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A Monte Carlo sampling procedure for rare events applied to power system reliability analysis

Erlend Sandø Kiel
Department of Energy Systems
SINTEF Energy Research
Trondheim, Norway

Gerd Hovin Kjølle
Department of Energy Systems
SINTEF Energy Research
Trondheim, Norway

***Abstract*—This paper presents a semi-analytical Monte Carlo method for rare event sampling applied to power system reliability analyses, which combines traditional Monte Carlo-based methods with simplifications from analytical methods. The result is a procedure which assess the reliability of a system with the level of detail which a Monte Carlo method offers, combined with computation speed gains from analytical methods. A case study is included, comparing the performance of the proposed Semi-Analytical Monte Carlo method with a Sequential Monte Carlo method, and an analytical reliability evaluation technique using approximate equations. The case study verifies the method results, scalability and ability to incorporate uncertainty in outputs. The method presented has other potential applications, e.g., in the study of power system resilience.**

***Index Terms*—reliability, Monte Carlo, analytical, power systems, resilience**

I. Introduction

The power system is a critical infrastructure which provides essential services for the normal operation of modern day society [1]. The introduction of complexity, interdependencies and uncertainties due to the integration of variable renewable energy resources, new components, and the merging of the information and communication system infrastructure and power system into the cyber physical power system (CPPS) are just some of the current challenges which can affect the security of electricity supply [2]–[4]. It is therefore important to understand how these challenges affect the capacity of the power system to withstand strains, and its ability to perform its intended function. This is typically studied through vulnerability/resilience analyses, and reliability analyses, respectively [5]–[7]. The method developed in this paper is applied to reliability analyses of the power system but has potential applications in other areas which are subject to rare events, e.g., power system resilience analyses.

This paper describes a hybrid procedure which combines a Monte Carlo Simulation (MCS) and analytical reliability evaluation technique building on the strengths of analytical and simulation approaches. The core difference between these traditional approaches and the Semi-Analytical Monte Carlo (SAMC) method presented in this paper is that the latter simulates the most likely time and duration of an unwanted event, e.g. the simultaneous outage of multiple components, before analytically calculating the associated failure frequency. The backwards reasoning is inspired by how extraordinary events in power systems are studied in [7], [8] and the progression of an Event Tree Analysis (ETA) [9]. The proposed method always produces realizations of unwanted events in each iteration of the simulation, thus making it time-efficient.

The remainder of the paper is organized as follows: Section II introduces key terms and modelling approaches relevant for the study of power system reliability, while the proposed method is presented in Section III. A case study is presented in Section IV to illustrate the applicability of the method, before the paper is concluded in Section V.

II. Modelling of power system reliability

An introductory discussion of analytical and simulation based methods of calculating power system reliability indices can be found in [10]. An advantage of the analytical approaches is that they are fast to calculate and produce long term expected values of system performance which may be sufficient for many purposes. The disadvantages are that they rely on simplifications, and have a limited ability to model complex systems. The reliance on expected values both as inputs and outputs in analytical approaches can be a potential source of under-communicated risks, where unwanted events with very large consequences but small probabilities is consumed into the expected value [11]. The alternative approach is to try to simulate the actual behavior of the system, incorporating uncertainties through probability distributions, which can give a more detailed understanding of the system reliability for complex systems. The primary challenge of the simulation approach is computational time [12].

The power system is both a complex and reliable infrastructure [6], which suggests that simulation approaches should be used to quantify the power system reliability but also that this may be a difficult task: Traditional power system operation is often based on the N-1 criterion, where the system should be able to withstand any credible single contingency at all times in such a way that the system is capable of accommodating the new operational situation without violating operational security limits [13]. A single contingency can be understood as *an outage occurrence of one system component*, while a

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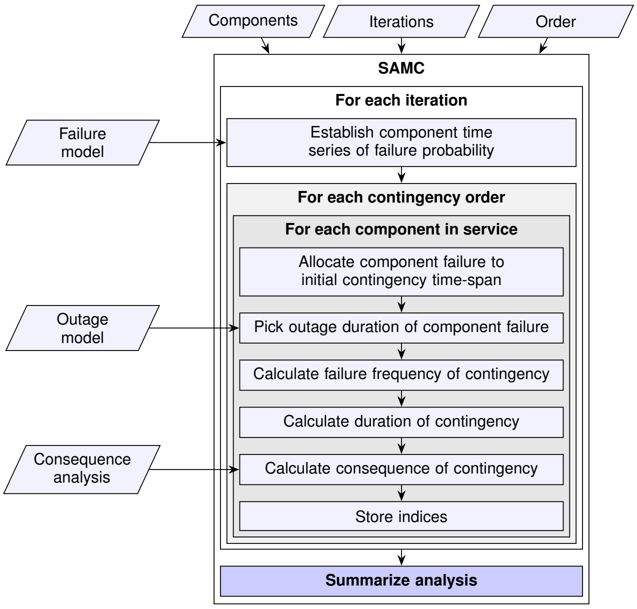

Fig. 1. Flowchart describing the method.

multiple contingency refers to *the concurrent outages of two or more system components* [14]. The number of components included in a contingency is sometimes referred to as its *order*.

Considering that the power system is a highly reliable infrastructure, higher order contingencies could be termed rare events. It may be difficult to simulate a sufficient number of samples of such events to provide good estimates of the quantity of interest [15], [16]. Variance reduction techniques such as importance sampling (IS), the cross entropy (CE) method, and multilevel Monte Carlo (MLMC) are some possible solutions to this challenge [17]–[19]. These methods adapt how samples are picked in order to generate sufficient samples of the quantity of interest, and in turn to describe it by its expected value and variance. The method proposed in this paper, on the other hand, simulates the system *conditional* on a rare event occurring, which has been identified as a problem which deserves more research attention [20].

III. Method

Fig. 1 gives a simplified description of the proposed SAMC method. The initial step is input data, where the number of components and iterations is specified, as well as the order of the contingencies evaluated. Following this, three nested loops are used to calculate the final reliability indices. The outer loop tracks the iterations of the algorithm and establishes a time series of failure probability for each component. The middle loop tracks the order of the contingencies evaluated, such that new realizations of higher order contingencies are based on information about previous order contingencies. Combinations of an initial contingency and additional component outages are constructed in the inner loop, which is used to generate samples of new contingencies of the given order, where information about the contingency sample is used to calculate the final reliability indices.

A single component failure is allocated to a point in time in the inner loop. This is done probabilistically based on the component time-series of failure probability, e.g., by picking the failure event from a categorical distribution with the relevant time-series as input. The failure can be allocated to any point in time covered by the time-series if there are no previous outages in the contingency, however, if this component failure is part of a contingency with an order higher than one, then the time-span is limited by the time-span of the initial contingency. The new component failure is assigned an outage duration. This is used to calculate a failure rate, outage duration and consequence for each realization of a contingency, which is stored and later summarized.

To find the failure frequency of the overlapping outage of two components, it is necessary to find the probability of the second component experiencing a failure during the outage of the first component. This is done in (1) where $p_j^{[t_i,t_i+r_i]}$ is the series-system failure probability of the second component j during the outage duration, r, of the first component, i, in the time-span after the initial fault happened, $[t_i, t_i + r_i]$.

$$p_j^{[t_i,t_i+r_i]} = 1 - \prod_{k=t_i}^{t_i+r_i} (1 - p_j^k) \tag{1}$$

For one iteration, the failure frequency, λ, of the produced second order contingency is the product of the failure frequency of the initial contingency and the probability of failure of the additional component j, in the time-span of the initial contingency, i (2). The total failure frequency of a second order contingency consisting of the two components i and j for the iteration is the sum of this sequence of events, starting with either component being the initial failure (3).

$$\lambda_{i\to j} = \lambda_i \cdot p_j^{[t_i,t_i+r_i]}; \quad \lambda_{j\to i} = \lambda_j \cdot p_i^{[t_j,t_j+r_j]} \tag{2}$$

$$\lambda_{i,j} = \lambda_{i\to j} + \lambda_{j\to i} \tag{3}$$

The outage duration, r, of a second order contingency is equal to the time-period of concurrent outages of component i and j (4).

$$r_{i,j} = [t_i, t_i + r_i] \cap [t_j, t_j + r_j] \tag{4}$$

Energy Not Supplied (ENS), as a measure of the consequence due to a contingency at a given time, is the product of the associated outage duration and interrupted power, $P_{i,j}^t$ (5).

$$ENS_{i,j}^t = r_{i,j} \cdot P_{i,j}^t \tag{5}$$

The distribution of sequences of events leading to an outage duration and ENS will be naturally weighted for by the sampling frequency of the event in traditional MCS. This is not the case in the SAMC-approach where the average value of outage durations and ENS must be weighted by failure frequency, as for $r_{i,j}$ in (6), where ι is the iteration number.

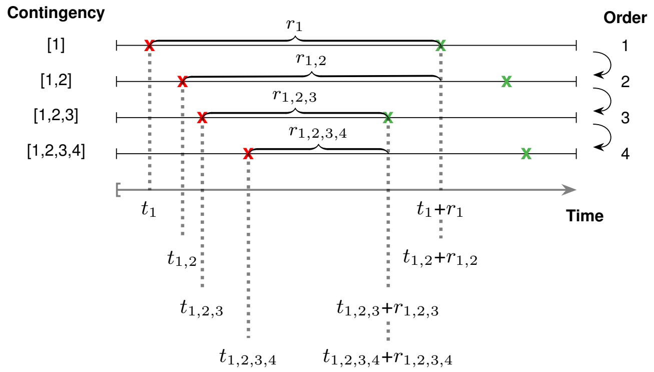

Fig. 2. Recursive logic of the method. Red crosses signifies component failures, leading to a "down" state. Green crosses signifies that the component is once again in an "up" state. Brackets show the associated outage durations.

$$\overline{r_{i,j}} = \frac{1}{\sum_{\iota} \lambda_{i,j}} \cdot \sum_{\iota} \sum_{i \neq j} \left[\lambda_{i,j}^{\iota} \cdot r_{i,j}^{\iota} \right] \tag{6}$$

The recursive logic of the method – appending an additional component failure to an existing contingency – can be extended to any number of concurrent outages of n system components. This is illustrated for a system with four overhead transmission lines – used in the Section IV case study – in Fig. 2. An initial contingency of line 1 is the basis of a second order contingency involving line 1 and 2: A failure of line 2 is allocated to a point in time bounded by the time-span of the initial contingency. Line 2 is then assigned an outage duration. The resulting second order contingency is the concurrent outage of the two components, and a failure frequency, outage duration is calculated for the new contingency. This contingency can be further extended by appending an additional component outage to the contingency to construct a contingency of a higher order. The dashed lines illustrate the time-span of a given contingency.

There are various forms of uncertainties that could be considered in the analysis, such as uncertainty due to lack of knowledge or due to natural variability, as discussed in e.g. [21]. This should be considered on a case-by-case basis, and is reflected in the way failure- and outage data is served as input from external models into the method.

Failure model. The method operates on a time series of failure probability. Time series of failure probabilities for the components can be constructed from a failure model. These could either be assumed constant or time varying, depending on modelling capability and which threats are incorporated into the analysis, see e.g. [22], [23]. The failure model could produce a fixed or uncertain output. Introducing the failure model at this stage of the analysis, as seen in Fig. 1, allows for re-calculation of the time series of failure probability for each iteration, thus incorporating different types of uncertainty related to the failure probability of the components.

Outage model. The outage duration due to a component failure can similarly be entered into the analysis in a number of different ways, which allows for incorporating uncertainty of this parameter into the analysis. The outage duration can be assumed to be a predefined scalar value, picked from a probability distribution, or generated by a logical model [24].

Consequence analysis. The consequence of a given contingency can be evaluated in different ways. DC- or AC power-flow is used to identify the interrupted power due to a contingency at a specific point in time in similar reliability evaluation tools [25], [26].

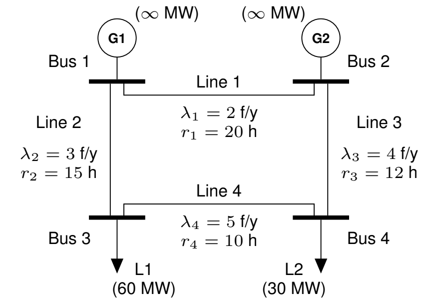

Fig. 3. 4-bus test network [28]. Including component reliability data for overhead transmission lines: Annual failure rate in failures/year. Outage durations in hours. Line ratings are 135 MW.

IV. Case Study

The performance of the proposed method is compared through a case study to that of more well known analytical approximate equations (see e.g. [10]) and an implementation of a Sequential Monte Carlo (SMC) method inspired by [27] and [26]. A 4-bus test network [28], seen in Fig. 3, is used in the case study. This test network has a high failure rate of the overhead transmission lines, which makes it easier to generate realizations of contingencies for the SMC comparison method. Only one operating state is considered, with both generators able to supply infinite demand. Interrupted power due to a contingency is decided by the transmission line ratings. The case study considers up to 2$^{\text{nd}}$ order contingencies. 10000 iterations of the SAMC and SMC implementations are performed.

A scaled beta distribution for the failure rate and outage duration is used to add uncertainty in the input and output variables. Both distributions have $\alpha = \beta = 3$ parameters, and minimum/maximum values of the failure rate distributions are $\pm$ 40 percent, while it is limited to $\pm$ 25 percent for the outage duration. The mean values are equal to that found in the test system. The distributions of values are arbitrarily chosen.

A failure model generates time-series of constant component failure probabilities from annual failure rates: The relationship between the annual failure rate of a component, λ_c, and the failure probability in a given hourly time step, p_c^t, is given in [23], where γ is the number of years covered by the times-series (7). Each iteration of the SAMC and SMC procedures corresponds to a simulation-year in the case study.

$$\lambda_c \approx \frac{1}{\gamma} \sum_{t \in T} p_c^t \tag{7}$$

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The implementation of the SMC method follows four main steps: A failure rate is picked from a probability distribution for each iteration. Time series of failure probabilities for components are constructed based on this failure rate. Component failures are picked from a binomial distribution by looping over the time series of failure probabilities. Component outage durations are picked from an associated probability distribution. The results are parsed in order to identify overlapping outages, and reliability indices are calculated.

The case study aims to uncover if the proposed SAMC method is efficient, accurate, and able to capture important information about the distribution of key reliability indices.

A. Results

TABLE I
COMPARISON OF PERFORMANCE: SMC AND SAMC PROCEDURES [a]

Method	Iterations	Realizations of contingencies			Computation time (seconds)
		Line 2, Line 3	Line 2, Line 4	Line 3, Line 4	
SMC	10 000	359	385	486	4.163
SAMC	10 000	10 000	10 000	10 000	0.256

[a] Python implementations using Numba [29]. CPU parallel processing of algorithm iterations. Computation time includes summation and parsing of results.

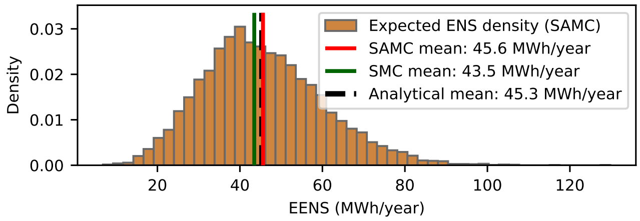

Fig. 4. Annual expected ENS for the system.

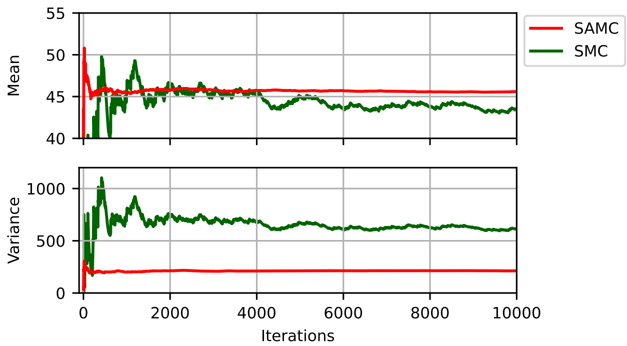

Fig. 5. Convergence. Annual expected ENS for the system.

A comparison of the two MCS methods included in the case study is presented in Table I. It shows that the SMC method produces less realizations of the second order contingencies causing interrupted power in the test system than what is the case when using the SAMC method, where a realization of the quantities of interest is produced in every iteration. The two approaches contain different calculations and summations, and the SAMC computation time was only six percent of that of the SMC implementation for the same number of iterations.

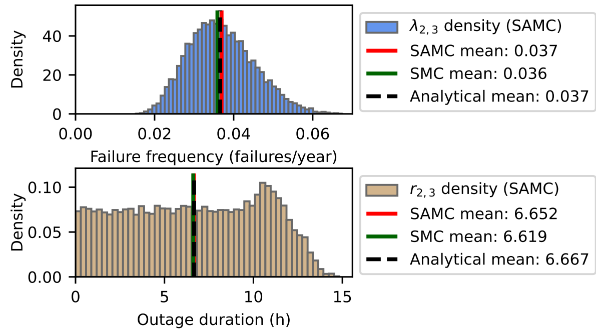

Fig. 6. Expected failure frequency and weighted outage duration for a contingency containing line 2 and 3.

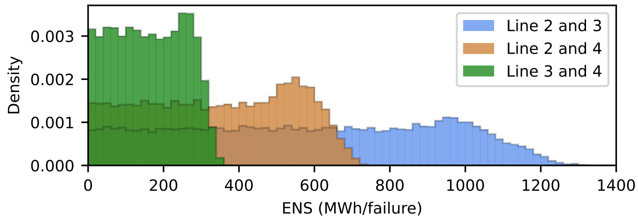

Fig. 7. Simulated distribution of ENS should a given contingency occur.

Fig. 4 shows the calculated annual expected ENS of the system. The orange histogram shows the distribution of expected ENS across all iterations using the SAMC method. The vertical lines show the average expected ENS for the three implemented methods. The SAMC-method (red line) yields an expected value above to that found by using analytical method (dashed black line) by less than one percent. The SMC implementation yields a result almost 5 percent below results found using the analytical method.

Fig. 5 show that both the mean and variance of the annual expected ENS for the system converges more rapidly with the SAMC-method, than when using the SMC method. An important point here is that the SMC method produces simulation years with either unwanted event(s) occurring which produces ENS, or no unwanted events which produces no ENS. The SAMC method, however, always produces an expected ENS for the system. This causes the variance of the SMC estimate to be considerably higher than for the SAMC estimate.

A closer inspection of results related to the contingency containing the overlapping outage of line 2 and 3 can be found in Fig. 6. The SAMC mean values are closer to that found by the analytical method also in this case, and there is an added benefit of having a distribution of possible outage durations – weighted by failure frequency – for that particualar contingency. Fig. 7 show the simulated distribution of ENS should different contingencies occur.

The 4-bus test network is not suitable to investigate the scaleability of the method, so an an additional analysis of the method is presented in Fig. 8. The figure shows the computation time of the SAMC algorithm for a test-system consisting of a different number of components, where each component is given an unique failure rate and outage duration distribution.

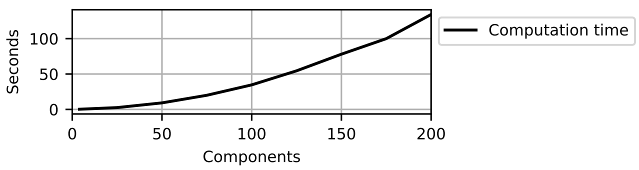

Fig. 8. Computation time for systems containing different number of components. 10000 iterations, 2nd order contingencies. Average of 3 executions.

The computation time follows what is expected trajectory of complexity due to increased components. For a fixed number of iterations and multiple contingencies containing up to two components this would be $O(n^2 - n)$. The figure shows that the method can be applied to larger systems.

V. Conclusion

The SAMC-method produces results which is verified against other methods in the case study. Avoiding simulating non-events leads to a quicker convergence in output parameters than what is the case when using a SMC method, which together with the analytical simplifications have a positive effect on the computation speed. The output is also in the form of distributions, which communicate a more complete risk picture than expected values. The proposed method is relatively easy to implement, is extendable to any n^{th} order contingencies, and scaleable to larger systems.

Some suggested further work is the inclusion of dependent events in the SAMC-modeling framework to further benefit from the MCS ability to model complex relationships, and improved communication of analysis results to decision makers. The use of the method in resilience or vulnerability analysis, through its extension to higher order contingencies, would also be a welcome addition to the analysis.

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References

[1] M. Ouyang, "Review on modeling and simulation of interdependent critical infrastructure systems," *Reliability Engineering & System Safety*, vol. 121, pp. 43–60, 2014.
[2] V. Singh, T. Moger, and D. Jena, "Uncertainty handling techniques in power systems: A critical review," *Electric Power Systems Research*, vol. 203, p. 107633, 2022.
[3] Y. Wang, A. O. Rousis, and G. Strbac, "On microgrids and resilience: A comprehensive review on modeling and operational strategies," *Renewable and Sustainable Energy Reviews*, vol. 134, p. 110313, 2020.
[4] R. V. Yohanandhan, R. M. Elavarasan, P. Manoharan, and L. Mihet-Popa, "Cyber-physical power system (cpps): A review on modeling, simulation, and analysis with cyber security applications," *IEEE Access*, vol. 8, pp. 151019–151064, 2020.
[5] J. Johansson, H. Hassel, and E. Zio, "Reliability and vulnerability analyses of critical infrastructures: Comparing two approaches in the context of power systems," *Reliability Engineering & System Safety*, vol. 120, pp. 27–38, 2013.
[6] M. Panteli and P. Mancarella, "The Grid: Stronger, Bigger, Smarter?: Presenting a Conceptual Framework of Power System Resilience," *IEEE Power and Energy Magazine*, vol. 13, pp. 58–66, May 2015.
[7] I. B. Sperstad, G. H. Kjølle, and O. Gjerde, "A comprehensive framework for vulnerability analysis of extraordinary events in power systems," *Reliability Engineering & System Safety*, vol. 196, Apr. 2020.
[8] G. H. Kjølle and O. Gjerde, "Vulnerability analysis related to extraordinary events in power systems," in *2015 IEEE Eindhoven PowerTech*, pp. 1–6, IEEE, June 2015.
[9] M. Rausand and A. Høyland, *System reliability theory: models, statistical methods, and applications*. Wiley series in probability and statistics, Hoboken, NJ: Wiley-Interscience, 2nd ed ed., 2004.
[10] R. Billinton and R. N. Allan, *Reliability Evaluation of Power Systems*. New York, NY, USA: Plenum Press, second edition ed., 1996.
[11] Y. Y. Haimes, "Risk of extreme events and the fallacy of the expected value," *Control and Cybernetics*, vol. 22, no. 4, pp. 7–31, 1993.
[12] M. Aien, A. Hajebrahimi, and M. Fotuhi-Firuzabad, "A comprehensive review on uncertainty modeling techniques in power system studies," *Renewable and Sustainable Energy Reviews*, vol. 57, pp. 1077–1089, 2016.
[13] GARPUR Consortium, "D1.1: State of the art on reliability assessment in power systems," 2014. Available at https://www.sintef.no/globalassets/project/garpur/deliverables/.
[14] International Electrotechnical Commission (IEC), "Outage occurrence, IEV ref 692-05-01." Electropedia: International Electrotechnical Vocabulary (IEV) Online. http://www.electropedia.org/, accessed 2023-02-09.
[15] D. P. Kroese and R. Y. Rubinstein, "Monte Carlo methods," *Wiley Interdisciplinary Reviews: Computational Statistics*, vol. 4, pp. 48–58, Jan. 2012.
[16] F. Bouchet, J. Rolland, and J. Wouters, "Rare Event Sampling Methods," *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 29, p. 080402, Aug. 2019.
[17] R. Y. Rubinstein and D. P. Kroese, *Simulation and the Monte Carlo method*. Wiley series in probability and statistics, Hoboken, New Jersey: John Wiley & Sons, Inc, third edition ed., 2017.
[18] R. Y. Rubinstein, "Optimization of computer simulation models with rare events," *European Journal of Operational Research*, vol. 99, pp. 89–112, May 1997.
[19] S. Tindemans and G. Strbac, "Accelerating system adequacy assessment using the multilevel Monte Carlo approach," *Electric Power Systems Research*, vol. 189, p. 106740, 2020.
[20] D. P. Kroese, T. Brereton, T. Taimre, and Z. I. Botev, "Why the Monte Carlo method is so important today," *WIREs Computational Statistics*, vol. 6, pp. 386–392, Nov. 2014.
[21] F. O. Hoffman and J. S. Hammonds, "Propagation of Uncertainty in Risk Assessments: The Need to Distinguish Between Uncertainty Due to Lack of Knowledge and Uncertainty Due to Variability," *Risk Analysis*, vol. 14, pp. 707–712, Oct. 1994.
[22] S. Perkin, G. Bjornsson, I. Baldursdottir, M. Palsson, R. Kristjansson, H. Stefansson, P. Jensson, E. Karangelos, and L. Wehenkel, "Framework for threat based failure rates in transmission system operation," in *2016 Second International Symposium on Stochastic Models in Reliability Engineering, Life Science and Operations Management (SMRLO)*, pp. 150–158, IEEE, 2016.
[23] Ø. R. Solheim, T. Trötscher, and G. Kjølle, "Wind dependent failure rates for overhead transmission lines using reanalysis data and a bayesian updating scheme," in *2016 International Conference on Probabilistic Methods Applied to Power Systems (PMAPS)*, pp. 1–7, IEEE, 2016.
[24] E. S. Kiel, M. D. Catrinu-Renstrøm, and G. H. Kjølle, "A transformer outage duration model with application to asset management decision support," in *The 33rd European Safety and Reliability Conference (ESREL)*, 2023. Accepted.
[25] G. H. Kjølle, I. B. Sperstad, and S. H. Jakobsen, "Interruption costs and time dependencies in quality of supply regulation," in *2014 International Conference on Probabilistic Methods Applied to Power Systems (PMAPS)*, pp. 1–6, 2014.
[26] Ø. R. Solheim, L. Warland, and T. Trötscher, "A holistic simulation tool for long-term probabilistic power system reliability analysis," in *2018 IEEE International Conference on Probabilistic Methods Applied to Power Systems (PMAPS)*, pp. 1–6, 2018.
[27] W. Wangdee and R. Billinton, "Bulk electric system well-being analysis using sequential Monte Carlo simulation," *IEEE Transactions on Power Systems*, vol. 21, no. 1, pp. 188–193, 2006.
[28] G. H. Kjølle and O. Gjerde, "Integrated approach for security of electricity supply analysis," *International Journal of Systems Assurance Engineering and Management*, vol. 1, pp. 163–169, June 2010.
[29] S. K. Lam, A. Pitrou, and S. Seibert, "Numba: A LLVM-Based Python JIT Compiler," in *Proceedings of the Second Workshop on the LLVM Compiler Infrastructure in HPC*, LLVM '15, Association for Computing Machinery, 2015.