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# CALCULATION OF PRESSURE OF CONCRETE ON FORMS 

R. Schjödt, ${ }^{*}$ M.ASCE

## SYNOPSIS

A formula for the calculation of pressure of concrete on forms is developed mathematically.

The setting time, consistency and weight of the concrete, the smoothness, permeability and cross-section of the forms, the rate of filling and the depth Which the working penetrates are the factors considered.

The material constants needed for the calculation are discussed, and safe values indicated.

The pressures obtained by using the formulas are compared with building practice and with tests.

## INTRODUCTION

The correct design of formwork is a necessary part of the design of any concrete structure, both from the point of view of safety and economy.

For a rational design, the size of the concrete pressure is needed. But reliable formulas for this pressure, taking into account all important factors, are not available, as far as the author knows.

Several investigators, such as Teller ${ }^{(1)}$ and Rodin, ${ }^{(2)}$ have made experimental determinations of the pressure. But in order to apply the results, formulas are needed.

Several empirical formulas have been proposed. They all have the usual weakness of such formulas, however, that they fit the tests and conditions from which they have been built up and not others.

Various authors such as Hoffmann ${ }^{(3)}$ and Ljungberg ${ }^{(4)}$ have developed theoretically formulas for the concrete pressure. Hoffmann's formula can be found in the above mentioned paper by Rodin. But they have used the exponential
mula for bin pressure as a starting point, as it was developed by Koenen ${ }^{(5)}$ and Janssen, ${ }^{(6)}$ without taking into account that an essential condition for the development of this formula is a constant friction angle, which is very far from being the case with setting concrete.

This paper will give a rational and simple formula for the pressure on formwork, taking into account what the author believes to be all important factors. The results are shown to check with the known tests, but differ frequently very much from the usually assumed values.

## I. Bases for the Calculation.

When concrete is placed in the forms, it is usually worked over, by spading, puddling, vibrating or other means, down to a certain depth. During this process the concrete is constantly moved and deformed, and must be considered

[^0]as a liquid. All experience supports this point of view, the pressure curve begins always as a tangent to the line of hydrostatic pressure.

As soon as the concrete is left alone, it develops a certain cohesion and friction.

As to the first, the slump test gives a good idea of its importance in fresh concrete. If the mixture can stand up with a vertical wall of say 8 in . corresponding to a slump of 4 in ., it has a cohesion (shear resistance) of $\tau=1 / 2$ $\mathrm{x} 0.087 \times 8=0.35 \mathrm{psi}$, if 0.087 is the weight of concrete in lb per cu. in.

Tests of the strength of fresh concrete give values of the same order of magnitude, according to Öfjord. ${ }^{(7)}$ Four to five hours later, the cohesion will be about twice the above amount.

The cohesion is not taken into account in the following. This provides an additional factor of safety, but is usually of little importance. Very dry and well vibrated concrete ( $\mathrm{w} / \mathrm{c}=0,35$ ), however, will stand with a vertical wall of 3 to 4 feet height, if the forms are taken away immediately after vibrating. Vacuum treated concrete will stand in the same way with up to 15 feet heig' In these cases, of course, the cohesion is important and the pressure will less than calculated.

A solution taking into account the cohesion is of theoretical interest, and may in some cases be of practical importance. This case is therefore considered in the appendix.

The angle of interior friction will have a relatively small value to begin with, but will increase during setting. It can be considered to have reached $90^{\circ}$ at the end of the setting time, when the horizontal pressure is equal to zero. For very dry concrete and very rigid forms, the latter may be under a certain pressure even after the final set, because the contraction of the concrete is not sufficient to free the forms. The remaining pressure may be considered a "passive pressure," a pressure of the forms on the concrete, and is of no importance for the following considerations.

The setting time, $\mathrm{t}_{\mathrm{s}}$, can be defined for our purpose as the time from the concrete is left at rest in the forms until it has taken the final set, and changes from a friction mass into a solid body.

The pore water pressure, finally, is a part of the total pressure against the forms, see for instance Terzaghi. (8) As we know, at least approximately, the amount of water and the voids in the mix, we can calculate the theoretical pore water stand. But from this must be subtracted the loss through the forms, which in many cases is difficult to estimate. However, in the two extreme cases of impermeable forms, and of thin walls with forms made with ordinary formboards, we know the pore water pressure with good approxin tion. Based on these, it is not difficult to arrive at safe values in the other cases.

The concepts developed above will serve as bases for the calculation. The actual values of the physical constants, and the safe values to be used in calculations, will be discussed later in this paper.
II. Pressure Without Friction Between Concrete and Forms.

In this case, the horizontal pressure below the reach of the working will be, using the earth pressure theories (Terzaghi Art. 24), and assuming rapid internal drainage

$$
\begin{align*}
& \mathrm{p}=\left[\gamma\left(\mathrm{h}_{1}+\mathrm{h}\right)-\gamma_{0} \mathrm{~h}_{\mathrm{w}}\right] \lambda_{1}+\gamma_{0} \mathrm{~h}_{\mathrm{w}}  \tag{1}\\
& =\left[\left(\gamma-\gamma_{0} \kappa\right) \lambda_{1}+\gamma_{0} \kappa\right]\left(\mathrm{h}_{1}+\mathrm{h}\right)
\end{align*}
$$

Here $h_{W}=\kappa\left(h_{1}+h\right)$ is the pore water pressure, and $\kappa$ is a coefficient giving the pore water pressure as a function of the height of concrete.
$\gamma$ is the unit weight of the mix, $\gamma_{0}$ of water and $h_{1}$ the depth to which the effect of the working reaches down. $h$ is the distance from this level to the point where the pressure shall be calculated, see Fig. 1.

The coefficient $\lambda_{1}$ is a function of the stiffness (internal friction) of the concrete, and varies with time. For a common friction mass, we have

$$
\lambda=\tan ^{2}\left(45-\frac{\phi}{2}\right) .
$$

Here, in order to take into account the setting, we write

$$
\begin{equation*}
\lambda_{1}=\tan ^{2}\left(45-\frac{\phi}{2}\right)\left(1-\frac{\mathrm{h}}{\mathrm{~h}_{\mathrm{S}}}\right)=\lambda\left(1-\frac{\mathrm{h}}{\mathrm{~h}_{\mathrm{S}}}\right) \tag{2}
\end{equation*}
$$

where $h_{S}$ is the height to which the concrete has risen when it has taken the Vinal set, counting from the depth $h_{l}$. $\phi$ is therefore the initial friction angle of the concrete, after it has been worked over and is left at rest.

Assuming slow internal drainage, it might be more correct to write instead of (1):

$$
\mathrm{p}=\left(\gamma-\gamma_{0}\right)\left(\mathrm{h}_{\mathrm{l}}+\mathrm{h}\right) \lambda_{1}+\gamma_{0} \mathrm{~h}_{\mathrm{w}}
$$

When the concrete does not rise too quickly in the forms, the equation chosen will be the best, and it is the one most in agreement with the tests. When $K<1$, the alternative formula gives smaller pressures.

If the concrete rises at the speed $v$ in the forms, we can write

$$
\begin{equation*}
h_{S}=v t_{s} \tag{3}
\end{equation*}
$$

where $\mathrm{t}_{\mathrm{S}}$ is the setting time.
(1) can now be written:

$$
\begin{equation*}
\mathrm{p}=\left[\left(\gamma-\gamma_{0} \kappa\right) \lambda\left(1-\frac{\mathrm{h}}{\mathrm{~h}_{\mathrm{S}}}\right)+\gamma_{0} \kappa\right]\left(\mathrm{h}_{\mathrm{l}}+\mathrm{h}\right) . \tag{4}
\end{equation*}
$$

If we want to find how the pressure varies when $h$ is constant, during pauses in the pouring or after it is completed, we can write:

$$
\begin{equation*}
\mathrm{p}=\left[\left(\gamma-\gamma_{0} \kappa\right) \lambda\left(1-\frac{\mathrm{t}}{\mathrm{t}_{\mathrm{S}}}\right)+\gamma_{0} \kappa\right]\left(\mathrm{h}_{\mathrm{l}}+\mathrm{h}\right) \tag{5}
\end{equation*}
$$

The maximum value of the pressure can be found by derivation of (4). We find

$$
\begin{equation*}
\mathrm{p}_{\mathrm{m}}=\left[\frac{\gamma \lambda}{2}\left(1+\frac{\mathrm{h}_{1}}{\mathrm{~h}_{\mathrm{S}}}\right)+\frac{\gamma_{0} \kappa}{2}\left(1-\lambda-\lambda \frac{\mathrm{h}_{\mathrm{l}}}{\mathrm{~h}_{\mathrm{S}}}\right)\right]\left[\mathrm{h}_{\mathrm{l}}+\frac{\mathrm{h}_{\mathrm{S}}}{2}\left(1-\frac{\mathrm{h}_{1}}{\mathrm{~h}_{\mathrm{S}}}+\frac{\kappa}{\lambda} \frac{\gamma_{0}}{\gamma+\gamma_{0} \kappa}\right)\right] \tag{6}
\end{equation*}
$$

at a distance from the surface $=\frac{\mathrm{h}_{\mathrm{S}}}{2}\left[\left(1-\frac{\mathrm{h}_{1}}{\mathrm{~h}_{\mathrm{s}}}\right)+\frac{\kappa}{\lambda} \frac{\gamma_{0}}{\gamma-\gamma_{0} \kappa}\right]$
If we take $\mathrm{h}_{\mathrm{l}}=\kappa=0$ and $\lambda=1$, we find $\mathrm{p}_{\mathrm{m}}=\frac{1}{4} \gamma \mathrm{~h}_{\mathrm{S}}$, a formula which has been much used for calculation of the pressure.

The formulas (4) and (6) take into account the weight and consistency (through $\gamma$ and $\lambda$ ), the rate of pouring, the pore water pressure and the
working of concrete. They can be used in many cases where the forms are smooth or the dimensions ample.

The formula (2) and the following are based on the assumption that the value of $\lambda_{1}$ decreases following a straight line, and that the effect of working the concrete ceases abruptly at a certain depth, as shown in Fig. 2.

What happens, of course, is that the effect of the working disappears gradually, and that the value of $\lambda_{1}$ approaches zero tangentially instead of at an angle, as shown with the dotted line in Fig. 1.

The effect of these approximations is not very important. The first has as a result that a jump will appear in the value of $p$ at the point $h_{l}$, whereas it usually will follow a smooth curve, see Fig. 5. Where the working stops suddenly, however, such a jump really appears in the pressure curves. Tellers Fig. 7 gives a typical example of this.

To evaluate the importance of the second approximation, we can write for instance

$$
\lambda_{1}=\frac{\lambda}{2}\left(1+\cos \frac{\pi \mathrm{h}}{\mathrm{~h}_{\mathrm{S}}}\right)
$$


instead of (2). This gives a smooth curve, with a very reasonable curvature. For the sake of simplicity, we consider here the case of $h_{1}=0$. Now the calculation for $\kappa=0$ gives a maximum $\mathrm{p}_{\mathrm{m}}=0,26 \gamma \lambda \mathrm{~h}_{\mathrm{S}}$ at a depth of $0,42 \mathrm{~h}_{\mathrm{S}}$ as compared to $\mathrm{p}_{\mathrm{m}}=0,25 \gamma \lambda \mathrm{~h}_{\mathrm{S}}$ at a depth of $0,50 \mathrm{~h}_{\mathrm{S}}$ for the straight line assumption. Other reasonable curves also give only slightly different results, proving that the error of the approximation is not great.
III. Pressure with Exterior Friction.

In narrow walls and in columns, especially when the forms have the rough surface of ordinary form boards, the friction of concrete against forms has a preponderant influence and cannot be left out.

In order to find an expression for the pressure in this case, we consider the equilibrium of a horizontal section in the concrete below $h_{1}$ (Fig. 2), and find

$$
\mathrm{dp}_{\mathrm{v}} \mathrm{~F}=\gamma_{1} \mathrm{Fdh}-\mathrm{p}_{1} \mathrm{Udh} \tan \phi_{1}
$$

$\mathrm{p}_{1}$ is the horizontal pressure not counting the pore water pressure, $\gamma_{1}=\dot{\gamma}-\gamma_{0} \kappa$, see equation (1), F is the concrete area in a horizontal section, and $U$ its circumference. $\phi_{1}$ is the angle of friction between concrete and forms. We write $\frac{F}{U}=R$, and find

$$
\begin{equation*}
\frac{\mathrm{dp}_{\mathrm{V}}}{\mathrm{dh}}=\gamma_{1}-\frac{\tan \phi_{1}}{\mathrm{R}} \mathrm{p}_{1} \tag{7}
\end{equation*}
$$

In the paper by Koenen referred to above, the relation between horizontal and vertical pressure was given as:

$$
\frac{\mathrm{p}_{1}}{\mathrm{p}_{\mathrm{v}}}=\lambda
$$

As mentioned before, this is not a constant for setting concrete. Instead of this expression, we write as before:

$$
\begin{equation*}
\frac{\mathrm{p}_{1}}{\mathrm{p}_{\mathrm{v}}}=\lambda\left(1-\frac{\mathrm{h}}{\mathrm{~h}_{\mathrm{S}}}\right) \tag{8}
\end{equation*}
$$

and equation (7) becomes

$$
\begin{equation*}
\frac{\mathrm{dp}_{\mathrm{v}}}{\mathrm{dh}}=\gamma_{1}-\frac{\mathrm{a}}{\mathrm{~h}_{\mathrm{S}}}\left(1-\frac{\mathrm{h}}{\mathrm{~h}_{\mathrm{s}}}\right) \mathrm{p}_{\mathrm{v}} \tag{9}
\end{equation*}
$$

with

$$
\mathrm{a}=\lambda \tan \phi_{1} \frac{\mathrm{~h}_{\mathrm{S}}}{\mathrm{R}}
$$

We find as a solution of (9), using $\mathrm{p}_{\mathrm{v}}=\gamma_{1} \mathrm{~h}_{\mathrm{l}}$ for $\mathrm{h}=0$ for determination of the constant:

$$
\begin{equation*}
\mathrm{p}_{\mathrm{v}}=\gamma_{1}\left(\mathrm{Ah}_{\mathrm{l}}+K \mathrm{~h}_{\mathrm{s}}\right) \tag{10}
\end{equation*}
$$

Here is:

$$
\begin{gather*}
A=e^{-a \frac{h}{h_{S}}\left(1-\frac{h}{2 h_{S}}\right)} \\
K=e^{\frac{a}{2}\left(1-\frac{h}{h_{S}}\right)^{2}} \sqrt{\frac{2}{a}} \int_{\sqrt{\frac{a}{2}}\left(1-\frac{h}{h_{S}}\right)}^{e=h^{2} d h} \tag{11}
\end{gather*}
$$

The numerical values of A can be found from the diagram in Fig. 3, and of K from Fig. 4.

The second part of formula (10), but with $\gamma$ instead of $\gamma_{1}$ has already been given in another paper by this author. ${ }^{(9)}$
$A$ and $K$ can evidently be calculated with the help of the tables of exponential functions and the error integral. Unfortunately, as the expression for K is found by multiplying the great value of the exponential function with the small value of the error integral, the usual tables do not have the necessary number of decimals for most values of $\leq \mathrm{a}$."
$K$ can also of course be evolved in series. But again, the convergence is bad for most values of "a."

Fortunately, therefore, tables for error integrals with the necessary number of decimals have been calculated by v. Oppolzer. ${ }^{(10)}$ The diagram in Fig. 4 is drawn with the help of these tables.

The pressure against the forms is now found from (8) and (10), and adding the pore water pressure:

$$
\begin{equation*}
\mathrm{p}=\gamma_{1} \lambda\left(\mathrm{Ah}_{\mathrm{l}}+\mathrm{Kh}_{\mathrm{S}}\right)\left(1-\frac{\mathrm{h}}{\mathrm{~h}_{\mathrm{S}}}\right)+\gamma_{0} \kappa\left(\mathrm{~h}_{\mathrm{l}}+\mathrm{h}\right) \tag{12}
\end{equation*}
$$

If $\phi_{1}$ is small, or if the wall is thick so that $R$ is great, " $a$ " will have a small value. In this case, we find $A \sim 1$ and $K \sim h / h_{S}$, and (12) becomes identical with (4).
IV. Material Constants.

All factors of importance for the calculation of the concrete pressure seem to be included in the formula (12). But in order to use it, it is necessary to have an idea of the safe values of the various material constants used. These shall now be discussed. The following is not meant to give the exact values of the constants, but to give an idea of the limits within which they will be found, and to indicate safe values based on this.

The setting time $\mathrm{t}_{\mathrm{S}}$ can be found from curves for the early strength of the concrete, see for instance Öfjord. ${ }^{(7)}$ We seethat it depends very much on the temperature of the concrete, the cement quality also is important, but the water-cement factor and the quality of the sand and stone make little difference.
$\mathrm{t}_{\mathrm{s}}$ will be nearly constant and about 4 hours with cement of usual quality and temperatures above $50^{\circ} \mathrm{F}$. At $40^{\circ}$ we must expect a value of 6 hours, and at $35^{\circ} \mathrm{F}$ it is necessary to reckon with more than 10 hours.

Some cements are slower than indicated above, and $\mathrm{t}_{\mathrm{S}}$ can reach 5 and up to 6 hours for usual temperatures. Sand-rich mixtures also tend to have a somewhat longer setting time.

It may be of interest in this connection to mention that the ASTM specifications require that cement shall have acquired its final set, according to the Gillmore test, in not more than ten hours.

If one has no information about the cement, it is safe to reckon with a $\mathrm{t}_{\mathrm{S}}$ of 5 hours for normal temperatures.

The value of $h_{1}$, the depth to which the working reaches down, can vary very much. If the surface is only tamped, as is frequently the case, $h_{1}$ is equal to zero. If the concrete is spaded or puddled, $\mathrm{h}_{1}$ is equal to the penetration of the spade or bar into the concrete, usually one to three feet. Under laboratory conditions, and occasionally elsewhere, puddling may reach deeper.

When vibration is used, it is difficult to define $h_{1}$. According to L'Hermite, ${ }^{(11)}$ fresh concrete which is being vibrated with a 3000 rev. per min. vibrator acts as a liquid until it is subjected to a pressure of 12 psi . In other words, it can be considered as a liquid while vibrated if the vibration does not reach more than about 11 ft . down.

With that pressure, the concrete begins to develop an interior friction even during vibration. This constitutes the upper limit for $h_{1}$, but generally of course the penetration of the vibration depends on the concrete section, the construction of the forms, and the size of the vibrator.

For walls, it seems that a reasonable figure is the penetration of the vibrator plus two feet.

The friction angle $\phi$ for ordinary qualities of concrete is $20^{\circ}$ to $30^{\circ}$, but may for dry concrete go up to $35^{\circ}$. For design purposes, $20^{\circ}$ is on the safe side. After vibration the angle can increase considerably, and reach $50^{\circ}$, according to L'Hermite.

The friction angle, $\phi_{1}$, between concrete and forms, is equal to $\phi$ for ordinary rough formboards. For smooth, oiled boards placed vertically, or for steel forms, figures for $\phi$ down to $5^{\circ}$ can be calculated from the tests.
$\phi_{1}=\frac{1}{2} \phi$ to $\phi_{1}=\frac{1}{3 \phi}$ are safe figures in most cases.
$\kappa$ can be estimated from the water content of the mix, the quality of the sand and the permeability of the forms.

A good concrete will have a $\kappa$ of $0,70-0,90$, supposing that the voids content of the concrete when set and the shrinkage during setting are both small, and that no water is lost through the forms. The amount of water which must be considered to be already chemically bound to the concrete is uncertain.

Usually, however, water is lost through the forms, $\kappa$ will decrease correspondingly and is, strictly, a variable. In this case, the pore water pressure may reach a high value locally, when a large batch of concrete has just been dumped, but will have decreased long before the height of concrete has reached the value giving maximum pressure.

Fig. 6 will give an idea of the variations of the pore water pressure during filling, for a column with vertical form boards. For a thin wall with horizontal boards in the formwork, the pore water pressure can hardly be more than corresponding to the width of two or three boards.

How the loss of water through the forms and the resulting pore water stand will influence the final strength of the concrete is an interesting question, but seems to have been little considered.
. Applications.
The formulas will be tried on some cases where the results can be checked. We will first consider a wall, six inches thick, being filled with concrete at the rate of 10 feet per hour. The concrete is only tamped, so $\mathrm{h}_{\mathrm{l}}=0$. Let us take $\mathrm{t}_{\mathrm{S}}=5$ hours. We find

$$
\mathrm{R}=\frac{0,5 \mathrm{sq} \mathrm{ft}}{2 \mathrm{ft}}=0,25 \mathrm{ft}, \mathrm{~h}_{\mathrm{S}}=10,5=50 \mathrm{ft}
$$

We will further assume $\kappa \mathrm{h}=1$ and $\phi=\phi_{1}=20^{\circ}$, which gives $\lambda=0,5$ and $\tan \phi_{1}=0,36$, and find

$$
a=0,5 \cdot 0,36 \frac{50}{0,25}=36
$$

For five feet of concrete, we get $h / h_{S}=0,10$ and formula (12) and Fig. 4 give:

$$
\mathrm{p}=(150-0,2 \cdot 62) \times 0.5 \times 0.025 \times 50(1-0.10)=139 \mathrm{psf}
$$

In order to find the maximum pressure, three or four points must be calculated and the pressure curve drawn. In this case 144 psf is the maximum pressure.

Most empirical formulas would give more than 500 pounds in this case. And yet the above result is well checked by practical experience.

Six inch concrete walls have namely been much used in building construction in Norway in the last twenty years, and the formwork, which never seems to have been calculated by engineers, has become standardized by use and habit. It is secured by 0,35 square inch strips, with a tensile strength at yield point of 1300 pounds, carrying about 8 square feet.

Evidently, the tension in the strips cannot be much more than the $144 \times 8$ $=1150$ pounds calculated above. This gives a good check for the formulas.

Rodin ${ }^{(2)}$ has given the results of tests reported by Roby.
Roby used a $2^{\prime} 6^{\prime \prime}$ square column, 15 feet high, with a rate of filling from one to ten feet per hour. We get

$$
\mathrm{R}=\frac{2.5^{2}}{4 \cdot 2.5}=0.625 \mathrm{ft}
$$

His "normal" mixture, with a rate of filling of four feet per hour will be compared with the formulas. From the shape of the pressure curves, the puddling seems to have reached about 4,5 feet down. We take $t_{S}=5$ hours, $\phi=20^{\circ}, \phi_{1}=7^{\circ}$, which give $\lambda=0,5, \tan \phi_{1}=0,12, \mathrm{~h}_{\mathrm{S}}=4.5=20$ feet, and $\mathrm{a}=$ 1,9. $k\left(h_{1}+h\right)$ is estimated at 5 feet. For $h=5$ feet, we find $h / h_{S}=0,25$ and from formula (12) and Fig. 3 and 4,

$$
\mathrm{p}=(150-62 \cdot 0,53) 0,5(0,7 \cdot 4,5+0,21 \cdot 20)(1-0,25)=634 \mathrm{psf}
$$

The pressure line is shown on Fig. 5, where also the result of the test found in Rodin's Fig. 1 is shown.

When the formulas are compared to other known tests, the same good agreement as in the two examples above is found.

## CONCLUSION

The aim of this paper has been to give a theoretical foundation for the calculation of the pressure of fresh concrete.

Taking into account all factors, the formula (12) has been arrived at. If the friction against the forms is neglected, the simpler formula (4) may be used.

In order to facilitate the application of these formulas, the material constants of fresh concrete have been discussed. This is a much neglected field, therefore the values given must not be considered as final. But the safe values indicated for design work may be used with confidence.

## APPENDIX I

Calculation of Pressure on Forms, Taking into Account the Cohesion.
The cohesion of a mass of fine particles (such as a clay) will increase with the pressure it is subjected to. We can here write

$$
\mathrm{c}=\frac{1}{2}\left(\mathrm{p}_{\mathrm{v}}-\mathrm{p}_{\mathrm{w}}\right) \mathrm{k}+\mathrm{k}_{0}
$$

where $c$ is the cohesion, $p_{V}$ the vertical pressure of the mass, $p_{w}$ is the pore water pressure, and K is a constant which for clays has values between 0,2 and $0,6 . \mathrm{k}_{0}$ is a constant which we here will consider equal to zero. We write $\gamma_{0} K\left(\mathrm{~h}_{1}+\mathrm{h}\right)$ for the pore water pressure, and find

$$
\mathrm{c}=\frac{1}{2}\left[\mathrm{p}_{\mathrm{v}}-\gamma_{0} \kappa\left(\mathrm{~h}_{1}+\mathrm{h}\right)\right] \mathrm{k}
$$

For concrete, the cohesion evidently also varies with time. For the short time, usually 4 to 6 hours, here in question, we can consider it a linear function of $t$. This gives finally:

$$
\mathrm{c}=\frac{1}{2}\left[\mathrm{k}_{1} \mathrm{t}+\mathrm{k}_{2}\left(\mathrm{p}_{\mathrm{v}}-\gamma_{0} \kappa \mathrm{~h}-\gamma_{0} \kappa \mathrm{~h}_{\mathrm{l}}\right)\right]
$$

The pressure against a vertical wall, (without the horizontal pressure from the pore water) is for an ordinary cohesive mass:

$$
\mathrm{p}_{1}=\mathrm{p}_{\mathrm{v}} \lambda-2 \mathrm{c} \sqrt{\lambda}
$$

For concrete, we write corresponding to (8)

$$
\begin{gather*}
\mathrm{p}_{1}=\mathrm{p}_{\mathrm{v}} \lambda\left(1-\frac{\mathrm{h}}{\mathrm{~h}_{\mathrm{S}}}\right)-2 \mathrm{c} \sqrt{\lambda} \\
=\mathrm{p}_{\mathrm{v}} \lambda\left(1-\frac{\mathrm{h}}{\mathrm{~h}_{\mathrm{S}}}\right)-\sqrt{\lambda} \mathrm{k}_{1} \mathrm{t}-\sqrt{\lambda \mathrm{k}_{2}}\left(\mathrm{p}_{\mathrm{v}}-\gamma_{0} \kappa \mathrm{~h}-\gamma_{0} \kappa \mathrm{~h}_{\mathrm{l}}\right) \tag{13}
\end{gather*}
$$

In $\sqrt{\lambda}$ the factor ( $1-\mathrm{h} / \mathrm{h}_{\mathrm{S}}$ ) is not added, considering that the effect of consolidation is already taken care of in the expression for $c$.

In the case of no frictions against the forms, $p_{v}=\gamma_{1}\left(h_{1}+h\right)$, and we find

$$
\begin{equation*}
\mathrm{p}_{1}=\gamma_{1}\left[\lambda\left(1-\frac{\mathrm{h}}{\mathrm{~h}_{\mathrm{s}}}\right)-\sqrt{\lambda \mathrm{k}_{2}}\left(1-\frac{\gamma_{0}}{\gamma_{1}} \kappa\right)\right]\left(\mathrm{h}_{1}+\mathrm{h}\right)-\sqrt{\lambda} \mathrm{k}_{1} \mathrm{t} \tag{14}
\end{equation*}
$$

Equation (14) corresponds to equation (4) for the case of no cohesion. When friction against the forms has to be taken into account, we can use equation (7) and find with the help of (13)

$$
\begin{gathered}
\frac{\mathrm{dp}_{\mathrm{v}}}{\mathrm{dh}}=\left(\gamma_{1}-\frac{\sqrt{\lambda} \tan \phi_{1}}{\mathrm{R}} \mathrm{k}_{2} \kappa \mathrm{~h}_{\mathrm{l}} \gamma_{0}\right)+\frac{\sqrt{\lambda} \tan \phi_{1}}{\mathrm{R}}\left(\frac{\mathrm{k}_{1}}{\mathrm{v}}-\mathrm{k}_{2} \kappa \gamma_{0}\right) \mathrm{h}-\frac{\lambda \tan \phi_{1}}{\mathrm{R}} \\
\left(1-\frac{\mathrm{k}_{2}}{\sqrt{\lambda}}-\frac{\mathrm{h}}{\mathrm{~h}_{\mathrm{S}}}\right) \mathrm{p}_{\mathrm{v}}=\gamma_{1}^{\prime}+m \mathrm{mh}-\frac{\mathrm{a}}{\mathrm{~h}_{\mathrm{S}}}\left(\mathrm{~b}-\frac{\mathrm{h}}{\mathrm{~h}_{\mathrm{S}}}\right) \mathrm{p}_{\mathrm{v}}
\end{gathered}
$$

The solution of this equation is:

$$
\begin{gather*}
p_{v}=\left(\gamma_{1}^{\prime} h_{1}+\frac{m}{a} 2 h_{s}^{2}\right) e^{-a \frac{h}{h_{s}}\left(b-\frac{h}{2 h_{S}}\right)}+\left(\gamma_{1}^{\prime} h_{s}+m b h_{s}^{2}\right) \sqrt{\frac{2}{a}} e^{\frac{a}{2}\left(b-\frac{h}{h_{s}}\right)^{2}} \\
\int_{\sqrt{\frac{a}{2}\left(b-\frac{h}{h_{S}}\right)}}^{e^{-h^{2}} d h-\frac{m}{a} 2 h_{s}^{2}} \tag{15}
\end{gather*}
$$

with

$$
\begin{gathered}
\gamma_{1}^{\prime}=\gamma_{1}-\frac{\sqrt{\lambda} \tan \phi_{1}}{\mathrm{R}} \mathrm{k}_{2} \kappa \mathrm{~h}_{1} \gamma_{0} \\
\mathrm{~m}=\frac{\sqrt{\lambda} \tan \phi_{1}}{\mathrm{R}}\left(\frac{\mathrm{k}_{1}}{\mathrm{v}}-\mathrm{k}_{2} \kappa \gamma_{0}\right) \\
\mathrm{b}=1-\frac{\mathrm{k}_{\mathrm{v}}}{\sqrt{\lambda}}
\end{gathered}
$$

Equation (15) can be made more manageable with the help of nomograms. But these will contain three variables, $a, b$, and $h / h_{S}$, instead of the two contained in (10) and (11). It will be correspondingly more complicated to use, and in view of the relatively small importance of the cohesion in most cases, and the uncertainty of the values of $\mathrm{k}_{1}$ and $\mathrm{k}_{2}$, it does not seem worth while to work nomograms at the present stage.

The value of $\mathrm{k}_{1}$ is easy to determine in any given case, but the material for finding its upper and lower limits in the general case is meagre. If, for instance, we find that the concrete has a compressive strength of 400 psf after 5 hours, then $\mathrm{k}_{1}=400 / 5=80 \mathrm{psf}$ per hour.
$\mathrm{k}_{2}$ is an abstract number. As mentioned before, for clays it varies between 0,2 and 0,6 . Here we must expect a smaller value. It will, of course, depend very much on the quality of the concrete, but apparently values near 0,1 may be expected.

## APPENDIX II

The pore water pressure in fresh concrete was recently measured in a test made by the Norwegian Geotechnical Institute. This is probably the first time such a measurement has been carried out. The pressure was measured in a $7^{\prime \prime} \times 40^{\prime \prime}$ column, the concrete had a water-cement ratio of 0,7 , and the forms were made with $1^{\prime \prime} \times 4^{\prime \prime}$ vertical boards. The results are shown on Fig. 6.

This test, of course, only gives a first indication of the size of the pore water pressure, and of the coefficient $\kappa$ used in this paper. It appears that we can get $\kappa>1$ near the surface, before the pressure from the recently dumped concrete has been equalized. But when the height of concrete giving the maximum pressure is approached, values of $\kappa$ near those used in this paper are found.

As the curves show, much water was lost through the forms.

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fig. 1
Variation of $\lambda_{1}$ with time

fig. 2
Increase of vertical pressure with depth


Fig. 4. Diagram for $K$


Pore water pressure in fresh concrete

## PROCEEDINGS-SEPARATES

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VOLUME 80 (1954)
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