



3rd Trondheim Gas Technology Conference, TGTC-3

Optimization Model to Analyse Optimal Development of Natural Gas Fields and Infrastructure

Kjetil Trovik Midthun^{a*}, Marte Fodstad^b, Lars Hellemo^a

^aSINTEF Technology and Society, Department for Applied Economics and Operations Research, NO-7465 Trondheim, NORWAY

^bSINTEF Energy Research, Department for Electrid Power Systems, NO-7465 Trondheim, NORWAY.

Abstract

We present an optimization model for analysis of system development for natural gas fields, processing and transport infrastructure. In this paper we present our experience from performing analyses for the natural gas industry with the optimization model. We also present a model extension in the form of continuous investment decisions. This extension allows the capacity in pipelines, processing facilities and compressors to be determined within a given range by the model. We also give a partial model description along with a case example that demonstrates the importance of using continuous investment decisions when considering design in natural gas systems.

© 2015 Published by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

Peer-review under responsibility of the Scientific Committee of TGTC-3

Keywords: Natural gas, investments analysis, optimization, decision support

1. Introduction

Thorough infrastructure design and investment analysis is crucial to the decision makers in the natural gas industry due to the large costs associated with production fields, processing facilities, compressor stations, pipelines and other infrastructure elements. The ability to value flexibility and identify bottlenecks in the system is also of importance due to the large value created by the production of natural gas. The decision maker needs to decide on which elements to invest in at what time and with what capacity, which gives a large combinatorial decision space that is almost impossible to explore by hand. Combinatorial optimization models are well suited to analyse such

* Corresponding author. Tel.: +47 97176881
E-mail address: Kjetil.Midthun@sintef.no

problems. The optimization model that we will use as a basis for our discussion in this paper is developed with the Norwegian Continental Shelf as a motivating case. The results and discussions will however be valid for other gas production systems. The Norwegian system consists of approximately 7800 km of subsea pipelines with large diameters operated at high pressure levels. Another important aspect of natural gas production and transportation is the multi-commodity characteristic of natural gas. The gas consists of several different components, such as methane, propane, butane, CO₂ and H₂S that contribute to the properties of the gas properties in different ways. These properties will influence the need for processing capacity and the possibilities for blending gas to meet quality specifications in markets. The optimization model that we discuss in this paper can be used for analysis of multi-commodity flows, but in this paper we will simplify the presentation by focusing on single-component flows.

An optimization model that has been used by both authorities and companies that invest in natural gas infrastructure is presented in [1]. The model we use in this paper, Ramona, is presented in more detail in [2]. It extends the model presented in [1] with more details on the operations of the network (such as pressure-flow modelling and gas quality). This way it can be used to analyse projects such as branch-offs and the trade-off between processing plants and blending of natural gas from different fields. The basis for the modelling of natural gas transport is given in [3] and [4].

The problem of designing offshore oil and gas infrastructure has received considerable attention over several years. Some early examples, mainly focusing on the development of reservoirs and wells of single fields, are [5] and [6]. [1], [7], [8], and [9] take a network perspective coordinating multiple fields. Several papers also model the influence of uncertainty, see for instance [10] on market uncertainty and [11] on uncertainty in reservoir properties. [12] use an equilibrium model for the investment planning, in contrast to most other papers that use mixed-integer linear or non-linear models. [13] limits the scope to the transportation network and describe the problem of optimal pipeline dimensioning when the network structure is given. Related problems are treated by [14] who discuss the problem of network expansion for a given transportation demand and [15] who combine network expansion and liquefied natural gas terminal location. There are two traditions for how to describe network capacity choices, while for instance [1], [13] and [15] model discrete capacities, [8] and [14] allow for a continuous capacity choice.

The main contribution of this paper is a presentation of experiences with using an optimization model for providing decision support on natural gas infrastructure design. We will also discuss a model extension which allows for continuous capacity investments in pipelines, processing facilities and compressor stations.

In the next section we will shortly describe the optimization model that we have used in our analyses, and give the continuous capacity extension formulation. In Section 3, we present a numerical example to illustrate the importance of using continuous capacity decisions for the investments. In Section 4 we discuss some of the experiences from real world applications of the model in general and the continuous formulation in particular. We conclude the paper in Section 5.

2. The optimization model

Ramona is a mixed-integer linear optimization model that includes both investment decisions and operational decisions. The level of detail in the operational decisions will vary with the availability of data and the size of the problems. Pressure-flow relationships, compressors, operation of processing facilities as well as multi-commodity flows are optional features in the model. The modelling of multi-commodity flows allows for inclusion of gas quality management also in the design decisions. For more details on this aspect, see [2] and [16].

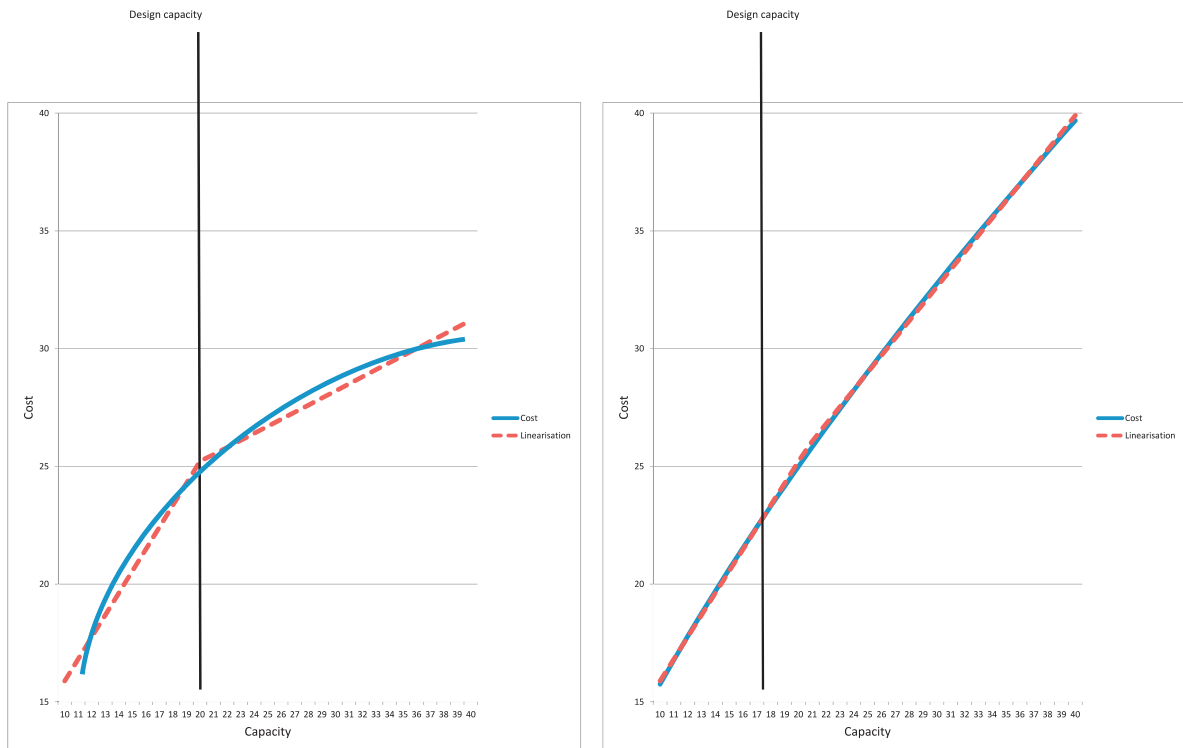
The investment decisions are modelled as binary variables for deciding between different alternative investments as well as timing for the investments. Fields have predefined production profiles, but are given some flexibility to withhold production capacity to adjust to the available capacities in the rest of the network, see [1] for details. For pipeline and processing capacity we have two alternative model formulations. The first alternative has predefined discrete capacity choices selected with the binary investment variables. The other formulation has a continuous capacity choice within given bounds.

2.1. Continuous capacity decisions

To build continuous capacity decisions into the model we need an estimate of how the costs of the installed component vary with respect to changes in installed capacity. One example of such a relationship is provided by [17]:

$$C_1 = C_0 \left(\frac{x_1}{x_0} \right)^{\frac{2}{3}}, \quad (1)$$

where C_0 is the total cost of installing a capacity of x_0 and C_1 is the total cost of installing a capacity of x_1 . In general, such relationships between costs and capacity will only be valid within certain bounds. In our model we have added upper and lower limits to the capacity investment to stay within the region where Equation (1) is valid (the bounds will vary with the type of project). This expression can be linearised in several ways, depending on the required accuracy. For the analyses we have performed in the natural gas systems, the capacity choice typically range from half the design capacity (x_0) to twice the design capacity. Given these bounds, we found that a linearisation with two line segments was sufficient for representing the cost function accurately (within 1% deviation). The two line segments were divided such that the first line segment covers the capacity between the minimum capacity and the design capacity, while the



other line segment covers the capacity between the design capacity and the maximum capacity. An illustration of the linearisation is shown in Figure 1.

Figure 1: An illustration of the linearisation of the relationship between capacity and investment cost for a facility. The figure on the left exaggerates the curvature to illustrate the linearisation technique, while the figure on the right shows the actual linearisation used in our analyses.

2.2. Mathematical Formulation

We now present the mathematical formulation used for the continuous capacity choices. This formulation provides the approximate relationship between capacity and cost, and also makes sure the timing of incurred cost and available capacity corresponds to the investment timing. We formulate the capacity choices with the help of additional binary variables that determine which line segment of the cost approximation is active. In addition, we use continuous variables that represent changes in capacity from one time period to the next. We use the following notation:

Nomenclature		
$k \in \mathbf{K}$	Line segment on approximate investment cost function, $\mathbf{K}\{Low, High\}$	index and set
$p \in \mathbf{P}$	Project/network element	index and set
$t, \tau \in \mathbf{T}$	Time	indexes and set
A_p^k, B_p^k	Regression parameters	parameters
$CP_{p,t}$	Share of investment cost t years after start	parameter
\bar{X}_p	Maximum capacity	parameter
$x_{p,t}$	Capacity in period t	decision variable
$\Delta x_{p,t}$	Changed capacity from previous period	decision variable
$c_{p,t}$	Cost in period t	decision variable
$\alpha_{p,t}, \beta_{p,t}$	Investment (start) and disinvestment (stop)	binary decision variables
$\Delta x_{p,t}^H$	Changed capacity from previous period (above design capacity)	decision variable
$\Delta x_{p,t}^L$	Changed capacity from previous period (below design capacity)	decision variable
γ_p	Indication on which line segment of the cost function that is active	binary decision variable

The constraints added to the mathematical model are:

$$\Delta x_{p,t}^k \leq \alpha_{p,t} \bar{X}_p, k \in \mathbf{K}, p \in \mathbf{P}, t \in \mathbf{T} \quad (2)$$

$$\sum_{t \in \mathbf{T}} \alpha_{p,t} \leq 1, p \in \mathbf{P} \quad (3)$$

$$x_{p,t} \leq x_{p,t-1} + \sum_{k \in \mathbf{K}} \Delta x_{p,t}^k, p \in \mathbf{P}, t \in \mathbf{T} \quad (4)$$

$$\Delta x_{p,t}^L \leq \bar{X}_p (1 - \gamma_p), p \in \mathbf{P}, t \in \mathbf{T} \quad (5)$$

$$\Delta x_{p,t}^H \leq \bar{X}_p \gamma_p, p \in \mathbf{P}, t \in \mathbf{T} \quad (6)$$

$$x_{p,t} \leq \bar{X}_p \left(1 - \sum_{\tau \in \mathbf{T} : \tau \leq t} \beta_{p,\tau} \right), p \in \mathbf{P}, t \in \mathbf{T} \quad (7)$$

Constraint (2) ensures that the change in capacity cannot be larger than zero in periods where the binary variable α_{pt} is not equal to 1. This binary variable can only take a non-zero value in one time period, as stated in Constraint (3), and this means that the full capacity decision must be made at one point in time. Constraint (4) gives the capacity in time period t as the sum of the capacity in time period $t-1$ plus the change in capacity in time period t . The change in capacity in time period t is divided into two parts due to the two line segments used to linearise the relationship between costs and capacity. Constraints (5) and (6) are used to provide the binary variable γ_p with a value that indicates whether or not the capacity extension was above the design capacity for the facility. When γ_p has a value of 1, the capacity extension is higher than the design capacity, whilst when it is 0, the capacity extension is lower than the design capacity. Constraint (7) is used to reduce the capacity in a facility to 0 from the time it is stopped (the variable β_{pt} equals 1) and in all subsequent periods. In the model we also include the delay between the investment decision and the actual start-up of production (corresponding to the time required to build and install a facility). This is omitted from our presentation here since it does not affect the linearisation formulation directly. To model the investment cost, $c_{p,t}$, we use the following expression:

$$c_{p,t} \geq \sum_{\tau \in \mathbf{T} : \tau \leq t} CP_{p,\tau-t+1} \sum_{k \in \mathbf{K}} \left(A_p^k + B_p^k \Delta x_{p,\tau}^k \right), p \in \mathbf{P}, t \in \mathbf{T} \quad (8)$$

3. Numerical Example

To illustrate the effect of using continuous capacity decisions in our model, we present a simple example. The network topologies that we consider are illustrated in Figure 2. The network consists of 3 fields that can be developed, a field hub that can tie the fields together, 2 potential processing plants, a compressor station and a natural gas market. In addition, there are pipelines that tie the installations together. For simplicity we have only made the investment in processing facilities discrete in this example. That means that the compressor station, as well as all pipelines in the model, will have continuous capacity choices. For the processing plants however, there are two possible choices: one facility with a processing capacity of 16.6 MSm³ per day, and another with a capacity of 37.45 MSm³ per day. The three fields have different production profiles and different time windows when they can be invested in. The largest field (Field A) has a maximum production rate of 16.6 MSm³ per day and can start production in 2019. Field B has a maximum production rate of 12.5 MSm³ per day and can start production in 2024, whilst the smallest field (Field C) has a maximum production rate of 8.3 MSm³ per day and can start production in 2029. Figure 3 illustrates the production profiles and the earliest production start for the 3 fields. The 2 processing plant alternatives are designed such that either a small capacity is started together with Field A, and then supplemented with the other fields later, or all fields can be started simultaneously filling the large processing capacity.

The investment decisions include which fields, pipelines and processing plants to invest in as well as the timing. The solution of the model is given in Figure 4. The figure illustrates the production from the different fields, as well as the timing of the investments which is given by the start of production. The chosen processing capacity is 37.45 MSm³ per day, and the facility is fully utilized when all fields are started. The net present value for this solution is 33.75 billion NOK.

As an alternative, we have also solved the same network with continuous capacity choices for the processing facility. This leads to the network shown to the right in Figure 2, and the corresponding solution is shown in Figure 4. The net present value in this case is 36.34 billion NOK that is 2.6 billion NOK more than in the instance where the discrete capacity choices were used. The capacity in the processing facility has been decreased from 37.45

M Sm^3 per day to 29.13 M Sm^3 per day. The main consequence of this in terms of timing is that the start of field C has been delayed for 2 years as compared to the solution with discrete capacity choices. The cash flow from Field C will then be delayed, but the cost of delaying this cash flow is more than compensated for by the lower investment cost in the processing facility.

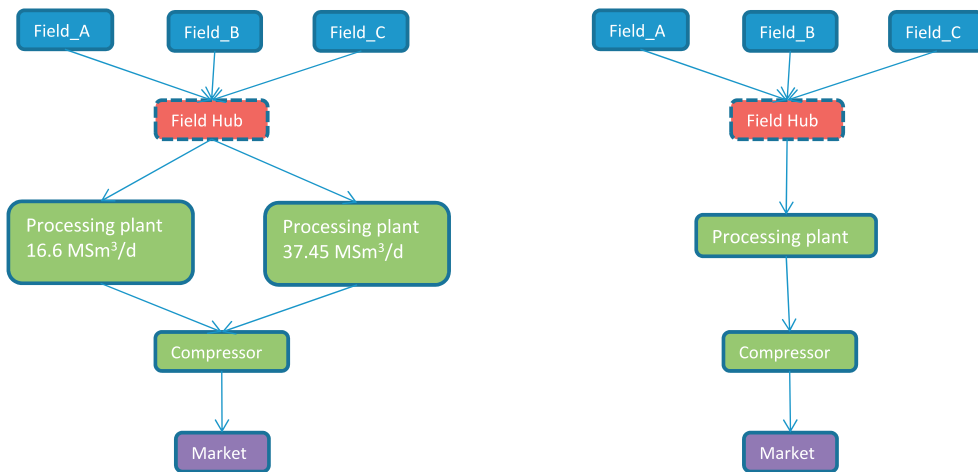


Figure 2: Illustration of the possible investments that can be made in the numerical example. The figure on the left illustrates the situation with two discrete choices for the processing plant, while the figure on the right shows the situation when the capacity in the processing plant is a continuous variable.

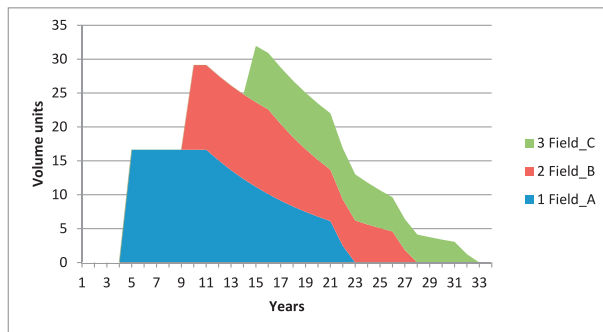


Figure 3: The production profile of the 3 fields in the numerical example. The illustration shows the resulting production profile if all fields were started as soon as possible and produce according to their maximal production profile.

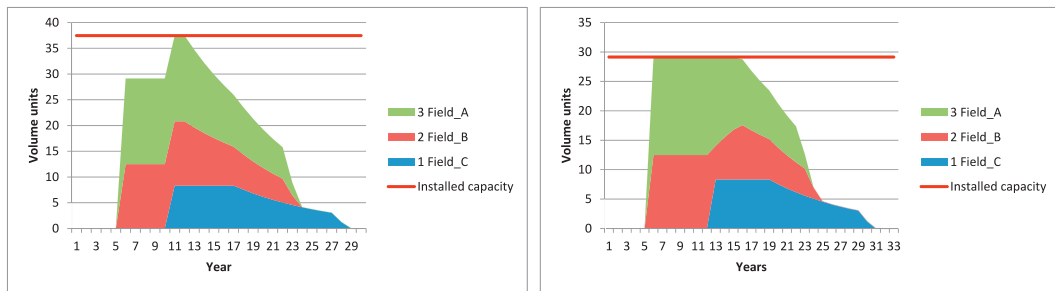


Figure 4: The solution for the numerical examples with discrete capacity (to the left) and continuous capacity (to the right) choices for the processing plant.

4. Application Experiences

In our simple numerical example it may be possible to find and test the different relevant capacity choices for the processing facility. That might also be possible for typical revision and expansion planning in real cases, where large parts of the network and production rates are fixed and the decision flexibility is limited. When designing the network in new areas on the other hand, a problem with a realistic size can contain substantially more investment options. The number of alternatives needed for such an analysis with discrete capacity choices grows rapidly in the number of projects. If, for instance, two pipelines supplying a processing facility have two different capacity alternatives each, the processing facility would need eight alternatives to match all pipeline options. Assuming yearly resolution with a 15 year startup window and 30 year horizon with shut-down possibilities these three network elements alone give 540 binary variables, as shown in Table 1. The table also shows the growth when the number of pipeline alternatives is increased and the resulting number of binary variables in the model given by multiplying the number of alternatives and the possible start or shut-down years. For the continuous formulation the corresponding number of network elements will be three and the number of binary variables 138. Describing discrete alternatives becomes even harder when fields are introduced, since the production profile of each field can be modified by the model (by decreasing the production rates and delaying production), which makes the theoretical number of possibly optimal processing and transportation capacities infinite.

Table 1: Model sizes with discrete capacity choices assuming two pipelines with different capacity alternatives supplying one processing plant, each with 15 year start window and 30 year horizon.

Alternatives per pipeline	Process alternatives	Start variables	Stop variables	Variables in total
1	3	75	150	225
2	8	180	360	540
3	16	330	660	990
4	32	600	1200	1800

5

64

1110

2200

3300

The consequence of adding a large number of alternatives are manifold. Firstly, it makes the model size, in terms of computational complexity, grow quickly (see Table 1). This can, to some extent, be counteracted by linking the variables with constraints. Secondly, a large number of alternative projects will make the visual representation of the investment cases almost unreadable for the users. Thirdly, adding a large number of projects will increase the probability of mistakes and misrepresentations in the data. Lastly, there is a large workload associated with building these cases, due to the large number of projects, and the need for quality assurance on all the input data. By using continuous capacity investments, the case specification becomes considerably more straightforward.

In addition to the problems of quality assurance and workload in terms of building the network, there is also a significant risk of excluding good solutions when specifying the discrete project alternatives. Given that the number of alternatives that can be handled is limited, one must choose carefully which ones to include. One possible strategy for increasing the number of alternatives is to consider the solution from one model run, and add new projects with higher capacity in the areas where the capacity limit is reached. This strategy will however not work when the optimal solution would have been structurally different if other capacity choices had been available elsewhere in the network. By using continuous capacity choices in the model, we make sure that the relevant range of capacity investments are considered for all projects. In practice, using discrete capacity choices implies that the user needs to a priori choose possible network patterns to make sure that capacities all through the network matches, rather than concentrating on describing the single network elements and letting the model take care of the network coordination.

Using continuous capacity decisions in a mixed-integer linear model implies approximating the investment costs rather than providing the exact cost, which is a disadvantage compared to discrete capacity alternatives. The cost functions and investment ranges used in our case gave a sufficient accuracy with only two line segments. Cost functions with a stronger curvature would give a poorer accuracy, as illustrated in the left part of Figure 1. As long as the cost function is concave this can be counteracted by using more line segments in the approximation. Naturally this increases the problem size in terms of more binary variables, but it should be noted that this increase is isolated to the element in question and would thereby give a lower overall growth rate than what is described in Table 1.

Finding the 'optimal' investment decision is an obvious motivation when working with optimization based investment models. When applying such models for decision support we experience a wider range of motivations. For multiple reasons, the comparison of model results from different input data assumptions is at least as important as finding a single best solution. A model will typically not capture all aspects that affect an investment decision. For instance, the choice between offshore or onshore processing and the location of onshore processing can have substantial knock-on effects outside the model scope. By changing input data, changing model assumptions or fixing some variables, the model can be used to compare such choices. Furthermore, some input data, for instance the size and location of natural gas reservoirs, can be highly uncertain, and solving the problem for several resource scenarios is a classical way to analyse the consequence of resource estimate errors. This is a much discussed strategy since each model run will not value solution properties like flexibility or robustness to withstand the uncertainty. On the other hand, such scenario analysis can evaluate the value of improved resource estimates and thereby the willingness to pay for further exploratory efforts.

5. Conclusions

We have presented a model extension to an existing investment analysis model for natural gas infrastructure, as well as experiences from using the model in real-life applications. The model extension is the use of continuous capacity investments in facilities in the natural gas network, such as processing plants, compressors and pipelines. This extension both makes the analysis more efficient and robust. The efficiency gain comes from the reduction in the number of projects that must be specified in the model, while the robustness comes from the improved specification in terms of covering the relevant ranges for capacity choices in all parts of the network. The larger system studies we consider, the larger the expected gain from introducing the continuous capacity investments are.

References

- [1] Nygreen, B., Christiansen, M., Haugen, K., Bjørkvoll, T. & Kristiansen, Ø. (1998), 'Modelling Norwegian petroleum production and transportation', *Annals of Operations Research* 82, 251-267.
- [2] Hellemo, L., Midthun, K., Tomasgard, A. & Werner, A. (2012a), 'Natural gas infrastructure design with an operational perspective', *Energy Procedia* 26, 67-73.
- [3] Rømo, F., Tomasgard, A., Hellemo, L. & Fodstad, M. (2009), 'Optimizing the Norwegian natural gas production and transport', *Interfaces* 39(1), 46-56.
- [4] Midthun, K. T. (2007), Optimization models for liberalized natural gas markets, PhD thesis, Department of Industrial Economics and Technology Management, Norwegian University of Science and Technology, Trondheim (Norway). Theses at NTNU, 2007:205.
- [5] Sullivan, J. (1988), 'The application of mathematical programming methods to oil and gas field development planning', *Mathematical Programming* 42, 189-200.
- [6] Haugland, D., Hallefjord, Å. & Asheim, H. (1988), 'Models for petroleum field exploitation', *European Journal of Operational Research* 37, 58-72.
- [7] Aboudi, R., Hallefjord, Å., Helgesen, C., Helming, R., Jørnsten, K., Pettersen, A. S., Raum, T. & Spence, P. (1989), 'A mathematical programming model for the development of petroleum field and transport systems', *European Journal of Operational Research* 43, 13-25.
- [8] van den Heever, S. A. & Grossmann, I. E. (2001), 'A Lagrangean decomposition heuristic for the design and planning of offshore hydrocarbon field infrastructure with complex economic objectives', *Industrial & Engineering Chemistry Research* 40, 2857-2875.
- [9] Carvalho, M. C. A. & Pinto, J. M. (2006), 'An MILP model and solution technique for the planning of infrastructure in offshore oilfields', *Journal of Petroleum Science and Engineering* 51, 97-110.
- [10] Jørnsten, K. O. (1992), 'Sequencing offshore oil and gas field under uncertainty', *European Journal of Operational Research* 58, 191-201.
- [11] Tarhan, B., Grossmann, I. E. & Goel, V. (2009), 'Stochastic programming approach for the planning of offshore oil or gas field infrastructure under decision-dependent uncertainty', *Industrial & Engineering Chemistry Research* 48(6), 3078-3097.
- [12] De Jonghe, C., Hobbs, B. & Belmans, R. (2011), Integrating short-term demand response into long-term investment planning, Technical report, Faculty of Economics, University of Cambridge.
- [13] Zhang, J. & Zhu, D. (1996), 'A bilevel programming method for pipe network optimization', *SIAM Journal of Optimization* 6(3), 838-857.
- [14] André, J. (2010), Optimization of investment in gas networks, PhD thesis, Université Lille Nord de France.
- [15] Zheng, Q. P. & Pardalos, P. M. (2010), 'Stochastic and risk management models and solution algorithm for natural gas transmission network expansion and LNG terminal location planning', *Journal of Optimization Theory and Applications* 147(2), 337-357.
- [16] Hellemo, L., Midthun, K., Tomasgard, A. & Werner, A. (2012b), Stochastic programming- Applications in finance, energy, planning and logistics, World Scientific, chapter Multi-stage Stochastic Programming for Natural Gas Infrastructure Design with a Production Perspective.
- [17] Moore, F. T. (1959), 'Economies of scale: Some statistical evidence', *Quarterly Journal* 73, 232-245.