Coordinated Voltage Support with Reactive Power from High-power Charging Stations for EVs

Rubi Rana
Dept. of Energy Systems
SINTEF Energy Research
Trondheim, Norway
rubi.rana@sintef.no

Bendik Nybakk Torsæter
Dept. of Energy Systems
SINTEF Energy Research
Trondheim, Norway
bendik.torsater@sintef.no

***Abstract*—High-power charging stations have recently drawn the attention of many researchers and electric vehicle (EV) infrastructure industries. However, the installation of fast chargers at various corridors of highways and cities can cause high peak loads and voltage deviations in distribution networks. In addition, the usage patterns can vary drastically from one fast charging station to another at any given instant. However, future high-power charging stations could be able to operate in any of the four P-Q quadrants. Thus, high-power charging stations for EVs have the capability of serving as a flexibility resource and minimising voltage deviations. This research introduces a method for mitigating voltage quality problems in distribution network by effective utilisation of the reactive power potential from fast charging stations. The suggested method introduces a second-order cone programming approach that is validated on the IEEE 69 bus distribution system. The performance of the method is analysed in a case study with measured charging profiles from real high-power charging stations in Norway.**

***Index Terms*—Fast charging station (FCS), Electric vehicle (EV), Reactive power, Voltage control, High-power charging**

I. Introduction

A network of fast charging stations (FCS) along major highways can provide a quick and convenient option to electric vehicle (EV) owners to replenish their EV's battery. The charging times of these high-power chargers could in the future be nearly equivalent to the duration of regular fuel stops [1]. As of today, an EV (electric car) with state-of-the-art battery technology could have a full recharge at an FCS in less than thirty minutes.

The sudden and high energy demand of FCS will possibly have a noticeable impact on the power quality in distribution grids. High-power chargers are being installed all over the world, in areas such as highway rest areas and convenient city refueling points. The aggregated load of an FCS could together with the base load in the grid create voltage drops and increase the load peaks during the day.

EVs with bidirectional on-board chargers have great potential to help with both frequency regulation and voltage control problems, as they can operate in any of the four P-Q quadrants. Thus, they have the ability to effectively consume/provide reactive power locally. Restrepo *et al.* [2] demonstrated that the reactive power capacity of EV chargers improves the operation of distribution feeders. The authors in [3] demonstrates the benefits of dispatching reactive power from on-board EV chargers to prevent undervoltage issues. Moreover, the authors consider EVs operating in the first and fourth P–Q quadrant, thus they can inject/withdraw reactive power while charging. Wang *et al* [4] used the on-board EV charger and coordinated the four quadrant operation of EVs and the distribution feeder to support voltage control in the grid.

The authors in [5] discuss off-board four quadrant EV chargers that have an independent and bidirectional reactive power control at the interface with the grid. EV chargers are often composed of a voltage converter circuit. Such circuits can adjust injected/absorbed reactive power to/from the grid by controlling the magnitude and phase angle of the ac-dc converter of the charger [6].

To investigate the grid impact and to see the flexibility of chargers in reactive power support, an optimisation framework that can solve power flow problems is needed. Power flow equations are quadratic and hence optimal power flow (OPF) can be formulated as a quadratically constrained quadratic program (QCQP). It is generally non-convex and non deterministic polynomial-time hard (NP-hard). There exists a rich literature of convex relaxations of power flow equations. They include second-order cone (SOC) [7], the semi-definite programming (SDP) [8], Convex-DistFlow (CDF) [9], and the quadratic convex (QC) [10]. However, out of the above-mentioned algorithms, the second-order cone programming method is widely accepted in power system applications such as; switching of shunt capacitors for minimisation of total power losses [11], voltage constraints management [12] and loss minimisation in distribution systems [13]. In [14], the authors proved that the second-order cone programming is computationally simpler than semi-definite programming, and also it fits well in the problems involving convex quadratic functions [15].

A. Motivation

In [3]-[6], the reactive power injection capabilities of EV chargers are explored and the benefits in terms of grid support are demonstrated. However, voltage control using high-power

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EV chargers is not explored in literature, as slow charging of EVs has been most prevalent up until today. The existing literature considers the flexibility in both active and reactive power during EV slow charging, as slow charging EVs usually have the flexibility to charge for a longer period of time. However, when it comes to high-power charging of EVs, the primary focus is to charge the EVs as fast as possible leaving little or no scope for delayed charging. Furthermore, the aggregated load demand of FCSs will vary depending on the location, day of the week and time of the day. Considering the variety of constraints with fast chargers to solve the optimal power flow models, second-order cone programming [11]-[15] appears to be a good option.

In this work, an optimisation framework that focuses on the reactive power dispatch from FCSs, while considering the spatial and temporal characteristics of the fast charging stations, is developed. Moreover, the proposed method is based on a second-order cone programming power flow model, which has the advantage of being amenable to integration of objective functions. Thus, by incorporating minimisation of voltage deviations as an objective function, a voltage support strategy for distribution networks from FCSs can be modelled.

B. Contributions and Structure

The major contributions in this paper are as follows:

- Analysis of the grid impact from FCSs in different locations, using actual measured load profiles from FCSs in Norway. In this paper, six FCSs are connected to the same radial distribution system.
- Method for modelling the load flow problem of a radial distribution system as a second-order cone optimisation problem with minimisation of voltage deviations as an objective function.
- An analysis of the coordinated operation of FCSs at different locations with and without charging occupancy, using actual measured load profiles from FCSs.

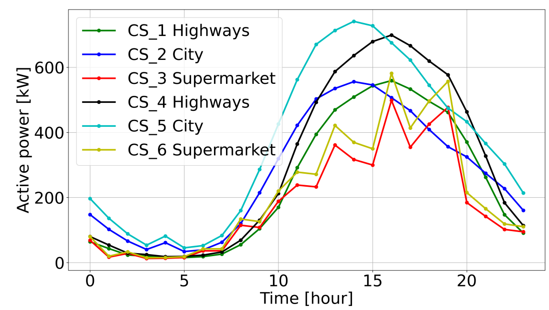
The paper has the following structure; the proposed methodology and voltage regulation strategy are elaborated in Section II, the simulation results and discussion are presented in Section III and Section IV, respectively, and the conclusion and suggested further work are presented in Section V.

II. Methodology

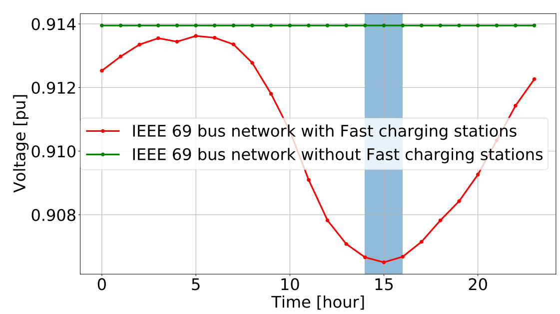
This section introduces the proposed methodology and discusses the grid impact from high-power charging stations.

A. Analysis of measurement data from FCS for EVs

Fig. 1(a) shows the hourly, average charging profile of six FCSs during weekend days in the year 2018. The profiles are based on active power measurements (kWh/h) from real high-power charging stations in Norway. The figure shows the differences in charging behaviour depending on the location of the FCSs. The FCS located near a highway (CS_1 and CS_4) has 8 chargers with a capacity of 125 kW per charger. The demand profiles are higher in weekends due to higher intercity travelling. The peak load in the weekend is stretched over several hours, which could be because EV users are travelling for larger distances in the weekends. The FCS located in a city (CS_2 and CS_5) has 20 chargers with a capacity of 50 kW per charger. The demand is lower due to less intracity travellers. The FCS located next to a supermarket (CS_3 and CS_6) has 7 charges with a capacity of 150 kW each. During the weekend, the demand is lower, but more evenly distributed over a period, from 13h to 20h. The charging hours are coinciding with normal shopping behaviour.



(a) Average load profiles for high-power charging stations during weekend days - Based on measurements from real FCSs in Norway



(b) Voltage drop at the vulnerable Bus 65 of IEEE 69 bus network

Fig. 1: Grid impact from different fast charging stations for EVs

B. Impact of high-power charging stations on the grid

In this section, the grid impact from high-power charging stations is discussed. In order to showcase this, six FCSs are connected to the IEEE 69 bus network [16], as shown in Fig. 2, and the voltage profiles are being observed.

From Fig. 1(a), it can be seen that the city FCS, highway FCS and supermarket FCS have consecutive peak loads at the 14th, 15th and 16th hour, respectively. As a result, voltage drops can be seen across these hours. Fig. 1(b) shows the voltage profile of the most vulnerable bus in the IEEE 69 bus system, where the highlighted hours correspond to the load peak of the FCSs. To reduce the grid impact caused by these FCSs, it is interesting to evaluate ancillary services such as reactive power support from the same FCSs. To investigate this, a voltage regulation strategy for FCSs in distribution networks is proposed in this paper.

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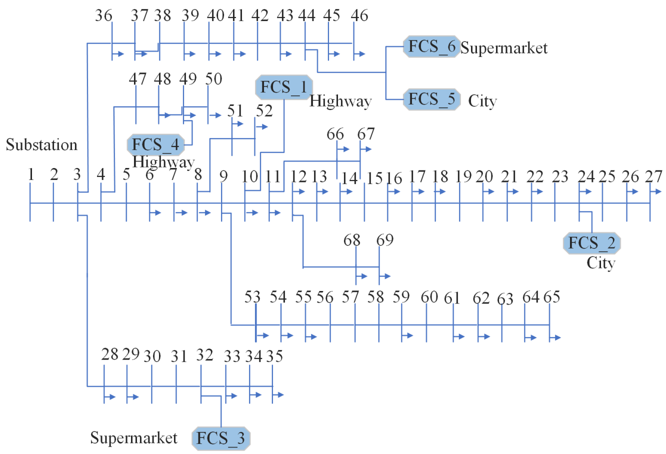

Fig. 2: IEEE 69 bus network with high-power charging stations

C. Optimisation Model for Voltage Support

In this section, the mathematical formulation required to develop an optimal voltage support strategy for FCSs is discussed. The proposed idea is to integrate the optimal voltage regulation problem with the load flow problem in a single optimisation framework. The author in [7] has modelled the load flow problem as a conic optimisation problem, which is more amenable to integration within an optimisation function. Thus, motivated from [7], the objective function in this work is formulated with a minimisation of voltage deviations, as given in (1). The formulation is such that the total voltage deviation, i.e. the difference between a voltage reference and an estimated value, is minimised. In this paper, the reference voltage is considered as 1 pu. The optimisation time horizon is 24 hours, and each time step (t) has a duration of 1 hour.

$$min \sum_{t\in\Omega_T} \sum_{i\in\Omega_b} (\sqrt{2} * u_{i,t} - 1)^2 \tag{1}$$

s.t.

$$\sum_{j\in k(i)} P_{ij,t} = -\sqrt{2}u_{i,t} \sum_{j\in k(i)} G_{ij} + \sum_{j\in k(i)} (G_{ij}R_{ij,t} - B_{ij}I_{ij,t})$$
$$= P_{d_{i,t}} + \sum_{cs\in m(i)} P_{cs,t} \forall\ i \in (2, ..., N_B), \forall\ t \in \Omega_T \tag{2}$$

$$\sum_{j\in k(i)} Q_{ij,t} = -\sqrt{2}u_{i,t} \sum_{j\in k(i)} B_{ij} + \sum_{j\in k(i)} (B_{ij}R_{ij,t} + G_{ij}I_{ij,t})$$
$$= Q_{d_{i,t}} + \sum_{cs\in m(i)} Q_{cs,t} \forall\ i \in (2, ..., N_B), \forall\ t \in \Omega_T \tag{3}$$

$$P_{ij,t} = V_{i,t}^2 G_{ij} - G_{ij} V_{i,t} V_{j,t} \cos\theta_{ij,t} + B_{ij} V_{i,t} V_{j,t} \sin\theta_{ij,t} \tag{4}$$
$$Q_{ij,t} = V_{i,t}^2 B_{ij} - B_{ij} V_{i,t} V_{j,t} \cos\theta_{ij} - G_{ij} V_{i,t} V_{j,t} \sin\theta_{ij,t} \tag{5}$$
$$V_{i,t}^2 = \sqrt{2}u_{i,t} \tag{6}$$
$$R_{ij,t} = V_{i,t} V_{j,t} \cos\theta_{ij,t} \tag{7}$$
$$I_{ij,t} = V_{i,t} V_{j,t} \sin\theta_{ij,t} \tag{8}$$

The constraints in (2) and (3) are the power injection constraints, derived from (4)-(8), where (4) and (5) denote the active and reactive power flows from node i to node j, respectively. (6), (7) and (8) denote the intermediate variables to convert the non-convex power flow problem to a convex power flow problem. Moreover, in (2) and (3), $k(i)$ denotes the set of nodes connected to node i. G_{ij} and B_{ij} are the real and imaginary parts of admittance. N_B is the total number of buses. Ω_T and Ω_b are the set of time intervals and buses respectively. $P_{d_{i,t}}$ and $Q_{d_{i,t}}$ denote the active and reactive power loads at node i, respectively. Moreover, $P_{cs,t}$ and $Q_{cs,t}$ denote the active and reactive power of the charging stations. $m(i)$ denotes the set of charging stations connected to node i.

To convert the problem to a second-order cone format, two auxiliary variables $R_{ij,t}$ and $I_{ij,t}$ are introduced as defined in (7) and (8) respectively. These variables are constrained with $u_{i,t}$ and $u_{j,t}$ and the rotated quadratic cone is formed as given in (9).

$$2u_{i,t}u_{j,t} \geq R_{ij,t}^2 + I_{ij,t}^2 \tag{9}$$

The constraint given in (10) is the voltage level of the substation bus, i.e. bus 1.

$$u_{1,t} = \frac{V_{1,t}^2}{\sqrt{2}} \tag{10}$$

The constraint in (11) ensures the positive real numbers.

$$R_{ij,t} \geq 0 \tag{11}$$

The constraint given in (12) is a nodal voltage constraint, which ensures that the voltage level must be within the maximum and minimum voltage limits of the distribution system:

$$V_{i,min} \leq V_{i,t} \leq V_{i,max} \tag{12}$$

$V_{i,min}$ and $V_{i,max}$ are the minimum and maximum allowable voltage limits in the distribution network, respectively. In this work, the considered IEEE 69 bus network has predefined minimum and maximum voltage limits of 0.9 and 1.1 pu respectively.

The constraint given in (13) is the branch current constraint, which ensures the current magnitude of each branch must lie within their allowable ranges.

$$0 \leq (G_{ij}^2 + B_{ij}^2)(\sqrt{2}u_{i,t} + \sqrt{2}u_{j,t} - 2R_{ij,t}) \leq I_{ij,max}^2 \tag{13}$$

D. Bidirectional FCS constraints

The schematic of the proposed work is given in Fig. 3. As shown in the figure, there are bidirectional off-board chargers installed at the FCS. The active and reactive power control of the charger is related to the capability curve of the charger, as illustrated in Fig. 3. The purpose of an FCS is to provide the highest amount of active power that an EV can accept. Reactive power, however, can be controlled without affecting the charging process of the EVs. Thus, in this work, the bidirectional chargers at the FCS are operating in quadrant I and IV. This means that reactive power can flow in both directions, while active power flow is considered to be unidirectional and controlled by the connected EVs.

The aim of the proposed work is to model the grid aspect,

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thus, it does not focus on modelling of individual chargers. Therefore, the apparent power capacity of the individual high-power chargers (converters) are aggregated into one single capacity. The aggregated apparent power rating of all the FCSs considered in this work is 1000 KVA. The reactive power available for voltage support from FCSs are limited by the apparent power rating of the FCSs S_{cs}, as well as the power consumption of the EVs, $P_{cs,t}$. $Q^{min}_{cs,t}$ and $Q^{max}_{cs,t}$ denote the allowable reactive power injection capability of the fast charging stations as given in (14) and (15).

$$Q^{min}_{cs,t} \leq -\sqrt{S^2_{cs} - P^2_{cs,t}} \tag{14}$$

$$Q^{max}_{cs,t} \leq \sqrt{S^2_{cs} - P^2_{cs,t}} \tag{15}$$

Thus, the constraint in (16) ensures that the optimal dispatch signals, $Q^{opt}_{cs,t}$, is within the reactive power capability of the FCSs. $Q^{opt}_{cs,t}$ is the optimal reactive power dispatched to the individual FCS, as shown in Fig. 3.

$$Q^{min}_{cs,t} \leq Q^{opt}_{cs,t} \leq Q^{max}_{cs,t} \tag{16}$$

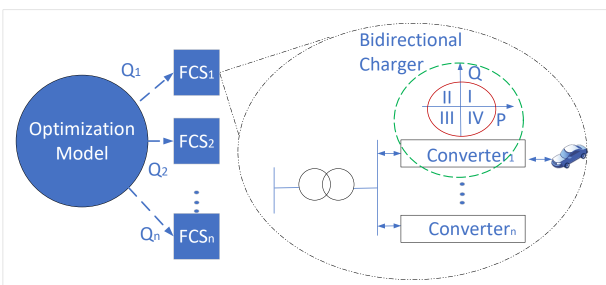


Fig. 3: Schematic Framework

III. Results

To validate the proposed model, the IEEE 69 bus radial network, as shown in Fig. 2, is considered. Six high-power charging stations are installed at the buses 10, 24, 32, 49 and 44. The weekend load data is gathered from 6 FCSs installed along a highway, in a city and at a supermarket in Norway, as shown in Fig. 1(a) and discussed in II-A. To simulate the coordinated operation of FCSs at different demographic locations, the city FCS and the supermarket FCS are connected using a common bus.

A. Validation with the Jabr Model

The proposed optimisation model is a modified version of the Jabr model [7]. The objective function, i.e. minimisation of voltage deviations, and some additional constraints are incorporated in the existing model. To validate the proposed optimisation model, the base case of both models are compared. Fig. 4 shows that the solutions are almost identical for both the cases.

B. Effective use of Reactive Power for voltage support

This case study demonstrates the scenario when FCSs with bidirectional converters are connected to the network with the load profiles as given in Fig. 1(a). A load peak is observed in the highway FCS during the 15th hour, as shown in Fig. 1(a), and this hour is considered as reference. The voltages across

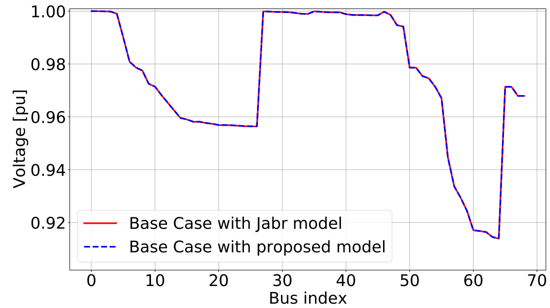
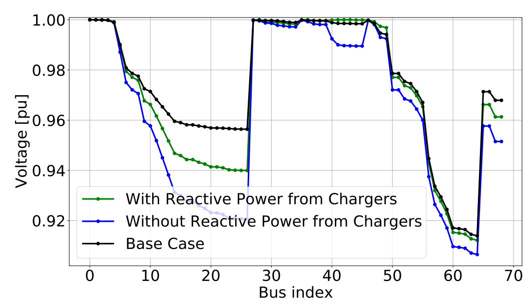
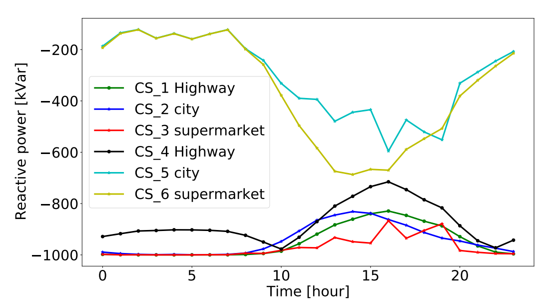


Fig. 4: Voltage profiles at all buses of IEEE 69 bus network

the network at the 15th hour are shown in Fig. 5(a). From the figure, it can be seen that the voltages of the vulnerable buses 27, 61, and 65 drop compared the base case. E.g., in the base case, the voltage of bus 27 is 0.956 pu. This voltage reduces to 0.920 p.u when the FCSs are connected to the system without allowing to inject/draw reactive power to/from the system. However, to prevent this, the optimisation model accommodate the load profile of the FCSs and calculates the optimal reactive power signals for the FCSs while considering the allowable reactive power limits. Fig. 5(b) clearly demonstrates that the optimised reactive signals for the FCSs have helped to improve the overall voltage profiles in the network. As an example, the voltage on bus 27 has increased from 0.920 pu to 0.939 pu. From this case study, we can conclude that the same FCSs can have both a positive and negative impact on the grid.



(a) Voltage profiles of all the buses at the 15^{th} hour with and without reactive power support



(b) Optimised Reactive Power Dispatch Signal for Charging Stations

Fig. 5: Reactive support Flexibility of Fast Charging Stations

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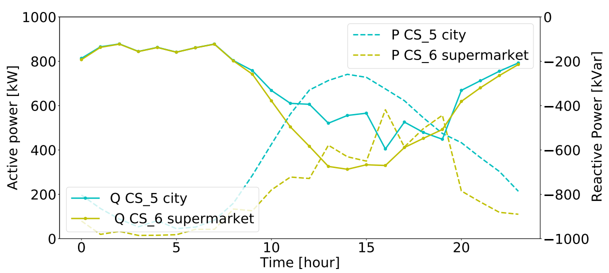
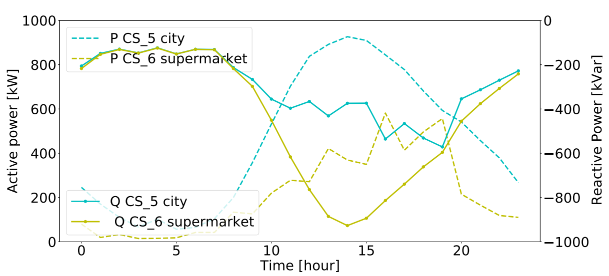
C. Coordinated operation of high-power charging Stations

1) With increased number of EVs at one of the FCSs: This case study investigates the coordinated operation of high-power charging stations with varying load profiles in both amplitude and time. As shown in Fig. 6(a), the city FCS has its peak at the 15th hour, whereas the supermarket FCS has its peak at the 16th hour. To minimise the overall voltage deviations, the optimisation algorithm suggests a lower reactive power support for cities and higher for supermarkets. At the 16th hour, there is equal reactive power sharing, as the active power load of these FCSs is same.

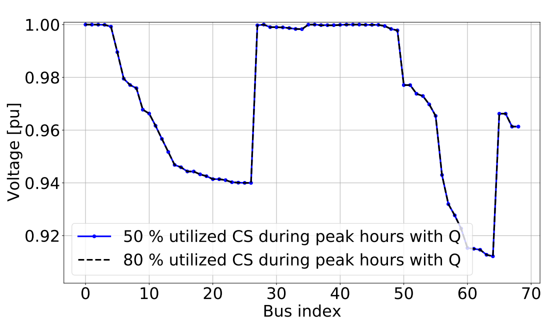
As shown in Fig. 6(b) the load of the city FCS (CS_5) at bus 44 is increased to 909 kW. As a result, the reactive power support from the supermarket FCS at the 15th hour is almost double compared to that of the city FCS, to ensure overall system stability. Fig. 6(c) shows that increased capacity utilisation of one FCS is compensated by the other FCSs, hence voltage deviations are the same in both cases.



(a) Active Power Consumption and Optimal Reactive Power Dispatch Signal with 50% utilised City Fast Charging Station



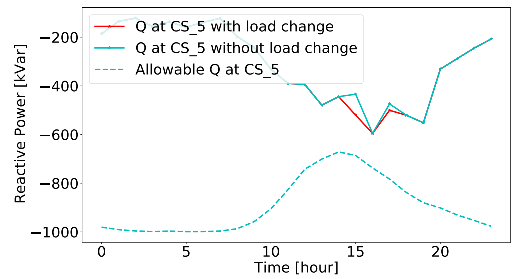
(b) Active Power Consumption and Optimal Reactive Power Dispatch Signal with 80% utilised City Fast Charging Station



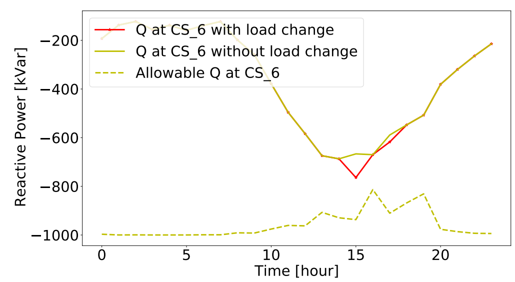
(c) Voltage profiles of the distribution network

Fig. 6: Coordinated Operation of Fast Charging Station with varying utilised capacity

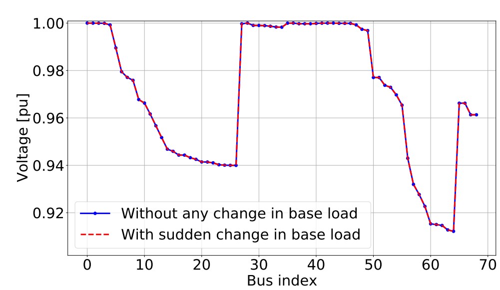
2) With a sudden change in base load: This case study demonstrates the scenario of a sudden change in the base load of the network. For example, at bus 44, the load is changed from 6 kW to 206 kW at the 15th hour, and further increased from 6 kW to 66 kW at the 17th hour. As shown in Fig. 7, CS_5 and CS_6 now have allowable reactive power of -837 and -954 KVar, respectively, at the 15th hour. Since there is less margin between the optimised and the allowable reactive power for the fast CS_5 at bus 44, CS_6 (-586 kVar) will contribute with more reactive power than CS_5 (-486 kVar) during the load change. However, the coordination of the FCSs helps in minimising the voltage deviations in the system as shown in Fig. 7(c).



(a) Optimised Reactive Power Dispatch Signal for City FCS



(b) Optimised Reactive Power Dispatch Signal for Supermarket FCS



(c) Voltage profiles of the distribution network with and without sudden change in base load

Fig. 7: Coordinated Operation of Fast Charging Stations with sudden change in base load

D. Grid support from bidirectional converters at FCSs in periods with no EVs connected

In this case study, a scenario with no EVs connected at the FCS is assumed. Thus a situation where there is minimum load

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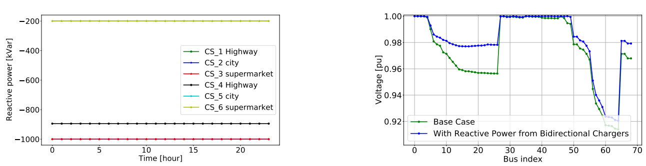
present at the FCSs is demonstrated. In this case, if there is a need of reactive power in the network, this can be provided by the bidirectional converters at the FCSs. The objective in this case study is to minimise the voltage deviations in the network across 24 hours. Therefore, the optimisation model suggests a reactive power support as shown in Fig. 8(a). As a result, the voltages of the vulnerable buses are higher than in the base case and are reaching to 1 pu, as shown in Fig. 8(b).

IV. Discussion

Table I presents the value of the objective function in (1), i.e. a summation of the voltage deviations of all the buses across 24 hours. This value is an index for discussing the advantages of reactive power support as an ancillary service from FCSs for EVs. Compared to the base case, the voltage deviation in the system decreases from 8.26 to 5.25, i.e. approximately 36 %, when reactive power support from FCSs is included (no EVs connected). The voltage deviation is increased to 11.28 when FCSs are operated only with active power. But with reactive power support from the FCSs in the system, the voltage deviation can be reduced to 7.32. In addition to this, coordinated operation of FCSs, can help a system in maintaining the voltage levels.



(a) Optimised Reactive Power Dispatch Signals for FCSs

(b) Voltage profiles of all the buses at the 15^{th} hour with no EVs

Fig. 8: Voltage Profiles with no EVs connected at the FCSs, but with reactive power support from the bidirectional converters

TABLE I: Case studies: Reactive power support from FCSs

Cases	Voltage deviations
Base Case	8.26
Underutilised FCS with Q	5.25
50 % utilised FCS during peak hours with no Q	11.28
50 % utilised FCS during peak hours with Q	7.32

V. Conclusion and Further work

This paper proposes a power flow optimisation model based on second-order cone programming, with the aim of minimising voltage deviations through reactive power support from grid-connected FCSs for EVs. Simulation results clearly show that the coordinated operation of FCSs with available reactive power can help in preventing voltage drop issues arising due to spikes in base load and active power consumption of FCSs. As a part of future work, reactive power sharing at the charger level will be analysed with two approaches. In the first approach, the optimisation framework will be improved to dispatch reactive power signals directly to an individual charger at the FCS. In the second approach, the hourly optimised reactive power signal received at the FCSs will be divided among the converters based on their Q-V droop characteristics. In addition, future work will focus on investigating the impact of voltage deviation minimisation on losses, as well as developing a multi-objective framework that can minimise losses and voltage deviations simultaneously in the network.

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