# Optimal Medium-Term Hydropower Scheduling Considering Energy and Reserve Capacity Markets

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***Abstract*—This paper describes a method for optimal scheduling of hydropower systems for a profit maximizing, price-taking and risk neutral producer selling energy and capacity to separate and sequentially cleared markets. The method is based on a combination of stochastic dynamic programming (SDP) and stochastic dual dynamic programming (SDDP), and treats inflow to reservoirs and prices for energy and capacity as stochastic variables.**

**The proposed method is applied in a case study for a Norwegian watercourse, quantifying the expected changes in schedules and water values when going from an energy-only market to a joint treatment of energy and reserve capacity markets.**

***Index Terms*—Hydroelectric power generation, Power generation economics, Linear programming, Stochastic processes.**

## NOMENCLATURE

### A. Index Sets

| | |
|---|---|
| $\mathcal{H}$ | Set of hydropower reservoirs/stations; |
| $\mathcal{S}_h$ | Set of discharge segments for station $h$; |
| $\mathcal{K}$ | Set of time steps within the week; |
| $\mathcal{K}_b$ | Set of time steps associated with block $b$; |
| $\mathcal{B}$ | Set of reservation blocks within the week; |
| $\mathcal{L}_{p,t}$ | Set of cuts for price node $p$ and week $t$; |
| $\Omega_h$ | Set of reservoirs upstream reservoir $h$. |

### B. Parameters

| | |
|---|---|
| $P_h^{max}$,$P_h^{min}$ | Max./Min. capacity in station $h$, in MW; |
| $V_{kh}^{max}, V_{kh}^{min}$ | Max./Min. limit for reservoir $h$, in Mm$^3$; |
| $R_h^{max}$ | Max. capacity reservation for station $h$, in MW; |
| $\tau_k$ | Duration of time step $k$, in hours; |
| $\tilde{\tau}_k$ | Relative duration of time step $k$, fraction; |
| $\tau_b$ | Total duration of reservation block $b$, in hours; |
| $\zeta_k^E$ | Energy price scaling coefficient for time step $k$; |
| $\zeta_b^C$ | Capacity price scaling coefficient for block $b$; |
| $\eta_{hs}$ | Energy equivalent for station $h$ and discharge segment $s$, in MWh/Mm$^3$; |
| $\gamma_h$ | Factor limiting the use of spinning reserves; |
| $\varphi$ | Cost of artificial water, in €/Mm$^3$; |
| $\pi_{ph\ell}$ | Coefficient for reservoir level for price node $p$, reservoir $h$ and cut $\ell$, in €/Mm$^3$; |
| $\mu_{pb\ell}$ | Coefficient for capacity sales for price node $p$, block $b$ and cut $\ell$, in €/MW; |
| $\beta_{p\ell}$ | Right-hand side for price node $p$ and cut $\ell$, in €; |
| $K$ | Last time step in week; |
| $T$ | Number of weeks in planning horizon; |
| $N_E$ | Number of energy price clusters; |
| $N_C$ | Number of reserve capacity price clusters. |

### C. Stochastic Variables

| | |
|---|---|
| $I_h$ | Sum weekly inflow to reservoir $h$, in Mm$^3$; |
| $\lambda_t$ | Vector of prices in week $t$, in €/MWh; |
| $\lambda_p^E$ | Weekly average energy price for price node $p$, in €/MWh; |
| $\lambda_p^C$ | Weekly average reserve capacity price for price node $p$, in €/MW; |

### D. Decision Variables

| | |
|---|---|
| $\alpha_{p,t+1}$ | Future expected profit for price node $p$ and week $t+1$, in €; |
| $\Phi(\ldots)$ | End value function, in €; |
| $e_{kh}$ | Generated electricity in time step $k$ for station $h$, in MWh; |
| $c_{b,t+1}$ | Sold capacity for block $b$ in week $t+1$, in MW; |
| $r_{kh}$ | Allocated capacity in time step $k$ for station $h$, in MW; |
| $v_{kh}$ | Volume in time step $k$ for reservoir $h$, in Mm$^3$; |
| $q_{khs}^D$ | Discharge in time step $k$ through station $h$ at segment $s$, in Mm$^3$; |
| $q_{kh}^S$ | Spillage in time step $k$ from reservoir $h$, in Mm$^3$; |
| $q_{kh}^B$ | Bypass in time step $k$ from reservoir $h$, in Mm$^3$; |
| $w_h$ | Artificial water supply to reservoir $h$, in Mm$^3$; |
| $\mathbf{X}_t$ | Vector of decision variables for week $t$; |
| $\mathbf{Z}_t$ | Vector of state variables for week $t$. |

## I. INTRODUCTION

The future Nordic electricity system will see stronger connections to the European system, and include a larger share of renewable, intermittent generation than what is the case today. This development will e.g. be driven by the building of new overseas cables, environmental targets set by the European Union and decisions on downscaling of nuclear generation capacity. Tighter market couplings and increased contributions from intermittent generation will call for efficient and reliable arrangements for balancing services. In the low-load (summer) period of the year a significant share of the large reservoir power stations will not be in operation, so that non-dispatchable intermittent generation will cover the load.

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In these periods it is important and costly for the system operator to procure sufficient amounts of spinning reserves to ensure stable operation of the system. Conversely, in the high-load period (winter) the system operator needs to procure sufficient up-regulation reserves to handle potential shortfalls in intermittent generation.

On the road towards an integrated European electricity market, regulators and system operators aim at establishing harmonized and economically efficient markets for balancing services [1]. For hydropower producers in the Nordic market, the importance of the different types of market products may change significantly from what they are familiar with. Today, the producers primarily benefit from selling energy in the day-ahead (spot) market. However, the flexibility of hydropower enables active contribution in balancing markets as well, which in turn will challenge the way hydropower systems traditionally are operated and scheduled. The choice between utilizing the hydropower for electricity generation or as reserve capacity will impact the strategic evaluation and scheduling of resources. That is, the consideration of multiple markets will impact the way the *water values* are calculated, and the scheduling tools and methods need to take this into account.

Numerous solution strategies have been applied to the hydropower scheduling problem, see e.g. [2] for a thorough review. Stochastic dynamic programming (SDP) has proved to be well suited for systems with relatively few reservoirs, but will suffer from extreme computation times when considering realistic multi-reservoir systems. In order to avoid the curse of dimensionality limiting the use of SDP, an approach known as stochastic dual dynamic programming (SDDP) was presented in [3]. Unlike SDP, there is no need to fully discretize the state variables with the SDDP algorithm. SDDP is a sampling-based variant of multi-stage Benders decomposition, where an outer approximation of a concave future expected profit function is constructed iteratively for each time-stage by adding Benders cuts. Thus, the overall optimization problem can be decomposed into small linear programming (LP) problems, one for each decision stage, that can be solved independently. Currently, SDDP seems to be the state-of-the-art method for solving the long- and medium-term scheduling problem in regions where hydropower is the dominant technology for generating electricity, see e.g. [4] and [5]. Recent research related to the SDDP algorithm has e.g. focused on convergence properties [6], parallel processing and computational performance [7], strategic bidding [8], risk aversion [9], emission constraints [10] and integration of pumped-storage and wind power [11].

In a liberalized market, hydropower producers will typically act as price-takers trying to optimize the utilization of hydro reservoirs to maximize their expected profit. For this purpose, a hybrid method combining the SDP and SDDP methods was presented in [12] and [13]. The basic idea is to combine the strengths of the SDP and the SDDP methods to retain the convexity of the problem while dealing with the curse of dimensionality. In this hybrid method, the energy price forecasts are treated as discrete Markov chains through SDP. Other state variables, such as reservoir levels and inflows, are continuously approximated in the SDDP scheme. Note that the price process can also be described by means of a scenario tree and combined with the SDDP method, as described in [14].

Approaches incorporating treatment of balancing markets in medium-term hydropower scheduling methods seems to be scarce. A detailed simulator allowing operational scheduling of a hydropower system in day-ahead and real-time markets was presented in [15], relying on water values computed in a separate and less detailed procedure. Some authors have decomposed the scheduling problem into intra- and inter-stage problems, as first suggested in [16] and further discussed in [17]. In this scheme the inter-stage problem will take care of the longer-term and strategic decisions, e.g. how much water to use in a given week, while intra-stage decisions concern the detailed operation using a much finer time-resolution. Based on this scheme [18] proposed a method for stochastic medium-term hydropower scheduling considering participation in both the spot and secondary reserve markets. Inter-stage decisions regarding operation of seasonal reservoirs are found by use of SDP, and the shorter term intra-stage decisions, e.g. related to sales of spinning reserves, are found by solving a multi-stage mixed-integer problem. This type of method will realistically capture short-term uncertainty by allowing stochastic intra-stage modeling. On the other hand, the approach depends on the assumption that inter-stage initial states and decisions cannot depend on realization of intra-stage uncertainty, and the use of SDP limits the number of reservoirs that can be included in the inter-stage part within reasonable computation time. A different and more uniform method for incorporating sales of spinning reserves in a medium-term hydropower scheduling model was presented in [19]. It extends the hybrid SDP/SDDP algorithm in [13] by allowing co-optimized sales of energy and reserve capacity, treating the capacity price as deterministic.

The novel contributions of this paper regard improvements of the model presented in [19] to more realistically address stochasticity, decision sequences and operational constraints seen in the multi-market scheduling. Firstly, the price model is updated to allow stochastic reserve capacity prices. Secondly, capacity sale is allowed the week ahead of operation, under uncertain energy prices and inflows. Although many power markets around the world are designed for co-optimized trade of energy and reserve capacity [20], the sequential decision sequence is more realistic in the Nordic market. It is elaborated how sales of reserve capacity enters as state variables in the decomposed problem structure and are included in the Benders cuts. Finally, linear constraints are added to discourage delivery of reserve capacity while running stations below their minimum power output, and to ensure that there is enough water in the reservoirs to deliver the committed up-regulation reserves.

The proposed method is applied in a case study where the optimal schedule for a Norwegian watercourse is found considering participation in both the day-ahead and reserve capacity markets.

The paper is outlined as follows. First, a brief description of the reserve capacity markets in Norway is provided in Section II. Subsequently, a basic mathematical description of the hydropower scheduling problem and the combined

SDP/SDDP algorithm is provided in Section III, emphasizing on the new features. The case study is presented in Section IV. Finally, conclusions are drawn in Section V.

## II. Markets for Reserve Capacity

In the Nordic system, the transmission system operators (TSOs) are responsible for matching supply and demand of electricity in real time. In order to ensure this balance the TSOs need to be able to acquire balancing services, both in terms of capacity and energy. The Norwegian TSO (Statnett) acquires primary, secondary and tertiary reserves through market-based approaches using the marginal pricing principle [21]. In this section the sequences and the basic properties of the various reserve markets in Norway are summarized.

In Table I the different energy and reserve capacity markets and their clearing is organized in a time-sequence, indicating the decision stages that a producer needs to relate to. The table shows the type of service, market name (explained below), the time period and the discrete time intervals for which the markets are cleared.

Momentary imbalances between production and demand will firstly be regulated by use of primary reserves, often referred to as frequency-controlled reserves for normal operation (FCR-N). FCR-N contributes to both upward and downward regulation in the frequency band 49.90-50.10 Hz. Such reserves are currently assured by the droop setting in the turbine governors for generators exceeding 10 MVA. That is, generators that do not participate in the primary reserve market will still participate in the primary regulation. In 2008 both a weekly and a daily primary reserve market were established.

If imbalances last for minutes, the secondary regulation reserves will take over, releasing the primary regulation reserves so that these are available in case of new imbalances. An arrangement for secondary reserves was initiated in Norway in 2008, and later on led to the introduction of a system service known as automatic frequency restoration reserves (FRR-A) in 2013.

If frequency deviations still persist after activation of objects in the primary and secondary markets, the manual frequency regulation reserves (FRR-M) will be activated by the TSO. The TSOs in the Nordic market have different arrangements for securing that sufficient amounts of reserve energy will be bid into the FRR-M market. The Norwegian regulating power option market (RKOM) was established in 2000 for this purpose. Both generators and consumers can bid to RKOM, but currently only for up-regulation. The accepted regulation offers for a given period receive an option payment.

The structure shown in Table I shows that reserve capacity is generally traded the week before actual operation and before the decisions on energy trade. This market design differs from the co-optimization of energy and reserves described e.g. [20].

## III. Model Description

The objective of the scheduling is to maximize the expected profit from sales to both the spot and the reserve capacity markets. It is assumed that the hydropower producer is a risk neutral price-taker in both markets.

### A. Model Overview

Stochastic medium-term hydropower scheduling models are typically used for generation scheduling and to provide input to short-term scheduling, and will normally take a scheduling horizon of 1-3 years. Three stochastic variables are considered in the presented model; the weekly inflow to each reservoir and the weekly average energy and reserve capacity prices. Decision stages are weekly; that is, realizations of the stochastic variables are known at the beginning of each week and for that entire week.

For a given week $t$ a vector $\mathbf{X}_t$ is defined, comprising all decision variables for that week. Associated with $\mathbf{X}_t$ there is a price vector $\lambda_t$ comprising all prices for that week. It is assumed that all costs and relationships are linear or piecewise linear and convex. The overall objective is then to find an operating strategy to obtain:

$$\max \mathbb{E}\left\{\sum_{t=1}^{T} \lambda_t^{\intercal} \mathbf{X}_t + \Phi(\mathbf{Z}_T)\right\} \tag{1}$$

The expectation is to be taken over the stochastic variables. The function $\Phi$ estimates the value of state variables $\mathbf{Z}_T$ at the end of the study period. The classification of state variables will be discussed in Section III-D.

Since water left in a reservoir at the end of a week is carried over to the next week, the water balances for the reservoirs are coupled across decision stages, making the optimization problem a dynamic one. Thus, the problem in (1) is a multi-stage stochastic optimization problem, which may be efficiently solved by decomposition techniques [22].

The overall optimization problem is solved by a combination of SDDP and SDP, using an approach which is close to that described in [13]. By using dynamic programming principles and representing the future expected profit functions by hyperplanes or *cuts*, the problem is decomposed into weekly subproblems with given values of inflows, energy and reserve capacity prices. The algorithm builds an operating strategy (represented by cuts) iteratively, by repeated forward and backward iterations through the sequence of weekly subproblems. The formulation of the decomposed weekly optimization problem is described in detail in Section III-D and the overall solution method is outlined in Section III-E.

Regarding the representation of stochastic variables, the price model is described in Section III-B. Inflows are sampled from a lag-1 autoregressive model both in the forward and backward iteration of the SDDP part of the algorithm, see e.g. [23] for further details.

### B. Price Modeling

Normally, the weekly average energy price will show a significant serial correlation. This seems also to be the case for the weekly average reserve capacity prices, although the correlation will depend on the type of market being considered. In this work we treat both prices as stochastic and present a price model capturing both the serial and cross correlation between the two price processes. Due to the serial correlation it is necessary to include the price state in the system state

TABLE I
TIME-SEQUENCE FOR THE DIFFERENT ENERGY AND RESERVE CAPACITY MARKETS IN NORWAY.

| | | | | | October | Thursday | Thursday | Friday | Day-1 | Day-1 | Hour-1 | Hour-0:45 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Type** | **Market** | **Period** | **Resolution** | **Commodity** | | 10:00 | 12:00 | 12:00 | 12:00 | 18:00 | | |
| Tertiary | RKOM season | Winter | Season | Capacity | ✓ | | | | | | | |
| Secondary | FRR-A | Week | Block | Capacity | | ✓ | | | | | | |
| Primary | FCR-N week | Weekend | Block | Capacity | | | ✓ | | | | | |
| Tertiary | RKOM week | Week | Block | Capacity | | | | ✓ | | | | |
| Primary | FCR-N week | Weekday | Block | Capacity | | | | ✓ | | | | |
| Day-ahead | ELSPOT | Day | Hour | Energy | | | | | ✓ | | | |
| Primary | FCR-N day | Day | Hour | Capacity | | | | | | ✓ | | |
| Intraday | ELBAS | Continuous | Hour | Energy | | | | | | | ✓ | |
| Tertiary | FRR-M | Hour | Hour | Energy | | | | | | | | ✓ |

description. As discussed in [13], a price state will violate the convexity requirement of the SDDP algorithm. The price processes are therefore modeled as a Markov chain using discrete states (price nodes), and embedded in the SDDP algorithm as in ordinary dynamic programming. Note that the combined SDDP/SDP algorithm generally requires the stochastic processes being modelled in the SDP part to be independent of those modelled in the SDDP part. Thus, the weekly price processes are assumed independent of the inflow. In our experience, for a regional system (e.g. a single watercourse within a price zone) it is difficult to find a significant correlation between local inflow during a week and the average spot price for the same week.

The method in [24] is used to establish the Markov model based on a set of energy and capacity price scenarios. First, scenarios from the two price processes are sorted individually into $N_E$ energy and $N_C$ reserve capacity price clusters for each stage, and average cluster prices $\lambda_i^E, i \in N_E$ and $\lambda_i^C, i \in N_C$ are found. A price node contains a pair of energy and capacity price clusters, as illustrated in Fig. 1, where price node $i$ in stage $t-1$ comprises cluster prices $\lambda_2^E$ and $\lambda_2^C$. The transition probability $\rho_{ij}$ in going from a node $i$ in week $t-1$ to node $j$ in week $t$ is computed by finding the ratio between the number of scenarios belonging to both node $i$ and $j$ and those belonging to node $i$.

For each week a maximum number of $N_E \times N_C$ price nodes and $(N_E \times N_C)^2$ transition probabilities shall be identified. Proper model identification requires a large number of scenarios. Such scenarios can e.g. be obtained from a fundamental long-term scheduling model.

### C. Decision Stages and Capacity Sales

The intra-week time resolution is illustrated in Fig. 2. Each week $t$ is divided into time steps $k$ in which energy can be sold and schedules for individual power plants are made.

Capacity is sold in blocks, where one block $b$ can cover multiple time steps, e.g. hours 0-8 on all weekdays. The capacity $c_{b,t}$ sold in one block should be a joint decision for all time steps belonging to that block. Moreover, it is assumed that the reserve capacity market is cleared the week before actual operation (week $t-1$), as illustrated in Fig. 2.

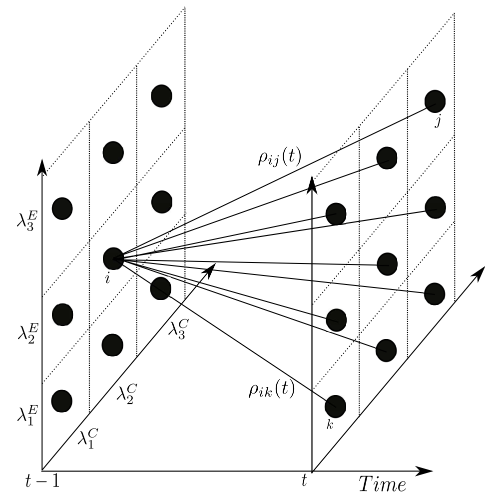


Fig. 1. Illustration of the price model.

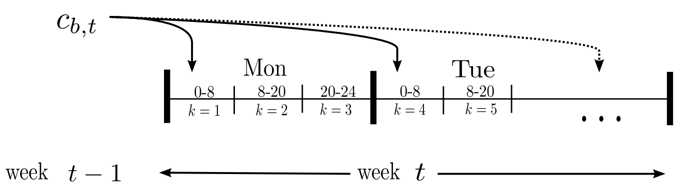


Fig. 2. Illustration of decision stages within a given week $t$.

At the beginning of each week $t$, the energy price is known for that week. Provided the capacity obligations $c_{b,t} \forall b \in \mathcal{B}$ from the week before, and realization of all stochastic variables, plans for electricity generation and capacity reservation can be decided per station in week $t$.

Capacity sales link any two consecutive weeks as in a two-stage stochastic problem. In the first decision stage representing week $t-1$, capacity sales is done based on a discretized probability distribution of energy price and inflows for week $t$. Subsequently, in the second decision stage representing week $t$, the energy price and inflows are known, and the system operation for each time step $k$ within week $t$ is found, given the capacity obligation from week $t-1$.

### D. Decomposed Weekly Decision Problem

In this section the decomposed weekly decision problem is formulated. Details on how it fits in to the combined SDDP/SDP algorithm are outlined in Section III-E.

The decomposed problem is formulated as an LP problem described by (2)-(12). For a given week $t$ the realization of weekly inflows $I_h$, the average energy price $\lambda_p^E$ and the average capacity price $\lambda_{p,t+1}^C$ for week $t+1$ are known. The amount of energy sold to the spot market and capacity to the reserve market is optimized for the whole water course, and it is assumed that there are no demand obligations. Note that for brevity of mathematical formulation, the week index is only used to indicate the next week ($t+1$) and the scenario index is only included to indicate prices and price node association of sets and parameters.

$$\max\left\{ \sum_{b\in\mathcal{B}} \tau_b \zeta_{b,t+1}^C \lambda_{p,t+1}^C c_{b,t+1} + \sum_{k\in\mathcal{K}} \sum_{h\in\mathcal{H}} \zeta_k^E \lambda_p^E e_{kh} - \sum_{h\in\mathcal{H}} \varphi w_h + \alpha_{p,t+1} \right\} \quad (2)$$

$$v_{kh} + \sum_{s\in\mathcal{S}_h} q_{khs}^D + q_{kh}^S + q_{kh}^B - w_h - \sum_{j\in\Omega_h} \left( \sum_{s\in\mathcal{S}_j} q_{kjs}^D + q_{kj}^S + q_{kj}^B \right) = v_{k-1,h} + \tilde{\tau}_k I_h \quad \forall k,h \quad (3)$$

$$e_{kh} - \sum_{s\in\mathcal{S}_h} \eta_{hs} q_{khs}^D = 0 \quad \forall k,h \quad (4)$$

$$\sum_{h\in\mathcal{H}} r_{kh} = c_{b,t} \quad \forall b, k \in \mathcal{K}_b \quad (5)$$

$$\gamma_h r_{kh} - \frac{1}{\tau_k} e_{kh} \leq 0 \quad \forall k,h \quad (6)$$

$$r_{kh} + \frac{1}{\tau_k} e_{kh} \leq P_h^{max} \quad \forall k,h \quad (7)$$

$$v_{kh} - \frac{\tau_k}{\eta_{hS}} r_{kh} \geq V_{kh}^{min} \quad \forall k,h \quad (8)$$

$$\alpha_{p,t+1} - \sum_{h\in\mathcal{H}} \pi_{ph\ell} v_{kh} - \sum_{b\in\mathcal{B}} \mu_{pb\ell} c_{b,t+1} \leq \beta_{p\ell}, \quad k = K, \ell \in \mathcal{L}_{p,t} \quad (9)$$

$$V_{kh}^{min} \leq v_{kh} \leq V_{kh}^{max} \quad \forall k,h \quad (10)$$

$$0 \leq c_{b,t+1} \leq \sum_{h\in\mathcal{H}} R_h^{max} \quad \forall b \quad (11)$$

$$0 \leq r_{kh} \leq R_h^{max} \quad \forall k,h \quad (12)$$

The objective (2) is to maximize the profit from both markets, subject to constraints (3-12). Energy and capacity prices corresponding to a specific time step or block, respectively, are found by scaling the weekly average values by pre-defined expected profiles.

The water balance equation for a specific reservoir $h$ and time step $k$ is formulated in (3). An auxiliary variable $w$ is introduced in (3) if $k=1$ allowing the model to artificially supply water to the reservoir at a high cost $\varphi$. This variable is needed to ensure that the stochastic model has complete recourse. Water discharge through the station is modeled using one variable per discharge segment in (4). These segments will be used in decreasing order according to their energy equivalent $\eta_{hs}$, provided that $\eta_{hs}$ decreases with $s$.

The capacity amount $c_{b,t}$ was sold in week $t-1$ and enters the optimization problem as an obligation in week $t$ in (5). Note that capacity obligation $c_{b,t}$ is tied to the entire watercourse. Electricity generation $e_{kh}$ in (4) and capacity allocation $r_{kh}$ in (6)-(7) are determined per station. Allocated capacity should be spinning and symmetric. The spinning requirement is taken care of in (6), ensuring that a station cannot offer more reserve capacity than what is already spinning. Eqn. (7) ensures that the generation capacity sold in the two markets does not exceed the station's installed capacity.

Eqn. (8) ensures that there is enough water in the reservoir to deliver up-regulation reserves at the lowest efficiency $\eta_{hS}$ for the entire time period in question. In the case of primary and secondary regulation reserves, this constraint may seem conservative, as the activation of these reserves will not span several consecutive hours.

The profit obtained for the current week is balanced against the future expected profit $\alpha_{p,t+1}$ for the given price node $p$. This variable is constrained by cuts in (9). The cuts should relate to all state variables, i.e. decision and stochastic variables that define the system state passed on to the subsequent week. In the presented model, the state variables are $\mathbf{Z}_{t+1} = [\mathbf{v}_K, \mathbf{c}_{b,t+1}, \mathbf{I_h}, \lambda^E, \lambda^C]$. The construction of cuts is described in Section III-E3. These cuts are built and stored for each week and price node in a set $\mathcal{L}_{p,t}$ in each backward iteration of the algorithm.

Being a linear model, one cannot guarantee operation above the station's minimum output $P_h^{min}$. To do so one would have to use binary variables, which conflicts with the convexity requirement of the SDDP algorithm. By introducing the parameter $\gamma_h$ in (6), as estimated in (13), the model is discouraged from operating below the station's minimum output for the purpose of delivering reserves.

$$\gamma_h = \max\left\{ \frac{P_h^{min}}{R_h^{max}}, 1.0 \right\} \quad (13)$$

Consider as an example a station with $P_h^{min}$ of 50 MW and maximum reserve delivery $R_h^{max}$ of 33 MW. Letting $\gamma_h = \frac{50}{33}$ according to (13), $r_{kh}$ cannot reach it's maximum value before the station produces at least 50 MW. Note that costs associated with starting and stopping stations are not considered in this work, but could be included as described in [11].

All variables have non-negativity constraints. The reservation variable $r_{kh}$ is of special interest in this study; it is constrained as shown in (12). The value of $R_h^{max}$ should be set by the modeler to realistically represent the amount of reserves required by the TSO, and will depend on the type of reserves being considered. In case of primary reserves, the capacity sold to the primary reserve market cannot exceed the physical limit dictated by the droop settings in the turbine governors, as described in [19].

Note that the presented formulation requires the reserves to be symmetric and spinning. This requirement is easily relaxed to adapt to reserve markets with different requirements, e.g. by adding separate variables for up and down regulation and omitting (6).

### E. Solution Strategy

A hybrid SDP/SDDP approach is applied to decompose the overall optimization problem. Repeated forward and backward iterations through the sequence of weekly subproblems are carried out as briefly described below, see e.g. [13], [23] for further details.

*1) Forward Iteration:* A set of scenarios are sampled for the stochastic variables. Weekly inflows are sampled from a lag-1 autoregressive model, and weekly average energy and reserve capacity prices are sampled based on the conditional transition probabilities in the discrete Markov chain. For a given scenario, the decomposed problem described by (2)-(12) is solved for a week. Subsequently, the simulated state at the end of the week is passed forward as an initial state for the next week. The forward simulation gives an updated set of state trajectories and an expected profit for the sampled scenarios, which serves as the lower bound.

*2) Backward Iteration:* Cuts at the end of the planning horizon $T$ can be obtained from a pre-defined final value function $\Phi$. For each state trajectory obtained in the forward simulation one starts from the state at the end of week $T-1$, and for each realization of stochastic variables one computes the optimal operation for week $T$. From the sensitivities of the objective function to the initial state values, new cuts at the end of week $T-1$ are obtained, and the process is repeated for week $T-1$, and so on. The upper bound is obtained from the solution of the first-week problem. Convergence can formally be declared when the upper bound is within a predefined confidence interval of the lower bound.

*3) Constructing Cuts:* Due to the time-couplings in equations (3) and (5), the decision variables $v_{kh,t}$ for $k = K$ and $c_{b,t+1}$ for $b \in \mathcal{B}$ will enter the decomposed optimization problem in week $t+1$, and must therefore be considered as state variables. Thus, these variables should enter the future profit function which is represented by cuts of type (9).

In the first time step in a given week $t+1$ in the backward iteration for a given inflow sample $i$ and price node $p$, the two equations (3) and (5) can be formulated as in (14) and (15), respectively. Dual values associated with constraints for the given sample are in parentheses.

$$v_{pikh,t+1} + (\cdots) = v_{Kh,t} + \tilde{\tau}_k I_{ih,t+1} \qquad (\pi_{pih}) \quad k = 1, \forall h \tag{14}$$

$$\sum_{h \in \mathcal{H}} r_{pikh,t+1} = c_{b,t+1} \qquad (\mu_{pib}) \quad k \in \mathcal{K}_b, \forall b \tag{15}$$

The dual values are together with the obtained objective value $\hat{\alpha}_{p,t+1}$ used to create cut for inflow sample $i$, price node $p$ and week $t$ in (16), where the starred variables represent the state passed from week $t$ to $t+1$.

$$\alpha_{p,t+1} - \sum_{h \in \mathcal{H}} \pi_{pih} \big( v_{piKh,t} - v^*_{piKh,t} \big) - \sum_{b \in \mathcal{B}} \mu_{pib} \big( c_{pib,t+1} - c^*_{pib,t+1} \big) \leq \hat{\alpha}_{p,t+1} \tag{16}$$

After separating variables and parameters and averaging coefficients over all inflow realizations, the cut takes the form in (9) and is stored in set of cuts $\mathcal{L}_{p,t}$.

It should be noted that inflow is also a state variable due to the time coupling in the autoregressive inflow model. As inflow is not a decision variable, its contribution to the cut will enter the right-hand side in (9), as described in [23].

## IV. Case Study

### A. Case Description

A computer model was established implementing the proposed method. The model was tested on a Norwegian watercourse comprising 7 hydropower reservoirs with corresponding power stations, with a total capacity of 986 MW. An illustration of the topology and technical characteristics is provided in Fig. 3. For each reservoir shown in the figure the average annual inflow and storage capacity are stated, both in $Mm^3$. Each power station is identified with a number and its installed capacity in MW. Stations 4 and 5 have a minimum production limit of 70 and 50 MW, respectively. The cost of artificial water ($\varphi$) was set to $10^6$ €/$Mm^3$.

A scheduling horizon of 2 years was applied with weekly decision stages. Each week was divided into 21 time steps and capacity sales were arranged in 3 blocks covering weekdays (night, day and evening), and 3 blocks covering weekends (night, day and evening). This definition of blocks is in line with the current market design for the Norwegian weekly primary reserve market and the secondary reserve market, as discussed in Section II. Each station is allowed to commit a maximum of 10 % of its installed capacity to the reserve market. Decisions regarding sales of energy are done for each time step, whereas decisions on sales of capacity are done per reserve block

A set of cuts of type (9) was used to ensure that state variables at the end of the scheduling horizon were valuated. These cuts were obtained as a result of a few initial model runs, and the same set of cuts was used for all simulated cases. An alternative approach would be to run the model with a longer scheduling horizon so that the results for the two-year period are less dependent on valuation of state variables at the end of the scheduling horizon.

Energy and reserve capacity price scenarios was obtained from the EMPS model, which is a fundamental hydrothermal market model [25]. The EMPS model was run on a system description of the Nordic power system, using 80 historical inflow years, and with reserve capacity constraints per price zone. We extracted 80 price scenarios from the simulation, and from these scenarios a discrete price model comprising 9 price nodes per stage (3 energy and 3 capacity price

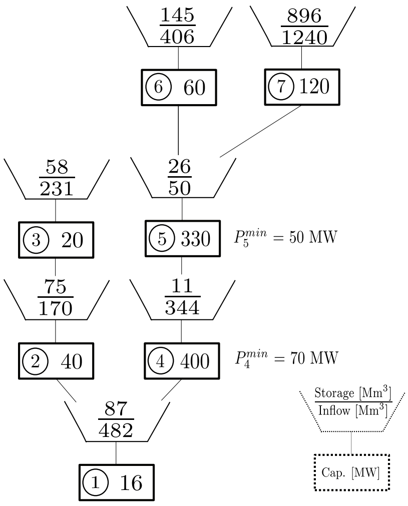


Fig. 3. Watercourse topology and technical data.

clusters) was identified by following the approach discussed in Section III-B. Note that the number of scenarios is small compared to the number of price nodes being identified. For this reason we slightly adjusted transition probabilities to ensure that the transition probability matrix was ergodic, as described in [26], chapter 3.3.

The lag-1 autoregressive inflow model was fitted using a single inflow series comprising 80 historical years, and the model error was sampled from a normal distribution.

A total of 200 scenarios of inflow and price were re-sampled in each forward iteration, and 12 discrete inflow error terms were sampled at each stage in the backward iterations in the SDDP-part of the algorithm. In our experience, this number of discrete inflow error terms should be sufficient to represent the stochasticity in inflow in this context. Recall that the inflow model is considered independent of the price levels. Therefore the same inflow error samples are used for each price node at a given stage in the backward iterations. A maximum number of 60 iterations were allowed. The model was implemented in C++, using the dual simplex algorithm from the Gurobi 6.0 library [27]. All tests were carried out on an Intel Core i7-4940MX processor with 3.30 GHz and 32 GB RAM. A single run required in the range of 55-60 hours, depending on the simulated case. Although not exploited in this work, the algorithm is well suited for parallel processing, see e.g. [7].

Three cases were defined as listed in Table II. Case A serves as a reference case considering the energy market only. This case was constructed by setting the upper bound on the capacity sales variable ($c_b$) to zero for all time blocks over the entire time horizon. Cases B and C both consider sequential sales to the capacity and energy markets, but they differ in the treatment of (8). Unlike case B, case C includes the volume requirement in (8), and the solution from case C will thus guarantee that there is sufficient amounts of water behind the turbines to support activation of the reserves.

### B. Results

The convergence characteristic of the algorithm is shown in Fig. 4. The cost gap gradually closes as the iteration number increases, but the sampling uncertainty in the lower bound is significant. By increasing the number of scenarios being re-sampled in every iteration from 200 to 400 we observed a slightly faster stabilization of the upper bound and less sampling uncertainty in the lower bound, but the corresponding strategy did not impact the results being presented in the following much.

TABLE II
SIMULATED CASES AND EXPECTED PROFITS.

| | Modeling feature | | Profit [M€] | |
|---|---|---|---|---|
| **Case** | **Markets** | **Volume req.** | **Total** | **From capacity** |
| A | Energy only | - | 278.2 | 0.0 |
| B | Energy and Capacity | - | 281.1 | 3.1 |
| C | Energy and Capacity | ✓ | 280.9 | 2.9 |

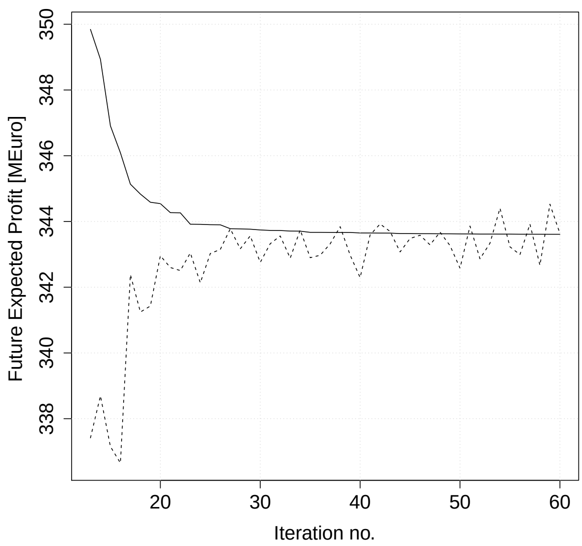


Fig. 4. Convergence properties of the algorithm applied to Case C.

The expected profits obtained from a final forward simulation using 1000 sampled scenarios are shown in Table II. These numbers are adjusted for the deviations between final and initial reservoir levels. As expected, when introducing the opportunity to sell capacity in cases B and C, the total profit increases compared to the energy-only case A. Furthermore, the additional volume constraint in case C results in a slightly lower expected profit than in case B. This constraint will primarily impact the operation of smaller reservoirs, such as no. 4 and 5 in Fig. 3. However, since these have large upstream reservoirs and the model does not consider time delays in the water course, the impact of the volume constraint is generally underestimated.

A significant part of the profit from capacity sales in cases B-C is obtained without changing the generation schedule compared to what would be found for case A in the same state. All hydropower stations have their best efficiency below the maximum generation level. Thus, when operated at the best efficiency, the stations will have room for both up- and down regulation. However, as will be clear when looking into more

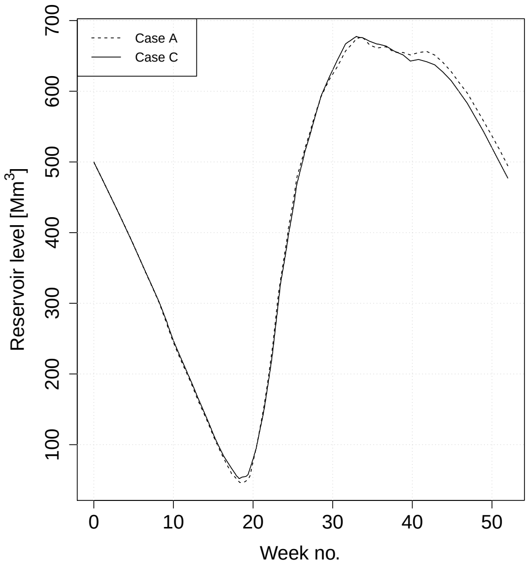


Fig. 5. Expected reservoir trajectories for reservoir 7 for cases A and C during the first year.

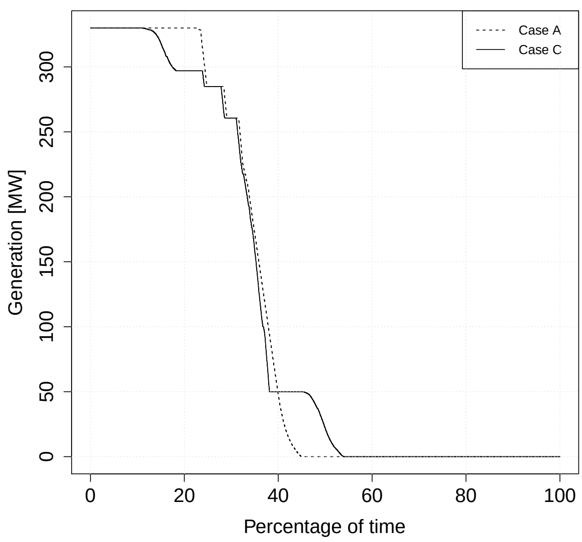


Fig. 6. Duration curves for generation in station 5 for cases A and C.

detailed results, the generation schedules for case A and cases B-C are significantly different, indicating that constraints (6) and/or (7) are frequently binding. In summary, the comparison presented in Table II is not only sensitive to the energy and capacity prices, but also certain system characteristics such as efficiency curves and reservoir volumes.

For the remainder of the result presentation we compare cases A and C, since case B only differs marginally from case C.

Fig. 5 shows the expected reservoir trajectories for reservoir 7 for cases A and C for the first year. Due to sales of reserve capacity, case C follows a slightly higher trajectory than case A until spring flood (around week no. 20). Furthermore, water is used more aggressively to keep downstream generators spinning during the low-load season in case C, giving a lower trajectory in autumn and early winter. Similar patterns were observed for the other large reservoirs (reservoir number 1, 2, 3 and 6).

Fig. 6 shows the duration curves for generation in station 5 for cases A and C. This station has an installed capacity of 330 MW, a minimum output of 50 MW, and is allowed to deliver at most 33 MW of reserve capacity. The impact of considering the sales of reserve capacity is evident in Fig. 6; the station is operated a significant portion of the time at 297 MW and 50 MW output in case C. Note that the modelling in (6) encourages the station to run at its minimum output (50 MW) rather than 33 MW for the purpose of delivering spinning reserves.

The expected water values at a given time stage and system state can be found as the coefficients ($\pi$) of the binding cut for that stage and state. Fig. 7 shows how the expected water values for week 20 differs between cases A and C for reservoir 3 for a given price node. These values are plotted as a function of the filling in reservoir 3, while fixing all other reservoir

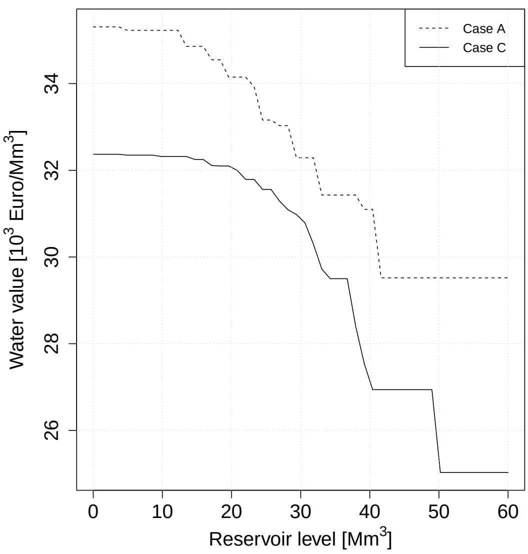


Fig. 7. Water values as functions of the filling in week 20 for reservoir 3 for cases A and C for a given price node.

levels to their corresponding expected values obtained from case A. In this case study the water values generally seems to be lower for cases B and C than in case A, as indicated by Fig. 7, which is due to the impact of withholding capacity for up-regulation in periods where one in case A would generate at maximum capacity. However, if prices where different one could end up with higher water values in cases B and C due to the additional use of water caused by the spinning requirement.

## V. Conclusions

A new method suitable for solving the medium-term hydropower scheduling problem for a profit maximizing and price-taking producer considering both the markets for energy and reserve capacity was presented. The method is based on a hybrid SDP/SDDP algorithm, treating inflow and prices for energy and reserve capacity as stochastic variables. In order to reflect decision stages seen in the Nordic power markets, the method allows allocating resources sequentially, selling reserve capacity prior to energy.

Traditionally, medium-term hydropower scheduling models only consider the energy market. This work demonstrates that a market for reserve capacity can be introduced as an extension of a previously presented scheduling model. By capturing the impact of an additional market on the water values, improved end-value settings can be provided to more detailed short-term scheduling tools and simulators.

The method was tested on a Norwegian watercourse considering sales to the spot and the spinning reserve markets. Emphasis was put on quantifying the expected changes in schedules and water values when going from an energy-only market to a joint treatment of energy and reserve capacity markets.

The error introduced when linearizing all relationships depends on the case and system being studied. Although the purpose of the proposed method is not to provide accurate commitment schedules, the linearization error may significantly impact expected profits and system operation for certain systems. This error will normally be more pronounced when considering sales of spinning reserve capacity in addition to energy. Thus, further work should focus on validating the proposed method against a method that allows integer variables and thus allows a more detailed system representation.

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