronous machine is disconnected from the system while producing close to target output. The abrupt disconnection is modelled by setting the field voltage to zero, deactivate the machine's control gear, and introducing a time-variable 'arc impedance' in series with the machine to be disconnected. The r-term of this impedance increases from zero to 150 pu via the formulation $r = k_{rx} \cdot 150$. The x-term of the impedance increases from 0 to 100pu via the formulation $x = k_{rx} \cdot 100$. k_{rx} is a pu factor varying as a ∫ - shaped time function from 0 to 1, over the here chosen time span of 0.02s, - i.e. the duration of one 50Hz cycle. In the present case with $\Delta t = 0.001s$, the stated increase of r and x takes place in the course of 0.02/0.001=20 integration time steps.

4.6 Islanding

Given the system layout of Figure 3.4 with an initial power flow implying delivery of about 0.56pu power from the main grid (i.e. infinite bus) to the local system. The generator of the local system produces at the outset≈ 0.2pu power. The line terminating on the infinite bus is suddenly disconnected, and we seek the response of the local system to this event.

Main characteristics of the initial system load flow :

The infinite bus:	Voltage	: 1.0pu (specified)
	Active power	: 0.5566pu (delivered to the study system)
	Reactive power	: -0.0089pu (the study system represents a capacitive load)
Load bus '1' :	Voltage	: 0.9858pu
	Active load '1'	: 0.2915pu
	Reactive load '1'	: 0.1943pu (inductive)
	Active power	: 0.2558pu (delivered to the transformer)
	Reactive power	: -0.2342pu (the transformer represents a capacitive reactive load)
Motor bus :	Voltage	: 1.0000pu (specified)
	Active SM power	: -0.1999pu (specified power supply to synchronous motor: –0.2pu)
	Reactive SM power	: -0.5358pu (the synchr. motor acts reactively as capacitor bank)
	Active AM power	: 0.2995pu (specified power supply to asynchronous motor: 0.3pu)
	Reactive AM power	: 0.3953pu (the asynchronous motor acts as inductive load)
	Capacitor load	: -0.7049pu (capacitive) (x_D=0.005pu is in series with the capacitor)
	Active load '2'	: 0.1500pu
	Reactive load '2'	: 0.6000pu (inductive)

Figure 4.64 – 4.72 investigate main electrical consequences to the local power system, of an abrupt disconnection of it from the main grid. The disconnection is afforded by introducing a time-variable 'arc impedance' in series with the transmission line (of impedance r_y+jx_y) to be opened *) . The r-term of this impedance increases from zero to 150pu via the formulation $r = k_{rx} \cdot 150$. The x-term of the impedance increases from 0 to 100pu via the formulation $x = k_{rx} \cdot 100$. k_{rx} is a pu factor varying as a ∫ - shaped time function from 0 to 1, over the here chosen time span of 0.02s, - i.e. the duration of one 50Hz cycle. In the present case with $\Delta t = 0.001s$, the stated increase of r and x takes place in the course of 0.02/0.001= 20 integration time steps.

---

*) A topological matrix denoted *system loop matrix* **B**, is generally applied in this report to formally describe the way components are tied together into a system. Among many prospective **B**-candidates, one was randomly chosen in Chapter 2.2 for analysis of the complete system of Figure 2.1 (or equivalently Figure 3.4). For the special purpose of our present study of islanding, we have to define a matrix **B** that in principle will allow us to increase (r_y + jx_y) to infinity, without jeopardizing the precision with which the remaining local power system is being modelled. The chosen graph description for our purpose of islanding analysis, is shown in Figure 4.73 and discussed thereafter.

We notice from Figure 4.64 that the (≈ 0.56pu) power initially imported from the grid, is replaced by local generation in the course of about 25s. The frequency however, recovers much more slowly ; after 75s the synchronous machine's rotor speed is still subsynchronous -and of the value≈ 0.9990 pu.This corresponds to a slip of≈ 0.1%. The local motor bus voltage dips incrementally right after disconnection of the line, but recovers in the course of the ensuing ≈ 15s. The saw-tooth presentation of the electrical motor angle follows from constraining its absolute value to being less or equal to (2 π).

In sum Figure 4.64-4.72 exemplify that system control responses are slower and less damped in a small local power system than in a (bigger) system that comprises a rigid voltage reference. See foregoing studies of integrated system performance. This holds true even when control gear settings are tailored to the system configuration at hand. In the present islanding case two parameter adjustments are made relative to the settings given in Chapter 3.11.1: *Transient droop time constant* T_t is alteted from 17s to 25s, and transient droop δ_t from 0.15pu to 0.04pu.

Further illustration of local vs. integrated system response is given in next Chapter 4.7.

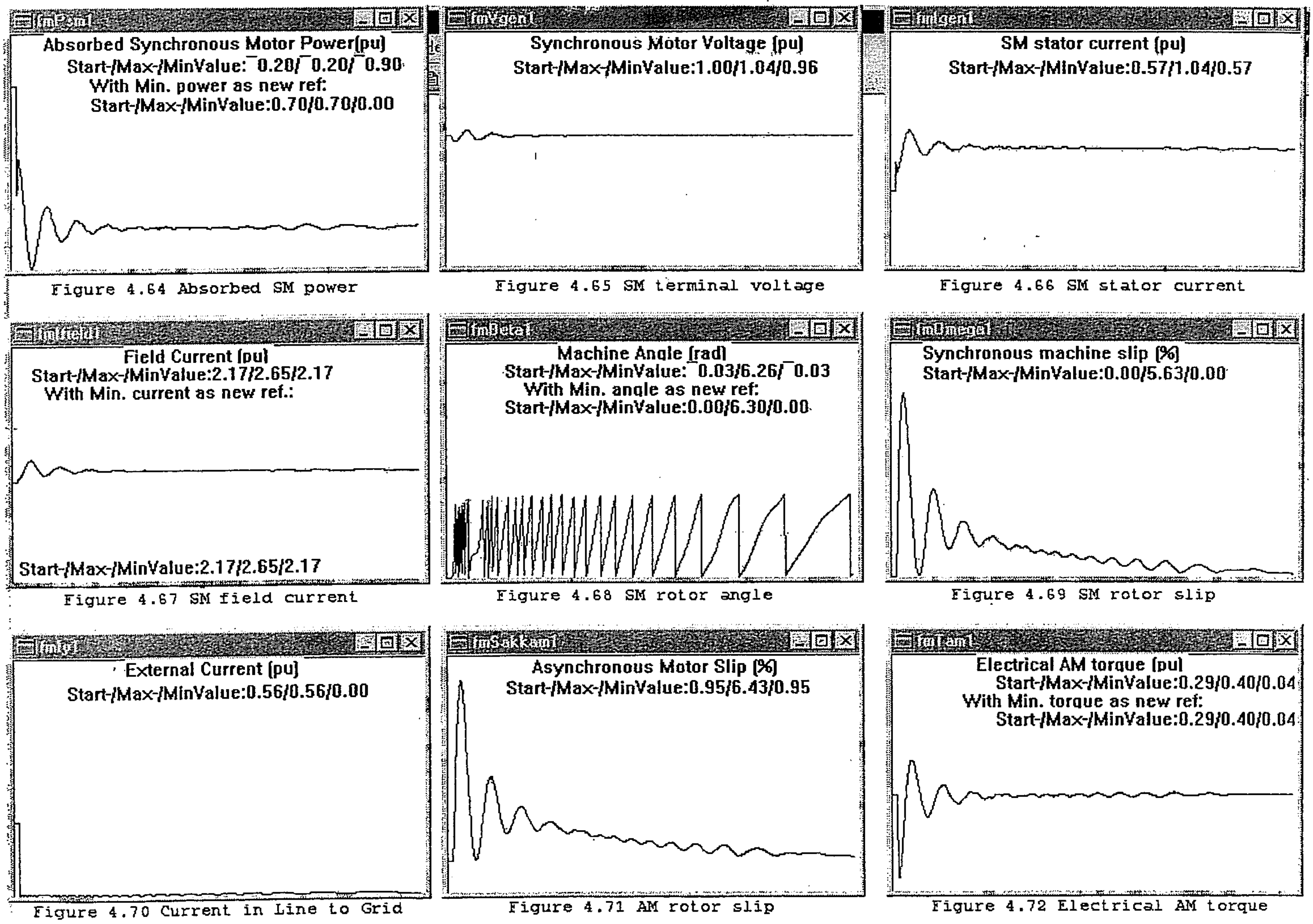
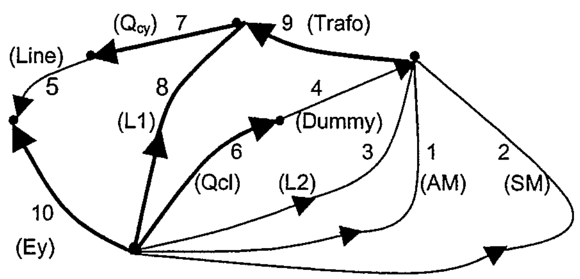


Figure 4.64 Absorbed SM power — Figure 4.65 SM terminal voltage — Figure 4.66 SM stator current

Figure 4.67 SM field current — Figure 4.68 SM rotor angle — Figure 4.69 SM rotor slip

Figure 4.70 Current in Line to Grid — Figure 4.71 AM rotor slip — Figure 4.72 Electrical AM torque

Figure 4.64 – 4.72 **Islanding into a local power system**, of the system of Figure 3.4
Sample results for an analysis period T_{max} **= 75s**. The local system- Initially importing≈ 0.56pu power from the grid- is disconnected at t=1s.



a) Oriented system graph

Element no →

Loop no ↓ (columns 1 … 5 | 6 … 10)

$$
\mathbf{B} = \left[\begin{array}{ccccc|ccccc}
1&0&0&0&0&0&0&-1&1&0\\
0&1&0&0&0&0&0&-1&1&0\\
0&0&1&0&0&0&0&-1&1&0\\
0&0&0&1&0&1&0&-1&1&0\\
0&0&0&0&1&0&1&1&0&-1
\end{array}\right] = [\mathbf{B}_S, \mathbf{B}_T]
$$

b) Loop matrix **B**

Figure 4.73 Graph description (of the system of Figure 3.4) adapted to 'Case Islanding'.

We see from Figure 4.73 that system loop '5' (which is defined by chord-element '5'), is the only loop that comprises the external emf E_y to be removed. Sudden removal is here afforded by quickly increasing the transmission line impedance ($r_y + x_y$) to a very large value. This will eliminate the effect of loop '5', without ruining the modelling of the remaining loop currents. (In the modelling of Chapter 2.2 the voltage E_y became an emf of each and every one of the defined system loops, and hence could not be disconnected in a simple way without destroying the network model itself).

4.7 Local vs. integrated system response to given disturbance.

We wish to investigate and compare main transient operational consequences of starting and loading up of an asynchronous motor, given the following two alternative system configurations: 1) The local system is 'on its own', i.e. the infinite bus of the system of Figure 3.4 is permanently disconnected, - and 2) the local system is tied to the grid, i.e. system configuration is 'integrated' as given in Figure 3.4. The initial system power flow is established the same in both cases, and control system target values are set so that final solutions also will be the same. Period of analysis: 45s. Integration time step: 0.001s

Main characteristics of the established initial system load flow(s) :

The infinite bus:	Voltage	: 0.9995pu
	Active power	: 0.0000pu (delivered to the local system)
	Reactive power	: 0.0000pu (" " " " ")
Load bus '1' :	Voltage	: 0.9995pu
	Active load '1'	: 0.2997pu
	Reactive load '1'	: 0.1998pu (inductive)
	Active power	: -0.2997pu (delivered to the transformer from bus 1)
	Reactive power	: -0.1998pu (")
Motor bus :	Voltage	: 1.017pu
	Active SM power	:-0.4612pu
	Reactive SM power	:-0.1031pu (The SM acts reactively as a capacitor)
	Active AM power	: 0.0000pu (AM not yet connected)
	Reactive AM power	: 0.0000pu (")
	Capacitor load	:-0.7286pu (capacitive) (x_D=0.005pu is in series with the capacitor)
	Active load '2'	: 0.1550pu
	Reactive load '2'	: 0.6202pu (inductive)

The asynchronous motor – initially at standstill – is connected to the system at t = 1s. During startup when the rotor speed of the asynchronous motor is in the range Ω_{AM} = 0 – 0.97pu, the mechanical torque -then reflecting the net effect of friction,- is set to 0.05 $(\Omega_{AM})^2$. When the motor speed (for the first time) exceeds the here chosen 0.97pu 'limit', the mechanical rotor torque is suddenly increased to the fixed value 0.3pu, to model the desired loading up of the asynchronous motor.

Figure 4.74 – 4.82 display main operational consequences of the motor start when taking place in the local/ isolated system *). To retain the frequency of the local network, the *frequency control mode* is chosen for the power control system of the local generator. Thus the generator will pick up whatever increase in power demand is incurred by commissioning of the asynchronous motor. The voltage regulator of the synchronous machine is set to retain the initial motor bus voltage of 1.017pu.

We notice that the asynchronous motor has picked up its load about 22s after being connected to the system. The voltage is recovered after ca. 30s. The frequency needs long time for recovery; at t = 45s it has reached back to ≈ 0.9965pu.

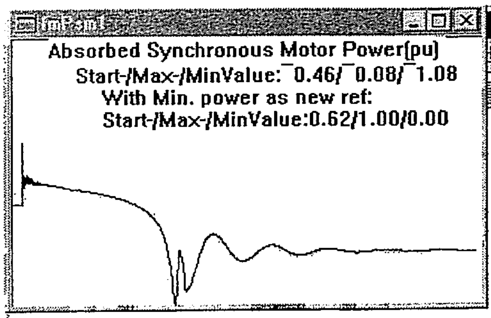


Figure 4.74 Absorbed SM power

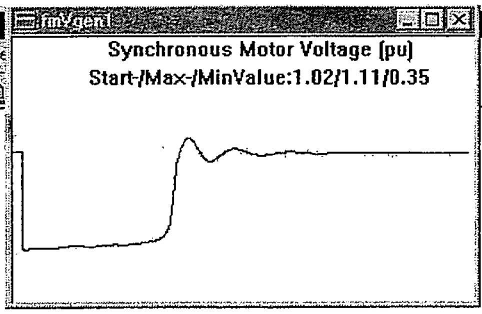


Figure 4.75 SM terminal voltage

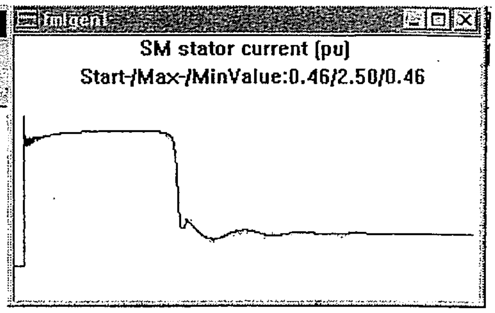


Figure 4.76 SM stator current

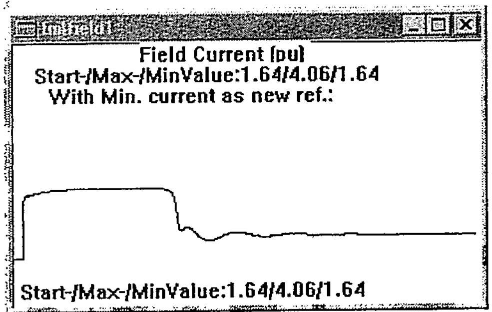


Figure 4.77 SM field current

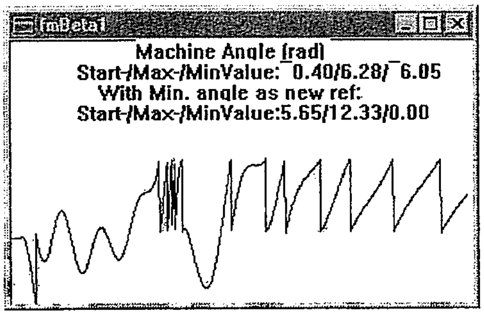


Figure 4.78 SM rotor angle

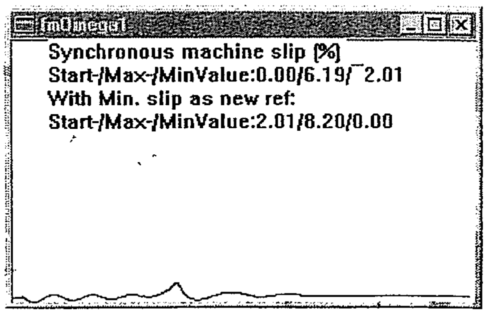


Figure 4.79 SM rotor slip

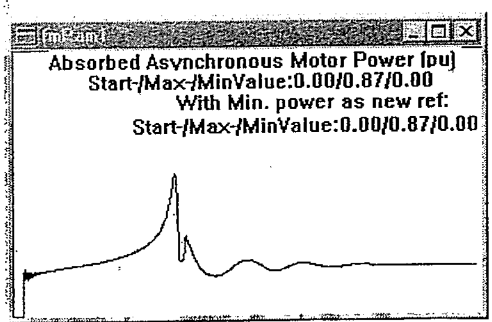


Figure 4.80 Absorbed AM power

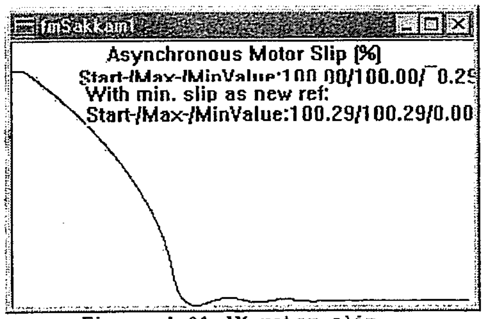


Figure 4.81 AM rotor slip

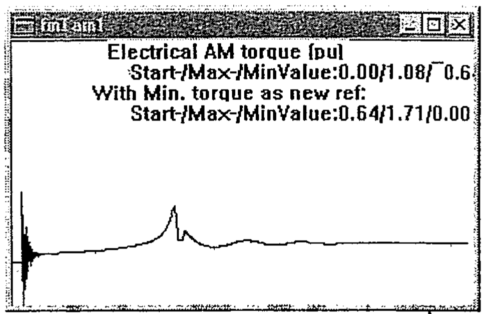


Figure 4.82 Electrical AM torque

Figure 4.74 – 4.82 **Local system respons to the start and loading up of an asynchronous motor.** The 'local system' is the system of Figure 3.4 when disconnected from grid. Analysis period: $\mathbf{T_{max} = 45s}$. The asynchronous motor - initially at standstill - is connected to the network at t =1s.

*) As the present study deals with analysing the local system disconnected from grid, system modelling should (to enhance precision as well as efficiency of computation) confine itself to only the local system. The chosen graph description for our present task of local system analysis, is given in Figure 4.83.

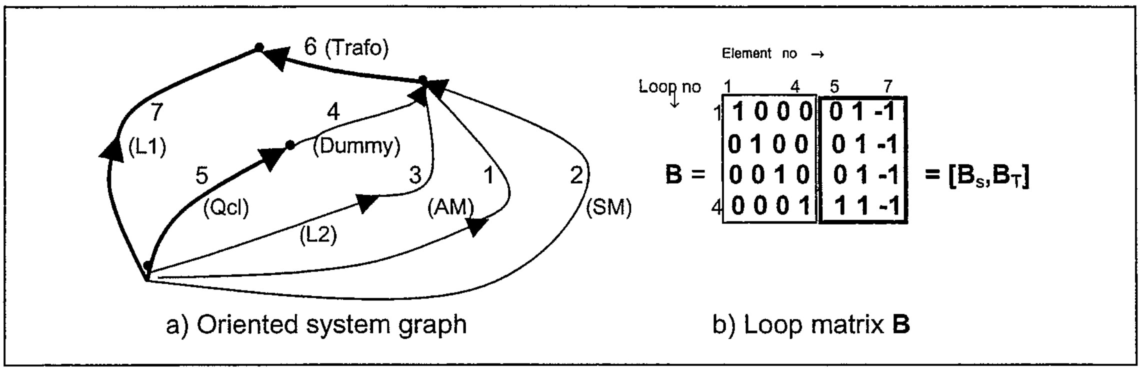


Figure 4.83 Graph description adapted to 'Case local system analysis'.

Figure 4.84 – 4.93 display main operational consequences of the asynchronous motor start, when taking place in the interconnected system of Figure 3.4. To converge on the same final solution as in

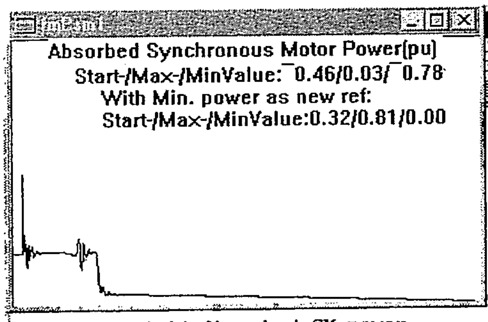


Figure 4.84 Absorbed SM power

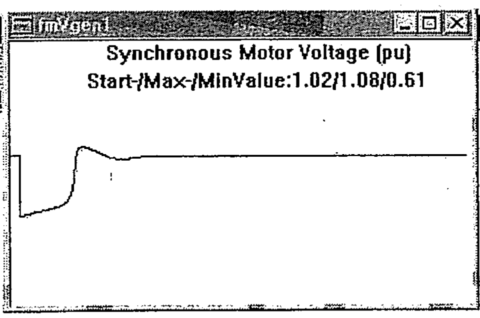


Figure 4.85 SM terminal voltage

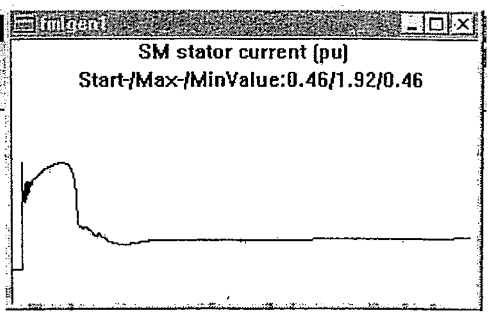


Figure 4.86 SM stator current

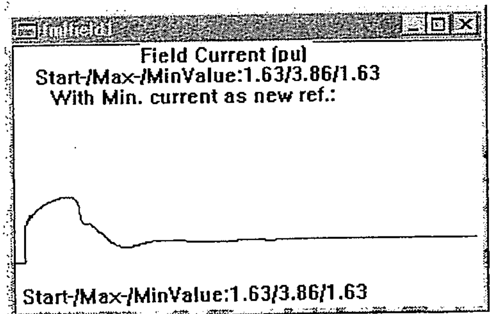


Figure 4.87 SM field current

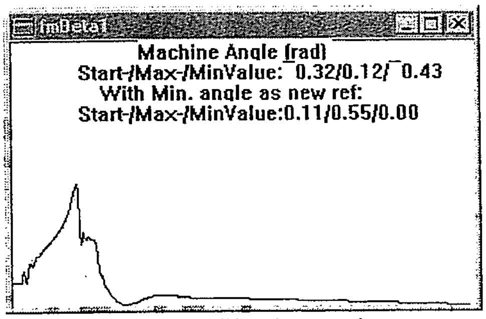


Figure 4.88 SM rotor angle

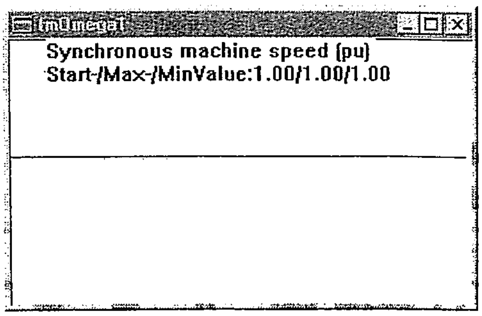


Figure 4.89 SM rotor speed

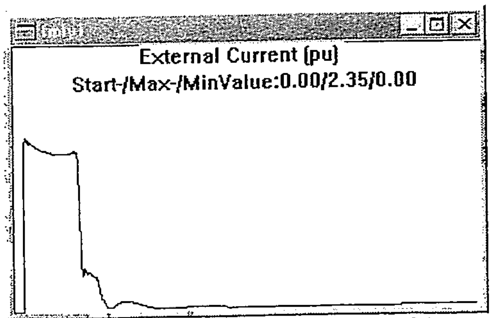


Figure 4.90 Current to/from grid

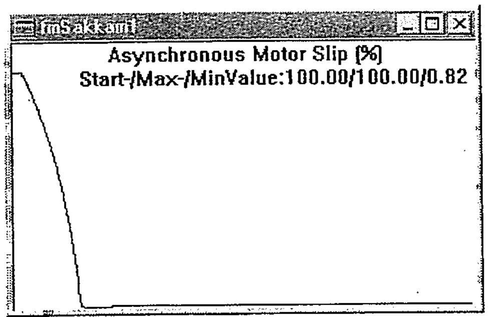


Figure 4.91 AM rotor slip

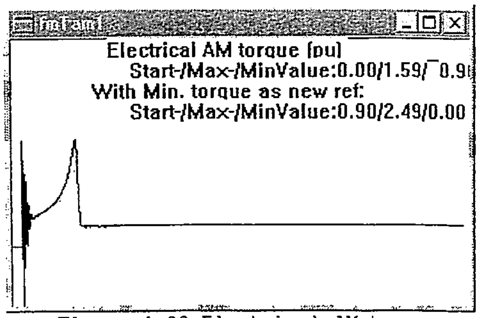


Figure 4.92 Electrical AM torque

Figure 4.84 – 4.92 **Integrated system respons to the start and loading up of an asynchronous motor.**
The 'integrated' system is the system of Figure 3.4. Analysis period: T_{max} **= 45s.**
The asynchronous motor – initially at standstill – is connected to the network at t = 1s.
Power control mode of local generator, with target output=final gen.prod. in previous case

the previous 'local case', the *power control mode* is chosen for the power control system of the local generator, and as target production setting is applied the local power output converged on at the exit of the previous motor start analysis. The voltage regulator of the local synchronous machine is again set to retain the initial motor bus voltage of 1.017pu.

We observe from the last set of results that the asynchronous motor now has picked up its load about 6s after being connected to the system. The motor bus voltage is recovered after ca 12s. Thanks to the stiff grid now prevailing, system frequency is retaineg throughout the analysis.

Initial load flow is the same in both the 'local' and 'integrated' study. Final load flow is also the same. Transiently, however, there is - in the 'integrated' case - a strong supply support from the grid, as evidenced from e.g. Figure 4.90. This transient support being the reason why the motor commissioning is much quicker and less detrimental to power supply quality, in case of *integrated* than in cases *of local* system operation.-

APPENDIX 1

The formal basis of modelling of power network loop currents

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The formal basis of modelling of power network loop currents

The *component model* concept

Chapter 1 develops a stock of four *component models* to apply for modelling of the commonly used power network components (like overhead lines, cables, capacitor banks, transformers, synchronous machines, asynchronous machines) in power system analysis. The four *component models* are *'The Lossy Inductor'*, *'The Lossy Capacitor Bank'*, *'The Synchronous Motor'*, and *'The Asynchronous Motor'* :

'The Lossy Inductor' models directly the three phase, inductive series impedance, and the three phase inductive impedance load. Transformers, overhead lines and cables are modelled by suitably arranging together component models of the type *'Lossy Inductor'* and *'Lossy Capacitor Bank'*, - see next.

'The Lossy Capacitor Bank' models directly the three phase, lossy series capacitor, and the three phase, lossy shunt capacitor. It also contributes to the modelling of other network components as stated above.

'The Synchronous Motor' models the two main modes of operation of the synchronous machine; the voltage controlled synchronous *motor*, and the voltage- and power controlled synchronous *generator*. For conceptual clearness, *motor* mode of operation is the 'default' modelling mode.

'The Asynchronous Motor' models *motor*- as well as *generator* mode of operation of the asynchonous machine. *Motor* mode of operation is the 'default' modelling mode.

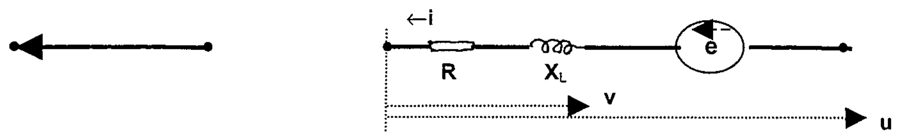
The *component model* is made up of one or more sets of *submodels*, the configuration of which depends on which *state variables* belong to the component at hand. The *submodels* that go into respective component models, are developed in Chapters 1.2 – 1.6. One of these submodels is the *electrical circuit model.* In terms of formal representation the *electrical circuit model* is made common to all four *component models*. A structural description of the *electrical circuit model* is given in Figure A1.1.

The electrical circuit model is common to all four component models. The *electrical circuit model* is a two-terminal model, where parameter interpretation depends upon type of component. The *electrical circuit models* interlink to describe integrated power network performance. *The electrical circuit model* comprises three main parts :

An oriented terminal graph showing positive direction of the circuit model variables (**i,e**) that connect electrically with the external power network. For the stock of two-terminal power components, the *oriented terminal graph* is an *oriented line segment*. An oriented terminal graph for a *two-terminal component* is shown in a) below. This graph fronts the standardized d-q axis circuit model labelled b) .

Impedance terms R and X_L describing the 'passive' properties of the circuit element. Index 'L' denotes *inductive* character of the reactance. If the component is the lossy capacitor, X_L=0. The effect of *capacitive* reactances appear in terms of separate state variables, see below. For further info, see Chapter 1.3. **v** is the voltage across the inductive impedance part of the electrical circuit model.

A voltage source vector e giving the defined source impact of the component. If the component is the infinite bus voltage, **R** and X_L are zero, and the voltage source **e** is equal to e_{dqo} given in (1-85). e_{dqo} reduces to e_{dq} in cases of symmetrical analyses. If the component is a synchronous motor, or an asynchronous motor, the voltage source **e** is respectively, ΔE_{SM} or ΔE_{AM}. See (1-106) and (1-124). If the component is the (lossy) capacitor, the voltage source **e** is ΔE_c. If the component is the (lossy) inductor, the voltage source **e** = 0. **u** is the voltage across the terminals of the circuit model :



$$v = R\,i + (1/\omega_b)\,X_L\,di/dt \quad \& \quad v - e = u$$

a) Oriented terminal graph

b) Content of circuit model fronted by a)

Figure A1.1 *The electrical circuit model* ; formal structure of submodel common to all component models

For the sake of basic illustration: Given the task of modelling the performance of the small power system to the left in Figure A1.2. The system comprises three power network components; *the infinite bus*, *a synchronous motor*, and *an impedance type inductive load*. The electrical circuit submodel of respective component models is given in Figure A1.2b. Moving from left to right in Figure A1.2b, the *oriented terminal graph* of respective circuit submodels is first given , followed by a description of the *content* of the *circuit models* fronted by the teminal graphs. For brevity of notation the oriented terminal graphs - and in this case also the corresponding network components - are identified by labels '1' to '3'.

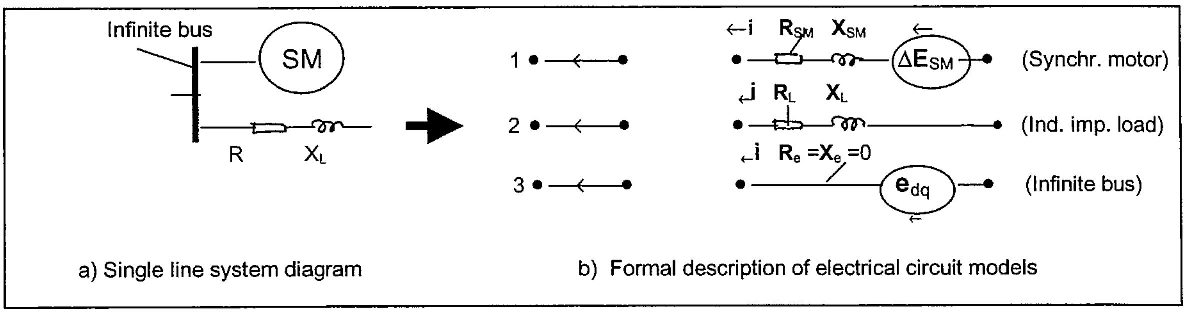


Figure A1.2 Simple three-component power system

The aggregate of separate power network component models - that interconnected constitute the network model -, may be said to form the *primitive network* of the system [2]. Once the *primitive network* is given and it is specified how the network components are tied together, a general basis for *power network modelling* is established.

The methodology of modelling of *power network loop currents* is next summarized. It is inherently a three-stage process to which the following subheadings may apply: *'The primitive network'*, *'Network topology'* and *'Network modelling'* :

The primitive network

The content of the primitive network is readily illustrated for the example system of Figure A1.2 : With the labelling '1' to '3' chosen, only a suitable re-arrangement of the given data is required to produce its *primitive network* shown in Figure A1.3.

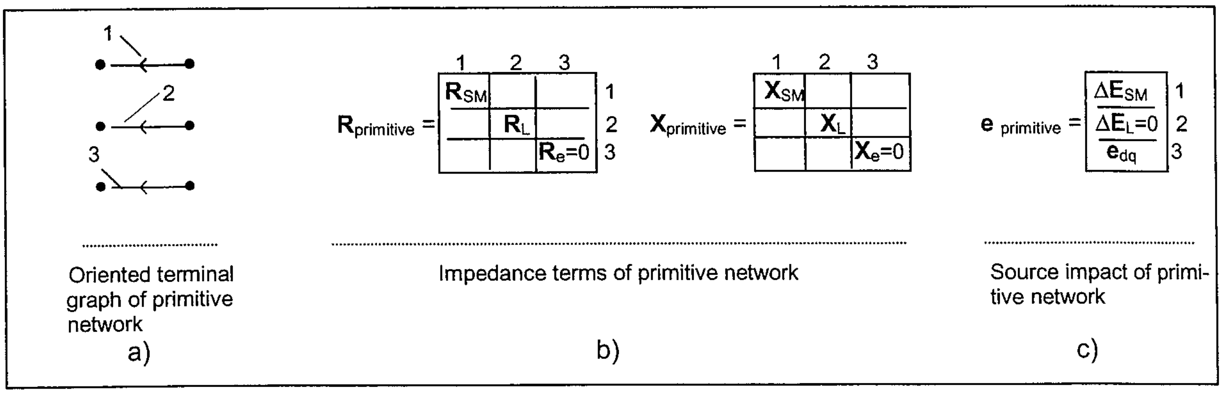


Figure A1.3 The primitive network of the three-component power system of Figure A1.2

Inherently, *the primitive network* comprises *three* main parts in the same way each of its contributing circuit models does:

- **An oriented terminal graph** showing defined positive direction of the primitive system variables ($\mathbf{i}_{primitive}$, $\mathbf{e}_{primitive}$) that will interconnect to form the appropriate power network variables. By convention, each of the graph's line segments fronts a standardized d-q axis circuit element as defined in Figure A1.1b.

- **Impedance terms** $\mathbf{R}_{primitive}$ and $\mathbf{X}_{Lprimitive}$ describing the 'passive' properties of the circuit elements that are contained in the network. $\mathbf{R}_{primitive}$ is diagonal. Index 'L' denotes that $\mathbf{X}_{Lprimitive}$ always is of *inductive* character. See comment on previous page and Chapter 1.1 for further details, concerning handling of *capacitive* reactances. If there is electromagnetic coupling between system components -as e.g. may be the case for parallell overhead lines close to each other -, $\mathbf{X}_{Lprimitive}$ may contain off-diagonal terms. Otherwise, $\mathbf{X}_{Lprimitive}$ will be diagonal as exemplified above.

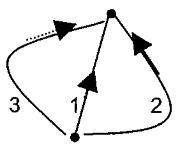
- **A voltage source vector** $\mathbf{e}_{primitive}$ giving the source impact of the defined set of voltage sources contained in the network.

For the primitive network described in terms of own variables, the following equations hold true, see definitions in Figure A1.1b :

$$\mathbf{v}_{primitive} = \mathbf{R}_{primitive}\, \mathbf{i}_{primitive} + (1/\omega_b)\, \mathbf{X}_{Lprimitive}\, d\mathbf{i}_{primitive}/dt \qquad \text{(A1-1)}$$
$$\mathbf{v}_{primitive} - \mathbf{e}_{primitive} = \mathbf{u}_{primitive} \qquad \text{(A1-2)}$$

Network topology

Graphwise the topology of a network is established by connecting together the graph elements of its primitive system, as directed by the single line diagram of the network at hand. The oriented graph of the small system in Figure A1.2a is formed by inteconnecting the primitive network graph elements of Figure A1.3a, as advised by the single line diagram of Figure A1.2a. The system graph is shown in Figure A1.4 :



1 : Synchronous motor
2 : Inductive impedance load
3 : Infinite bus

b = no. of nework graph elements = 3
N_{node} = no. of nodes of network graph = 2
N_{tree} = no. of network tree elements = (N_{node} –1) = (2-1) = 1
N_{loop} = no. of cotree elements (chords) = (b -N_{tree}) =(3 -1) =2

Figure A1.4 Oriented graph of the three-component power system of Figure A1.2a.

The formal modelling of interconnection of components may be afforded by different topological matrices comprising plus/minus '1', or '0' as matrix elements. In the present outline a *system loop matrix* ***B*** is used to formally describe how the power network components are tied together.

The system loop matrix ***B*** is conveniantly defined on the basis of a chosen *tree* and *cotree* of the oriented network graph:

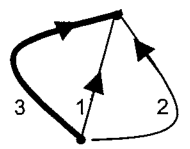
The *tree* is a set of N_{tree} graph elements that connects all nodes of the connected network graph without closing any circuit. N_{tree} = (N_{node} -1), where N_{node} is the total number of nodes in the network graph.

For the network graph of Figure A1.4, N_{tree} = (2 –1) = 1. The chosen tree of this graph is shown in thick line in Figure A1.5a.

The remaining N_{loop} =(b –N_{tree}) graph elements constitute the corresponding *cotree* of the oriented network graph. b is the number of elements of the network graph. Each cotree element - or chord - identifies a unique loop of the network graph. Thus the collection of chosen cotree elements identifies a necessary and sufficient set of independent system loops for evaluation of network flow solutions. When deciding on the sequence (numbering) of elements of the primitive network graph, the elements that are being defined as members of the cotree should conveniently be numbered first.

For the network graph of Figure A1.4 the number of cotree elements is N_{loop} = (3 –1) = 2. The chords are identified by thin lines in Figure A.5a.

The system loop matrix **B** gives the incidence of independent network loops as defined by the set of cotree elements, and the set of all graph elements of the network. See Figure A1.5b for illustration.



a) Oriented network graph

Element no →

Loop no ↓	1	2	3
1	1	0	-1
2	0	1	-1

$$\mathbf{B} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} = [\mathbf{B}_{chord}, \mathbf{B}_{tree}] \qquad \text{(A1-3)}$$

b) Loop matrix **B**

Figure A1.5 Graph description in terms of *tree, cotree (chords)* and *B-matrix.*

The labels (read numbers) attatched to the cotree elements can conveniently identify also the associated set of independent network loops. Furthermore, the chosen orientation of the cotree elements can suitably define positive direction of the network loop currents.

B can be partitioned into a submatrix $\mathbf{B}_{chord}$ that describes the incidence of *loops* and *cotree elements* or *chords*, and submatrix $\mathbf{B}_{tree}$ that gives the incidence of *loops* and *tree* elements. Given the con-ventions above, $\mathbf{B}_{chord}$ will always be a unit matrix. Figure A1.5 illustrates the definition of submatrices in our simple network case.

To prepare for the upcoming elaborations on *'Network modelling'*, three fundamental transformations associated with the loop matrix **B** are briefly pointed to:

Network loop currents = f(Primitive network currents)
B (of dimension N_{loop}xb) postmultiplied by a vector **i** (of dimension bx1) associated with respective elements of the network graph, produces a vector **I** (of dimension N_{loop}x1) associated with respective defined loops of the network. Mathematically :

$$\underset{(N_{loop}x1)}{\mathbf{I}} = \underset{(N_{loop}xb)}{\mathbf{B}} \cdot \underset{(bx1)}{\mathbf{i}} \qquad \text{(A1-4)}$$

In application directed terms: *The network loop currents **I** are produced from the primitive system element currents **i** via the linear transformation **B**.*

Resulting network loop voltages = f(Primitive network component voltages u) = 0
B (of dimension N_{loop}xb) postmultiplied by a vector **u** (of dimension bx1) associated with respective elements of the network graph, produces a vector **U** (of dimension N_{loop}x1) connected to respective defined loops of the network. Mathematically :

$$\underset{(N_{loop}x1)}{\mathbf{U}} = \underset{(N_{loop}xb)}{\mathbf{B}} \cdot \underset{(bx1)}{\mathbf{u}} = 0 \qquad \text{(A1-5)}$$

In application directed terms: *The resulting network loop voltages **U** are produced from the primitive system element voltages **u** via the linear transformation **B**. According to Kirchoff's second law (the*

Voltage Law), the voltage must add up to zero when traversing any loop in any circuit at any instant of time.

Primitive network currents = F(Network loop currents)

$\mathbf{B}^t$ (of dimension bxN_{loop}) postmultiplied by a vector **I** (of dimension $N_{loop}x1$) associated with respective loops of the network graph, produces a vector **i** (of dimension bx1) connected to respective elements of the network. Mathematically :

$$\underset{(bx1)}{\mathbf{i}} = \underset{(bxN_{loop})}{\mathbf{B}^t} \cdot \underset{(N_{loop}x1)}{\mathbf{I}} \qquad \text{(A1-6)}$$

In application directed terms: *The primitive system element currents* ***i*** *are produced from the network loop currents* ***I*** *via the linear transformation* $\boldsymbol{B}^t$.

Elaborating on (A1-6) :

$$\mathbf{i} = \mathbf{B}^t\,\mathbf{I} = \left[\frac{\mathbf{B}_{chord}{}^t}{\mathbf{B}_{tree}{}^t}\right] \cdot \mathbf{I} = \left[\frac{\mathbf{1}}{\mathbf{B}_{tree}{}^t}\right] \cdot \mathbf{I} = \left[\frac{\mathbf{I}}{\mathbf{B}_{tree}{}^t\,\mathbf{I}}\right] \quad \begin{matrix}\text{(cotree elements)}\\ \\ \text{(tree elements)}\end{matrix} \qquad \text{(A1.7)}$$

With *loop* currents **I** given it is noticed that the *tree* element currents $\mathbf{i}_{tree}$ are given as $\mathbf{B}_{tree}{}^t\,\mathbf{I}$. The cotree element currents $\mathbf{i}_{chord}$ are (per def.) equal to the network loop currents **I**.

Network modelling

For the primitive network described in terms of own variables, the following equations have been established earlier under the heading 'The primitive network' :

$$\mathbf{v}_{primitive} = \mathbf{R}_{primitive}\,\mathbf{i}_{primitive} + (1/\omega_b)\,\mathbf{X}_{Lprimitive}\,d\mathbf{i}_{primitive}/dt \qquad \text{(A1-1)}$$
$$\mathbf{v}_{primitive} - \mathbf{e}_{primitive} = \mathbf{u}_{primitive} \qquad \text{(A1-2)}$$

With (A1-1) as the basis for further development, the following 3 substitutions/transformations are introduced:

1. $\mathbf{v}_{primitive}$ is solved from (A1-2), and inserted on the left side of (A1-1)
2. From (A1-6) : $\mathbf{i}_{primitive} = \mathbf{B}^t\,\mathbf{I}$. The right hand side is introduced in (A1-1) instead of '$\mathbf{i}_{primitive}$'
3. Equation (A1-1) is finally premultiplied by **B**.

Following these actions, (A1-1) takes on this form:

$$\mathbf{B}\,\mathbf{e}_{primitive} + \mathbf{B}\,\mathbf{u}_{primitive} = \mathbf{B}\,\mathbf{R}_{primitive}\,\mathbf{B}^t\,\mathbf{I} + (1/\omega_b)\,\mathbf{B}\,\mathbf{X}_{Lprimitive}\,\mathbf{B}^t\,d\mathbf{I}/dt \qquad \text{(A1-8)}$$

From (A1-5): $\mathbf{B}\,\mathbf{u}_{primitive} = 0$. Setting $\mathbf{E}_{loop} = \mathbf{B}\,\mathbf{e}_{primitive}$, the following network model emerges:

$$\underline{\mathbf{E}_{loop} = \mathbf{R}_{loop}\,\mathbf{I} + (1/\omega_b)\,\mathbf{X}_{Lloop}\,d\mathbf{I}/dt} \qquad \text{(A1-9)}$$

By choice, the orientation of the loop currents **I** coincides with the orientation of the cotree elements of the network graph. For each graph element , arrow direction is by convention the direction in which the elements e.m.f (if present) will contribute to driving 'its own' current. Thus component currents will inherently be positive for components that act as sources. This contradicts our premise set earlier in the motor modelling part of analysis, where power (and current) supplied to a motor per definition was assumed positive. To retain and generalize this earlier premise, new loop currents

$$\mathbf{I}_{loop} = -\,\mathbf{I} \qquad \text{(A1-10)}$$

are defined. In summary the power network loop current model then becomes as follows:

$$\mathbf{E}_{loop} = \mathbf{R}_{loop}\, \mathbf{I}_{loop} + (1/\omega_b)\, \mathbf{X}_{Lloop} \cdot d\mathbf{I}_{loop}/dt \qquad \text{(A1-11)}$$

where;

$\mathbf{E}_{loop} = -\,\mathbf{B}\ \mathbf{e}_{primitive}$ = (N_{loop}x1) loop voltage vector comprising the net driving voltage of resp. N_{loop} loops of the network graph.

$\mathbf{e}_{primitive}$ = (bx1) voltage source vector. Comprises a specified value for each of the b elements (components) of the network graph.

$\mathbf{I}_{loop}$ = (N_{loop}x1) loop current vector comprising the current of resp. N_{loop} cotree elements of the network graph. Orientation of the currents is opposite the orientation of the cotree elements. Power from a source is negative. Power to a load is positive. See previous page.

$\mathbf{R}_{loop} = \mathbf{B}\ \mathbf{R}_{primitive}\ \mathbf{B}^t$ = (N_{loop}xN_{loop}) network loop resistance (A1-12)

$\mathbf{X}_{Lloop} = \mathbf{B}\ \mathbf{X}_{Lprimitive}\ \mathbf{B}^t$ = (N_{loop}xN_{loop}) network loop *inductive* reactance

$\mathbf{R}_{primitive}$ = (bxb) primitive network resistance matrix. $\mathbf{R}_{primitive}$ is the collection of resistances associated with resp. b elements (components) of the network graph. $\mathbf{R}_{primitive}$ is diagonal.

$\mathbf{X}_{Lprimitive}$ = (bxb) primitive network *inductive* reactance matrix. $\mathbf{X}_{Lprimitive}$ is the collection of inductive reactances associated with resp. b elements of the network graph. $\mathbf{X}_{Lprimitive}$ is normally diagonal. See text following Figure A1.3.

$\mathbf{B}$ = (N_{loop}xb) system loop matrix giving the incidence of network loops and elements of the network graph.

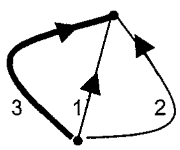
Figure A1.6 Modelling of power network loop currents.

Applying the algorithm of Figure A1.6, the *power network loop currents* of the simple system of Figure A1.2a is elaborated as follows:

From Figure A1.3 which presents the *primitive network* of the three component power system, we copy:

$$\mathbf{R}_{primitive} = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \left[\begin{array}{ccc} \mathbf{R}_{SM} & & \\ & \mathbf{R}_L & \\ & & \mathbf{R}_e{=}0 \end{array}\right] \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \quad \mathbf{X}_{primitive} = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \left[\begin{array}{ccc} \mathbf{X}_{SM} & & \\ & \mathbf{X}_L & \\ & & \mathbf{X}_e{=}0 \end{array}\right] \end{array} \quad \mathbf{e}_{primitive} = \left[\begin{array}{c} \Delta\mathbf{E}_{SM} \\ \Delta\mathbf{E}_L{=}0 \\ \mathbf{e}_{dq} \end{array}\right] \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \qquad \text{(A1-13)}$$

From Figure A1.5 which gives the graph description of the power network, we copy:



a) Oriented network graph

Element no →

$$\begin{array}{c} \text{Loop no} \\ \downarrow \end{array} \quad \mathbf{B} = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} \left[\begin{array}{cc|c} 1 & & -1 \\ & 1 & -1 \end{array}\right] \end{array} = [\mathbf{B}_{chord}, \mathbf{B}_{tree}] \qquad \text{(A1-14)}$$

b) Loop matrix **B**

The sought loop current model is given by (A1 -11). Inserting into the algorithms of (A1-12) from above:

$$E_{loop} = -\mathbf{B}\ e_{primitive} = -\begin{bmatrix} 1 & & -1 \\ & 1 & -1 \end{bmatrix} \begin{bmatrix} \Delta E_{SM} \\ \Delta E_L = 0 \\ e_{dq} \end{bmatrix} = \begin{bmatrix} e_{dq} - \Delta E_{SM} \\ e_{dq} \end{bmatrix} \qquad \text{(A1-15)}$$

$$R_{loop} = \mathbf{B}\ R_{primitive}\ \mathbf{B}^t = \begin{bmatrix} 1 & & -1 \\ & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} R_{SM} & & \\ & R_L & \\ & & R_e = 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & \\ & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} R_{SM} & \\ & R_L \end{bmatrix} \qquad \text{(A1-16)}$$

$$X_{Lloop} = \mathbf{B}\ X_{Lprimitive}\ \mathbf{B}^t = \begin{bmatrix} 1 & & -1 \\ & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} X_{SM} & & \\ & X_L & \\ & & R_e = 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & \\ & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} X_{SM} & \\ & X_L \end{bmatrix} \qquad \text{(A1-17)}$$

The network loop model:

$$E_{loop} = R_{loop} \cdot I_{loop} + (1/\omega_b)\ X_{Lloop} \cdot dI_{loop}/dt$$

$\supset$:

$$\begin{bmatrix} e_{dq} - \Delta E_{SM} \\ e_{dq} \end{bmatrix} = \begin{bmatrix} R_{SM} & \\ & R_L \end{bmatrix} \cdot \begin{bmatrix} I_{loop(1)} \\ I_{loop(2)} \end{bmatrix} + (1/\omega_b) \begin{bmatrix} X_{SM} & \\ & X_L \end{bmatrix} \cdot \begin{bmatrix} dI_{loop(1)}/dt \\ dI_{loop(2)}/dt \end{bmatrix} \qquad \text{(A1-18)}$$

Or conveniently for each *loop*, - which here also means for respective two system *components* :

$$e_{dq} = R_{SM}\ i_{SM} + (1/\omega_b)\ X_{SM}\ di_{SM}/dt + \Delta E_{SM} \qquad \text{where} \quad i_{SM} = I_{loop(1)}$$

$$e_{dq} = R_L\ i_L + (1/\omega_b)\ X_L\ di_L/dt \qquad \text{where} \quad i_L = I_{loop(2)} \qquad \text{(A1-19)}$$

Equations (A1-19) could intuively have been set up directly. Then however, the main point of illustrating the general process of *network loop current modelling*, would have been lost.-

[2] Gabriel Kron: 'Tensors for Circuits'. Dover Publications, Inc. New York, New York, 1959.

APPENDIX 2

The adjustable speed synchronous machine

APPENDIX 2

The Adjustable Speed Synchronous Motor

The concept of the 'extended' synchronous motor model

The basic scheme of development in this appendix is identical to that of Chapter 1.4 which deals with modelling of the *traditional* synchronous machine. In fact, a copy of Chapter 1.4 has been adjusted and extended to cover the marginally more complex task of modelling the performance of the *adjustable speed* synchronous machine. The extended model retains all saliency aspects, and thus can be applied also as a potentially more accurate description of the traditional machine. With proper parameter setting the extended model reduces to the model developed in Chapter 1.4.

The adjustable speed synchronous motor is in principle an asynchronous motor extended with a power electronic converter that supplies ac current to a three phase rotor winding, - in contrast to the traditional supply of dc current to a rotor field winding.

Formal basis for the ensuing model development is again the d-q *diagram of a generalised machine* as presented by B.Adkins [1]. To deal with the extended rotor circuitry it is deemed appropriate to specify a ***six****-coil, salient pole generalised machine* as main basis for analysis. See Figure A2.1.The three phase stator winding is assumed to be the rotating part, while the d-q axes with associated windings 'f', 'fq', 'kd' and 'kq' are considered fixed.

The 'pseudo-stationary' d- and q coils equivalence in a conveniant way the electromagnetic effects of the stator windings of the physical three phase machine. The currents, voltages and fluxes associated with these coils, are definitionwise related to their corresponding physical phase variables via their Park transformation, see equations (1-1) and (1-2) for illustration of formal definitions.
The 'pseudo-stationary' f- and fq coils equivalence in a conveniant way the electromagnetic effects of the three phase rotor winding of the machine. The currents, voltages and fluxes associated with these coils, are definiton-wise related to their corresponding physical phase variables via their Park transformation, see above comment.
The fixed coils denoted 'kd' and 'kq', aim at equivalencing the effects of all damper circuits in the machine.

The six-coil representation implies 6 state variables to describe the electrical performance of the synchronous machine. As such variables we choose to apply the stator current - represented by the two components (i_d,i_q), - and the flux linkages associated with respectively the f-, fq-, kd- and kq- coil.

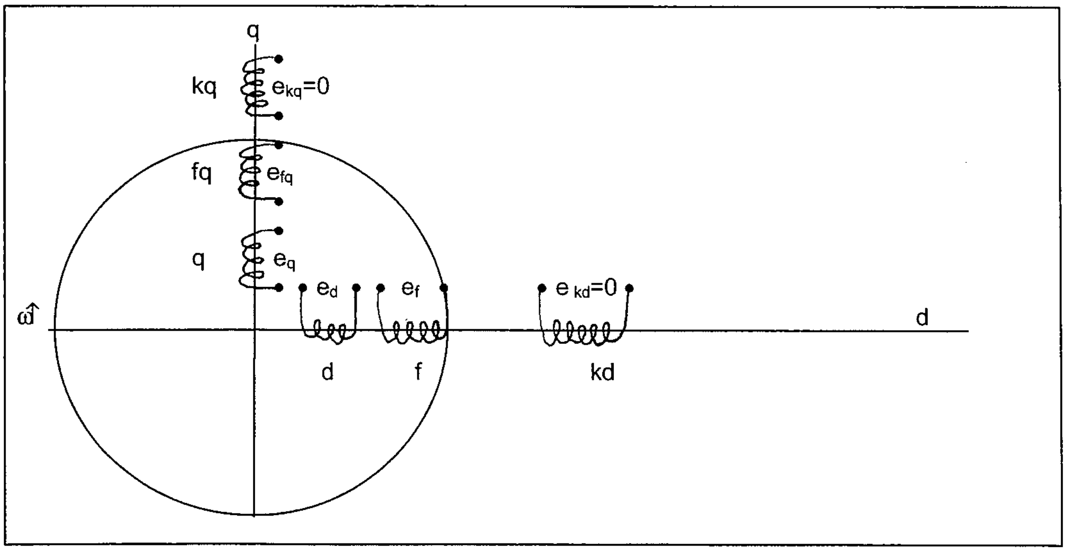


Figure A2.1 Diagram of six-*coil salient pole generalised machine.*

The elaboration of a practical synchronous motor model that – in the context of power network modelling – takes the form of a standardized d-q axis circuit element, is presented in four steps: Step 1 develops the *basic motor equations* that form the platform for the ensuing algorithmic development. Step 2 generates the *rotorflux model*, step 3 the *electrical circuit model*, and step 4 the *elecrtomechanical model*. Summary description of the extended synchronous motor model is given at the end of this appendix. Ilustrations of model application are also included.

Basic 'extended' synchronous motor model equations

The main, basic premises for the *voltage-* and *flux* equations to be developed, are given in Chapter 1.4, p.1/7, and are not repeated here.

In consistency with the premises referred to, the following inductivities are now defined for the model machine:

For the d-axis: $L_d = L_{a\sigma}+L_{ad}$, $L_f = L_{f\sigma}+L_{ad}$, $L_{kd} = L_{kd\sigma}+L_{ad}$

For the q-axis: $L_q = L_{a\sigma}+L_{aq}$, $L_{fq} = L_{fq\sigma}+L_{aq}$, $L_{kq} = L_{kq\sigma}+L_{aq}$ (A2-1)

Between flux linkages and currents within respective axes we have these defining relationships:

$$\begin{aligned}
\Psi_d &= L_d\, i_d+L_{ad}\, i_f+L_{ad}\, i_{kd}\\
\Psi_q &= L_q\, i_q+L_{aq}\, i_{kq}\\
\Psi_f &= L_f\, i_f+L_{ad}\, i_d+L_{ad}\, i_{kd}\\
\Psi_{fq} &= L_{fq}\, i_{fq}+L_{aq}\, i_{kq}+L_{aq}\, i_q\\
\Psi_{kd} &= L_{kd}\, i_{kd}+L_{ad}\, i_f+L_{ad}\, i_d\\
\Psi_{kq} &= L_{kq}\, i_{kq}+L_{aq}\, i_q+L_{aq}\cdot i_{fq}
\end{aligned}
\rightarrow \text{In matrix form:}
\begin{bmatrix}\Psi_d\\ \Psi_q\\ \Psi_f\\ \Psi_{fq}\\ \Psi_{kd}\\ \Psi_{kq}\end{bmatrix}
=
\begin{bmatrix}
L_d & & L_{ad} & & L_{ad} & \\
 & L_q & & L_{aq} & & L_{aq}\\
L_{ad} & & L_f & & L_{ad} & \\
 & L_{aq} & & L_{fq} & & L_{aq}\\
L_{ad} & & L_{ad} & & L_{kd} & \\
 & L_{aq} & & L_{aq} & & L_{kq}
\end{bmatrix}
\cdot
\begin{bmatrix}i_d\\ i_q\\ i_f\\ i_{fq}\\ i_{kd}\\ i_{kq}\end{bmatrix}
\rightarrow \Psi = L\cdot i \qquad \text{(A2-2)}$$

(matrix columns: d, q, f, fq, kd, kq)

For the d- and q-coil the sought voltage balance is readily established by starting from the 3-phase frame of reference (as also done previously,- see Chapter 1.1 and 1.2) : In the physical three phase (RST) reference frame we can for (say) phase 'R' of the motor, express the voltage balance as:

$$e_R = i_R\, r_a + d\Psi_R/dt \qquad \text{(A2-3)}$$

where e_R, i_R, Ψ_R and r_a is - respectively - impressed voltage, current, flux linkages and resistance of motor phase 'R'. The per phase variables e_R , i_R and Ψ_R are related to their respective d – q axis components in the following way, see (1-4) :

$$\begin{aligned}
e_R &= e_d \cos\theta - e_q \sin\theta + e_o\\
i_R &= i_d \cos\theta - i_q \sin\theta + i_o\\
\Psi_R &= \Psi_d \cos\theta - \Psi_q \sin\theta + \Psi_o
\end{aligned} \qquad \text{(A2-4)}$$

θ is the angular displacement of the axes of the (RST) reference frame relative to the axes of the (d-q) reference frame. Inserting expressions from (A2-4) into (A2-3), and observing that

$$d\Psi_R/dt = \cos\theta\, d\Psi_d/dt - \sin\theta\, d\Psi_q/dt + d\Psi_o/dt - \omega\, \Psi_d \sin\theta - \omega\, \Psi_q \cos\theta \qquad \text{(A2-5)}$$

we get the following 'd-q-o version' of (A2-3), where ω is angular speed of the rotating winding :

$$\begin{aligned}
0 = {} & [\, - e_d + r_a\, i_d + d\Psi_d/dt - \omega\, \Psi_q\,] \cos\theta\\
& + [\ e_q - r_a\, i_q - d\Psi_q/dt - \omega\, \Psi_d\,] \sin\theta\\
& + [\, - e_o + r_a\, i_o + d\Psi_o/dt\,]
\end{aligned} \qquad \text{(A2-6)}$$

For general validity of (A2-6), the following d-q conditions must be observed to equivalence (A2-3) :

$$\begin{aligned}
e_d &= r_a\, i_d + d\Psi_d/dt - \omega\, \Psi_q\\
e_q &= r_a\, i_q + d\Psi_q/dt + \omega\, \Psi_d\\
e_o &= r_a\, i_o + d\Psi_o/dt
\end{aligned} \qquad \text{(A2-7)}$$

We assume in the present outline that zero sequence phenomena are inconsequential. Hence the last equation of (A2-7) can be disregarded. In conclusion at this stage, we get the the following equations describing the voltage balance of the d- and q-coil of the synchronous motor:

$$\boxed{\begin{aligned}
e_d &= r_a\, i_d + d\Psi_d/dt - \omega\, \Psi_q\\
e_q &= r_a\, i_q + d\Psi_q/dt + \omega\, \Psi_d
\end{aligned}} \qquad \text{(A2-8)}$$

For the f- and fq-coil representing the three phase field winding, the voltage balance is readily established via a transformation process similar to the one described above by equations (A2-3) – (A2-8).

Thus, -analogous to (1-9) -, we get the following equations describing the voltage balance of the presumed fixed f- and fq-coils :

$$e_f = r_f \cdot i_f + d\Psi_f/dt - \omega_{fo} \cdot \Psi_{fq}$$
$$e_{fq} = r_{fq} \cdot i_{fq} + d\Psi_{fq}/dt + \omega_{fo} \cdot \Psi_f \qquad \text{(A2-9)}$$

where ω_{fo} is the reference angular speed associated with the 3-phase field winding. $\omega_{fo} = 2\pi f_{rotor}$, where f_{rotor} is the frequency of the impressed three phase field voltage.

For the remaining two 'ordinary' coils 'kd' and 'kq' of the generalised machine, the respective voltage balances can readily be defined in this way:

$$e_{kd} = 0 = r_{kd} \cdot i_{kd} + d\Psi_{kd}/dt \qquad \text{(A2-10)}$$
$$e_{kq} = 0 = r_{kq} \cdot i_{kq} + d\Psi_{kq}/dt$$

Equations (A2-8), (A2-9) and (A2-10) are put together in (A2-11). They form as a set the voltage equations of the model synchronous machine.

$$\begin{aligned}
e_d &= r_a \cdot i_d + d\Psi_d/dt - \omega \cdot \Psi_q \\
e_q &= r_a \cdot i_q + d\Psi_q/dt + \omega \cdot \Psi_d \\
e_f &= r_f \cdot i_f + d\Psi_f/dt - \omega_{fo} \cdot \Psi_{fq} \\
e_{fq} &= r_{fq} \cdot i_{fq} + d\Psi_{fq}/dt + \omega_{fo} \cdot \Psi_f \\
e_{kd} &= 0 = r_{kd} \cdot i_{kd} + d\Psi_{kd}/dt \\
e_{kq} &= 0 = r_{kq} \cdot i_{kq} + d\Psi_{kq}/dt
\end{aligned} \qquad \text{(A2-11)}$$

An exogenously specified, symmetrical three phase ac voltage E_{fRST} of given frequency f_f, is applied to the field winding for excitation. How this voltage transforms into the d-q axis voltage variables (e_f, e_{fq}) of (A2-11), is dealt with later under the heading 'Modelling of special voltages in the d-q axis frame of reference' , see pages A2/12-13.

The defining flux equations of the machine are given by (A2-2). In summary fashion, the voltage and flux equations are shown in Figure A2-2. They form the platform for the ensuing algorithmic development.

$$\begin{bmatrix} e_d \\ e_q \\ e_f \\ e_{fq} \\ e_{kd} \\ e_{kq} \end{bmatrix} = \begin{bmatrix} r_a &&&&& \\ & r_a &&&& \\ && r_f &&& \\ &&& r_{fq} && \\ &&&& r_{kd} & \\ &&&&& r_{kq} \end{bmatrix} \cdot \begin{bmatrix} i_d \\ i_q \\ i_f \\ i_{fq} \\ i_{kd} \\ i_{kq} \end{bmatrix} + \begin{bmatrix} d\Psi_d/dt \\ d\Psi_q/dt \\ d\Psi_f/dt \\ d\Psi_{fq}/dt \\ d\Psi_{kd}/dt \\ d\Psi_{kq}/dt \end{bmatrix} + \begin{bmatrix} & -\omega &&&& \\ \omega &&&&& \\ &&& -\omega_{fo} && \\ && \omega_{fo} &&& \\ &&&&& \\ &&&&& \end{bmatrix} \cdot \begin{bmatrix} \Psi_d \\ \Psi_q \\ \Psi_f \\ \Psi_{fq} \\ \Psi_{kd} \\ \Psi_{kq} \end{bmatrix}$$

(columns: d q f fq kd kq)

Voltage equations of the model synchronous machine. (Eqn (A2-11) on matrix form)

↓

$$\begin{bmatrix} \mathbf{e}_{dq} \\ \hline \mathbf{e}_{fk} \end{bmatrix} = \begin{bmatrix} \mathbf{r}_a & \\ & \mathbf{r}_{fk} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{i}_{dq} \\ \hline \mathbf{i}_{fk} \end{bmatrix} + \begin{bmatrix} d\mathbf{\Psi}_{dq}/dt \\ \hline d\mathbf{\Psi}_{fk}/dt \end{bmatrix} + \begin{bmatrix} \mathbf{H}_{dq} & \\ & \mathbf{H}_{fk} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{\Psi}_{dq} \\ \hline \mathbf{\Psi}_{fk} \end{bmatrix} \qquad \text{(A2-12)}$$

where;

$$\mathbf{r}_a = \begin{bmatrix} r_a & \\ & r_a \end{bmatrix} \quad \mathbf{e}_{dq} = \begin{bmatrix} e_d \\ e_q \end{bmatrix} \quad \mathbf{i}_{dq} = \begin{bmatrix} i_d \\ i_q \end{bmatrix} \quad \mathbf{\Psi}_{dq} = \begin{bmatrix} \Psi_d \\ \Psi_q \end{bmatrix} \quad \mathbf{H}_{dq} = \omega \cdot \begin{bmatrix} & -1 \\ 1 & \end{bmatrix} \qquad \text{(A2-13)}$$

$$\mathbf{r}_{fk} = \begin{bmatrix} r_f &&& \\ & r_{fq} && \\ && r_{kd} & \\ &&& r_{kq} \end{bmatrix} \quad \mathbf{e}_{fk} = \begin{bmatrix} e_f \\ e_{fq} \\ e_{kd}=0 \\ e_{kq}=0 \end{bmatrix} \quad \mathbf{i}_{fk} = \begin{bmatrix} i_f \\ i_{fq} \\ i_{kd} \\ i_{kq} \end{bmatrix} \quad \mathbf{\Psi}_{fk} = \begin{bmatrix} \Psi_f \\ \Psi_{fq} \\ \Psi_{kd} \\ \Psi_{kq} \end{bmatrix} \quad \mathbf{H}_{fk} = \omega_{fo} \cdot \begin{bmatrix} & -1 && \\ 1 &&& \\ &&& \\ &&& \end{bmatrix} \qquad \text{(A2-14)}$$

Figure continues…

Figure A2-2 (start of) Basic synchronous motor equations. The platform for the ensuing algorithmic development. Vector **e** comprises impressed voltages.

continuation of Figure A2-2..

$$\begin{bmatrix}\Psi_d\\ \Psi_q\\ \Psi_f\\ \Psi_{fq}\\ \Psi_{kd}\\ \Psi_{kq}\end{bmatrix} = \overset{\quad d \quad q \quad f \quad fq \quad kd \quad kq}{\begin{bmatrix} L_d & & L_{ad} & & L_{ad} & \\ & L_q & & L_{aq} & & L_{aq} \\ L_{ad} & & L_f & & L_{ad} & \\ & L_{aq} & & L_{fq} & & L_{aq} \\ L_{ad} & & L_{ad} & & L_{kd} & \\ & L_{aq} & & L_{aq} & & L_{kq}\end{bmatrix}} \begin{bmatrix} i_d\\ i_q\\ i_f\\ i_{fq}\\ i_{kd}\\ i_{kq}\end{bmatrix} \quad \rightarrow \quad \Psi = \mathbf{L}\cdot\mathbf{i}$$

(Equations copied from (A2-2))

$$\downarrow$$

$$\begin{bmatrix}\Psi_{dq}\\ \hline \Psi_{fk}\end{bmatrix} = \left[\begin{array}{c|c} \mathbf{L}_{dq} & \mathbf{L}_{(dq)(fk)} \\ \hline \mathbf{L}_{(fk)(dq)} & \mathbf{L}_{fk}\end{array}\right] \cdot \begin{bmatrix}\mathbf{i}_{dq}\\ \hline \mathbf{i}_{fk}\end{bmatrix} \qquad \text{(A2-15)}$$

where;

$$\mathbf{L}_{dq} = \overset{d \quad q}{\begin{bmatrix} L_d & \\ & L_q\end{bmatrix}} \qquad \mathbf{L}_{(dq)(fk)} = \overset{f \quad fq \quad kd \quad kq}{\begin{bmatrix} L_{ad} & & L_{ad} & \\ & L_{aq} & & L_{aq}\end{bmatrix}}\begin{matrix} d\\ q\end{matrix} \qquad \text{(A2-16)}$$

$$\mathbf{L}_{(fk)(dq)} = \begin{bmatrix} L_{ad} & \\ & L_{aq} \\ L_{ad} & \\ & L_{aq}\end{bmatrix} \qquad \mathbf{L}_{fk} = \begin{bmatrix} L_f & & L_{ad} & \\ & L_{fq} & & L_{aq} \\ L_{ad} & & L_{kd} & \\ & L_{aq} & & L_{kq}\end{bmatrix}\begin{matrix} f\\ fq\\ kd\\ kq\end{matrix} \qquad \text{(A2-17)}$$

Figure A2-2 (end of) Basic synchronous machine equations: The platform for the ensuing algorithmic development.

The rotor flux model

We seek the description of the flux variables $\Psi_{fk} = [\Psi_f \ \Psi_{fq} \ \Psi_{kd} \ \Psi_{kq}]^t$ (and - if desired - also their implied currents $\mathbf{i}_{ifk}$) . For brevity of expression, we denote the algorithms that are developed in this context, 'the rotor flux model'.

Two sets of equations from Figure A2-2 provide the appropriate basis of this analysis; the lower set of equations from respectively (A2-12) and (A2-15) :

$$\mathbf{e}_{fk} = \mathbf{r}_{fk}\cdot\mathbf{i}_{fk} + d\Psi_{fk}/dt + \mathbf{H}_{fk}\cdot\Psi_{fk} \qquad \text{(A2-18)}$$

$$\Psi_{fk} = \mathbf{L}_{(fk)(dq)}\cdot\mathbf{i}_{dq} + \mathbf{L}_{fk}\cdot\mathbf{i}_{ifk} \qquad \text{(A2-19)}$$

We wish to retain the flux variables Ψ_{fk} as state variables, while eliminating the currents $\mathbf{i}_{fk}$ from the 'surface' of analysis. Thus we eliminate $\mathbf{i}_{fk}$ from (A2-19) and insert the expression of it into (A2-18), yielding:

$$d\Psi_{fk}/dt = \mathbf{e}_{fk} + (-\mathbf{H}_{fk} - \mathbf{r}_{fk}\cdot \mathbf{L}_{fk}^{-1})\cdot\Psi_{fk} + (\mathbf{r}_{fk}\cdot\mathbf{L}_{fk}^{-1}\cdot\mathbf{L}_{(fk)(dq)})\cdot\mathbf{i}_{dq} \qquad \text{(A2-20)}$$

The currents $\mathbf{i}_{dq}$ are referred to the model machine's local d-q axes. We want generally to refer them to the chosen global system reference phasor. The shift from global to local description is given by the following transformation:

$$\mathbf{i}_{dq} = \mathbf{T}\cdot\mathbf{i}_{DQ} \qquad \text{where;} \qquad \mathbf{T} = \left[\begin{array}{c|c} \cos\beta & -\sin\beta \\ \hline \sin\beta & \cos\beta\end{array}\right] \qquad \text{(A2-21)}$$

Here small letters (dq) signal locally referred currents, and capital letters (DQ) globally referred. β is the angular displacement of the local reference axis relative to the global axis.

We insert $\mathbf{i}_{dq}$ from (A2-21) into (A2-20) and define for convenience new flux variables $\phi_{fk} = \omega_0\cdot\Psi_{fk}$. We then get the following sought form of the equations for modelling of the fluxes ϕ_{fk} :

$$d\phi_{fk}/dt = = \omega_o\cdot\mathbf{e}_{fk} + (-\mathbf{H}_{fk} - \mathbf{r}_{fk}\cdot\mathbf{L}_{fk}^{-1})\cdot\phi_{fk} + (\omega_o\cdot\mathbf{r}_{fk}\cdot\mathbf{L}_{fk}^{-1}\cdot\mathbf{L}_{(fk)(dq)}\cdot\mathbf{T})\cdot\mathbf{i}_{DQ} \qquad \text{(A2-22)}$$

(A2-22) is inconvenient to apply. By setting *in* the appropriate matrices from Figure A2-2, and doing some further reductions and definitions, we arrive at the following practical version of (A2-22), see Figure A2-3. For further on how *machine* parameters relate to basic *model* parameters, see later on page A2/11.

$$\underset{(4x1)}{d\phi_{fk}/dt} = \omega_0\cdot(\mathbf{e}_{fk} + \mathbf{F}_{fki}\cdot\mathbf{i}_{DQ} + \mathbf{F}_{fk\phi}\cdot\phi_{fk}) \qquad \text{(A2-23)}$$

where:

$$\mathbf{e}_{fk} = \begin{bmatrix} K_f\cdot E_f \\ K_{fq}\cdot E_{fq} \\ 0 \\ 0 \end{bmatrix} \begin{matrix} f \\ fq \\ kd \\ kq \end{matrix}$$

$E_f = \sqrt{2}\cdot(E_{f\text{-}eff(o)} + \Delta E_f)\cdot\cos\beta_f$ =voltage of field coil 'f'. See page A2/14 - 15

$E_{fq} = \sqrt{2}\cdot(E_{f\text{-}eff(o)} + \Delta E_f)\cdot\sin\beta_f$ = voltage of field coil 'fq'. See page A2/14 -15

$K_f = [(\sqrt{2}/(\omega_o\cdot T'_{do}\cdot\varepsilon_f))\cdot X_{ad}/(X_d - X'_d)]$

$K_{fq} = [(\sqrt{2}/(\omega_o\cdot T'_{qo}\cdot\varepsilon_{fq}))\cdot X_{aq}/(X_q - X'_q)]$

$(\varepsilon_f,\varepsilon_{fq})$ = factors =1, unless adjusted speed SM.

For the adjustable speed SM (ie. the symmetrical machine): $K_f = K_{fq}$ & $\varepsilon_f = \varepsilon_{fq}$, see p. A2/15.

ΔE_{qf} = voltage control response. (Voltage *phase* not a control variable here).

$X_{ad} = X_d - X_{a\sigma}$
$X'_{ad} = X'_d - X_{a\sigma}$
$X''_{ad} = X''_d - X_{a\sigma}$
$X_{aq} = X_q - X_{a\sigma}$
$X'_{aq} = X'_q - X_{a\sigma}$
$X''_{aq} = X''_q - X_{a\sigma}$

$\mathbf{F}_{fki}$ =

	D	Q	
	$(1/(\omega_o\cdot T'_{do}))\cdot(X_{ad}/X'_{ad})\cdot X''_{ad}\cdot\cos\beta$	$-(1/(\omega_o\cdot T'_{do}))\cdot(X_{ad}/X'_{ad})\cdot X''_{ad}\cdot\sin\beta$	f
	$(1/(\omega_o\cdot T'_{qo}))\cdot(X_{aq}/X'_{aq})\cdot X''_{aq}\cdot\sin\beta$	$(1/(\omega_o\cdot T'_{qo}))\cdot(X_{aq}/X'_{aq})\cdot X''_{aq}\cdot\cos\beta$	fq
	$(1/(\omega_o\cdot T''_{do}))\cdot X'_{ad}\cdot\cos\beta$	$-(1/(\omega_o\cdot T''_{do}))\cdot X'_{ad}\cdot\sin\beta$	kd
	$(1/(\omega_o\cdot T''_{qo}))\cdot X'_{aq}\cdot\sin\beta$	$(1/(\omega_o\cdot T''_{qo}))\cdot X'_{aq}\cdot\cos\beta$	kq

$\mathbf{F}_{fk\phi} = (1/\omega_o)\cdot$

f	fq	kd	kq
$\mathbf{F}_{fk\phi}(f,f)$	$\omega_0\cdot f_{rotor(pu)}$	$(X_{ad}/X'^2_{ad})\cdot(X'_d - X''_d)/T'_{do}$	
$-\omega_0\cdot f_{rotor(pu)}$	$\mathbf{F}_{fk\phi}(fq,fq)$		$(X_{aq}/X'^2_{aq})\cdot(X'_q - X''_q)/T'_{qo}$
$(1/T''_{do})\cdot(1/X_{ad})\cdot(X_d - X'_d)$		$-1/T''_{do}$	
	$(1/T''_{qo})\cdot(1/X_{aq})\cdot(X_q - X'_q)$		$-1/T''_{qo}$

$\mathbf{F}_{fk\phi}(f,f) = -(1/(T'_{do}\cdot X'_{ad}))\cdot[(X_{ad}/X'_{ad})\cdot(X'_d - X''_d) + X''_{ad}]$

$\mathbf{F}_{fk\phi}(fq,fq) = -(1/(T'_{qo}\cdot X'_{aq}))\cdot[(X_{aq}/X'_{aq})\cdot(X'_q - X''_q) + X''_{aq}]$

β = angular displacement of the local machine reference axes relative to the global axes
β_f = specified phase shift (relative to local axes) of applied three phase field voltage.
$f_{rotor(pu)}$ = pu frequency of applied 3-phase rotor voltage. (Base frequency : 50Hz. Not subject to sign shift).

X_d, X'_d, X''_d : direct-axis synchronous, transient and subtransient reactance (pu)
X_q, X'_q, X''_q : quadrature-axis synchronous, transient and subtransient reactance (pu)
$X_{a\sigma}$: stator leakage reactance (pu)
T'_{do}, T''_{do} : direct axis open stator transient and subtransient time constant (s)
T'_{qo}, T''_{qo} : quadrature axis open stator transient and subtransient time constant (s)

Model application alternatives:
- If adjustable speed SM: Symmetrical machine; $X_d=X_q$, $X'_d=X'_q$, $X''_d=X''_q$, $T'_{do}=T'_{qo}$, $T''_{do}=T''_{qo}$, $K_f=K_{fq}$. β_f to be set.
- If 'traditional' SM : Individual parameter setting. $\beta_f = 0$. $f_f = 0$ (i.e. dc to the field circuit)
- If 'traditional' AM : Symmetrical machine. No field voltage excitation : $E_f=E_{fq}=0$. $f_f=0$. No P&U-control.

Figure A2-3 **Rotor flux model** of the extended synchronous motor (incl. the adjustable speed version). Model part describing synchronous motor state variables $\phi_{fk}=[\phi_f\ \phi_{fq}\ \phi_{kd}\ \phi_{kq}]^t$

At any time during integration the rotor- and damper currents $\mathbf{i}_{fk}$ may be derived from equation (A2-19), after introducing $\phi_{fk} = \omega_o\cdot\Psi_{fk}$ and $\mathbf{i}_{dq} = \mathbf{T}\cdot\mathbf{i}_{DQ}$. See algorithm in Figure A2-4.

In the elaboration of equations (A2-23) and (A2-24), letter combinations like 'fk', 'dq' and 'DQ' have been used for indexing to (hopefully) ease understanding of the algorithmic development. From a systems analysis point of view (once the component models have been established), better notations should be applied. See end of this appendix for summary model description that aim at being user-oriented.

$$\mathbf{i}_{fk} = (\mathbf{X}_{fk})^{-1} \cdot (\phi_{fk} - \mathbf{X}_{DQr} \cdot \mathbf{i}_{DQ}) \qquad \text{(A2-24)}$$

where:

X_{fk} =

	f	fq	kd	kq	
	$X^2_{ad}/(X_d - X'_d)$		X_{ad}		f
		$X^2_{aq}(X_q - X'_q)$		X_{aq}	fq
	X_{ad}		$X_{ad}+X'_{ad} \cdot X''_{ad}/(X'_d - X''_d)$		kd
		X_{aq}		$X_{aq}+X'_{aq} \cdot X''_{aq}/(X'_q - X''_q)$	kq

$X_{DQr} = X_{(fk)(dq)} \cdot T$ =

D	Q	
$X_{ad} \cdot \cos\beta$	$-X_{ad} \cdot \sin\beta$	f
$X_{aq} \cdot \sin\beta$	$X_{aq} \cdot \cos\beta$	fq
$X_{ad} \cdot \cos\beta$	$-X_{ad} \cdot \sin\beta$	kd
$X_{aq} \cdot \sin\beta$	$X_{aq} \cdot \cos\beta$	kq

Figure A2-4 ***Rotor flux model.*** Model part describing (locally referred) currents $\mathbf{i}_{fk} = [i_f \; i_{fq} \; i_{kd} \; i_{kq}]^t$, given (locally referred) machine fluxes ϕ_{fk} and (globally referred) stator currents $\mathbf{i}_{DQ}$.

The electrical circuit model

In the context of power network analysis the task at hand is that of equivalencing the synchronous motor model of Figure A2-2, by a standardized d-q axis series element comprising an **R**-term, an inductive **X**-term, and an emf. Δ**E**. See Figure 1.1 of Chapter 1, and associated text.

Three sets of equations from Figure A2-2 form the basis for the ensuing analysis, namely the upper set from (A2-12), and both sets contained in (A2-15) :

$$\mathbf{e}_{dq} = \mathbf{r}_a \cdot \mathbf{i}_{dq} + d\Psi_{dq}/dt + \mathbf{H}_{dq} \cdot \Psi_{dq} \qquad \text{(A2-25)}$$
$$\Psi_{dq} = \mathbf{L}_{dq} \cdot \mathbf{i}_{dq} + \mathbf{L}_{(dq)(fk)} \cdot \mathbf{i}_{fk} \qquad \text{(A2-26)}$$
$$\Psi_{fk} = \mathbf{L}_{(fk)(dq)} \cdot \mathbf{i}_{dq} + \mathbf{L}_{fk} \cdot \mathbf{i}_{ifk} \qquad \text{(A2-27)}$$

$\mathbf{i}_{ifk}$ solved from (A2-27) is inserted into (A2-26), which then describes Ψ_{dq} as a function of $\mathbf{i}_{dq}$ and Ψ_{fk}. The expression thus found for Ψ_{dq} is inserted into (A2-25), yielding finally the applied stator voltage $\mathbf{e}_{dq}$ as a function of the machine's state variables $\mathbf{i}_{dq}$ and Ψ_{fk}. Introducing also the new flux variables $\phi_{fk} = \omega_o \cdot \Psi_{fk}$ and $\phi_{dq} = \omega_o \cdot \Psi_{dq}$, we find as a result from this process:

$$\begin{aligned}\mathbf{e}_{dq} = \; & \mathbf{r}_a \cdot \mathbf{i}_{dq} + (\mathbf{L}_{dq} - \mathbf{L}_{(dq)(fk)} \cdot \mathbf{L}_{fk}^{-1} \cdot \mathbf{L}_{(fk)(dq)}) \cdot d\mathbf{i}_{dq}/dt + (1/\omega_o) \cdot \mathbf{L}_{(dq)(fk)} \cdot \mathbf{L}_{fk}^{-1} \cdot d\phi_{fk}/dt \\ & + \mathbf{H}_{dq} \cdot (\mathbf{L}_{dq} - \mathbf{L}_{(dq)(fk)} \cdot \mathbf{L}_{fk}^{-1} \cdot \mathbf{L}_{(fk)(dq)}) \cdot \mathbf{i}_{dq} + (1/\omega_o) \cdot \mathbf{H}_{dq} \cdot \mathbf{L}_{(dq)(fk)} \cdot \mathbf{L}_{fk}^{-1} \cdot \phi_{fk}\end{aligned} \qquad \text{(A2-28)}$$

By introducing the appropriate submatrices from Figure A2-2 into (A2-28), and then elaborating some further on the equation, we find the following 'intermediate' state of it, see Figure A2-5. The state is termed intermediate since stator voltage $\mathbf{e}_{dq}$ and stator current $\mathbf{i}_{dq}$ still are referred to the machine's own d-q axes. It remains to replace these variables by their globally referred counter-parts $\mathbf{e}_{DQ}$ and $\mathbf{i}_{DQ}$, respectively. The machine fluxes ϕ_{fk} are locally referred, and will conveniently be kept so throughout all modelling processes.

$$\mathbf{e}_{dq} = \mathbf{r}_a \cdot \mathbf{i}_{dq} + (1/\omega_o) \cdot \mathbf{X}'' \cdot d\mathbf{i}_{dq}/dt + \Omega \cdot \dddot{\mathbf{X}}'' \cdot \mathbf{i}_{dq} + \mathbf{B}_1 \cdot d\phi_{fk}/dt + \Omega \cdot \mathbf{B}_2 \cdot \phi_{fk} \qquad \text{(A2-29)}$$

where;

$\Omega = (\omega/\omega_o)$ = pu rotor speed

$$\mathbf{X}'' = \begin{bmatrix} X''_d & \\ & X''_q \end{bmatrix} \quad \text{and} \quad \dddot{\mathbf{X}}'' = \begin{bmatrix} & -X''_q \\ X''_d & \end{bmatrix}$$

$\mathbf{B}_1 = (1/\omega_o) \cdot$

	f	fq	kd	kq	
	$(X''_{ad}/(X_{ad} \cdot X'_{ad})) \cdot (X_d - X'_d)$		$(X'_d - X''_d)/X'_{ad}$		D
		$(X''_{aq}/(X_{aq} \cdot X'_{aq})) \cdot (X_q - X'_q)$		$(X'_q - X''_q)/X'_{aq}$	Q

$\mathbf{B}_2 =$

	$-(X''_{aq}/(X_{aq} \cdot X'_{aq})) \cdot (X_q - X'_q)$		$-(X'_q - X''_q)/X'_{aq}$
$(X''_{ad}/(X_{ad} \cdot X'_{ad})) \cdot (X_d - X'_d)$		$(X'_d - X''_d)/X'_{ad}$	

From Figure A2-3 ;

$$d\phi_{fk}/dt = \omega_o \cdot (\mathbf{e}_{fk} + \mathbf{F}_{fki} \cdot \mathbf{i}_{DQ} + \mathbf{F}_{fk\phi} \cdot \phi_{fk}) \qquad \text{(A2-23)}$$

Figure A2-5 'Intermediate state 1' of equation (A2-28): It remains to replace locally referred stator voltage $\mathbf{e}_{dq}$ and stator current $\mathbf{i}_{dq}$ by their globally referred counterparts $\mathbf{e}_{DQ}$ and $\mathbf{i}_{DQ}$

As part of the basis for finalizing (A2-29), we point to a few premises and rules that are crucial to the process of shifting from *local* to *global* reference (or vice versa) :

For applied stator voltage and corresponding impressed current the following holds true:

$$\mathbf{e}_{dq} = \mathbf{T} \cdot \mathbf{e}_{DQ} \qquad \mathbf{i}_{dq} = \mathbf{T} \cdot \mathbf{i}_{DQ} \qquad \text{where; } \mathbf{T} = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} \quad \text{and} \quad \mathbf{T}^{-1} = \begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix} \qquad \text{(A2-30)}$$

Definitionwise we have for electrical rotor angle and angular rotor speed, see p. A2/11 :

$$\beta = \omega_o \cdot t - (\theta + \theta_f) \quad \rightarrow \quad d\beta/dt = (\omega_o - \omega - \omega_f) = \omega_o \cdot (1 - \Omega - \Omega_f) \qquad \text{(A2-31)}$$

From mathematics: $\quad d\mathbf{i}_{dq}/dt = d(\mathbf{T} \cdot \mathbf{i}_{DQ})/dt = (d\mathbf{T}/dt) \cdot \mathbf{i}_{DQ} + \mathbf{T} \cdot d\mathbf{i}_{DQ}/dt \qquad$ (A2-32)

From mathematics and (A2-31) : $\quad d\mathbf{T}/dt = (d\beta/dt) \cdot (d\mathbf{T}/d\beta) = \omega_o \cdot (1 - \Omega - \Omega_f) \cdot d\mathbf{T}/d\beta \qquad$ (A2-33)

Introducing the global variables $\mathbf{e}_{DQ}$ and $\mathbf{i}_{DQ}$ into (A2-29), and processing the set of equations in accordance with premises and rules above, we arrive at 'Intermediate state 2' of equation (A2-28). See Figure A2-6. It remains to develop more userfriendly expressions for the terms $\mathbf{R}_{DQ}$, $\mathbf{X}_{DQ}$ and $\Delta\mathbf{E}_{DQ}$, while abiding with the adopted definitions associated with *an electrical circuit model.* See Figure 1.1 of Chapter 1 for summary of definitions.

$$\mathbf{e}_{DQ} = \mathbf{R}_{DQ} \cdot \mathbf{i}_{DQ} + (1/\omega_o) \cdot \mathbf{X}_{DQ} \cdot d\mathbf{i}_{DQ}/dt + \Delta\mathbf{E}_{DQ} \qquad \text{(A2-34)}$$

where;

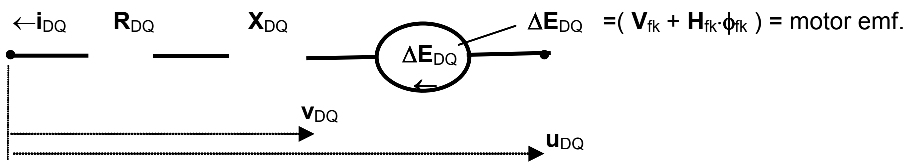
$$\mathbf{R}_{DQ} = \mathbf{r}_a + (1 - \Omega - \Omega_f) \cdot \mathbf{T}^{-1} \cdot \mathbf{X}'' \cdot d\mathbf{T}/d\beta + (\Omega \cdot \mathbf{T}^{-1} \cdot \dddot{\mathbf{X}}'' \cdot \mathbf{T}) + (\omega_o \cdot \mathbf{T}^{-1} \cdot \mathbf{B}_1 \cdot \mathbf{F}_{fki})$$

$$\mathbf{X}_{DQ} = \mathbf{T}^{-1} \cdot \mathbf{X}'' \cdot \mathbf{T}$$

$$\Delta\mathbf{E}_{DQ} = [\omega_o \cdot \mathbf{T}^{-1} \cdot \mathbf{B}_1] \cdot \mathbf{e}_{fk} + [(\omega_o \cdot \mathbf{T}^{-1} \cdot \mathbf{B}_1 \cdot \mathbf{F}_{fk\phi}) + (\Omega \cdot \mathbf{T}^{-1} \cdot \mathbf{B}_2)] \cdot \phi_{fk}$$

Figure A2-6 'Intermediate state 2' of equation (A2-28). It remains to laborate more userfriendly expressions for $\mathbf{R}_{DQ}$, $\mathbf{X}_{DQ}$ and $\Delta\mathbf{E}_{DQ}$, abiding with adopted model conventions.

After some straightforward but tedious laborations, we arrive at the *electrical circuit model* shown in Figure A2-7. At the end of the finalizing process sign conventions and terminology in line with that adopted for the formal *electrical circuit model* of Figure 1.1, are observed.



$$\underset{(2x1)}{\mathbf{v}_{DQ}} = \mathbf{R}_{DQ}\cdot\mathbf{i}_{DQ} + (1/\omega_o)\cdot\mathbf{X}_{DQ}\cdot d\mathbf{i}_{DQ}/dt \quad \& \quad \mathbf{v}_{DQ} - \Delta\mathbf{E}_{DQ} = \mathbf{u}_{DQ} \qquad \text{(A2-35)}$$

Electrical circuit model of the synchronous motor in d-q axis frame of reference

$\Delta\mathbf{E}_{DQ} = (\mathbf{V}_{fk} + \mathbf{H}_{fk}\cdot\phi_{fk})$ = synchronous motor emf.

$$\mathbf{R}_{DQ} = \begin{bmatrix} (R_a+\hat{X}''_r) + (\Omega_{SMref}+2\cdot\Delta\Omega_{SM})\cdot\bar{X}''\cdot\sin2\beta + \bar{X}''_r\cdot\cos2\beta & -\Omega_{SMref}\cdot\hat{X}''+(\Omega_{SMref}+2\cdot\Delta\Omega_{SM})\cdot\bar{X}''\cdot\cos2\beta - \bar{X}''_r\cdot\sin2\beta \\ \Omega_{SMref}\cdot\hat{X}''+(\Omega_{SMref}+2\cdot\Delta\Omega_{SM})\cdot\bar{X}''\cdot\cos2\beta - \bar{X}''_r\cdot\sin2\beta & (R_a+\hat{X}''_r) - (\Omega_{SMref}+2\cdot\Delta\Omega_{SM})\cdot\bar{X}''\cdot\sin2\beta - \bar{X}''_r\cdot\cos2\beta \end{bmatrix}$$

$\Omega_{SMref} = (1-\Omega_f)$, where Ω_f = pu angular speed of rotor mmf relative to rotor. See p. A2/11.

$$\mathbf{X}_{DQ} = \begin{bmatrix} \hat{X}'' + \bar{X}''\cdot\cos2\beta & -\bar{X}''\cdot\sin2\beta \\ -\bar{X}''\cdot\sin2\beta & \hat{X}'' - \bar{X}''\cdot\cos2\beta \end{bmatrix} \qquad \mathbf{V}_{fk} = \begin{bmatrix} C_f\cdot\bar{E}_f\cdot\cos(\beta-\beta_f) \\ -C_f\cdot\bar{E}_f\cdot\sin\beta(\beta-\beta_f) \end{bmatrix}$$

$\Delta\Omega_{SM} = (\Omega_{SM} - \Omega_{SMref})$. Ω_{SM} = pu rotor speed.

$\bar{E}_f = (\sqrt{2}\cdot E_{f\text{-}eff} + \Delta E_f)$ = peak field excitation.

ΔE_f = voltage regulator response. See p.1/29.

$C_f = (\sqrt{2}/(\omega_o\cdot T'_{do}\cdot\varepsilon_f))\cdot(X''_{ad}/X'_{ad})$. See *)

β = synchronous motor angle, see Fig.A2-3.

β_f = phase shift of magnetizing ac voltage. See Figure A2-3.

$$\mathbf{H}_{fk} = \begin{array}{c} \\ \\ \end{array}\begin{array}{cccc} f & fq & kd & kq \end{array}$$

	f	fq	kd	kq	
$\mathbf{H}_{fk}$ =	$(\Omega_{SM}\cdot f_1 - f_{rotor(pu)}\cdot f_7)\cdot\sin\beta + f_2\cdot\cos\beta$	$(\Omega_{SM}\cdot f_7 - f_{rotor(pu)}\cdot f_1)\cdot\cos\beta + f_8\cdot\sin\beta$	$\Omega_{SM}\cdot f_3\cdot\sin\beta + f_4\cdot\cos\beta$	$\Omega_{SM}\cdot f_5\cdot\cos\beta + f_6\cdot\sin\beta$	D
	$(\Omega_{SM}\cdot f_1 - f_{rotor(pu)}\cdot f_7)\cdot\cos\beta - f_2\cdot\sin\beta$	$-(\Omega_{SM}\cdot f_7 - f_{rotor(pu)}\cdot f_1)\cdot\sin\beta + f_8\cdot\cos\beta$	$\Omega_{SM}\cdot f_3\cdot\cos\beta - f_4\cdot\sin\beta$	$-\Omega_{SM}\cdot f_5\cdot\sin\beta + f_6\cdot\cos\beta$	Q

$\hat{X}'' = 0.5(X''_d + X''_q)$ $\quad \hat{X}''_r = 0.5(X''_{rd} + X''_{rq})$ $\quad \leftarrow \quad X''_{rd} = (1/(\omega_o\cdot T'_{do}))\cdot(X''_{ad}/X'_{ad})^2\cdot(X_d - X'_d) + (1/(\omega_o\cdot T''_{do}))\cdot(X'_d - X''_d)$

$\bar{X}'' = 0.5(X''_d - X''_q)$ $\quad \bar{X}''_r = 0.5(X''_{rd} - X''_{rq})$ $\quad \leftarrow \quad X''_{rq} = (1/(\omega_o\cdot T'_{do}))\cdot(X''_{aq}/X'_{aq})^2\cdot(X_q - X'_q) + (1/(\omega_o\cdot T''_{qo}))\cdot(X'_q - X''_q)$

$f_{rotor(pu)} = (f_{rotor}/50)$ = pu frequency of 3-phase voltage applied to field winding. $f_{rotor(pu)}$ is *not* subject to sign shift.

$f_1 = (X_d - X'_d)\cdot(X''_{ad}/(X_{ad}\cdot X'_{ad}))$ $\quad \leftarrow \quad X_{ad} = X_d - X_{a\sigma}$

$f_2 = f_1\cdot[(X'_d - X''_d)\cdot[(1/(\omega_o\cdot T''_{do}))\cdot(1/X''_{ad}) - (1/(\omega_o\cdot T'_{do}))\cdot(X_{ad}/X'^2_{ad})] - (1/(\omega_o\cdot T'_{do}))\cdot(X''_{ad}/X'_{ad})]$ $\quad \leftarrow \quad X'_{ad} = X'_d - X_{a\sigma}$

$f_3 = (X'_d - X''_d)/X'_{ad}$ $\quad \leftarrow \quad X''_{ad} = X''_d - X_{a\sigma}$

$f_4 = f_3\cdot[(1/(\omega_o\cdot T'_{do}))\cdot(X''_{ad}/X'^2_{ad})\cdot(X_d - X'_d) - (1/(\omega_o\cdot T''_{do}))]$

$f_5 = -(X'_q - X''_q)/X'_{aq}$ $\quad \leftarrow \quad X_{aq} = X_q - X_{a\sigma}$

$f_6 = -f_5\cdot[(1/(\omega_o\cdot T'_{qo}))\cdot(X''_{aq}/X'^2_{aq})\cdot(X_q - X'_q) - (1/(\omega_o\cdot T''_{qo}))]$ $\quad \leftarrow \quad X'_{aq} = X'_q - X_{a\sigma}$

$f_7 = -(X_q - X'_q)\cdot(X''_{aq}/(X_{aq}\cdot X'_{aq}))$ $\quad \leftarrow \quad X''_{aq} = X''_q - X_{a\sigma}$

$f_8 = -f_7\cdot[(X'_q - X''_q)\cdot[(1/(\omega_o\cdot T''_{qo}))\cdot(1/X''_{aq}) - (1/(\omega_o\cdot T'_{qo}))\cdot(X_{aq}/X'^2_{aq})] - (1/(\omega_o\cdot T'_{qo}))\cdot(X''_{aq}/X'_{aq})]$

Figure A2-7 **Electrical circuit model** of the 'extended' synchronous motor (incl. the adjustable speed version)

*) In the general, salient pole development we find that $C_f = (\sqrt{2}/(\omega_o\cdot T'_{do}\cdot\varepsilon_f))\cdot(X''_{ad}/X'_{ad})$ and $C_{fq} = (\sqrt{2}/(\omega_o\cdot T'_{qo}\cdot\varepsilon_{fq}))\cdot(X''_{aq}/X'_{aq})$. Also: $\mathbf{V}_{fk}^t = [(C_f\cdot E_f\cdot\cos\beta + C_{fq}\cdot E_{fq}\cdot\sin\beta), (-C_f\cdot E_f\cdot\sin\beta + C_{fq}\cdot E_{fq}\cdot\cos\beta)]$. See pages 12-16 of this appendix. Since the adjustable speed synchronous machine is symmetrical of design, parameter symmetry prevails, leading to the more convenient description of $\mathbf{V}_{fk}$ given in Figure A2-7.

In the elaboration of equation (A2-35), letter combinations like 'fk' and 'DQ' have been applied for indexing to (hopefully) enhance understanding of the algorithmic development. From an ensuing application point of view, better notations could be devised. See end of this appendix for summary model description that aims at being more *user*oriented.

The *rotor flux model* of Figure A2-3 & A2-4, and the *electrical circuit model* of Figure A2-7, are based on the *six*-coil generalised machine. By specifying premises/data in such a way that the fq-coil is eliminated, these models reduce to those of Chapter 1.4 which were based on the *five*-coil generalised machine. The following specific assumptions provide the stated reduction:

- Time constant T'_{qo} is set to a very large value – as a consequence of presuming that the resistance of the fq-circuit is zero and thus of no 'torque-providing' influence.
- X'_q is set equal to X_q to observe the assumption that the self-reactance of the fq-circuit is set infinitely large and thus of no 'current-providing' effect. See Addendum on p. A2-11.
- Matrix rows and columns associated with the fq-coil are deleted.

The electromechanical model

We seek here the description of the final two synchronous motor state variables, - namely pu rotor speed Ω, and rotor's electrical angle β relative to some chosen synchronous reference. For brevity of characterization, the algorithms developed in this context is denoted *'the electromechanical model'.*

The algorithm that governs motor speed performance is the torque equation of the machine unit. A brief elaboration of this equation on pu form referred to common system base, is given in Chapter 1.4. We fetch from that chapter this practical pu form of the motor torque equation :

$$((S_{Motor}/S_{Bas})\ T_a \cos\varphi_{Motor})\ d\Omega/dt = (T_{(el)} - T_{(mec)}) \qquad [pu] \qquad \text{(A2-36)}$$

Part of *electromechanical model*: The description of synchronous motor rotor speed Ω. Acceleration time T_a used for characterizing total moment of inertia of rotating masses.

Another widely used normalized inertia figure is the H-constant. H is defined as *stored kinetic energy at synchronous speed divided by machine voltampere rating,* - i.e.: $H = 0.5\ J\ \omega_{mec(0)}^2/S_{Motor}$. This implies the following relationship between T_a and H : $H = 0.5\ T_a \cos\varphi_{Motor}$.

The electrical motor torque $T_{(el)}$ is developed next. Per definition we have the following expression for power supplied to the synchronous motor, see Figure A2-2 and comments on basic premises in Chapter 1.4 :

$$P_{(el)} = 0.5\ \mathbf{e}_{dq}^t\ \mathbf{i}_{dq} \qquad \text{(A2-37)}$$

Setting in for $\mathbf{e}_{dq}$ and $\mathbf{i}_{dq}$ from (A2-12) and (A2-13), and observing that $\phi_{dq} = \omega_b\ \Psi_{dq}$, we find that;

$$P_{(el)} = 0.5\ r_a\ (i_d^2 + i_q^2) + 0.5\ \Omega\ \mathbf{i}_{dq}^t\ \bar{\mathbf{1}}^t\ \phi_{dq} + (0.5/\omega_b)\ \mathbf{i}_{dq}^t\ d\phi_{dq}/dt \qquad \text{(A2-38)}$$

$\bar{\mathbf{1}}$ is defined earlier, see Figure 1.4 and also below. Replacing the locally referenced current $\mathbf{i}_{dq}$ by its globally referenced counterpart $\mathbf{i}_{DQ}$ according to the transformation $\mathbf{T}$ of (A2-21), we find:

$$P_{(el)} = \underbrace{0.5\ r_a\ (i_D^2 + i_Q^2)}_{\text{Losses in stator resistance}} + \underbrace{0.5\ \Omega\ \mathbf{i}_{DQ}^t\ \mathbf{T}_1\ \phi_{dq}}_{\text{Airgap power}} + \underbrace{(0.5/\omega_b)\ \mathbf{i}_{DQ}^t\ \mathbf{T}^t\ d\phi_{dq}/dt}_{\text{Oscillating power (zero power over time)}} \qquad \text{(A2-39)}$$

The *electrical torque* is found by dividing the expression for *airgap power* by Ω. $\mathbf{T}_1 = (\bar{\mathbf{1}} \cdot \mathbf{T})^t$. The flux vector ϕ_{dq} is determined as a function of $\mathbf{i}_{DQ}$ and ϕ_{fk} from equations (A2-26), (A2-27), (A2-21). After some elaborations the following practical algorithm emerges for determining the electrical motor torque $T_{(el)}$:

$$T_{(el)} = 0.5\ \mathbf{i}_{DQ}^{t}\ \mathbf{T}_1\ \boldsymbol{\phi}_{dq} \qquad \text{(A2-40)}$$

where;

$$\boldsymbol{\phi}_{dq} = \mathbf{X}''\cdot \mathbf{T}\ \mathbf{i}_{DQ} + \mathbf{f}\ \boldsymbol{\phi}_{fk}$$

and

$$\mathbf{i}_{DQ} = \begin{bmatrix} i_D \\ i_Q \end{bmatrix}$$ = Synchronous motor current, global reference.

$$\boldsymbol{\phi}_{fk} = \begin{bmatrix} \phi_f \\ \phi_{fq} \\ \phi_{kd} \\ \phi_{kq} \end{bmatrix}$$ = Field- and damper flux linkages, local reference.

(Both together: Synchronous motor state variables)

$$\mathbf{T} = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}$$ = Transformation that shifts stator current from global to local reference axis : $\mathbf{i}_{dq} = \mathbf{T}\cdot \mathbf{i}_{DQ}$ See (A2-21).

$$\mathbf{T}_1 = (\bar{1}\ \mathbf{T})^t = \left[\begin{bmatrix} & 1 \\ -1 & \end{bmatrix} \cdot \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}\right]^t = \begin{bmatrix} \sin\beta & -\cos\beta \\ \cos\beta & \sin\beta \end{bmatrix}$$

$$\mathbf{X}'' = \begin{bmatrix} X''_d & \\ & X''_q \end{bmatrix}$$

$$\mathbf{f} = \begin{bmatrix} f_1 & & f_3 & \\ & -f_7 & & -f_5 \end{bmatrix}$$

where;

$f_1 = (X_d - X'_d)\ X''_{ad}/(X_{ad}\ X'_{ad})$

$f_3 = (X'_d - X''_d)/(X'_{ad})$

$f_5 = -(X'_q - X''_q)/(X'_{aq})$

$f_7 = -(X_q - X'_q)\ X''_{aq}/(X_{aq}\ X'_{aq})$

Figure A2-8 Practical algorithm for computing synchronous motor electrical torque $T_{(el)}$ in (A2-36)

The *mechanical torque* $T_{(mec)}$ will take on different forms, depending on the operational regime; whether motor or generator mode of operation :

For motor operation (which per definition implies a positive sign of the mechanical torque), the following premises may in many cases prevail:

$T_{(mec)} = T_{(mec(o))}\ \Omega^{\kappa}$, where $T_{(mec(0))}$ and exponent κ depend on the 'rotational status' of the motor at $t = -0$; whether already up and running, or to be started from $\Omega = 0$: (A2-41)

If the motor is up and running ;

$T_{(mec(o))} = T_{(el(o))}$ = electrical motor torque at $t = -0$. The proper value is found by applying equation (1-60) to data from the initial power system load flow.

κ = exponent that depends on the load torque's sensitivity to rotational speed for Ω close to 1.0 . In many cases: κ = (say) 1.5 – 3.5.

If the motor is to be started from stillstand (as e.g. an asynchronous motor) ;

$T_{(mec(o))}$ = coefficient that contributes to modelling the effect of mechanical friction, air resistance, etc, during the startup phase. Expected range: (say) 0.02 – 0.05

κ = exponent reflecting speed dependency of $T_{(mec)}$. Prospective area of variation: κ = (say) 1-5. κ as well as $T_{(mec(o))}$ may change over the range $\Omega = 0 \rightarrow 1$.

For generator operation (which per definition implies a neg. sign of the mechanical torque):

$T_{(mec)} = (T_{(el(o))} + \Delta T_{(mec)})$, where $T_{(el(0))}$ is initial electrical motor torque, and $\Delta T_{(mec)}$ is given by the respons of the *power control system.* For details, see 'model stock' of Chapter 1.7. (A2-42)

Figure A2-9 Example algorithms for computing synchronous motor load torque $T_{(mec)}$ in (1-57)

The algorithm that governs the variation of the rotating part's electrical angle β relative to some chosen global synchronous reference phasor, can definitionwise be given as follows :

$$\beta = \omega_o \cdot t - (\theta + \theta_f) \qquad \text{(A2-43)}$$

where interpretation of defined and derived terms are summarized next to finalizing of current laboration on (A2-43). Taking the derivative of this equation we arrive at the sought differential equation governing the electrical positioning of the motor's rotating part relative to the synchronous reference phasor :

$$d\beta/dt = \omega_o - \omega - \omega_f = \omega_o \cdot (1 - \omega/\omega_o - \omega_f/\omega_o)$$

$\supset$:

or equivalently;

$$d\beta/dt = \omega_o \cdot (1 - \Omega_f - \Omega)$$

$$d\beta/dt = \omega_o \cdot (\Omega_{SMref} - \Omega_{SM}) = - \omega_o \cdot \Delta\Omega_{SM} \qquad \text{(A2-44)}$$

Part of *electromechanical model*: The description of synchronous motor electrical angle β in radians

Interpretations:

$\omega_o \cdot t$ = angular electrical displacement of global synchronous reference phasor.

$\theta = \omega \cdot t = \omega_{SM} \cdot t$ = angular electrical displacement of rotating part. $\omega = \omega_{SM}$ is angular electrical speed of rotating part. $\Omega = \Omega_{SM}$ = pu angular speed of rotating part.

$\theta_f = \omega_f \cdot t = 2\pi f_{rotor}$= angular electrical displacement of rotating mmf set up by the three phase voltage of frequency f_{rotor}, applied to the distributed three phase field winding. See additional comment below. θ_f is measured relative to the field winding. Whether the field winding is on the rotating or fixed part of the motor, $(\theta + \theta_f)$ will - with suitable choice of sign conventions - describe the electrical displacement of the rotating mmf relative to the global electrical system

$\Omega_f = \omega_f/\omega_o$=pu angular speed of field winding mmf, relative to the field winding. Ω_f is subject to +/-

$\Omega_{SMref} = (1-\Omega_f)$ = 'target' pu speed of rotating winding when 3-phase voltage of frequency f_{rotor} is applied to the field winding.

$\Delta\Omega_{SM} = (\Omega_{SM} - \Omega_{SMref})$ = rotating winding's pu speed deviation from previously stated 'target' value.

As additional comment to the interpretation of terms, it is noted that a three phase, symmetrical voltage applied to an appropriately distributed three phase winding located to a machine's rotor or stator, will create an m.m.f. that is fixed in space and rotates with an angular speed of $\omega_f = 2 \cdot \pi \cdot f_{rotor}$ relative to the field winding itself.

Summary synchronous motor model description is presented at the end of this appendix. Illustrations of model application are also presented.

Addendum

To enlighten the laborations towards compact motor model descriptions, the interrelationship between *'commercial' machine parameters* like $(X_d, X'_d, X''_d, X_q, X'_q, X''_q, T'_{do}, T''_{do}, T''_{qo}, T''_{qo})$ and *basic model parameters* like $(L_{a\sigma}, L_{ad}, L_{aq}, L_{f\sigma}, L_{fq\sigma}, L_{kd\sigma}, L_{kq\sigma}, r_a, r_{kd}, r_f, r_{fq}, r_{kq})$, are briefly summed up:

$X_d = X_{a\sigma} + X_{ad}$

$X_f = X_{f\sigma} + X_{ad}$

$X_{kd} = X_{kd\sigma} + X_{ad}$

$X_q = X_{a\sigma} + X_{aq}$

$X_{fq} = X_{fq\sigma} + X_{aq}$

$X_{kq} = X_{kq\sigma} + X_{aq}$

$X'_d = X_{a\sigma} + X'_{ad}$ where $1/X'_{ad} = (1/X_{ad}) + (1/X_{f\sigma})$

$X''_d = X_{a\sigma} + X''_{ad}$ where $1/X''_{ad} = (1/X_{ad}) + (1/X_{f\sigma}) + (1/X_{kd\sigma}) = (1/X'_{ad}) + (1/X_{kd\sigma})$

$X'_q = X_{a\sigma} + X'_{aq}$ where $1/X'_{aq} = (1/X_{aq}) + (1/X_{fq\sigma})$

$X''_q = X_{a\sigma} + X''_{aq}$ where $1/X''_{aq} = (1/X_{aq}) + (1/X_{fq\sigma}) + (1/X_{kq\sigma}) = (1/X'_{aq}) + (1/X_{kq\sigma})$

$T'_{do} = L_f/r_f = X_f/(\omega_o \cdot r_f)$ (Open stator. 'Seen' from the f- circuit)

$T''_{do} = L/r_{kd} = X/(\omega_o \cdot r_{kd})$ where $X = X_{kd\sigma} + 1/((1/X_{ad}) + (1/X_{f\sigma}))$ (Open stator. 'Seen' from the kd-circuit)

$T'_{qo} = L_{fq}/r_{fq} = X_{fq}/(\omega_o \cdot r_{fq})$ (Open stator. 'Seen' from the fq- circuit)

$T''_{qo} = L/r_{kq} = X/(\omega_o \cdot r_{kq})$ where $X = X_{kq\sigma} + 1/((1/X_{aq}) + (1/X_{fq\sigma}))$ (Open stator. 'Seen' from the kq-circuit)

Modelling of specal voltages in the d-q axis frame of reference

Four voltage aspects will be dealt with under this heading; the transformation of the three phase voltage at some reference/infinite system bus, the transformation of the three phase voltage applied to the three phase field winding, the voltage excitation of the *Rotor Flux Model*, and the voltage excitation of the *Electrical Circuit Model* of the extended synchronous machine.

The infinite bus voltage of nominal system frequency

Depending on the type and scope of task at hand the three phase, symmetrical voltage across the terminals of e.g. a motor or a load, may (at the outset of analysis) be a specified phasor quantity. Alternatively, a remote 'infinite bus' may be declared, and the voltage phasor specified at that bus. In specific terms : Given the symmetrical three phase voltages

$$\mathbf{E}_{RST} = \sqrt{2}\cdot E_{eff}\cdot \begin{bmatrix} \cos(\alpha) \\ \cos(\alpha-2\pi/3) \\ \cos(\alpha-4\pi/3) \end{bmatrix} \qquad \text{(A2-45)}$$

at some specified bus of the system. E_{eff} is the *root mean square* (*r.m.s.*) value of the three phase voltage. $\alpha = (\omega_o\cdot t+\gamma)$, where $\omega_o = 2\cdot\pi\cdot f_o = 2\cdot\pi\cdot 50$. γ accounts for an arbitrary phase shift of the voltages relative to zero time. For convenience of final expressions – see (A2-47) – we further define $\gamma = (\gamma_{ref} + \pi/2)$. The transition of $\mathbf{E}_{RST}$ in (A2-45) to $\mathbf{e}_{dqo}$ of the d-q-o axis frame of reference, is afforded by the Park transformation – see equation (1-1):

$$\mathbf{e}_{dqo} = \mathbf{P}\cdot\mathbf{E}_{RST} \qquad \text{(A2-46)}$$

where;

$$\mathbf{P} = 2/3\cdot \begin{bmatrix} \cos\theta & \cos(\theta-2\pi/3) & \cos(\theta-4\pi/3) \\ -\sin\theta & -\sin(\theta-2\pi/3) & -\sin(\theta-4\pi/3) \\ \tfrac{1}{2} & \tfrac{1}{2} & \tfrac{1}{2} \end{bmatrix} \begin{matrix} d \\ q \\ o \end{matrix} \qquad \text{(1-2)}$$

(columns R, S, T)

In the present 'synchronous reference phasor' context, $\theta=\omega_o\cdot t$. Evaluating the product in (A2-46), the sought source voltage description in the d-q-o frame of reference is established:

$$\mathbf{e}_{dqo} = \begin{bmatrix} e_d \\ e_q \\ e_o \end{bmatrix} = \sqrt{2}\cdot E_{eff}\cdot \begin{bmatrix} -\sin\gamma_{ref} \\ \cos\gamma_{ref} \\ 0 \end{bmatrix} \qquad \text{(A2-47)}$$

d-q axis model of given symmetrical system voltage $\mathbf{E}_{RST}$ of the three phase frame of reference. See given voltage in (A2-45). E_{eff}=r.m.s. value of given voltage. γ_{ref}= arbitrary chosen phase shift. Often conveniant choice: γ_{ref} =0.

Simple examples of a specified voltage $\mathbf{e}_{dqo}$ in analysis, are given in Chapter 1.7. The treatment of $\mathbf{e}_{dqo}$ in arbitrary complex power networks, is covered in systems modelling Chapter 2.

The three phase voltage of frequency f_f applied to the field winding

The symmetrical three phase voltage $E_{f\text{-}RST}$ of frequency f_f Hz applied to the field winding, is given by (A2-48):

$$\mathbf{E}_{f\text{-}RST} = \sqrt{2}\cdot E_{f\text{-}eff}\cdot \begin{bmatrix} \cos(\alpha_f) \\ \cos(\alpha_f-2\pi/3) \\ \cos(\alpha_f-4\pi/3) \end{bmatrix} \qquad \text{(A2-48)}$$

Subscript 'f-' signals association with the field winding. $E_{f\text{-}eff}$ is the *root mean square* (*r.m.s.*) value of the three phase field voltage. $\alpha_f = (\omega_f\cdot t+\gamma_f)$, where $\omega_f = 2\cdot\pi\cdot f_f$. γ_f accounts for an arbitrary phase shift of the voltages relative to zero time.

The transition of $\mathbf{E}_{f\text{-}RST}$ in (A2-48) to $\mathbf{E}_{f\text{-}dqo}$ of the d-q-o axis frame of reference, is afforded by the Park transformation :

$$\mathbf{E}_{f\text{-}dqo} = \mathbf{P}_f \cdot \mathbf{E}_{f\text{-}RST} \quad \text{(A2-49)}$$

where;

$$\mathbf{P}_f = 2/3 \cdot \begin{array}{|c|c|c|l} \multicolumn{1}{c}{R} & \multicolumn{1}{c}{S} & \multicolumn{1}{c}{T} & \\ \hline \cos\theta_f & \cos(\theta_f - 2\pi/3) & \cos(\theta_f - 4\pi/3) & d \\ -\sin\theta_f & -\sin(\theta_f - 2\pi/3) & -\sin(\theta_f - 4\pi/3) & q \\ ½ & ½ & ½ & o \\ \hline \end{array} \quad \text{(A2-50)}$$

The angular displacement θ_f of the axes of the m.m.f set up by the field winding- relative to the displacement $(\omega_f \cdot t)$ of the axes of a 'local' field related reference phasor, - can be expressed as

$$\beta_f = \omega_f \cdot t - \theta_f. \qquad *) \quad \text{(A2-51)}$$

See p. A2/11 for description of the angular displacement of the rotating electrical axes relative to the global system. Applying the matrices (A2-48) and (A2-50) into (A2-49), the field voltage description in the d-q-o frame of reference is established:

$$\mathbf{E}_{f\text{-}dqo} = \begin{bmatrix} E_f \\ E_{fq} \\ E_o \end{bmatrix} = \sqrt{2} \cdot E_{f\text{-}eff} \cdot \begin{bmatrix} \cos(\theta_f - \alpha_f) \\ -\sin(\theta_f - \alpha_f) \\ 0 \end{bmatrix} \quad \text{(A2-52)}$$

From the definition following equation (A2-48), and from equation (A2-51), we have ;

$$\left.\begin{array}{l} \alpha_f = (\omega_f \cdot t + \gamma_f) \\ \theta_f = (\omega_f \cdot t - \beta_f) \end{array}\right. \longrightarrow (\theta_f - \alpha_f) = -(\beta_f + \gamma_f) \quad \text{(A2-53)}$$

Inserting these expressions into (A2-52) and choosing $\gamma_f = 0$ **), we arrive at the sought d-q axis description of the exogenously applied three phase field voltage :

$$\boxed{\mathbf{E}_{f\text{-}dqo} = \begin{bmatrix} E_f \\ E_{fq} \\ E_o \end{bmatrix} = \sqrt{2} \cdot E_{f\text{-}eff} \cdot \begin{bmatrix} \cos\beta_f \\ \sin\beta_f \\ 0 \end{bmatrix}} \quad \text{(A2-54)}$$

d-q axis model of given symmetrical field voltage $\mathbf{E}_{f\text{-}RST}$ in the three phase frame of reference. See given voltage in (A2-48). $E_{f\text{-}eff}$ = r.m.s. value of given voltage. β_f is an exogenously specified field voltage phase shift, relative to the local field reference phasor.

') Taking the derivative of (A2-51) we get ; $d\beta_f/dt = \omega_f - d\theta_f/dt = \omega_f - \omega_f = 0$, since the field m.m.f. rotates synchronous with the defined field related reference phasor. There may however be a steady state displacement between the two local axis systems, reflected by the angle β_f .

**) As defined at the bottom of page A2/12, the γ_f accounts for an arbitrary phase shift of the impressed field voltage relative to zero time. Choosing $\gamma_f = 0$ is convenient ; we then conceptually have a 'smooth' transition from the *adjustable speed synchronous machine* model description of Figure (A2-54), to the *traditional synchronous machine* model description of Chapter 1.4, where $e_f = E_f$ is the applied dc field voltage. To excitation-wise 'reduce' the 3-phase field winding into the dc field winding, we specify $\beta_f = 0$ in (A2-54). The algorithm then yields; $e_f = \sqrt{2} \cdot E_{f\text{-}eff}$ and $e_{fq} = 0$. The latter equation eliminates the excitation of the fq-winding. In addition the fq-winding itself has to be eliminated. See page A2/9 for model settings that will afford that.

Excitation (e_f, e_{fq}) to the Rotor Flux Model expressed in tems of excitation variable (E_f,E_{fq}) scaled to the machine's pu phasor diagram

For the linear machine it is reasonable to expect a proportional relationship between the rotor flux model's field voltage excitations (e_f,e_{fq}) and the d-q components of the applied three phase field voltage as given in (A2-54). Thus the task at hand is to seek to determine the factors (K_f,K_{fq}) displayed in (A2-55) :

$$\mathbf{e}_{fk} = \begin{bmatrix} e_f \\ e_{fq} \\ e_{kd}=0 \\ e_{kq}=0 \end{bmatrix} \begin{matrix} f \\ fq \\ kd \\ kq \end{matrix} = \begin{bmatrix} K_f\cdot(\sqrt{2}\cdot E_{f\text{-}eff}\cdot\cos\beta_f) \\ K_{fq}\cdot(\sqrt{2}\cdot E_{f\text{-}eff}\cdot\sin\beta_f) \\ 0 \\ 0 \end{bmatrix} \tag{A2-55}$$

The following line of reasoning forms a basis for determining the two factors:

> In idle, steady state operation with zero stator current flowing, the machine's phasor diagram reduces to a single voltage phasor representing pu r.m.s. machine bus voltage u_{rms}. At the same time it would seem convenient also to interpret this phasor as the pu r.m.s. magnetizing or field voltage $e_{f\text{-}rms}$, yielding in this particular operational case the pu condition $u_{rms} = e_{f\text{-}rms}$.
>
> This simple operating state provides a convenient basis for identifying K_f and K_{fq} : The electrical circuit model of the motor as given by (A2-35), is applied in describing the particular idle operating state of the machine. From the two equations (one for the d- and one for the q-axis) that arise, K_f and K_{fq} are solved so as to fulfill the required pu equality conditions.

In the following the process of determining the content of K_f and K_{fq} is outlined in some detail: Starting point is modelling of the motor connected to (say) an 'infinite bus' where the voltage $\mathbf{e}_{dq}$ is as given by (A2-45). The *electrical circuit model* of the motor is given in (A2-35). The simple loop equation comprising the serial effect of the infinite bus voltage and the synchronous motor, is shown in (A2-56).The equation could be set up directly by intuition, or developed in accordance with the full machinery of system analysis, as demonstrated at the end of Appendix 1. See text leading up to equation (A1-19) there.

$$\mathbf{e}_{dq} = \mathbf{R}_{SM}\cdot\mathbf{i}_{SM} + (1/\omega_o)\cdot\mathbf{X}_{SM}\cdot d\mathbf{i}_{SM}/dt + \Delta\mathbf{E}_{SM} \tag{A2-56}$$

Assuming idle, steady state operation with $\mathbf{i}_{SM} = 0$, we get the following set of two equations for further reduction and application, observing -from Figure A2-7 - that $\Delta\mathbf{E}_{DQ}= \Delta\mathbf{E}_{SM} =(\mathbf{V}_{fk} + \mathbf{H}_{fk}\cdot\phi_{fk})$;

$$\mathbf{e}_{dq} = \mathbf{V}_{fk} + \mathbf{H}_{fk}\cdot\phi_{fk} \tag{A2-57}$$

Here;

$$\mathbf{e}_{dq} = \sqrt{2}\cdot E_{eff}\cdot \begin{bmatrix} -\sin\gamma_{ref} \\ \cos\gamma_{ref} \end{bmatrix} \tag{A2-58}$$

Copied from (A2-47)

$$\mathbf{V}_{fk} = (\omega_o\cdot\mathbf{T}^{-1}\cdot\mathbf{B}_1)\cdot\mathbf{e}_{fk}$$

See Figure A2-6. $\mathbf{T}^{-1}$ is copied from equation (A2-30), and $\mathbf{B}_1$ from Fig. A2-5. Noting that $\mathbf{i}_{SM}$=0 implies β=0, we find after some laboring:

$$= \omega_o \begin{bmatrix} B(D,f)\cdot\cos\beta\cdot e_f + B(Q,fq)\cdot\sin\beta\cdot e_{fq} \\ -B(D,f)\cdot\sin\beta\cdot e_f + B(Q,fq)\cdot\cos\beta\cdot e_{fq} \end{bmatrix} = \omega_o\cdot \begin{bmatrix} B(D,f)\cdot e_f \\ B(Q,fq)\cdot e_{fq} \end{bmatrix} \tag{A2-59}$$

$$\mathbf{H}_{fk}\cdot\phi_{fk} = \begin{bmatrix} i_f\cdot(f_2\cdot x_f+f_4\cdot X_{ad}) + i_{fq}\cdot(\Omega_{SM}\cdot(f_7\cdot X_{fq}+f_5\cdot X_{aq}) - f_{rotor(pu)}\cdot f_1\cdot X_{fq}) \\ i_{fq}\cdot(f_8\cdot x_{fq}+f_6\cdot X_{aq})+ i_f\cdot(\Omega_{SM}\cdot(f_1\cdot X_f+f_3\cdot X_{ad}) - f_{rotor(pu)}\cdot f_7\cdot X_f) \end{bmatrix} \tag{A2-60}$$

$\mathbf{H}_{fk}$ is taken from Figure A2-7, with β=0. $\phi_{fk} =\omega_o\cdot\Psi$ is developed from equation (A2-2)

Inserting expressions (A2-58) – (A2-60) into (A2-57), we get ;

$$\begin{bmatrix} -\sqrt{2}\cdot E_{eff}\cdot \sin\gamma_{ref} \\ \sqrt{2}\cdot E_{eff}\cdot \cos\gamma_{ref} \end{bmatrix} = \overbrace{\begin{bmatrix} \omega_o\cdot B(D,f)\cdot e_f \\ \omega_o\cdot B(Q,fq)\cdot e_{fq} \end{bmatrix}}^{=0} + \begin{bmatrix} i_f\cdot(f_2\cdot x_f+f_4\cdot X_{ad}) + i_{fq}\cdot(\Omega_{SM}\cdot(\overbrace{f_7\cdot X_{fq}+f_5\cdot X_{aq}}^{-X_{aq}}) - f_{rotor(pu)}\cdot f_1\cdot X_{fq}) \\ i_{fq}\cdot(f_8\cdot x_{fq}+f_6\cdot X_{aq}) + i_f\cdot(\Omega_{SM}\cdot(\underbrace{f_1\cdot X_f+f_3\cdot X_{ad}}_{X_{ad}}) - f_{rotor(pu)}\cdot f_7 X_f) \end{bmatrix} \quad \text{(A2-61)}$$

(The upper "=0" arrow spans the first row and the lower "=0" arrow spans the second row of the $\omega_o\cdot B$ term.)

In steady state operation we generally have, see equations (A2-12) and Addendum, p. A2/11 ;

$$i_f = e_f/r_f \quad = e_f\cdot\omega_o\cdot T'_{do}\cdot(X_d - X'_d)/X^2_{ad} \quad \text{(A2-62)}$$
$$i_{fq} = e_{fq}/r_{fq} = e_{fq}\cdot\omega_o\cdot T'_{qo}\cdot(X_q - X'_q)/X^2_{aq} \quad \text{(A2-63)}$$

From Addendum on page A2/11, and Figure A2-7 we deduct ;

$$f_1\cdot X_{fq} = [(X_d - X'_d)/(X_q - X'_q)]\cdot(X''_{ad}\cdot X^2_{aq})//(X'_{ad}\cdot X_{ad}) \quad \text{(A2-64)}$$
$$f_7\cdot X_f = -[(X_q - X'_q)/(X_d - X'_d)]\cdot(X''_{aq}\cdot X^2_{ad})//(X'_{aq}\cdot X_{aq}) \quad \text{(A2-65)}$$

The above expressions for i_f, i_{fq} , $(f_1\cdot X_{fq})$ and $(f_7\cdot X_f)$ are applied to (A2-61), resulting in the following equations for describing the specified, idle operating state, observing that in steady state operation $\Omega_{SM} = (1- \Omega_f)$;

$$E_{eff}\cdot\sin\gamma_{ref} = [(1/\sqrt{2})\cdot\omega_o\cdot T'_{qo}\cdot(X_q-X'_q)/X_{aq}]\cdot(1-\Omega_f + f_{rotor(pu)}\cdot[(X_d - X'_d)/(X_q - X'_q)]\cdot(X''_{ad}\cdot X_{aq})/(X'_{ad}\cdot X_{ad})\cdot e_{fq} \quad \text{(A2-66)}$$
$$E_{eff}\cdot\cos\gamma_{ref} = [(1/\sqrt{2})\cdot\omega_o\cdot T'_{do}\cdot(X_d-X'_d)/X_{ad}]\cdot(1-\Omega_f + f_{rotor(pu)}\cdot[(X_q - X'_q)/(X_d - X'_d)]\cdot(X''_{aq}\cdot X_{ad})/(X'_{aq}\cdot X_{aq})\cdot e_f \quad \text{(A2-67)}$$

Solved with respect to e_f and e_{fq} we finally arrive at the sought 'equations of scaling' :

$$e_{fq} = (\sqrt{2}/(\omega_o\cdot T'_{qo}\cdot\varepsilon_{fq}))\cdot X_{aq}/(X_q-X'_q)\cdot(E_{eff}\cdot\sin\gamma_{ref}) = K_{fq}\cdot(E_{eff}\cdot\sin\gamma_{ref}) \quad \text{(A2-68)}$$
$$e_f = (\sqrt{2}/(\omega_o\cdot T'_{do}\cdot\varepsilon_f))\cdot X_{ad}/(X_d-X'_d)\cdot(E_{eff}\cdot\cos\gamma_{ref}) = K_f\cdot(E_{eff}\cdot\cos\gamma_{ref}) \quad \text{(A2-69)}$$

The terms $(E_{eff}\cdot\sin\gamma_{ref})$ and $(E_{eff}\cdot\cos\gamma_{ref})$ on the right hand side of the latter equations, are pu field excitations referred to the machine's pu phasor diagram. Qualitatively, these excitations that are fetched from the machine's initial condition phasor diagram, are the initial values of field circuit excitation to apply in analysis. Via the factors K_{fq} and K_f they transform into appropriate 'process variables' e_{fq} and e_f that go into the rotor flux model. From (A2-68) and (A2-69) the factors are found to be as follows:

$$K_f = (\sqrt{2}/(\omega_o\cdot T'_{do}\cdot\varepsilon_f))\cdot X_{ad}/(X_d-X'_d) \quad \text{where} \quad \varepsilon_f = 1-\Omega_f + f_{rotor(pu)}\cdot[(X_q - X'_q)/(X_d - X'_d)]\cdot(X''_{aq}\cdot X_{ad})/(X'_{aq}\cdot X_{aq})$$
$$K_{fq} = (\sqrt{2}/(\omega_o\cdot T'_{qo}\cdot\varepsilon_{fq}))\cdot X_{aq}/(X_q-X'_q) \quad \text{where} \quad \varepsilon_{fq} = 1-\Omega_f + f_{rotor(pu)}\cdot[(X_d - X'_d)/(X_q - X'_q)]\cdot(X''_{ad}\cdot X_{aq})/(X'_{ad}\cdot X_{ad}) \quad \text{(A2-70)}$$

Factors that relate pu phasor diagram excitations to excitation variables in the rotor flux model.

For the adjustable speed synchronous machine symmetry prevails. For this practical case the K-factor description to apply can be summarized as follows :

$$e_f = K_f\cdot E_f = K_f\cdot(\sqrt{2}\cdot E_{f\text{-}eff}\cdot\cos\beta_f) \qquad K_f = (\sqrt{2}/(\omega_o\cdot T'_{do}\cdot\varepsilon_f))\cdot X_{ad}/(X_d-X'_d) = K_{fq} = (\sqrt{2}/(\omega_o\cdot T'_{qo}\cdot\varepsilon_{fq}))\cdot X_{aq}/(X_q-X'_q) \quad \text{(A2-71)}$$
$$\text{where} \qquad \&$$
$$e_{fq} = K_{fq}\cdot E_{fq} = K_{fq}\cdot(\sqrt{2}\cdot E_{f\text{-}eff}\cdot\sin\beta_f) \qquad \varepsilon_f = 1-\Omega_f + f_{rotor(pu)}\cdot(X''_{aq}/X'_{aq}) = \varepsilon_{fq} = 1-\Omega_f + f_{rotor(pu)}\cdot(X''_{ad}/X'_{ad}) \quad \text{(A2-72)}$$

For the symmetrical synchronous motor with a 3-phase field winding: Excitation (e_f, e_{fq}) to the Rotor Flux Model given in terms of excitation variables (E_f, E_{fq}) scaled to the machine's pu phasor diagram. For definition of terms, see p. A2/8.

Comment re. model application : To sustain generality in machine modelling the field coils 'f' and 'fq' have been defined as separate coils with individual excitations, - which results in *separate* factors K_f and K_{fq}. From the point of view of practical utilization, three main synchronous machine cases would seem to be met with :

The adjustable speed synchronous machine: The three phase rotor winding means symmetrical conditions, leading to $K_f = K_{fq}$, see (A2-70). Equations (A2-71) and (A2-72) apply, with β_f exogenously given. See the Rotor Flux Model of Figure A2-3 into which the equations are applied.

The ordinary synchronous machine, modelled on the basis of a 6-coil generalised machine (for enhanced precision in modelling): K_f applies as given by (A2-70). K_{fq} is not of interest, since the fq-coil is not excited. With $\beta_f=\Omega_f=f_{rotor(pu)}=0$ in this 'dc-case', the voltage peak $E_f=\sqrt{2}\cdot E_{f\text{-}eff}$ in (A2-71) is interpreted as the dc field voltage. See consistency with the '5-coil model' of Chapter 1.4.

The ordinary synchronous machine, modelled on the basis of a 5-coil generalised machine : K_f applies as given by (A2-70). K_{fq} is not defined as no fq-coil is present. With $\beta_f = \Omega_f = f_{rotor(pu)}=0$ in this 'dc-case', the voltage peak $E_f=\sqrt{2}\cdot E_{f\text{-}eff}$ in (A2-71) is interpreted as the dc field voltage. See consistency with the '5-coil model' of Chapter 1.4.

Excitation contribution $\mathbf{V_{fk}}$ to the electrical circuit model

The electrical circuit model of the synchronous motor is given in Figure A2-7. Part of this model is the motor's e.m.f. $\Delta\mathbf{E}_{DQ}$:

$$\Delta\mathbf{E}_{DQ} = (\mathbf{V}_{fk}+\mathbf{H}_{fk}\cdot\phi_{fk}) \qquad \text{(A2-73)}$$

It is the task of the current section to develop a compact/useful expression for the contribution $\mathbf{V}_{fk}$ to this e.m.f. The term $\mathbf{H}_{fk}\cdot\phi_{fk}$, is conveniently processed directly as the matrix product it is.

On page A2/14 we have already started processing of the expression for $\mathbf{V}_{fk}$: From (A2-59) we copy that ;

$$\mathbf{V}_{fk} = (\omega_o\cdot\mathbf{T}^{-1}\cdot\mathbf{B}_1)\cdot\mathbf{e}_{fk} = \omega_o\cdot \begin{bmatrix} B(D,f)\cdot\cos\beta\cdot e_f + B(Q,fq)\cdot\sin\beta\cdot e_{fq} \\ -B(D,f)\cdot\sin\beta\cdot e_f + B(Q,fq)\cdot\cos\beta\cdot e_{fq} \end{bmatrix}_{\begin{smallmatrix}D\\Q\end{smallmatrix}} \qquad \text{(A2-74)}$$

From the definition of $\mathbf{B}_1$ which is given in Figure A2-5, we fetch;

$$B(D,f) = (X''_{ad}/(X_{ad}\cdot X'_{ad}))\cdot(X_d - X'_d) \qquad \text{(A2-75)}$$
$$B(Q,fq) = (X''_{aq}/(X_{aq}\cdot X'_{aq}))\cdot(X_q - X'_q) \qquad \text{(A2-76)}$$

Applying the two latter equations together with equation (A2-71) and (A2-72) into (A2-74), we can summarize as follows for the extended non-symmetric synchronous motor model ;

$$\mathbf{V}_{fk} = \begin{bmatrix} C_f\cdot E_f\cdot\cos\beta + C_{fq}\cdot E_{fq}\cdot\sin\beta \\ -C_f\cdot E_f\cdot\sin\beta + C_{fq}\cdot E_{fq}\cdot\cos\beta \end{bmatrix}_{\begin{smallmatrix}D\\Q\end{smallmatrix}} = C_f\cdot\bar{E}_f\cdot\mu\cdot \begin{bmatrix} \cos(\beta-\delta) \\ -\sin(\beta-\delta) \end{bmatrix}_{\begin{smallmatrix}D\\Q\end{smallmatrix}} \qquad \text{(A2-77)}$$

where;

$$C_f = (\sqrt{2}/(\omega_o\cdot T'_{do}\cdot\varepsilon_f))\cdot(X''_{ad}/X'_{ad}) \quad \text{and} \quad E_f = \sqrt{2}\cdot E_{f\text{-}eff}\cdot\cos\beta_f$$
$$C_{fq} = (\sqrt{2}/(\omega_o\cdot T'_{qo}\cdot\varepsilon_{fq}))\cdot(X''_{aq}/X'_{aq}) \qquad E_{fq} = \sqrt{2}\cdot E_{f\text{-}eff}\cdot\sin\beta_f \qquad \text{(A2-78)}$$

$$\mu = (\cos^2\beta_f + \sin^2\beta_f\cdot(C_{fq}/C_f)^2)^{0.5} \quad \text{and} \quad \bar{E}_f = \sqrt{2}\cdot E_{f\text{-}eff}$$
$$tg\delta = tg\beta_f\cdot(C_{fq}/C_f) \qquad \text{(A2-79)}$$

The e.m.f. $\mathbf{V}_{fk}$ contributing to the voltage source $\Delta\mathbf{E}_{DQ} = (\mathbf{V}_{fk}+\mathbf{H}_{fk}\cdot\phi_{fk})$ of the electrical circuit model of the extended non-symmetric synchronous motor model. For further parameter definitions, see Fig. A2-7 and p. A2/13 & -/15

Again it should be noted that separate field coils 'f' and 'fq' - for maximum flexibility reasons, - have been presumed in the modelling processes. From a practical model application viewpoint we once more point to three main synchronous machine modelling cases, - plus a fourth which is the asynchronous machine 'coming for free'.:

The adjustable speed synchronous machine: The three phase rotor winding means symmetrical conditions, leading to $C_f = C_{fq}$, see (A2-78). Equations (A2-77) to (A2-79) take on the form given by (A2-80) - (A2-81), with β_f exogenously given. See the Electrical Circuit Model of Figure A2-7 into which the equations are applied.

$$V_{fk} = C_f \cdot \bar{E}_f \begin{bmatrix} \cos(\beta - \beta_f) \\ -\sin(\beta - \beta_f) \end{bmatrix} \begin{matrix} D \\ Q \end{matrix} \qquad \text{(A2-80)}$$

where;

$$C_f = C_{fq} = (\sqrt{2}/(\omega_o \cdot T'_{do} \cdot \varepsilon_f)) \cdot (X''_{ad}/X'_{ad}) \quad \text{and} \quad \bar{E}_f = \sqrt{2} \cdot E_{f\text{-}eff} \qquad \text{(A2-81)}$$

The e.m.f. V_{fk} contributing to the voltage source $\Delta E_{DQ} = (V_{fk} + H_{fk} \cdot \phi_{fk})$ of the electrical circuit model of the *adjustable speed* synchronous motor.

The ordinary synchronous machine, modelled on the basis of a 6-coil generalised machine: C_f applies as given by (A2-78). C_{fq} is not of interest, since the fq-coil is not excited. With $\beta_f = \Omega_f = f_{rotor(pu)} = 0$ in this 'dc-case', the equations (A2-77) to (A2-79) take on the form given by (A2-82) - (A2-83). Thus the formal description of V_{fk} reduces to that developed in Chapter 1.4 for the traditional synchronous motor, when the voltage peak $E_f = \sqrt{2} \cdot E_{f\text{-}eff}$ in (A2-83) is interpreted as the dc field voltage. See the synchronous motor model on p.1/31.

$$V_{fk} = C_f \cdot \bar{E}_f \begin{bmatrix} \cos\beta \\ -\sin\beta \end{bmatrix} \begin{matrix} D \\ Q \end{matrix} \qquad \text{(A2-82)}$$

where;

$$C_f = (\sqrt{2}/(\omega_o \cdot T'_{do})) \cdot (X''_{ad}/X'_{ad}) \quad \text{and} \quad \bar{E}_f = \sqrt{2} \cdot E_{f\text{-}eff} \qquad \text{(A2-83)}$$

The e.m.f. V_{fk} contributing to the voltage source $\Delta E_{DQ} = (V_{fk} + H_{fk} \cdot \phi_{fk})$ of the electrical circuit model of the *traditional* synchronous motor.

The ordinary synchronous machine, modelled on the basis of a 5-coil generalised machine : C_f applies as given by (A2-78). C_{fq} is not defined since no fq-coil is present. . With $\beta_f = \Omega_f = f_{rotor(pu)} = 0$ in this 'dc-case', the equations (A2-77) to (A2-79) take on the form given by (A2-82) - (A2-83). Thus the formal description of V_{fk} reduces to that developed in Chapter 1.4 for the tradititional synchronous motor, when the voltage peak $E_f = \sqrt{2} \cdot E_{f\text{-}eff}$ in (A2-83) is interpreted as the dc field voltage. See the synchronous motor model on p.1/31.

The ordinary asynchronous machine, modelled on the basis of a 6-coil generalised machine: Modellingwise, this implies using the full model of the 'extended' synchronous motor, while observing the following when specifying data input :

Symmetry in machine parameter setting, since the rotor winding is 3-phase on a round rotor.

Zero rotor voltage $E_{f\text{-}eff}$ (and thus elimination of field voltages (e_f, e_{fq}), as well as voltage contributions V_{fk} to the electrical circuit model), since the three phase field winding is short-circuited. Also zero value of the *frequency* f_f associated with the applied rotor voltage.

Setting of parameters so that power- and voltage control no longer is operative.

In terms of computertime this modelling of the asynchronous motor is less efficient, since the model retains the machine angle - being only a dummy variable in this case - as one of the state variables.

The extended synchronous motor model applied in example analyses

Introduction

As basis for analysis is applied the 'extended' synchronous motor model founded on the d-q diagram of the six-coil generalised machine. The d- and q-coils equivalence the three phase stator winding. The kd-and kq-coils describe the net effect of damping circuits. Two distinct coils, namely coil 'f' of the d-axis and coil 'fq' of the q-axis, equivalence the effect of the magnetizing circuitry. Depending first of all on the interpretation and settings for the latter two coils, different types of rotating machines may be dealt with. Illustrations:

If the model accounts for an *ordinary synchronous machine*, the f-coil is excited with dc, while the fq-coil is without excitation. Whether the fq-coil as such should be eliminated alltogether (as described on p. A2/9), may depend on considerations related to modelling precision and availability of data. In the example analyses to follow, the full 6-coil description is used throughout for modelling of the ordinary synchronous machine. In all of the examples the ordinary synchronous machine is applied as 'initial machinery' for establishing the desired initial power flow balance.

If the model accounts for an *adjustable speed synchronous machine*, the 'f'- and 'fq'-coils are interpreted as the axis-equivalent coils of a three phase rotor winding. The 'f' -and 'fq' -coils are excited by respective components of the Park-transformed three phase voltage of frequency f_f (=0.05x50=2.5Hz in our case study) applied to the field winding.

If the model accounts for an *ordinary asynchronous machine*, the 'f'- and 'fq'-coils are interpreted as the axis-equivalent coils of a three phase rotor winding. The 'f' -and 'fq' -coils are shortcircuited. In addition parameters are set so as to eliminate voltage- and power control.

Three example analyses are conducted in the following, - all having the same initial power balance :

First, the steady state operation of an *adjustable speed synchronous machine* is settled and described. It is based on suddenly (at t=0.3s) modellingwise changing interpretation and excitation of the rotating unit from being an ordinary synchronous generator running in steady state operation, to being an adjustable synchronous generator running at a non-synchronous speed (here at slip 5%), retaining power output as well as terminal voltage. The only disturbance involved is the one caused by replacing the dc field voltage by a 3-phase ac voltage of 2.5Hz (while assuming for convenience that pu machine reactances and time constants are the same before and after the 'switch').

Next, the up and running *adjustable speed synchronous generator* is exposed to a temporary three phase short circuit, to illustrate rotor- , voltage- as well as other dynamics associated with this type of machine. Both field voltage *magnitude*, *-phase* and *-frequency* are conceivable control variables for effectively contributing to retaining system stability as well as desired machine voltage. However, only the field voltage *magnitude* is used for this purpose in the current basic study. In fact, both 'initial' control systems are for simplicity reasons retained unaltered after the abrupt shift of machine type.

Finally, the ordinary synchronous generator operated in a steady state mode, is switched into being an *asynchronous machine in generator mode of operation.* System transients are pursued until new steady state conditions are reached for the asynchronous machine. The turbine torque is throughout retained constant, and there is no longer local control of the machine's bus voltage.

System data/ Initial load flow

The single-line diagram of the simple power system is shown in Figure A2-10. All pu data given are referred to common system basis. 'SM' stands at the outset for an *ordinary synchronous motor.*

External system : An infinite bus of per phase r.m.s. voltage $E_{y(ref)}$=1.0pu. Angle γ_{ref} arbitrarely set to zero, see p.A2/12.

Load flow constraints: As operational conditions for the system we specify the following:

Power supplied to the synchronous motor $P_{SM(target)}$ = - 0.8pu. I.e. generator mode of operation.

Synchronous motor voltage $E_{SM(target)} = 1.0pu$.

Series impedance $(r_y + j \cdot x_y)$: $r_y = 0.03pu$ $x_y = 0.1pu$

Transformer $(r_t + j \cdot x_t)$: $r_t = 0.01pu$ $x_t = 0.07pu$

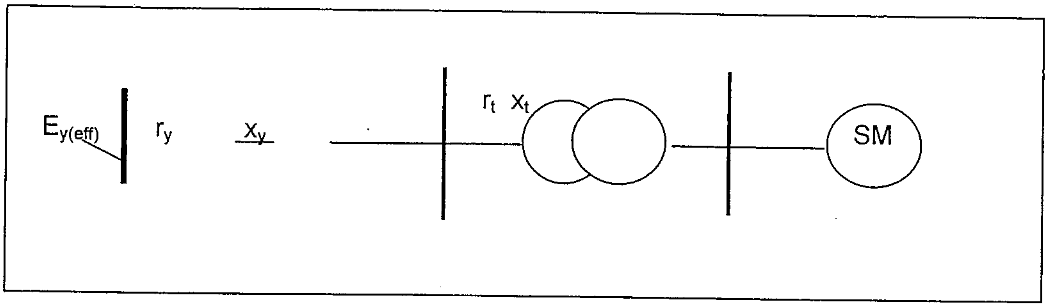


Figure A2-10 Example power system under study

The extended synchronous motor model (here in generator mode of operation) :

$X_{a\sigma} = 0.10pu$ $X'_d = 0.35pu$ $R_a = 0.008pu$ $T''_q = 0.04s$ $Cos\varphi_N = 0.9pu$
$X_d = 1.40pu$ $X''_d = 0.22pu$ $T'_{do} = 1.0s$ $T'_{qo} = 1.0s$ $T_a = 10s$
$X_q = 1.40pu$ $X''_q = 0.22pu$ $T''_d = 0.04s$ $X'_q = 0.35pu$ $C_D = 12pu$

Synchronous motor voltage control system:

$T_f = 0.1s$; field circuit time constant
$K_R = 40pu$; resulting forward amplification
$T_R = 0.1s$; regulator time constant
$K_D = 0.25pu$; transient feedback amplification
$T_D = 0.25s$; transient feedback time constant
$K_\Omega = 1.0pu$; power stabilizer amplification
$E_{qf(max)} = 3.0pu$; ceiling field voltage
$E_{qf(min)} = -2.0pu$; floor field voltage
$T_\Omega = 2s$; power stabilizer time constant

(See equations (1-106)- (1-121) for full parameter interpretation)

Synchronous motor power control system (when in hydro generator mode of operation):

$T_r = 0.3s$; Time constant for hydraulic system
$T_c = 0.5s$; Time constant for main servo (eg 0.08s)
$T_t = 17s$; Transient droop time constant
$\delta_t = 0.15pu$; Transient droop
$\delta_p = 0.00pu$; Permanent droop. (0-0.04) (The value 0.0 apply if the frequency is to be sustained by this unit alone)
$P_{target} = -0.8$; Target value of of absorbed motor power. (Applicable when loading up automatically, following synchronization)

With initial conditions specified as stated above, the iterative solution process described in section 2.4 of systems chapter 2, is called upon for targeting the given operating point to required accuracy. As arbitrary starting values for the present set of *'load flow control variables'* ($\beta_{SM(0)}$, $E_{f(0)}$) discussed in section 2.4, we choose $\beta_{SM(0)} = 0$ and $E_{f(0)} = 1.2pu$. Applying the three-step logic that comprises equations (2-36) to (2-39), and using the default value 1.0 of the factor k of (2-39), - we arrive at a feasible solution after 8 iterations. Exit from the iterative process is made when

$$Res < 0.0001pu \qquad \text{('Res' is abbreviation for 'Residual')} \qquad (A2\text{-}84)$$

where Res = ($ResV_{SM}$ + $ResP_{SM}$). The contributions in parenthesis are the deviations in absolute (pu) terms from target value, of respectively *synchronous motor voltage* and *power supplied to the synchronous motor.*

The iteratively determined *'load flow control variables'* contributing to giving a valid initial load flow, are :

$$\beta_{SM(0)} = -1.077808\text{rad.}$$
$$E_{f(0)} = 1.392082\text{pu} \qquad \text{(A2-85)}$$

Main characteristics of the established initial load flow for the system in Figure A2-10 are as follows:

The infinite bus: Voltage : 1.0000pu (specified)
Active power : - 0.7737pu (received from the local study system)
Reactive power : 0.2410pu (the study system acts reactively as an *inductuctor*)

Generator bus : Voltage : 1.0000pu (specified)

Active SM power : -0.8000pu (= specified power to motor)
Reactive SM power : 0.1293pu (the SM acts reactively as an inductive load)

Study 1:
The adjustable speed synchronous generator in steady state operation at 5% reduced rotor speed

Sample study results are shown in Figure A2-11 – A2-19. The period of analysis T_{max} is 10s. Time increment during integration; Δt=0.005s.

In the initial stage of operation, - i.e. for t less or equal to 0.3s, - an ordinary synchronous machine with a dc field voltage computed to 1.3921pu, contributes to sustaining the required initial power system balance described above.

From t=0 to t=0.3s, steady state conditions prevail and all state- as well as other variables remain at initial value.

At t=0.3s the *ordinary synchronous machine* is instantaneously switched into being an *adjustable speed synchronous machine*, desired to contribute to sustaining the same load flow balance as before, - but now at 5% reduced rotor speed. $\Omega_f = f_f/f_o = 0.05$ means impressed rotor frequency $f_f = 50 \cdot 0.05 = 2.5$Hz. For reasons just of convenience, all machine reactances and time constants are kept unaltered throughout the analysis. The following four model changes are introduced at t=0.3s, to adapt to the new version of the machine:

- *The Rotor Flux Model requires new excitation*: For the *ordinary* synchronous machine, the e.m.f.-vector exciting the flux model in initial/steady state operation, was as directed by equation (1-114);

$$e_{SMr(ordinary)} = \begin{bmatrix} K_f \cdot E_{fo} \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} f \\ fq \\ kd \\ kq \end{matrix} \qquad \text{(A2-86)}$$

  where the initial load flow analysis gave dc voltage $E_{fo} \approx 1.3921$pu. For the *adjustable speed* unit the corresponding e.m.f. vector is generally given by equations (A2-71)-(A2-72), while observing d-q axis symmetry. In the present case the equations may at the outset purposely be expressed as follows, when applying the more *user*oriented machine model terminology from the model summary description on p. A2-26 – A2-28;

$$e_{SMr(adjustable)} = \begin{bmatrix} K_f \cdot (\sqrt{2} \cdot E_{f\text{-}eff(o)} + \Delta E_f) \cdot \cos\beta_f \\ K_f \cdot (\sqrt{2} \cdot E_{f\text{-}eff(o)} + \Delta E_f) \cdot \cos\beta_f \\ 0 \\ 0 \end{bmatrix} \begin{matrix} f \\ fq \\ kd \\ kq \end{matrix} \qquad \text{(A2-87)}$$

  Here $\sqrt{2} \cdot E_{f\text{-}eff(o)}$ is the amplitude of an ac voltage of r.m.s. value $E_{f\text{-}eff(o)}$ applied to the 3-

phase field winding. It would seem reasonable to set pu starting value of this ac amplitude equal to the value of the dc field voltage E_{fo} ($\approx$ 1.3921pu), and then let the 'residual' voltage ΔE_f -which is provided by the voltage control system-, assure that target machine voltage is accurately retained. Applying the foregoing special premises to equation (A2-87), we get the following new e.m.f. vector to the Rotor Flux Model:

$$e_{SMr(adjustable)} = \begin{bmatrix} K_f\cdot(E_{f(o)} + \Delta E_f)\cdot\cos\beta_f \\ K_f\cdot(E_{f(o)} + \Delta E_f)\cdot\cos\beta_f \\ 0 \\ 0 \end{bmatrix} \begin{matrix} f \\ fq \\ kd \\ kq \end{matrix} \quad \text{where;} \quad \begin{matrix} E_{f(o)} = 1.3921\text{pu} \\ \beta_f = 0 \ \text{(by choice)} \\ \Delta E_f = \text{AVR-response} \end{matrix} \qquad \text{(A2-88)}$$

The e.m.f. e_{SMr} for $t \geq 0.3$s (ie. for the adjustable speed SM)

β_f is a potential control parameter which here arbitrarely is set to zero. See p.A2/13 for special comment on β_f.

- *The electrical circuit model requires new contribution* V_{SM} to its e.m.f. $\Delta \mathbf{E}_{SM} = (\mathbf{V}_{SM} + \mathbf{H}_{SM}\cdot\phi_{SM})$. For the ordinary synchronous machine the contribution $\mathbf{V}_{SM}$ in initial/steady state operation was, as directed by equation (1-109);

$$V_{SM(ordinary)} = \begin{bmatrix} C_f\cdot E_{f(o)}\cdot\cos\beta_{SM} \\ -C_f\cdot E_{f(o)}\cdot\sin\beta_{SM} \end{bmatrix} \begin{matrix} D \\ Q \end{matrix} \qquad \text{(A2-89)}$$

where β_{SM} is initial rotor angle of the ordinary synchronous machine, relative to the global synchronous reference phasor. For the adjustable speed unit the corresponding e.m.f. vector is generally given by equations (A2-80)-(A2-81), when d-q axis symmetry is pre-sumed._ In the present case the equations may suitably be expressed as follows, when setting $\bar{E}_{f'} = (\sqrt{2}\cdot E_{f\text{-}eff(o)} + \Delta E_f) = (E_{f(o)} + \Delta E_f)$;

$$V_{SM(adjustable)} = \begin{bmatrix} C_f\cdot\bar{E}_{f'}\cos(\beta_{SM} - \beta_f) \\ -C_f\cdot\bar{E}_{f'}\sin(\beta_{SM} - \beta_f) \end{bmatrix} \quad \text{where;} \quad \begin{matrix} \bar{E}_{f'} = (E_{f(o)} + \Delta E_f) \\ E_{f(o)} = 1.3921\text{pu} \\ \beta_f = 0 \ \text{(by choice)} \end{matrix} \qquad \text{(A2-90)}$$

The e.m.f. V_{SM} for $t \geq 0.3$s (ie. for the adjustable speed SM)

- *The machines electrical angle β_{SM} is governed by a new differential equation*: For the ordinary synchronous machine, equation (1-119) was the valid one;

$$d\beta_{SM}/dt = \omega_o\cdot(1 - \Omega_{SM}) \qquad \text{(A2-91)}$$

For the adjustable speed machine, the variation of β_{SM} is governed by (A2-44);

$$d\beta_{SM}/dt = \omega_o\cdot(1 - \Omega_f - \Omega_{SM}) \qquad \text{(A2-92)}$$

where;

Ω_{SM} = pu rotor speed

$\Omega_f = f_f/f_o = f_f/50$ = pu speed of field winding m.m.f. relative to rotor. Positive value of Ω_f implies reduced rotor speed

Equation governing machine angle β_{SM} for $t \geq 0.3$s .

- *The power control system's reference signal $\Delta\Omega_{ref}$ must change* : Relative to the presumed still valid model description on p. 1/29, the reference signal applied for $t \geq 0.3$s is as follows;

$$\Delta\Omega_{ref} = c\cdot(P_{target} - P_{SM}) + \Omega_f \qquad \text{(A2-93)}$$

A few comments to the results follow next. For interpretation of diagram variables, see p.4/2-3.

Figure A2-12 shows r.m.s. machine bus voltage which appears to recover in about 3.5s, following the 'shift' in machine type. The residual voltage ΔE_f approaches ≈13.9pu as the bus voltage closes in on target value 1.0pu. (ΔE_f -which is a state variable -is not shown explicitly in the diagram). The high ΔE_f -value illustrates the 'trouble' caused by inductances when fieldwise replacing dc with ac.

Figure A2-15 shows rotor slip relative to synchronous speed : The slip changes from its initial value (0.0) to its new, desired value (5%) in less than 6s.

Figures A2-14 and A2-17 relate to the three phase field winding; the first shows the actual 2.5Hz current of the winding, while the second depicts the current i_{fq} that reside in the equivalent q-axis coil ('fq') of the field winding.

Figures A2-19 and A2-20 show the current of damper coil 'kd' , respectively damper coil 'kq'. The transients incurred by the simulated shift from synchronous to subsynchronous rotor speed, impacts in the present case relatively little on the damping circuitry ; the currents are zero up to t= 0.3s, and settle quickly to zero again, after some initial, diminutive ripple following the 'switch' in field excitation ; from dc voltage fed to coil 'f' of the motor model, to three phase ac voltage parktransformed and fed to coils 'f' and 'fq' of the model.

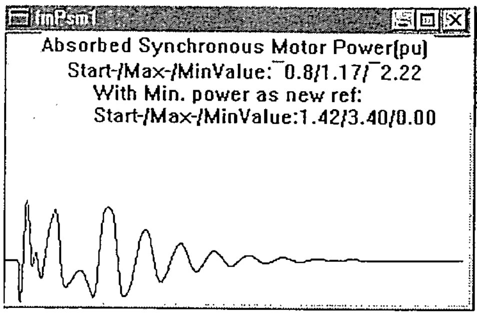


Figure A2-11 Absorbed SM power

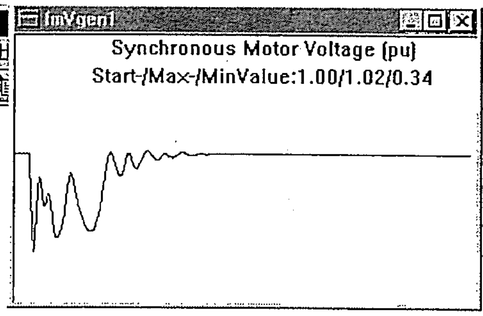


Figure A2-12 SM bus voltage

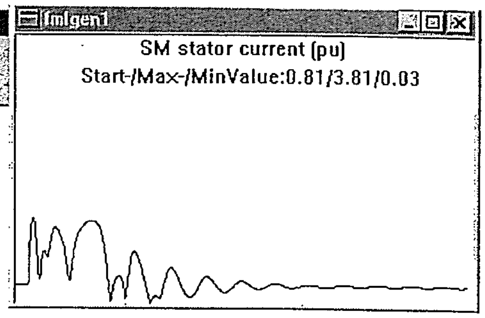


Figure A2-13 SM stator current

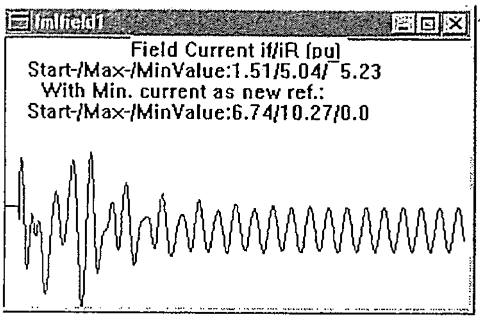


Figure A2-14 Field current-phase R

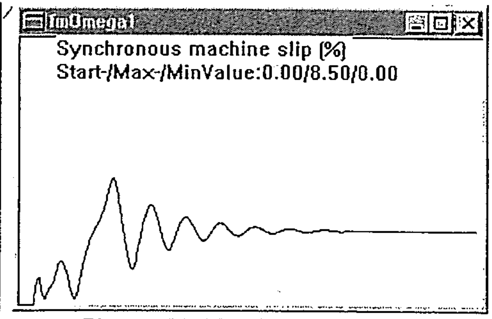


Figure A2-15 SM rotor slip

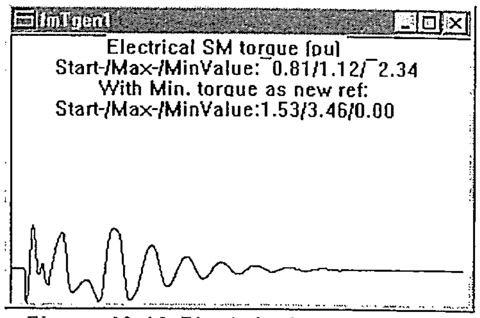


Figure A2-16 Electrical motor torque

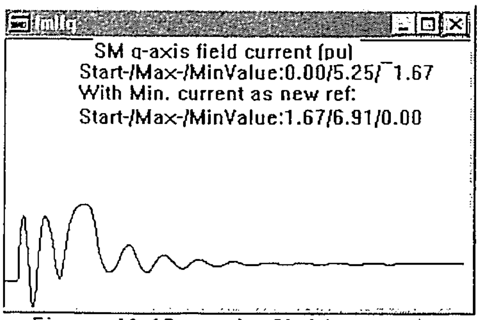


Figure A2-17 q-axis field current

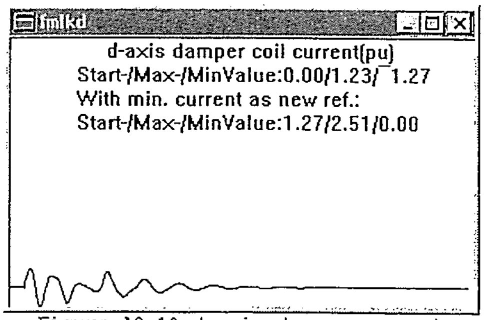


Figure A2-18 d-axis damper current

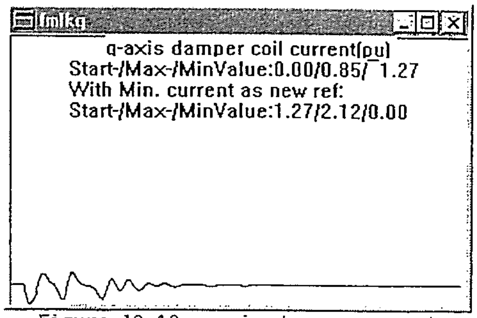


Figure A2-19 q-axis damper current

Figure A2-11 to A2-19 **The adjustable speed synchronous generator** at 5% reduced rotor speed
An *ordinary* synchronous motor set to absorb –0.8pu power at machine bus voltage 1.0pu, is suddenly, - at t=0.3s, - redefined to be an *adjustable speed* unit, retaining power output as well as machine bus voltage. Period of analysis: 10s. Integration time step: 0.005s.

Study 2 :

The adjustable speed synchronous generator exposed to a temporary short circuit

Sample study results are shown in Figure A2-20 - A2-28. Period of analysis: 10s. Time increment during integration: 0.005s.

The 'early history' of the process is as in the previous study: For t < 0.3s steady state conditions prevail for an ordinary synchronous motor working in generator mode of operation. Throughout the study the unit's control gear is set to keep bus voltage at 1.0pu and absorbed motor power = -0.8pu. At t=0.3s the machine is 'switched' into an adjustable speed synchronous motor, and before another (say) 3.5s has passed, the 'new' adjustable speed unit is in steady state operation at 5% reduced rotor speed, see Figure A2-24.

At t = 6.0s the adjustable speed synchronous machine is exposed to a three phase short circuit at its terminals. Fault duration: 0.25s. For t > 6.25s, normal network conditions again prevail.

It is observed from the figures below that normal operation at 5% reduced rotor speed, is restored in the course of about 3.5s after removing the short circuit. The voltage control system is the same as for the 'old'/ordinary synchronous generator. Additional local variables (such as *rotor frequency* and *rotor phase shift*) could potentially be utilized to enhance the restoring of power system control.

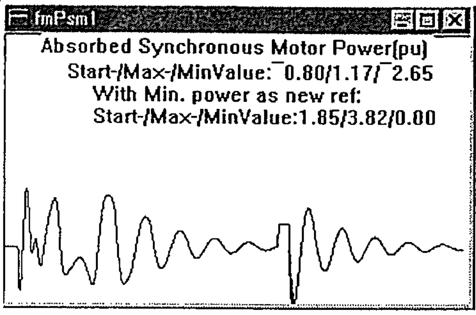


Figure A2-20 Absorbed SM power

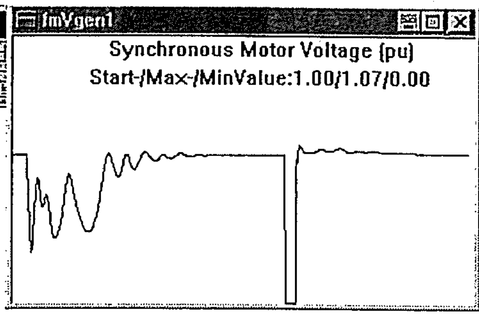


Figure A2-21 SM bus voltage

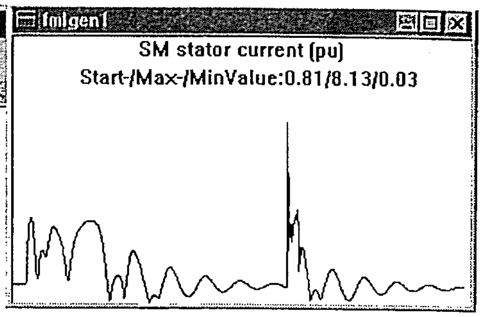


Figure A2-22 SM stator current

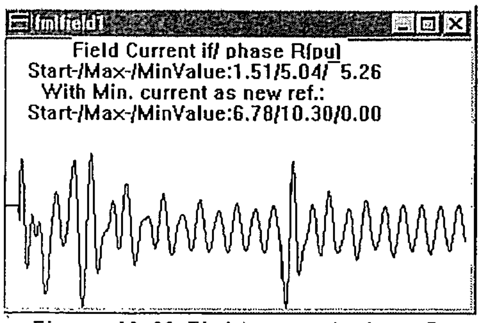


Figure A2-23 Field current-phase R

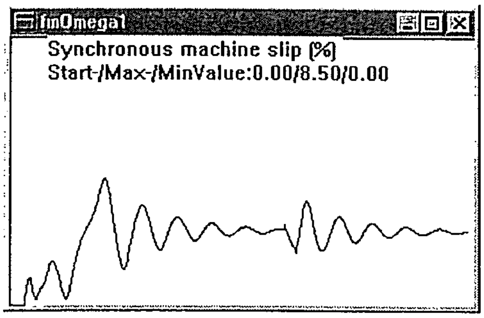


Figure A2-24 SM rotor slip

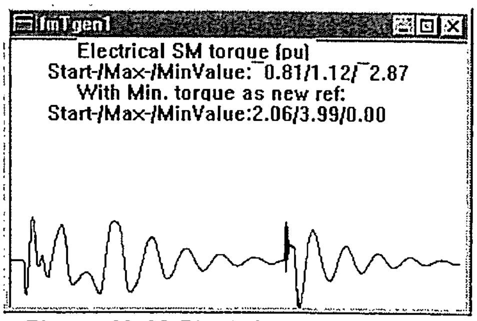


Figure A2-25 Electrical motor torque

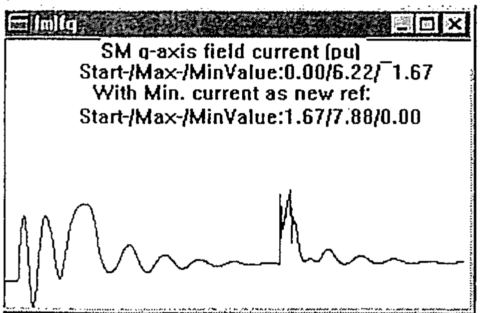


Figure A2-26 q-axis field current

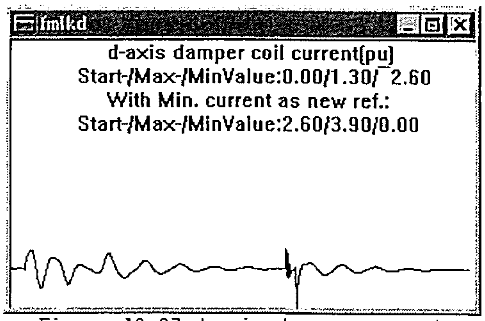


Figure A2-27 d-axis damper current

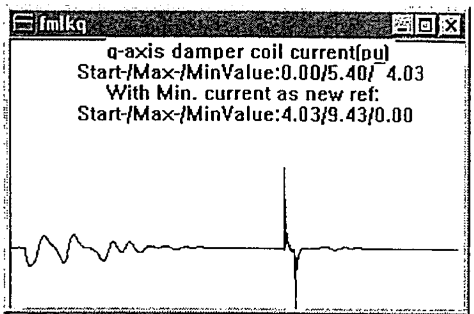


Figure A2-28 q-axis damper current

Figure A2-20 to A2-28 **The adjustable speed synchronous generator exposed to 3-phase fault**
The motor operated at 95% of synchronous rotor speed, is set to absorb –0.8 pu power at machine bus voltage 1.0pu. A three phase short circuit applied at t=6s.Fault duration:0.25s. Period of analysis:10s. Integration time step:0.005s

Study 3 :

Switch from *ordinary synchronous* generator operation to *asynchronous* generator operation

Sample study results are shown in Figure A2-29 – A2-37. Period of analysis:10s. Time increment during integration: 0.005s. Scope of analysis: To ilustrate adaptability of component modelling concept.-

At the outset the stated synchronous machine is modelled using the *extended* synchronous motor model, - which is based on the *6*-coil generalised machine. See p. A2/17 for comments on the limited, but special parameter settings required, to adapt the extended model also for detailed *asynchronous* machine simulations.

The 'early history' of the process is identical to that of the previous two studies: For t<0.3s steady state conditions prevail for an ordinary synchronous motor working in generator mode of operation; absorbed motor power is –0.8pu, and the machine is keeping the bus voltage at 1.0pu.

At t=0.3 the *synchronous* generator is instantly 'switched' into being an *asynchronous* generator by resetting parameters as referred to above. Initial shaft torque corresponding to absorbed motor power of –0.8pu, is retained throughout the analysis. It appears that new, asynchronous generator operation is established in about 4s after the 'machine shift' is initiated. Other observations to notice:

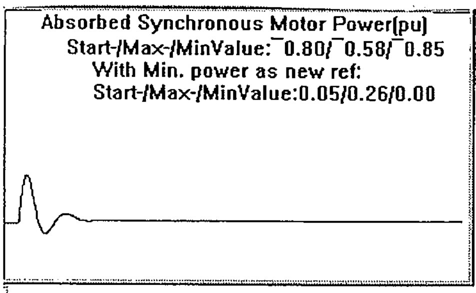


Figure A2-29 Absorbed AM power

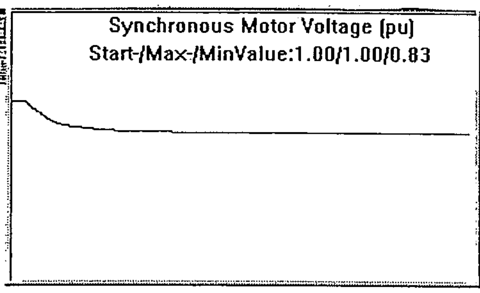


Figure A2-30 AM bus voltage

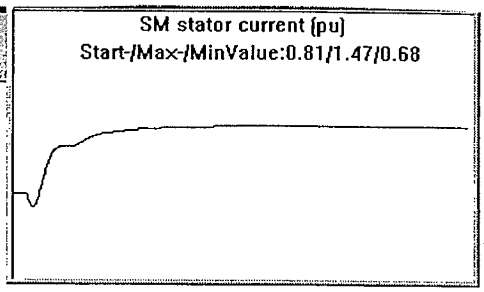


Figure A2-31 AM stator current

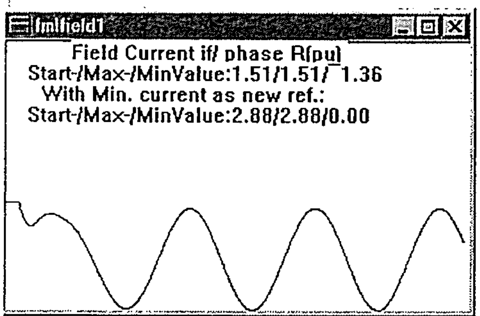


Figure A2-32 Field current-phase 'R'

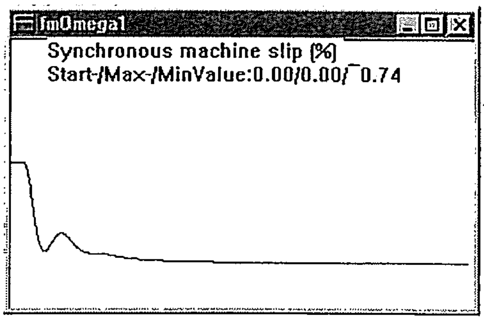


Figure A2-33 AM rotor slip

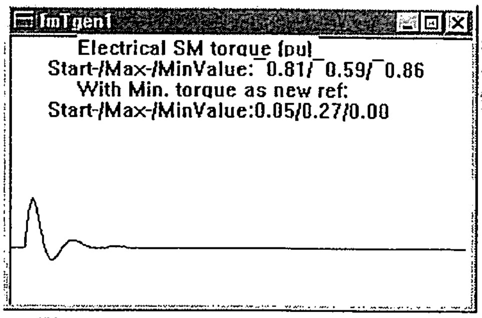


Figure A2-34 Electrical AM torque

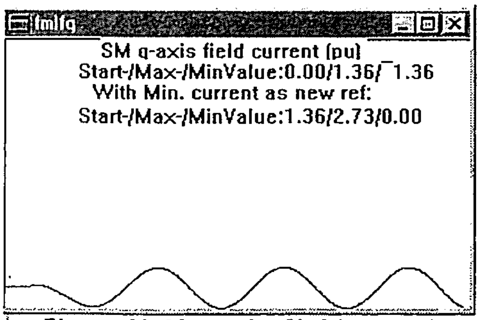


Figure A2-35 q-axis field current

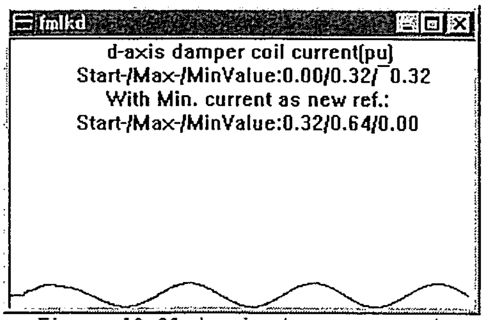


Figure A2-36 d-axis damper current

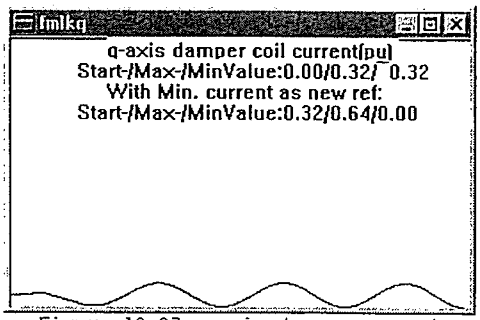


Figure A2-37 q-axis damper current

Figure A2-29 to A2-37 **Switch from *synchronous* generator- to *asynchronous* generator operation**
An ordinary *synchronous* motor (described by the *extended* SM model) set to absorb –0.8pu power at 1.0pu voltage, is suddenly, - at t=0.3s, - redefined to to be an *asynchronous* motor, retaining the torque implied by $P_{absorbed}$= -0.8pu. Period of analysis:10s. Integration time step: 0.005s.

Since local voltage control now is absent, and the asynchronous machine reactively acts as an inductive load, machine bus voltage drops from the initial 1.0pu setting, to a level of 0.83pu. See Figure A2-30 for qualitative confirmation.

Rotor slip reduces from 0.0 at initial synchronous operating state, to its new value of - 0.74272% to sustain the constant generator torque implied by $P_{absorbed}$= -0.8pu in the initial load flow balance. See Figure A2-33.

Figure A2-32 and A2-35 show that an ac current of low frequency reside in the rotor field winding. Figure A2-36 and A2-37 show that a similar, but smaller current reside in the damper coils. Some reflections and observations related to these appearances, follow:

For the synchronous- as well as the asynchronous machine it is a prerequisite for useful power transfer over the machine airgap, that the rotating m.m.f. set up by the stator winding, respectively the rotor winding, are synchronous.

For the *adjustable speed synchronous machine* an applied (d-q axis-attached) 3-phase field voltage of given frequency f_f, provided a rotor m.m.f. that rotated at pu angular speed $\Omega_f = f_f/f_o = f_f/50$ vis-a-vis rotor. With pu rotor speed Ω, the following equation was then to be fulfilled in order to secure synchronism between the two stated fields: $(\Omega + \Omega_f) = \Omega_o = 1$. See presentation on page A2/11. With specified frequency f_f, the rotor of the *adjustable speed synchronous machine* then attains the speed directed by the just given equation.

For the *asynchronous* machine, rotor currents are not produced from an impressed/external voltage source, but from electromagnetic coupling to the stator winding. The frequency of the currents thus induced in the field winding, is such that rotor angular speed plus the angular speed of the m.m.f. produced by the rotor currents relative to rotor, add up to synchronous speed. In our example we register that $\Omega_f = f_f/f_o = f_f/50 = -0.0074272$. I.e. $f_f = -0.0074272 \cdot 50 \approx -0.37$Hz. This is the settled frequency that can be observed from Figure A2-32. (Ill.: From the diagrams we measure that a simulation period of 10s is 'covered' by a paper length of ≈6.15 cm. From Figure A2-32 we observe that the 'last' period of the current trace measure ≈ 1.65 cm. From these registrations we get the estimate $f_f = 6.15/(1.65 \times 10) \approx 0.373$Hz). *Generator* operation means supersynchronous rotor speed which implies a negative f_f in order to comply with the above binding condition $(\Omega + \Omega_f) = 1$. What is qualitatively commented on above for the field circuitry also holds true for the damper circuits: The machine's data input implies special but equal parameter setting for the two symmetrical damper circuits. This governs the special damper circuitry response which will be characterized by a size-wise similarity of the two damper currents.

The 'Extended' Synchronous Motor

(based on the d-q diagram of a 6-coil generalised machine)

Synchronous motor/generator behaviour is described in terms of 8 state variables :

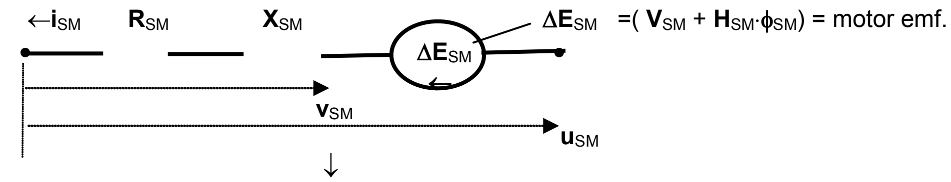
- 2 stator current components $\mathbf{i}_{SM} = [i_{SM(d)}\ \ i_{SM(q)}]^t$ (where 't' stands for 'transpose ')
- 4 rotor flux components $\phi_{SM} = [\phi_f\ \ \phi_{fq}\ \ \phi_{kd}\ \ \phi_{kq}]^t$ (fieldw.'f' fieldw.'q damperw. 'kd' damperw. 'kq')
- 1 speed variable $\Omega_{SM} = \omega_{SM}/\omega_o$
- 1 rotor angle variable β_{SM}

To handle *voltage control* 4 state variables are introduced. *Power control* in (hydro) generator mode of operation is afforded via 3 additional state variables. Altogether (8+7) =15 state variables to model generator mode of operation:

Synchronous Motor parameters to be specified (with example adjustable speed machine data in parenthesis) :

$X_{a\sigma}$ (0.1pu)	X'_d (0.35pu)	R_a (0.008pu)	T''_q (0.04s)	$\cos\varphi_N$ (0.9pu)
X_d (1.4pu)	X''_d (0.22pu)	T'_{do} (1.0s))	T'_{qo} (1.0s))	T_a (10.0s)
X_q (1.4pu)	X''_q (0.22pu)	X'_q (0.35pu)	T''_d (0.04s)	C_D (12pu)

Electrical Circuit Model



$$\underset{(2x1)}{\mathbf{v}_{SM}} = \mathbf{R}_{SM}\cdot\mathbf{i}_{SM} + (1/\omega_o)\cdot\mathbf{X}_{SM}\cdot d\mathbf{i}_{SM}/dt \quad \& \quad \mathbf{v}_{SM} - \Delta\mathbf{E}_{SM} = \mathbf{u}_{SM} \qquad \text{(A2-35)}$$

Electrical circuit model of the extended synchronous motor in d-q axis frame of reference

$\Delta\mathbf{E}_{SM} = (\mathbf{V}_{SM} + \mathbf{H}_{SM}\cdot\phi_{SM})$ = synchronous motor emf.

$$\mathbf{R}_{SM} = \begin{bmatrix} (R_a+\hat{X}''_r) + (\Omega_{SMref}+2\cdot\Delta\Omega_{SM})\cdot\bar{X}''\cdot\sin2\beta_{SM} + \bar{X}''_r\cdot\cos2\beta_{SM} & -\Omega_{SMref}\cdot\hat{X}'' + (\Omega_{SMref}+2\cdot\Delta\Omega_{SM})\cdot\bar{X}''\cdot\cos2\beta_{SM} - \bar{X}''_r\cdot\sin2\beta_{SM} \\ \Omega_{SMref}\cdot\hat{X}'' + (\Omega_{SMref}+2\cdot\Delta\Omega_{SM})\cdot\bar{X}''\cdot\cos2\beta_{SM} - \bar{X}''_r\cdot\sin2\beta_{SM} & (R_a+\hat{X}''_r) - (\Omega_{SMref}+2\cdot\Delta\Omega_{SM})\cdot\bar{X}''\cdot\sin2\beta_{SM} - \bar{X}''_r\cdot\cos2\beta_{SM} \end{bmatrix}$$

$\Omega_{SMref} = (1-\Omega_f)$, where Ω_f = pu angular speed of rotor mmf relative to rotor. See p.A2/11

$\Delta\Omega_{SM} = (\Omega_{SM} - \Omega_{SMref})$, where Ω_{SM} =pu rotor speed

$$\mathbf{X}_{SM} = \begin{bmatrix} \hat{X}''+\bar{X}''\cdot\cos2\beta_{SM} & -\bar{X}''\cdot\sin2\beta_{SM} \\ -\bar{X}''\cdot\sin2\beta_{SM} & \hat{X}''-\bar{X}''\cdot\cos2\beta_{SM} \end{bmatrix} \qquad \mathbf{V}_{SM} = \begin{bmatrix} C_f\cdot\bar{E}_f\cdot\cos(\beta_{SM}-\beta_f) \\ -C_f\cdot\bar{E}_f\cdot\sin(\beta_{SM}-\beta_f) \end{bmatrix}$$

$\bar{E}_f = (\sqrt{2}\cdot E_{f\text{-}eff} + \Delta E_f)$ = peak field excitation.

ΔE_f = voltage regulator response. See p.1/29.

$C_f = (\sqrt{2}/(\omega_o\cdot T'_{do}\cdot\varepsilon_f))\cdot(X''_{ad}/X'_{ad})$. See p.A2/8.

β_{SM} = synchronous motor angle, see Fig.A2-3.

β_f = phase shift of magnetizing ac voltage. See Figure A2-3.

$$\mathbf{H}_{SM} = \begin{array}{c} \\ D \\ Q \end{array}\begin{bmatrix} f & fq & kd & kq \\ (\Omega_{SM}\cdot f_1 - f_{pu}\cdot f_7)\cdot\sin\beta_{SM}+f_2\cdot\cos\beta_{SM} & (\Omega_{SM}\cdot f_7 - f_{pu}\cdot f_1)\cdot\cos\beta_{SM}+f_8\cdot\sin\beta_{SM} & \Omega_{SM}\cdot f_3\cdot\sin\beta_{SM}+f_4\cdot\cos\beta_{SM} & \Omega_{SM}\cdot f_5\cdot\cos\beta_{SM}+f_6\cdot\sin\beta_{SM} \\ (\Omega_{SM}\cdot f_1 - f_{pu}\cdot f_7)\cdot\cos\beta_{SM}-f_2\cdot\sin\beta_{SM} & -(\Omega_{SM}\cdot f_7 - f_{pu}\cdot f_1)\cdot\sin\beta_{SM}+f_8\cdot\cos\beta_{SM} & \Omega_{SM}\cdot f_3\cdot\cos\beta_{SM}-f_4\cdot\sin\beta_{SM} & -\Omega_{SM}\cdot f_5\cdot\sin\beta_{SM}+f_6\cdot\cos\beta_{SM} \end{bmatrix}$$

$\hat{X}'' = 0.5(X''_d + X''_q)$ $\quad \hat{X}''_r = 0.5(X''_{rd} + X''_{rq})$ $\quad \leftarrow \quad X''_{rd} = (1/(\omega_o\cdot T'_{do}))\cdot(X''_{ad}/X'_{ad})^2\cdot(X_d - X'_d) + (1/(\omega_o\cdot T''_{do}))\cdot(X'_d - X''_d)$

$\bar{X}'' = 0.5(X''_d - X''_q)$ $\quad \bar{X}''_r = 0.5(X''_{rd} - X''_{rq})$ $\quad \leftarrow \quad X''_{rq} = (1/(\omega_o\cdot T'_{qo}))\cdot(X''_{aq}/X'_{aq})^2\cdot(X_q - X'_q) + (1/(\omega_o\cdot T''_{qo}))\cdot(X'_q - X''_q)$

f_{pu} = shortened notation for $f_{rotor(pu)}$ = pu frequency of 3-phase voltage applied to field winding. Not subject to sign shift.

$f_1 = (X_d - X'_d)\cdot(X''_{ad}/(X_{ad}\cdot X'_{ad}))$ $\qquad \leftarrow \quad X_{ad} = X_d - X_{a\sigma}$

$f_2 = f_1\cdot[(X'_d - X''_d)\cdot[(1/(\omega_o\cdot T''_{do}))\cdot(1/X''_{ad}) - (1/(\omega_o\cdot T'_{do}))\cdot(X_{ad}/X'^2_{ad})] - (1/(\omega_o\cdot T'_{do}))\cdot(X''_{ad}/X'_{ad})]$ $\qquad \leftarrow \quad X'_{ad} = X'_d - X_{a\sigma}$

$f_3 = (X'_d - X''_d)/X'_{ad}$ $\qquad \leftarrow \quad X''_{ad} = X''_d - X_{a\sigma}$

$f_4 = f_3\cdot[(1/(\omega_o\cdot T'_{do}))\cdot(X''_{ad}/X'^2_{ad})\cdot(X_d - X'_d) - (1/(\omega_o\cdot T''_{do})]$

$f_5 = -(X'_q - X''_q)/X'_{aq}$ $\qquad \leftarrow \quad X_{aq} = X_q - X_{a\sigma}$

$f_6 = -f_5\cdot[(1/(\omega_o\cdot T'_{qo}))\cdot(X''_{aq}/X'^2_{aq})\cdot(X_q - X'_q) - (1/(\omega_o\cdot T''_{qo})]$ $\qquad \leftarrow \quad X'_{aq} = X'_q - X_{a\sigma}$

$f_7 = -(X_q - X'_q)\cdot(X''_{aq}/(X_{aq}\cdot X'_{aq}))$ $\qquad \leftarrow \quad X''_{aq} = X''_q - X_{a\sigma}$

$f_8 = -f_7\cdot[(X'_q - X''_q)\cdot[(1/(\omega_o\cdot T''_{qo}))\cdot(1/X''_{aq}) - (1/(\omega_o\cdot T'_{qo}))\cdot(X_{aq}/X'^2_{aq})] - (1/(\omega_o\cdot T'_{qo}))\cdot(X''_{aq}/X'_{aq})]$

The 'Extended' Synchronous Motor, cont…

Rotor Flux Model

$$\underset{(4x1)}{d\phi_{SM}}/dt = \omega_0 \cdot (\mathbf{e_{SMr}} + \mathbf{F}_{SMi} \cdot \mathbf{i}_{SM} + \mathbf{F}_{SM\phi} \cdot \phi_{SM}) \qquad \text{(A2-94)}$$

where:

$\mathbf{e}_{SMr}$ =

$K_f \cdot E_f$	f
$K_{fq} \cdot E_{fq}$	fq
0	kd
0	kq

$E_f = (\sqrt{2} \cdot E_{f\text{-eff}(o)} + \Delta E_f) \cdot \cos\beta_f$ =voltage of field coil 'f'. See page A2/14 - 15

$E_{fq} = (\sqrt{2} \cdot E_{f\text{-eff}(o)} + \Delta E_f) \cdot \sin\beta_f$ = voltage of field coil 'fq'. See page A2/14 -15

$K_f = [(\sqrt{2}/(\omega_o \cdot T'_{do} \cdot \varepsilon_f)) \cdot X_{ad} / (X_d - X'_d)]$

$K_{fq} = [(\sqrt{2}/(\omega_o \cdot T'_{qo} \cdot \varepsilon_{fq})) \cdot X_{aq} / (X_q - X'_q)]$

$(\varepsilon_f, \varepsilon_{fq})$ = factors =1, unless adjusted speed SM.

For the adjustable speed SM (ie. the symmetrical machine): $K_f = K_{fq}$ & $\varepsilon_f = \varepsilon_{fq}$, see p. A2/15.

ΔE_f = voltage control response. (Voltage *phase* not a control variable here).

$X_{ad} = X_d - X_{a\sigma}$
$X'_{ad} = X'_d - X_{a\sigma}$
$X''_{ad} = X''_d - X_{a\sigma}$
$X_{aq} = X_q - X_{a\sigma}$
$X'_{aq} = X'_q - X_{a\sigma}$
$X''_{aq} = X''_q - X_{a\sigma}$

$\mathbf{F}_{SMi}$ =

D	Q	
$(1/(\omega_o \cdot T'_{do})) \cdot (X_{ad}/X'_{ad}) \cdot X''_{ad} \cdot \cos\beta$	$-(1/(\omega_o \cdot T'_{do})) \cdot (X_{ad}/X'_{ad}) \cdot X''_{ad} \cdot \sin\beta$	f
$(1/(\omega_o \cdot T'_{qo})) \cdot (X_{aq}/X'_{aq}) \cdot X''_{aq} \cdot \sin\beta$	$(1/(\omega_o \cdot T'_{qo})) \cdot (X_{aq}/X'_{aq}) \cdot X''_{aq} \cdot \cos\beta$	fq
$(1/(\omega_o \cdot T''_{do})) \cdot X'_{ad} \cdot \cos\beta$	$-(1/(\omega_o \cdot T''_{do})) \cdot X'_{ad} \cdot \sin\beta$	kd
$(1/(\omega_o \cdot T''_{qo})) \cdot X'_{aq} \cdot \sin\beta$	$(1/(\omega_o \cdot T''_{qo})) \cdot X'_{aq} \cdot \cos\beta$	kq

$\mathbf{F}_{SM\phi} = (1/\omega_o) \cdot$

f	fq	kd	kq
$\mathbf{F}_{fk\phi}(f,f)$	$\omega_0 \cdot f_{rotor(pu)}$	$(X_{ad}/X'^2_{ad}) \cdot (X'_d - X''_d)/T'_{do}$	
$-\omega_0 \cdot f_{rotor(pu)}$	$\mathbf{F}_{fk\phi}(fq,fq)$		$(X_{aq}/X'^2_{aq}) \cdot (X'_q - X''_q)/T'_{qo}$
$(1/T''_{do}) \cdot (1/X_{ad}) \cdot (X_d - X'_d)$		$-1/T''_{do}$	
	$(1/T''_{qo}) \cdot (1/X_{aq}) \cdot (X_q - X'_q)$		$-1/T''_{qo}$

$\mathbf{F}_{fk\phi}(f,f) = -(1/(T'_{do} \cdot X'_{ad})) \cdot [(X_{ad}/X'_{ad}) \cdot (X'_d - X''_d) + X''_{ad}]$

$\mathbf{F}_{fk\phi}(fq,fq) = -(1/(T'_{qo} \cdot X'_{aq})) \cdot [(X_{aq}/X'_{aq}) \cdot (X'_q - X''_q) + X''_{aq}]$

β = angular displacement of the local machine reference axes relative to the global axes

β_f = specified phase shift (relative to local axes) of applied three phase field voltage.

$f_{rotor(pu)}$ = pu frequency of applied 3-phase rotor voltage. (Base frequency: 50Hz. Not subject to sign change)

X_d, X'_d, X''_d : direct-axis synchronous, transient and subtransient reactance (pu)
X_q, X'_q, X''_q : quadrature-axis synchronous, transient and subtransient reactance (pu)
$X_{a\sigma}$: stator leakage reactance (pu)
T'_{do}, T''_{do} : direct axis open stator transient and subtransient time constant (s)
T'_{qo}, T''_{qo} : quadrature axis open stator transient and subtransient time constant (s)

Model application alternatives:

- If adjustable speed SM: Symmetrical machine; $X_d = X_q$, $X'_d = X'_q$, $X''_d = X''_q$, $T'_{do} = T'_{qo}$, $T''_{do} = T''_{qo}$, $K_f = K_{fq}$. β_f to be set.
- If 'traditional' SM : Individual parameter setting. $\beta_f = 0$. $f_f = 0$ (i.e. dc to the field circuit)
- If 'traditional' AM : Symmetrical machine. No field voltage excitation : $E_f = E_{fq} = 0$. $f_f = 0$. No P&U-control.

At any time during integration the rotor currents may be derived from (A2-19) :

$$\mathbf{i}_{SMr} = (\mathbf{X}_{rr})^{-1} \cdot (\phi_{SM} - \mathbf{X}_{DQr} \cdot \mathbf{i}_{SM}) \qquad \text{(A2-95)}$$

where:

$\mathbf{X}_{rr}$ =

f	fq	kd	kq	
$X^2_{ad}/(X_d - X'_d)$		X_{ad}		f
	$X^2_{aq}(X_q - X'_q)$		X_{aq}	fq
X_{ad}		$X_{ad} + X'_{ad} \cdot X''_{ad}/(X'_d - X''_d)$		kd
	X_{aq}		$X_{aq} + X'_{aq} \cdot X''_{aq}/(X'_q - X''_q)$	kq

$\mathbf{X}_{DQr} = \mathbf{X}_{(fk)(dq)} \cdot \mathbf{T}$ =

D	Q	
$X_{ad} \cdot \cos\beta$	$-X_{ad} \cdot \sin\beta$	f
$X_{aq} \cdot \sin\beta$	$X_{aq} \cdot \cos\beta$	fq
$X_{ad} \cdot \cos\beta$	$-X_{ad} \cdot \sin\beta$	kd
$X_{aq} \cdot \sin\beta$	$X_{aq} \cdot \cos\beta$	kq

The 'Extended' Sybchronous Motor, cont...

Electromechanical Model

$$\frac{d\Omega_{SM}}{dt}_{(1x1)} = (S_{Bas}/S_{SM})\cdot(1/(T_a\cdot\cos\varphi_N))\cdot(T_{SMel} - T_{SMmec}) \qquad \text{(A2-96)}$$

Here:

T_{Smel} = $0.5\cdot i^t_{SM}\cdot T_{SM1}\cdot\phi_{dq}$ = electrical motor torque , - where $\phi_{dq} = X''_{SM}\cdot T_{SM}\cdot i_{SM} + f_{SM}\cdot\phi_{SM}$ (A2-97)

T_{SMmec} = $T_{SMmec(o)}\cdot\Omega_{SM}^{\kappa}$ = mechanical torque in **motor** mode of operation. (Motor operation implies pos. sign of mech. torque)

If the motor is up and running at t=-0: $T_{SMmec(o)} = T_{SMel(o)}$ = electrical motor torque at t = -0. This is found from equation (1-117) applied to the initial power system load flow. κ= (say) 1.5-3.5

If the motor is to be started from stillstand (as e.g. an asynchronous motor) : $T_{SMmec(o)}$ = coefficient to model mechanical friction, air resistance, etc. during startup. Probable range: 0.02-0.05

T_{SMmec} = $(T_{SMel(o)} + \Delta T_{mec})$ = mechanical torque in **generator** mode of operation. ΔT_{mec} is the response from the *power control system.* See below for a sample hydro generator power control system.

S_{Bas}, S_{SM} = Chosen VA system power base, and rated VA motor capacity, respectively

T_a, $\cos\varphi_N$ = Dynamical system's inertia constant, and motor's rated power factor, respectively

$$T_{SM1} = \begin{bmatrix} \sin\beta_{SM} & -\cos\beta_{SM} \\ \cos\beta_{SM} & \sin\beta_{SM} \end{bmatrix} \quad T_{SM} = \begin{bmatrix} \cos\beta_{SM} & -\sin\beta_{SM} \\ \sin\beta_{SM} & \cos\beta_{SM} \end{bmatrix} \quad X''_{SM} = \begin{bmatrix} X''_d & \\ & X''_q \end{bmatrix} \quad f_{SM} = \begin{bmatrix} f_1 & & f_3 & \\ & -f_7 & & -f_5 \end{bmatrix}$$

$f_1 = (X_d - X'_d)\cdot X''_{ad}/(X_{ad}\cdot X'_{ad})$
$f_3 = (X'_d - X''_d)/X'_{ad}$
$f_5 = -(X'_q - X''_q)/X'_{aq}$
$f_7 = -(X_q - X'_q)\cdot X''_{aq}/(X_{aq}\cdot X'_{aq})$
(A2-98)

The electrical angle of the rotor is defined as, see equation (A2-43);

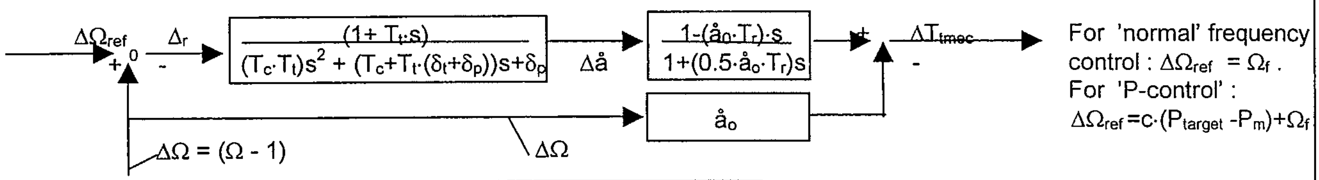
$$\beta_{SM} = (\omega_o\cdot t - (\theta_{SM} + \theta_f)$$

giving rise to the following differential equation describing the angular movement of the 'Extended' Synchronous Motor:

$$\frac{d\beta_{SM}}{dt}_{(1x1)} = \omega_o\cdot(1 - \Omega_f - \Omega_{SM})$$

where; Ω_{SM} = pu angular speed of rotor (A2-99)
$\Omega_f = f_f/f_o$ =pu angular speed of rotor m.m.f. relative to rotor. See p. A2/11.

Power Control System Model for generator mode of operation (here: *hydro* generator as illustration.)



For 'normal' frequency control : $\Delta\Omega_{ref} = \Omega_f$.
For 'P-control' :
$\Delta\Omega_{ref} = c\cdot(P_{target} - P_m) + \Omega_f$

$$d\Delta å/dt = K_1\cdot(\Delta\Omega_{ref} - (1 - \Omega_{SM}) + \Delta w) - K_2\cdot\Delta å \qquad \text{Regulator system}$$
$$d\Delta w/dt = K_3\cdot\Delta å - K_4\cdot\Delta w \qquad \text{Regulator system}$$
$$d\Delta g/dt = (3K_o/\Omega_{SM})\cdot\Delta å - K_o\cdot\Delta g \qquad \text{Hydraulic system} \qquad \text{(A2-100)}$$
$$\downarrow$$
$$\Delta T_{mec} = \Delta g - å_o\cdot(1 - \Omega_f - \Omega_{SM}) - (2/\Omega_{SM})\cdot\Delta å \qquad \text{Net change of mechanical torque}$$

$K_o = 2/(å_o\cdot T_r)$
$K_1 = 1/T_c$
$K_2 = (\delta_p + \delta_t)/T_c$ ←
$K_3 = \delta_t/T_t$
$K_4 = 1/T_t$
P_m = absorbed motor power $=0.5\cdot i_{SM}^t\cdot u_{SM}$
P_m is negative in generator mode.
P_{target} = target value of P_m
c = per unit scaling factor (eg.: c=0.1)

Control system parameters:

T_r : Time constant for hydraulic system (eg 0.3s)
T_c : Time constant for main servo (eg 0.08s)
T_t : Transient droop time constant (eg 17s)
δ_t : Transient droop (eg 0.15pu)
δ_p : Permanent droop (eg 0.0 – 0.03. The value 0.0 if frequency sustained by a single unit)
$å_o$: Initial pu turbine opening ($t_å$). (if $t_å$ <0.3 then $å_o$=0.3)

Voltage Control System Model → See chosen example system on p. 1/29

APPENDIX 3

The Park transformation P

APPENDIX 3
The Park transformation P

Formal basis for the ensuing development is again the d-q *diagram of a generalised model machine*. Figure A3-1 shows the main structure of the generalised machine that we apply, where the three phase stator winding is assumed to be the rotating part, while the d-q axes with associated windings related to field- and damper circuits, are considered fixed. The phasors 'R', 'S' and 'T' symbolize the m.m.f.-axes of respective phase windings.

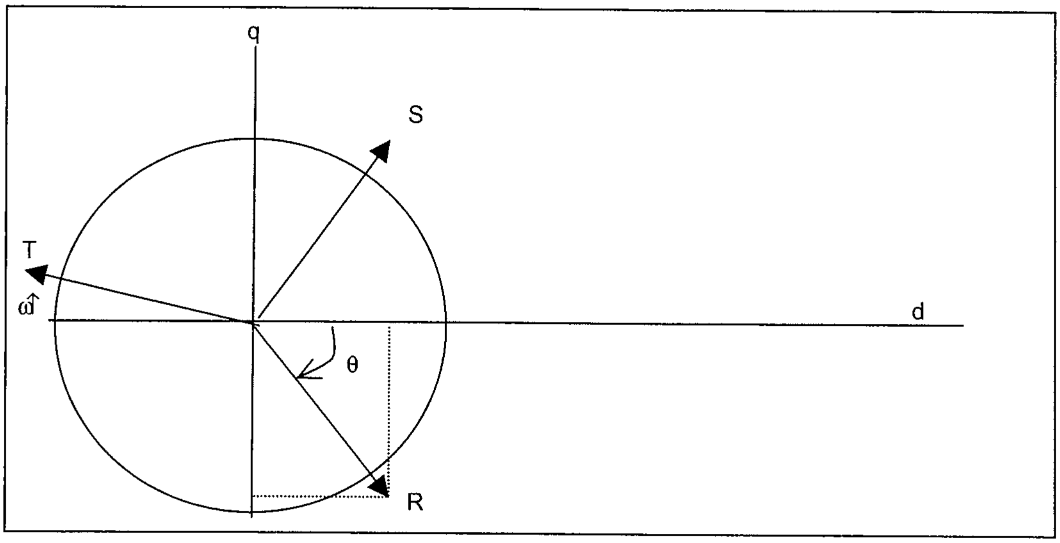


Figure A3.1 Main structure of *salient pole generalised machine.*

We wish to transform currents, fluxes and voltages associated with the rotating 3-phase winding to equivalent d- and q axis quantities that are fixed in space. By inspection of the diagram of Figure A3-1, we readily see that;

$$i_d = k\,(i_R \cos\theta + i_S \cos(\theta - 2\pi/3) + i_T \cos(\theta - 4\pi/3))$$
$$i_q = -\,k\,(i_R \sin\theta + i_S \sin(\theta - 2\pi/3) + i_T \sin(\theta - 4\pi/3)) \qquad \text{(A3-1)}$$

where k is a constant to be determined. To make the transformation reversible it is necessary to define a third variable that relates to the phase currents. To this end the zero sequence current i_o does suitably;

$$i_o = 1/3)\ (i_R + i_S + i_T) \qquad \text{(A3-2)}$$

Equations (A3-1) and (A3-2) are put together to produce the following tentative form of the Park transformation;

$$\begin{bmatrix} i_d \\ i_q \\ i_o \end{bmatrix} = k \begin{bmatrix} \cos\theta & \cos(\theta - 2\pi/3) & \cos(\theta - 4\pi/3) \\ -\sin\theta & -\sin(\theta - 2\pi/3) & -\sin(\theta - 4\pi/3) \\ 1/(3k) & 1/(3k) & 1/(3k) \end{bmatrix} \begin{bmatrix} i_R \\ i_S \\ i_T \end{bmatrix} \qquad \text{(A3-3)}$$

On compact form:

$$\mathbf{i}_{dqo} = \mathbf{P}\ \mathbf{i}_{RST} \qquad \text{(A3-4)}$$

Before turning to determining the coefficient k, it is practical to already be in possession of the inverse of (A3-3). We find that;

$$\begin{bmatrix} i_R \\ i_S \\ i_T \end{bmatrix} = (2/(3k)) \begin{bmatrix} \cos\theta & -\sin\theta & 3k/2 \\ \cos(\theta - 2\pi/3) & -\sin(\theta - 2\pi/3) & 3k/2 \\ \cos(\theta - 4\pi/3) & -\sin(\theta - 4\pi/3) & 3k/2 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_o \end{bmatrix} \qquad \text{(A3-5)}$$

or compactly;

$$\mathbf{i}_{RST} = \mathbf{P}^{-1} \cdot \mathbf{i}_{dqo} \qquad \text{(A3-6)}$$

Final task is determining k: We require that the voltages transform in the same way as the currents. It is also logical that pu power is 1 (or close to 1) when voltage and current of the main circuits are 1pu. Thus in the *phase frame of reference* pu power supplied to the motor is in principle to be expressed as;

$$p = (1/3)\ (e_R\, i_R + e_S\, i_S + e_T\, i_T) = (1/3)\ \mathbf{e}^t_{RST}\, \mathbf{i}_{RST} \qquad \text{(A3-7)}$$

$$= (1/3)\ (\mathbf{P}^{-1} \cdot \mathbf{e}_{dqo})^t \cdot (\mathbf{P}^{-1} \cdot \mathbf{i}_{dqo})$$

$$= \underline{(1/3)\ \mathbf{e}^t_{dqo}\ ((\mathbf{P}^{-1})^t \cdot \mathbf{P}^{-1})}\ \mathbf{i}_{dqo} \qquad \text{(A3-8)}$$

while the expression in the *d-q frame of reference* becomes;

$$p = (1/2)\ (e_d\, i_d + e_q\, i_q) + e_o\, i_o = \underline{(1/2)\ \mathbf{e}^t_{dq}\, \mathbf{i}_{dq} + e_o\, i_o} \qquad \text{(A3-9)}$$

These constraints that 'boil down' to the equations (A3-8) and (A3-9), set the value of k : Using the matrix expression for $\mathbf{P}^{-1}$ from (A3-5) into (A3-8), this latter equation takes on this form;

$$p = (2/(9k^2))\ \mathbf{e}^t_{dq}\, \mathbf{i}_{dq} + e_o\, i_o \qquad \text{(A3-10)}$$

which - according to the premises - should be the same as (A3-9). Equality requires that $½ = 2/(9k^2)$, from which is found that k=2/3.

Setting k =2/3 in (A3-3) and (A3-5), we finally arrive at the sought Park transformation and its inverse:

$$\mathbf{P} = (2/3) \begin{bmatrix} \cos\theta & \cos(\theta - 2\pi/3) & \cos(\theta - 4\pi/3) \\ -\sin\theta & -\sin(\theta - 2\pi/3) & -\sin(\theta - 4\pi/3) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \qquad \text{(A3-11)}$$

$$\mathbf{P}^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta & 1 \\ \cos(\theta - 2\pi/3) & -\sin(\theta - 2\pi/3) & 1 \\ \cos(\theta - 4\pi/3) & -\sin(\theta - 4\pi/3) & 1 \end{bmatrix} \qquad \text{(A3-5)}$$

The Park transformation **P** and its inverse, with present modelling premises .

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