

A Comprehensive Simulator for Hydropower Investment Decisions

1st Hans Olaf Hågenvik
SINTEF Energy Research
Trondheim, Norway
hans.hagenvik@sintef.no

2nd Siri Mathisen
SINTEF Energy Research
Trondheim, Norway
siri.mathisen@sintef.no

3rd Arild Helseth
SINTEF Energy Research
Trondheim, Norway
arild.helseth@sintef.no

4th Birger Mo
SINTEF Energy Research
Trondheim, Norway
birger.mo@sintef.no

Abstract—Due to a higher share of power production from renewable sources with high short-term variation, hydro systems must more often operate closer to their components' physical limits. To simulate system behaviour, a hydropower system simulator must therefore include most physical details. We present a simulator for hydropower investment analysis that combines a medium-term production planning model based on stochastic dual dynamic programming principles with a detailed and deterministic short-term hydro scheduling model. To reduce computation times, the system description for the short-term model may include only a snipped subset of the plants and reservoirs without deteriorating the results. The simulator is verified in a case study where an investment decision has been analysed for a Norwegian hydropower producer. The combination of medium-term optimization and short-term, detailed simulation is a useful decision support tool and provides both economic results and detailed physical information about the system behaviour.

Index Terms—Investment analysis, Hydropower Scheduling, Simulator, Stochastic Optimization

I. INTRODUCTION

The European electricity system, which Norway and Sweden are parts of, is developing to a system with much more new renewable production, in particular wind and solar production, giving more short-term variation in market prices and an increased need for short-term balancing. To benefit from this development the hydro system must more often operate near its limits and it will consequently be more important to include all the physical detailed properties of the hydro system in investment analysis. This is the motivation for the development of the simulation tool described in this paper. The purpose is to develop a comprehensive simulator that can calculate profits from different investment alternatives where detailed physical properties are important for the evaluation. The simulator should provide results that are as close to optimal as possible with regards to both short and medium-term variations and uncertainties in market prices and inflows. In Scandinavia most hydro production systems along a water course consist of a combination of large and small reservoir systems.

The stochastic dual dynamic programming (SDDP) method is state of the art for solving stochastic reservoir optimization problems, and requires a convex optimization problem to allow for reasonable convergence rates. Both the SDDP convexity

requirement and the presence of multiple uncertainties limit the amount of physical details that can be represented within acceptable computation times. Research is ongoing to improve on the properties that *can* be modelled. Examples typically include linearization and convexifications [1], i.e. the physical problem is simplified to fit the requirements of the SDDP method.

To provide simulation results that are more in line with the physical and judicial reality some SDDP implementations include a final simulation, performed after convergence, where the model properties are changed to be more in line with the plant properties [2]. For example, in the SDDP implementation used in Scandinavia, the maximum discharge limit is made dependent on the reservoir level at the beginning of each decision stage and the final simulation can be done using unit start-up costs. Furthermore, all forward simulations are done for inflow scenarios constructed from historical observations. With these differences between the forward simulation and backward recursion, the final simulation results are closer to the reality, but there is no longer any guarantee that they are optimal (according to the applied strategy).

The simulator described in this paper takes the idea of a final simulation a large step further. With the final set of cuts from the SDDP method as input, an applied short-term hydro scheduling model (SHOP), referred to as the short-term model (STM), is used to optimize the hydro scheduling problem at each decision stage, while providing all details and physical properties, see [3], [4] and [5]. The STM is deterministic and based on a combination of successive linearization and mixed-integer linear programming. To generate a strategy for operation of a hydropower production river system, the medium-term SDDP method ProdRisk is used [6], referred to as the medium-term model (MTM). ProdRisk is in operational use in Scandinavia for medium-term hydro scheduling by most large hydropower producers. The producer is assumed to be a risk-neutral price taker and each river system can therefore be optimized independently. The input price forecasts are typically calculated by the market players using fundamental market models, see [7] and [8]. For the simulator development we assume that the price is given by a set of scenarios for the future. An example of detailed simulation of a system with substantial share of hydropower is presented in [9], where the technical details are emphasized and the hydropower strategy

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is computed by a separate and less detailed procedure. A simulator similar to that presented here but with a different model setup was applied for benchmarking the operation of a hydropower system was documented in [10].

Because of the methodology and because it is used operationally by many customers throughout the Nordics and Central Europe, the MTM includes almost all features needed to give a near implementable solution for any physical hydro production system. A combination of optimization and simulation often gives the best decision support tools, and many generalized tools combine optimization and simulation [11]. The contributions of this paper are twofold: First, we describe a new simulator framework coupling an STM and an MTM model, and thus combining optimization and detailed simulation. The simulator is validated in a realistic case study in the southwestern part of Norway. Second, the concept of system downscaling (or "snipping") is mathematically elaborated and validated in the same case study. We show how computation times can be substantially reduced by snipping the system prior to running the STM. The simulator is especially useful for the profit calculations that are part of an investment analysis.

II. SIMULATOR FRAMEWORK

The production planning simulator presents a unified user interface for a consecutive process, illustrated in Fig.1. As input, it requires a set of static system descriptions for the MTM and the STM, as well as forecasts and updates for inflow and price given as a set of scenarios (typically ≥ 30). The variation between the scenarios corresponds to the sample space that is normally found in the MTM. The MTM produces a strategy (a set of cuts) for the optimization horizon, which is used as input to the STM. The STM then performs an optimized production planning simulation with fine time resolution and physical realistic details for each stage, using the cuts as end value. The output from the simulator are the simulation results from both the MTM and STM.

The simulator is implemented in the Python programming language, using Python APIs of the latest official versions of the STM and MTM. The MTM and STM both rely on a optimization solver to solve linear and mixed integer programming problems. In this work the commercial optimization solver CPLEX is applied [12].

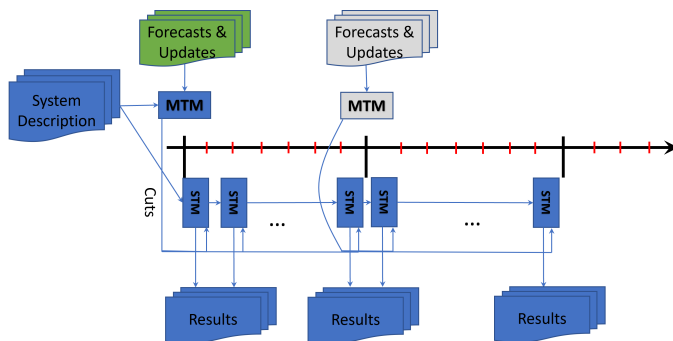


Fig. 1. Simulator Framework

A. Medium-term Scheduling

The medium-term scheduling task solved by the MTM has a planning horizon that is usually between 104 and 208 weeks. In the forward simulations, the recorded (or predicted) inflow and price series are used as input to the model, whereas in the backward recursion, stochastic models describe the inflows and prices [13], [6]. The model uses weekly decision stages, assuming known inflows and prices for each time period of the coming week. For a given market price and stage, the cuts from the MTM have the following form (we refer to [13] for a more thorough description of the cuts and how they are calculated):

$$\alpha + \pi_j^\top (v - v_j^*) + \mu_j^\top (z - z_j^*) \leq \alpha_j^*, \quad j = 1, \dots, J \quad (1)$$

where π_j and μ_j are the cut coefficients, α_j^* , v_j^* and z_j^* are the initial profits, reservoir volumes and inflows, for cut j , α is the future expected profit, v are the reservoir volume variables and z are the inflow variables, which are inputs to the algorithm and known for a given week and scenario.

When the medium-term scheduling finishes, the cuts are stored and made available as input for the short-term scheduling.

B. Short-term Scheduling

The STM solves weekly deterministic decision problems using the cuts from the MTM to represent the future value of water in the reservoir at the end of the week. The STM weekly decision problem differs from the MTM weekly decision problem as it is more detailed and contains features that are simplified in the MTM to reduce computation time and have a convex problem. Once the operation of one week in a simulation scenario has been found using the STM, the end values are used as start values in the next week and the procedure is repeated. This is very similar to how the short-term model is applied for operational decisions. However, when applied for operational decisions, only the decisions for the first day are implemented. The whole model chain would typically be rerun with updated information (prices and inflows, planned revisions etc) the next day. This paper presents a basic version of the simulator, running the MTM only once and thereafter the STM sequentially week by week. This is repeated for all scenarios simulated without any updating of the strategy.

III. SNIPPED SYSTEM

The simulation computation times might be very long for large hydro production systems consisting of more than 30 reservoirs and plants along the same water course, due to the detailed modelling used in the short-term model. Therefore, it can be practical for some analyses to be able to run the simulator for only a snipped part of the whole system. The MTM is run for the whole water course, but the detailed and realistic final simulation with the STM is run on the part of the system directly connected to the investment, which is referred to as a snipped system. Investment type analyses are typically done for many different cases and assumptions about the future and shorter computation times are beneficial. This section

describes how to use the strategy or cuts that are calculated for the whole system in a final detailed simulation for a snipped system. The snipping part is based on the following assumptions:

- All simulation results (reservoirs, discharges) from the MTM are available to the snipped system.
- Discharge/bypass/overflow from modules upstream the snipped system are treated as inflow to the snipped system.
- The simulated reservoir volumes from the MTM are not significantly different from the optimized reservoir volumes from the STM. This can to some extent be controlled by invoking the updates (grey box) in Fig.1.

Assume now that the whole water course consists of N reservoirs. The snipped system consists of M reservoirs, with D reservoirs downstream the snipped system and U reservoirs upstream the snipped system (where $N=M+D+U$), using subscript m to represent a reservoir included in the snipped system, d to represent a reservoir downstream the snipped system and subscript u to represent a reservoir upstream the snipped system. The original cuts for the whole system (1) can be rearranged and described by:

$$\alpha + \pi_{j,u}^\top v_u + \pi_{j,m}^\top v_m + \pi_{j,d}^\top v_d + \mu_{j,u}^\top z_u + \mu_{j,m}^\top z_m + \mu_{j,d}^\top z_d \leq \alpha_j^{**}, \quad (2)$$

where

$$\alpha_j^{**} = \alpha_j^* + \pi_{j,u}^\top v_u^* + \pi_{j,m}^\top v_m^* + \pi_{j,d}^\top v_d^* + \mu_{j,u}^\top z_u^* + \mu_{j,m}^\top z_m^* + \mu_{j,d}^\top z_d^*, \quad j = 1, \dots, J.$$

It is desirable to utilize as much as possible of the information given by the cuts calculated for the whole system when simulating for the snipped system. To do this the simulation results from the MTM are used. The snipped cut description contain MTM simulation results from the reservoirs and therefore become scenario specific. Moving the parts that are known to the right hand side, this gives:

$$\alpha + \pi_{j,m}^\top v_m \leq \alpha_j^{***}, \quad (3)$$

$$\alpha_j^{***} = \alpha_j^{**} - \pi_{j,u}^\top v_{u,sim} - \mu_{j,u}^\top z_u - \pi_{j,d}^\top v_{d,sim} - \mu_{j,d}^\top z_d - \mu_{j,m}^\top z_m.$$

Here v_u , v_d and all inflows z_u , z_m , z_d are assumed known from the full model and can be moved to the right hand side and included in α_j^{***} . The cut represented by (3) has the correct dimension, i.e. only the m reservoirs that are part of the snipped system are included. However, because the downstream reservoirs are removed from the snipped system, the dual values for the short-term problem do not have the correct reference to sea level. The STM shall only decide on decisions from modules m down to the first downstream reservoir(s) that are not part of the snipped system. Equation (3) can be rewritten as follows:

$$\alpha + (\pi_{j,m}^\top - \pi_{j,dm}^\top) v_m + \pi_{j,dm}^\top v_m \leq \alpha_j^{***}. \quad (4)$$

Here $\pi_{j,dm}^\top$ is a vector of cut coefficients representing the first reservoir that is downstream and not part of the snipped system. If the system is a straight cascade of reservoirs, $\pi_{j,dm}^\top$ is a vector with dimension equal to the number of reservoirs in the snipped system, all with the same cut coefficient from the first reservoir downstream that are not part of the snipped system. Again, simulated values from the MTM were used and the part of the cuts that comes from utilisation of water downstream the snipped modules were moved to the right hand side. This gives:

$$\alpha + (\pi_{j,m}^\top - \pi_{j,dm}^\top) v_m \leq \alpha_j^{***} - \pi_{j,dm}^\top v_{m,sim}, \quad (5)$$

where $v_{m,sim}$ are the reservoir storages belonging to the snipped system, simulated in the MTM for the same week and inflow year. The final expression for the reduced cuts of the snipped system is:

$$\alpha + (\pi_{j,m}^\top - \pi_{j,dm}^\top) v_m \leq \alpha^{****}, \quad (6)$$

$$\alpha_j^{****} = \alpha_j^{***} - \pi_{j,dm}^\top v_{m,sim} = \alpha_j^{**} - \mu_{j,u}^\top v_u - \pi_{j,u}^\top v_{u,sim} - \mu_{j,d}^\top v_d - \pi_{j,d}^\top v_{d,sim} - \mu_{j,m}^\top v_m - \pi_{j,dm}^\top v_{m,sim}$$

where $v_{d,sim}$ and $v_{u,sim}$ are the reservoir storages downstream and upstream respectively, simulated in the MTM for the same week and inflow year. In (6), the reduced cuts for the snipped system based on the original cuts from the whole water course are shown. If the STM for the snipped system delivers the exact same reservoir levels as the results from the MTM for the whole system, then the exact same cuts are binding. In that case, the cut description would be superfluous, as fixed end-of-week reservoir volumes could be used in the STM optimization. However, the STM includes more details and it might be beneficial to deviate from the MTM. The reduced cut description allows for this flexibility and utilize as much as possible of the information from the strategy that is optimized for the whole water course.

IV. CASE STUDY

A. Case Description

The simulator is demonstrated in a case study concerning an investment in the Sira-Kvina hydropower system, which is owned and operated by the Sira-Kvina power producing company on the southwest part of Norway. The system consists of multiple reservoirs and power plants along two main branches, which meet in a tunnel junction above Tonstad power plant (see Fig.2). The investment in question is to upgrade the runners of the two reversible pump turbines in the Duge pumped storage plant [14]. The simulator is used to provide profit estimates that may be included in an evaluation

of the profitability of an upgrade. The simulation results may also be used to investigate potential consequences in the rest of the system caused by the upgrade.

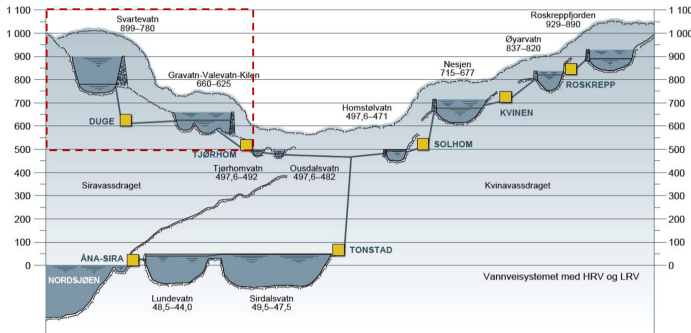


Fig. 2. Sira-Kvina watercourse with framed snipped system. Figure obtained with permission from Sira-Kvina Kraftselskap (2020)

A constraint in the operation of the pump in Duge is of particular interest. Downstream Duge, there are three reservoirs that are connected when the water level is sufficiently high: Gravvatn, Valevatn and Kilen, see Fig.2. The STM models this as two separate reservoirs “Kilen” and “Gravvatn”, where the physical reservoir Valevatn is included in Kilen in the model. The combined tunnel and channel flow from Kilen to Gravvatn is made dependent on the water level in the respective reservoirs, to get a more physically accurate system description. In the MTM such head dependent water flow may not be modelled, and all three reservoirs are therefore modelled as one reservoir. Duge may only pump water from Kilen to Svartevatn when the reservoir level in Kilen is above a minimum level (in this analysis assumed to be 651 masl. throughout the year, although the real constraint description is more complicated). This constraint is not modelled in the MTM, but is included in the STM. Due to this difference, the operation of the pump in Duge is expected to depend on which simulation results are used.

Future inflows are based on the historical period 1958-2017, assuming a 5% increase in inflows due to climate change [15], [16]. The power price forecast is a result of the research done in [17], where the balance between supply and demand in Northern Europe was simulated with a fundamentally based market model using a low-emission scenario for Europe in 2030. As elaborated in Section II, the simulator first runs the MTM for the whole Sira-Kvina hydro production system to produce a strategy. The strategy produced in the MTM is simulated using the STM for two separate cases:

- 1) The whole Sira-Kvina system
- 2) A snipped system containing the reservoirs Svartevatn, Gravvatn and Kilen, and the power plants Duge and Tjørnrom. The area in the snipped system is framed in red in Fig.2.

B. Results

The reservoir operation of *Svartevatn* without the investment, for all three simulation alternatives, is shown in Fig.3.

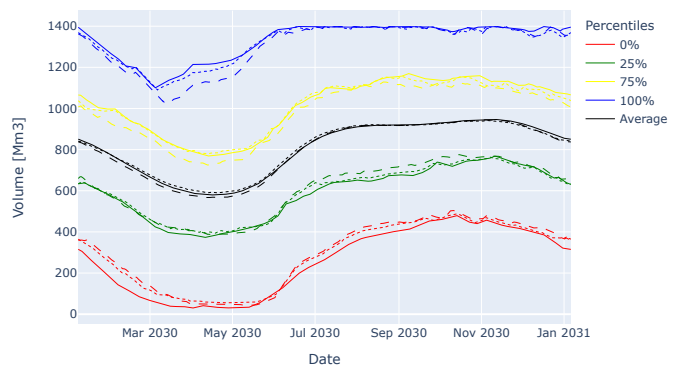


Fig. 3. Reservoir operation of Svartevatn using results from the MTM (solid lines), STM with full model (dashed lines) and STM with snipped model (dotted lines).

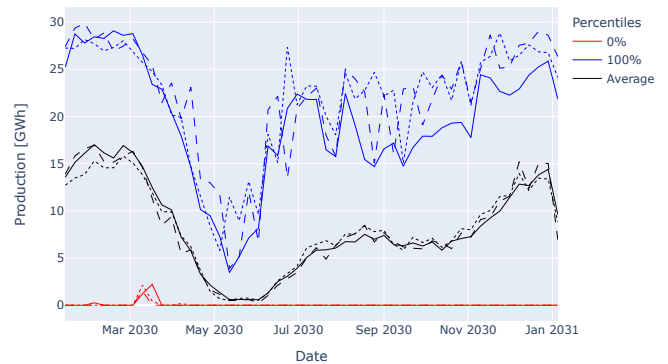


Fig. 4. Production at Duge without the investment, using results from the MTM (solid lines), STM with full model (dashed lines) and STM with snipped model (dotted lines).

The plot shows the 0, 25, 75 and 100 percentiles and the average reservoir volume during 52 weeks. The simulation methods show qualitatively the same results, although there is a perceptibly lower 100-percentile for the STM-results for the full model in April 2030 and lower 0-percentile for the MTM-results in February 2030. The production from *Duge* without the investment is shown for all three simulation alternatives in Fig.4. The plot shows two percentiles and the average production during 52 weeks. The 100-percentile shows a significant difference between the MTM-results and the STM-results after June, but the averages of the simulation methods are closer to one another.

1) *Investment decision:* The main economic results from the simulations with and without the planned investment are presented in Table I. As the results from the snipped STM simulation include results from the snipped area only, these results are compared to the results from the same plants from the simulations using the MTM and the full STM system description. The estimated income increase from the upgrade may be calculated as the difference between the net income of the upgraded compared to the current system.

Based on this calculation, the three different simulation

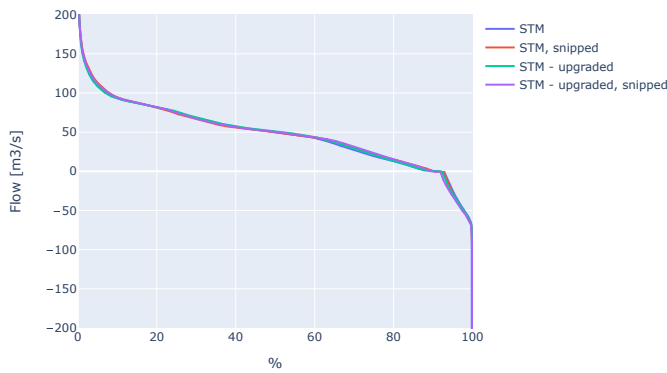


Fig. 5. Duration curve showing the discharge from Kilen to Gravatn relative to the share of time this discharge is kept

methods (MTM, STM and STM with the snipped system description) estimate the increased annual income from the upgrade to be 0.4 MEUR, 1.3 MEUR and 0.4 MEUR, respectively. Table I shows that all three simulations give approximately the same estimate of an increased net income from Duge (0.3 – 0.4 MEUR). The STM simulation also indicates a significantly increased income from the largest power plant Tonstad as a result of the upgrade in Duge. A potential explanation of this increase could be that the upgraded pumping capacity in Duge will provide extra flexibility to the operation of the largest reservoir Svartevatn.

2) *Flow between Kilen and Gravatn*: The flow between Kilen and Gravatn serves as an example of a physical detail that may be modelled in the STM, but that is challenging to model in the MTM (due to the convexity requirement of the SDDP method). According to the power producer, a limited flow does not suppress the daily operation of the nearby power plants significantly. It may still be reassuring that the profit estimates from the simulator includes physical details such as this flow in the calculations. It could also be of interest to evaluate how the given upgrade affects the flow between Kilen and Gravatn. The duration curve for the flow is shown in Fig.5 for both investment alternatives and for both STM simulation alternatives. The plot shows that there is little qualitative difference between the two simulation alternatives. This suggests that if the simulator was run to get estimates of physical details such as this flow, it would in this case be sufficient to run the STM with a snipped system description (thus saving some computation time). The plot also indicates that the flow is not expected to change significantly if the investment in Duge is carried out.

3) *Pump activity in Duge*: Fig.6 is another illustration of how the simulator provides more realistic and physical accurate results for the system operation. The plot shows duration curves for the pump activity in Duge from Kilen to Svartevatn for both investment alternatives and for both STM simulation alternatives. In the STM the two pumping turbines are modelled separately, allowing for pumping with only one turbine ($Q \leq 40$ m³/s) or two turbines ($Q \geq 60$

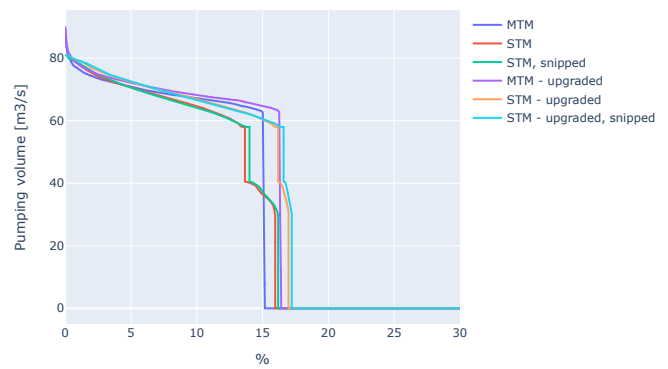


Fig. 6. Duration curve showing the pump activity in Duge.

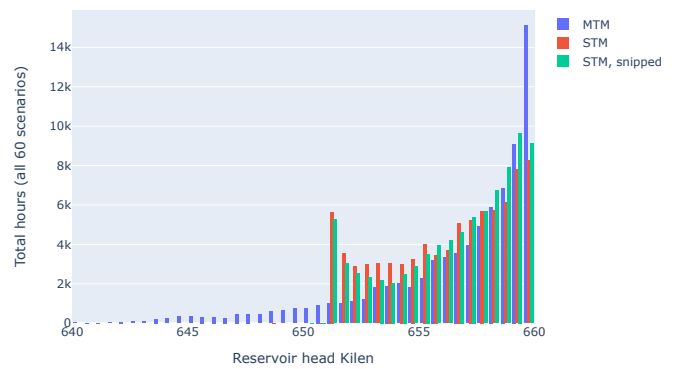


Fig. 7. Histogram of pumping hours at Duge for different reservoir levels in Kilen.

m³/s). The consumption of each turbine depends on pumping volume and head, tunnel loss in the plants main tunnel, the turbines' penstocks and the start-up costs of each turbine. The pump description in the MTM is simpler: The two turbines are modelled as one pumping unit, where maximum pumping volume decreases linearly with pumping head, consuming a constant power if the pump is on. Fig.6 indicates that the snipped STM simulation could be sufficient to show more accurate simulation of the pumping units. All three simulation alternatives indicate the same qualitative (intuitive) result that investing in more efficient pumping turbines will result in increased pump flow in Duge.

Fig.7 shows a histogram of the total number of pumping hours in Duge for different reservoir levels in Kilen, for the three simulations of the current system (without upgrade). The figure shows how the imposed restriction of no pumping if the reservoir level of Kilen is below 651 masl. is taken into account by the STM simulations. The MTM cannot model this restriction due to the convexity requirements in the SDDP.

For this system, the simulator computation time is distributed over the MTM run, lasting 26 hours, and the STM run, lasting 58 hours for the full Sira-Kvina system and 10 hours for the snipped system. The reported run times were from the simulations of the upgraded system, which were run on a

TABLE I
ECONOMIC RESULTS

[MEUR]	Current			Upgraded			Diff ^a		
Plant	MTM	STM	STM snipped	MTM	STM	STM snipped	MTM	STM	STM snipped
Duge production	19.1	18.8	18.9	19.8	19.9	20.0	0.9	1.1	1.1
Duge pumping	-5.0	-5.3	-5.2	-5.6	-6.1	-6.0	-0.5	-0.8	-0.8
Tjørhom	26.1	25.2	26.0	26.2	25.4	26.0	0.1	0.2	0.0
Rosskrepp	5.6	5.6		5.5	5.6		0.0	0.0	0.0
Kvinen	11.7	11.8		11.7	11.8		0.0	0.0	0.0
Solhom	38.3	36.7		38.3	36.7		0.0	0.1	0.0
Tonstad	189.7	185.3		189.6	185.9		-0.1	0.6	0.0
Åna-Sira	30.0	28.8		30.0	28.9		0.0	0.1	0.0
Total start-up costs	0.0	-1.4	-0.4	0.0	-1.4	-0.4	0.0	0.0	0.0
NET INCOME	315.3	305.5	39.2	315.7	306.7	39.6	0.4	1.3	0.4
NET INCOME SNIPPED^b	40.2	38.7	39.6	40.6	39.2	40.0	0.4	0.5	0.4

^a Diff is Upgraded-Current

^b Does not include start-up costs, as they are incomparable for different systems

server with an Intel(R) Xeon(R) Silver 4116 processor with maximum frequency of 2,1 GHz, with two kernels (8 virtual processors) and 16 GB RAM. The MTM was parallelized to run in 7 processes.

V. DISCUSSION AND CONCLUSION

The presented new simulator tool may provide income estimates that are based on more physically accurate modelling in the STM than in the MTM. On the other hand, due to the difference between modelling in the simulation (using the STM) and in the strategy calculation (using the MTM), there is no guarantee of the optimality of the simulation results. The simulated increased income caused by the investment is therefore uncertain. In total, the STM and MTM results supplement each other and the combination provides more information than each model would alone.

The case study indicates that the proposed methodology for simulating only a snipped part of the system using the STM may provide physically accurate results for this snipped part with a significant reduction of computation time compared to an STM simulation of the full system. For some purposes this functionality might thus be useful. In this case, the simulation results from the MTM must be used for the remaining part of the system. Testing indicates that the boundaries for the snipped system seem to be of importance: one should have reservoirs of a satisfactory capacity in each end of the snip to avoid that the MTM results restrain the snipped STM simulations.

Recommended future work is to investigate whether the use of updated cuts from re-runs of the MTM will give improved income predictions. It is suggested that the initial reservoir state may play an important role for certain systems and external conditions (price and inflow). This may be verified either through realistic case studies, or by designed cases where we expect that updating the cuts based on the initial state provides better end value setting for the short term model.

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