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Parametric Study on Masonry Arches Using 2D Discrete Element Modeling

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Abstract:	Maintenance of existing structures requires a solid understanding of their structural behavior with their varying geometry, boundary conditions, and load cases. For the case of the masonry arches, this understanding is still lacking in the field, yet, masonry arches are crucial parts of the railway and highway network systems in many countries. In this paper, two dimensional numerical models were prepared to simulate the nonlinear response of masonry arches under static loading without soil-structure interaction effects. A custom-made Discrete Element Method (DEM) software was employed for this research, such that the models represent a discontinuous medium of rigid blocks. Different scenarios were generated on a hypothetical masonry arch model to observe the influence of different parameters on the structural behavior of masonry arches. Investigated parameters include: effect of soil infill and spandrel walls, bond pattern at the arch barrel for double layer arches, and boundary conditions. In addition, the discrete element approach and the software were validated by an experimental work from literature. The results of the analyses show that discrete element modeling is a powerful technique, which demonstrates the development of collapse mechanisms of masonry arch structures. Parametric analyses also indicated that soil infill and spandrel walls, if intact, can have beneficial effects on the load carrying capacity of arches. Based on the results of this study, the bond pattern between arch layers does not make a significant difference in the overall behavior. As expected, boundary conditions matter, and should be taken into careful consideration for each masonry arch bridge through detailed observations on site.
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Suggested Reviewers:	Carlo Citto ccitto@ana-usa.com Mr.Citto is a professional engineer in Atkinson-Noland and Associates. One of the motivation of this paper to provide a link between advanced numerical models and engineering practice. In this context, Mr.Citto would make a contribution to this article to improve the link between both sides.
	Jose Lemos vlemos@lnec.pt Dr.Lemos has a great contribution to the literature on discrete element modeling of masonry structures. In this context, numerical modeling part of this paper would be strengthened with his comments and opinions.
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Parametric Study on Masonry Arches Using 2D Discrete Element Modeling

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ABSTRACT: Maintenance of existing structures requires a solid understanding of their structural behavior with their varying geometry, boundary conditions, and load cases. For the case of the masonry arches, this understanding is still lacking in the field, yet, masonry arches are crucial parts of the railway and highway network systems in many countries. In this paper, two dimensional numerical models were prepared to simulate the nonlinear response of masonry arches under static loading without soil-structure interaction effects. A custom-made Discrete Element Method (DEM) software was employed for this research, such that the models represent a discontinuous medium of rigid blocks. Different scenarios were generated on a hypothetical masonry arch model to observe the influence of different parameters on the structural behavior of masonry arches. Investigated parameters include: effect of backfill and spandrel walls, bond pattern at the arch barrel for double layer arches, and boundary conditions. In addition, the discrete element approach and the software were validated by an experimental work from literature. The results of the analyses show that discrete element modeling is a powerful technique, which demonstrates the development of collapse mechanisms of masonry arch structures. Parametric analyses also indicated that backfill and spandrel walls, if intact, can have beneficial effects on the load carrying capacity of arches. Based on the results of this study, the bond pattern between arch layers does not make a significant difference in the overall behavior. As expected, boundary conditions matter, and should be taken into careful consideration for each masonry arch bridge through detailed observations on site.

KEYWORDS: Masonry, Discrete element model, Masonry Arch, DEM, Collapse Mechanism

Introduction

Throughout history, masonry arches were used to span relatively large distances as a common structural form dating back to Roman Empire. Many of them are still in use in road and railway networks in Europe and northeast United States. Until the first half of the nineteenth century, design and analysis of these structures had been performed by using empirical methods and common construction techniques (Brencich and Morbiducci 2007). With recent developments in modern mechanics and the increased computational capacity, more detailed structural analysis of masonry arches became possible.

In the last three decades, discrete element based modeling, including DEM, discontinuous deformation analysis (DDA) and combined discrete/finite element analysis, has become a widely used solution to model masonry structures. It has an advantage over continuum-based methods, due to its inherently discontinuous medium and compatibility with the nature of masonry structures (e.g. composed of separate or distinct units). In this study, DEM, which can be classified as a simplified micro modeling approach, was used to investigate the response of a typical masonry arch under static loading (Lourenço 2009).

The goal of the paper is to simulate the realistic behavior of masonry arches and demonstrate the impact of different parameters: boundary conditions, bond patterns for double ring arches and existence of spandrel walls or backfill. A detailed discrete element model was generated to validate the custom-made software with previous experimental work from literature. In addition, the obtained results were compared with limit analysis for two case studies including arch with and without backfill.

Background

The modern mechanics of masonry arches begins with the introduction of plastic analysis by Heyman (1966) in order to assess the load carrying capacity. The collapse of the masonry arches is considered as a geometrical problem and series of assumptions are employed: *(i)* stone has no tensile strength; *(ii)* friction between voussoirs is high enough to prevent sliding failure; *(iii)* the masonry has infinite compressive strength. Heyman indicated that plastic hinges appear where the line of thrust touches either extrados or intrados of the arch and turns out a collapse mechanism. This is often referred to as the mechanism method. Therefore, plastic design method, previously used as a technique to analyze rigid-plastic structural frames, was applied to unreinforced masonry structures by Heyman and limit analysis became a widely employed tool to analyze masonry structures. Later, important contributions were made on the Heyman's theorem by limiting the infinite compressive strength to material crushing strength and accounting for the possibility of sliding (Gilbert 2007).

Linear programming (LP) technique, first applied by Livesley (1978) on masonry arches formed by rigid blocks, has been exhaustively used in the literature to solve equilibrium and mechanism formulations of limit analysis. Recently, using dual LP framework, rigid elements and homogenized interfaces, where deformations were lumped at the joints, were used to analyze masonry arches and double curvature shell structures (Milani et al. 2008, Milani 2015). Still, limit analysis is one the most common methods. It is easy to use and requires fewer number of input parameters. It can provide the maximum load carrying capacity and related failure mechanisms for an arch structure, despite spending negligible computational time compared to advanced numerical techniques, such as nonlinear finite element analysis (FEA). Hence, it appears that it is an efficient and appropriate technique to analyze masonry arches and vaults (Tralli et al. 2014).

In the last three decades, more detailed and comprehensive numerical approaches were developed, which are used to analyze both modern and historical masonry structures (Lourenço 2002). These are often referred to as continuum and discontinuum (or discrete) models. Analysis may involve “micro” or “macro” modeling, depending on the level of accuracy required. The micro modeling focuses on each part of the masonry by taking into account the unit, mortar and unit/mortar interfaces. In literature, there exits 2D and 3D strategies utilized for micro modeling on different type of masonry structures (Lourenço and Rots 1997; Milani and Lourenço 2012). It is important to note that there are also hybrid and meso-scale models, falling between micro and macro modeling strategies (Zhang et al. 2016). On the other hand, in macro modeling, masonry is described as an equivalent continuum model and nonlinear models capturing the overall structural behavior are used. Unit and mortar are implicitly represented by following a continuity condition at the nodes, as in the case of standard finite element method (FEM) procedure.

Methodology: Numerical modeling

In this study, a custom-made software utilizing discrete element method (DEM) was employed for all the numerical simulations to demonstrate failure mechanism of masonry arches. The software was first employed to analyze masonry dams and then it was used to simulate out-of-plane behavior of masonry walls. For further description about software, the reader is referred to (Bretas et al. 2013, 2014, 2016). DEM falls within the classification of discontinuum analysis. This approach, originally proposed by Cundall (1971), provides an opportunity to model structures as composed of 2D and 3D polygonal blocks that may be rigid or deformable. This method is successfully applied by many researchers on different masonry structures (Bui et al. 2017; Isfeld and Shrive 2015; Lemos 2007; Pulatsu et al. 2017; Simon and Bagi 2014; Tóth et al. 2009)

For the numerical models, the masonry units were modeled as rigid blocks and mortar joints were represented as zero thickness interfaces between each block. The main reason to employ rigid blocks was to take advantage of high compressive strength of stone and brick masonry units (in comparison to mortar) and low computational cost in the analysis. Thus, nonlinear response of masonry arch models was only controlled by the joints where normal and shear springs were assigned in two orthogonal directions (Fig. 1). In the custom-made software, the governing differential equations for translational and rotational motions were integrated through each time step, using an explicit finite-difference method. The static solutions are obtained by dynamic relaxation, using scaled masses. Furthermore, out-of-balance forces are checked in each calculation step and additional load is applied after the stability of the structure is ensured.

A force-displacement law was assigned to each spring and used to calculate stress increments for normal stress ($\Delta\sigma$) and shear stresses ($\Delta\tau$), depending on the considered constitutive model, as given in equations (1) and (2).

$$\Delta\sigma_i = k_n\,(\Delta u_n)_i \tag{1}$$

$$\Delta\tau_i = k_s\,(\Delta u_s)_i \tag{2}$$

Where k_n and k_s are normal and shear stiffness at the joints, respectively; and u_n and u_s are relative displacements in the normal and tangential directions, respectively.

To illustrate a simple case, the external force, *F*, acting on a rigid block (Fig. 2a) and the corresponding stress distribution are given in Figure 2b where σ_t indicates the tensile stress and σ_c shows compressive stress at the joint.

Stresses, calculated at each time step, are corrected according to the given failure criteria as presented in Equations (3) and (4), where *'c'* stands for cohesion and 'θ' indicates the friction angle. In this study, it is assumed that masonry has zero tensile strength and the Coulomb model is used to determine the shear stress (σ_s).

$$\sigma_n < Tensile\ strength \tag{3}$$

$$|\sigma_s| < c + \sigma_n tan\theta \tag{4}$$

New contact forces, F_n for the normal direction and F_s for the tangential direction, are calculated, using the contact lengths, $l_{contact}$ depending on the tension or the compression part of the contact, as shown in Figure. 2b by l_t *and* l_c, respectively. Finally, new position and displacement of the blocks are found in an explicit way. The discontinuous representation of blocks allows to model joint sliding and openings that determine the ultimate load carrying capacity of the structure. An external load is increased until the movements increase without bound, which demonstrates a non-equilibrium state for the structure.

It is important to note that implemented contact type (“face to face” or “edge to edge” in 2D), allows for the use of different stress integration schemes in order to find the resultant contact forces, F_t and F_c, as shown in Figure 2b. Therefore, the contact type, to model mechanical interaction between blocks, provides linear stress distribution along the contact length. These are statically consistent with the stress diagrams and bending stiffness in the linear elastic range. Therefore, different from standard point contact model, commonly used in DEM codes, accurate results are obtained with less number of blocks. However, it is efficient to use simple contact models (e.g. point contact), in the parts where failure is not expected and more rigorous contact assumptions may be employed among the blocks actively participates in failure mechanism.

Material Properties

The strength of masonry units and mortar vary remarkably, especially in case of historical masonry structures. Heterogeneous and composite characteristics of masonry make it further difficult to select a representative number for each of the mechanical characteristics for the constitutive laws employed in the numerical model. Here, zero tensile strength and cohesion at the joints were assigned to replicate dry-joint masonry, where there is no mortar to bind masonry units. Different contact stiffness for both orthogonal directions and related joint properties were given in Table 1. Representative unit weights for the stone units and backfill material were taken from the literature, as 24 kN/m^3 and 20 kN/m^3, respectively (Oliveira et al. 2010).

Validation of the Methodology

In this section, the custom-made software and the numerical approach, used in this study are benchmarked against an experimental study conducted in the Technical University of Catalonia (UPC). Two short span true-scale brick masonry arch bridges were tested under quasi-static loads, applied at quarter span, and the ultimate load carrying capacities were predicted by limit analysis (Roca and Molins 2004). Here, the semi-circular arch, named BA2, spanning 3.2 meter was selected from that study to validate our custom-made software and modeling strategy. The geometrical and material properties were taken from the related article, given in Table 2 and Table 3, respectively. In order to perform static analysis in DEM, contact stiffnesses (k_n and k_s) were calculated according to Lourenço and Rots (1997) using the expressions below.

$$k_n = \frac{E_u E_m}{t_m(E_u - E_m)} \tag{5}$$

$$k_s = \frac{G_u G_m}{t_m(G_u - G_m)} \tag{6}$$

Where E_u and E_m are Young's modulus; G_u and G_m are shear modulus for unit and mortar, respectively and t_m indicates the thickness of joint. Shear modulus was calculated using linear elastic relationship; E / 2(1+v).

The calculated mechanical properties of contacts, tensile and cohesion strength at the joints were shown in Table 4. Specific weight of the backfill (sand) and masonry units (bricks) were considered as 18 kN/m^3 and 20 kN/m^3. Backfill load was taken as dead weight acting on extrados of the arch-barrel and external load dispersion through the backfill was applied at the quarter span according to the Boussinesq distribution model with an angle of $\pi/6$. Hence, the spandrel walls were not used actively during the load application. Both ends of spandrel wall and bottom part of the abutments were restrained during loading.

Through incremental loading, the damage procession on the masonry arch bridge was observed and a point at the intrados of the arch ring, located at $1/4$ of the span, was monitored. First, separation of the spandrel wall and arch ring was noticed and failure occurred because of the formation of 4-hinge mechanism. The capacity curve obtained by DEM and limit analysis results from Roca & Molins (2004) are given in Figure 3 together with the experimental peak load. It is worth noting that the observed failure mechanism matched experimental observations very closely as shown in Figure 4a. Then, successive plastic hinges developed starting from extrados of the arch barrel where the load was applied. Discrete element model did not only capture the experimental peak load, but also demonstrated the damage progression up to failure, as shown in Figure 4b.

According to the numerical simulations and experimental work, collapse mechanism may change significantly depending on load path. In other words, separation between arch-ring and spandrel wall may occur where the loads are not applied on the spandrel wall.

Parametric Study

In this parametric study, the validated numerical modeling method is used on a base model. The model is varied to test the effect of the following parameters: backfill, spandrel wall, boundary conditions, and the morphology of the arch barrel. Morphology will be varied in the form of layers of stones in the arch (single versus double layer) and the bond pattern (running versus stack bond).

Geometry of the Base Model

Historic masonry construction was mostly a manual trade, therefore the geometry, bond patterns, materials and other considerations vary greatly with local traditions and the architectural styles (roman era semi-circular arches to more recent shallower arches, for instance). As a result of this, it is not straightforward to find a "typical" masonry arch configuration to build a parametric study upon. However, one can hypothesize that within the constraints of a specific style, the effect of varying the common elements of a masonry arch will have similar response on the overall structural behavior. This study utilizes this hypothesis to examine the effect of spandrel walls, boundary conditions, arch thickness, number of arch rings, and bond pattern on one hypothetical masonry arch model. Authors strongly emphasize that the readers will benefit from the understanding developed from this study greatly, but for more accurate results, each masonry arch bridge should be examined considering its own geometrical characteristics. This is because, the structural response is strongly controlled by the geometrical properties (Block et al. 2006).

The base DEM model for the hypothetical arch was generated (Fig. 5), adapting some of the typical geometrical parameters presented in the literature (de Arteaga and Morer 2012; Conde et al. 2016;

Oliveira et al. 2010). Then, each structural component was studied individually, and different boundary conditions were simulated.

Effect of the Backfill and Spandrel Wall

For this analysis, first, an isolated semi-shallow arch (without any backfill material and spandrel wall), having a 0.4 rise to span ratio with 0.6 m thickness, was modeled. A vertical point load of F was applied incrementally at quarter-span. The load carrying capacity, found as 65 kN, was compared by limit analysis approach, using an open-source software ArchNURBS, as shown in Figure. 6a (Chiozzi et al. 2016). Less than 2% difference was obtained for ultimate load and identical collapse mechanisms were observed (Fig. 6 b-c).

This is informative of masonry arch behavior under such point loading and the collapse mechanism is well known from the literature. However, while estimating the actual load carrying capacity of masonry arches, one needs to take into account other parameters than only the arch itself, as masonry arches are rarely, if ever, in this vulnerable condition. For instance, for a masonry arch bridge, the backfill material is one of the important parts of the system, which provides more strength to the arch barrel by applying compression forces around the extrados of the arch that counteract the flexural tension forming on the arch due to any point load. To present this phenomenon, the dead load from the backfill material was applied on each rigid block as an external load. Then, a point load was incrementally applied on the arch extrados without considering the load dispersion angle. At this stage of the study, soil-structure interaction was not taken into account which allows for the study of parameters one at a time using DEM. While this simplified approach has merit, authors are aware that, backfill and arch barrel has a complex relationship. The soil can be mobilized and play an active role in the load carrying capacity and failure mechanism rather than applying a static weight (Callaway et al. 2012). Future work will incorporate this complex relationship to the models. In addition to that, there are several approaches in discrete element frame work, such as modeling the backfill material as deformable blocks to represent backfill material (Bićanić et al. 2003). There are also different approaches, mentioned earlier that considers finite element limit analysis approach based on the kinematic theorem to take into account the arch-fill interaction (Cavicchi and Gambarotta 2007). Here, for the sake of simplicity, backfill loads were considered as dead loads applied as external forces on each block.

In Figure 7, the contribution coming from the backfill material to the load carrying capacity of masonry arch is presented in terms of the load-displacement curve. Limit analysis is also performed to benchmark the results (Fig. 7). According to results, more than three times higher capacity was obtained under given geometrical form when the backfill compressive forces are considered. It can be deduced that, if backfill material is not taken into consideration the capacity of masonry arches is underestimated. Therefore, it is crucial to consider individual parts of arch bridges to assess their overall behavior (Brookes 2010). This is in fact an important problem in the present day analysis and rehabilitation of masonry arch bridges.

The above analysis considers a cross-section at the middle of the arch bridge. If there is backfill, there has to be masonry spandrel walls on either end of the arch barrel to contain the backfill. To simulate the behavior one of these outer sections of the arch bridge, the same masonry arch is modeled with varying thickness of spandrel walls. Contributions of the backfill and the spandrel walls depend on their geometrical characteristics. Spandrel wall thicknesses are varied as 0.15, 0.25 and 0.375 meters on each side were modeled together with a 1 meter width of backfill material in each case. If the backfill material was not included, this demonstration would represent the case of a 3-D masonry arch bridge model that has a spandrel wall without backfill material (Lemos 1995). Concentrated load was applied at quarter span on spandrel wall. Each displacement was monitored after getting an equilibrium state under every incremental loading. From the results of the analyses, it is concluded that, similar to the backfill, the spandrel wall also provides a significant contribution to the capacity depending on its thickness (Fig. 8). However, it should be noted that these in-plane discrete element models assume no out-of-plane action, which needs 3-D analysis, and may not be necessary in all cases.

Furthermore, both sides of the spandrel wall were left free in these examples. Boundary conditions could provide extra capacity to the structure, which will be discussed in the next sections.

Effect of Boundary Conditions on Load Carrying Capacity

In engineering practice, soil characteristics and exact boundary conditions are among the most difficult parameters to determine for an existing masonry arch bridge. In this case, to see the influence of boundary conditions on the load carrying capacity of the masonry arch including backfill material and spandrel walls, two case studies were prepared. Both backfill and spandrel wall have a significant contribution to the capacity of the arch, but, which one has the higher impact on the arch bridges is an ongoing research topic (Sarhosis et al. 2016). In this context, it should be underlined that, the results are obtained for a unit width of backfill and 0.15 m spandrel wall thickness. Therefore, readers should keep in mind that, the obtained results may increase or decrease depending on the thickness of the spandrel wall and backfill properties.

In the first case, there are no assigned constraints at the spandrel wall boundaries. In the second model, a passive earth pressure is employed at both sides of the masonry spandrel walls, as shown in Figure 9. In both models, the bottom of the models has fixed.

As expected, there is a significant change on the load carrying capacity of the models depending on the boundary condition assigned to rigid blocks (Fig. 10a). Collapse mechanisms are identical for both cases; however, the capacities at which this mechanism is achieved, are different. The formation of the plastic hinges first follows the line of action of the force, and other plastic hinges appear at both extrados and intrados on the arch. After the arch develops these hinges cracking in the spandrel wall follows, as demonstrated in Figure 10b. The contribution of the boundary conditions may have even further impact when the arch geometry is varied from shallow to deep arches, since the horizontal thrust distribution would be vastly different. However, for a comparison with the same arch geometry, it is clear that the model with free ends would underestimate the capacity of the structure and generate conservative results.

Effect of the Morphology of the Arch Barrel (Number of layers and Bond Pattern)

Construction practice of masonry structures varies from one country to another, or even within one country by region or time of construction. Characteristics of masonry structures may affect their structural behavior depending on the arch configuration (Pulatsu et al. 2016).

To account for such variability, this simulation experiment is expanded to include the analysis of arches with double layer arches and bond patterns (stack versus running bond). For comparison, the previously examined model with a single layer arch (0.6 m thick) was modified to create a double layer arch (0.3 m thickness each layer) with two different bond patterns (Fig. 11).

Several comparisons were carried out with these models. First, double layer masonry arch was compared with the single layer arch. Both of these models included masonry spandrel wall and backfill material. DEM analysis results show that single layer of arch with a thickness of 0.6 m has almost double strength than one that has two stack bonded arch layers of 0.3 m thickness (Fig. 12a), despite the fact that the total arch thickness is the same in these models. Since, the model does not have any bonding material (i.e. mortar), no tensile and cohesion strengths were presented at the joints. As such, this analysis demonstrates the worst case situation for the double layer arch structure. The collapse mechanism starts when the thrust line touches the intrados of the arch barrel and tensile forces appear at the contact that yields to opening between two adjacent blocks. To understand the influence of the tensile strength considered at the arch barrel, a sensitivity analysis was performed by adding tensile strength to the joints. Since, mortar joints are the weak planes for masonry construction, low tension capacity (f_t) and a cohesion value of ($1.5f_t$) were employed.

Although this affects the stiffness and the strength of the structure, shown in Figure 13, it seems difficult to reach single layer arch capacity (0.6 m thickness). The failure mechanism generally triggers by the lack of tensile strength at the joints due to aged and damaged mortar. Due to such poor mechanical properties at the joints, in existing historical masonry structures, tensile strength, if any, would be very small. This is why as a part of sensitivity analysis, the effect of a small range of tensile strengths was studied. Figure 13 shows the overall capacity is only marginally affected. It should be noted, however, that a higher tensile strength for contact in DEM for the mortar joints may result in over estimation, especially for historical masonry structures.

As a second step, different bond patterns were considered to see the possible inter-locking effects that may have an influence on the overall structural behavior and the capacity. However, there was no considerable difference observed neither at the load carrying capacity (Fig. 12b) nor at the failure mechanism of the structure (Fig. 12c). Therefore, without the binding effects of mortar in the joints, the bond pattern does not have a significant influence on the ultimate load capacity of the structure. Table 5 summarizes the maximum load carrying capacities for single and double layer arches along with the effect of the different structural components. The results clearly indicate the positive influence of backfill and spandrel walls.

Conclusions

Using two-dimensional rigid block DEM models, the load carrying capacity and the nonlinear response of a family of masonry arch structures were studied. The following conclusions are drawn from this parametric study:

- The discrete element method, which was the numerical strategy employed in this project, is a powerful technique that can capture the nonlinear behavior of masonry arches and the complex relationship between its structural components and assigned boundary conditions. It allows to visualize the progression of damage and formation of hinges.
- Among the parameters studied, the boundary conditions have the most significant contribution to the load carrying capacity of masonry arch structures.
- A single layer of thicker units presented a higher capacity than an arch of same thickness but formed of two layers. This is partially because the analyses assumed no mortar in the joints and presented the most vulnerable case. However, in general, mortar joints are the weak planes for masonry structures and does not provide remarkable strength.
- For the cases investigated, there was no significant impact due to the bond pattern between the two arch layers.
- Masonry arch with a thicker spandrel wall emerged as the stiffest case scenario. This makes sense, especially for the case of a well-preserved structure with an intact masonry wall. The backfill above the arch barrel also had a significant effect on the arch's capacity, however, modeling backfill accurately is more difficult than spandrel walls since most of the time there is a lack of knowledge about the status of the backfill material. Nevertheless, these analyses show that analyzing existing masonry arch bridges without any consideration of the backfill or the spandrel wall vastly underestimates their inherent strength.
- The custom-made application was validated via an experimental study and results found demonstrated realistic collapse mechanism that matched published experimental results. It was noted that the application of load through the spandrel wall may yield different conclusions and results. The load transfer influences the damage progression of the

structure. For example, when loads are applied through the backfill, separation between arch ring and spandrel wall would be observed. Thus, the strength contribution coming from spandrel wall should be evaluated carefully via parametric studies on the numerical models of real structures.

In future work, each of the parameters will be further studied using 3D models and considering the soil-structure interactions. With a 3D analysis, the effect of the spandrel walls as a boundary condition in the orthogonal direction, transverse cracking in the arch barrel and other 3D effects will be captured.

References

de Arteaga, I., and Morer, P. (2012). "The effect of geometry on the structural capacity of masonry arch bridges." *Construction and Building Materials*, Elsevier Ltd, 34, 97–106.

Bićanić, N., Stirling, C., and Pearce, C. J. (2003). "Discontinuous modelling of masonry bridges." *Computational Mechanics*, 31(1–2 SPEC.), 60–68.

Block, P., Dejong, M., and Ochsendorf, J. (2006). "As hangs the flexible line: Equilibrium of masonry arches." *Nexus Network Journal*, 8(2), 13–24.

Brencich, A., and Morbiducci, R. (2007). "Masonry Arches: Historical Rules and Modern Mechanics." *International Journal of Architectural Heritage*, 1(2), 165–189.

Bretas, E. M., Lemos, J. V., and Lourenço, P. B. (2013). "Hydromechanical Analysis of Masonry Gravity Dams and their Foundations." *Rock Mechanics and Rock Engineering*, 46(2), 327–339.

Bretas, E. M., Lemos, J. V., and Lourenço, P. B. (2014). "A DEM based tool for the safety analysis of masonry gravity dams." *Engineering Structures*, Elsevier Ltd, 59, 248–260.

Bretas, E. M., Lemos, J. V., and Lourenço, P. B. (2016). "Seismic Analysis of Masonry Gravity Dams Using the Discrete Element Method: Implementation and Application." *Journal of Earthquake Engineering*, 20(2), 157–184.

Brookes, C. (2010). "Application of the finite/discrete element method to arches." *Proceedings of the ICE - Engineering and Computational Mechanics*, 163(3), 203–211.

Bui, T. T., Limam, A., Sarhosis, V., and Hjiaj, M. (2017). "Discrete element modelling of the in-plane and out-of-plane behaviour of dry-joint masonry wall constructions." *Engineering Structures*, Elsevier Ltd, 136(January), 277–294.

Callaway, P., Gilbert, M., and Smith, C. (2012). "Influence of backfill on the capacity of masonry arch bridges." *Proceedings of the ICE-Bridge Engineering*, 165(3), 147–158.

Cavicchi, A., and Gambarotta, L. (2007). "Lower bound limit analysis of masonry bridges including arch-fill interaction." *Engineering Structures*, 29(11), 3002–3014.

Chiozzi, A., Malagù, M., Tralli, A., and Cazzani, A. (2016). "ArchNURBS: NURBS-Based Tool for the Structural Safety Assessment of Masonry Arches in MATLAB." *Journal of Computing in Civil Engineering*, 30(2), 4015010.

Conde, B., Díaz-Vilariño, L., Lagüela, S., and Arias, P. (2016). "Structural analysis of Monforte de Lemos masonry arch bridge considering the influence of the geometry of the arches and fill material on the collapse load estimation." *Construction and Building Materials*, Elsevier Ltd, 120, 630–642.

Cundall, P. A. (1971). "A computer model for simulating progressive, large-scale movements in blocky rock systems." *The International Symposium on Rock Mechanics*, 47–65.

Gilbert, M. (2007). "Limit analysis applied to masonry arch bridges: state-of-the-art and recent developments." *5th International Conference on Arch Bridges (ARCH'07)*, 13–28.

Heyman, J. (1966). "The stone skeleton." *International Journal of Solids and Structures*, 2(2), 249–279.

Isfeld, A., and Shrive, N. (2015). "Discrete Element Modeling of Stone Masonry Walls With Varying Core Conditions: Prince of Wales Fort Case Study." *International Journal of Architectural Heritage*, 9(5), 564–580.

Lemos, J. (1995). "Assessment of the ultimate load of a masonry arch using discrete elements." *Computer methods in structural masonry*, 294–302.

Lemos, J. V. (2007). "Discrete Element Modeling of Masonry Structures." *International Journal of Architectural Heritage*, 1(2), 190–213.

Livesley, R. K. (1978). "Limit analysis of structures formed from rigid blocks." *International Journal for Numerical Methods in Engineering*, 12(12), 1853–1871.

Lourenço, P. B. (2002). "Computations on historic masonry structures." *Progress in Structural Engineering and Materials*, 4(3), 301–319.

Lourenço, P. B. (2009). "Recent advances in masonry structures: Micromodelling and homogenisation." *Multiscale modeling in solid mechanics: computational approaches*, 251–294.

Lourenço, P. B., and Rots, J. G. (1997). "Multisurface interface model for analysis of masonry structures." *Journal of Engineering Mechanics-Asce*, 123(7), 660–668.

Milani, E., Milani, G., and Tralli, A. (2008). “Limit analysis of masonry vaults by means of curved shell finite elements and homogenization.” *International Journal of Solids and Structures*, 45(20), 5258–5288.

Milani, G. (2015). “Upper bound sequential linear programming mesh adaptation scheme for collapse analysis of masonry vaults.” *Advances in Engineering Software*, 79, 91–110.

Milani, G., and Lourenço, P. B. (2012). “3D non-linear behavior of masonry arch bridges.” *Computers and Structures*, Elsevier Ltd, 110–111, 133–150.

Oliveira, D. V., Lourenço, P. B., and Lemos, C. (2010). “Geometric issues and ultimate load capacity of masonry arch bridges from the northwest Iberian Peninsula.” *Engineering Structures*, Elsevier Ltd, 32(12), 3955–3965.

Pulatsu, B., Bretas, E. M., and Lourenço, P. B. (2016). “Discrete element modeling of masonry structures: Validation and application.” *Earthquakes and Structures*, 11(4), 563–582.

Pulatsu, B., Sarhosis, V., Bretas, E. M., Nikitas, N., and Lourenço, P. B. (2017). “Non-linear static behaviour of ancient free-standing stone columns.” *Proceedings of the Institution of Civil Engineers - Structures and Buildings*, 170(6), 406–418.

Roca, P., and Molins, C. (2004). “Experiments on arch bridges.” *Arch Bridges IV-Advances in Assessment, Structural Design and Construction*, P. Roca and C. Molins, eds., Barcelona, 365–374.

Sarhosis, V., De Santis, S., and de Felice, G. (2016). “A review of experimental investigations and assessment methods for masonry arch bridges.” *Structure and Infrastructure Engineering*, 2479(March), 0–26.

Simon, J., and Bagi, K. (2014). "Discrete element analysis of the minimum thickness of oval masonry domes." *International Journal of Architectural Heritage*, 3058(August).

Tóth, A. R., Orbán, Z., and Bagi, K. (2009). "Discrete element analysis of a stone masonry arch." *Mechanics Research Communications*, Elsevier Ltd, 36(4), 469–480.

Tralli, A., Alessandri, C., and Milani, G. (2014). "Computational methods for masonry vaults: A review of recent results." *Open Civil Engineering Journal*, 8(1), 272–287.

Zhang, Y., Macorini, M., and Izzudin, B. A. (2016). "Mesoscale Partitioned Analysis of Brick-Masonry Arches." *Engineering Structures*, Elsevier Ltd, to appear, 142–166.

Table 1. Joint properties

Normal Stiffness (kn)	Shear Stiffness (ks)	Friction Angle (degrees)	Cohesion	Tensile strength
50 GPa/m	20 GPa/m	40	0	0

Table 2. Geometrical Properties (in meters)

Span	Spandrel Wall Thickness	Total Length	Width
3.2	0.15	5.2	1
Rise	Arch Thickness	Backfill Depth on Crown	Loaded Point
1.6	0.14	0.1	1/4 of span

Table 3. Material Properties (BA2)

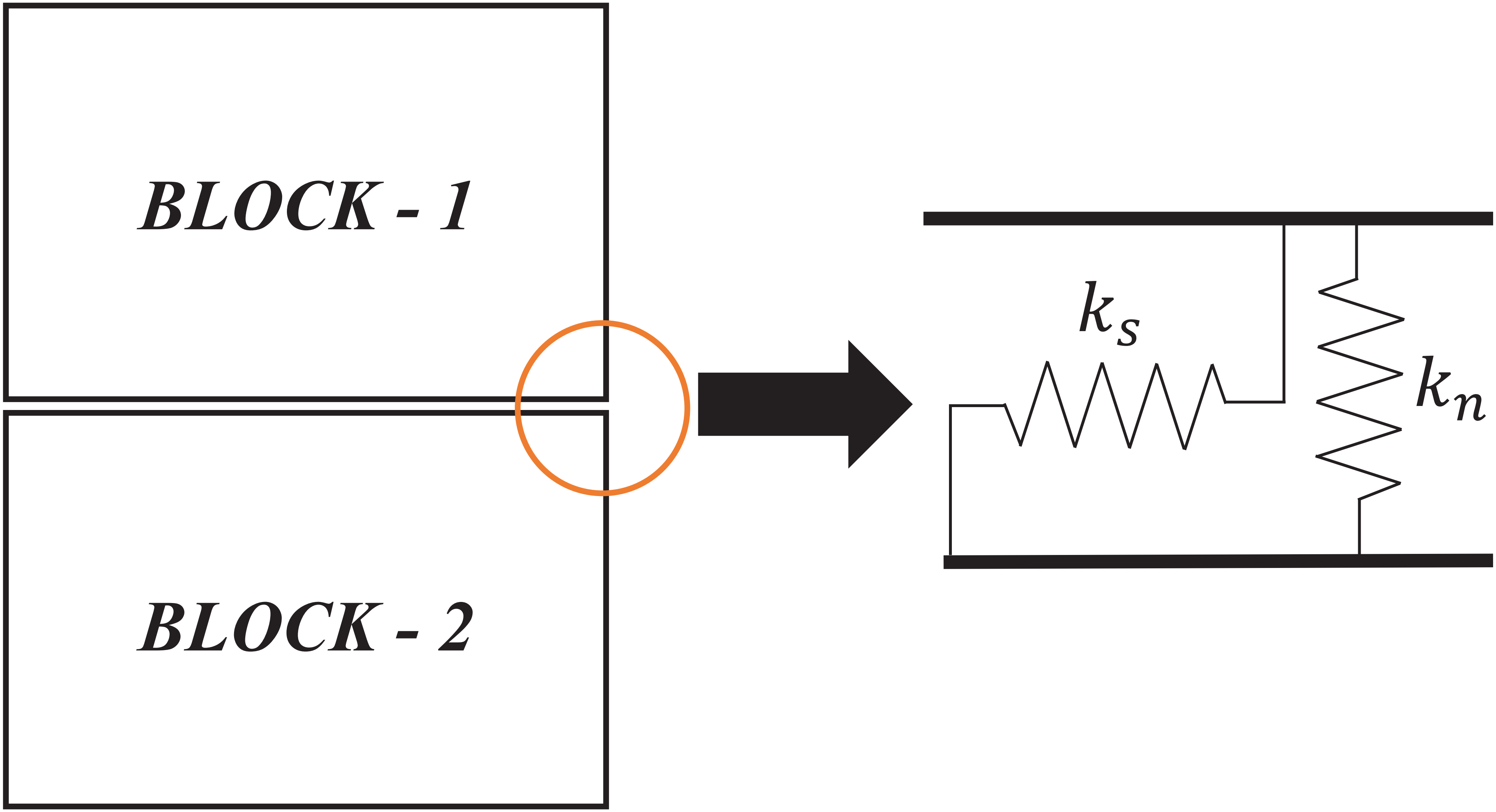
E_{unit} (GPa)	E_{mortar} (GPa)	G_{unit} (GPa)	G_{mortar} (GPa)	t_m (m)	Poisson's ratio
10.45	0.81	4.35	0.34	0.02	0.2

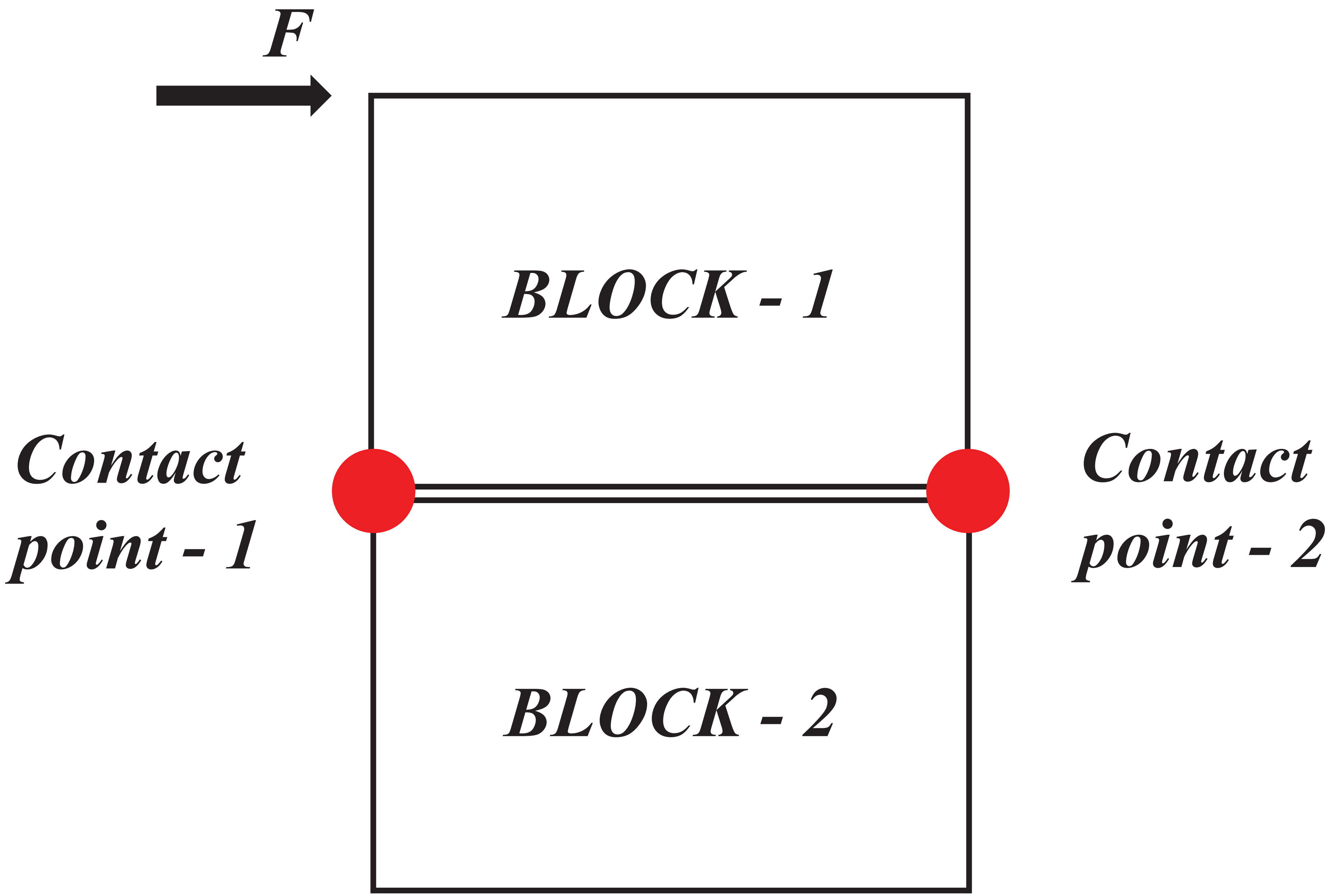
Table 4. Contact Properties

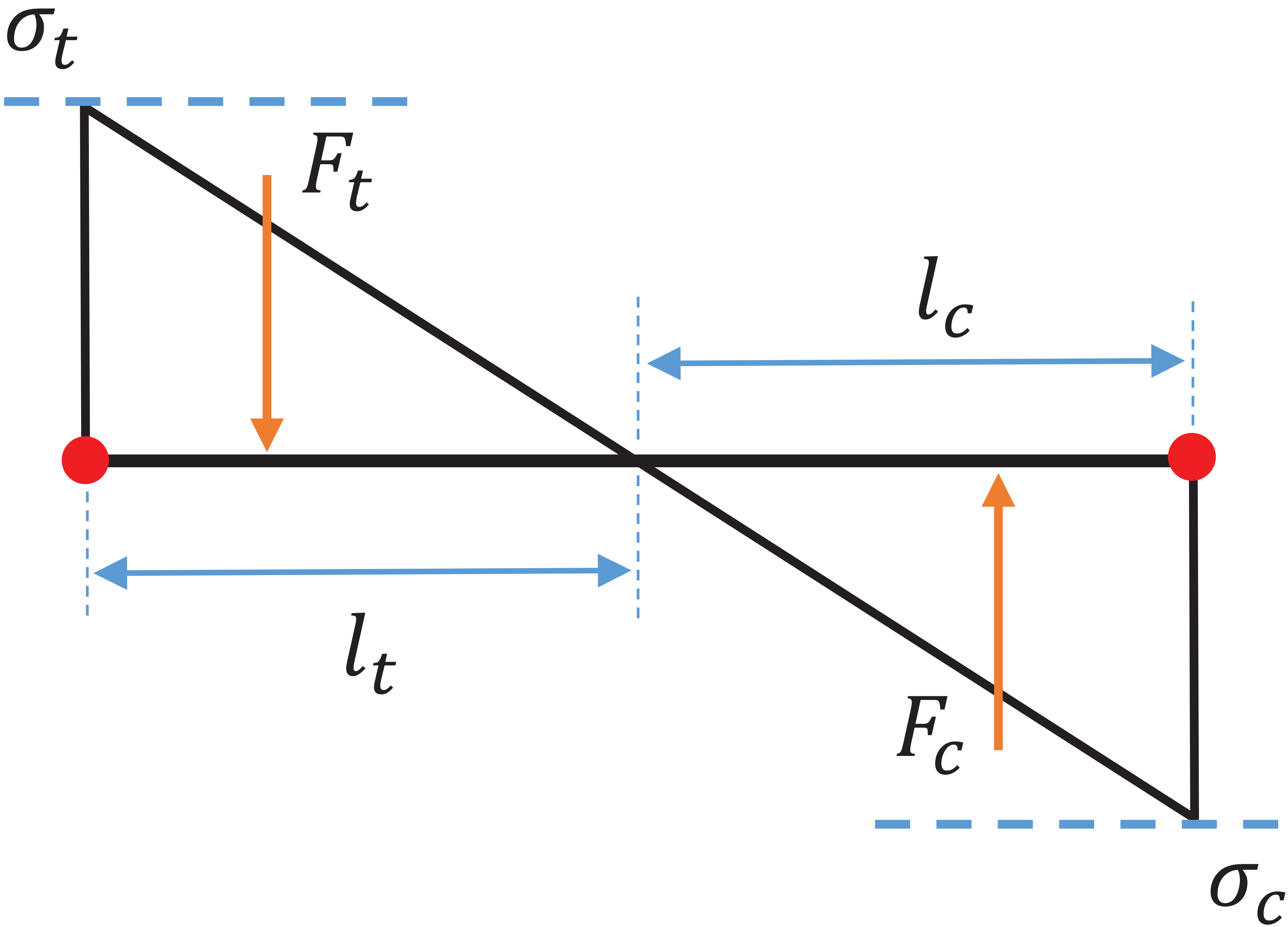
k_n (GPa/m)	k_s (GPa/m)	f_t, *Tensile Strength* (kPa/m)	*c, Cohesion* (kPa/m)	*Friction Angle (deg.)*
43.9	18.29	40	1.5 f_t	35

Table 5. Load carrying capacities (kN) for single layer and double layer masonry arch structures

Arch-Type (thickness)	Arch	Arch+Infill	Arch+Infill+Spandrel Wall
Single Layer (0.6 m)	65	217	600
Double Layer (0.3x2 m)	26	100	300


BLOCK - 1
BLOCK - 2
k_s
k_n


F
BLOCK - 1
Contact
point - 1
Contact
point - 2
BLOCK - 2


σ_t
F_t
l_c
l_t
F_c
σ_c

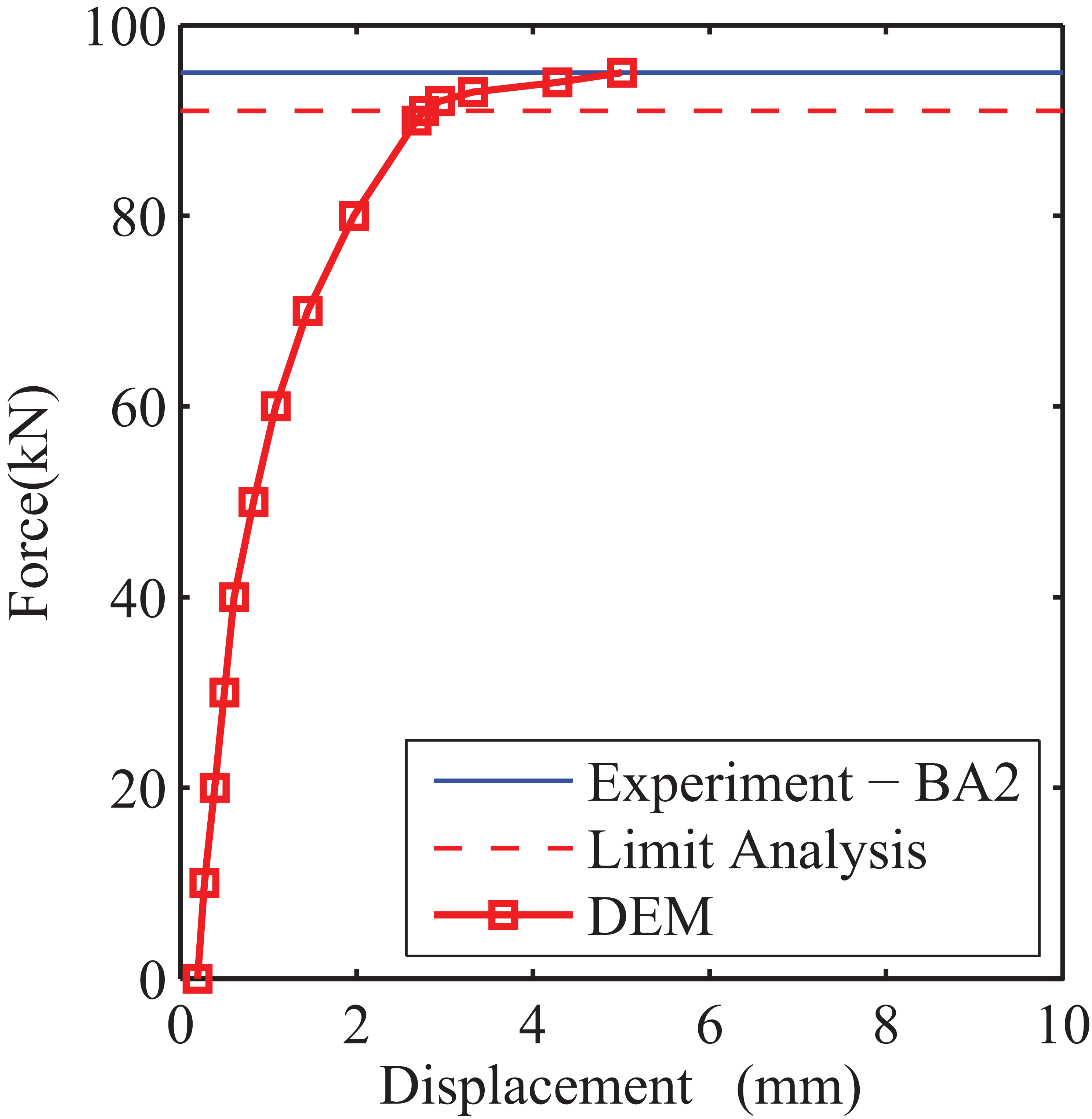
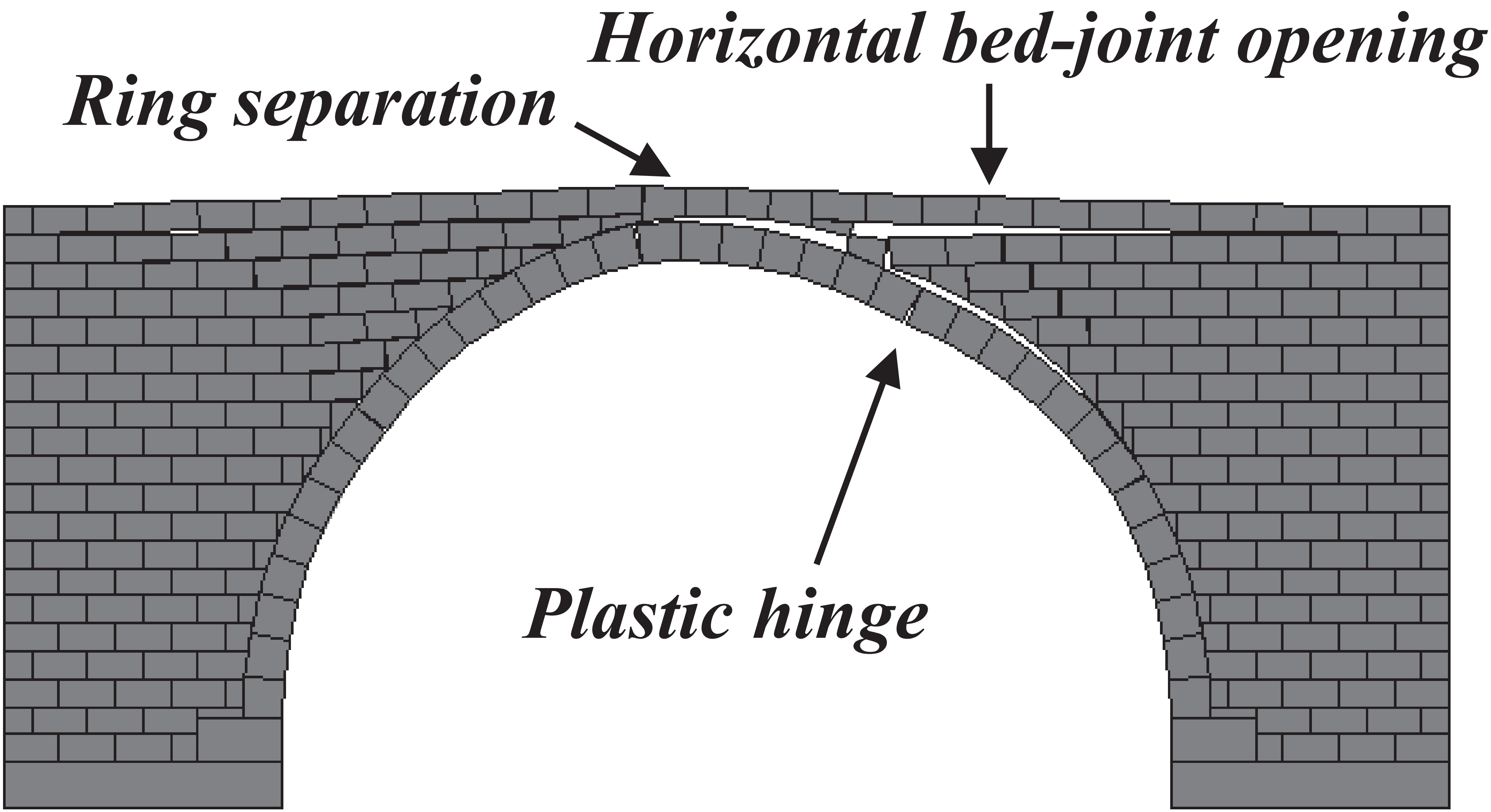
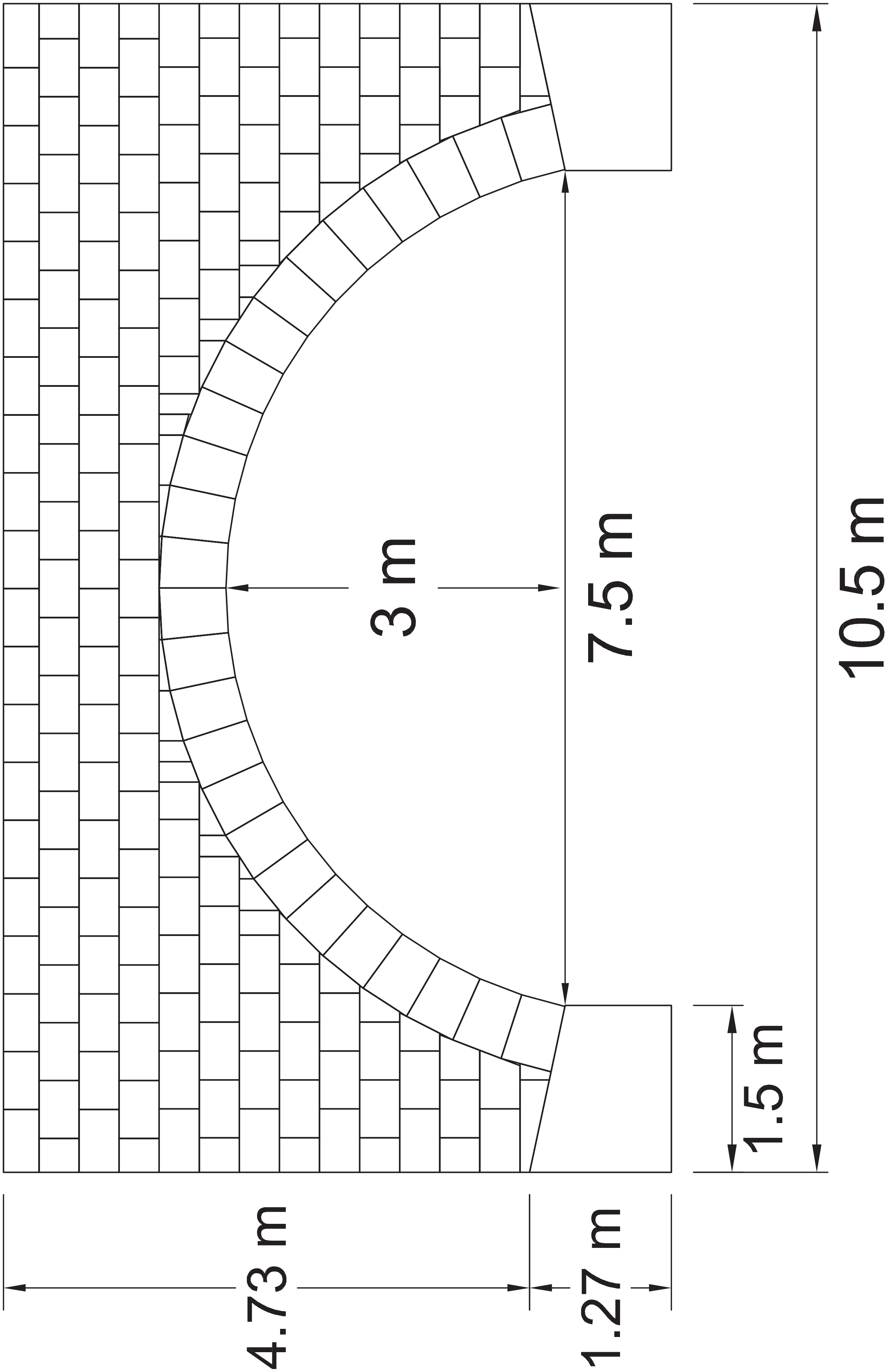

100
80
60
40
20
0
Force(kN)
0
2
4
6
8
10
Displacement (mm)
Experiment − BA2
Limit Analysis
DEM

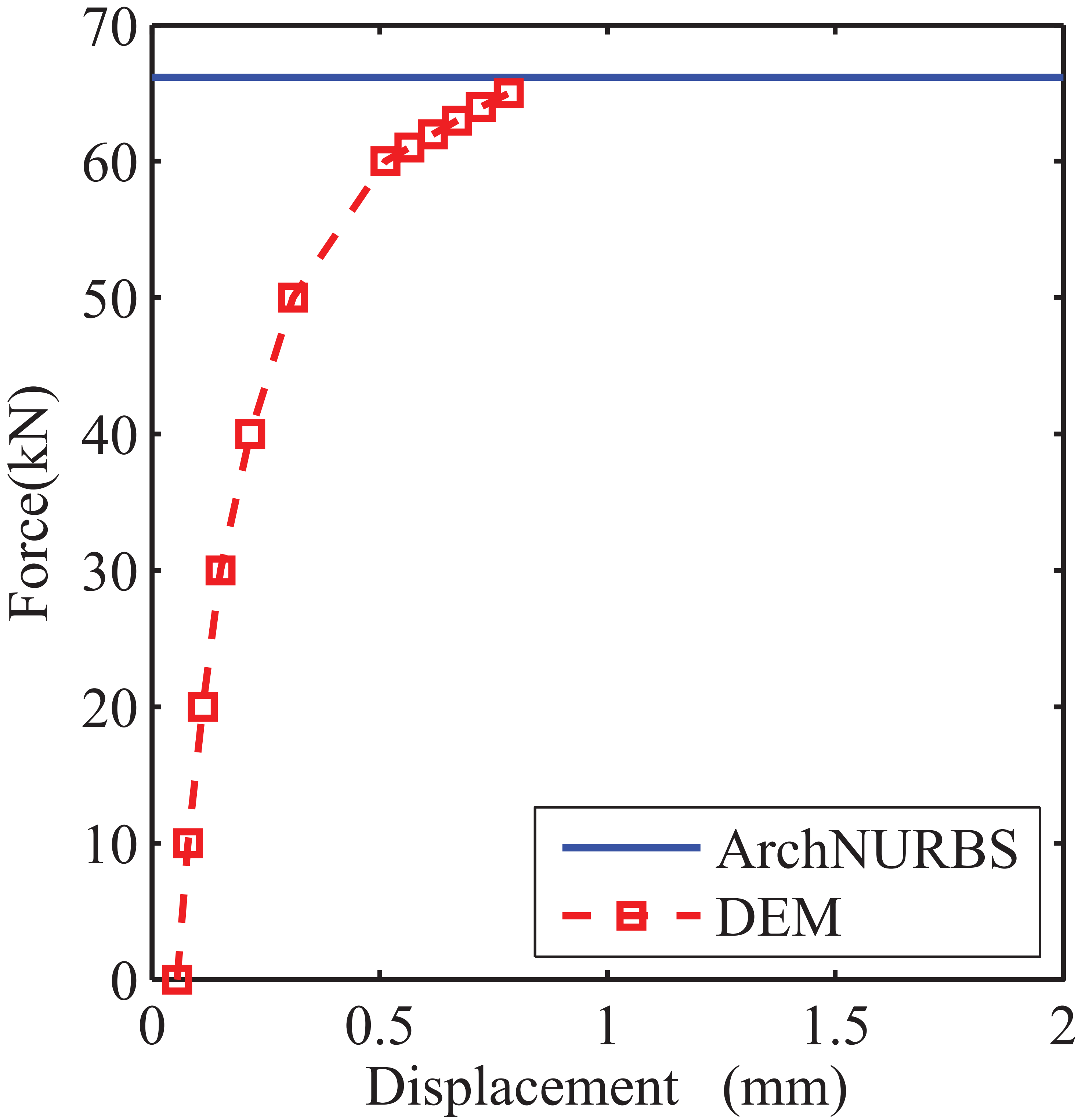
Figure	Click here to download Figure Fig. 4a.pdf

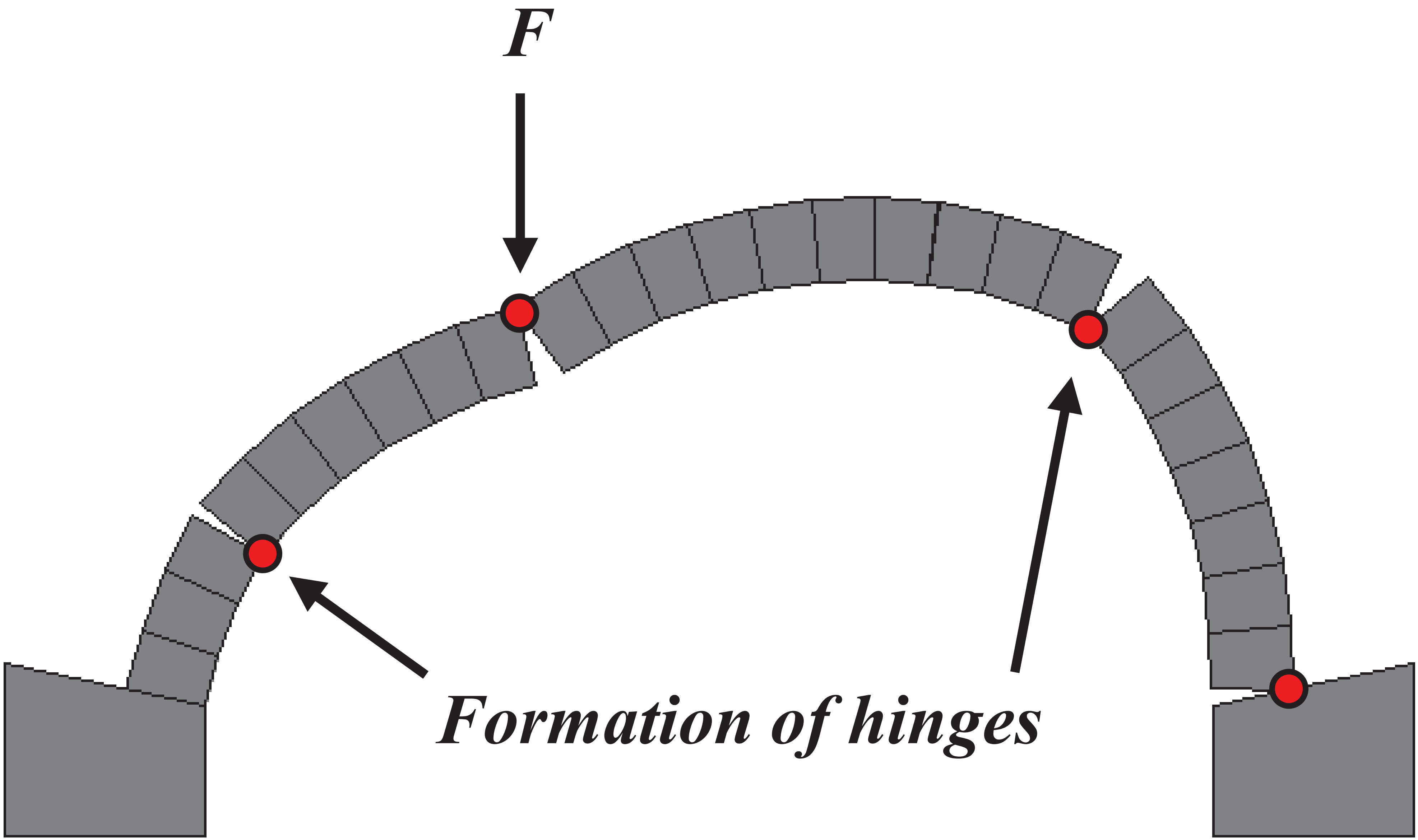

Horizontal bed-joint opening
Ring separation
Plastic hinge

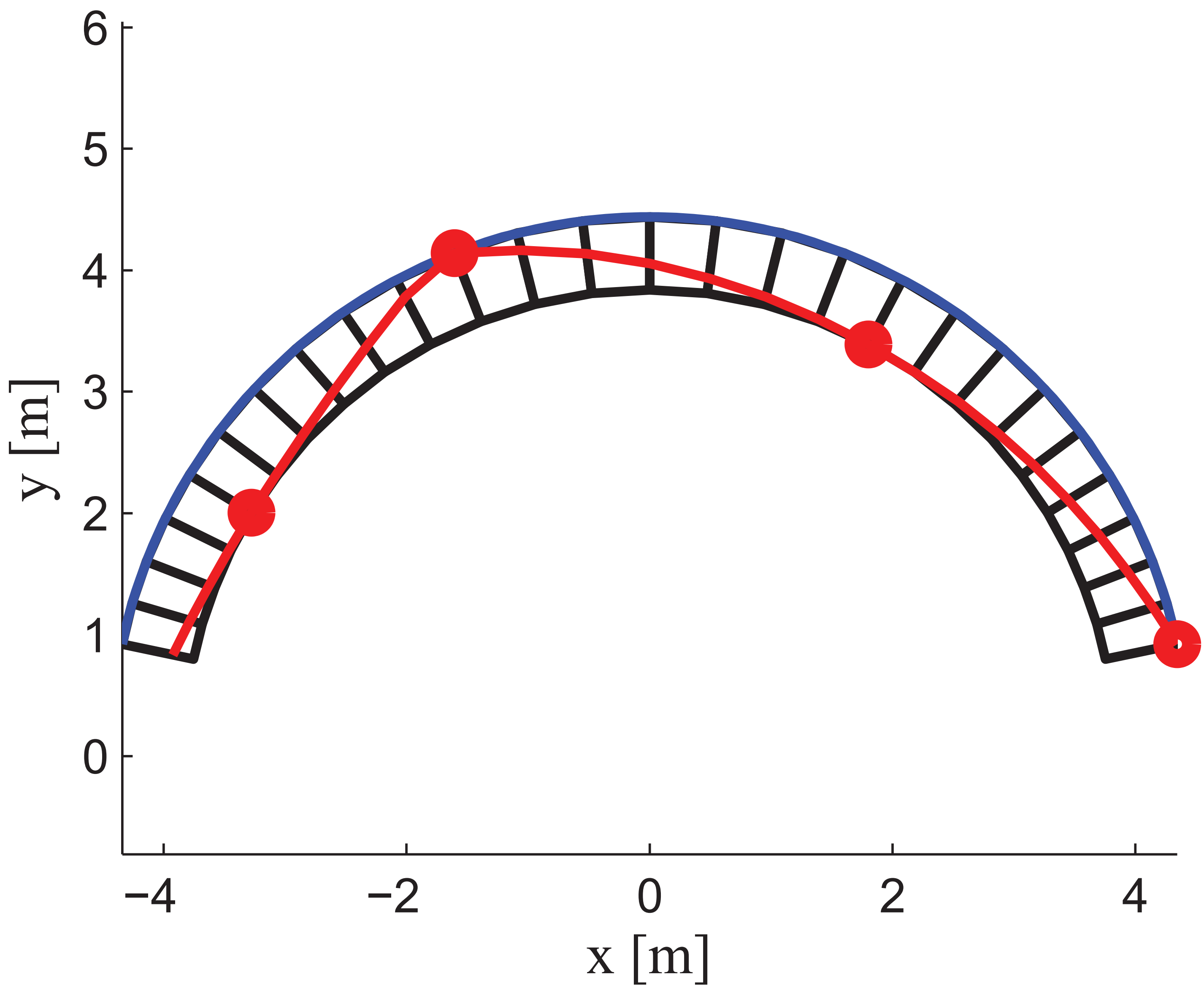

3 m
7.5 m
10.5 m
1.5 m
4.73 m
1.27 m

Figure

Click here to download Figure Fig. 6a.pdf

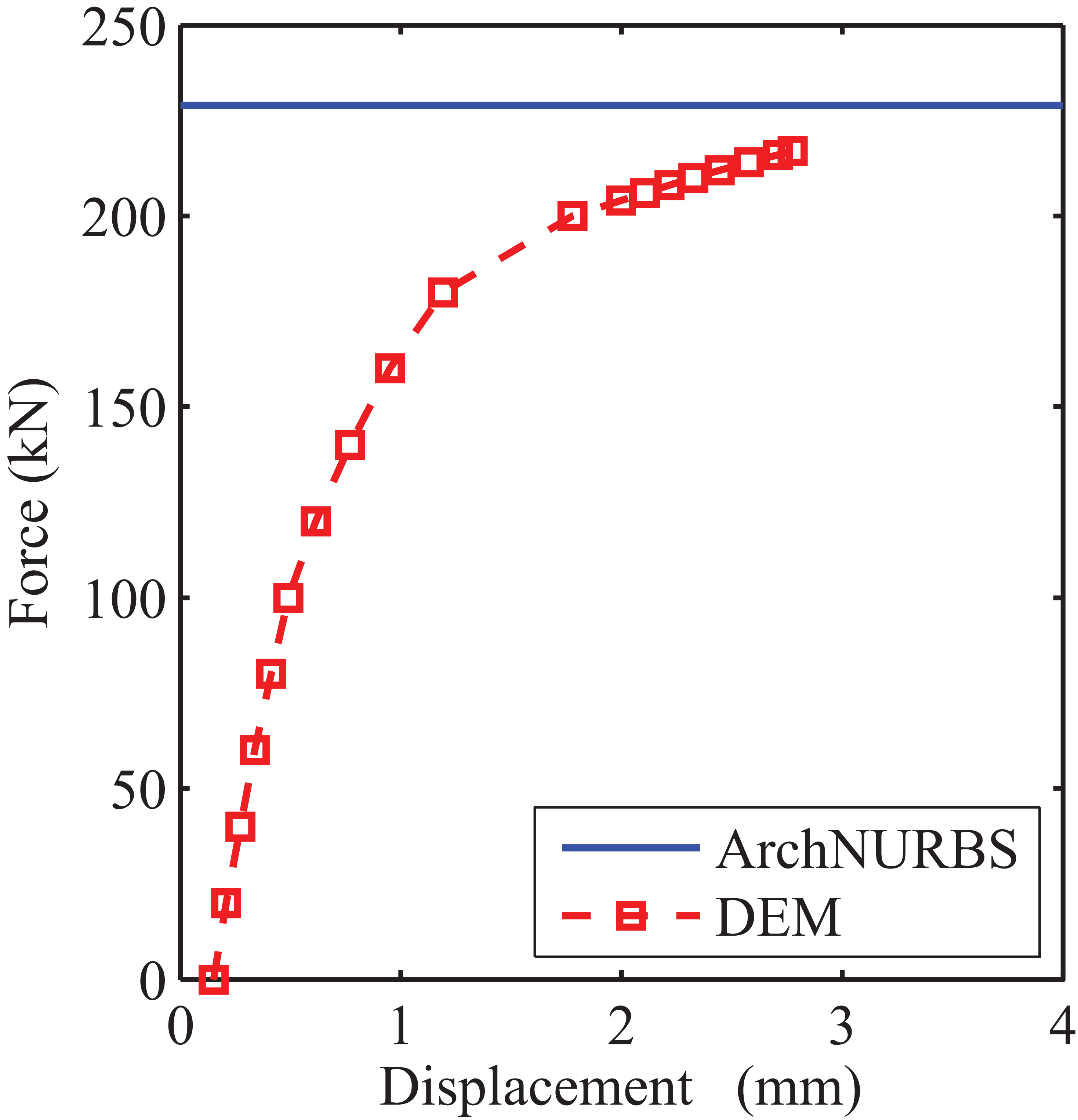


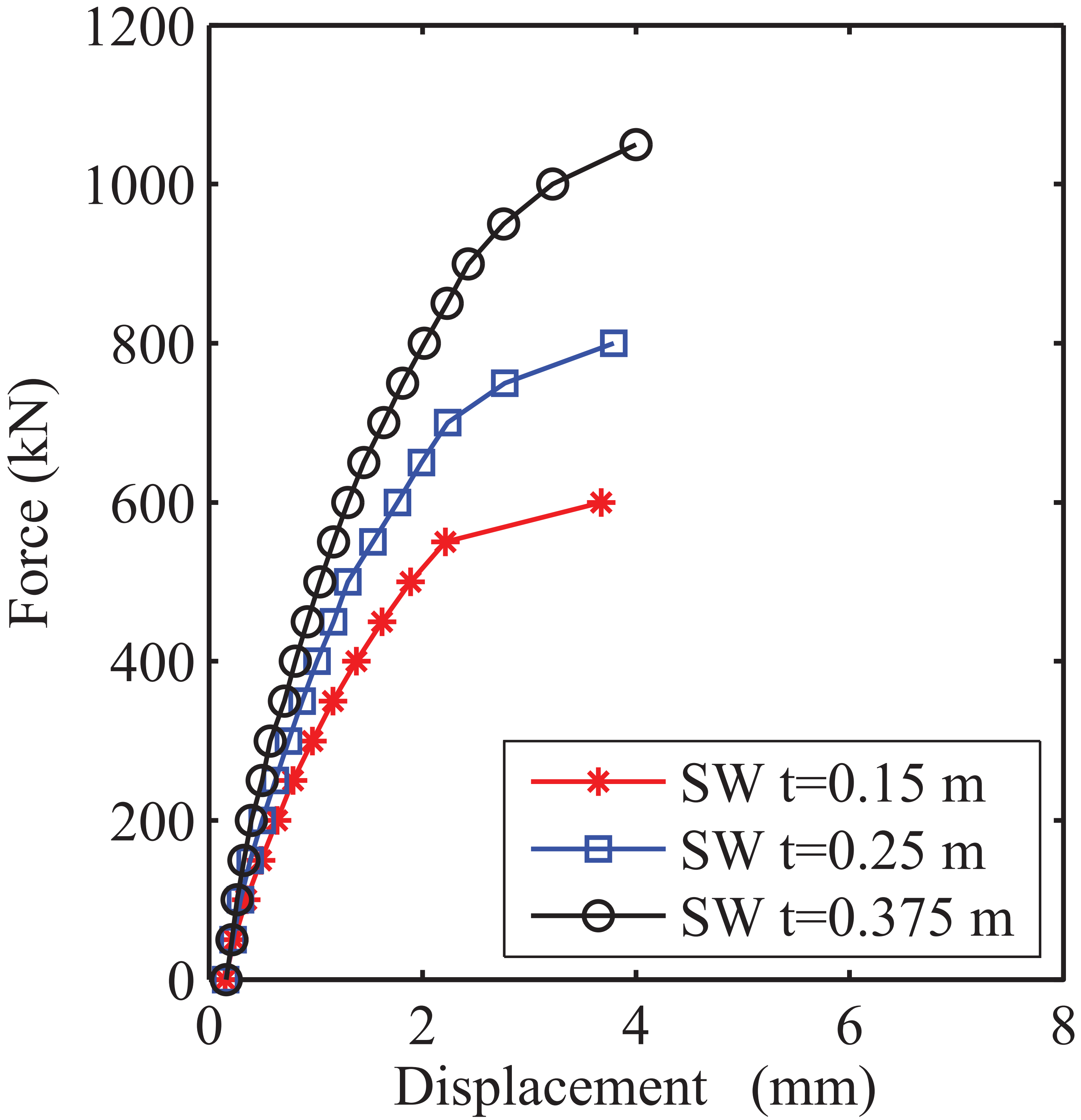

F
Formation of hinges


6
5
4
3
2
1
0
y [m]
−4
−2
0
2
4
x [m]

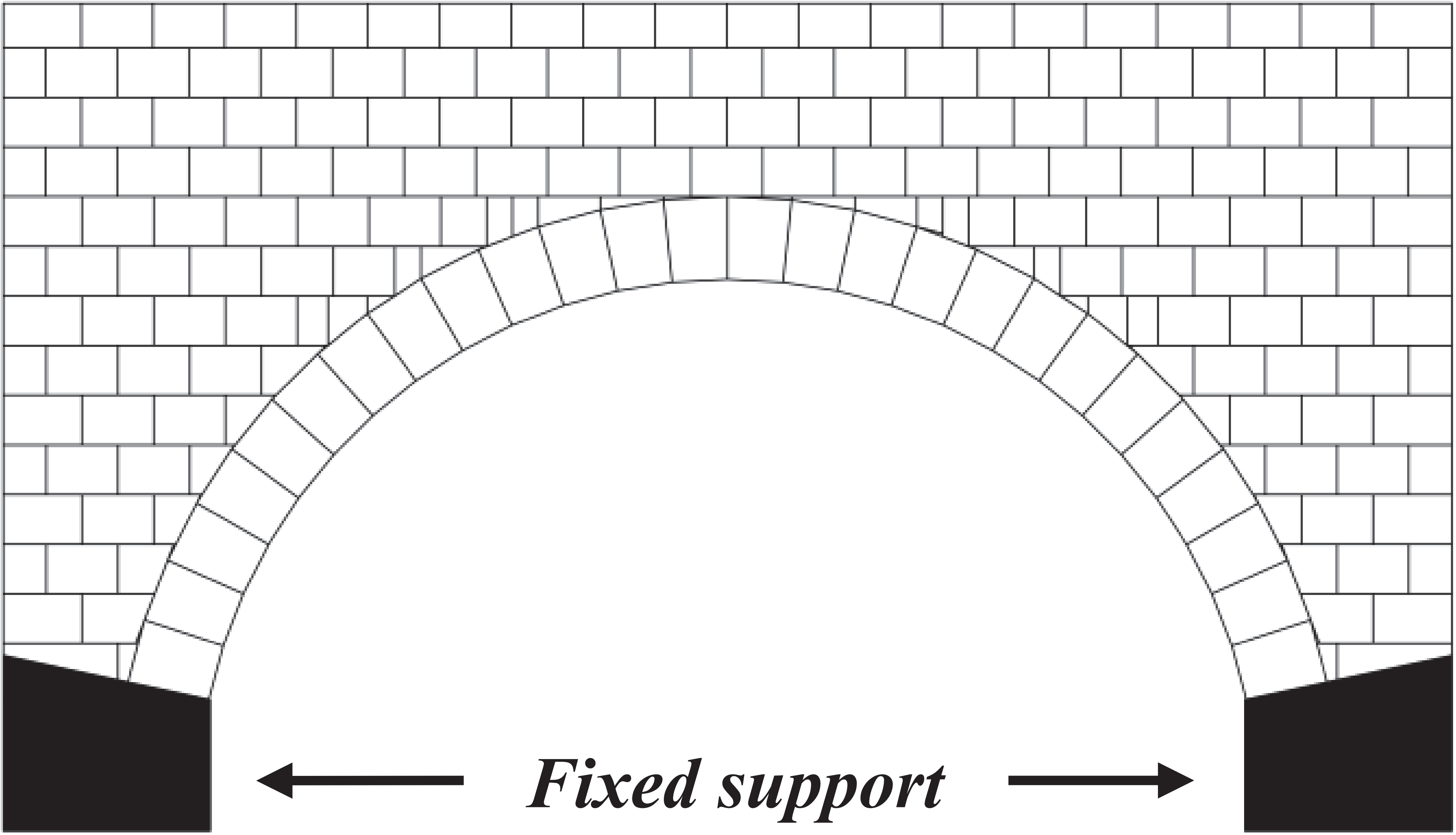
Figure

Click here to download Figure Fig. 7.pdf

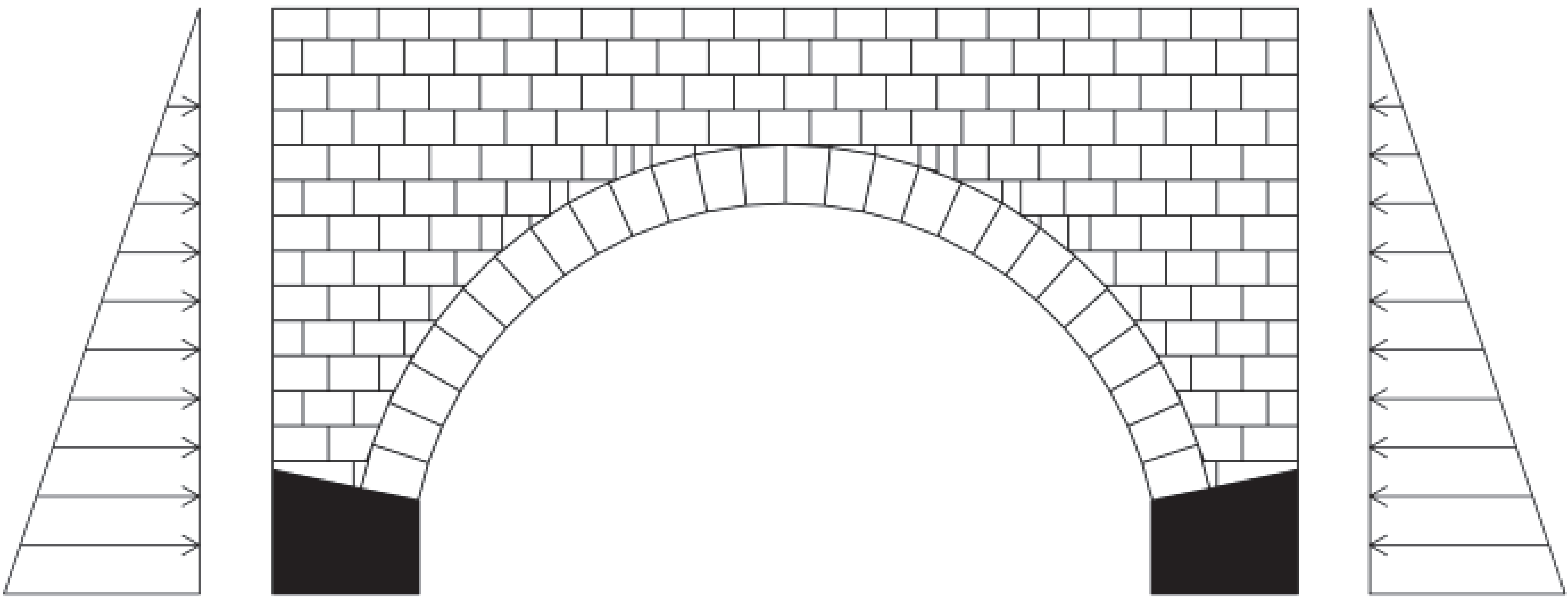


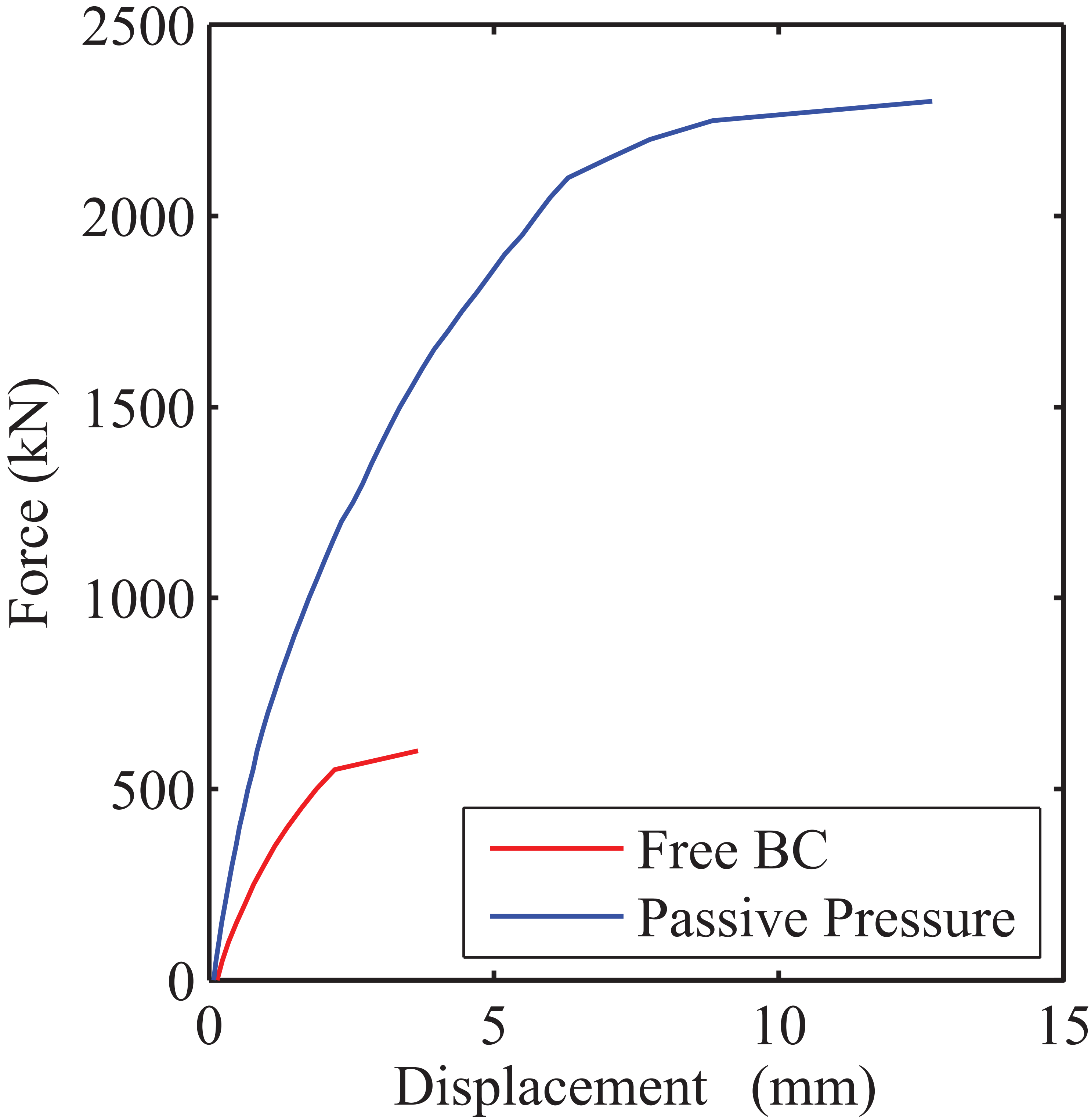


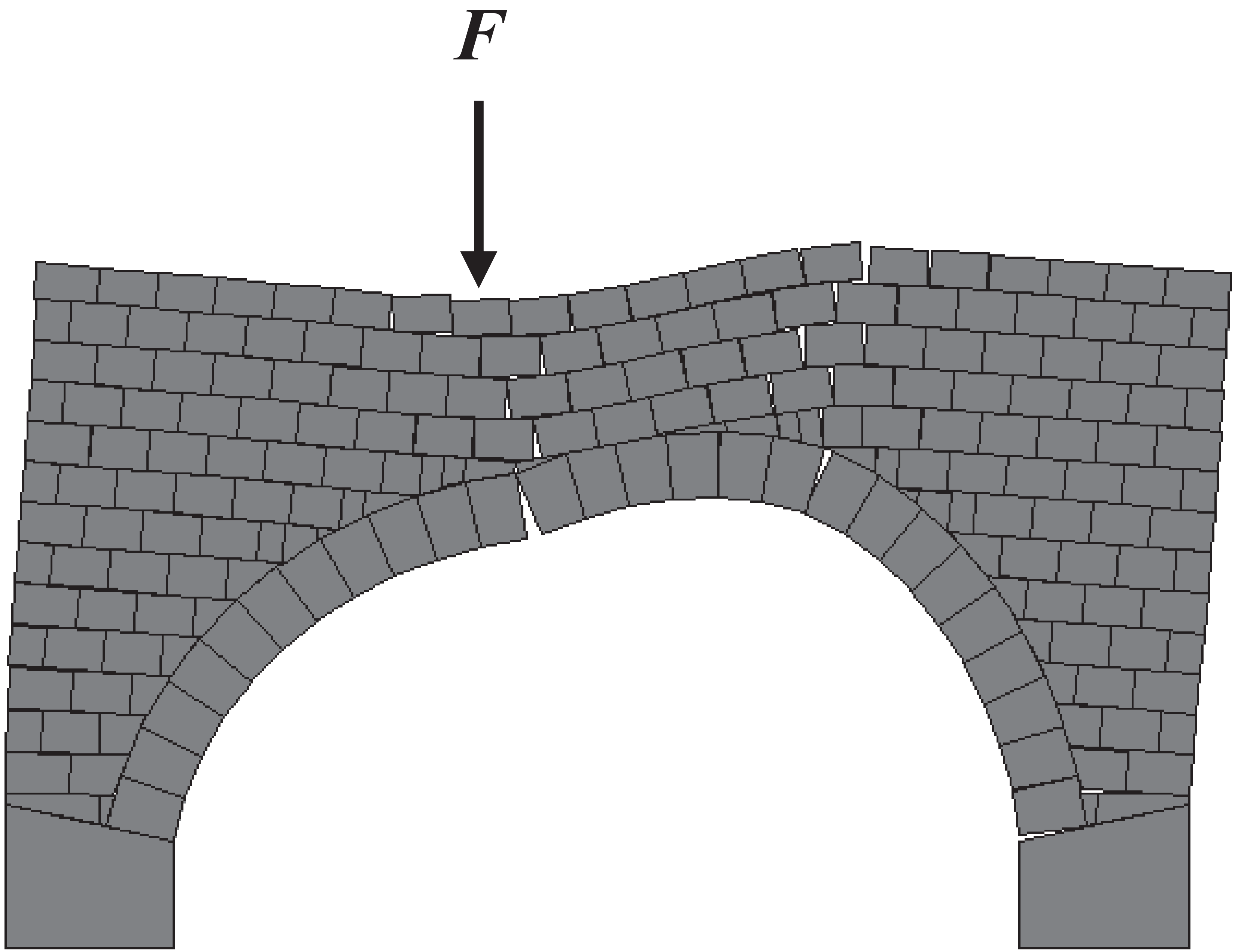
1200
1000
800
600
400
200
0
Force (kN)
0
2
4
6
8
Displacement (mm)
SW t=0.15 m
SW t=0.25 m
SW t=0.375 m

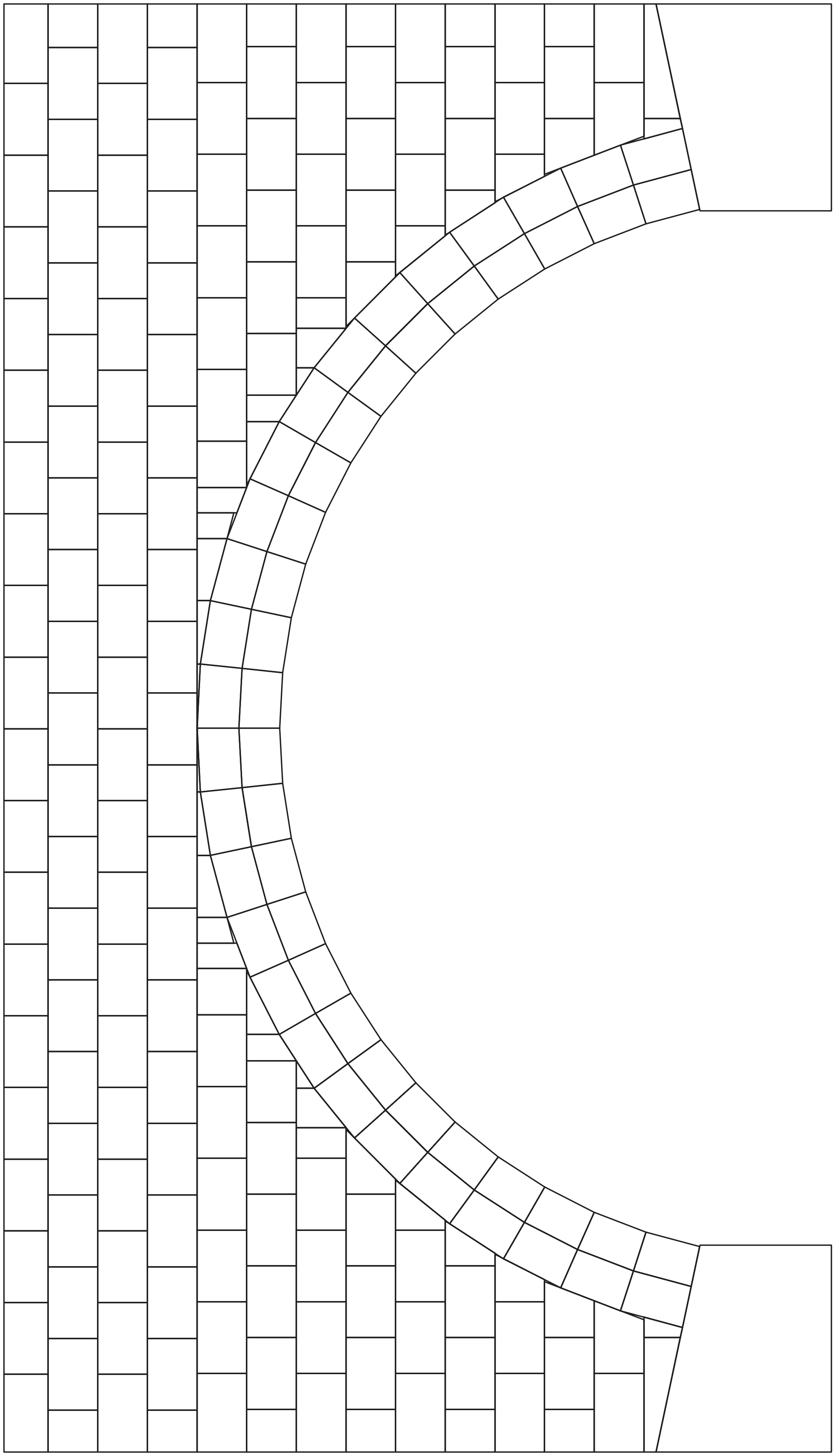

Fixed support

Passive soil pressure




2500
2000
1500
1000
500
0
0
5
10
15
Force (kN)
Displacement (mm)
Free BC
Passive Pressure


F



Figure

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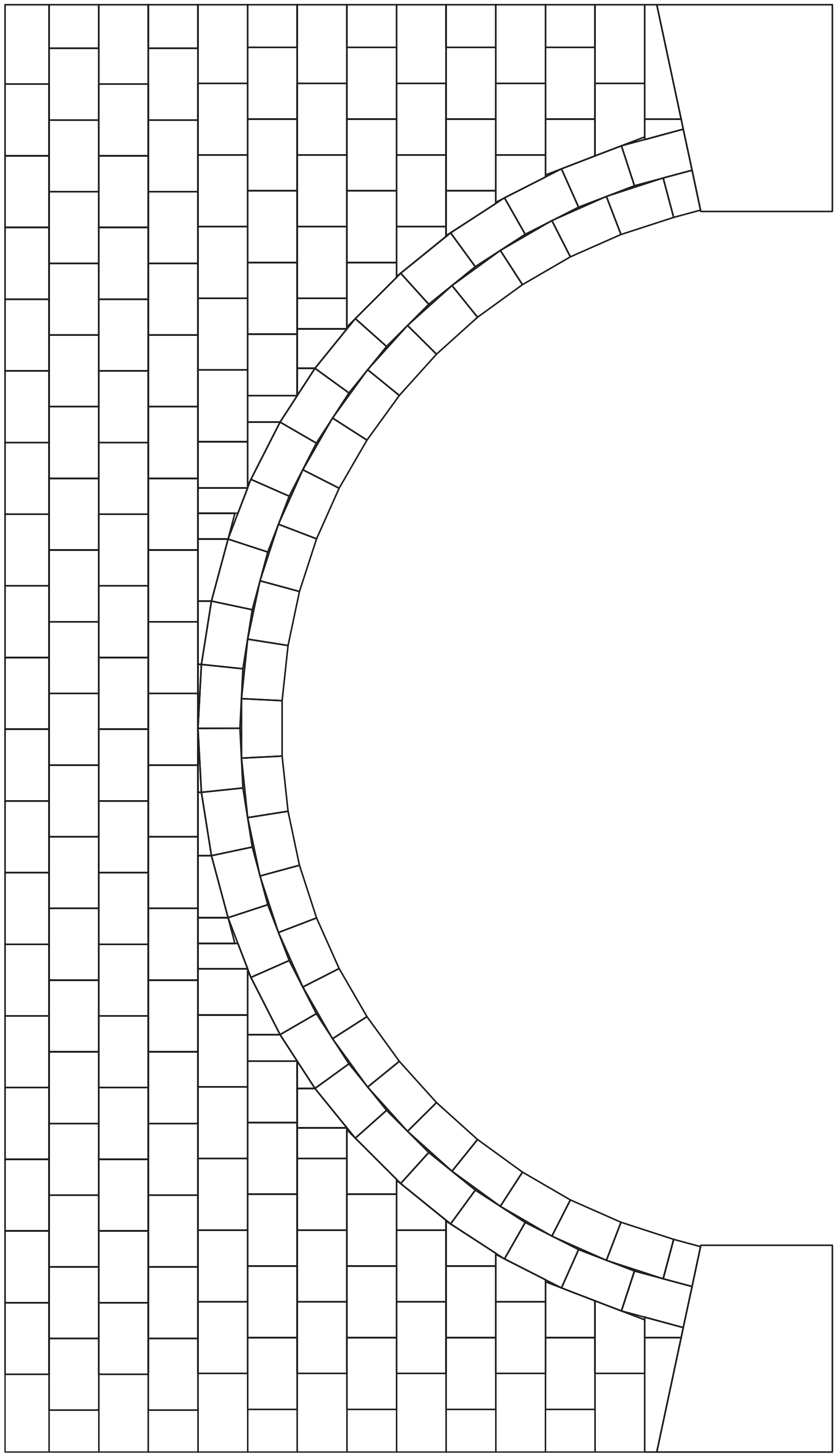
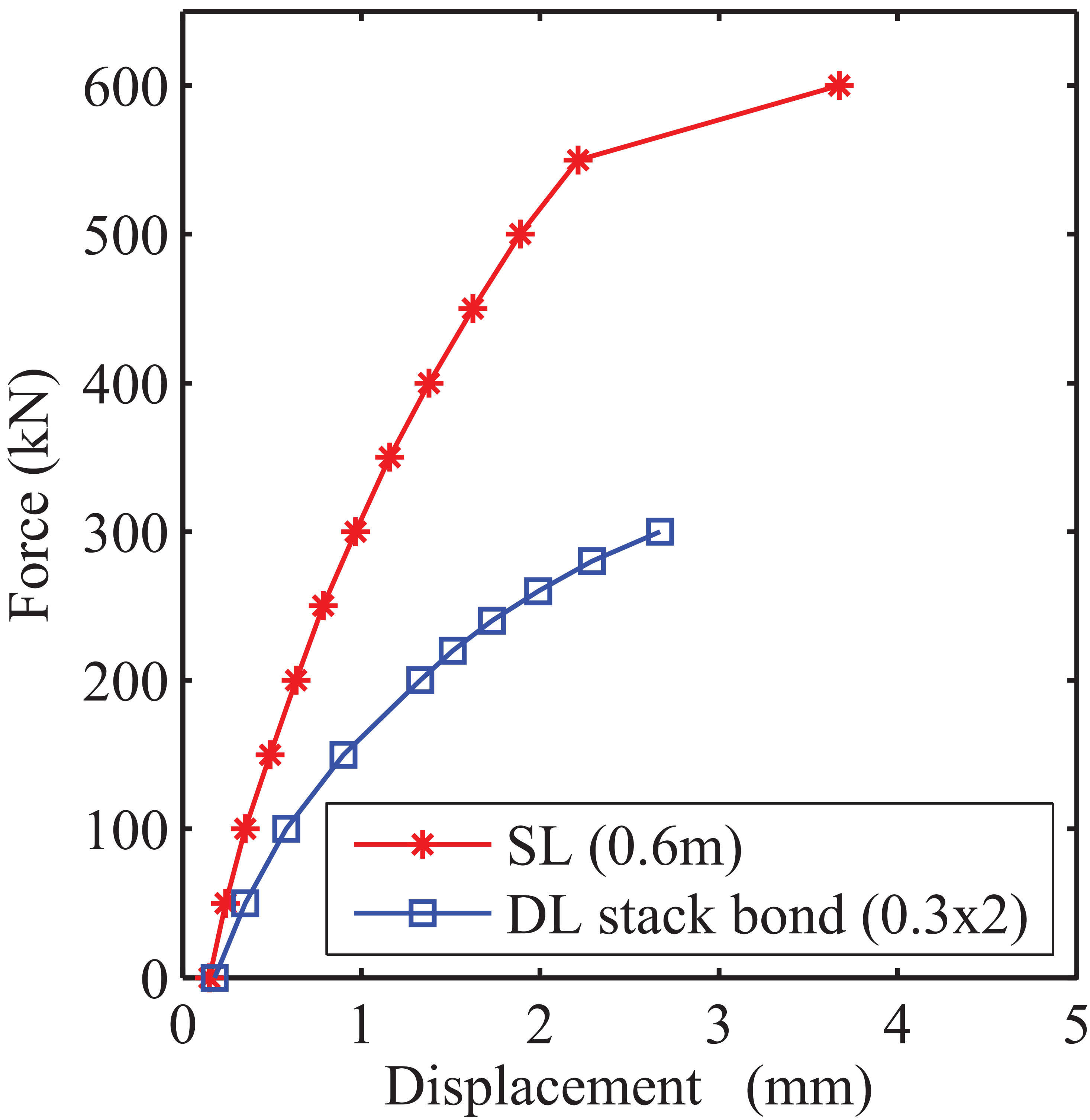
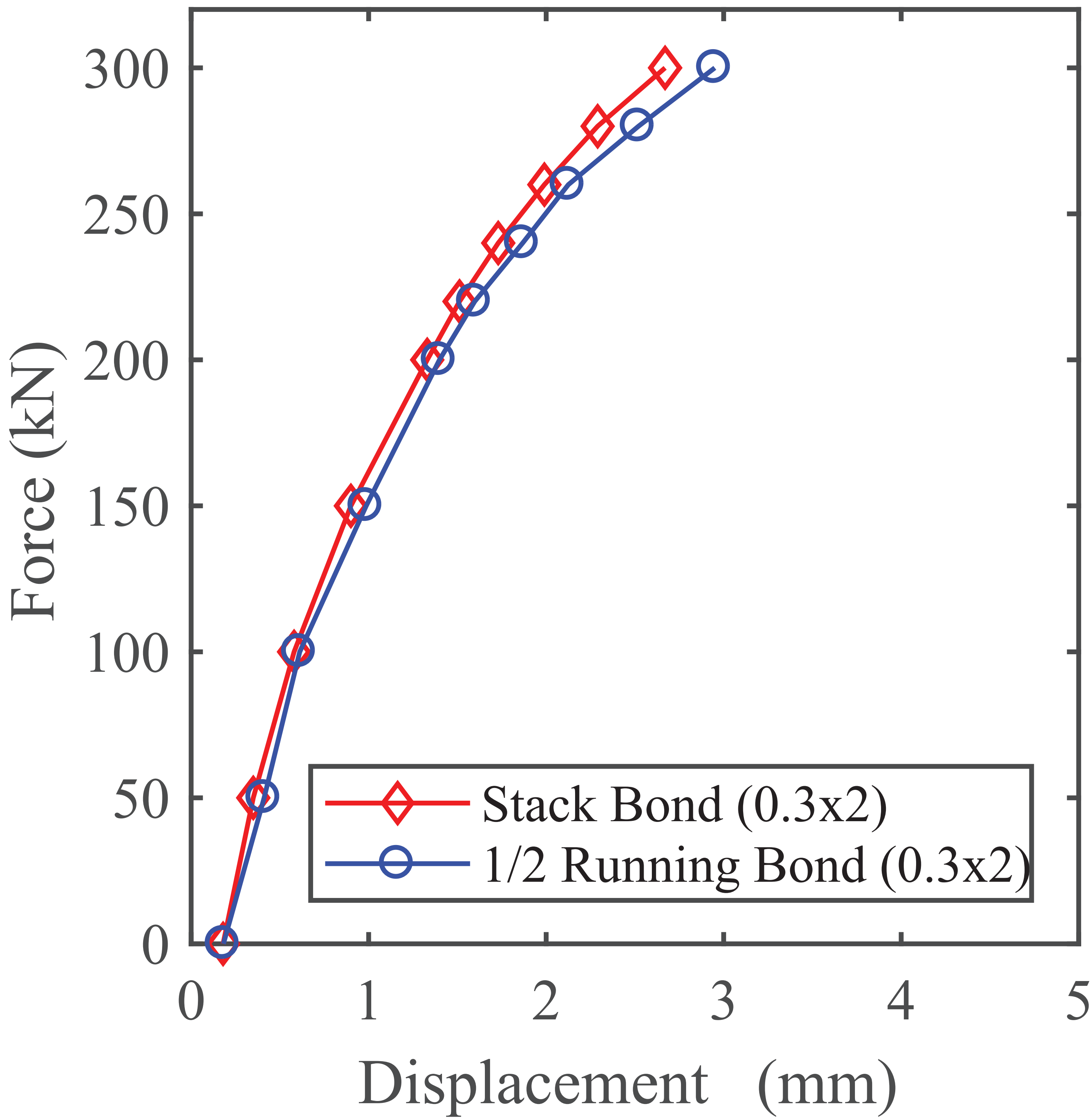


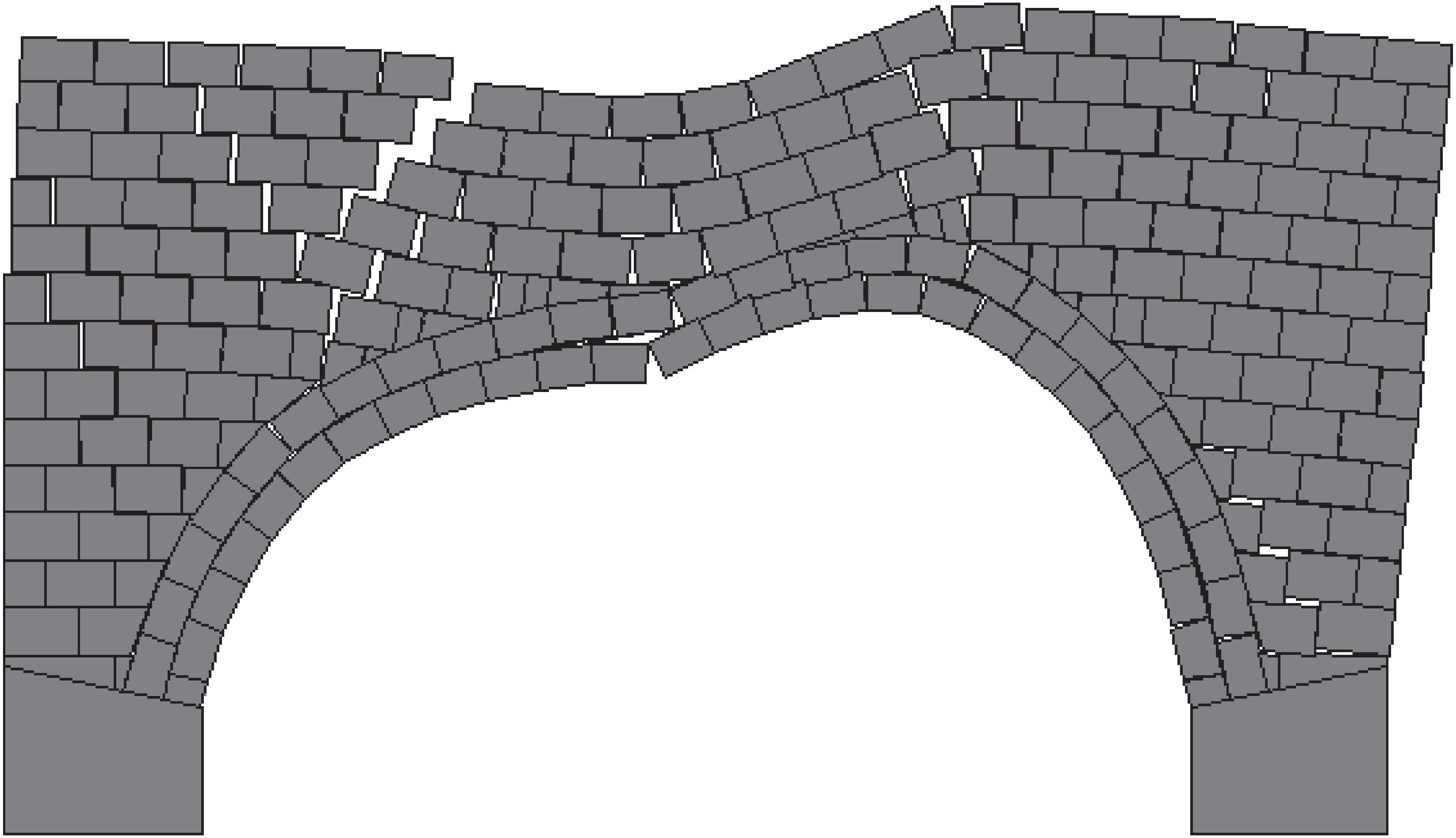

600
500
400
300
200
100
0
Force (kN)
0
1
2
3
4
5
Displacement (mm)
SL (0.6m)
DL stack bond (0.3x2)

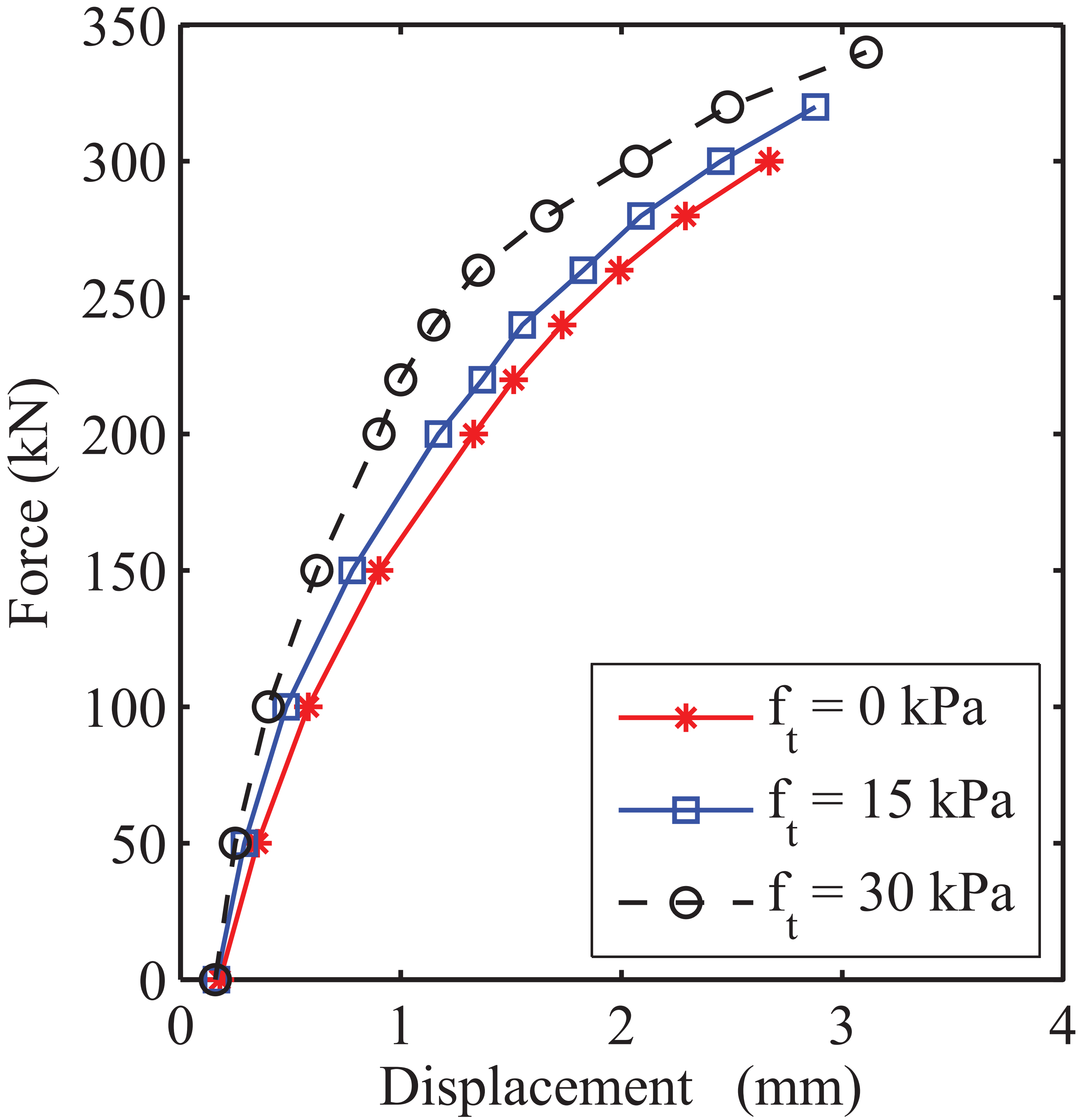


300
250
200
150
100
50
0
Force (kN)
0
1
2
3
4
5
Displacement (mm)
Stack Bond (0.3x2)
1/2 Running Bond (0.3x2)

Figure

Click here to download Figure Fig. 12c.pdf




Force (kN)
Displacement (mm)
0
50
100
150
200
250
300
350
0
1
2
3
4
f_t = 0 kPa
f_t = 15 kPa
f_t = 30 kPa

List of Figure Captions

Fig. 1. Discrete blocks and assigned springs in two orthogonal directions.

Fig. 2. Contact points and stress distribution.

Fig. 2a Contact points and external force, F

Fig. 2b Stress distribution at the contact.

Fig. 3. Capacity curve, obtained via discrete element code and comparison with limit analysis and experimental peak load.

Fig. 4. Experimental and numerical modeling of true scale brickwork masonry arch bridge.

Fig. 4a Damaged state of masonry arch bridge (BA2) loaded at ¼ of the clear span.

Fig. 4b Damaged state of masonry arch bridge using DEM incrementally loaded at ¼ of the clear span.

Fig. 5. Dimensions of the base model and the arrangement of rigid blocks in the numerical model.

Fig. 6. Collapse mechanism of masorny arch under vertical eccentric loading.

Fig. 6a Comparison between DEM and Limit Analysis.

Fig. 6b Discrete Element Model.

Fig. 6c Limit Analysis (ArchNURBS).

Fig. 7. Force-Displacement curve for masonry arch with in-fill material using DEM and Limit Analysis (ArchNURBS).

Fig. 8. Force-Displacement curves for different thickness of spandrel wall (SW) thicknesses.

Fig. 9. Boundary conditions.

Fig. 9a Fixed at the supports and both sides of the structure are free to move.

Fig. 9b Fixed at the supports and passive earth pressure applied at both sides.

Fig. 10. Capacity curves and corresponding failure mechanism.

Fig. 10a Load vs. displacement for different boundary conditions.

Fig. 10b Collapse mechanism.

Fig. 11. Double layer masonry arch models with different bond pattern.

Fig. 11a Masonry arch (double layer stack bond) with spandrel wall.

Fig. 11b Masonry arch (1/2 running bond) with spandrel wall.

Fig. 12. Force vs. displacement curves for two different arch layer and corresponding collapse mechanism.

Fig. 12a Force vs. displacement of single (SL) and double layer (DL) arch with spandrel and back-fill material.

Fig. 12b Force vs. displacement curve of two different bond pattern used for arch barrel

Fig. 12c Collapse state of the structure with double layer and ½ running bond.

Fig 13. Parametric study for tensile strength at the contacts located at the arch barrel.

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Answer to reviewer comments:

Authors would like to acknowledge the reviewers for their valuable comments and opinions. We are also grateful anonymous reviewers whose comments provided important improvements to this manuscript.

Reviewer #1:

1) DEM is not the most appropriate tool to analyze arches, even interacting with the backfill. The most appropriate way (also for vaults and 2D curvature structures) is limit analysis because it provides immediately collapse loads and failure mechanisms. In this regard, this should be acknowledged at least in introduction with a proper discussion. Good references to corroborate such idea are the following:
- Milani E., Milani G., Tralli A. (2008). Limit analysis of masonry vaults by means of curved shell Finite Elements and homogenization. International Journal of Solids and Structures, 45(20): pp. 5258-5288.
- Milani G., Simoni M., Tralli A. (2014). Advanced numerical models for the analysis of masonry cross vaults: A case-study in Italy. Engineering Structures, 76, pp. 339-358.
- Milani G. (2015). Upper Bound Sequential Linear Programming mesh adaptation scheme for collapse analysis of masonry vaults. Advances in Engineering Software, 79, pp. 91-110.
- Chiozzi A., Milani G., Tralli A. (2017). A Genetic Algorithm NURBS-based new approach for fast kinematic limit analysis of masonry vaults. Computers & Structures, 182, pp. 187-204.
the following general review paper would be also very beneficial to improve the discussion:

Related valuable references were added and further explanation of limit analysis in the introduction part was given as suggested (line 67 – line 76)

2) In the examples treated it would be interesting to plot the thrust-line at collapse, to see what really changes in the load carrying capacity, especially for the spandrel model.

The proposed discrete element models were analyzed using limit analysis and thrust line was obtained at the stage of collapse. All the limit analysis was done by ArchNURBS, cited in the text.

3) I do not understand why authors use such a simplified way for the backfill. There is a classic paper by Bicanic (that should be cited) where DEM is used taking into account backfill in a proper way. Again limit analysis is more straightforward and there are at least two classic papers by Cavicchi and Gambarotta discussing this issue that should be cited.

The motivation was to model infill material in a simplified way computationally without including the details of the infill into discrete element method (e.g. deformable blocks for back-fill material). Suggested references were included to acknowledged, there are other methods for more detailed analysis.

4) The numerical approach used should be discussed in detail. This is a static approach but traditionally DEM works in dynamics. Is it a "slow" dynamic solver? There is a special issue in International Journal of Masonry Research and Innovation (Vol 1 Issue 4) in Memory of Prof. Bicanic where some details are provided. Also, in the same Journal there is a paper by Drei, Sincraian and Milani that study two masonry aqueducts with UDEC. Maybe a new Section could be beneficial.

Authors of the paper greatly appreciate from your suggestions. Since, discrete element method has a dynamic procedure to solve differential equations of motion, we used dynamic relaxation method to obtain static solutions. Therefore, we can model damaging process of the masonry arches. Improvements in the text was done to clarify the employed numerical approach.

5) Is there any possibility to compare DEM prediction with that provided by Limit Analysis? In all the force-displacement curves provided there are horizontal lines that I imagine indicate the collapse load. However in some cases the capacity curve at collapse still does not have a horizontal first derivative. This is theoretically

expected in any elasto-plastic model. Is there any reason about this issue or simply it indicates the first point of lack of convergence? Maybe using Ring software by Gilbert or ArchNurbs software by Chiozzi authors could have an idea of the expected collapse load, also to corroborate results.

A new section (Validation of the Methodology) was added to validate the custom-made software. Experimental results were from Roca & Molins (2004) compared with the limit analysis and DEM solutions. All of the capacity curves were revised and the last point where we have the successful convergence was selected as maximum load carrying capacity.

Reviewer #2:

1) line 72. It states that "custom-made software" was used. There should be a reference to some paper (or thesis) with more information about the software.

The references were added.

2) line 104 and Fig. 2b. The figure shows a linear stress variation. Is this diagram used to calculate the contact forces? Most DEM codes are based on simple vertex-face contacts. The linear diagram is an interesting approach that should be better explained.

The stress diagram is used for contact forces. This issue is clarified further in the text.

3) line 121. Instead of "numerical stability is lost", perhaps it would be more accurate to say "equilibrium is lost", or "movements increase without bound".

The sentence is changed to "movements increase without bound".

4) The results obtained for the various conditions are quite credible. The relative values of the carrying capacity appear realistic. The weak point of the paper is that there is no quantitative comparison of any failure load to an analytical solution (for a simple arch), or to other numerical models. The arch shape is similar to the one analyzed by Lemos (1995) with UDEC/3DEC. Is it possible to compare any loads? Or at least to confirm the conclusions drawn about the arch behavior?

A whole new section was added to validate the custom-made software as 'Validation of the Methodology'. Experimental results from Roca & Molins (2004) were compared with the limit analysis and DEM solutions. Similar collapse mechanism and load carrying capacities were found with experimental study.

5) line 225. The study of the model with a 2 layer arch is very interesting. There is not much in the literature about this issue, so these results are valuable. They show clearly the much lower capacity of the double layer arch. The authors comment that in this model the joint between the layers is only frictional. But, would the consideration of a bond between the 2 layers (assuming mortar cohesion and tensile strength) really increase significantly the capacity? An extra analysis to check this would be a good enhancement of the paper.

A sensitivity analysis was performed to check how much the capacity will be influenced by the tensile strength at the joints (for the arch-ring). Although, we found that there is a contribution coming from limited tensile strength, it is still difficult to achieve single layer arch-ring capacity. This is because, strength is mostly governed by the geometrical properties of the arch form and distinct composition of the masonry units. (line 318-325)

Associate Editor's comments:

1. Line 34: Consider changing "Throughout the history" to "Throughout history";

It is corrected.

2. Line 41: Consider changing "Last three decades" to "In the last three decades,";

It is corrected.

3. Line 52: Consider changing "including arch and arch with infill" to "including arch with and without infill";

It is corrected.

4. Line 56: Consider changing "The collapse of the masonry structures" to "The collapse of the arch" as the referenced for is pertinent to arches;

It is corrected.

5. Line 58: This assumption is improperly states: "(ii) friction between voussoirs is high enough to get sliding failure". Heyman's assumption it that sliding failure cannot occur. Consider changing to "friction between voussoirs is high enough to prevent sliding failure";

The statement is revised and put into the form as suggested.

6. Line 64: Consider changing "by extending the infinite compressive strength to material crushing strength and the possibility of sliding" to "by limiting the infinite compressive strength to material crushing strength and accounting for the possibility of sliding";

The related correction is made.

7. Line 203: "Masonry construction is mostly a manual trade,.." This may be true for historic masonry construction, but not for modern construction which follows current standard codes for design and construction. Consider changing to "Historic masonry construction was mostly a manual trade,...";

Authors agree with your comment and changed as suggested.

8. Line 215: Consider changing "masonry" to "arch";

It is changed.

9. Line 231: Use either "in-fill" or "infill" throughout the paper, but not both. Also, backfill would be more appropriate than infill for masonry arches;

In the revised version of the text, the term 'backfill' is used.

10. Line 233: Consider deleting "asymmetric" to make this statement more general. One would expect that even a symmetrical point load, i.e. point load at the arch crown, will produce tensile flexural stresses in the arch barrel.

The word, 'asymmetric' is taken out from the related sentence.

11. Line 248, Figure 7: It would be helpful to combine Fig 6a and Fig 7 into one figure in order to directly compare the effect of infill on the arch capacity;

Due to the organization of the other figures and the text, we decided to keep as it is with all due respect to your comment.

12. Line 258: Consider changing "To simulate behavior one of these outer planes of the arch bridge,.." to "To simulate the behavior of one of these outer sections,..."

It is changed as suggested.

13. Line 311: Use either "back-fill" or "backfill" throughout the paper, but not both.

In the revised version of the text, it is changed to 'backfill'.

14. Line 318: Consider changing "..., sensitivity analysis...." to "..., a sensitivity analysis...."

It is changed as suggested.

15. Line 324: "The failure mechanism generally triggers by the lack of tensile strength at the joints due to aged and damaged mortar." Please revise or delete this sentence as it seems that it is not supported by the numerical results obtained in this study. Results presented in Fig 13 suggest the opposite: that the failure load and, consequently, the failure mechanism are marginal affected by the masonry tensile strength.

The related part is revised, and new explanations are added to clarify that the range of assigned tensile strength is quite small which does not lead to any significant difference on the overall load carrying capacity of the structure.