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SINTEF REPORT

TITLE

A Survey of Heuristics for the Vehicle Routing Problem
Part II: Demand Side Extensions

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ABSTRACT

This survey paper presents a review of the recent heuristic and metaheuristic solution techniques for different extensions of the well-known capacitated Vehicle Routing Problem (VRP) that are related to the demand type. Among the discussed topics are VRP with backhauls, pickup and delivery problems, dial-a-ride problems, period- and inventory routing, time window constraints, split deliveries as well as dynamic and stochastic routing problems. An introduction to each extension is provided and the recent heuristic and metaheuristic solution methods are shortly described. For earlier approaches, we refer to previous survey articles. The Vehicle Routing Problem (VRP) is one of the most well-known combinatorial optimization problems, and holds a central place in distribution management and logistics. The objective of the VRP is to deliver or supply a set of customers with known demands on minimum-cost vehicle routes originating and terminating at a central depot. Motivated by significant practical importance as well as considerable computational difficulty, there has been a huge amount of research on different practical extensions of the VRP. The purpose of this two-part survey is to review the recent heuristic and metaheuristic solution methods for different multi-vehicle variants of the VRP. We concentrate on papers written in 1995 or after that. For earlier methods, we refer to previous survey papers. This second part focuses on extensions related to demand side, i.e., the customer.

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1 Introduction

A typical VRP can be described as the problem of designing least cost routes from one depot to a set of geographically scattered points (cities, stores, warehouses, schools, customers etc). The routes must be designed in such a way that each point is visited only once by exactly one vehicle, all routes start and end at the depot, and the total demands of all points on one particular route cannot exceed the capacity of the vehicle. In practice, however, this basic model can be extended with various side-constraints, such as mixed pick-ups and deliveries, hard and soft time windows, route duration constraints etc.

The complexity of the side constraints dealt with in the literature to a large extent determines the applicability of academically developed methods and concepts to real-world vehicle routing problems. The aim of this two-part survey paper is to give an overview of the heuristic and metaheuristic solution methods for the most relevant types of extended vehicle routing problems tackled by researchers and that are important to industry.

There are numerous extensions to the basic VRP that deal with the nature of the demand-point, i.e., the customer. The first type of extensions includes the combination of linehauls and backhauls. Instead of having only drop-off or pick-up points, many real-life problems have both pick-up and delivery stops. Other specific situations occur when the quantity to be delivered is stochastic or revealed in real-time, or it can be split over more than one route. A lot of publications have been dedicated to the inventory routing problems. There is some industrial interest also to period routing problem because a delivery schedule for several days or weeks can be developed. This is particularly useful for medium-term planning. Often also different types of side constraints are considered. Most research attention has been directed at time-windows at the customer site because it remains an important topic for practical applications.

This article is organized as follows. In the next section we review problems having both pickup and delivery type customers, i.e., the VRP with backhauls, pickup and delivery problem and dial-a-ride problem. Section 3 is devoted to split deliveries whereas Section 4 focuses on longer term period and inventory routing problems. VRPs with time window constraints restricting the time of visiting a customer have been under intensive research during the past few years. They are discussed in section 5. The dynamic and stochastic nature of demands is considered in Section 6.

2 Pickup and delivery problems

The General Pickup and Delivery problem (GPDP) is defined as follows: One has a fleet of vehicles for serving a set of transportation requests with given capacity and start and end locations. Each request specifies the size of the load to be transported, the locations (one or more) where it is to be picked up, and the locations where it is to be delivered. Each load has to be transported by one vehicle from its set of origins to its set of destinations, without any transshipment at other locations. The fleet may be based at a single or multiple depots. The goal is to minimize total transportation costs.

In the literature, the General Pickup and Delivery problem is also referred to as the VRP with Pickup and Delivery (VRPPD). For the VRPPD, it is generally assumed that each transportation request involves a single pickup location and a single delivery location. The basic version of the GPDP has several intrinsic constraints that stem from assumptions on the structure of the pickup and delivery operation. Each partial load, i.e. the load associated with a single transportation request and a single pickup or delivery location, must be serviced only once. The partial loads of a given transportation request must be serviced by the same vehicle. A partial load must of course be picked up before it can be delivered. The vehicles have capacities that must be adhered to. Some of the above constraints may be relaxed (or not present) in richer models based on less restrictive assumptions. To accommodate various real-world complications, the basic GDP may be extended with several types of side constraints, such as time windows and compatibility constraints.

Five well-known and extensively studied routing problems are special cases of GPDP. The capacitated VRP is a PDP in which either all the origins or all the destinations are located at the depot. In VRP with Backhauls (VRPB), the delivery of all or most of the requests must precede the pickups to the depot. In the Pickup and Delivery problem (PDP), each transportation request specifies a single origin and a single destination and all vehicles depart from and return to a single depot. The Dial-A-Ride problem (DARP) is a PDP in which the loads to be transported represent people and all load sizes are equal to one. In the VRP with Simultaneous Pickup and Delivery (VRPSPD), a subset of the transportation requests all have their pickups at a central depot and corresponding deliveries at customer locations. The remaining requests all have their pickups at customer locations and the corresponding deliveries to the central depot. Simultaneous pickups and deliveries are allowed. Savelsbergh and Sol [1] present a survey of the GPDP problem types and solution methods found in the literature. Various types of VRP with pickup and delivery are treated also in the book by Toth and Vigo [2].

2.1 VRP with Backhauls

In cases where the transportation operations involve both pickups and deliveries, requests may be partitioned into *linehauls* and *backhauls*. Linehauls are deliveries from the depot to customers, and backhauls are pickups from customers to the depot. Many companies tend to avoid pickups before completing deliveries even when no capacity violation is involved, because of the inconvenience and extra costs involved in rearranging on-board loads. For details, see for example Mosheiov [3]. The classical VRP with Backhauls (VRPB) is based on pickup and delivery transportation where linehauls for some reason must precede backhauls.

The VRPB can be stated as follows: Find a set of routes that service linehaul and backhaul customers such that vehicle capacity is not violated. For each route, linehauls must precede backhauls. Objectives typically include minimizing number of routes, total distance traveled, or route time. In the standard VRPB, there is a single depot, where loading is performed for linehauls, and unloading is performed for backhauls.

The VRPB occurs frequently in some branches of industry. Examples are the distribution of beverages where some stops require a delivery and others a pickup, and the distribution in grocery industries where goods are picked up at the suppliers and delivered at the grocery stores. The literature on the VRPB can be classified into two main categories: (i) the case where backhauling must succeed linehauling, and (ii) less restrictive models where pickups and deliveries may be intermingled to a certain extent.

For an early account of backhauling in practice, VRPB models, and a survey of algorithmic procedures based on construction heuristics, we refer to Casco et al. [4] and Goetschalckx and Jacobs-Blecha [5]. Algorithms for the VRPB at that time were mostly basic extensions of the previous work on the VRP. Methods involved adaptations of the Clarke and Wright savings heuristic, space-filling curves, and cheapest insertion heuristics. Below we briefly describe more recent studies. Most of these have also been extensions of approaches for the VRP and the VRPTW. In general, they include model enhancements to accommodate the desired level of precedence between linehauls and backhauls, and algorithms that exploit this added structure to the problem. Since 1996, meta-heuristics, including genetic algorithms and tabu search, have been investigated.

Thangiah et al. [7] designed route construction heuristic and several route improvement heuristics for the Vehicle Routing Problem with Backhauls and Time Windows (VRPBTW). In their model, linehauls must precede backhauls on each route. Furthermore, the time of beginning the service at each customer must occur within a particular time interval. Their route improvement heuristics stem from exchange heuristics previously developed for the VRPTW. In a computational study partly based on problem instances developed by Gélinas et al. [8], variants of the proposed heuristics were compared. For small instances, results were compared with exact solutions. On average, the heuristics produced solutions within 2.5 % of optimum at a lower computational cost.

Potvin et al. [9] combined a greedy insertion heuristic based on Solomon's construction heuristic for the VRPTW with a GA-based generator of tour sequences. The construction method uses a predefined ordering of customers for insertion. A simple GA is used to create good customer sequences to be fed to the construction heuristic. An empirical study based on the Gélinas et al. [8] benchmark showed that solutions within 1.6 % of optimum were produced in a few minutes on 100-customer instances.

Duhamel et al. [10] describe a tabu search heuristic for the VRPBTW. The approach combines a greedy insertion heuristic similar to Solomon [11] for VRPTW with iterative improvement based on 3 local search operators that exchange customer sequences. The 3 operators are selected randomly. A short term memory and an arc-based tabu criterion is used to prevent cycling and increase search aggressiveness. The authors conclude that their approach produces results within 0.5% of the optimum, on average, on a standard set of test problems.

Mosheiov [3] proposes two tour-partitioning type heuristics for VRPB with mixed pickups and deliveries. The heuristics are based on the idea of partitioning a "grand tour" servicing all customers into segments that are feasible. A heuristic where every vehicle serves the maximum number of both pickup and delivery customers by skipping locations on the basic tour that cause a capacity violation was found to perform best on a set of 50 problems generated by the author.

Toth and Vigo [12] present a cluster-first-route-second heuristic for VRPB and use it also to solve problems with asymmetric cost matrices. The approach exploits the information of the normally infeasible VRPB solutions associated with a Lagrangian relaxation lower bound as a basis for heuristics. The final set of feasible routes is built through a modified traveling salesman problem heuristic and improved by inter-route and intra-route arc exchanges. An experimental study based on symmetric and asymmetric test sets from the literature showed that the approach was competitive. Salhi and Nagy [13] investigate an extension to the classical insertion-based heuristic for the VRPB with mixed pickups and deliveries. It is based on the idea of inserting more than one backhaul (a cluster) at a time and on the approach by Casco et al. [4]. The method is tested on data sets with single and multiple depots.

Dethloff [14] investigates the relation between different variants of the VRPB. In the VRP with Backhauls and Mixed loads (VRPBM), there may be a certain overlap of linehauls and backhauls. In the VRP with Simultaneous Delivery and Pickup (VRPSDP, see also 2.2 below), customers may receive deliveries and provide goods to be picked up at the same time. The two problems are closely related. The VRPSDP may be modeled as a VRPBM, and the VRPBM may be regarded as a relaxation of the VRPSDP. The author shows that solution methods for the VRPSDP may be used with good results on the VRPBM. Wade and Salhi [15] present an insertion-type heuristic for a new variant of the VRPB where the user experience is used to define the positions of the first backhaul customers.

Osman and Wassan [16] have developed a tabu search meta-heuristic for the VRPB. Two alternative construction heuristics based on savings insertion and savings assignment combined with 2-opt and 3-opt are proposed. They are used to construct a number of initial solutions. The best initial solution is selected for iterative improvement using λ-interchange operators, short-term memory, and tabu restrictions based on consecutive customer moves. An empirical investigation shows that variants of the proposed approach are competitive for medium-sized and large instances.

2.2 The Pickup and Delivery Problem

Many applications of VRP involve pickup and delivery services between two locations (depots, warehouses, stores, stations etc.), not just transportation between the depot and peripheral locations. In the General Pickup and Delivery problem (GPDP) introduced above, each transportation request involves a set of pickup locations and a set of delivery locations. A feasible solution to a GPDP must of course adhere to the causal precedence relationship between the pickup tasks and their corresponding delivery tasks. In the more restrictive Pickup and Delivery Problem (PDP), also referred to as the VRPPD, all transportation requests specify a single origin and a single destination and all vehicles depart from and return to a central depot. Practical applications of PDP include the dial-a-ride problem, airline scheduling, bus routing, tractor-trailer problems, helicopter support of offshore oil field platforms and logistics and maintenance support. They also arise in less obvious situations such as VLSI circuit design, flexible manufacturing systems, and evaluating casualties. For a survey of many variants of the GPDP, see Savelsbergh and Sol [1].

Nanry and Barnes [17] present a reactive tabu search approach to solve the Pickup and Delivery Problem with Time Windows (PDPTW) using three distinct customer-pair insertion and swapping neighborhoods. The authors did also construct a new set of benchmark problems based on Solomon's [131] VRPTW benchmark. Sigurd et al. [18] consider a new variant of the GPDP called the Pickup and Delivery Problem with Time Window and Precedence Constraints (PFPTWP). PDPTWP has applications in the transportation of live animals, where veterinary rules demand that the farms be visited in a given sequence in order not to spread specific diseases. The authors use a Dantzig-Wolfe decomposition based approach, where the master problem is a set-covering problem and subproblem involves generating legal routes for a single vehicle. The LP-relaxation of the master problem is solved through delayed column generation. For the subproblem, both exact and heuristic approaches are presented and discussed. A comprehensive computational study demonstrated the practical utility of the approaches on test problems inspired by the transportation of live pigs in Denmark.

Xu et al. [19] consider a real-life pickup and delivery vehicle routing problem involving multiple vehicle types, multiple time windows, compatibility constraints, and driving time regulations. The authors use a standard column generation procedure to solve the linear relaxation of a set partitioning type formulation in which the resulting master problem is a linear program and solved by an LP-solver, while the resulting subproblems are solved by fast heuristics. Near-optimal solutions are found for 200-order instances in a few minutes of CPU time. Li and Lim [20] propose a multi-restart simulated annealing meta-heuristic to solve the Pickup and Delivery Problem with Time Windows. The authors also propose a set of new benchmark problems based on the Solomon benchmark.

In many practical applications occurring in environmentally motivated distribution systems, customers have both a pickup and delivery demand. Examples of such applications are soft drink industry where empty bottles have to be returned. It also occurs at grocery stores where reusable specialized pallets/containers are used for the transportation of merchandise. Here customers want to be served with a single stop only. This planning situation is referred to as PDP with Simultaneous Delivery and Pickup (VRPSDP). The VRPSDP is a special case of PDP, where either the origin or the destination of each transport request is the depot and, in addition, transport requests occur only pairwise, to and from the same customer. Examples of VRPSDP are the distribution of parcels from railway stations, the distribution of books, films, video tapes etc. from a central library, the grocery industry, and distribution of clean and dirty clothes from a laundry.

The VRPSDP was introduced in the literature by Min [21]. The problem was studied later by Irnich [22] who deals with a problem in the context of mail delivery in Germany, and considers also transportation between hubs and final customers. Doerner et al. [23] investigated similar problems with multiple depots and performed a thorough investigation of approaches based on ant colony optimization. Gronalt [24] proposed three modified versions of the savings algorithm by Clarke and Wright [6] with and without introducing opportunity costs into the solution procedure. Dethloff [25] suggests a heuristic construction procedure based on the cheapest insertion-concept and different insertion criteria for VRPSDP. Doerner et al. [26] consider PDP with multiple periods and time windows, where the demand of each customer equals full truckload. The practical applications are routing of containers, pallets or freight cars. The authors propose a MIP model and embed a simple construction heuristic into an Ant Colony Optimization algorithm.

2.3 The Dial-a-Ride Problem

In a typical Dial-a-Ride Problem (DARP) a carrier that is providing transportation service receives calls for door-to-door transportation of people. Origin, destination and time windows for the beginning of service at each end characterize every request for transportation. Often, there is a general constraint on the maximum ride time for each person. People may require vehicles with special equipment. The carrier has to design vehicle routes so as to maximize the number of requested trips, minimize the total traveled distance, number of vehicles needed, detour time, and excess ride time of a customer. Each transportation request consists of a pickup and a delivery task with a precedence relationship. In the literature, there is a distinction between static and dynamic DARPs. Applications of DARP include telebuses, taxis, ambulances, and parcel pickup and delivery service in urban areas. An early survey can be found in Bodin et al. [27]. Cordeau and Laporte [28] have written a recent DARP survey.

Madsen et al. [29] describe an insertion-based heuristic for the static DARP with time windows (DARPTW) in the context of the Copenhagen fire-fighting service for scheduling elderly and disabled persons in a dynamic environment. Healy and Moll [30] propose an extension to traditional local search improvement algorithms for the DARP that varies between using the real objective and a secondary metric defined on neighborhood size. The experimentation focused on 2-opt and 3-opt procedures. Hart [31] presents a MIP model of the DARP and develops a simulated annealing based heuristic solution method. Promising results are reported for small and medium sized instances. Liaw et al. [32] describe a decision support system for a DARP with two transportation modes involving paratransit vehicles and fixed route buses. Testing on actual data shows 10 % improvement over manual routing. Horn [33] introduces a decision support system for the dynamic DARP.

Fu [34] has studied the DARP in the context of paratransit scheduling with tight service time constraints and time-dependent, stochastic travel times. Extensions of well-known heuristics for the static DARP were developed to cater for dynamic and stochastic travel times and multiple optimization criteria. A series of numerical experiments on hypothetical problems were conducted. Colorni and Righini [35] study rich variants of the static and dynamic DARP. They develop a model with a variety of hard and soft constraints and a corresponding heuristic solution algorithm based on iterative local search. The approach has been tested on two cases inspired from transportation systems in Italy. De Paepe et al. [36] classify the computational complexity of a large set of DARP variants, and Hunsaker and Savelsbergh [37] demonstrate how to test the feasibility of the maximum wait time and ride time restrictions in linear time. Cordeau and Laporte [38] describe a tabu-search heuristic for a specific version of the static DARP where the total routing cost is minimized. Results from an extensive computational study on randomly generated and real-life instances are given.

3 Split delivery

Sometimes the demand of a customer may be fulfilled by more than one vehicle. This can occur for instance when the demand required by a customer is larger than the capacity of a vehicle, or when it is less costly to service a customer more than once. In practice the customers with a split delivery are usually the ones with above-average load. Distance savings may usually be obtained by splitting demands among customers with large distances to the depot.

For earlier studies, we refer to survey article by Van Breedam [39]. Frizzell and Giffin [40] consider a realistic problem with numerous constraints, such as multiple time windows, variable service times and grid network distances. The solution approach consists of a construction heuristic based on splitting the problem into time slots, and relocating and exchanging customers between routes. Asano et al. [41] present a new approximation algorithm with several reforming operators for a vehicle routing problem on a tree-shaped network with split deliveries.

Ho and Haugland [42] consider a variant of the VRP with split deliveries where the service at each customer must start within specified time windows, and describe a tabu search based on four well-known local search neighborhoods. Archetti et al. [43] describe a competitive tabu search algorithm that is based on simple cheapest insertions of customers to other existing routes or empty routes. Mullaseril et al. [44] present several heuristics for a feed distribution problem in a cattle ranch, where split deliveries are allowed and time windows are set on arcs. The presented heuristics include modifications of well-known nearest neighbor, savings and 2-opt heuristics, and operator for splitting routes. The authors report 30% reduction in the distance traveled.

4 Period and Inventory routing

4.1 Period Routing

The real-world planning situation requires quite often a weekly schedule in addition to the daily planning, for instance for fuel, oil and industrial gas distribution, and for garbage collection and beer and soft drink distribution. The related VRP is called the Period Vehicle Routing Problem (PVRP). It can be defined as the problem of finding routes for all days of a given T-day period. It is assumed that the number of customer visits per period is lower than or equal to the number of days of the period. On the other hand each customer must be visited at least once during the considered period. In classical PVRP the vehicle fleet is assumed to be homogenous and the vehicles must start from and end their journeys at depot. Each customer has a known demand and each vehicle has a capacity that cannot be exceeded. These types of problems are also called allocation/routing problems. The allocation part consists of the assignment of customers to days of the period, while the routing part governs the daily planning.

Although the PVRP has been used in many applications, it has not been extensively studied in related literature. Early heuristics were adaptations of methods previously proposed for the VRP. One of the best methods for PVRP is that proposed recently by Chao et al. [45]. The authors use integer linear programming to assign a visit combination to each customer. They then solve a VRP for each day by means of modified version of Clarke and Wright [6] savings heuristic. Finally local improvements are tried and re-initializations are performed to diversify the search.

Cordeau et al. [46] propose a tabu search heuristic for PVRP. The used insertion scheme borrows from Gendreau et al. [47]. The approach allows intermediate infeasible solutions, and it employs a diversification scheme based on a penalized function. They also indicate that the MDVRP (Multiple-depot) can be regarded as a PVRP by substituting days with depots. In Ochi and Rocha [48], a hybrid evolutionary metaheuristic (HEM) based on genetic algorithm, scatter search and local search method is introduced. This work is parallelized (PAR-HEM) in Drummond et al. [49] and Vianna et al. [50]. The algorithm is based on the Island model with a low migration frequency, and it is implemented on a cluster of workstations. Cordeau et al. [51] study the Periodic Vehicle Routing Problem with Time Windows (PVRPTW). The authors propose a tabu search based on simple customer reinsertion neighborhood and allowance of intermediate infeasible solutions that are then penalized with dynamically changing factors.

Hadjiconstantinou and Baldacci [52] consider a multi-depot PVRP arising in the utilities sector in the context of preventive maintenance service. The problem consists of determining the boundaries of the geographic areas served by each depot, a list of customers visited each day and the routes followed by the gangs. The authors use a tabu search with well-known simple neighborhood moves to solve the VRP for each day and simple insertion heuristics to assign customers to depot and planning period. Simple interchanges of these assignments are also tried. Yang and Chu [53] consider a multi-depot, single-commodity problem where the customers need to be serviced with a fixed periodicity. They solve a set of randomly generated instances with a greedy constructive heuristic, followed by a greedy refinement algorithm.

A practical waste-paper collection problem is described in Baptista et al. [54]. They describe a model and heuristic solution methods for the city of Almada, including many real-world considerations like the asymmetry of the distance matrix. The heuristics is an extension of the one proposed by Christofides and Beasley [55]. One extension of the PVRP is when there are intermediate facilities where the vehicles can unload, and where the depot is returned to only at the end of the shift. This is typically the case with waste collection, where the intermediate facilities are waste treatment plants. This model is called PVRP-IF, and is described in Angelli and Speranza [56], where a tabu search heuristic is applied to a set of randomly generated problems. Matos and Oliveira [57] present a waste collection case study with a data from Portugal. The authors suggest a simple ant system with 3-opt and λ-interchange heuristics. The assignment of collections to days is obtained through solving a graph coloring problem with an exchange mechanism.

Campbell and Hardin [58] propose a greedy scheduling heuristic for minimizing the number of vehicles in special case where each customer requires a full day. Francis et al. [59] extend the PVRP to allow service frequency to become a decision of the model. An exact combination of Lagrangean and branch and bound algorithms is suggested. For larger problems the authors propose a heuristic obtained by limiting the time given for different parts of the exact method. An extension of PVRP is to consider also inventory management. This issue is discussed in the next section.

4.2 Inventory routing

Recently the business practice called Vendor Managed Inventory replenishment (VMI) has been adopted by many companies. VMI refers to the situation in which a vendor monitors the inventory levels at its customers and decides when and how much inventory to replenish at each customer. This contrasts with conventional inventory management, in which customers monitor their own inventory levels and place orders when they think that it is the appropriate time to reorder. VMI has several advantages over conventional inventory management. Vendors can usually obtain more uniform utilization of production resources that leads to reduced production and inventory holding costs. Similarly, vendors can often obtain a more uniform utilization of transportation resources that in turn leads to reduced transportation costs. Additional savings can be obtained by increasing the use of low-cost full-truckload shipments, and by using more efficient routes by coordinating the replenishment at customers close to each other. VMI also has advantages for customers. Service level may increase, measured in terms of reliability of product availability. Also, customers do not have to devote as many resources to monitoring their inventory levels and placing orders.

Practical applications of VMI include logistics in the petrochemical and industrial gas industry. More recently, the automotive industry (parts distribution) and the soft drink industry (vending machines) have entered this arena. One reason for increased attention to VMI is the rapidly decreasing cost of technology that allows monitoring customers' inventories. VMI requires accurate and timely information about the inventory status of customers. It is very difficult to develop a distribution strategy that minimizes the number of stock outs and at the same time realizes the potential savings in distribution costs. The task of developing such a distribution strategy is called Inventory Routing Problem (IRP). The IRP is concerned with the repeated distribution of a single product from a single facility, to a set of customers over a given planning horizon. Each customer consumes the product at a given rate and has a known capacity to maintain a local inventory of the product. A fleet of homogenous vehicles with known capacity is available for the distribution of the product. The objective is to minimize the average distribution costs during the planning horizon without causing stock outs at any customer. Three issues are central to all IRPs: 1) when to serve a customer, 2) how much to deliver to a customer, and 3) which delivery routes to use.

The real-life IRPs are stochastic and dynamic, i.e., the customers' usage rates or future demand are not known. These stochastic variants are called Stochastic Inventory Routing Problems (SIRP). In SIRP one has only a probability distribution of the demands. A good treaty of this is in Trudeau and Dror [60]. Campbell et al. [61] present an excellent survey of the IRP and methodologies developed for its solution. Another survey can be found in Federgruen and Simchi-Levi [62]. Thomas and Griffin [63] review related work addressing the coordination of various operations in the supply chain, such as production, inventory and distribution. Baita et al. [64] review IRPs in a dynamic environment. The authors propose a classification scheme and summarize results obtained in the area. Here we focus only on some key issues and recent solution approaches.

The IRP is a long-term dynamic control problem that is almost impossible to solve. Therefore, almost all solution approaches solve only a short-term planning problem. Regarding this, two key issues have to be solved: how to model the long-term effect of short-term decisions, and which customers to include in the short-term planning period. One can distinguish two short-term approaches. In the first, it is assumed that all customers included in the short-term planning period have to be visited. In the second, it is assumed that customers included in the short-term planning period may be visited, but the decision whether or not to actually visit them still has to be made. Two basic principles are often used: always try to maximize the quantity delivered per visit and always try to send out trucks with a full load. When the short-term planning period consists of a single day, the problem can be viewed as an extension of the VRP and corresponding solution techniques can be adapted. Typically the short-term problems are however formulated as mathematical programs and solved using decomposition techniques, such as Langrangean relaxation.

Webb and Larson [65] study a variation of IRP, called strategic IRP. The strategic IRP seeks to find the minimum fleet size to service the customers from a single depot. Bramel and

Simchi-Levi [66] consider another variant of IRP in which customers can hold unlimited amount of inventory. To obtain a solution, they transform the problem to a Capacitated Concentrator Location Problem (CCLP), solve the CCLP, and transform the solution back to a solution to the IRP. Carter et al. [67] proposed a Lagrangean heuristic to solve single supplier IRP with Time Windows (IRPTW). Bard et al. [68] consider a variant, in which vehicles can be loaded also in satellite facilities in addition to depot. The authors propose three heuristics for VRP with satellite facilities, namely randomized savings heuristic by Clarke and Wright [6] GRASP (Kontoravdis and Bard [69]), and modified sweep (Gillett and Miller [70]). The heuristics were combined with arc- and node-exchange improvement procedures. Campbell et al. [71] consider a real-life variation of IRP faced by an industrial gases company. The introduced variations to the basic model include time windows and start and end time of customer usage on each day. The authors use a rolling-horizon framework, and an integer programming model (with several speedup techniques) is used to allocate customers to specific days. Then for the daily routing a greedy randomized insertion heuristic is proposed.

Lau and Liu [72propose an approach for the IRPTW with tight interaction between the subproblems, and propose both integer programming and a tabu search for the inventory management problem. Kleywegt et al. [73 consider IRP with direct deliveries in which multiple customers can be visited on a route. They formulate IRP as a Markov decision process, and develop efficient dynamic programming based on approximation methods (based on non-linear knapsack problem) for the problem. The work was motivated from a real-life distribution of air products such as liquid nitrogen and oxygen. The objective is to maximize the expected discounted value, incorporating sales revenues, production costs, transportation costs, inventory holding costs, and shortage penalties, over an infinite horizon. Flatberg et al. [74] study a real-world sea transportation application of transporting single commodity between a set of separate factories and consumers. The inventory levels at both the factories and the consumers must be kept within acceptable limits. The solution methods consist of two parts. An iterative improvement method is used to solve the combinatorial problem of finding the vessel routes, and an LP solver is then used on the subproblem to find the actual time for calls and quantity to load and discharge. The same problem was studied in Christiansen [75] using more traditional methods. Jaillet et al. [76] develop a priori strategies based on expected consumption of customers, as opposed to reactive strategies based on accurate current knowledge of all local inventories, for a rolling horizon framework. Campbell et al. [71] study a variant with flexible delivery amounts, where one prescribes lower and upper bounds for the delivery volumes. The authors propose a polynomial time algorithm for optimizing delivery volumes and insertion heuristics for routing, and combine them with GRASP (Kontoravdis and Bard [69]). Other recent studies can be found in Bassok and Ernst [77], Rubio [78], Dror and Trudeau [79], Barnes-Schuster and Bassok [80], Campbell et al. [61], Chan et al. [81], Berman and Larson [82], Fumero and Vercellis [83], Reiman et al. [84] and Cetinkaya and Lee [85], who present an analytical model for a variant in which demands should be shipped immediately, but the vendor has the autonomy of holding small orders until an agreeable dispatch time. Another variant is the Metered IRP (MIRP), described by Herer and Levy [86], where the supplier pays the holding cost, and the user pays for what he actually uses, when he uses it.

5 Time Windows

The Vehicle Routing Problem with Time Windows (VRPTW) has received a lot of attention in the literature. This is probably due to the wide applicability of these types of side constraints in real-world cases. Compared to basic VRP model, in VRPTW each customer has one or more time windows in which the service must start.

Basically two types of time windows can be distinguished: hard and soft time windows. Hard time windows restrict the delivery time at the customer's site. Violation of this window implies that the customer cannot be serviced. In the case of soft time windows, the customer can be serviced outside time windows, but the violation induces a penalty cost. Almost all types of heuristics have been provided to handle hard time windows easily. For details, see for example surveys by Desrosiers et al. [87], Cordeau et al. [88], Bräysy et al. [89] and recent two-part survey by Bräysy and Gendreau [90,91]. For recent most successful applications of heuristic solution techniques, see classical studies of Rochat and Taillard [92] and Taillard et al. [93] who proposed a tabu search approach using adaptive memory to record the best routes produced during the search. In the latter paper the authors suggest also a new neighborhood structure, based on exchanges of consecutive customers. Chiang and Russell [94] suggest a reactive tabu search that dynamically varies the size of the list of forbidden moves and uses λ-interchanges as its neighborhood structure. Cordeau et al. [51] introduce simple tabu searches that allow infeasible solutions during the search process. Cordeau et al. [95] present a modification of the unified tabu search of Cordeau et al. [51] that considers also forward time slack at depot when evaluating moves. The authors report significant performance improvements. Russell and Chiang [96] examine a scatter search metaheuristic framework. The basic idea of scatter search is to combine solutions from a reference set to achieve improved solutions. Within the framework two tabu searches are applied: a reactive tabu search with dynamic diversifications and intensifications and enhanced tabu search utilizing a recovery strategy that samples from a pool of elite solutions. In addition, a global route combination approach that uses a set covering algorithm to select the minimum cost solution using the routes in a set of improved trial solutions, is also applied. The authors report good solutions within 1% from the best-known, though at the cost of quite high computational effort.

Lau et al. [97] study a variant with limited number of vehicles where some customers may remain unserviced or they are not serviced within their time windows (soft time windows). The authors suggest a tabu search approach characterized by a holding list and a mechanism to force dense packing within a route. The holding list provides an extended neighborhood and all local search moves are done by relocating and exchanging customers back and forth from the holding list. Another study on the soft time windows is reported by Ioannou et al. [98]. The authors couple the nearest neighbor heuristic with a problem generator that provides, in a structured manner numerous instances that result from the manipulation of the level of time window violations and the population of customers that allow such violations. The reported results outperform the previous approaches for VRPs with soft time windows.

Homberger and Gehring [99, 100] and Gehring and Homberger [101] propose evolutionary metaheuristics based on the class of evolution strategies and three well-known route improvement procedures. Mester et al. [102] applied also evolution strategies. Here of the search is mainly driven by mutation based on a remove-insert mechanism and a composite of standard improvement heuristics. In addition, two embedded decomposition schemes are proposed to speed up the search. The authors report competitive results and several best-known solutions. Mester and Bräysy [103] present an improved version of the method of Mester et al. [102]. The improved method combines the evolution strategies with guided local search metaheuristic [104], resulting in iterative two-stage procedure. The suggested method yields the best-known solution to 86% of tested 302 benchmark instances. Berger et al. [105] propose a hybrid genetic algorithm that uses well-known route construction heuristics and local searches within a ruin and recreate scheme. A parallel version of the heuristic is presented in Berger and Barkaoui [106]. Le Bouthillier and Crainic [107] introduce a parallel co-operative methodology in which several agents communicate through a pool of feasible solutions. The agents consist of simple construction and improvement heuristics and four different evolutionary and tabu search algorithms. The authors report very competitive results. Alvarenga et al. [108] report excellent results in terms of distance objective by using a fast genetic algorithm as route generator for a column generation heuristic that is used to solve a set partitioning formulation of the problem. Hwang [109] considers a variant with multiple depots and present a sector clustering model and a genetic algorithm for its solution.

Liu and Shen [110] suggest a fast two-phase metaheuristic focusing on the relationship between routes and nodes. A feasible solution is constructed with nested parallel strategy based on lower bounds for routes, and improved with standard reinsertions and λ-interchanges. Gambardella et al. [111] test a technique based on multiple colonies of artificial ants, and Bräysy

[112] propose a new deterministic variable neighborhood scheme that uses cyclically four new improvement procedures proposed in the paper. Bräysy et al. [113] continue the work of [112] by suggesting a multi-start local search approach together with threshold accepting [114] post-optimization procedure. Another variable neighborhood study, focusing on large neighborhood search strategies and ejection chains [115] can be found in Roussseau et al. [116]. The idea of large neighborhood search for introduced for the VRPTW originally by Shaw [117]. Chaovalitwongse et al. [118] suggest a GRASP heuristic with new lower bounding procedures and local search ideas such as restricted candidate list and consideration of distance in a route elimination procedure. Pisinger and Röpke [119] present a unified heuristic where search is based on adaptive large neighborhood search that adaptively chooses among a number of removal and insertion heuristics to intensify and diversify the search. The authors report excellent results with numerous new best-known solutions with competitive computing times. Ioannou and Kritikos [120] suggest a 3-phase method. In the first phase the Hungarian method is used to obtain the optimal customer matching for an assignment approximation of the VRPTW. In the second phase the approximate assignment matching is transformed into feasible routes with a simple decoupling heuristic. In the third phase the final solution is formed by combining the best assignment routes and by applying two insertion heuristics to the remainder of the customers.

Li and Lim [121] consider a Generalized Vehicle Routing Problem with Time Windows. The authors propose a tabu-embedded simulated annealing metaheuristic for the problem. The proposed method combines different customer relocation and exchange neighborhoods and cheapest insertion heuristic with the simulated annealing procedure that is compelled to restart from the current best solution after several iterations without any improvement. Another simulated annealing implementation can be found in Bent and Van Hentenryck [122], where the simulated annealing is hybridized with a Large Neighborhood Search (Shaw, [117]) to minimize the number of routes. Arbelaitz and Rodriquez [123] compare several different combinations of simulated annealing and evolutionary algorithms. The algorithms are based on random insertions and exchanges.

Ibaraki et al. [124] introduce three methods for VRP with general time windows. The time window constraint for each customer is treated as a penalty function that can be non-convex and discontinuous as long as it is piecewise linear. After fixing the order of customers for a vehicle to visit, a dynamic programming method is used to determine the optimal start times to serve the customers so that the total penalty is minimized. The proposed approaches are based on iterated multi-start local searches and standard move neighborhoods.

Vaidyanathan et al. [125] consider and VRP for just-in-time delivery. The problem is to determine the route for each vehicle that will permit just-in-time delivery of parts to the demand locations in the production system, while minimizing the requirements for material handling equipment. The quantity delivered per trip should be just enough to meet the demand for the duration of the trip so that a vehicle will have no idle time between trips and the inventories at the demand points are minimized. The authors propose a heuristic procedure, a modified version of nearest neighbor algorithm (Rosenkrantz et al. [126]) and 3-opt heuristic by Golden et al. [127].

6 Dynamic and stochastic vehicle routing problems

Most of the existing VRP research has been focused on problems of a deterministic and static nature, although a substantial number of dynamic and stochastic models have been formulated and studied. The increasing role of on-line freight marketplaces, the more demanding customer service expectations and the availability of real-time information on traffic network conditions have all led to a dramatic rise in the number of freight and fleet management systems that are operating under explicit dynamic conditions. In dynamic and stochastic routing models decisions must be made before all information needed is known. The stochastic nature of these problems takes many forms. The timing, location and level of demand may vary. The availability of resources may vary as well due to service times, dock times and operations in other zones, regions or areas of operation.

There are two main classes of models. The first class of models deals with a priori optimization in which a solution is generated for stochastic problems prior to the receipt of information regarding the realization of its random elements. The general approach is to generate an a priori solution that has the least cost in the expected sense. This approach is discussed in the next subsection. The second set of applications involves making decisions and observing outcomes on a continuous, rolling horizon basis. It is discussed in section 6.2.

6.1 Stochastic Vehicle Routing Problems

The Stochastic VRP (SVRP) arises when some of the elements of the problem are stochastic. The stochastic component could for instance be uncertain travel times, unknown demands and the existence of the customers. Applications of SVRP include delivery of money from central banks to branches and automatic teller machines, delivery of home heating oil, sludge disposal, design of hot meals delivery system, waste collection and routing of forklifts in a cargo terminal or in a warehouse. Also in less-than-truckload operations the amount of goods to be collected on any given day from a set of customers is generally uncertain.

SVRPs are usually modeled as mixed or pure integer stochastic programs, or as Markov decisions processes. The two basic formulations of stochastic programs are chance constrained programs (CCP) and stochastic programs with recourse (SPR). For the CCP the probability of failure in the first stage is considered to be below a given threshold. The CCP solution does consider the costs of failure. In the SPR, a planned or a-priori route that minimizes the expected costs of second stage and recourse, is first designed and then a recourse is used in the second stage modify the routes according to changes, for instance to tackle infeasibilities. Usually, the recourse generates a cost (for example due to returning to depot) or a saving that should be considered when the first stage solution is designed. For a further discussion of issues concerning stochastic vehicle routing problems, the reader is referred to the survey paper by Dror et al. [128].

Gendreau et al. [129] divide SVRPs in several classes and state in their survey paper that VRP with Stochastic Demands (VRPSD) is the most studied one. In VRPSD customer demands are random variables usually assumed to be independent. The main earlier contributions in heuristic context involve adaptations of traditional construction heuristics for static VRPs. For recent application, see for example Teodorovic et al. [130] who consider a variant of heterogeneous fleet and proposed a route-first cluster-second approach. A space-filling curve heuristic is used to produce a giant TSP tour and then this tour is divided into smaller parts using generalized Floyd algorithm. Bertsimas et al. [131] test several graph-based a priori heuristics and compare the quality of the produced solutions to sample averages of a posteriori solutions of the deterministic realizations, obtained with clustering heuristic by Christofides [132] and savings heuristic by Clarke and Wright [6]. The authors conclude gaps between 1-5%. Savelsbergh and Goetschalkx [133] propose a new heuristic and provided estimates for travel distance benefits of route re-optimization. The classical VRPSD presupposes that the distribution of the demands at the nodes is known. Popovic [134] neglects this assumption, and uses a Bayesian approach to solve the real problem. The suggested approach translates the uncertain problem to VRPSD and the use a simulated annealing based approach to tackle the problem. Haughton [135] develop a model for estimating whether route modification yields greater logistical efficiency than fixed routes, using statistical calibration. The author focuses on VRPSD and estimates also the transportation and inventory effects of persuading customers to stabilize their ordering patterns. The author used modified version of Clarke and Wright [6] savings heuristic.

Ong et al. [136] studied a variant with multiple time periods and proposed a sequential insertion heuristic inspired from Solomon [11] and selection criteria for customers to be served. Haughton and Stenger [137] compare different strategies for addressing delivery shortages in stochastic demand settings and develop accurate distance prediction models to tackle the problem. The authors suggest that direct and indirect costs of route re-optimization may make it desirable to use a fixed set of routes each day and an inventory buffer on each route. Another key contribution of the research is that it determines the information cost threshold for accepting/rejecting route re-optimization. The authors use well-known heuristic procedures by Clarke and Wright [6] and

Gendreau et al. [47] to evaluate different strategies. Haughton [138] present a similar study, and provide analytically derived and statistically calibrated models for determining the economic benefit of route re-optimization in the context of SVRP. The author concludes that superiority of route re-optimization depends on the specific combination of cost coefficients and other inputs. Protonotarius et al. [139] tackle a large-scale real-life problem with limited travel time, and stochastic multiple product demand and time windows at customers. A genetic algorithm with cluster-first, route-second framework is used to solve the relaxed problem, where violation of capacity and time constraints is penalized in the objective function. Teodorovic and Lucic [140] describe an intelligent modification of well-known Sweep heuristic that learns from the solutions obtained before, assuming that the future situations are known.

Secomandi [141] proposes a set of rollout algorithms to improve any initial tour, and Secomandi [142] compares the performance of two neuro-dynamic programming (reinforcement learning) algorithms, optimistic approximate policy iteration and a rollout policy, for VRPSD with single vehicle. Instead of adopting the simple recourse action of returning to the depot whenever the vehicle runs out of stock, Yang et al. [143] propose an approach where points along the route at which restocking is to occur are designed into the route. More precisely, instead of waiting for route failures to occur, the authors propose ”preventive breaks” at strategic points the planned route, when the vehicle is close to the depot and is near full capacity. Another form of recourse could be to re-optimize the remaining portion of the route upon each failure. The authors develop two heuristic algorithms for the problem. The first algorithm uses the route-first cluster second procedure, whereas the second uses the cluster-first, route-second approach.

Another well-studied SVRP is VRP with Stochastic Customers (VRPSC). Here customers are present with some probability but have deterministic demands. Finally, VRP with Stochastic Customers and Demands (VRPSCD) combines the above two problems, VRPSD and VRPSC. The VRPSCD is extremely hard problem. Even computing the value of the objective function is hard. For recent applications, see for example Gendreau et al. [144] who applied tabu search and proposed a proxy function for calculating costs of candidate solutions. For more details regarding all mentioned problem types, we refer to survey by Gendreau et al. [129]. Labermeier et al. [145] investigate VRPSCD in the real-life application of book pickup and deliveries, and present a simulated annealing metaheuristic, coupled with clustering heuristic and classical arc exchange neighborhoods.

Cheng et al. [146] consider a time-constrained routing problem with fuzzy due-times, where in addition to traditional objectives of number of vehicles and total distance or waiting time, also average grade of customer satisfaction with respect to service time is considered as objective. To tackle the problem, the authors propose a genetic algorithm coupled with insertion heuristic and push-bump-throw heuristic procedure to determine the best service time.

6.2 Dynamic or Real-time routing

Above we have assumed that routes are planned based on stochastic/probabilistic information, before the vehicle leaves the depot in the morning. Since all information, deterministic as well as stochastic is incorporated in the model before determination of the routes, these models are static. Another possibility is to construct routes during the day, i.e., in real-time while the vehicle is on-route. Dynamic vehicle routing problems are important for several reasons. Real-time distribution scenarios are becoming more and more common. In the past few years, there has been a rapid growth in communication and information technologies (e.g., global positioning satellites, cellular phones, geographic information systems, geosynchronous satellite-based systems, etc.) These recent advances afford opportunities for using real-time information to enhance the performance of decision systems in the area of vehicle routing.

Psaraftis [147] defines the Dynamic VRP (DVRP) as follows: A VRP is dynamic if information on the problem is made known to the decision maker or is updated concurrently with the determination of the set of routes. By contrast, if all inputs are received before the determination of the routes and do not change thereafter, the problem is termed static. Practical applications of DVRP include delivery of petroleum products and industrial gases, courier services, inter modal services, tramp ship operations, combined pickup and delivery services, management of container terminals, routing of production robots, paratransit, emergency services and share-a-cab services.

Because real-time VRPs are hard and quick response times are required, exact algorithms are not yet capable of handling problems of realistic sizes. This justifies the use of heuristics. Most of the first approaches for solving dynamic VRPs were straightforward adaptations of static procedures. In these approaches, a static VRP is solved either exactly or heuristically each time an input update occurs. For details, see surveys by Psaraftis [147,148], and PhD thesis by Larsen [149]. Psaraftis' [147] review article on DVRP focuses on three issues: real-world examples, technological advances that enhance the interest on DVRP and methodological advances of solution techniques. Other surveys can be found in Lund et al. [150] and in Powell et al. [151], who concentrate on stochastic programming based models, but do also provide an excellent survey on various dynamic vehicle routing problems. Gendreau and Potvin [152] conclude in their recent survey that more attention should be given to demand forecasting, and uncertainty regarding for example cancellation of requests and service delays. Bianchi [153] presents a review on main properties of the DVRP and the main methodological advances for the problem. Recent adaptations of static algorithms for DVRP can be found in Gendreau et al. [154] and Shen and Potvin [155], who used a neural network to elaborate an expert consulting system for a dispatcher working in a courier service company. In the same context, Benyahia and Potvin [156] proposed an approach based on genetic programming.

Rego and Roucairol [157] describe two-phase algorithm for a real-life dynamic tank truck dispatching problem with multiple depots. The initial routes are created using a decomposition method and subsequently improved using a tabu search method. The tabu search procedure is based on specific compound moves that attempt to improve two or three routes at each step. Savelsbergh and Sol [158] present a branch-and-price algorithm combined with approximation and incomplete optimization techniques as well as a heuristic column management scheme (utilizing construction and improvement algorithms) for dynamic real-life routing problem faced by a large road transportation company. Hall [159] demonstrates how the constraints of overnight delivery affect the design of dynamic pickup and delivery systems involving transport between and within metropolitan regions. Lund et al. [150] study dynamic VRPTW inspired from a real-life oil delivery problem. The authors used modified version of Solomon's insertion heuristic to solve current problem from scratch every time an input revision has occurred. Simulation is used to evaluate the procedure. The authors conclude that the traditional strategy of solving static VRP each time input update has occurred is appropriate only in situations where the degree of dynamism, defined as how much input of a problem changes over time, is very weak.

Kilby et al. [160] look how the cost of the system is affected by the degree of dynamicity, what they define as the proportion of parcels known beforehand. They consider also cost of maintaining a commit horizon where the schedule is fixed ahead of time for communications or other business reasons. The authors introduce some new benchmark data derived from classic VRP benchmarks and use a cheapest insertion method combined with well-known move neighborhoods, such as 2-opt and Or-opt for tackling the problem. Shieh and May [161] present online (that accepts data one by one in their input sequence) local search heuristics that always produce result, no matter how much deliberation time is available (anytime algorithm). The initial solution is created with modified Solomon's insertion heuristic and improved with well-known 2-opt and Or-opt procedures.

Regan et al. [162] introduce a simulation framework for evaluating dynamic fleet management of truckload carrier fleet operations. The framework is used to evaluate a number of operational strategies including load acceptance, and assignment and reassignment strategies and assess the impact of recent technological advances. Gendreau et al. [163,164] test different heuristics embedded in one strategy for DVRP with time windows and DVRP with time windows and pickups and deliveries. In both papers the authors use adaptive tabu search algorithm with chain exchange-based neighborhood and parallel implementation. Larsen et al. [165] extend the work by Lund et al. [150] for DVRP with Time Windows (DVRPTW). Lund et al. [150] were the first to consider the degree of dynamism and its influence on solution quality. They adapted the well-known insertion heuristic proposed by Solomon [11] and simulated its behavior for varying dynamic levels. The authors conclude that when only a limited number of dynamic requests were introduced, the solution quality was preserved relatively well. Kohout and Erol [166] consider a highly dynamic PDPTW arising in real-life airport shuttle operations. The authors combine a modified version of Solomon's [11] insertion heuristic with an agent-based stochastic improvement technique.

Teodorovic and Radivojevic [167] develop two approximate reasoning algorithms for dynamic DARP, using fuzzy or vague logical statements in the formation of the algorithms. The first reasoning algorithm is used to allocate requests for vehicles and the second reasoning algorithm is used to design new route and schedule for the chosen vehicle. Other contributions on dynamic dial-a-ride systems can be found in Madsen et al. [29] and Dial [168]. Yang et al. [169] consider a real-time multi-vehicle truckload PDP. In this problem, every job's arrival time, duration and deadline are known. The authors allow dispatcher some time to report the final decision on job acceptance to each customer. Based on off-line formulations, the authors introduce various rolling horizon real-time policies for tackling the problem. Mahmassani et al. [170] present strategies for real-time PDPTW with full truckloads and computing time constraints. The proposed strategies combine dynamic local heuristic rules for quick initial assignment with procedures for load reassignment. The authors conclude that reassignment strategies have a significant potential to improve real-time system performance.

Ichoua et al. [171] propose a new strategy for assigning customer requests, including diversion motivated from a courier service application. More precisely, by diversion the authors mean that it is possible to consider diverting a vehicle away from its current destination in response to a new customer request. An empirical evaluation of the proposed approach is performed within a parallel tabu search approach with an adaptive memory. The objective is to minimize weighted sum of total distance and total lateness over all customers. Ichoua et al. [171] propose a strategy for exploiting information about future demands (e.g. historical averages, probability distributions, etc.) in real-time routing context. More precisely, the proposed strategy introduces dummy customers (representing forecasted requests) in vehicle routes to provide a good coverage of the territory. The effectiveness of the proposed strategy was assessed by implementing it within a parallel tabu search heuristic, previously reported in Gendreau et al. [164]. Slater [172] specifies a dynamic vehicle routing system for e-commerce environment that allows customers to select their own delivery time windows. The methodology is based on demand forecasting that leads to the generation of phantom orders and routes.

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