Low-complexity Risk-averse MPC for EMS

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***Abstract*— A data-driven stochastic MPC strategy is presented as an EMS for the Skagerak Energilab microgrid. Uncertainties, introduced due to the intermittent nature of RES and load demands, are systematically incorporated into the MPC problem via adaptive chance-constraints. These chance-constraints promote admissible probabilistic operation of the microgrid within the stipulated SOC bounds of an ESS. For computational tractability, these chance-constraints are approximated by solving the inverse cumulative distribution function of a disturbance innovation sequence. This disturbance innovation sequence defines the difference between forecast and realized disturbances, and is sampled for a sliding window as disturbances are revealed over closed-loop operation. No a-prior assumptions are made on the distribution function of the disturbance innovation sequence; instead, solving the Maximum Spacings Estimation problem (off-line), we adapt some parametrized distribution function to fit this disturbance innovation sequence. The proposed strategy has computational complexity comparable to nominal deterministic MPC, promote the satisfaction of constraints in a probabilistic sense, and, decrease closed-loop operational costs by 26%.**

I. Introduction

The Energy Management System (EMS) for microgrids is primarily concerned with the energy balance between generation and consumption in the most efficient manner. This entails managing energy deficits/excess that satisfy both economical and operational criteria; whilst, ensuring stable power delivery to local load consumers [1]. The proliferation of Renewable Energy Sources (RES) within microgrids implies intermittent power generation. To manage this intermittent nature of RES, Energy Storage Systems (ESS) have been advocated as a viable solution [2]. The introduction of ESS, however, increases the computational complexity of the underlying optimal energy balance problem with EMS's. That is, to optimally operate ESS within an EMS, one is often concerned with solving a dynamic optimal control problem over a time horizon of operation.

Model Predictive Control (MPC) is considered a promising optimal control strategy for EMS's that include ESS [3]. MPC defines a receding-horizon control strategy which solves a constrained (deterministic) Optimal Control Problem (OCP) iteratively. The popularity behind MPC is that of anticipating, and counteracting, disturbances over the receding-horizon of operation in order to maximize performance and satisfy system constraints. Although MPC does provide some form of robustness against disturbances, unforeseen large forecast errors may cause the MPC strategy to lose recursive feasibility. That is, for the state at the next time step, there may not exists a feasible control policy [4].

Recently, Robust MPC (RMPC) and Stochastic MPC (SMPC) have received considerable attention due to their attractive ability to incorporate uncertainties in the OCP, in a systematic manner. RMPC methods rely on bounded deterministic descriptions of uncertainties where early work led to min-max OCP formulations, and more recently, tube-based approaches [5]. RMPC has not been widely adopted in industry due to the inherent computational complexity, and the overestimation of disturbances often results in unnecessary performance deterioration [6]. Furthermore, uncertainties for real-world systems are often characterized by their probabilistic nature and does not align with bounded deterministic descriptions. SMPC exploits probabilistic uncertainty descriptions in the form of chance-constraint formulations. The latter either need to be satisfied in the expectation, or at least to some a-prior specified probability level. Key challenges for SMPC are the computational complexity associated with uncertainty propagation through complex system dynamics and cost; and, chance constraints in general results in a non-convex optimization problem whose explicit evaluation may be intractable [7]. Scenario based approaches can be used to approximate and satisfy the chance constraints to provide probabilistic guarantees. The latter strategy is popular in the sense that scenario trees can be constructed from data and the assumption of independence a not imposed; however, it is still unclear how to identify the appropriate number of scenarios [7], [8]. Another common strategy to handle chance constraints is to reformulate stochastic programs as a deterministic problem with constraint tightening. Satisfying these tightened constraints, in a deterministic setting, equates to satisfying the chance constraints in a probabilistic setting. Some strategies calculate constraint tightening policies explicitly by solving the inverse cumulative distribution function (CFD) for a known underlying distribution while other strategies enforce Chebyshev type inequalities to guarantee chance constraint satisfaction (see [9] and references therein).

This work draws inspiration from earlier contributions made in [10] in the sense that we solve the inverse CDF for some underlying distribution to obtain constraint tightening policies. Enforcing these constraint policies in a deterministic setting implicitly satisfy the concerned chance constraints in

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a probabilistic setting. What distinguishes this work from [10] is that we do not assume the distribution to be known a-priori. Instead, in this work we consider the disturbance innovation sequence. In a data-driven approach we fit a distribution function, using Maximum Spacings Estimation [11], to the historical disturbance innovation sequence over a receding horizon, and subsequently update a constraint tightening policy to be applied when we solve the open-loop MPC problem. This, in essence, defines a systematic data-driven approach to seek trade-offs between fulfilling control objectives and satisfying the chance constraints in a probabilistic sense, while adaptively adjusting constraint tightening policies as input disturbances are being revealed during process operation. As case study to illustrate introduced concepts, we consider the Skagerak Energilab pilot [12] which is a fully functional microgrid located in Skien Norway.

II. Theory

We consider the stochastic LTI system

$$x_{k+1} = Ax_k + Bu_k + B_d d_k(\xi) + Gw_k \tag{1a}$$
$$y_k = Cx_k \tag{1b}$$

in which (1a) defines the evolution of the discrete state $x_k \in \mathbb{R}^{n_x}$ primarily driven by: control input $u_k \in \mathbb{R}^{n_u}$; measured input disturbance, $d_k(\xi) \in \mathbb{R}^{n_d}$; and, unmeasured process noise $w_k \in \mathbb{R}^{n_w}$. $d_k(\xi)$ is the disturbance realization for a particular scenario denoted by the index $\xi \in \Xi$, in which the set Ξ defines the representative set of forecast scenarios under which (1a) can evolve. For compact notation, we henceforth adopt d_k to describe measured input disturbances with the a-prior estimate $\hat{d}_k := \mathbb{E}(d_k)$ such that we can define the measured disturbance error as $\Delta d_k := d_k - \hat{d}_k$ for all $k \in \mathbb{I}_{\geqslant 0}$. Stochasticity of (1) for the single process measurement $y_k \in \mathbb{R}$ is defined in (1b). $A \in \mathbb{R}^{n_x \times n_x}$, $B \in \mathbb{R}^{n_x \times n_u}$ and $C \in \mathbb{R}^{1 \times n_x}$ defines the respective process and measurement matrices, whereas $B_d \in \mathbb{R}^{n_x \times n_d}$ and $G \in \mathbb{R}^{n_x \times n_w}$ models the influence of measured and unmeasured disturbances on the system state.

Assumption 1 (Boundedness on innovation uncertainty): The innovation sequence of measured input disturbance errors Δd_k, and system disturbances w_k, are assumed i.i.d zero mean variables with unknown probability distributions being P_d and P_w, respectively. Their CFD's satisfy the generalized inverse distribution function.

Let $\hat{\mathbf{d}}_{N|k} := [\hat{d}_{k|k}, \hat{d}_{k+1|k}, ..., \hat{d}_{N-1|k}]$ define the N-successive measured input disturbance forecasts as predicted from time $t = k$. Then, for some given control input sequence $\mathbf{u}_{N|k} := [u_{k|k}, u_{k+1|k}, ..., u_{N-1|k}]$, we define the uncertain system with state prediction by

$$x_{j+1|k} = Ax_{j|k} + Bu_{j|k} + B_d\hat{d}_{j|k+} + \tilde{G}\tilde{w}_{j|k} \tag{2a}$$
$$y_{j|k} = Cx_{j|k} \tag{2b}$$

for all $j \in \mathbb{I}_{k:k+N-1}$. For compact notation, $x_{j+1|k}$ defines the value of x at future time t_{j+1}, predicted from time t_k and $x_{k|k} = x_k$. In (2a), we define $\tilde{G} := [G, B_d]$ and $\tilde{w}_{j|k} := [w_j, \Delta d_{j|k}]^\top$. As common in SMPC, the predicted state $x_{j|k}$ can be split into its equivalent deterministic nominal part, $z_{j|k} := \mathbb{E}(x_{j|k})$; and, zero mean stochastic error part, $e_{j|k}$ [13] such that

$$x_{j|k} = z_{j|k} + e_{j|k},\ \forall j \in \mathbb{I}_{k:k+N} \tag{3}$$

To simplify optimization, parametrization of the control is widely used when the system of interest is linear [5]. Let $K \in \mathbb{R}^{n_x \times n_u}$ be some stabilizing feedback gain such that $A_{cl} := A + BK$ is Schur. The control parametrization can consider either feedback from the predicted state, or, the predicted error [14]. Resorting to the latter, we adopt the parametrized predicted input

$$u_{j|k} := Ke_{j|k} + v_{j|k},\ \forall j \in \mathbb{I}_{k:k+N-1} \tag{4}$$

where $v_{j|k} \in \mathbb{R}^{n_u}$ becomes the new free SMPC control optimization variable. Substituting (3) and (4) into (2) allows us to define the respective predictions of the nominal state and stochastic error as

$$\begin{aligned} z_{j+1|k} &= Az_{j|k} + Bv_{j|k} + B_d\hat{d}_{j|k},\ z_{0|k} = x_k \\ e_{j+1|k} &= A_{cl}e_{j|k} + \tilde{G}\tilde{w}_j,\ e_{0|k} = 0 \end{aligned} \tag{5}$$

In SMPC, the control objective is to (approximately) minimize the expected value of the quadratic cost over the given N-step prediction horizon i.e.,

$$\mathbb{E}\left(\sum_{j=k}^{k+N-1} x_{j|k}^\top Q x_{j|k} + u_{j|k}^\top R u_{j|k}\right) \tag{6}$$

in which $Q \in \mathbb{R}^{n_x \times n_x}$, $Q \succ 0$ and $R \in \mathbb{R}^{n_u \times n_u}$, $R \succ 0$ are some defined penalty weights on the quadratic terms. From (3), and having $\mathbb{E}(e_{j|k}) = 0$, the deterministic equivalent to the probabilistic cost value (6) can be expressed as

$$J_N(x_k) = \sum_{j=k}^{k+N-1} z_{j|k}^\top Q z_{j|k} + v_{j|k}^\top R v_{j|k} \tag{7}$$

For the stochastic measured output (1b), we concern us with the following inequality constraint

$$\lfloor H \rfloor_c\, y_{j|k} \leq \lfloor h \rfloor_c, \forall j \in \mathbb{I}_{k:k+N-1} \tag{8}$$

to be enforced for the uncertain predicted states (2) in which $c \in \mathbb{I}_{1:n_s}$ being the c-th inequality constraint. The notation $\lfloor H \rfloor_c$ and $\lfloor h \rfloor_c$ define the c-th row and entry of the matrix H and vector h, respectively. Constraint (8) enforced for a stochastic setting, characterized by multiple scenario realizations $\xi \in \Xi$, can either be evaluated by expectation type constraints, or more comprehensively, as chance constraints [14]. In the latter, we consider the following probabilistic chance constraint formulation

$$\mathbb{P}_k\left(\lfloor H \rfloor_c\, y_{j|k} \leq \lfloor h \rfloor_c\right) \geq 1 - \lfloor \epsilon \rfloor_c, \forall j \in \mathbb{I}_{k:k+N-1} \tag{9}$$

which states the probability of not violating the constraint $c \in \mathbb{I}_{1:n_s}$ with a given probability of $\epsilon \in (0, 1]$.

Proposition 1: For the random variable $\lfloor H \rfloor_c Ce_{j|k}, \forall c \in \mathbb{I}_{1:n_s}$, $j \in \mathbb{I}_{k:k+N-1}$ with corresponding CFD function $F_{j|c}$ defined for the j-th prediction stage given the c-th inequality constraint, there $\exists \eta_{j|c} \in \mathbb{R}$ such that the probabilistic chance constraint (9) can equivalently be stated as

$$\lfloor H \rfloor_c Cz_{j|k} \leq \lfloor h \rfloor_c + \eta_{j|c} \tag{10}$$

Proof: The CDF $F_{j|c}$ and its inverse for the random variable $\lfloor H \rfloor_c Ce_{j|k}$ exists by Assumption 1 [13]. Substituting (3) into (9) and grouping stochastic and deterministic sides together yields

$$\mathbb{P}_k \left(\lfloor H \rfloor_c Ce_{j|k} \leq - \lfloor H \rfloor_c Cz_{j|k} + \lfloor h \rfloor_c \right) \geq 1 - \lfloor \epsilon \rfloor_j \tag{11}$$

(11) is equal to stating $\exists \eta_{j|c} \in \mathbb{R}$ such that $\lfloor H \rfloor_C Cz_{j|k} - \lfloor h \rfloor_c \leqslant \eta_{j|c}$ and $\mathbb{P}_k \left(\lfloor H \rfloor_c Ce_{j|k} \leq -\eta_{j|c} \right) \geq 1 - \lfloor \epsilon \rfloor_c$. By the existence of $F_{j|c}$, it follows that from $F_{j|c} \left(-\eta_{j|c} \right) \geq 1 - \lfloor \epsilon \rfloor_c$ we can define $\eta_{j|c} := -F_{j|c}^{-1} \left(1 - \lfloor \epsilon \rfloor_j \right)$. ∎

Based on the nominal cost index (7) (approximate equivalent to the stochastic cost objective (6)), and deterministic constraints (10) (approximate equivalent to the probabilistic chance constraints (9)), the deterministic OCP equivalent to the stochastic approach of interest for this work is formulated as

$$\min_{\mathbf{v}_{N|k}} J_N \left(x_k \right) \tag{12a}$$

$$s.t. \; z_{j+1|k} = Az_{j|k} + Bv_{j|k} + B_d \hat{d}_{j|k} \tag{12b}$$

$$\lfloor H \rfloor_c Cz_{j|k} \leq \lfloor h \rfloor_c + \eta_{j|c} \tag{12c}$$

$$Du_{j|k} \leq g \tag{12d}$$

$$z_{0|k} = x_k, \; \forall j \in \mathbb{I}_{k,k+N-1}, \; c \in \mathbb{I}_{1,n_s} \tag{12e}$$

where in (12c) we define $\eta_{j|c}$ according to Proposition 1. We define the control vector $\mathbf{v}_{N|k} := \left[v_{k|k}, v_{k+1|k}, ..., v_{N-1|k} \right]$ and (12d) defines additional control constraints. The optimal solution of (12) is evaluated by the optimal control vector $\mathbf{v}_{N|k}^0 := \left[v_{k|k}^0, v_{k+1|k}^0, ..., v_{N-1|k}^0 \right]$ where the optimal receding horizon control law at time t_k is defined $\kappa_N(x_k) := v_k^0$. The next time step evolution of (1) using control law $\kappa_N(x_k)$ defines the next evolved system state at time $t = k+1$ as x_t.

Note 1: $\eta_{j|c}$ in (12c) promotes a form of constraint tightening (backoff), such that when satisfied, it implies that (8) is satisfied in a probabilistic sense with a probability of $1 - \epsilon$. For the case where we enforce constraint (8), opposed to (12c), in (12) (i.e., $\eta_{j|c} \to 0$), then (12) reduces to the standard deterministic MPC formulation.

III. Adaptive data-driven chance-constraints

Initial assumptions on the probability distributions of P_d and P_w, characterizing model and exogenous disturbance uncertainty, may impose overly conservative chance constraints. The ambition here is to adapt these constraints (implicitly updating the statistics of P_d and P_w) by incorporating measured disturbance input uncertainties, as revealed over the receding horizon of operation. The strategy is to learn and construct less conservative probabilistic chance constraints that will increasingly promote economic incentives during closed-loop operation.

Note 2: We assume here the initial assumptions on P_d and P_w always results in more conservative constraint tightening in the form of $\eta_{j|c}$. The latter can be artificially introduced by scaling $\alpha \eta_{j|c}$ in which $\alpha(t_0) > 1, \lim_{t \to T} \alpha(t) \to 1$.

We proceed presenting a strategy for adapting some parametrized function approximation of the CFD functions used in (12c). That is, for some parameter matrix $\theta \in \mathbb{R}^{n_p \times (N-1)}$; $\lfloor \theta \rfloor_i \in \mathbb{R}^{n_p}$ defines the $n_p \in \mathbb{I}_{\geqslant 1}$ parameters associated with the parametrized CFD function approximation F_j^θ ($\approx F_{j|c}$) defined for the $j \in \mathbb{I}_{k:k+N-1}$-th stage along the prediction horizon. These parameters are adapted sequentially over a receding horizon of operation as uncertainty reveals itself and taking into account historic open-loop predictions. Let $L := k - N - M + 1$. Suppose at time t_k we have stored M-historical optimal open-loop trajectories $\mathbf{z}_{N|j}^0 := \left[z_j^0, z_{j+1|j}^0, ..., z_{j+N|j}^0 \right]$ and forecast measured input disturbance trajectories $\hat{\mathbf{d}}_{N|j} := \left[\hat{d}_j, \hat{d}_{j+1|j}, ..., \hat{d}_{j+N-1|j} \right]$ for the time interval $j \in \mathbb{I}_{L:k-N+1}$. In addition, consider $M + N - 1$ historical measurements on input disturbances and system states $\mathbf{d}_{L|k} := [d_L, d_{L+1}, ...d_k]$ and $\mathbf{x}_{L|k} := [x_L, x_{L+1}, ..x_k]$ for the time interval $j \in \mathbb{I}_{L:k}$. Then, we can construct the innovation sample sequences $\mathcal{X}_i \in \mathbb{R}^M$, $\forall i \in \mathbb{I}_{0:N}$ (associated with the N respective MPC stages). Each sequence contains M random samples drawn from system state and measured disturbance input observations as revealed over the receding horizon. The i-th sample innovation sequence is defined

$$\mathcal{X}_i := \left[X_{0|i}, X_{1|i}, ..., X_{M-1|i} \right] \tag{13}$$

where the random innovation sample $X_{s|i}$ in (13) is defined

$$X_{s|i} := \tilde{G} \begin{bmatrix} x_{L+i+s} - z_{L+i|L+s}^0 \\ d_{L+i+s} - \hat{d}_{L+i|L+s} \end{bmatrix} \tag{14}$$

for all $i \in \mathbb{I}_{0:N}$, $s \in \mathbb{I}_{0:M-1}$. The parameter vector $\lfloor \theta \rfloor_i \in \mathbb{R}^{n_p}$ associated with the parametrized CFD function approximation F_i^θ can be evaluated by solving the Maximum Spacings Estimator (MSPE) problem [11]. Suppose the random samples of (13) has order statistics denoted by $X_{(0|i)} < X_{(1|i)} < ... < X_{(M-1|i)}$, then the optimal parameters $\lfloor \theta \rfloor_i^0$ supporting the CFD function F_i^θ can be evaluated by minimizing the product of

$$\lfloor \theta \rfloor_i^0 := \operatorname*{argmin}_\theta \prod_{j=0}^{M} \left\{ F_\theta \left(X_{(j|i)} \right) - F_\theta \left(X_{(j-1|i)} \right) \right\} \tag{15}$$

in which $F_\theta \left(X_{(-1|i)} \right) := 0$ and $F_\theta \left(X_{(M|i)} \right) := 1$. Solving $(N+1)$ optimization problems of the form (15) provides and updated parametric matrix θ which can be incorporated into the chance constraints formulation of (12).

IV. Skagerak Energilab

As case study for illustrating the concepts introduced in Sections II-III, we consider the Skagerak Energilab pilot located in Skien, Norway. Skagerak Energilab is a fully

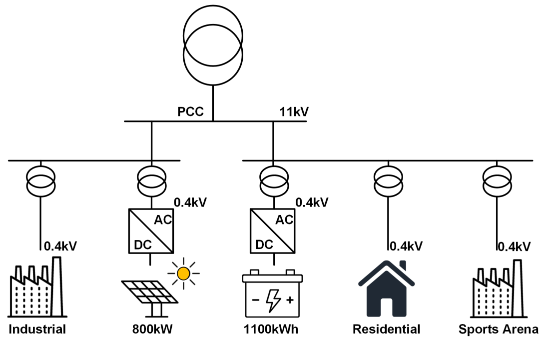

Fig. 1. Skagerak Energilab with adjacent distribution network in Skien, Norway.

functional microgrid pilot installation for local production, storage and distribution of electrical energy. In addition, Skagerak Energilab has a strong research prerogative which allows the validation and testing of new technologies and services [12]. In general, the control operational objective for intelligent energy management within this pilot is to utilize BESS capacity as means to effective load shifting given uncertain load demands and distributed RES. The network, as illustrated by Figure 1, consists of two segments supplied by two different distribution transformers connected to the upstream network. In the first segment, there is a PV plant with peak installed capacity of 800 kW and a large industrial demand. In the second segment, the BESS is connected with two other load demands, one being residential loads and the other being the Skagerak Arena football stadium. The BESS has a capacity of 1100 kWh and 800 kW peak charging and discharging power. We denote the produced photo-voltaic power as P_{PV}. Power consumption and production during charging and discharging of the BESS is denoted P_{Bc} and P_{Bd}, respectively. The net energy flow of the BESS is subsequently defined by

$$P_B = P_{Bc} - P_{Bd};\ P_{Bc} \geqslant 0,\ P_{Bd} \geqslant 0 \tag{16}$$

The industrial load associated with segment 1 is defined by P_{L1} where as the combined residential and sports arena loads associated with segment 2 is P_{L2}. The net energy balance of concern is formulated as

$$P_{PCC} + P_{PV} - P_B - P_{L1} - P_{L2} = 0 \tag{17}$$

The state-of-charge (SOC) of the BESS dynamics are defined by the following discrete ordinary differential equation

$$SOC_{k+1} = SOC_k + \eta_a P_{Bc_k} - \eta_b P_{Bd_k} \tag{18}$$

in which $\eta_a := \frac{\eta_{Bc}\Delta t}{C_{\max}}$ and $\eta_b := \frac{\Delta t}{\eta_{Bd} C_{\max}}$. $\eta_{Bc} = 0.82$ and $\eta_{Bd} = 0.86$ defines the charging and discharging efficiency coefficients; $C_{max} = 1100$ kWh is the maximum energy capacity and $\Delta t = 1$ h the charging time constant. To translate the previous problem description into the formulation of (12), we adopt the notation for the system state $x := SOC$; for control vector $u := [u_1, u_2, u_3]^\top$, we consider control variables $u_1 := P_{PCC}$, $u_2 := P_{Bc}$, $u_3 := P_{Bd}$; and, for vector $d := [d_1, d_2, d_3]^\top$ the measured disturbance input variables are $d_1 := P_{L1}$, $d_2 := P_{L2}$ and $d_3 := P_{PV}$. For the latter, we assume a-prior estimates to be defined $\hat{d}_i := \mathbb{E}(d_i)\ \forall i \in \{1,2,3\}$ such that $\Delta d_i := d_i - \hat{d}_i \backsim \mathcal{N}(0, \sigma_i^2)\ \forall i \in \{1,2,3\}$ defines the normally distributed measured disturbance errors with σ_i being the standard deviation Δd_i. Operational objectives for this case is associated with minimizing operational costs during BESS operation as well as import or selling energy from the grid via PCC. The value function of interest is defined

$$J_N(x_k) = \sum_{j=k}^{k+N-1} u_{j|k}^\top R u_{j|k} \tag{19}$$

in which $R \succ 0$, when chosen appropriately[1], will penalize BESS degradation cost and numerically prevent simultaneous charge/discharge while also reduce reliance on energy import/export via the PCC. Although the structure of (19) differs from (7), it does not change the generality of translating the probabilistic cost evaluation of the former into its deterministic equivalent. For the state dynamics (12b) we define $B := \begin{bmatrix} \eta_b & \eta_a - \eta_b & 0 \end{bmatrix}$, $B_d := \begin{bmatrix} -\eta_b & -\eta_b & \eta_b \end{bmatrix}$ and system matrix $A = 1$. For the chance constraint (12c) we have $C = 1$ and critical constraints preventing over and under charging the BESS, dictates we define $H := [1, -1]^\top$ and $h := [SOC_{max}, -SOC_{min}]^\top$. Constraints on control inputs (12d) let us define the diagonal block matrix $D := \text{diag}([1,-1]^\top, [1,-1]^\top, [1,-1]^\top)$ and $g := [PCC_{max}, -PCC_{min}, Bc_{max}, 0, Bd_{max}, 0]^\top - Ke$.

V. Performance assessment and results

We are interested assessing the closed-loop performance for two variants of (12), iteratively solved over a receding horizon of operation. For both variants, we choose $N = 24$. In the context of SMPC with probabilistic chance constraints (here denoted problem $\mathbf{P}_1$), we have adaptive constraint tightening by means of enforcing (12c) and choosing $\eta_{j|c}$ according to Proposition 1. This constraint implicitly satisfies the conditions under which the probabilistic chance constraints (9) are satisfied subject to the probability of $1 - \epsilon$. For $\mathbf{P}_1$, we choose $M = 200$ and $\epsilon = 0.2$ where the latter is not a very restrictive probability on constraint violations but instead a probabilistic incentive to operate the process within feasible SOC boundaries 80% of the time. The second problem of interest, denoted $\mathbf{P}_2$, does not consider probabilistic chance constraints; hence, by setting $\eta_{j|c} \to 0$ in (12c), $\mathbf{P}_2$ is reduced to a version similar to standard nominal deterministic MPC where we are only concerned in adhering to hard output constraints as formulated by (8). Probabilistic forecast models have been trained on historical power generation and load demand data, as obtained from the Skagerak Energilab pilot [15]. These Markov process models allow sampling a forecast prediction $d_k(\xi)$ for a given scenario $\xi \in \Xi$. We resort to defining $\hat{d}_k := \mathbb{E}(d_k(\Xi))$ for problems $\mathbf{P}_1$ and $\mathbf{P}_2$ given some representative set of forecast

[1] choosing non-zero off-diagonal elements corresponding to u_2 and u_3 will prevent simultaneous BESS charge/discharge during the optimal evaluation of (12)

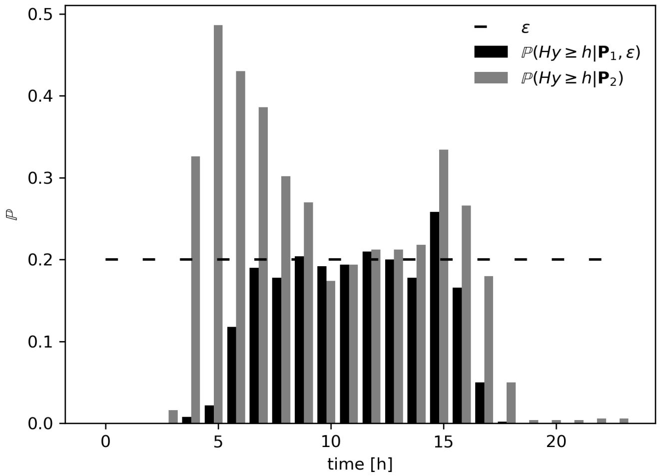

Fig. 2. Probability of violating the BESS SOC constraint (9) for closed-loop $\mathbf{P}_1$ and $\mathbf{P}_2$ operation, given the violation probability of $\epsilon = 0.2$ chosen for the SMPC chance constraint (9)

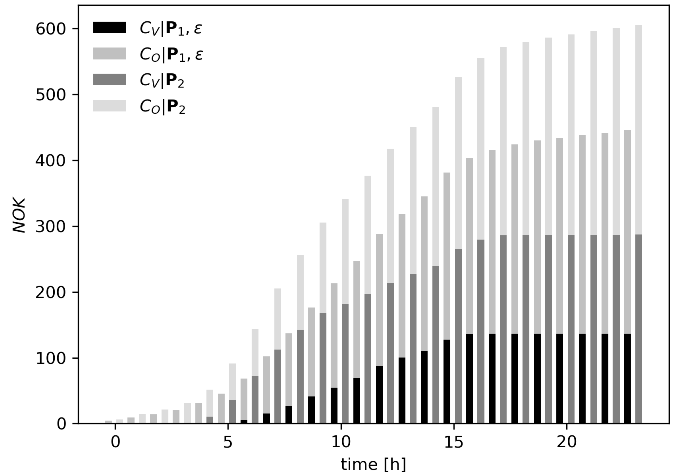

Fig. 3. Cumulative average closed-loop cost (stacked) associated with uncertainty violation (C_V), and, operational cost C_O.

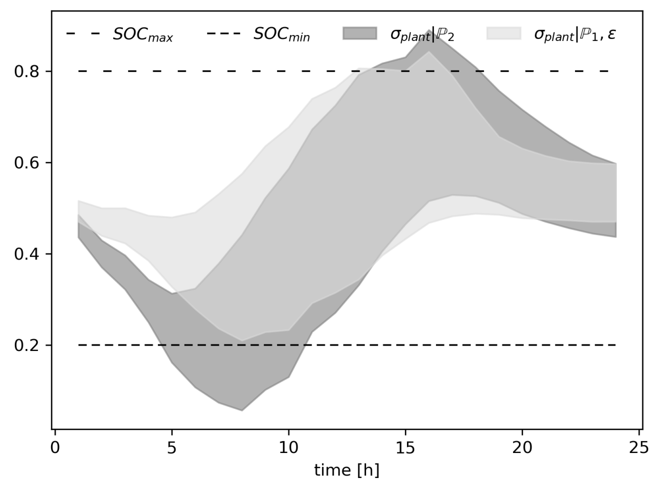

Fig. 4. Expected operational envelope for closed-loop plant evolution, evaluated for the optimal control law $\kappa_N(x_k)$ obtained from problems $\mathbf{P}_1$ and $\mathbf{P}_2$, subject to forecast scenarios $\xi \in \Xi$.

scenarios, Ξ. For performance assessment, Key Performance Indicators (KPI)'s, in general, will be related to the probability of violating output process constraints (SOC for the BESS under consideration), and cost associated with the closed-loop operation for $\mathbf{P}_1$ and $\mathbf{P}_2$. The KPI's considered here can be summarized as: (i) for a sampled set of disturbance scenarios $\xi \in \Xi$, the probability $\mathbb{P}$ of violating BESS SOC constraints, when for a forecast scenario $d_k(\xi)$ driving (1a), we have that $\hat{d}_k := \mathbb{E}(d_k(\Xi)) \neq d_k(\xi),\ \forall \xi \in \Xi$; (ii) violation of SOC constraints implies overcharging/undercharging and will incur operational cost penalties in terms of: (a) increased battery degradation cost; (b) potential load shedding (cost associated with consumer discomfort); (c) penalty cost on importing/exporting energy from/to the grid not cleared by utilities. For (ii), we adopt the following cost structure to evaluate constraint violation observed during closed-loop operation:

$$C_V := \begin{cases} \alpha P_{Bd}^{\text{recourse}} & SOC > SOC_{max} \\ \alpha P_{Bc}^{\text{recourse}} & SOC < SOC_{min} \end{cases} \tag{20}$$

$\alpha \gg \|R\|_\infty$ is the penalty cost coefficient for violating constraints, and $P_{Bd}^{\text{recourse}} := \frac{SOC - SOC_{max}}{\eta_b}$ and $P_{Bc}^{\text{recourse}} := \frac{SOC_{min} - SOC}{\eta_a}$ define the corrective recourse battery discharge/charge power flow. These recourse actions are corrections taken outside the MPC to ensure a feasible SOC operation prior to solving $\mathbf{P}_1$ and $\mathbf{P}_2$ at time t_{k+1}. For the last KPI of interest: (iii) defines operational cost associated with normal battery degradation and grid import/export defined $C_O := [P_{PCC}, P_{Bc}, P_{Bd}]^\top R\,[P_{PCC}, P_{Bc}, P_{Bd}]$.

Figure 2 illustrates the probability of violating SOC constraints during closed-loop operation for problems $\mathbf{P}_1$ and $\mathbf{P}_2$ when evaluated for 500 scenarios, respectively. In reference to Figure 4, process evolution near the SOC boundaries are observed during the morning and afternoon sessions. For adaptive constraint tightening in $\mathbf{P}_1$, we note that for all considered scenarios $\xi \in \Xi$, we do not violate SOC constraints more than the specified $\eta = 0.2$ probability. In contrast, the standard MPC formulation $\mathbf{P}_2$ does not enforce constraint tightening with almost 50% of scenarios leading to constraint violations.

Figure 3 illustrates the average cumulative cost as evaluated during closed-loop operation for problems $\mathbf{P}_1$ and $\mathbf{P}_2$. For both problems, we have evaluated: i) cumulative cost penalties associated with violating constraints, C_V (see (20)), given the set of disturbance predictions $d_k(\Xi)$) realized over the receding horizon; and, ii) the operational cost, C_O, which in this context is normal BESS degradation costs and costs/revenue from importing/exporting from/to the grid. The cumulative operation and uncertainty cost have been stacked to get a better cost performance comparison between $\mathbf{P}_1$ and $\mathbf{P}_2$. We note that for $\mathbf{P}_1$, constraint tightening in the chance constraint formulation implies naturally that costs associated with violating constraints are lower, however at the trade-off that operational costs are increased. The latter is due to that more energy needs to be imported/sold and the BESS are operated more conservatively. This trade-off, irrespective of this conservative strategy, still implies lower total average cumulative costs evaluated for $\mathbf{P}_1$ when compared to $\mathbf{P}_2$.

Figure 4 depicts the operational envelope of the plant (1) when operated in closed-loop control for all considered scenarios $\xi \in \Xi$. The envelopes, for the respective MPC strategies $\mathbf{P}_1$ and $\mathbf{P}_2$, encapsulates all the closed-loop trajectories evaluated for corresponding receding horizon control obtained by solving $\mathbf{P}_1$ and $\mathbf{P}_2$, over all input disturbance realizations defined for the set $d_k(\Xi)$.

VI. Concluding remarks

MPC is now consider as a mature control strategy which have widely been adopted in industry. Despite well established theoretical results and insights in the MPC academic literature, the systematic treatment of uncertainty in MPC by means of RMPC and SMPC strategies still necessitate further investigations to be considered viable and pragmatic solutions for general uncertain processes in industry [16]. RMPC may promote overly conservative control laws due to uncertainties that are often overly overestimated. SMPC, in contrast, systematically treat uncertainties in a probabilistic sense by including chance constraints in the underlying optimal control problem. Introducing chance constraints may in general promote non-convex intractable formulations, hence the formulation of computationally tractable methods to propagate uncertainty is essential to the underlying success of SMPC. Sample based approaches used for approximating the stochastic optimization have received considerable attention due to the natural appeal to promote a data driven method for constructing representative scenario trees that capture uncertainty. Despite extensive work done in scenario based MPC for uncertain systems, the primary challenge for the latter remains in how to appropriately select, or reduce, the set of scenarios that not only guarantee an admissible level of constraint satisfaction, but also promote computationally tractable solutions [7].

In this work, a computationally tractable SMPC strategy is presented. This strategy entails a data-driven chance constraint formulation where the latter is adapted on-line. Adaptation is done by incorporating a sliding window effect over a realized innovation sequence of historical measured disturbance samples and subsequently solves the MSPE problem given this window of samples. This approach removes the need to implement some heuristic for scenario reduction. Instead, the user can default to a conservative sampling based approach of scenarios (i.e., expected mean over a set of scenarios realizations $\hat{d}_k = \mathbb{E}(\Xi)$) and by observing the innovation sequence generated during uncertainty realization, one can systematically update the probabilistic uncertainty representation in the underling problem by adapting the chance constraints appropriately. This, among other, addresses the biased uncertainty representation introduced due to conservative sampling of scenarios. Calculations involved for updating the chance constraints can be done off-line, therefore not adding any computational complexity to the underlying MPC problem. The computational complexity of the proposed SMPC is similar to that of standard deterministic MPC problem. The proposed strategy has been validated by simulation (given measurement data taken from the Skagerak Energilab) where it has been shown that constraint violation of the ESS SOC can be ameliorated in a probabilistic sense to some a-prior degree of probability.

Future work considered for the current strategy would be to extend the individual chance constraint framework to that of a joint chance constraints. The former are of particular interest where one is typically concerned with multiple ESS in a microgrid setting. Although the computational complexity for solving the MSPE problem (15) is low, an interesting question would be how to reduce the number of historical samples used during this problem and instead use methods within reinforcement learning to adapt the underlying parameters between successive solutions of (15).

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