Report

Speeding up the hydrothermal power market model FanSi

Project results

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SUMMARY

This report summarises the work performed in the RAKETT-project. The project goal is to reduce the calculation time of the FanSi model. The FanSi model is a power market simulator for hydro dominated large-scale power systems. The FanSi model formulates optimisation problems for the solution of the Economic Dispatch Problems.

The primary means for reducing calculation time is to apply decomposition techniques on deterministic Linear Programming problems from FanSi. Two methods for decomposition in space, Lagrangian Relaxation and Benders decomposition, were studied. The computational experiments showed that results were not of high enough quality to be used with the FanSi model.

Benders decomposition was applied for decomposition in time. Three decomposition schemes were tested, one serial implementation and two schemes applying parallel processing. One synchronous and one approximate asynchronous implementation. The asynchronous parallel implementation showed reduced calculation time and adequate result quality, but further study is recommended for use in FanSi.

A heuristic to relax the reservoir balance constraints was proposed and studied. The computational experiments showed a significant reduction in calculation time with minimal reduction in result quality.

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1 Introduction

This report is written as part of the Rakett project. The project is an "Innovation project for the Industrial Sector" financed by partners in the industrial sector and the research council of Norway.

A large share of the electricity in the Scandinavian electricity system is supplied by hydropower plants, consisting of more than a thousand reservoirs. The utilisation of fundamental power market models, the EMPS model, has been used in Scandinavia for decades [1]. The EMPS model is developed for hydro dominated systems and requires a detailed description of the power system as input. The model formulates stochastic dynamic optimisation problems with the objective of minimising the expected future cost of operating the power system. The stochasticity is represented by uncertain weather (inflow, wind, temperature, and solar production). The model is run to forecast the future expected cost of operation, power prices and the power system behaviour for a selected period, typically several years into the future.

The increase in production from non-controllable power sources and a stronger coupling between the hydro dominated Scandinavian power system to the European system, is expected to increase the short-term variation in power production. The EMPS model is not designed to handle short term variability, thus a new fundamental power market model was designed in the SOVN-project, called The FanSi model [2]. This prototype model handles the short-term variability, and its results are promising, but its drawback is the long calculation time. To reduce the calculation time the Rakett-project was initiated in 2020.

1.1 The Rakett project

The description in this section is based on the work in [3].
Our main concern in the Rakett project is reduction of calculation time for the FanSi model using decomposition techniques. The FanSi model formulates mathematical optimisation problems of a large power system with detailed description of hydropower, for example the Nordic power system. The Nordic power system span a large geographical area, in addition, the optimisation problem has a long time-horizon stretching over many timesteps, for example 8760 hours of a year. The resulting optimisation problem is large and computationally expensive and time consuming. A know technique to reduce calculation time is to divide the large optimisation problem into many smaller optimisation problems, for example one can split the problem into smaller geographical areas and/or in time. This requires the coordination of the solutions from the smaller problems into a complete solution of the problem. This allows for reduced calculation time and large-scale parallel processing which in turn will reduce calculation time.

Other means for the reduction of calculation time is also of interest. We have considered relaxation of reservoir balance constraints, tuning parameters of the optimisation problem solver and parametrisation of the FanSi-model.

Project goals

The goals described here are taken from the project application sent to the Norwegian Research Council. The primary project goal is to investigate and assess the potential for reduction of calculation time for the FanSi model by applying decomposition techniques. The assessment shall give clear recommendations to speed-up the FanSi model and quantify the potential speed-up with the goal of making the FanSi model applicable for daily operational use and for investment analysis. For relevant datasets the computation time should be no longer than

1. 4 hours for daily operational use, or
2. 48 hours for investment analysis.

The secondary goals are:

1. Implement decomposition techniques in a prototype model which solves similar optimisation problems as the FanSi model and find the potential reduction in calculation time.
2. Run this prototype model on relevant datasets
3. Documentation of project work through presentations, scientific publications, and reports.

Project partners

The project involved four partners, NVE, Statnett, Statkraft and SINTEF Energy. NVE contributed to the project through their study of Benders decomposition in time. They provided insight and new ideas worth proper attention. The work has been references in this document on the relevant sections. The collaboration with NVE was fruitful for the project.

1.2 The FanSi model

This section is based on the work in [3] when it comes to descriptions and figures.

The FanSi prototype model is a stochastic optimisation model used for long term planning of hydrothermal power systems, where the objective is to minimise system costs. The FanSi model can simulate large hydrothermal power systems with a detailed description of hydropower and individual water values for all hydroelectric reservoirs of the system and high shares of intermittent energy source such as wind- and solar power. The operation of each individual hydroelectric reservoir is based on the result of formal stochastic optimisation in which all relevant physical attributes of the market are represented. Weekly decisions are determined by solving scenario fan problems (SFP) considering uncertainty in weather and exogenous market prices. The overall scheduling problem is obtained by solving a sequence of SFPs spanning a chosen period. The SFP is illustrated in Figure 1, the SFP is decomposed in a first stage master problem and second stage scenario problems. The scenario problems provide averaged Benders cuts to the master problem. The SFPs are formulated as a set of deterministic linear programming (LP) problems. The master problem is a weekly problem, while the scenarios typically cover a longer horizon from half a year to several years.

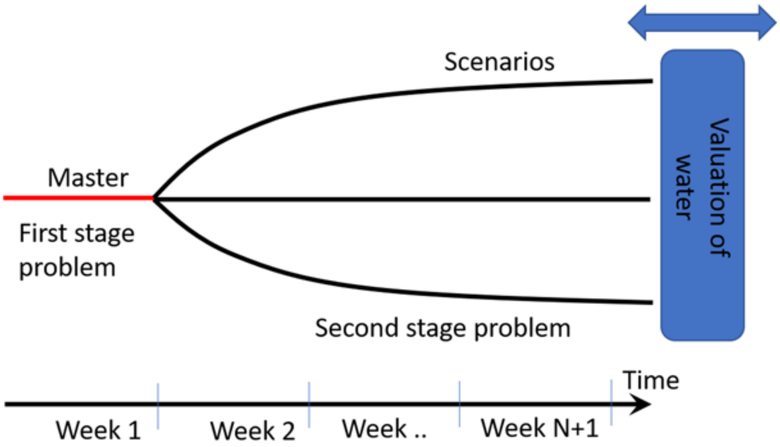

Figure 1: Illustration of the FanSi scenario fan. The first stage (master) problem is a weekly decision problem, the multi-week second stage (scenario) problem consists of many scenarios and the valuation of water at the scenario end.

The main challenge of the FanSi model is the calculation time. Reducing the calculation time is a prerequisite for the model to be applicable by the model users.

1.3 Dataset

The dataset used in the Rakett project is the "HydroCen 2030 Low emission" unless otherwise is stated. The dataset is a scenario representation of a 2030 power system, covering Northern-Europe and Western-Europe separated in 57 price areas. The dataset is hydro dominated with more than 1000 hydropower modules, many of them have PQ-curves representing different efficiency segments and contains high shares of energy sources with short term variability. More details of the dataset are described in [4]. The FanSi model is designed for use on hydro dominated power systems with high shares of energy sources with short-term variability [2]. Therefore, the dataset is a good fit for the intended use of the FanSi model.

1.4 Computational framework

The description in this section is based on the work in [5].
All computational experiments are conducted using the programming language Julia and the mathematical modelling language JuMP together with the optimization solver CPLEX (v 12.10). Julia/JuMP facilitates flexible model building and experimentation and allows updating LP problems in memory. Thus, it is well suited for the type of tests conducted here. The Dual Simplex algorithm and the Barrier algorithm were used in all computational experiments. Pre-solve functionality is activated in all tests and "warm starting" the solution processes using a previous basic solution is applied when possible.

Computational specifications are available in the referenced documents.

1.5 Report Structure

The report starts with an introduction of the Rakett project, the FanSi model and the data being used in computations in section 1.

In section 2 use of decomposition techniques applied on the SFP for reducing the overall calculation time of the FanSi model are presented. The applied decomposition techniques are:

1. Decomposition in time by Benders decomposition
2. Spatial decomposition by Lagrangian relaxation
3. Spatial decomposition by Benders decomposition

The decomposition techniques allow for the splitting of the full problem into several subproblems, where the subproblem solutions are collected and coordinated into one problem solution for the full system. The splitting into subproblems allows for both serial and parallel problem solution. Both solution options are covered in this section. The result quality is assessed by total system costs, marginal cost of electricity per area and individual reservoir water values and calculation time.

Section 3 covers the work performed on other means (other than applying decomposition techniques) for reducing calculation time of the FanSi model.

1. First, a heuristic for relaxing reservoir balance constraints is presented. The result from applying the heuristic is assessed with focus on calculation time reduction and result quality.
2. Then a subsection on tuning of parameters of the optimisation problem solver (CPLEX) follows

3. A subsection on parametrisation of FanSi. By "parameterisation of FanSi" we refer to the specification of the time resolution, the scenario length, and the functionality of the FanSi scenario fan.

In section 4 we test the consequences by applying a method to make individual water values from aggregated water values, the so-called "target reservoir calculations" and we report the calculation time from the current implementation of FanSi for several cases.

In section 5 we summarise the work in the Rakett project and give recommendations for applying project findings to reduce the calculation time of the FanSi model.

2 Decomposition techniques

The Rakett project primary goal is to reduce the calculation time of the FanSi model by investigating the use of decomposition techniques. The FanSi model applies Benders decomposition in the SFP. The SFP contains a deterministic LP-problem in the first stage master problem and set of deterministic LP-problems in the second stage scenario problems. In this report we will investigate further decomposition of the master and scenario problems, they can be decomposed in time or space. We introduce the decomposition techniques for spatial and temporal decomposition and report on the result quality and the calculation time of the applied decomposition. Finally, we give recommendations for the use of applied decomposition techniques in the FanSi model.

2.1 Temporal Decomposition

This section is based on the work in [5] when it comes to descriptions, figures and conclusions, and presented in compact form. Temporal decomposition is decomposition along the time-axis. The deterministic LP-problems formulated by the FanSi model are the master problem and the scenario problems. Both are hydrothermal scheduling problems formulated along the time-axis with a specified resolution of discrete time-steps within a specified time horizon. The master problem has one week time horizon with typically 3-hours or 1-hour intraweek time resolution. The scenario problems normally have a longer time horizon, they can be set according to user needs in the range from 1 week to several years. The scenarios provide a signal for operating individual reservoirs of the master problem through Benders cuts. The length and time resolution of the scenarios must be chosen carefully, weighting the computational burden against the quality of the signal for operating individual reservoirs [6], [7], [8].

In the following we will briefly present serial and parallel Benders decomposition.

Serial Benders decomposition

Temporal decomposition in the form of Benders decomposition in is illustrated in Figure 2. The original problem is decomposed in multiple stages (four stages in the illustration). Initial conditions are the system state and the end valuation. One starts from a known system state and simulate forward along the time axis, stage-by-stage, passing on the system state as input to the next stage. When reaching the last stage, backward simulation begins, and Benders cuts are passed backward in time stage-by-stage. In this decomposition the stages are solved in series. Convergence is found by defining an acceptable tolerance on the difference of the upper and lower bound on minimum costs. The upper bound is the minimum cost for all stages and the lower bound is the minimum cost for the first stage.

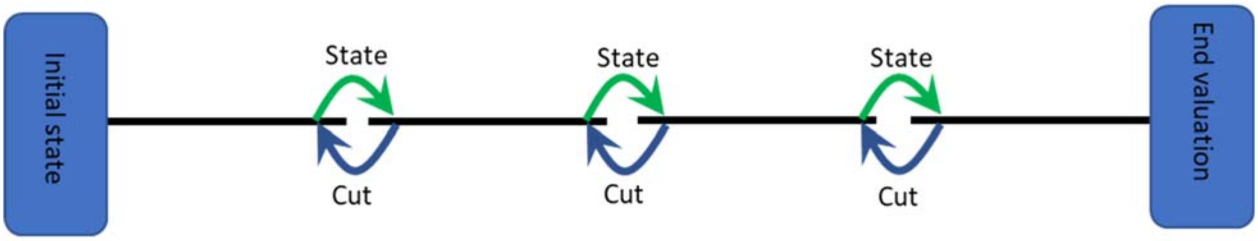

Figure 2: Temporal decomposition by Benders decomposition.

In the project PRIBAS (NFR project number 59;3472H53), a simplified decomposition algorithm was applied to speed up the computation time for solving a weekly optimization problem. Each stage (day) was optimized separately interpolating in the end-valuation (Benders cuts) available for the end and beginning of the week. This process is described in detail in [9]. In this work we will evaluate the simplified version of this algorithm illustrated in Figure 3. The stage problems are simulated in a single forward simulation, each seeing the end-valuation. Note that we do not interpolate in cost functions as was done in PRIBAS. We refer to this approach as a *PRIBAS-iteration*[1].

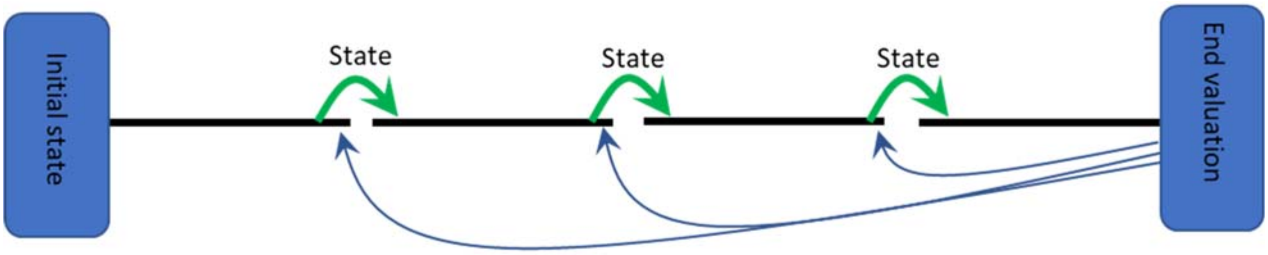

Figure 3: Illustration of a PRIBAS iteration.

Parallel Benders decomposition

An alternative scheme for the parallel solution of multi-stage Benders decomposition is covered in [10]. The scheme is illustrated in Figure 4.

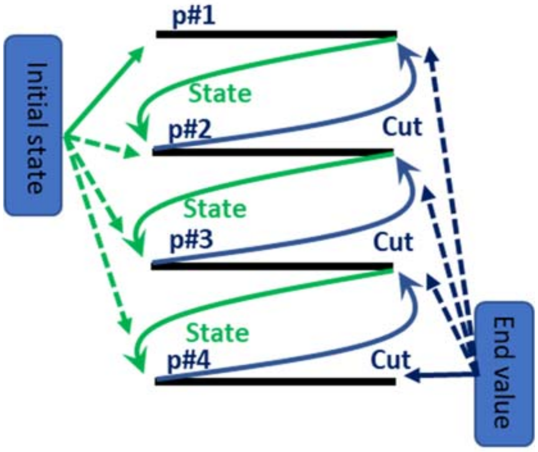

Figure 4: Benders decomposition solved in parallel.

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[1] Note that PRIBAS interpolated in cost functions (Benders cuts) available at the beginning and end of the week. Here, we only use cuts at the end of the week without interpolation.

The initial state and end value are sent to each stage. Each stage is solved by one allocated process(p#1-p#4), in the illustration four stages requires four allocated processes which are solved in parallel. This scheme avoids waiting for each stage to be solved in series and re-starts the solution process when updated system state and cuts are available for all stages (still requires waiting for solutions for each stage). Note that the system state and cuts are not update serially, which will affect the convergence. Only a synchronous parallel implementation of this scheme was tested as part of this work.

The solution time of each stage can vary from a fraction of a second to 10 seconds. Thus, always waiting for the slowest stage to solve is computationally inefficient. Therefore, we introduce a heuristic to approximate an asynchronous scheme where the solution time of each stage is limited to 3 seconds. More details are available in [5].

2.1.1 Serial Benders decomposition – results

Selected results from the computational experiments with the serial Benders decomposition are presented. The results are system cost bounds, convergence properties and calculation time.

The optimisation problem is a weekly decision problem with hourly time resolution (week 9, weather year 1969). The optimisation problem is divided into multiple stages, the number of stages N vary between configurations. We have four configurations where N=2,4,7 or 168. The number of iterations is limited to 20. The limit is set because going beyond 20 iterations increases computation time relative to solving the full problem.

In Figure 5 the upper and lower bound on system costs per iteration is shown for all configurations. The legend shows the number of divisions, N2, N4, N7 and N168. The convergence properties depend on the number of divisions into stage. Fewer stages give better convergence rate.

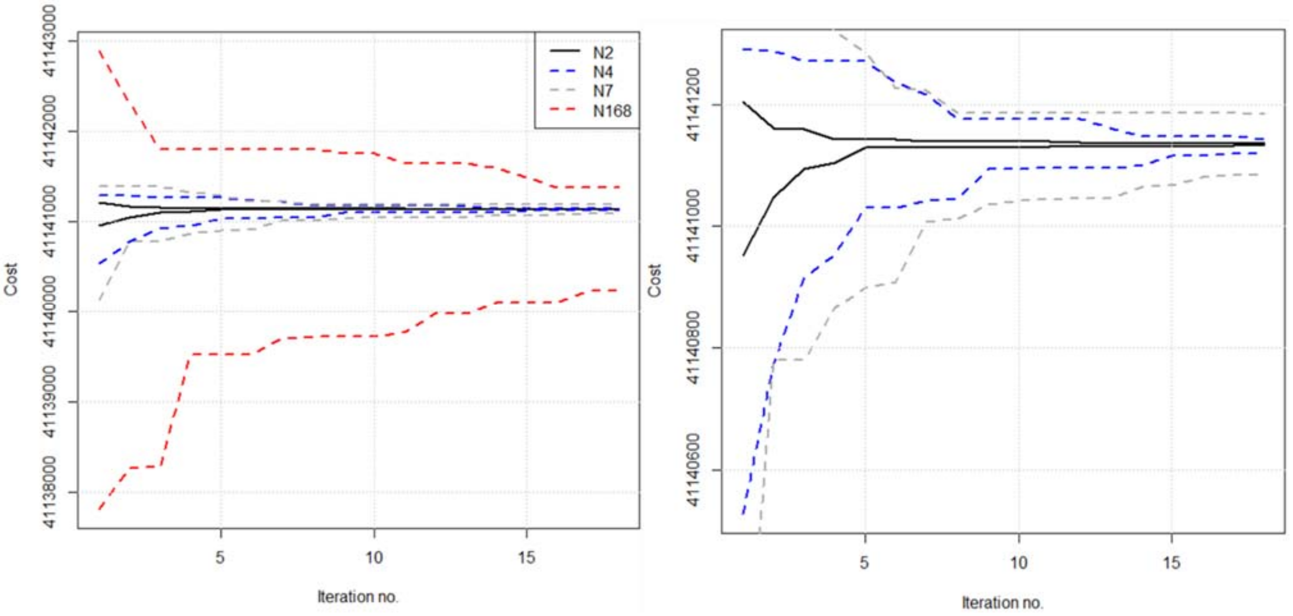

Figure 5: Upper and lower bound on system costs per iteration. Right figure zooms in.

Figure 6 shows the computation time per iteration for all configurations. Computation time refers to the accumulated time spent by CPLEX to solve all problems. The yellow dotted line shows the computation time of solving the full LP-problem. Only the case N168 is below the yellow dotted line and thus faster than solving the full LP. However, the low computation time comes at the cost of slower convergence and lower solution quality.

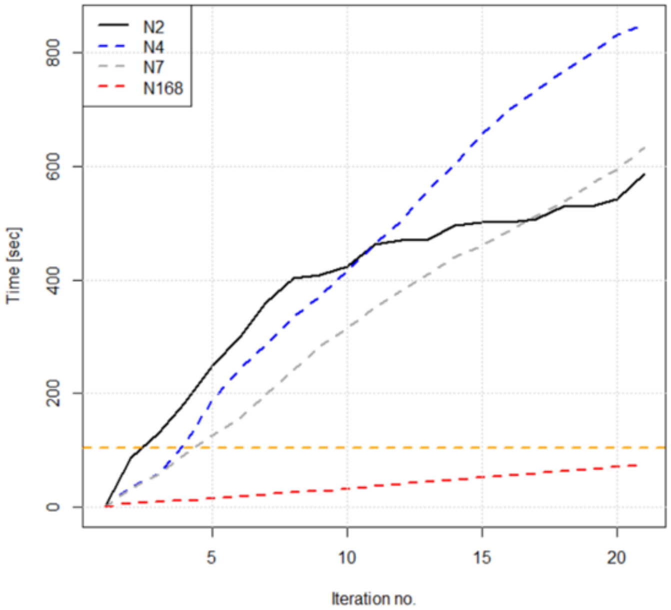

Figure 6: Computation time per iteration.

For comparison, the computational times for a PRIBAS iteration are shown in Table 1. This heuristic is faster at solving the decomposed problem but comes at the cost of lower precision in simulation results when compared to the solution of the full LP problem.

Table 1: Computation time (seconds) for a PRIBAS iteration.

	N2	N4	N7	N168
Week 9	44	39	35	9
Week 31	61	50	47	9

2.1.2 Parallel Benders decomposition – results

Selected results from the experiments with the parallel Benders decomposition is presented. The results are system cost bounds, convergence properties and calculation time. The optimisation problem is a weekly decision problem with hourly time resolution (week 31, weather year 2013). The optimisation problem is divided into multiple stages, the number of stages is N=4 and 7. The number of iterations is limited to 60. The serial decomposition is run for 20 iterations and used as benchmark. In Figure 7 the lower and upper cost bound per iteration is shown. PAR-SYNC refers to the parallel synchronous scheme, the PAR-ASYNC refers to the approximate asynchronous scheme (solver time limit 3 seconds imposed after the first 7 iterations) and SER refers to the serial decomposition. The obtained cost gaps are similar except for the PAR-ASYNC on the left which has a wider gap.

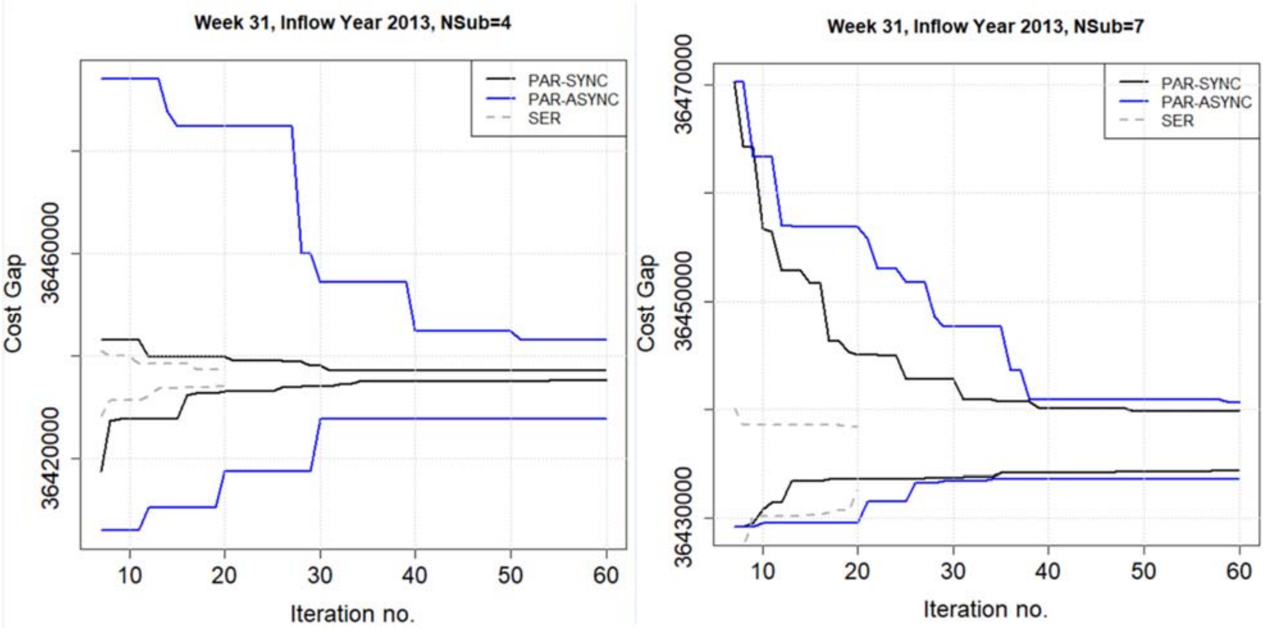

Figure 7: Convergence of lower and upper cost bound. Left:Nsub=4 and Right:Nsub=7.

Accumulated time spent by the solver per iteration is shown in Figure 8. The time consumption is significantly reduced for the parallel synchronous scheme when also considering the slower convergence rate for the parallel scheme. The parallel asynchronous scheme is the fastest and faster than solving the full LP.

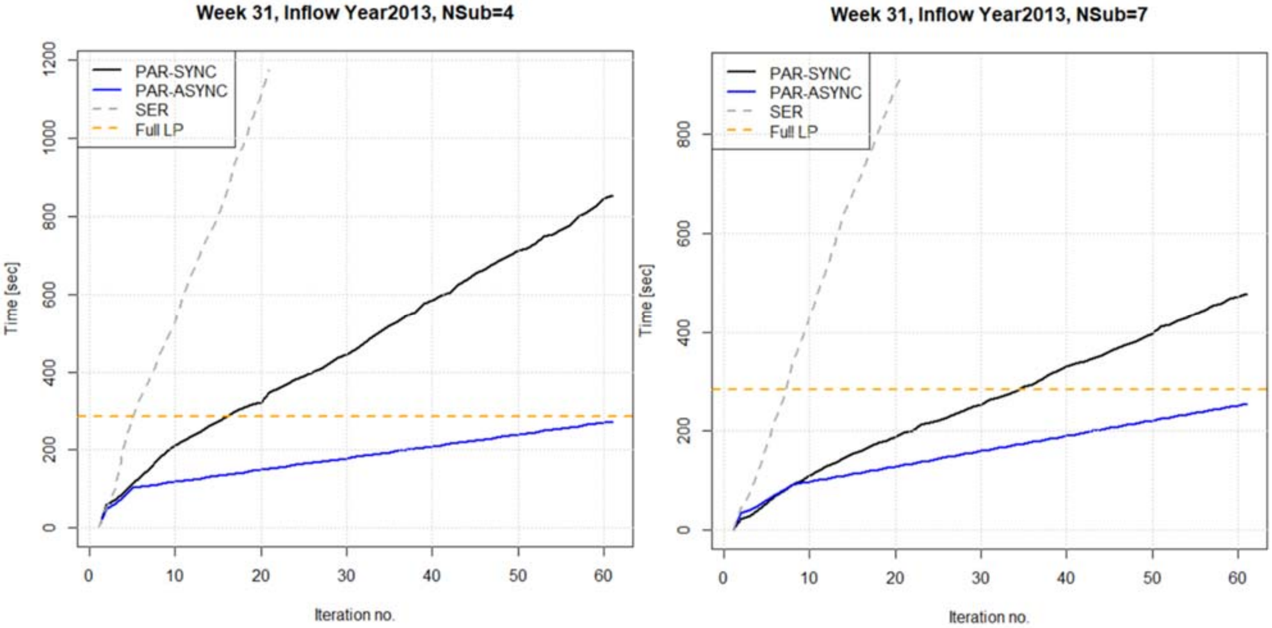

Figure 8: Accumulated time spent by the optimisation solver. Left:Nsub=4 and Right:Nsub=7. The dotted orange line shows the solution time for the full LP-problem.

2.1.3 Discussion and conclusions

We investigated the potential for saving computation time by applying temporal decomposition of the hydrothermal scheduling problem formulated as a deterministic LP. Initial testing indicated that the potential is higher on scenarios of shorter time horizon [8], this is explained by the storage levels of larger reservoirs being subject to relatively small changes. Thus, all tests were conducted on LP problems with a weekly time horizon.

The spatial decomposition is sequential in nature, meaning that one must solve each subproblem in sequence. The first tests with serial Benders decomposition show that many iterations are required to meet a standard convergence criterion, and that the overall computation time exceeds the solution time for the full LP problem without decomposition, this find was confirmed in [8]. On the other hand, if one allows the cost gap remaining after a limited number of iterations, computational savings can be achieved with modest reduction in solution quality. In particular, the so-called 'PRIBAS iteration' provide promising results and is worth further exploration. We emphasize that the PRIBAS iteration only makes sense when applied to the master problem in FanSi (obtaining primal results) and not for the scenarios (obtaining dual results). The PRIBAS iteration provides an interesting heuristic for decomposing the Fansi master problem. It allows for significant computational speedup, and leads to differences in results, particularly for the operation of small-scale storages.

Finally, the temporal decomposition algorithm was arranged in parallel according to a synchronous parallel processing scheme known from the existing literature. Parallel processing allowed breaking the sequential nature of the temporal decomposition algorithm and led to notable improvements in computation time. Through parallelization, the computation time per iteration was severely reduced on the one hand, while a higher number of iterations were needed to achieve a target cost gap on the other. Overall, the computational time was improved compared to the serial decomposition algorithm, but obviously the improvement comes at the cost of additional use of computational resources. To further improve the parallel implementation, we experimented with an approximate asynchronous parallel processing scheme were subproblems were allowed a maximum time for reaching a solution, and in the case of no solution, their "contributions" were not updated. This scheme showed promising results and could be worthwhile further investigations.

2.2 Spatial Decomposition

In this section we present two decomposition techniques. The use of decomposition techniques allows for dividing the full problem into many smaller subproblems but requires the collection and coordination of the subproblem solutions into a full problem solution. Lagrangian Relaxation and Benders decomposition are applied on deterministic LP-problems representing large hydro-thermal power systems, for example northern Europe. These systems span large geographical areas, consisting of many price areas with local power generation interconnected by transmission power lines. The subproblems can represents the local power production, typically per price areas. This separation into subproblems allows for parallel solution of all local power generation. The goal of the decomposition is to reduce the calculation time while maintaining a good results quality.

2.2.1 Lagrangian Relaxation

The content and results presented in this subsection is based on [3] when it comes to descriptions, figures and conclusions. Lagrangian Relaxation (LR) is applied to facilitate spatial decomposition of hydro-thermal power systems. LR can be used to compute a lower bound on system costs for minimization problems. A review of LR can be found in [11]. LR is a known technique which can be used for solving the hydrothermal scheduling problem for large power systems [12], [13].

2.2.1.1 Illustration of the decomposition

LR is attractive due to the high potential for parallelization, it allows for splitting the large regional hydrothermal scheduling problem in one market subproblem, several separable local hydro and thermal subproblems. In addition, a problem providing updated power prices to these subproblems is utilized, this problem is called the dual problem. The solution process starts by providing initial area power prices to the subproblems, the solution process is an iterative process following four steps in sequence per iteration

1. Local power generation problems are solved for given power prices
2. The power market is cleared in the market subproblem for given power prices
3. Collect the solution from subproblems into a full problem solution
4. Update power prices based on aggregated area power generation from subproblems
5. Check if convergence criterion is within acceptable tolerance

The LR solution process is illustrated in Figure 9.

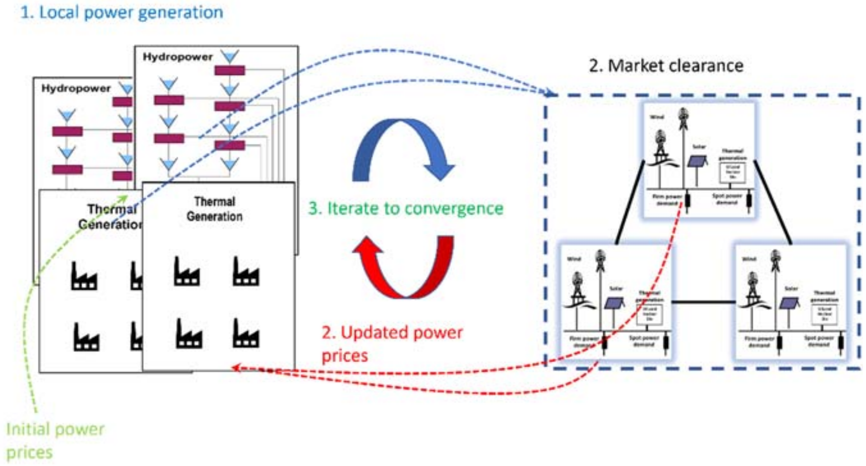

Figure 9: Illustration of the solution mechanism of the decomposition by LR.

The decomposition by LR is applied by relaxing the power balance constraints and introducing Lagrangian multipliers per timestep and price area into the objective function. The multipliers penalize deviation from the power balance constraint.

The dual problem providing power prices can be solved by using different algorithms, such as the subgradient or Bundle method [14], we use the Bundle method, because of its accuracy. It should however be noted that it is not the ideal choice for computational speed. The Bundle method has a quadratic objective function constrained by linear cuts. The cuts are obtained from the collected solution of the subproblems, by calculation of the mismatch of aggregated power generated per area and timestep.

The LR subproblems can be formulated as LP problems, i.e., all functional relationships are linear. The local hydro and thermal generation LP-problems maximise the generation revenue for given hydro and thermal constraints. The market problem minimizes the remaining system costs for aggregated hydropower and thermal generation and future costs of water while subject to power balance constraints.

We will show results from decomposition by LR of the weekly problem and the scenario problems of the FanSi model. The results are the computation time, the area prices, and the convergence properties of the decomposition by LR on the weekly problem. The computation time from the LR process is the accumulated time spent by the LP-solver. The convergence properties are displayed as the remaining *convergence gap* on system costs:

$$Convergence\ gap = System\ cost\ of\ solution\ from\ LP - system\ cost\ of\ solution\ from\ LR$$

where the benchmark for system costs is taken from the solution of the full LP-problem. Ideally, the convergence gap should be zero.

2.2.1.2 Results – Decomposing the weekly problem

The weekly problem is built for week 1 in weather year 1961 taken from historical weather records. The problem has 168 timesteps, at the first timestep the initial reservoir volumes sets the system state. At the end of the week the future cost function is set by Benders cuts provided by the FanSi model. Included constraints per time step is shown in Table 2.

Table 2: Included constraints per timestep.

Constraints per timestep
Hydro balance
Power balance
Minimum and maximum bypass
Minimum and maximum discharge
Up and down ramping on discharge
Minimum and maximum generation on thermal units
Start-up costs for thermal generation units
Shut-down costs for thermal generation units
Up and down ramping on flow on transmission lines

The weekly problem contains 1 114 000 constraints and 1 967 000 variables.

The calculation time for the LP and the LR process is shown in Table 3. The LR process is faster than solving the full LP-problem. Solving the subproblems in parallel is faster than in series.

Table 3: Comparing the solution time of the LR iteration process with the LP.

	LP	LR	
		Parallel	Serial
Solution time (s)	645	299	412

The convergence gap per iteration for the LR iteration process is shown in Figure 10. The convergence gap seems to stabilise after approximately 60 iterations. The gap is approximately 20 000 k€ at termination after 116 iterations. The solution of the problem by LR underestimates the system costs by approximately 20 000 k€. This value is very large due to the use of slack variables with high penalty costs in the problem constraints. If the constraints are not met, the slack variables ensure feasible solutions with added penalty costs. The slack variable coefficient is 10^5 k€ per unit. Slack variables are added to the constraints on bypass and discharge. These constraints are added for all timesteps on all hydro modules.

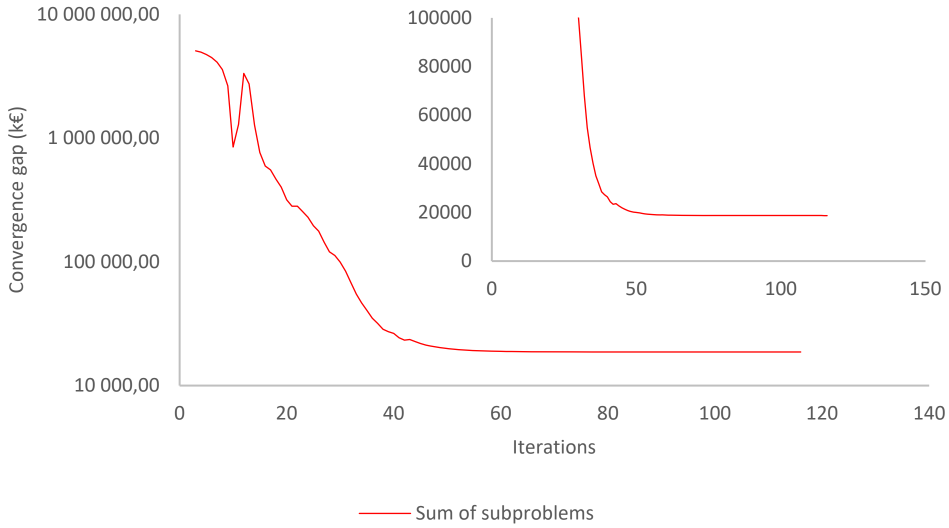

Figure 10: The convergence gap per iteration for the problem solved by Lagrangian Relaxation, note the logarithmic scale on the vertical axis. The embedded figure has linear scale on the vertical axis.

The area power prices for areas Østland and Norgemidt is shown in Figure 11. The green curve is the power prices obtained from LR and the red curve is the power prices obtained from the LP. Power price deviations are present.

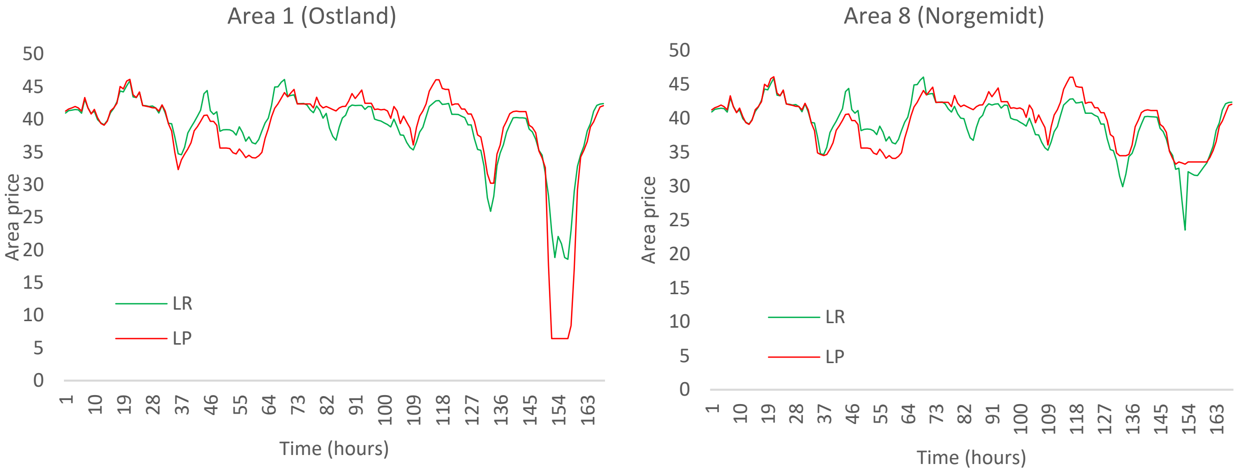

Figure 11: Area prices obtained by the LR solution and the LP solution.

2.2.1.3 Results – decomposing the scenario problems

The scenario problem length is 52 weeks, (week 1 to 52) with 7 intraweek timesteps and a total of 364 time-steps for the weather year 1962. At the first timestep the initial reservoir volumes sets the system state and the end of the last week the future cost function is set by Benders cuts provided by the FanSi model. The scenario problem does not include start-up and shut-down costs for thermal generation or ramping constraints as the weekly problem. The number of constraints is 1 326 785 and the number of variables is 3 624 713 for the full LP-problem.

The objective function value of the full LP-problem is 1 291 970 441 752 k. This value is very large due to the use of slack variables with high penalty costs in the problem constraints. If the constraints are not met, the slack variables ensure feasible solutions with added penalty costs. The slack variable coefficient is 10^5

k€ per unit. Slack variables are added to the constraints on bypass and discharge. These constraints are added for all timesteps on all hydro modules.

The convergence gap per iteration of the LR solution process is shown in Figure 12. The convergence gap is 462 215 k€ (3.5E-5 %) at termination. The convergence gap seems to stabilize after approximately 200 iterations. The solution is still improving after 200 iterations, but much slower. Adjusting the termination of the iteration process would reduce the calculation time, but also the result quality. Before attempting to adjust the termination point of the LR iteration process we inspect the obtained results in more detail.

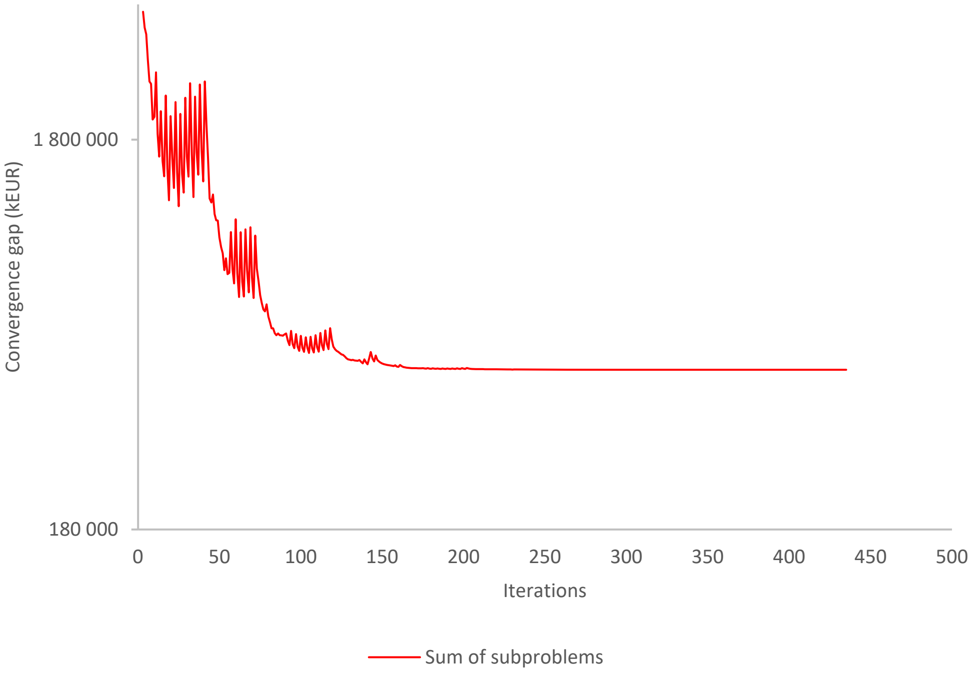

Figure 12: The convergence gap versus the iteration number.

In the FanSi model, Benders cuts are obtained from the solution of the scenario problems by extracting the dual values of the reservoir balance restrictions (marginal water values) and the corresponding reservoir volumes of the first time-step, in addition to the objective function value. High quality Benders cuts is required for optimal operation of the power system.

The quality of the Benders cuts is assessed by comparing the LP and LR solutions, where the LP solution is the benchmark. In Figure 13 a scatter plot shows the marginal water values for all reservoir in the price area Ostland for the LP and the LR solution. Deviating water values are observed for the two solutions where LP values are higher and seem to follow a consistent ratio. This is explained by deviations in area prices. The area price obtained from the LP is 187.95 €/MWh and the area price from the LR is 122.97 €/MWh. The price ratio between the LP and LR solution 188/123 is approximately 1.5 and will define the ratio between the LP and LR marginal water values. There is a linear relation between production and discharge, in the so-called pq-curves, which leads to a linear relation between the area power price and the marginal water values.

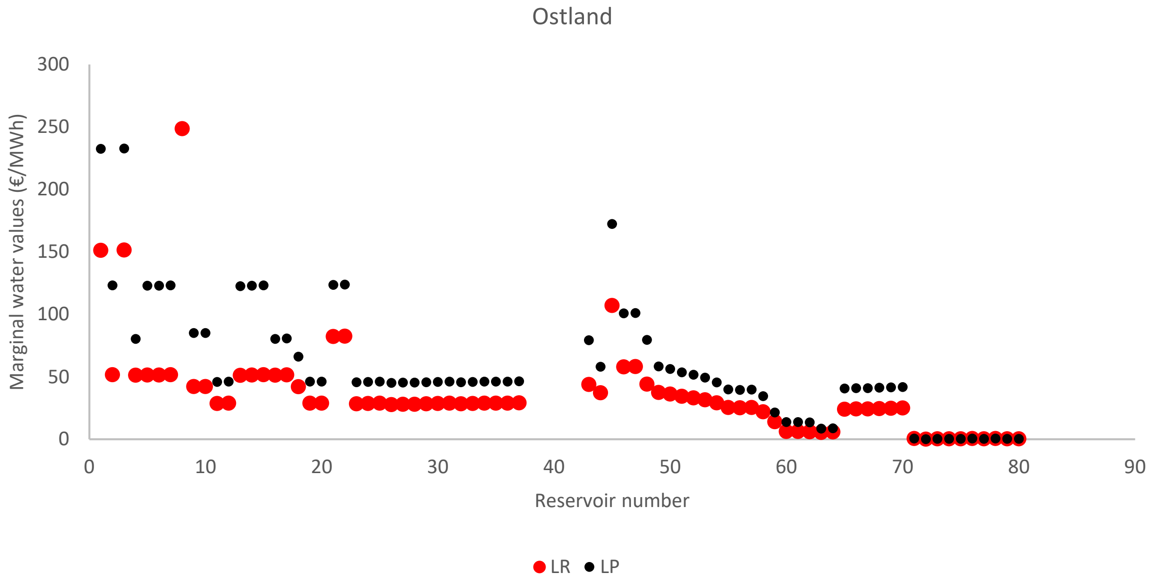

Figure 13: Scatter plot of the marginal water values for all reservoirs in price area Ostland.

The scatter plot in Figure 14 shows the reservoir volumes of all reservoirs in price area Ostland. Most of the red and black markers overlap, but there are some noteworthy exceptions. For example, reservoir 1 has a much lower volume in the LR solution than the LP solution.

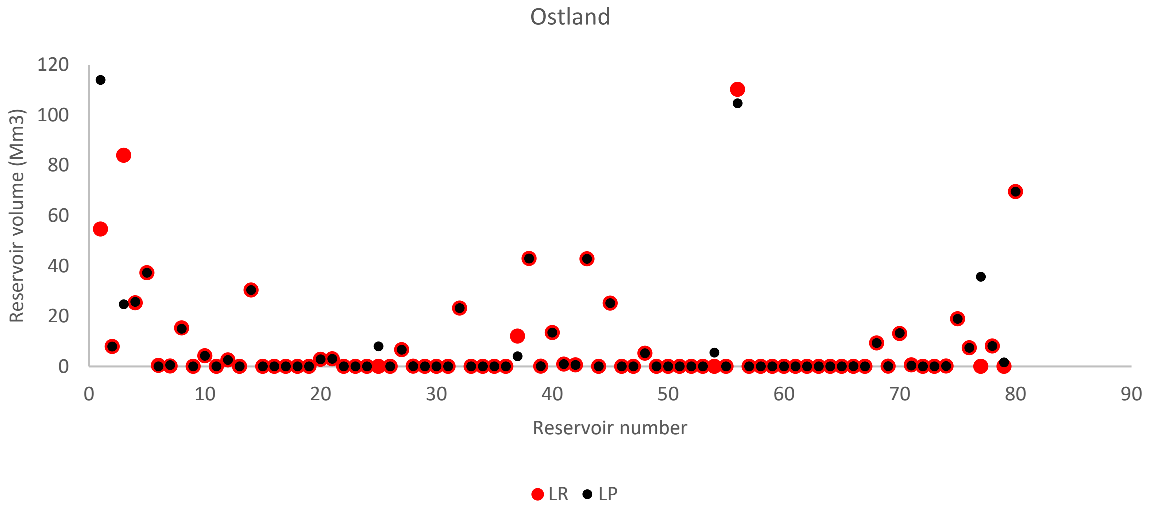

Figure 14: Scatter plot of the reservoir volumes for the area Ostland.

In the FanSi model a weighted Benders cuts is calculated based on results from several scenarios and the scenario probability. Therefore, a better test is to calculate the weighted marginal water values from a set of scenarios. We solved 6 additional scenarios from the years (1962-1967) and the weighted marginal water values from there scenarios are shown in Figure 15.

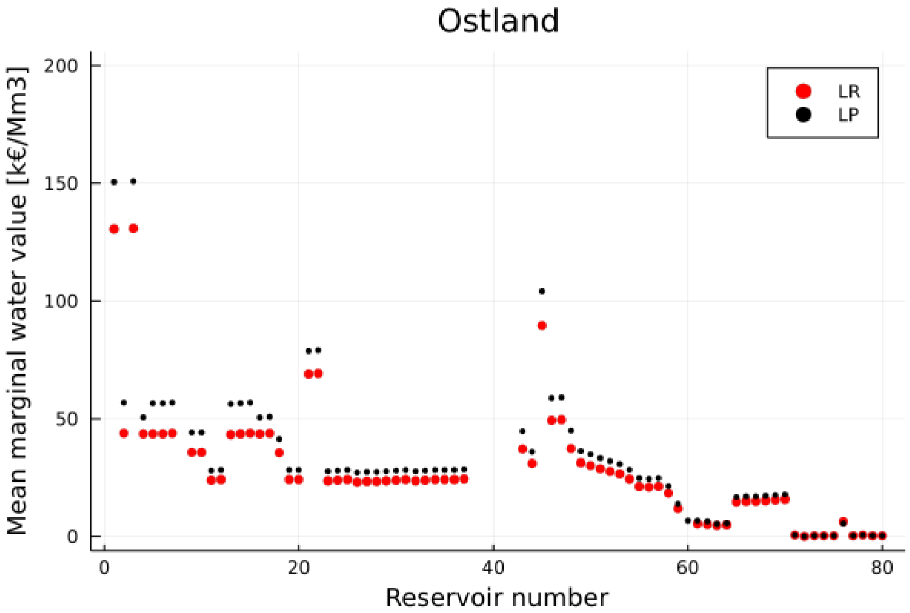

Figure 15: The weighted marginal water values for all reservoirs of the Ostland area.

The red points show the results from the solution of the LR, and the black point are results from the solution of the LP-problem. The water values from the LR are lower than those from the LP. The ratio between the weighted water values, for the first time-step in area 1, from the LP and LR solutions are approximately 1.15. The Benders cuts obtained from LR is not considered to be of sufficient quality for use in the FanSi model. The mean calculation time for these 7 scenarios were very high at 13436 seconds, which is too high for practical use. It is possible to reduce the solution time significantly by terminating the LR iteration process earlier, but that would only reduce the results quality further and was not pursued in this work.

2.2.1.4 Conclusion and discussion

The spatial decomposition of the hydro-thermal scheduling problem by applying LR was studied. The communication of area prices and power generation (mismatches) between the dual problem and the subproblems facilitates the solution. The decomposition was tested on a large-scale power system. The purpose of the tests was to investigate the quality of the obtained results.

A conclusion from our tests of using LR both on the master and scenario problems formulated by Fansi is that the obtained result quality of the solution found by LR is not high enough. Thus, based on the presented results, we will not further pursue the use of LR decomposition within the Rakett project. In addition, the relaxation of the power balance equation may result in a power balance equation which is not fulfilled and thus will require an additional primal recovery phase to attain valid results. And the Benders cuts extracted from the solution may not be stable between iterations.

However, we recognize that improvements are likely, and list below a set of possible avenues for further studies:

The solution of the dual problem (quadratic LP) was found to be a bottleneck with respect to computational time. Techniques such as cut relaxation and linearization of the quadratic terms in the objective functions could be further explored.

We chose the Bundle method to update the Lagrangian multipliers. This method is known to be accurate, but computationally slow. Faster methods, such as the subgradient method, could be tested.

The hydropower subproblems were divided based on price areas, with large variations in the number of hydropower reservoirs within each area. A finer division to ensure a more balanced sizes of the hydropower subproblems would likely lead to better performance when applying parallel processing.

The details related to updating the Bundle method's penalty parameter were not fully explored. This is a complex area where problem-specific knowledge is needed. For example, the relationship between the numerical scaling of the optimisation problem, the objective value and the size of the penalty could be further explored. Moreover, the heuristic rules for choosing between null and descent-steps could be revisited.

2.2.2 Benders decomposition

This subsection is based on [15] when it comes to descriptions, figures and conclusions. A detailed description of spatial decomposition by Benders decomposition applied on the FanSi model is presented.

In a similar manner to the decomposition of the hydrothermal scheduling problem by LR, the application of Benders decomposition (BD) facilitates the separation of the full problem into separate subproblems. One advantage of BD over LR is that the power balance equations are not relaxed.

2.2.2.1 Illustration of the decomposition

Again, we are concerned with the application of decomposition on problems from the FanSi model, these problems are deterministic multiperiod LP-problems. Benders decomposition is applied to reduce the solution time. The decomposition by BD is conceptualized in Figure 16, where the detailed hydro generation is separated from the full problem into subproblems, keeping only an aggregated description of hydropower in the master problem. The aggregated hydro is described as an energy model and is introduced in the master problem to improve the convergence properties by reducing the flexibility of the hydro generation.

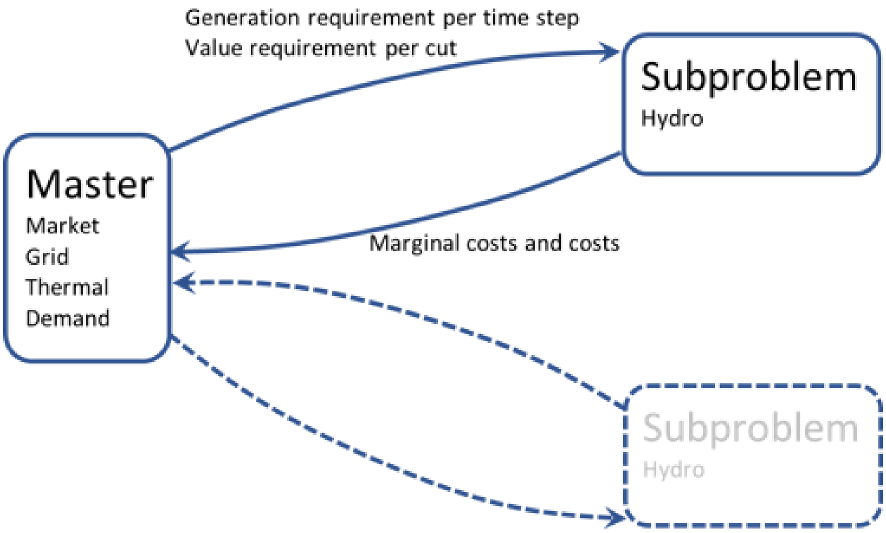

Figure 16: Spatial Benders decomposition illustrated.

The overall solution requires communication of generation requirements per time step and energy storage requirements per cut to the subproblems, and marginal hydro generation costs and hydro generation costs to the master problem to form Benders cuts. The cuts are constructed around the generation requirement as illustrated in Figure 17.

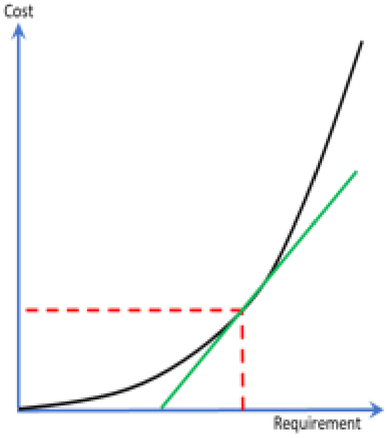

Figure 17: Illustration of the Benders cuts. Cost of generation versus the generation requirement on the axes and the marginal value of generation cost in green.

2.2.2.2 Results – decomposing the scenario problem

The scenario has one month horizon (30 consecutive days). The full LP problem has 241.000 variables and 98.000 constraints and solves in 7 seconds. Running the decomposition algorithm, the master problem takes approximately M seconds to solve while the subproblems range from N1-N2 seconds. The solution of the full LP-problem is used as benchmark. The focus of the results are the objective function of the master problem and the detailed water values for the first timestep.

The convergence of the master problem is shown in Figure 18, where the objective function value (red line) gradually converges towards the benchmark. The decomposition was stopped after 400 iterations.

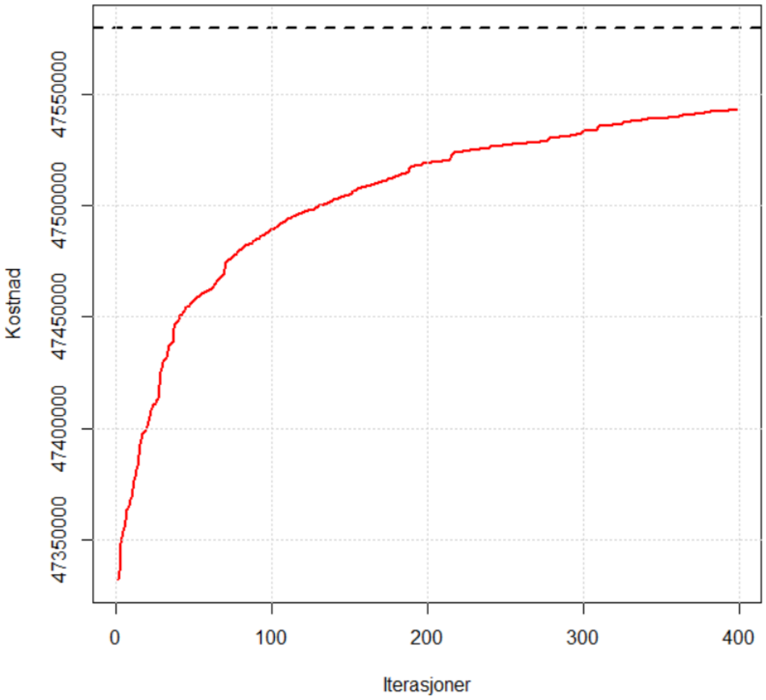

Figure 18: System costs per iteration. The red line shows the objective function value for the master problem and the black dotted line for the full LP-problem.

In the following the water values from selected reservoirs listed in Table 4 will be presented. We show how the computed water values (black curve) gradually improve towards the correct water value from the benchmark (blue curve).

Table 4: Studied reservoirs.

#	Price area	Module number	Reservoir name	Storage capacity [Mm3]
2	**6 – Vestsyd**	16	Blåsjø	3105
3	**6 – Vestsyd**	51	Sauda1	7.9
5	**7 – Vestmidt**	20	Langavatn	160
8	**14 – SVER-NN1**	1	Ransaren	414

In Figure 19 the water values for Blåsjø and Langavatn is shown and in Figure 20 the water values for the Sauldal and Ransaren are shown.

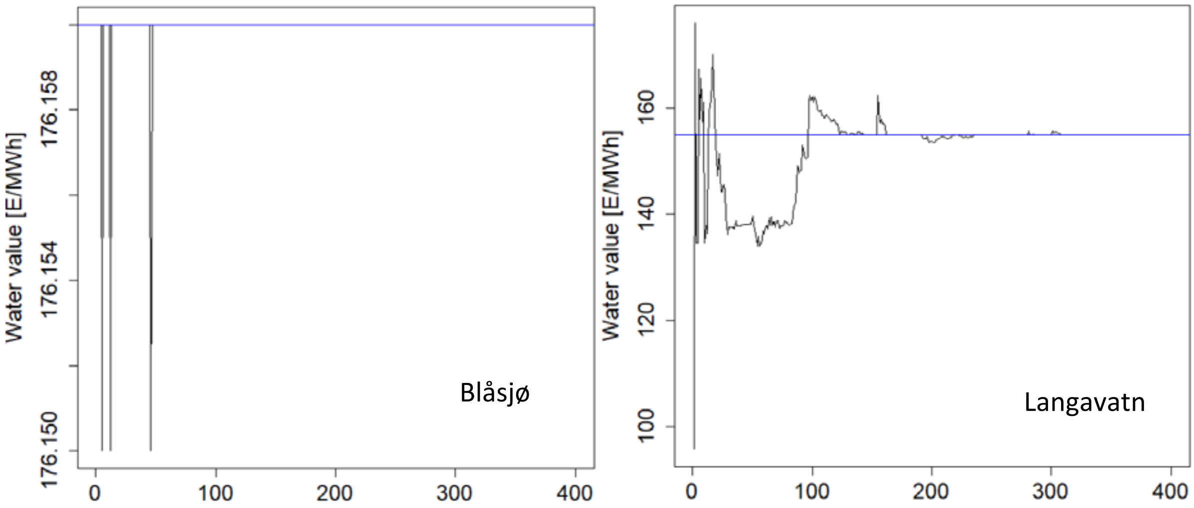

Figure 19: Water values per iteration for the Blåsjø and Langavatn reservoirs.

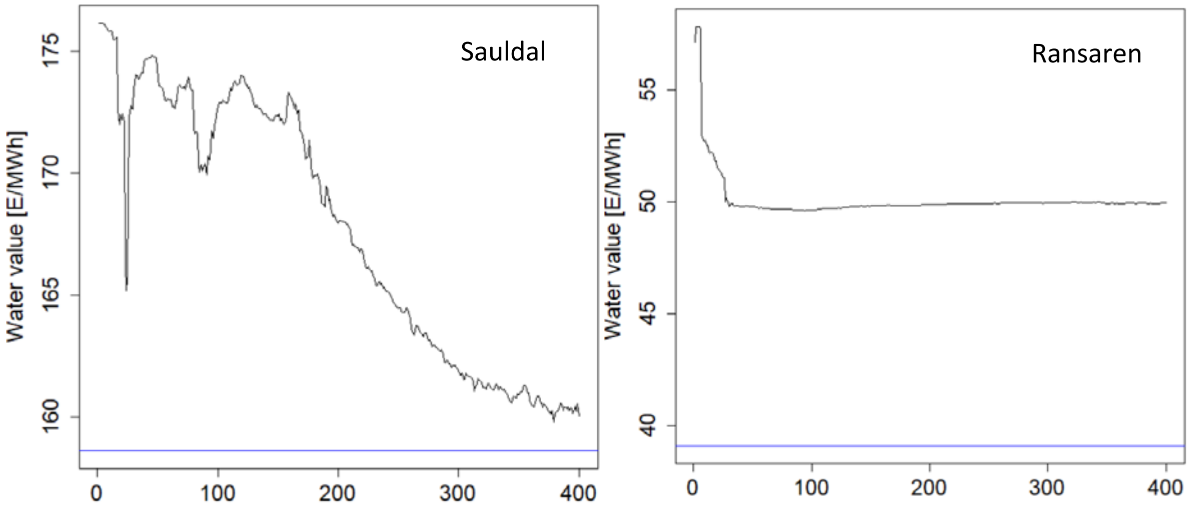

Figure 20: Water values per iteration for the Sauldal and Ransaren reservoirs.

The figures show that the water values for the Blåsjø reservoir converges early. The reservoir Langavatn has a more gradual convergence. The Sauldal reservoir also has a gradually improved water value per iteration but does not fully converge after 400 iterations. The Ransaren reservoir does not converge to the expected value.

2.2.2.3 Conclusions and discussion

Benders decomposition was proven as a possible method for spatial decomposition of the deterministic hydrothermal scheduling problem. A master problem with aggregated hydropower was coordinated with subproblems representing detailed hydropower subsystems. The coordination was facilitated by Benders cuts. We find that this decomposition approach has some interesting properties, enabling separated treatment of aggregated and disaggregated hydropower in a manner that has similarities to the EMPS model. This was further investigated in [16].

The presented method was tested on a large-scale system, emphasizing on its ability to find water values for the detailed hydropower system. While water values can be extracted from the computations, results revealed that these did not demonstrate adequate accuracy to be used within the Fansi model. For this reason, we did not further investigate the potential of this decomposition method within the RAKETT project.

3 Other means for reducing calculation time

3.1 Relaxing reservoir balance constraints

The relaxation of reservoir balance constraints is documented in detail in [5] and the content of this subsection, in terms of descriptions, figures and conclusions, is based on the findings in this report.

The FanSi model is designed for use on power systems dominated by hydropower such as the Nordic power system with more than 1000 hydropower modules. The hydropower modules will have varying characteristics and varying degree of regulation. Well-regulated reservoirs have low risk of reaching its boundaries (full or empty reservoirs) while poorly regulated reservoirs have higher risk of reaching its boundaries. Therefore, many (well regulated) reservoirs will never reach its boundaries and the reservoir balance constraints can be relaxed.

The LP-problems formulated by the FanSi model are economic dispatch problems with the objective of minimising system costs. The problems cover multiple time steps and include power and reservoirs balance constraints and water value coupling constraints at the end of the horizon. The detailed representation of the hydropower gives rise to the large size of the LP-problem. Reducing the LP-problem size will generally reduce the computation time.
The number of reservoir balance constraints per week with hourly time resolution is 1000 reservoirs * 168 time-steps = 168 000 constraints. By relaxing the reservoir balance constraints, applying them on a weekly basis the number of constraints is reduced to 1000 constraints. This leads to a lighter LP-problem which can be solved faster but it reduces solution quality. Means of retaining high quality solution while keeping calculation time low is necessary.

High quality can be ensured through a simple heuristic, solving the LP-problem in two iterations. The heuristics discriminate on reservoirs size by selecting the smaller reservoirs in the system and giving them constraints on reservoir balance for all time steps. The reservoir size of the smaller reservoirs is defined by a threshold value, VThres. The remaining reservoirs with volumes higher than VThres has reservoir balance constraints on a weekly basis. The LP-problem is then solved in a first iteration, and the solution is used to control if reservoirs with weekly constraints reach their boundaries. If so, reservoir balance constraints are added for all time-steps to the LP-problem and re-solved.

3.1.1 Results

Following this heuristic, a set of computational experiments were initiated where the results were evaluated by total system costs, power prices and calculation time. Only selected results from the complete work are extracted here. Cases 3 and 4 are defined in Table 5. The cases refer to the starting weeks 9 and 31. The initial system state is obtained from a FanSi run. The weather years are 1969 (dry) and 2011 (wet). The number of weeks in the scenario is 8 and the timesteps are 3 hours long.

Table 5: Case description.

Case no	Number of weeks	Time resol. [h]	Start week	Weather year
3	8	3	9	1969
4	8	3	31	2011

The computation time for different values of VThres is shown in Figure 21. The legend of the figure shows the cases (3,4) colour coded to separate the curves.

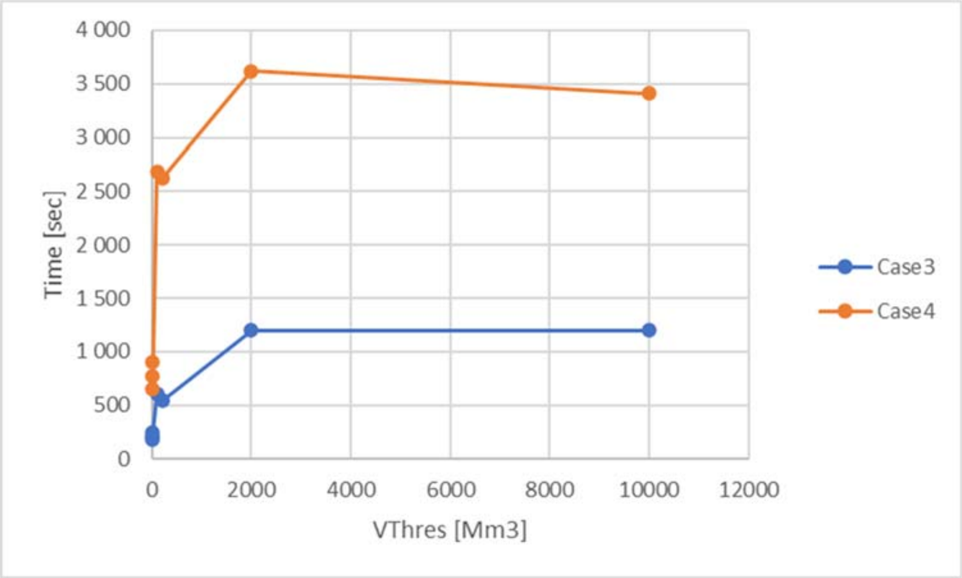

Figure 21: Total time spent by solver for varying threshold values.

The potential calculation time by varying VThres is significant.

The objective function values (minimum system costs) for varying values of VThres for both iterations are shown in Figure 22. The blue points show the objective values achieved in the first iterations, while the orange points show objective values from the second iteration. A lower value of VThres results in a high difference in objective value between the first and second iteration.

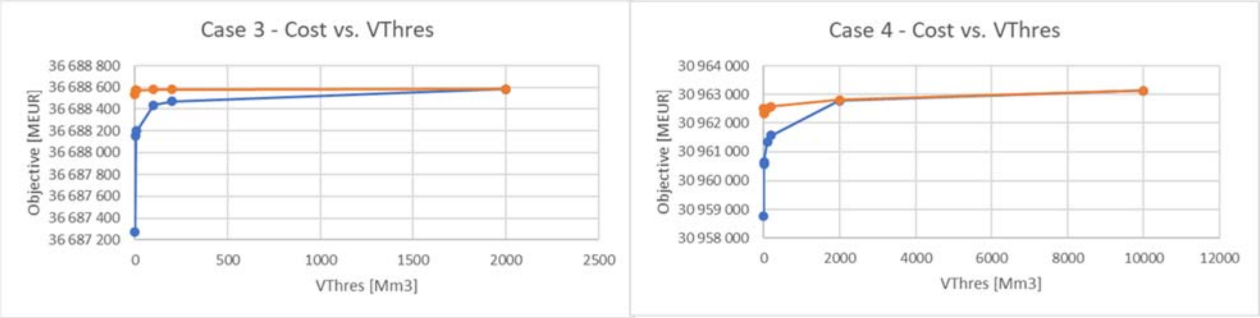

Figure 22: Objective function value for varying threshold values.

The simulated power price for the first and second iteration is compared with the solution of the full problem in Figure 23. In the full problem all reservoir balance constraints are included. After the first iteration the prices shows some deviations, but after the second iteration the deviations have lessened.

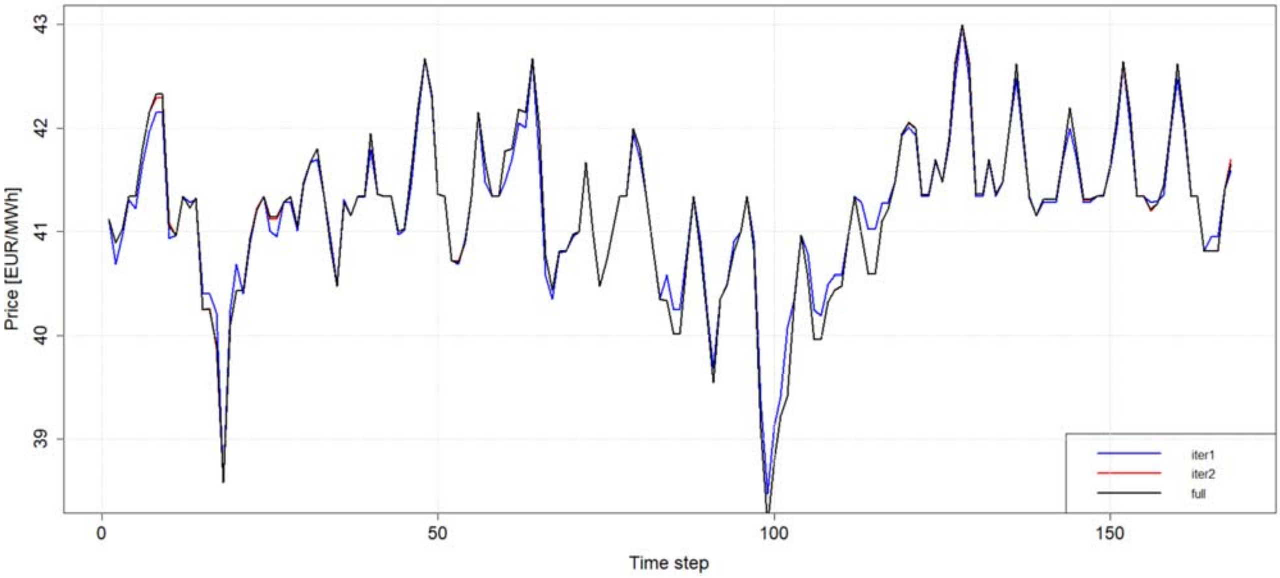

Figure 23: Simulated power prices in the first and second interation for Norgemidt in case 3.

3.1.2 Summary

These experiments show that relaxation of reservoir balance restrictions can significantly reduce calculation time while keeping high result quality. The reduction of calculation time by a factor of 2 (and more) was observed. Including the reservoir balance constraints for the smallest reservoirs (low threshold value) is important to maintain the result quality. In addition, the initial state of the reservoirs and the length of the scenario affects the impact of this heuristics.

3.2 Parametrisation of FanSi

By "parameterisation of FanSi" we refer to the specification of the time resolution, the scenario length, and the functionality of the FanSi scenario fan. The parametrisation affects the calculation time and the result quality. Simple testing of the parametrisation of the FanSi scenario fan was documented in the Rakett project memo [6]. By simple testing we refer to simulations over one weather year with the FanSi prototype model.

The tests show that running the FanSi-model with the base constraints power balance and reservoir balance, scenario fan length of one year with weekly power and reservoir constraints gives reasonable results and acceptable calculation times. Further, considering the length of the scenario fan is important due to seasonal variations of water values from EMPS. The water values are normally lower in the summer and higher in the winter. Choosing a scenario length of half a year will couple to water values in the wrong season and may under/overestimate water values. This is especially important for well-regulated reservoirs.

A simple study of the FanSi SFP for a given week and year was performed to quantify the solution time per iteration [17]. The variation in solution time of the scenarios is significant and the slowest scenario defines each iterations calculation time. A transition to the use of a multi-cut decomposition algorithm may reduce the calculation time by removing the synchronisation point between the master and scenarios of the SFP. Solving the first iteration use most of the solution time of the SFP and up to 70% of the total time.

The tests revealed that FanSi solves individual LP-problems slower than if solved in the CPLEX interactive optimizer, indicating that the source code communicating with CPLEX API can be improved and made faster.

3.3 Tuning optimization problem solver

This section is based on the documents [18], [19] and presented in condensed form.

The FanSi model builds LP-problems which is sent to a solver. The solver used by FanSi is primarily CPLEX, but the COIN-OR LP-solver has been used. The default algorithm in FanSi is the dual simplex. The advantage of the dual simplex algorithm is the possibility of using an existing solution basis to accelerates the solution process. However, other algorithms can be used and may reduce the solution time. In this work the CPLEX solver is used.

The FanSi model can write LP-problems to files on a format readable by the CPLEX interactive optimizer. By writing a set of LP-problems to file from the FanSi model the CPLEX interactive optimizer can be used to investigate the impact of tuning solver parameters on the solution time for the same LP-problems.

3.3.1 Comparing the Barrier and the Dual Simplex algorithms

The work in [8] showed a speed-up when switching from the Dual Simplex to the Barrier algorithm for cold started problems, especially when solving large problems. Inspired by that work we investigated the use of the Barrier algorithm for potential reduction in solution time. The Barrier algorithm can utilise parallel processing to speed-up the solution time, but it requires an additional cross-over phase to obtain a solution basis (necessary to minimise calculation time in FanSi).

LP-problems with varying size (scenario problems written to files by FanSi) were solved using the CPLEX interactive optimizer. Both the Dual simplex and the Barrier algorithm used only one thread. The LP-problem sizes and their mean solution time are shown in Figure 24, where the points represent the size of the LP-problems studied. The mean solution time is calculated from the solutions of 10 scenarios and the error bars represent one standard deviation, all problems are cold started.

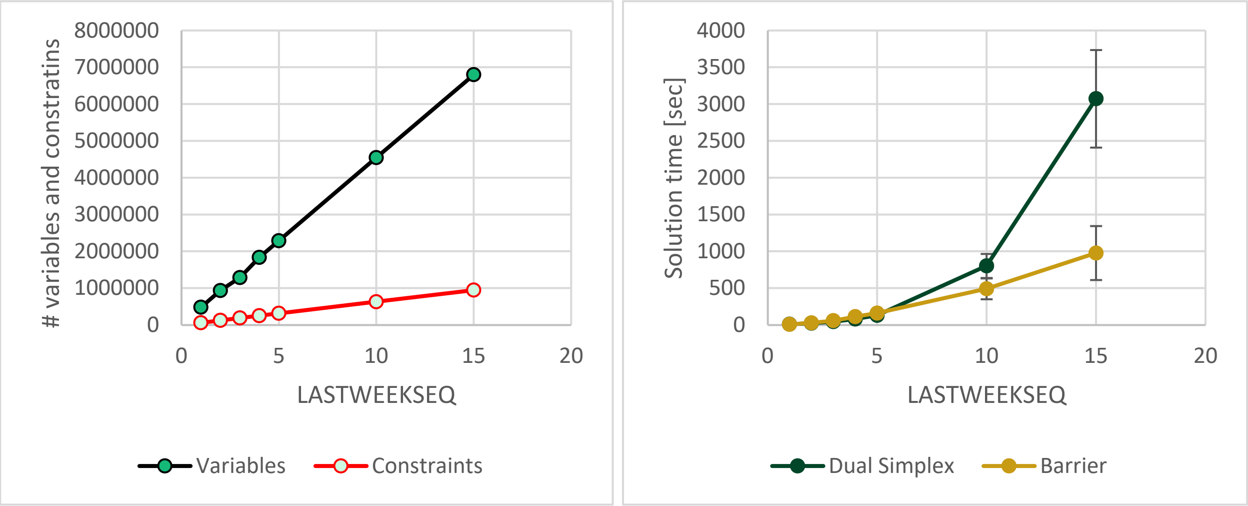

Figure 24: Problem size of the LP-problems from FanSi (left) and their solution time (right).

For the two larger problems the Barrier algorithm is significantly faster than the Dual Simplex algorithm, but for the remaining smaller problems the Dual Simplex is faster.

The use of parallel processing can significantly reduce the calculation time when using the Barrier algorithm. Our results show a time reduction by a factor larger than 2 when allocating 10 threads or more for both the weekly master problem and the scenario problems. However, the FanSi model is already computationally heavy and contains 2 layers of parallel processing. A fully parallelised FanSi, parallel simulation with 50 scenarios, each with 50 scenarios in the SFP requires 50X50 = 2500 threads. The benefit of further parallel processing of the Barrier algorithm is limited by available computational resources.

Some benefit can be obtained when solving the weekly master problem of the SFP in the first iteration. The master problem is always solved with no solution basis in the first iteration and computational resources are available (the allocated threads for solving the scenarios in the SFP are idle and waiting for the solution from the master problem).

We investigated the impact on computation time by using the Barrier algorithm with parallel processing. The FanSi model was adapted to switch to parallel Barrier algorithm when no solution basis was available. Tests were conducted on four different parametrisations of the FanSi SFP. The reported calculation times are obtained from simulations for the weather year 1962. Scenario reduction was applied and set to 5 scenarios of the available 58. The results show a reduced calculation time of between 16 and 26 % for these tests.

3.3.2 Summary

We compared the calculation times obtained by using the Barrier or the Dual Simplex algorithm of the CPLEX solver for cold started problems. The tests show that the fastest of the two algorithms depends on the characteristics of the LP-problems. Parallel processing of the Barrier algorithm can significantly reduce the calculation time.

4 Summary and recommendations

4.1 Summary

The following activities have been part of this project:

- Studies of decomposition techniques and their application on large-scale hydrothermal scheduling problems, specifically deterministic LP problems. Both decomposition in time and space were studied with the focus on calculation time and result quality.
- The establishment of a computational framework capable of performing computational experiments to reveal the effects of applied decomposition techniques on relevant large-scale power systems. The computational framework is flexible and can readily be adapted to future modelling needs.
- Studies of parallel and serial schemes for Benders decomposition in time. The results show potential for reduced computation time, but future study is recommended.
- Development of a new method applying spatial Benders decomposition in to the hydrothermal scheduling problem. Although not directly applicable within the FanSi concept, the spatial Benders decomposition algorithm fits well into the SDDP algorithm as shown in [16]. This seems like an interesting pathway for further research.

- Lagrangian relaxation was applied for spatial decomposition of the hydrothermal scheduling problem. The computational experiments showed that the results did not hold adequate quality to be used with the FanSi model.
- Other means for reducing calculation time include:
    - Reducing the problem size by relaxation of reservoir balance constraints. The results show a significant reduction of calculation time with a modest reduction in result quality.
    - Selection of the fastest of the Dual Simplex and the Barrier algorithm for solving the optimisation problems. The parallel processing opportunities for the Barrier algorithm can significantly reduce calculation time at the cost of higher computational burden.

The project has improved the knowledge of decomposition techniques among the project partners. The project has given recommendations on how to reduce the computation time of FanSi.

4.2 Recommendations

The recommendations provided here are based on the combination of RAKETT project results, existing literature reviewed as part of this project and growing experience with use of the FanSi prototype model.

Our recommendations for improved decomposition within FanSi to facilitate reduced computation time:

1. The so-called 'PRIBAS iteration' provides a heuristic that is particularly interesting for the solution of the master problem within FanSi. We demonstrated that it substantially reduces computation time. The implications on results were discussed and analysed in the PRIBAS project but needs to be further assessed.

2. Asynchronous parallel processing of the Benders decomposition along the time axis is a promising path for further exploration and is documented in existing literature. Our computational experiments point to a potential reduction in computation time when solving a single scenario. However, the decomposition scheme adds complexity which should be weighed against the gain in computation time.

3. Techniques for spatial decomposition did not provide satisfactory results for use in FanSi. Both Lagrangian Relaxation and spatial Benders decomposition were tested. Further studies are however encouraged, taking a broader view on the principles of such techniques.

4. Hydropower aggregation by spatial decomposition can be applied efficiently within the SDDP algorithm. We believe this principle is worthwhile further investigations and could form the basis for a model that can provide end-valuation of water in the SFP (alternative to the EMPS).

5. In the current version of FanSi, the coupling point between the master and scenario problems of the SFP is a synchronisation point since cut parameters from the scenario problems are aggregated. The variation in solution time of the scenarios is typically large and one must wait for the slowest scenario. This synchronization point can be removed by introducing a multi-cut algorithm, which most likely leads to reductions in overall computation time. In addition, the transition to multi-cut algorithm is supported theoretically in [21].

Other recommendations related to the FanSi modelling concept that are based on tests and discussions carried out in the RAKETT project:

1. The FanSi model should be flexible in the choice of LP algorithm. Originally, the model was set-up with the dual simplex method, but throughout the project the barrier solver was tested in several cases and provided notable speed-up in some of them. In addition, the option to use the parallelized Barrier algorithm when computational resources are available is recommended.

2. Careful parametrization of the SFP is important for keeping calculation within acceptable computation times. The trade-off between computation time and solution quality as function of FanSi parametrization was assessed in [6] and [22].

3. Relaxation of reservoir constraints is essential for efficient computations when considering the detailed Nordic in FanSi. A relaxation heuristic was suggested, providing a significant reduction in computation time and only a modest reduction in result quality. This heuristic fits well with the current FanSi model and the iteration logic of the Benders decomposition.

Finally, it is worth mentioning that the RAKETT project has facilitated discussions related to the design and parametrization of the FanSi model. One of the possible designs discussed is a two-phase SFP solution.

- First, the SFP is solved with a reasonably detailed technical description of the system and its time discretization. This first phase serves to provide cuts.
- Second, and for the same week, a detailed simulation of the first week is performed using the cuts from the first phase. This allows simulation results with a much finer level of detail and time discretization. In this second-phase simulation, the PRIBAS iteration can be used, similar to the framework developed in the PRIBAS project [23].

Finally, we repeat the list of pros and cons for further decomposing the SFP solved in FanSi along the time axis, as originally presented in [5]:

Pros:

- Temporal decomposition is already a part of the FanSi concept and is relatively straightforward to implement and maintain. In contrast, spatial decomposition (e.g. by Lagrangian Relaxation) involves a deeper integration in the problem structure and may need additional functionality to recover primal results.
- There is no need for tailor-made model builders. All optimization problems (subproblems) cover the same system and system boundary.
- Techniques for decomposition in time using parallel processing has been studied in the existing literature and applied to similar type of problems with success [10].

Cons:

- The algorithm described for solving deterministic LP problems by decomposing along the time axis is sequential in nature. This is different from the spatial decomposition which is more intuitively suited for parallel computations.
- The importance of representing temporal couplings is likely to increase in the future power market models. Functionalities such as ramping (power flows, generation, water discharges, etc.), start-stop costs on power generation units, time-delays in rivers and end-user flexibility across time-scales all represent temporal couplings. Ideally, all such couplings should be represented in the Benders cuts, but from a practical point of view it is worth asking if the improved accuracy is worth the additional computational complexity.

The work described has included quantifiable reduction in calculation time for each individual means to reduce calculation time. It is very difficult to quantify the accumulated effect of these means on calculation time without implementing them in the FanSi model.

A list of the applied means for reduction of calculation time and their importance for overall reduction of calculation time of FanSi is shown in Table 6.

Table 6: The achieved reduction in calculation time and the techniques tested.

	Calculation time			
	Substantial reduction	**Modest reduction**	**No reduction**	**Promising**
Technique	***Relaxation of reservoir balance constraints*** (section 3.1)	***Benders decomposition applied in time*** – asynchronous parallel scheme (further work necessary) (section 2.1.2)	***Benders decomposition applied in space*** (section2.2.2)	***Multi-cut algorithm*** (section 4.2)
	PRIBAS – iteration (master problem only) (section 2.1)	***Selection of the fastest of the Barrier and the Dual Simplex algorithms*** (section 3.3)	***LR applied in space*** (section 2.2.1)	

The **relaxation of the reservoir balance constraints** provided a reduction in calculation time by a **factor of 2 and more**. The effect on the FanSi model is probably highest for the first iterations when solving the SFP. We consider the improvement potential in FanSi as high.

The **PRIBAS – iteration** provided a speed-up by a **factor of 2 and more** (master problem only). The improvement in FanSi depends on the parametrisation of the SFP.

We expect that the implementation of the **multi-cut algorithm** will reduce calculation time.

We also expect that the project recommendations will provide a substantial reduction to the over calculation time of FanSi. This has been assessed by numerical studies on realistic datasets in a framework based on Julia/JuMP.

FanSi is a flexible simulator, allowing multiple parametrization schemes and uses. The magnitude of computation time reductions as result of our suggested measures will depend heavily on the use and parametrization of FanSi, e.g.:

- The dataset and type of study being conducted
- The functionality used

- The parametrization of FanSi, including time discretization in master and scenario problems as well as coupling point to exogenously defined end-values.

The project brings the FanSi model a substantial step forward when competing for the role as the next generation long-term market model.

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