Available online at www.sciencedirect.com

ScienceDirect

Energy Procedia 80 (2015) 349 – 356

Energy Procedia

12th Deep Sea Offshore Wind R&D Conference, EERA DeepWind'2015

Grid model reduction for large scale renewable energy integration analyses

Harald G. Svendsen

SINTEF Energy Research, Trondheim, Norway

Abstract

This paper provides a brief description of an algorithm for generating equivalent, reduced power flow models. It presents the underlying principles and the various steps of the reduction process. The procedure is applied to two different cases: The Moroccan power system and the Norwegian power system. The derived reduced, equivalent models are briefly discussed and compared with the full models. The model reduction procedure and its implementation as Matlab/Python scripts is generic and may be applied for reduction of any grid model described by a PSSE load flow file.

Peer-review under responsibility of SINTEF Energi AS

Keywords: grid model reduction, power flow analysis,

---

1. Introduction

It is often useful to perform analyses with simplified models in order to gain a clearer understanding of specific phenomena, avoiding "noise" from less relevant issues. Additionally, simplifications means reduced effort in terms of time or computational power. When working with simplified models it is of course important that the simplification keeps the most important characteristics of the real case. In other words, good reduction procedures are essential.

This document outlines a reduction procedure for an electricity grid model that is suitable for optimal power flow analyses of large interconnected power systems. The idea of this reduction procedure is to reduce the number of nodes and branches starting from a detailed grid model and arriving at an *equivalent* model with an arbitrarily chosen number of nodes. Equivalence in this case means that the reduced model exhibits similar power flow characteristics.

The reduction procedure is based on and previously described in refs. [1] and [2]. It includes a well-defined algorithm for bus aggregation based on power transfer distribution factors (PTDFs), and computation of branch reactances in the reduced model based on similarity of PTDFs and voltage angles, with appropriately chosen weighting.

The application of such reduced grid models may be several, but the main one foreseen at present is the analysis of the impacts of large scale integration of renewable energy in the European and North African power systems. Similar

---

E-mail address: harald.svendsen@sintef.no

Peer-review under responsibility of SINTEF Energi AS
doi:10.1016/j.egypro.2015.11.439

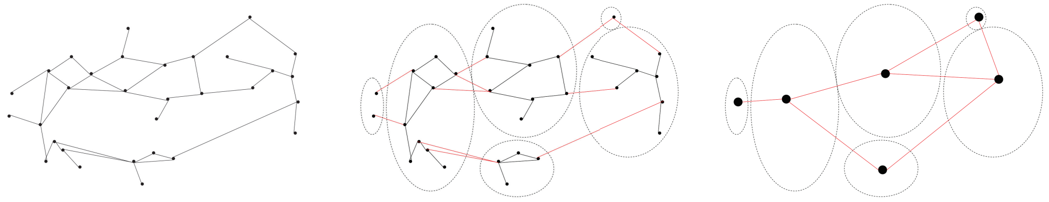

(a) Original grid (b) Clustering (c) Reduced grid

Fig. 1: Principle of grid reduction. a) The original full model, for which the full PTDFs are computed. b) Zones are uniquely defined explicitly, or by a clustering algorithm. Zone-crossing lines are shown in red. c) In the reduced model, each zone is represented by one node, and connections between zones by single lines.

analyses have been done in the past with emphasis on future scenarios for wind energy in Northern Europe [3,4]. Such analyses may give valuable insight into power system bottlenecks and price variations. The results are useful in understanding power system impact of large scale integration of renewable energy, and for timely planning of grid reinforcements.

The reduction procedure described and applied here is one of several proposed approaches, see e.g. [5,6,7,8,9,10]. It has been chosen since it is particularly suitable for power flow analyses of large power systems.

2. Grid reduction algorithm

This section describes a generic algorithm for creating a reduced model that pertains power flow characteristics of the original model. The purpose of this reduction is to reduce computational demands, and to arrive at a model more suitable for high level analyses involving multiple countries. An added benefit may be that the reduced model can more easily be be shared publicly or at least amongst research partners, as the model reduction involves an aggregation and obscuration of sensitive grid data. The procedure is based on work published by others [1,2], and so the following description is kept concise. For more thorough treatment of the concepts presented, the reader is referred to the cited literature.

This reduction process is a *static* reduction based on a snapshot of the power system at one particular point in time. This of course means that the reduced model is less accurate for operating conditions differing significantly from what they are at this time. Figure 1 shows the principle of reducing the full grid to a smaller, equivalent model.

2.1. Power transfer distribution factors (PTDF)

Power transfer distribution factors (PTDFs) are a way to express power flow characteristics of a grid model. They are represented as an $N \times L$ matrix, where N equals the number of nodes (buses) and L equals the number of branches (connections between nodes). The elements of the PTDF matrix specifiy how power flows from a given injection point to a chosen extraction point (sink). That is, the value of the element in row $n \in N$ and column $l \in L$ specifies how large a fraction of the power goes through branch l when one power unit is injected at node n and extracted at the sink node. The sink node is given, and is typically taken to be the slack bus used for power flow analyses.

The PTDF matrix is used in the grid model reduction algorithm both for clustering of nodes, and as a key element for computing equivalent reactances in the reduced model. The idea is that the PTDF matrix from the reduced model should be as similar as possible to the PTDF matrix of the original, full model. Basing the model reduction on the PTDF equivalence is appropriate when creating models that will be used for power flow analyses.

2.2. Node clustering

If the input model is already separated into areas that represent geographical zones, it may be possible to use one cluster per zone. Or if zone borders are given beforehand, the clustering can be achieved by identifying all zone-crossing connections. However, if information about the grid model is limited, performing such explicit clustering may be difficult, or at least time consuming.

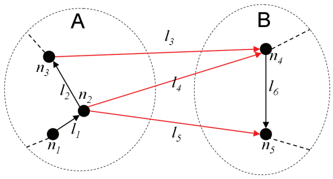


Fig. 2: Two zones within a large grid model.

An alternative to explicit clustering is to apply a clustering algorithm such as some variant of the well-known *k-means* algorithm, as proposed in ref. [2]. In this algorithm, nodes are clustered into the same zone based on a Euclidean distance function. The distance function used here takes selected columns of the PTDF matrix, representing important transmission corridors that have been identified upfront. The distance d_{nm} between two nodes n and m is defined as

$$d_{nm} = \sqrt{\sum_{j\in\mathcal{L}} (p_{nj} - p_{mj})^2}, \tag{1}$$

where p_{nj} is an element of the PTDF matrix, and $\mathcal{L}$ is the set of important branches. If power injection at two nodes gives similar power flow on these branches, then the "distance" will be small and the nodes are clustered together. The number of zones or clusters is specified upfront. The k-means algorithm iterates a process whereby it distributes cluster centroids such that the total distance from nodes to its nearest centroid is minimised.

When nodes have been clustered into zones, the reduced model grid layout is obtained by replacing each cluster by a single node. Connections between nodes in the reduced model are included if there are any connections between the associated zones in the original model. Similarly, power demand and generation at the reduced model node is given by summing up demand and generation within the associated zone in the full model.

Computing equivalent branch capacities in the reduced model is not straightforward because the power flow is in reality distributed along multiple branches. Here, the simple assumption is made that the branch capacity in the reduced model is given by the sum of capacities of all corresponding branches in the full model. In reality, this sum is an upper limit to the reduced model capacity, based on the optimistic assumption that power flow is distributed such as to utilise each inter-zonal branch in the best possible way.

2.3. *Zonal PTDFs*

Zonal PTDFs describe the aggregated power flow between zones. Computation of zonal PTDFs by aggregating the full PTDF is straightforward, but requires one choice to be made: How to weigh power injection contributions from different nodes within each zone? This may be done uniformly (equal weight) or e.g. based on generation capacity or generation output at each node.

To illustrate the process of obtaining the zonal PTDF matrix from the full PTDF matrix, consider two zones A and B of a larger system, see Figure 2. Assume further that the relevant part of the PTDF matrix is Ψ, and given as

$$\Psi = \begin{array}{cc} & \begin{array}{cccccc} l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \end{array} \\ \begin{array}{c} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \end{array} & \begin{pmatrix} 0.7 & 0.2 & 0.15 & 0.3 & 0.2 & 0.3 \\ -0.2 & 0.2 & 0.1 & 0.3 & 0.3 & 0.35 \\ 0 & -0.1 & 0.7 & 0.05 & 0.05 & 0.5 \\ -0.05 & -0.1 & -0.2 & -0.1 & 0.05 & 0.4 \\ -0.05 & -0.1 & -0.05 & 0 & -0.15 & 0.2 \end{pmatrix} \end{array} \Rightarrow \quad \Psi' = \begin{array}{c} l_{AB} \\ \begin{pmatrix} 0.65 \\ 0.7 \\ 0.8 \\ -0.35 \\ -0.2 \end{pmatrix} \end{array}. \tag{2}$$

In this case, lines 1, 2, and 6 are inside a single zone and can be ignored, whereas the lines 3, 4 and 5 are all connecting the two zones A and B. Summing up the power flow between the zones and ignore flows within the zones, the resulting matrix is Ψ', as shown above.

The next step is to merge contributions from nodes within each zone into a single value per zone. That is, to compute the weighted average for power injection within each zone. In the present example this means the weighted average for n_1, n_2, n_3 and for n_4, n_5 separately. Using uniform weighting factors, the result is

$$\Psi'' = \begin{array}{cc} & l_{AB} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} 0.717 \\ -0.275 \end{pmatrix} \end{array}. \tag{3}$$

This means that if one unit of power is injected in zone A (and extracted at the sink node), 75% of the power will flow through the connection to zone B. And if one unit of power is injected at zone B, then 27.5% will flow through the connection to zone A.

2.4. Reduced model reactances

The clustering of nodes determines the topology of the reduced grid including capacities of branches. But for power flow analyses it is essential to also have the branch impedances. For linearised power flow ("DC power flow") equations, only reactances are considered.

Since power flow, and therefore the PTDF matrix, depends on branch impedances, reduced model reactances can be derived from the PTDFs by demanding that the reduced model PTDF matrix equals the zonal PTDF matrix derived from the full model. Or to be more precise, that the deviation is minimal.

Using this criterion alone, however, tends to produce reactance values that are very far away from what is considered physical, and gives voltage angle differences that are inconsistent with the linearised power flow assumption. To remedy this problem, an additional term in the objective function used to compute reduced model reactances is introduced [2]. This term ensures that the voltage angle difference between nodes in the reduced model is similar to the average voltage angle difference between all nodes in the associated zones in the full model.

Omitting any detail, and working with the linearised dc power flow equations, the following expressions sketch how the reactances are computed. Define

$$C = \Psi A - I_L, \tag{4}$$

where Ψ is the $L \times N$ zonal PTDF (computed from the full model), A is the $N \times L$ incidence matrix which describes the grid topology, and I_L is the $L \times L$ identity matrix. The matrix C is then an $L \times L$ matrix. Now, write $A = [W_1, \ldots, W_N]^T$, where W_n are row vectors in A, and define the $LN \times L$ matrix F' as

$$F' = \begin{pmatrix} C \cdot diag(W_1) \\ \vdots \\ C \cdot diag(W_N) \end{pmatrix} \tag{5}$$

The matrix equation $F'\, b = 0$ then expresses the equivalence between the zonal PTDF computed from the full model and from the reduced model, where b is the susceptance vector and $x = b^{-1}$ are the desired reactances. However, this equation is over-determined and generally has no solution, so it must be expressed as a minimisation problem,

$$\min ||F' b||^2. \tag{6}$$

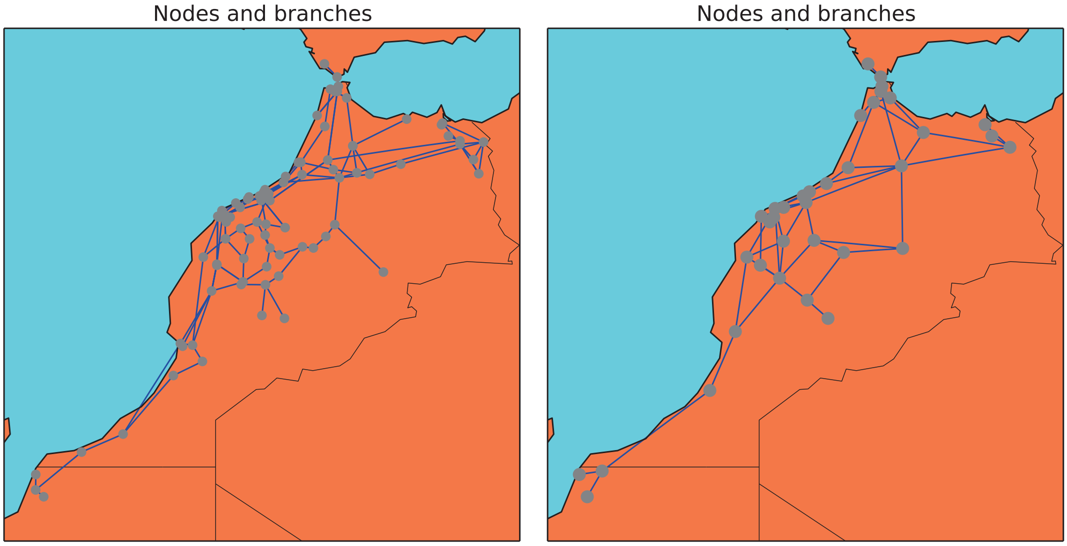
To include voltage angle similarity, this expression is amended with an additional term. Voltage angle differences are computed by the power flow equations and also depend on the reactances. The relationship is

$$P_j = b\, \delta\theta_j, \tag{7}$$

where P_j is the power flow on branch j and $\delta\theta_j$ is the voltage angle difference at the branch endpoints. If we define

$$F'' = I_N, \tag{8}$$

where I_N is the $N \times N$ identity matrix, and use the full model to compute aggregated power flow and average voltage angle differences we arrive at the equation $F''_{jk} b_k = \frac{P_j}{\delta\theta_j}$.



(a) Full model, N=857 (showing ≥ 200 kV) (b) Reduced model, N=40.

Fig. 3: Morocco grid model reduction

To combine these criterions, we define

$$F = \begin{pmatrix} w_1 F' \\ w_2 F'' \end{pmatrix}, \qquad y = \begin{pmatrix} 0 \\ w_2 \frac{P_j}{\delta\theta_j} \end{pmatrix}. \tag{9}$$

where w_1 and w_2 are weighting factors. The final optimisation problem is then

$$\min ||Fb - y||^2, \tag{10}$$

whose solution b gives the best fit susceptances. The inverse of the susceptances gives the reduced model equivalent reactances.

It should be noted that even with the voltage angle similarity criterion, the computed reactances are not physical in the sense that they correspond to a particular type of cable. The purpose of the reactance values are to recreate the same or similar power flow as in the original, full model.

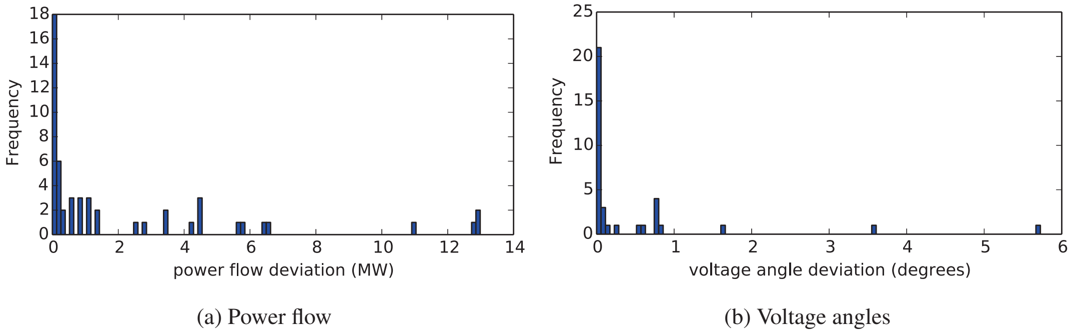
The weights w_1 and w_2 have to be chosen. If $w_2 = 0$ the system has to be separated along cut-nodes, and reactance values must be found for each sub-system independently [1]. The weights used in the examples presented in this paper are based on some experimentation and were finally chosen to be $w_1 = 1$ and $w_2 = 10$.

3. Examples

3.1. Morocco

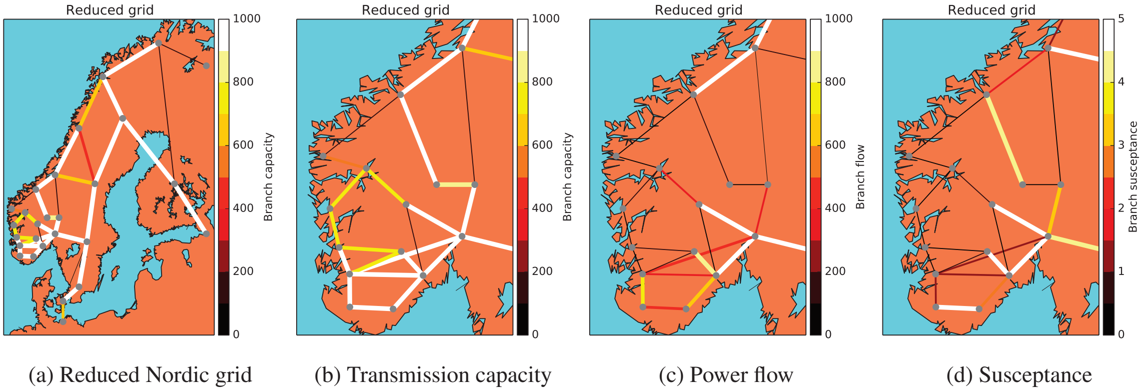
The following section illustrates the application of the above described procedure on the case of Morocco, with the aim to generate a 40 node equivalent of the full Moroccan power system. This reduced model will subsequently be merged with an existing reduced model for Europe to create a Western Mediterranean grid model for studying large scale renewable energy integration in the region.

The original full grid model has 867 nodes and 1232 branches, and includes the high voltage (≥ 60 kV) part of the Moroccan power system. The highest voltage levels of this model is plotted on a map in Figure 3a. The model was provided as a PSSE raw format file, corresponding to a 2016 scenario of the power system. It is unknown which



(a) Power flow (b) Voltage angles

Fig. 4: Morocco model: Histograms showing absolute deviation between branch power flow (a) and nodal voltage angles (b) in full and reduced model, using the same operating state and computed by linearised power flow equations



(a) Reduced Nordic grid (b) Transmission capacity (c) Power flow (d) Susceptance

Fig. 5: Reduced Norway grid model.

precise operating condition it represents, but the total power demand of 4980 MW corresponds roughly to the annual average value.

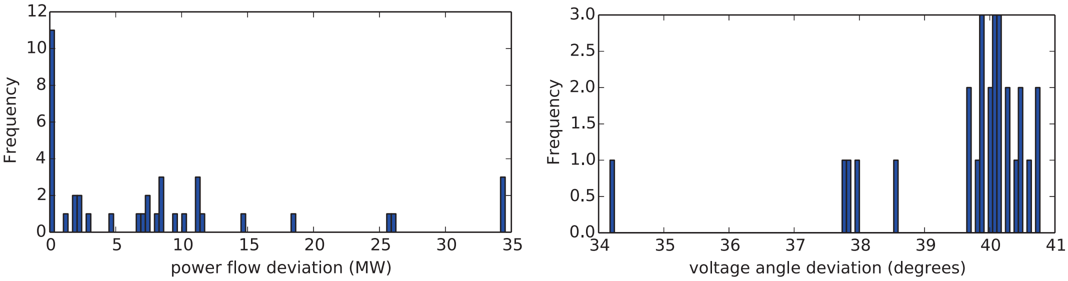
Following the procedure described above, a 40 node grid model was obtained, using a node in Spain as the slack bus. All branches at voltage levels above 200 kV were used as selected“important” branches in the node clustering algorithm. Reduced model reactances were computed using weighting factor of 10:1 for the voltage angle versus PTDF similarity conditions. For the computation of zonal PTDF, power injection contribution was scaled according to generator output. A plot of the reduced model is shown in Figure 3.

For validation of the reduction process, Figure 4a shows a comparison of power flow computed with the reduced model, and aggregated zone crossing power flow computed with the full model. Similarly, Figure 4b shows a comparison of nodal voltage angles in the reduced model and zonal average values for nodal voltage angles in the full model. All these computations are based on the linearised dc power flow approximation.

It is concluded from these comparisons that the match is good, at least in the particular operating condition used in the reduction process. As noted previously, the match will be less good with other operating condition. This has not been checked due to unavailability of full model data for other conditions.

3.2. Norway

The reduction procedure has also been applied to the Norwegian grid, starting from a detailed grid model of the Nordic system with 6133 nodes. In this case, the nodes were explicitly grouped into 27 zones by identifying all country and zone crossing connections. The zones correspond roughly to administrative regions in Norway. Only a few zones were defined for Sweden and Finland, as the emphasis was the Norwegian grid. The detailed model has a



(a) Power flow

(b) Voltage angles

Fig. 6: Norway model: Histograms showing absolute deviation between branch power flow (a) and nodal voltage angles (b) in full and reduced model, using the same operating state and computed by linearised power flow equations

Norwegian demand of 11609 MW, which is somewhat less than the annual average net consumption of 14900 MW (2012 statistics). Again, a weighting ratio of 10:1 was applied between the angle and PTDF similarity conditions.

The resulting 27 node reduced grid is shown in Figure 5. The Norwegian part consists of 18 nodes. An overview of how the reduced model power flow and voltage angles compare to the full model is shown in Figure 6. There is a big difference in voltage angles, about 40 degrees on average. However, since the reference for voltage angles can be chosen at will, the important thing is that the variation is small, which is indeed the case.

4. Application of reduced models in large scale integration analyses

As argued in the introduction, the type of reduced models obtained by the presented reduction procedure are particularly useful for high level analysis of large, interconnected power systems in situations when the detailed grid data are not available.

One such application of the reduced Morocco model found above. The study [11] concerns a 2030 scenario with large scale renewable energy in north western Africa. The main objective is to assess power system impact on a high level, i.e. to identify regional grid bottlenecks, power flows between countries and areas within countries, power supply cost variations, and the value of increased interconnection capacities. Simulations are run for a year such that variations in demand and generation, including wind and solar energy availability, is accounted for. Figure 7 illustrates the extent and detail of the simulated grid, with preliminary results highlighting annual average generation/demand balance in each node, and average utilisation of branches.

5. Conclusion

This paper has provided a brief description of an algorithm for generating equivalent, reduced power flow models. It has presented the underlying principles and the various steps of the reduction process. The procedure has been demonstrated on two cases, the Moroccan and the Norwegian power systems. The reduced Morocco model is being used in analyses of renewable energy integration in the Western Mediterranean region.

The procedure has been implemented both in Matlab and Python, taking a PSSE raw format power flow file as input. It is generic and applicable to any grid model. It is assumed to be particularly relevant for large systems where manual grid modelling is infeasible.

The model reduction is done based on a particular operating state, and using linearised power flow equations. This implies that the reduced model will not be accurate on a detailed level. However, on larger scales and when averaging over time, these approximations are less critical. Such models are therefore mostly useful for high-level analyses such as the Western Mediterranean integration study that was outlined.

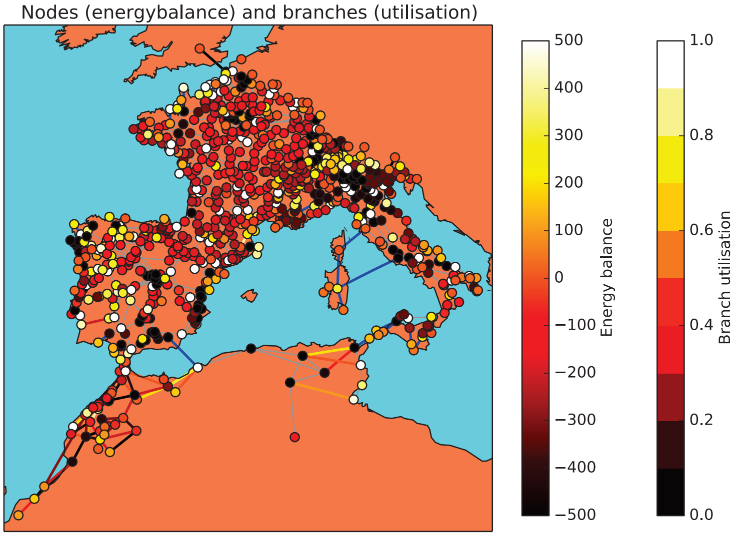


Fig. 7: Western Mediterranean case study

Acknowledgements

The research leading to these results has received funding from the European Union Seventh Framework Programme (FP7/2007-2013) under grant agreement no. 608593 (EuroSunMed, `www.eurosunmed.eu`), and from the Norwegian Research Centre for Offshore Wind Technology (NOWITECH, `www.nowitech.no`).

References

[1] X. Cheng, T. Overbye, Ptdf-based power system equivalents, Power Systems, IEEE Transactions on 20 (4) (2005) 1868–1876. `doi:10.1109/TPWRS.2005.857013`.

[2] D. Shi, Power system network reduction for engineering and economic analysis, Ph.D. thesis, Arizona State University, `http://hdl.handle.net/2286/R.A.97598` (accessed 2014-06-24) (2012).

[3] M. Korpås, L. Warland, J. O. G. Tande, K. Uhlen, K. Purchala, S. Wagemans, Tradewind – d3.2 grid modelling and power system data, Tech. Rep. TR F6604, SINTEF Energy Research (2007).

[4] Tradewind – integrating wind (final report), Tech. rep., EWEA (2009).

[5] J. B. Ward, Equivalent circuits for power-flow studies, American Institute of Electrical Engineers, Transactions of the 68 (1) (1949) 373–382. `doi:10.1109/T-AIEE.1949.5059947`.

[6] S. Deckmann, A. Pizzolante, A. Monticelli, B. Stott, O. Alsac, Studies on power system load flow equivalencing, Power Apparatus and Systems, IEEE Transactions on PAS-99 (6) (1980) 2301–2310. `doi:10.1109/TPAS.1980.319798`.

[7] W. Jang, S. Mohapatra, T. Overbye, H. Zhu, Line limit preserving power system equivalent, in: Power and Energy Conference at Illinois (PECI), 2013 IEEE, 2013, pp. 206–212. `doi:10.1109/PECI.2013.6506059`.

[8] D. Shi, D. Shawhan, N. Li, D. Tylavsky, J. Taber, R. Zimmerman, W. Schulze, Optimal generation investment planning: Pt. 1: network equivalents, in: North American Power Symposium (NAPS), 2012, 2012, pp. 1–6. `doi:10.1109/NAPS.2012.6336375`.

[9] N. Li, D. Shi, D. Shawhan, D. Tylavsky, J. Taber, R. Zimmerman, W. Schulze, Optimal generation investment planning: Pt. 2: Application to the ercot system, in: North American Power Symposium (NAPS), 2012, 2012, pp. 1–6. `doi:10.1109/NAPS.2012.6336374`.

[10] A. Papaemmanouil, G. Andersson, On the reduction of large power system models for power market simulations, in: 17th Power Systems Computation Conference, Stockholm, Sweden, 2011, `http://www.eeh.ee.ethz.ch/uploads/tx_ethpublications/PSCC2011_Antonios.pdf` (accessed 2014-06-24).

[11] H. G. Svendsen, O. C. Spro, O. Alstad, K. Loudiyi, A. S. Sennou, Modelling and analysis of large scale solar energy integration in the moroccan power system, in: 4th Solar integration workshop, Berlin, Germany, Nov 2014.