Report

Veres3D User Manual

for Work in Progress Release 0.2.0

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ABSTRACT
When a ship is moving forward in waves it will experience an increase in resistance in waves. Traditionally, ship designs are optimized for calm water performance. However, this condition does not reflect the real condition the vessel will experience at sea. The International Maritime Organization (IMO) has established the Energy Efficiency Design Index (EEDI) as the most important policy measure to reduce greenhouse gas emissions from shipping. The current implementation of the EEDI is based on calm water conditions. Depending on the type and size, vessels are not allowed to exceed a threshold for emitted CO_2. However, when vessels operate under real-sea conditions with effects of wind and waves, the emissions are higher than assumed for a given speed according to the calculations following the EEDI. This is due to the fact that the required power in real sea exceeds the power requirement in calm-water conditions. Hence, calm-water optimization, currently applied by the EEDI, does not necessarily produce the ship with the lowest energy consumption over its lifetime, leading to increased fuel consumption and raising a safety concern for the operation of the vessel in extreme weather conditions. To better address the IMO 2050 greenhouse gas reduction target, the practical numerical tool VERE3D for the prediction of seakeeping qualities of ships with forward speed has been developed.

As a linear three-dimensional (3D) frequency-domain formulation, VERES3D is based on boundary element methods (BEM). The code shall be used for practical applications in the early ship design process and will be further developed and extended over the next years, considering ongoing validation activities and the feedback received from users within the industry and academia.

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Document History

VERSION	DATE	VERSION DESCRIPTION
0.1	November 4, 2020	Manual following Work in Progress Release 0.1
0.1.1	November 6, 2020	Updating instructions for license
0.2.0	December 01, 2020	Including far-field method, following VERES3D Release 0.2.

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1 Introduction

This document describes the use of VERES3D following the current *work in progress* release version 0.2 and the basic theory applied.

VERES3D is a linear, three-dimensional, frequency domain radiation/diffraction code, developed for the analysis of the hydrodynamic interaction of waves with large volume structures for zero and forward speed. Large volume structures are defined as structures where diffraction is important.

VERES3D is based on potential flow theory applying a panel method in the solution procedure. The fluid is assumed ideal and the flow time harmonic. The free-surface condition is linearised so that only terms proportional to the wave amplitude are retained. The second-order mean wave drift forces are included in the analysis as they may be obtained from the linear solution.

The radiation and diffraction velocity potentials on the mean wetted surface of the body are determined from the solution of an integral equation obtained from Green's theorem. Without current, a Green function satisfying the linear free-surface condition is used, and only the mean wetted surface of the body is discretised into panels. A numerical formulation using the Green function satisfying the far-field free-surface condition in the outer domain is applied.

This formulation is often referred to as the direct method since the potential is obtained directly from the solution of the integral equation. Surface elevation, pressure and fluid velocities in the surrounding fluid without wave-current interaction are obtained by applying an equivalent source distribution on the mean wetted surface of the body. This source distribution is calculated by using the computed potential distribution on the body.

VERES3D uses a low-order panel method where the surface of the body is defined by flat quadrilateral panels and the solution for the velocity potential (and thus also source strengths) are assumed constant over each panel.

The program may be applied to single bodies in waves with forward speed. The bodies may be located on the free-surface, or be fully submerged.

Viscous forces may be applied in the motion response calculation. On ship structures, viscous roll damping may be specified. The viscous forces contain quadratic terms which are linearised by equivalent linearisation. The motion responses are then obtained by iteration.

The following quantities can be evaluated in VERES3D:

- Hydrostatic coefficients.

- Added mass and damping coefficients for all modes of motion.

- Wave excitation forces and moments.

- Motion RAOs for a freely floating body.

- Horizontal mean drift forces and mean yaw moment.

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Additional mass, damping and restoring forces may be specified. These additional matrices may be used to tune the natural periods against model test or full scale results. Additionally, viscous roll damping may be included.

The horizontal drift forces and moments are obtained through the direct pressure integration method and control surface integration based on conservation of momentum.

The present version of the program imposes some limitations on the geometry modelling and the size of the problem to be analysed. The program uses allocated memory for data storage and thus the size of the geometrical model in terms of number of panels is restricted by available RAM. Panels with a very large aspect ratio should be avoided because these panels may require special treatment in the code which is not yet implemented.

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2 Using VERES3D

VERES3D is distributed as one executable where both the potentials, resulting forces and motions are calculated in a complete calculation and stored in files. It is also possible to rerun VERES3D for new response calculations by reading the previously calculated potentials from the file.

The present version of VERES3D reads a geometry file in the WAMIT gdf-file format and writes output files compatible with the corresponding files from WAMIT. VERES3D also writes output files compatible with the output files from the SINTEF Ocean vessel response strip theory program VERES, making it possible to use the VERES post-processor to calculate short and long-term statistical values, carry out operability studies with imposed seakeeping criteria, etc.

2.1 Installation and setup

The present version of VERES3D is distributed as one executable file without graphical user interface. The executable is compiled with the Intel Visual Fortran Compiler adapted to a MS Windows 64-bits architecture. The program is run by entering the name of the input file as a command line argument such as

```
VERES3D.exe $input_folder\input.inp
```

The user must have a valid license. The location of the license file is defined in the Windows Registry key $HKEY\_CURRENT\_USER\backslash Software\backslash FLEXlmLicenseManager$. Putting the license file in the same location as the VERES3D.exe file does not work automatically.

The license file should be placed in one location on the computer and the reference to that location is set in the Windows Registry. This is a one-time operation. Once that is done, you can put/move the VERES3D.exe file wherever you like. Every time the VERES3D.exe is run it will look in the registry for the license file location and then read that license file.

There are a few ways of adding the license file location into the registry:

- If the user starts VERES3D without having run any SINTEF Ocean software (typically VeSim) earlier, a dialog will be presented where the user can define the location of the license file:
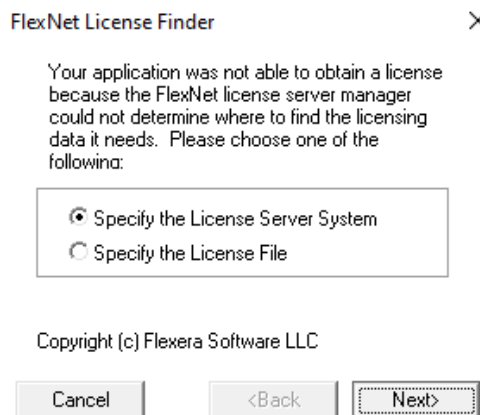

Figure 1: Dialogue window to register the license

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Select *Specify the License File*, click *Next* and locate the file.

- If a software (for example VeSim) is installed the license key will exist in the registry, but it will point to the VeSim/ShipX license file. That does not include the VERES3D license. Here the solution is to add the VERES3D license file to the existing registry key:

  - Start regedit.exe
  - Navigate to $HKEY\_CURRENT\_USER$ -> Software -> FLEXlm License Manager
  - Right-click the $MARINTEK\_LICENSE\_FILE$ Key, select *Modify*
  - Add a semicolon to the end of the existing string and then add the path to the VERES3D.lic location, for example, $C：\backslash ProgramFiles(x86)\backslash VERES3D\backslash 20201104\_Release_v.0.1\backslash Release\backslash VERE$ The new complete string may now look like this:

    $C：\backslash ProgramFiles(x86)\backslash ShipX\backslash bin\backslash ShipX.lic; C：\backslash ProgramFiles(x86)\backslash ShipX\backslash bin; C：$ $\backslash Program\ Files\backslash VERES3D\backslash 20201104\_Release\_v.0.1\backslash Release\backslash VERES3D.lic$
  - Click OK and exit the registry editor.
  - Start VERES3D.exe to check that it works correctly.

- When a new license file is issued, the user simply needs to overwrite the existing license file with the new one.

2.2   Standard test examples

A set of standard test examples is included with the software delivery. These test examples are used to illustrate different types of applications and features of the program and may be used to verify the installation and set-up of the program.

2.3   Memory and storage restrictions

VERES3D is written in Fortran90 and uses allocatable arrays for temporary storage at run time. Large data sets will require sufficient memory installed. Running the program on a PC will in general require a 64 bit architecture in order to access sufficient memory for practical cases. The memory requirements may be reduced by using geometrical symmetry properties.

2.4   Run Options

VERES3D features two run options: The complete calculation and the rerun for response calculations given that a complete calculation has been carried out in advance. In a complete calculation, the velocity potentials as well as the source strength (when calculated) are written to a binary file [file_hdr_p].pot. The resulting forces and motions are calculated simultaneously and written to output files. In a rerun, the velocity potentials as well as source strength (when existing) are read from the .pot file during the execution of the program. The source distribution is only calculated when drift forces are obtained by the control surface integration method. If the source strength is not stored on the .pot file during the complete calculation, it will be calculated in the rerun and stored on the .pot file if needed. In that case, the computational time of the rerun will increase significantly.

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If a .pot file with an identical name as the name specified in the input file already exists when a complete calculation is initiated, the user will be asked whether or not to overwrite the file. If overwriting is not accepted, the execution of the program stops.

The same input file is used for all run options. It is hence important to note that the input to the "potential solver" must not be changed before a rerun is made. However, the input to the potential solver is also read from the .pot file during a rerun, and changes made in the input file will not be allowed. The only exception is the specification of wave headings and frequencies. If these are modified, the execution of the code continues by applying the input read from the .pot file. Additionally, the input connected to the removal of the irregular frequencies option does not affect the response calculation.

Input to the "response solver" may be modified before each rerun. It should be noted that the file name header [file_hdr_r] of the output files is specified in the input file, so that for each rerun it is possible to obtain new output files. If the file name header of the output files is not changed between reruns, the files are overwritten.

2.5 Input files

VERES3D requires two types of input files; one set of files containing the panel models of body geometries, and one file describing the environment and body data, specifying run options, etc. [filename].inp. VERES3D presently only supports the WAMIT .gdf file format for the geometry description.

If viscous forces are to be included in the motion response calculation of a body, an additional file with information about the viscous force calculation on the body is required. If viscous roll damping is specified, the required input file to VERES3D is an output file [filename].re7 from the SINTEF Ocean ship motion program VERES, containing frequency-dependent linear and quadratic viscous roll damping coefficients. Coefficients are interpolated w.r.t frequency and velocity.

The input file may contain commented lines. A line in the input file is treated as a comment if the first entry on the line is an apostrophe ('). The number of comments in the file is arbitrary. VERES3D will successfully read the input file and check for valid values and combinations of input data when the sequence of required input is given correctly. If the sequence of required input is incorrect, an error message will normally be returned, but it should be noted that the input error may have occurred one or more lines before the program returns the error message.

The format of the input file is given below. The type of input data is specified as:

I   -   Integer
R   -   Real
Ch   -   Character

Input line 1

file_version		
file_version	**Version compatibility of input file**	Ch
Note:	The present file version number is 0.1.0	

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Input line 2

output_format			
output_format	**Output file format**		**Ch**
	wamit	Results written on WAMIT format files	
	veres	Results written on VERES format files (only .re1 and .re2 files)	
	all	Results written on both WAMIT and VERES format files	
Note:	When output_format = veres or all, additional input is required for each body.		

Input line 3

file_hdr_r		
file_hdr_r	**Header for output file names for responses**	**Ch**

Input line 4

ical			
ical	**Run option**		**I**
	0	Data check only	
	1	Complete calculation, using previously calculated potentials and responses	
	2	Rerun for new response calculation	

Input line 5

file_hdr_p		
file_hdr_p	**File header for potential file name**	**Ch**

Input line 6

nohead nofreq icomfr			
nohead	**Number of wave headings**		I
nofreq	**Number of wave 'frequencies'**		I
icomfr	**Code for input form of wave 'frequencies'**		I
	1	Wave frequencies are given (rad/sec)	
	2	Wave periods are given (sec)	
	3	Min and max wave periods are given and periods are calculated evenly distributed	

Input line 7

whead(i), i=1,nohead		
whead(i)	**Wave headings (deg)**	R
Note:	All wave headings must be given on one line, separated by a space. The wave headings are given relative to the global coordinate system with `whead = 0` representing waves traveling along the positive x-axis and `whead = 90` representing waves traveling along the positive y-axis.	

Input line 8

wfreq(i), i=1,nofreq		
wfreq(i)	**Wave frequencies/periods**	R
Note:	All wave frequencies must be given on one line, separated by a space.	

Input line 9

vel		
vel	**Ship velocity (m/s)**	R
Note:	The ship velocity must be positive.	

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Input line 10

n_bodies		
n_bodies	**Number of bodies**	I
Note:	If n_bodies > 1, the following block of properties must be repeated as an input. Bodies have same velocity, same heading. Results have not been validated yet.	

The following sequence of lines (lines 11 - 12) must be given for each body $i = 1, \dots,$ n_bodies.

Input line 11

xpos_body(i) ypos_body(i) zpos_body(i) rot_body(i) (deg)		
xpos_body	*X* **coordinate of origin of local coordinate system for body** i	R
ypos_body	*y* **coordinate of origin of local coordinate system for body** i	R
zpos_body	*z* **coordinate of origin of local coordinate system for body** i	R
rot_body	**Orientation of local x-axis of body** i **with respect to the x-axis of the global coordinate system, (rotation in horizontal plane)**	R
Note:	Coordinates are given in the global coordinate system.	

Input line 12

remirr(i)			
remirr	**Removal of irregular frequencies for body** i		I
	0	Do not remove irregular frequencies for this body	
	1	Do remove irregular frequencies for this body	

INPUT TO RESPONSE SOLVER. (Can be modified for new response calculation.)

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Input line 13

idrift_dpi idrift_cs idrift_ff			
idrift_dpi	**Use direct pressure integration to compute added resistance**		I
	0	No	
	1	Yes	
idrift_cs	**Use control surface method to compute added resistance**		I
	0	No	
	1	Yes	
idrift_ff	**Use far-field method to compute added resistance**		I
	0	No	
	1	Yes	
Note:	Far-field method may be applied only at zero speed.		

Input line 14

useAsympFormula			
useAsympFormula	**Use Asymptotic Formula for short waves**		I
	0	No	
	1	Use Faltinsen's method	
	2	Use Liu et al.'s method	
Note:	Asymptotic formulas are not applied on far-field method results.		

Input line 15

ivisc			
ivisc	**Include viscous forces**		I
	0	No viscous forces	
	1	Viscous forces included in motion response calculation	
Note:	When `ivisc` = 1, option iviscf and additional .re7 input file is required for each body, and wave amplitude must be given.		

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Input line 16

wave_amp		
wave_amp	Representative wave amplitude for viscous force calculation	R
Note:	Only given for `ivisc = 1`	

Input line 17

rho		
rho	Density of water	R

The following sequence of lines must be given for each body $i = 1, ..., $ `n_bodies`.

Input line 18

Input data, $L_{PP}(i)$ draft(i) breadth(i) xpos_AP(i)		
L_{PP}	Length between perpendiculars of body i	R
draft	Draft of body i	R
breadth	Breadth of body i	R
xpos_AP	X coordinate of aft perpendicular of body i	R
Note:	Only given when output_format = veres or all. Coordinate is given in local coordinate system for body. xpos_AP < 0 because local x-axis is assumed to point towards bow.	

Input line 19

zcog		
zg	Vertical position of the centre of gravity of body i	R
Note:	Position is given in local coordinate system for body. Horizontal position of centre of gravity is given by centre of buoyancy of body, assuming freely floating bodies.	

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Input line 20

imass(i) idamp(i) istif(i)			
imass	**Mass input**		I
	1	Mass matrix given, including mass matrix reference point	
	2	Radii of gyration given. Mass calculated from submerged volume	
idamp	**External damping input**		I
	0	No external damping	
	1	External damping matrix given	
istiff	**External stiffness input**		I
	0	No external stiffness	
	1	External stiffness matrix given	

Input line 21

xpos_mass(i) ypos_mass(i) zpos_mass(i)		
xpos_mass	*X* coordinate of mass matrix reference point for body i	R
ypos_mass	*Y* coordinate of mass matrix reference point for body i	R
zpos_mass	*Z* coordinate of mass matrix reference point for body i	R
Note:	Only given when `imass(i) = 1`. Coordinates are given in local coordinate system for body. For freely floating bodies, mass should correspond to displaced volume.	

Input line 22

mass_matr_inp(j,k)(i)		
mass_matr_inp(j,k)	**Element of mass matrix for body i, given at mass matrix reference point**	R
Note:	Only given when `imass(i) = 1`. The elements of the mass matrix are given with one row per line, that is six entries on six lines.	

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Input line 23

r44(i) r55(i) r66(i) r46(i)		
r44	**Radius of inertia in roll**	R
r55	**Radius of inertia in pitch**	R
r66	**Radius of inertia in yaw**	R
r46	**Coupled radius of inertia between roll and yaw**	R
Note:	Only given when `imass(i) = 2`. The radii of inertia are given about the center of gravity. WARNING: in VERES3D, these values are relative to the centre of gravity In WAMIT they are relative to the origin of coordinate system in the gdf file.	

Input line 24

damp_matr_inp(j,k)(i)		
damp_matr_ext(j,k)	**Element of external damping matrix for body** i	R
Note:	Only given when `idamp(i) = 1`. Given at origo in local coordinate system of body. The elements of the damping matrix are given with one row per line, that is six entries on six lines.	

Input line 25

stiff_matr_inp(j,k)(i)		
stiff_matr_ext(j,k)	**Element of external stiffness matrix for body** i	R
Note:	Only given when `istif(i) = 1`. Given at origo in local coordinate system of body. The elements of the stiffness matrix are given with one row per line, that is six entries on six lines.	

Input line 26

iviscf(i)			
iviscf	**Include viscous forces on body** i		I
	0	No viscous forces	
	2	Viscous roll damping from VERES calculation included	
Note:	Only given when `ivisc = 1`.		

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Input line 27

vdf_file_visc(i)		
vdf_file_visc	Input file for viscous forces, .re7 file	Ch
Note:	Only given when `iviscf = 2`.	

The end of the sequence of lines which must be given for each body $i = 1, \ldots,$ `n_bodies`

Input line 28

ctrl_a ctrl_b		
ctrl_a	**Ratio of ship length defining the control surface**	R
ctrl_b	**Ratio of ship breadth defining the control surface**	R
Note:	Should be higher than 1.	

2.6 Example of input file

```
'file_version
0.1.0
'------------------------------------------------------------------------
'output_format
VERES
'------------------------------------------------------------------------
'file_hdr
kvlcc2
'========================================================================
'PROGRAM RUN OPTIONS
'========================================================================
' ical    - Calculation option
'           = 0 -> Data check only
'           = 1 -> Complete calculation (both potential and responses)
'           = 2 -> Rerun for new response calculation
1
'------------------------------------------------------------------------
' File header for potential: file_hdr_p
kvlcc2
'------------------------------------------------------------------------
' nohead, nofreq, icomfr
'    * nohead - Number of wave headings
'    * nofreq - Number of wave frequencies
'    * icomfr - Code for input form of wave frequencies
'        = 1 -> Wave frequencies are given (rad/sec)
'        = 2 -> Wave periods are given (sec)
'        = 3 -> Min and max wave period are given and periods are calculated evenly distributed
1 3 2
'------------------------------------------------------------------------
' whead(ihead), i=1,nohead - Wave headings
0.0
'------------------------------------------------------------------------
' wfreq(ifreq), i=1,nofreq - Wave frequencies or periods
6.0 8.0 10.0
'------------------------------------------------------------------------
'vel
8.11
```

```
'------------------------------------------------------------------------
'n_bodies - same velocity, same heading (not validated)
1
'------------------------------------------------------------------------
'
'--------------------------------------------------------------------------------
'----------------------------- Loop over bodies 1 -----------------------------
' xpos_body, ypos_body, zpos_body, rot_body
0,0,0,0
'gdf_file
kvlcc2.gdf
' remirr
0
'----------------------------- End loop over bodies 1 -----------------------------
'
'--------------------------------------------------------------------------------
'======================================================================
'INPUT TO RESPONSE SOLVER (can be modified for new response calculation)
'======================================================================
' idrift  - Calculation of added resistance (0: No, 1:Yes)
' Direct pressure integration, Control surface, Far-field
1 1 0
'------------------------------------------------------------------------
' Use asymptotic formula for short waves
' 0 : No
' 1 : Faltinsen method
' 2 : Liu et al. method
0
'------------------------------------------------------------------------
'ivisc
0
'------------------------------------------------------------------------
'wave_amp    / Only for ivisc = 1  (Comment if ivisc = 0)
'1.0
'------------------------------------------------------------------------
'rho
1025
'
'--------------------------------------------------------------------------------
'----------------------------- Loop over bodies 2 -----------------------------
' Lpp, draft, breadth, xpos_ap
320, 0, 58.0, 0.8098
'------------------------------------------------------------------------
' zcog - Center of gravity
-2.2
'------------------------------------------------------------------------
' imass, idamp, istiff
' imass  - Mass input
'        = 0 -> No mass given (not implemented)
'        = 1 -> Mass matrix given, including mass reference point
'        = 2 -> ZG and radii of gyration given. Mass == submerged volume * rho.
' idamp  - External damping input
'        = 0 -> No external damping
'        = 1 -> Damping matrix given
' istiff  - External stiffness input
'        = 0 -> No external stiffness
'        = 1 -> Stiffness matrix given
2, 0, 0
'------------------------------------------------------------------------
' r44,r55,r66,r46
'      * r44 - Radii of inertia in roll (about center of gravity)
'      * r55 - Radii of inertia in pitch (about center of gravity)
'      * r66 - Radii of inertia in yaw (about center of gravity)
'      * r46 - Coupled radii of inertia between roll and yaw (about center of gravity)
18.5 80.0 80.0 0.0
'------------------------------------------------------------------------
'Damping matrix    (Comment if idamp = 0)
'
'------------------------------------------------------------------------
'Stiffness matrix    (Comment if istiff = 0)
' 0.0    0.0    0.0    0.0    0.0    0.0
' 0.0    0.0    0.0    0.0    0.0    0.0
' 0.0    0.0    0.0    0.0    0.0    0.0
```

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```
' 0.0     0.0     0.0     0.0     0.0     0.0
' 0.0     0.0     0.0     0.0 1.e+30     0.0
' 0.0     0.0     0.0     0.0     0.0     0.0
'-----------------------------------------------------------------------
' Viscous drag force: (Comment if ivisc = 0)
' If ivisc = 1:
' iviscf   = 0 -> No viscous drag forces on body
'          = 2 -> Viscous roll damping from VERES calculation included
'2
'-----------------------------------------------------------------------
' If iviscf = 2:vFile from VERES calculation (re7) giving roll damping coefficients (Comment if ivisc = 0)
'input.re7
'------------------------------ End loop over bodies 2 --------------------------
'
'-----------------------------------------------------------------------
' control_surface_parameters
1.2 1.5
'-----------------------------------------------------------------------
```

3 Main program reference

3.1 Basic assumptions

The basic assumptions for the theoretical formulation in the VERES3D program are

- The body is assumed to oscillate harmonically with the frequency of the incident waves. No transient effects due to initial conditions are accounted for. No hydro-elastic effects are accounted for.

- A linear relation is assumed between the responses and the incident wave amplitude. This may not be correct in high sea states. This assumption also implies that the body should be close to wall-sided at the mean free-surface.

- Potential theory can be applied. The fluid is assumed to be homogeneous, non-viscous, irrotational and incompressible.

- The wave frequency is assumed high $\omega \gg 1$ and forward speed is moderate such that the Froude number is $Fn < 0.4$.

3.2 Coordinate systems

In the coordinate system (X, Y, Z), X-axis is pointing towards the bow. The Z-axis is vertical and positive upwards. The origin of this coordinate system may in principle be chosen arbitrarily, but it is recommended to locate it on the mean free-surface. The resulting motions and forces are output in the local coordinate system for each body.

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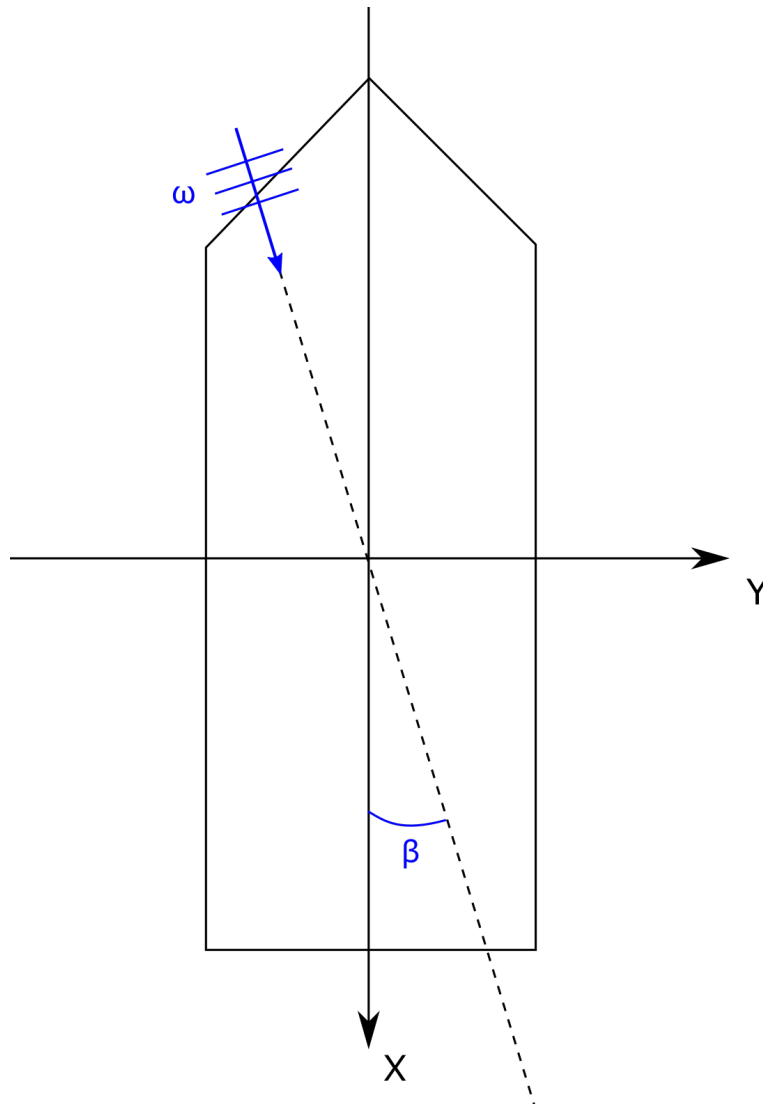OC2020 A-128

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Figure 2: Definition of the coordinate system showing wave heading angle. Viewed from above, along positive z-axis.

The translatory displacements in the X, Y and Z directions with respect to the local coordinate system are denoted η_1, η_2 and η_3, respectively, where η_1 is the surge, η_2 is the sway and η_3 is the heave displacement. Furthermore, the angular displacements of the rotational motion about the X, Y and Z axes are denoted η_4, η_5 and η_6, respectively, where η_4 is the roll motion, η_5 is the pitch motion and η_6 is the yaw motion, respectively.

3.3 Description of body geometry and free-surface

3.3.1 Geometry files

The wetted part of the body is described by a number of quadrilateral panels. For each panel the x, y and z coordinates of the four corner points are specified. Triangular panels may be specified by letting the coordinates of two of the corners coincide. The order in which the panels are presented in the geometry file does not matter, but each panel must be described by 12 real numbers, which

are the three coordinates (x, y, z) for each of the four corner points. The data for each panel are separated by a line break in the input file. The four corner points of each panel in the geometry file must be specified in the counter-clockwise direction when the panel is seen from a point in the fluid outside of the body. By following this convention of numbering, the unit normal vector is pointing outwards into the fluid.

In the geometry file, symmetry may be utilized by discretising only one half or one quarter of the wetted surface of the body. However, symmetry properties are utilised in the numerical solution only for cases with single bodies.

The displaced volume of the body is calculated by integrating the volume of the discretised panel model with respect to the different coordinate axes. This yields three different computations of the displaced volume, $volx$, $voly$ and $volz$, which should be practically identical. Small variations will arise due to numerical inaccuracies and the panel discretisation, but large differences indicate possible errors in the geometry file. VERES3D uses the median value of the three evaluated volumes in the computations.

The body geometry is currently specified in a file format compatible with that of the WAMIT gdf file. VERES3D only accepts ULEN=1, which is specified in the gdf-file. The acceleration of gravity is not read from the gdf-files, but from the main input file.

3.3.2   Panel discretization on the body

A sufficient number of panels must be used in order to catch the variation of the potential over the wavelength. As a general rule of thumb, the typical dimension of the panel should be less than 1/8 of the wavelength.

Normally, the panel size should be small in regions with large variations in the potential. For bodies with sharp corners, this implies that small panels may be required close to the corners.

Steady potential is not computed at the moment in VERES3D due to the high sensitivity of the results. For a body in waves and forward speed, locating small panels close to sharp corners is not favourable, as the gradient of the steady potential would be infinite at the corner and the solution for the steady potential would not converge. This also implies that the solution for the first-order potentials does not converge. Thus a too fine panel discretisation close to sharp corners may introduce inaccuracies in the numerical solution of the potential. A more pragmatic approach should then be applied in that larger panels are used so that the potential is not calculated too close to the sharp corners.

It should be observed that the present version of VERES3D implements analytical integration of the Rankine components in the Green function and applies the midpoint formula for the wave component. This implies that panels with a large aspect ratio (i.e. panels with large length to breadth ratio) may be inaccurately treated in the present version.

3.4   Irregular frequency option

VERES3D includes a method for removing the effect of irregular frequencies on the calculation of the radiation and diffraction potentials for zero-speed. This method is referred to as the extended

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boundary condition method, as the computational domain is extended to include the interior free-surface of the body. This requires that the interior free-surface of the body is discretised into panels. This is done automatically by the program. For multiple bodies, the removal of irregular frequencies is treated individually for each body.

3.5   Added resistance option

The mean horizontal drift forces and moment in infinite water depth may be calculated by two different approaches. The first approach is a near-field method where added resistance is obtained by integrating the pressure over the instantaneous submerged body and then taking the time average. In the other approach, referred to as control surface integration, the potential is calculated and integration is performed numerically at a control surface which may be located close to the body.

The control surface integration requires a control surface that is vertical and sufficiently deep. The control surface is created automatically by the program, but with shape and dimensions specified by the user. A cylindrical control surface is used. The program controls that the specified size of the chosen control surface is large enough not to intersect with the body. The depth of the generated control surfaces is controlled by the wave length. The advantage is that the program generates a new control surface for each wave frequency, adjusting both the depth and panel resolution for each frequency.

3.6   Viscous force calculation

VERES3D supports viscous forces in the motion response calculations. For ship structures, viscous roll damping can be included by reading viscous damping coefficients from a VERES (VERES 2D software using strip-theory) re7-output file. VERES is a ship motion strip-theory code, where also viscous roll damping coefficients can be calculated by empirical formulas. Both frictional damping, eddy damping as well as bilge keel damping are included. The re7-file contains linear and quadratic frequency- and speed dependent viscous roll damping coefficients. VERES3D applies only the damping coefficients for a given forward speed. VERES3D interpolates the viscous roll damping coefficients for the frequencies and velocities used in the VERES3D calculation. If the frequency or velocity is outside the interpolation domain, truncated values are used.

The quadratic terms of the viscous forces are linearized by equivalent linearization. For the viscous roll damping, we may write the linearized quadratic term as:

$$B_{VNL} = \frac{8\omega}{3\pi}\eta_a B_{VN} \tag{1}$$

with

$$\eta_a = H_\eta \zeta_a \tag{2}$$

where H_η is the transfer function between wave amplitude ζ_a and roll motion η_a.

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4   Guidelines for use

4.1   Gridding of body

4.1.1   Panel size

A sufficient number of panels must be used in order to catch the variation of the potential over the wavelength. As a general rule of thumb, the typical dimension of the panel should be less than 1/8 of the wavelength. The wavelength of the incident waves is given by the wave number, which is related to the wave frequency through the dispersion relation, which for deep water may be written as:

$$k = \frac{\omega_0^2}{g}.$$

(3)

The wave frequency ω_0 may be related to the encounter frequency ω through the relation

$$\omega = \omega_0 + \frac{\omega_0^2 U_c}{g} \cos \beta,$$

(4)

where g is the acceleration of gravity, U is the forward speed and β is the angle between the direction of wave propagation and forward speed. Then $\beta = 180$ corresponds to head sea waves.

4.1.2   Regions with strong curvature

Very narrow or very small panels close to large panels should in general be avoided in a panel model, as the derivatives of the Rankine source will be singular at the panel boundary due to a jump in the potential, and this may introduce local errors in the numerical solution.

Small or narrow panels are customary in regions with strong curvature. It should however be observed that the body boundary condition is satisfied only at the centre point of each panel, and that small gaps between the panels have minor importance provided that the calculated volume is acceptable.

The present version of VERES3D applies analytical integration of the Rankine components in the Green function but uses the midpoint formula for the wave component. This implies that panels with large aspect ratios (i.e. panels with large length to breadth ratio) may be inaccurately treated.

4.2   Summary

4.2.1   Limitations

- The program is limited to moderate ship velocities, such that Froude number $Fn < 0.4$.

- Removing irregular frequencies requires that a panel model for the internal free-surface is provided and should be used for zero-speed.

- The horizontal position of the centre of gravity is given by the centre of buoyancy, assuming freely floating bodies.

- Infinite water depth is assumed.

- Neumann-Kelvin assumption is used ($\bar{\phi} = 0$)

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4.2.2 Checklist for use

- Check that the error of the calculated volume from the panel model is acceptable. Typically the error should be less than 1% from the target value. Further, the three calculated volumes should agree. Differences in these values indicates that the unit normal on parts of the panel model, points in the wrong direction.

- Check that the specified wave periods will give sufficiently long waves relative to the typical panel size of the geometry model. Typically it is recommended that the length of the incident waves should be at least eight times a typical panel dimension.

- The wave periods should be selected so that important characteristics of the motion responses are captured.

- When radii of inertia are used to define the mass matrix, check that the specified values refer to the centre of gravity.

- When specifying additional matrices, check that the given values correspond to the specified water density. If the density is given as $tons/m^3$, then the specified additional matrices must be given in kN etc.

- Parameter studies with varying mass properties, additional matrices, or viscous effects calculations may be carried out using calculation option *ICAL=2*. Then, the file header for the output files may be changed for each run using the same potential file.

5 Theory

VERES3D is based on linear potential flow theory. The basic assumptions of linear theory are:

- The wave-amplitudes are small compared to the characteristic dimension of the body. The resulting motions will then be proportionally small.

- The wave-steepness is small, i.e. the waves are far from breaking.

In linear theory, the wave forces and motions are linearly proportional to the wave amplitude. As a consequence of this, results in irregular waves are obtained by adding together results from regular waves of different amplitudes, wavelengths, propagation directions and phases.

In order to further simplify the problem, steady-state conditions are assumed, i.e. there are no transient effects present due to initial conditions. This implies that the linear dynamic forces on the body are harmonically oscillating with the same frequency as the wave force that excites the body. Thus the calculations may be carried out in the frequency domain.

5.1 The hydrodynamic boundary-value problem

The position of the body is given in the global coordinate system (X, Y, Z).

The description of the wave environment is given in the coordinate system, with the wave heading β_G defined so that the waves travel along the positive X-axis and $\beta = 90$ degrees corresponds to waves traveling along the positive Y-axis.
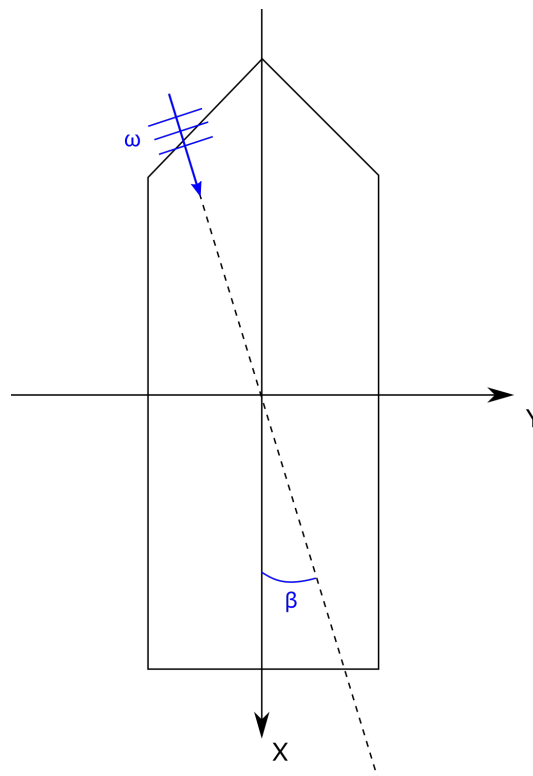

Figure 3: Definition of the coordinate system in which the problem is formulated, and wave heading angle. Viewed from above, along positive z-axis.

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The objective of VERES3D is to evaluate the unsteady hydrodynamic forces and motions on the body, the free-surface elevation around the body and unsteady pressure and fluid velocities at points in the fluid domain. Infinite or finite water depth is assumed and hydrodynamic interaction between multiple bodies will be accounted for. The body boundary conditions and free-surface conditions are linearised, and the effect of flow separation is neglected. The time-dependence is assumed to be harmonic. Then the hydrodynamic boundary value problem may be formulated within potential flow theory. The fluid velocity is then obtained as the gradient of the velocity potential Φ satisfying the Laplace equation

$$\nabla^2 \Phi = 0, \tag{5}$$

in the fluid domain.

For a single body, the velocity potential may be decomposed as

$$\Phi = \bar{\phi} - Ux + \varphi_0 + \varphi_7 + \sum_{j=1}^{6} \varphi_j \eta_j, \tag{6}$$

where U is the forward speed of the body, $\bar{\phi}$ is the disturbance of the incident steady flow due to the presence of the body, φ_0 is the incident wave potential which for infinite water depth may be written as

$$\varphi_0(x, y, z, t) = \frac{g\zeta_a}{\omega_0} e^{\nu z} e^{i(\nu x \cos\beta + \nu y \sin\beta + \omega t)}, \tag{7}$$

where the frequency of encounter ω is defined by

$$\omega = \omega_0 + \nu U \cos\beta. \tag{8}$$

The wave frequency ω_0 and wave number ν is related through the dispersion relation

$$\frac{\omega_0^2}{g} = \nu. \tag{9}$$

The velocity potential φ_7 represents the scattered disturbance of the incident wave by the body fixed in the mean position. The potential

$$\varphi_D = \varphi_0 + \varphi_7, \tag{10}$$

will be referred to as the diffraction potential. φ is then the solution of the diffraction problem for which the body boundary condition is given by

$$\frac{\partial \varphi_D}{\partial n} = 0, \tag{11}$$

or

$$\frac{\partial \varphi_7}{\partial n} = -\frac{\partial \varphi_0}{\partial n}. \tag{12}$$

The radiation potentials φ_j represent the flow created when the body is oscillating about the mean position in otherwise calm water without incident waves, with unit motion amplitude. The body boundary condition for the radiation potentials are then

$$\frac{\partial \varphi_j}{\partial n} = i\omega n_j + M_j, \quad j = 1, \dots, 6. \tag{13}$$

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Here n_j is the generalized unit normal

$$[n_1, n_2, n_3]^T = \mathbf{n}, \tag{14}$$

$$[n_4, n_5, n_6]^T = \mathbf{x} \times \mathbf{n}, \tag{15}$$

$$\mathbf{x} = [x, y, z]^T. \tag{16}$$

The unit normal $\mathbf{n}$ is normal to the mean position of the body and is pointing out of the body and into the fluid domain. The terms M_j in the body boundary condition for the radiation potentials are due to the interaction with the steady flow. The M_j coefficients may be defined as:

$$M_1 = m_1, \tag{17}$$

$$M_2 = m_2, \tag{18}$$

$$M_3 = m_3, \tag{19}$$

$$M_4 = m_4 - \left(n_2 \bar{\phi}_z - n_3 \bar{\phi}_y\right), \tag{20}$$

$$M_5 = m_5 - \left(n_3(\bar{\phi}_x - U) - n_1 \bar{\phi}_z\right), \tag{21}$$

$$M_6 = m_6 - \left(n_1 \bar{\phi}_y - n_2(\bar{\phi}_x - U)\right). \tag{22}$$

The m_j components are defined by

$$[m_1, m_2, m_3]^T = \mathbf{m}, \tag{23}$$

$$[m_4, m_5, m_6]^T = \mathbf{x} \times \mathbf{m}, \tag{24}$$

where

$$\mathbf{m} = -\mathbf{n}\nabla \left(\nabla \bar{\phi} - U\mathbf{i}\right). \tag{25}$$

Assuming that the body is slender we may include only the free-stream velocity $-U\mathbf{i}$ in the M_j-terms of the body boundary condition:

$$M_1 = 0, \tag{26}$$

$$M_2 = 0, \tag{27}$$

$$M_3 = 0, \tag{28}$$

$$M_4 = 0, \tag{29}$$

$$M_5 = +Un_3 \tag{30}$$

$$M_6 = -n_2 U \tag{31}$$

Neglecting the contribution from the disturbance part of the steady flow, the free-surface condition may be written as

$$\phi_{tt} - 2U\phi_{xt} + U^2\phi_{xx} + g\phi_z = 0, \tag{32}$$

on $z = 0$. With the time dependence taken as $e^{i\omega t}$, we will obtain

$$-\omega^2\phi - 2i\omega U\phi_x + U^2\phi_{xx} + g\phi_z = 0. \tag{33}$$

Further assuming that the encounter frequency is large and the forward speed is moderate, we may neglect the speed-dependent terms in the free-surface condition and then the free-surface condition reduces to

$$-\omega^2\phi + g\phi_z = 0. \tag{34}$$

Then the zero speed Green function may be used with the encounter frequency.

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5.2   Integral equations for the velocity potential

Applying Green's second identity to the control volume Ω bounded by the control surface $S = S_B + S_1 + S_F + S_\infty$ where S_B is the mean wetted surface of the body and S_F is the free-surface at $z = 0$. S_1 as a small sphere enclosing the singular point $r = 0$. S_∞ is a control surface at $r = \infty$. Then

$$\iiint\limits_{\Omega} (G\nabla^2\phi - \phi\nabla^2 G)\,dV = \iint\limits_{S} \left(G\frac{\partial\phi}{\partial n} - \phi\frac{\partial G}{\partial n} \right) ds. \tag{35}$$

Here n is the outward unit normal pointing into the fluid. We will further assume that both the potential ϕ and the Green function G satisfies the Laplace equation, that is $\nabla^2\phi = 0$ and $\nabla^2 G = 0$. The Green function will further satisfy the free-surface condition on S_F and the radiation condition at infinity. Neglecting the speed-dependent terms in the free-surface conditions, the contribution from S_F and S_∞ will then vanish.

At S_1 it will be assumed that the leading behaviour of the Green function is $G \approx 1/r$, and then we can write $\partial(1/r)/\partial n = \partial(1/r)/\partial r = -1/r^2$, and thus

$$\iint\limits_{S_1} \frac{\partial\phi}{\partial n}\frac{1}{r}\,ds \approx 0, \tag{36}$$

$$\iint\limits_{S_1} \phi\frac{\partial}{\partial n}\frac{1}{r}\,ds \approx 4\pi r^2 \left(-\frac{1}{r^2} \right)\phi(x_1, y_1, z_1) = -4\pi\phi(x_1, y_1, z_1), \tag{37}$$

where (x_1, y_1, z_1) is the singular point where $r = 0$. We may then write

$$\iint\limits_{S_1} \left(G\frac{\partial\phi}{\partial n} - \phi\frac{\partial G}{\partial n} \right) ds = 4\pi\phi(x_1, y_1, z_1). \tag{38}$$

We will then obtain

$$4\pi\phi(x_1, y_1, z_1) + \iint\limits_{S_B} \left(G\frac{\partial\phi}{\partial n} - \phi\frac{\partial G}{\partial n} \right) ds = 0. \tag{39}$$

$\partial\phi/\partial n$ is obtained from the body boundary conditions and the integral equations for the different radiation problems and diffraction problem is then obtained as

$$-4\pi\phi_j(x_1, y_1, z_1) + \iint\limits_{S_B} \phi_j\frac{\partial G}{\partial n}\,ds = \iint\limits_{S_B} G\frac{\partial\phi_j}{\partial n}\,ds, \quad j = 1, ..., 7. \tag{40}$$

Introducing the body boundary conditions, we will then obtain

$$4\pi\phi_j(x_1, y_1, z_1) - \iint\limits_{S_B} \phi_j\frac{\partial G}{\partial n}\,ds = -\iint\limits_{S_B} G\frac{\partial\phi_j}{\partial n}\,ds, \quad j = 1, ..., 7, \tag{41}$$

or

$$-4\pi\phi_j(x_1, y_1, z_1) + \iint\limits_{S_B} \phi_j\frac{\partial G}{\partial n}\,ds = \iint\limits_{S_B} G\frac{\partial\phi_j}{\partial n}\,ds, \quad j = 1, ..., 7. \tag{42}$$

A numerical formulation of the zero speed Green function is given by Telste and Noblesse [1986]. Non-dimensional coordinates are formed in terms of some reference length ℓ.

$$\hat{\mathbf{x}} = [x/\ell, y/\ell, z/\ell]^T, \tag{43}$$

$$\hat{\boldsymbol{\xi}} = [\xi/\ell, \eta/\ell, \zeta/\ell]^T. \tag{44}$$

The non-dimensional frequency parameter is determined in terms of the acceleration of gravity as

$$f = \frac{\omega^2 \ell}{g}. \tag{45}$$

The non-dimensional Green function may be written as,

$$\hat{G} = \frac{1}{\hat{r}} + \frac{1}{\hat{r}_1} + 2f\left(R_0(h, v) - i\pi J_0(h)e^v\right), \tag{46}$$

where

$$\rho = \sqrt{(\hat{x} - \hat{\xi})^2 + (\hat{y} - \hat{\eta})^2}, \tag{47}$$

$$\hat{r} = \sqrt{\rho^2 + (\hat{z} - \hat{\zeta})^2}, \tag{48}$$

$$\hat{r}_1 = \sqrt{\rho^2 + (\hat{z} + \hat{\zeta})^2}, \tag{49}$$

$$h = f\rho, \tag{50}$$

$$v = f(\hat{z} + \hat{\zeta}), \tag{51}$$

$$d = \sqrt{h^2 + v^2} = f\hat{r}_1. \tag{52}$$

Further, J_0 is the Bessel function of the first kind.

The Green function may be written as

$$\hat{G} = \frac{1}{\hat{r}} + \frac{1}{\hat{r}_1} + \hat{G}_w, \tag{53}$$

where

$$\hat{G}_w = 2f\left[R_0(h, v) - i\pi J_0(h)e^v\right]. \tag{54}$$

The non-dimensional derivatives of $\hat{G}_w$ are then expressed as

$$\frac{\partial \hat{G}_w}{\partial \hat{x}} = \frac{\partial \hat{G}_w}{\partial h}\frac{\partial h}{\partial \rho}\frac{\partial \rho}{\partial \hat{x}}, \tag{55}$$

$$\frac{\partial \hat{G}_w}{\partial \hat{y}} = \frac{\partial \hat{G}_w}{\partial h}\frac{\partial h}{\partial \rho}\frac{\partial \rho}{\partial \hat{y}}, \tag{56}$$

$$\frac{\partial \hat{G}_w}{\partial \hat{z}} = \frac{\partial \hat{G}_w}{\partial v}\frac{\partial v}{\partial \hat{z}}, \tag{57}$$

where

$$\begin{aligned}
\frac{\partial \hat{G}_w}{\partial h} &= 2f\left[\frac{\partial}{\partial h}R_0(h, v) - i\pi\frac{\partial J_0(h)}{\partial h}e^v\right] \\
&= 2f\left[-R_1(h, v) - i\pi J_1(h)e^v\right] \\
&= -2f\left[R_1(h, v) + i\pi J_1(h)e^v\right],
\end{aligned} \tag{58}$$

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where we have set

$$R_1(h,v) = -\frac{\partial}{\partial h}R_0(h,v). \tag{59}$$

$J_0(h)$ and $J_1(h)$ are the Bessel functions of the first kind. The functions $R_0(h,v)$ and $R_1(h,v)$ are real functions which are expressed in terms of different integral representations, asymptotic expansions, ascending series, Taylor series and Haskind integral representations by Telste and Noblesse [1986].

5.3 Discretization of the integral equations

The mean wetted surface of the body is approximated by a number of quadrilaterals. Each quadrilateral is defined by four points located on the body surface. The corner points are numbered in the clockwise direction when viewed from a point in the fluid domain outside of the body. Triangular panels may be input by collapsing the third and fourth corner points. VERES3D may also read WAMIT geometry files. In this case, the corner points of the quadrilaterals are numbered in the counter-clockwise direction.

The four corner points on the input quadrilaterals will in general not lie in a plane. Plane quadrilateral panels are formed by projecting the four corner points onto a plane passing through the centre of the input quadrilateral. For bodies of complex geometrical shape, gaps may be present between the plane panels. The errors induced on the calculated potentials and resulting forces due to this is insignificant. By assuming that the potentials are constant over each panel, a discretised version of the hydrodynamic boundary value problem may be formed.

A sufficient number of panels must be used in order to catch the variation of the potential over the wavelength. As a general rule of thumb, the typical dimension of the panel should be less than 1/8 of the wavelength.

5.4 Solution of the linear equation systems for the potentials

VERES3D applies a direct equation solver from the LAPACK library to solve the resulting equation systems for the hydrodynamic boundary value problems.

5.5 Calculation of first order forces and motions

The total fluid pressure is given by

$$p = -\rho g z - \rho\frac{\partial \phi_1}{\partial t} - \rho\nabla(\bar{\phi} - Ux)\cdot\nabla\phi_1 - \frac{1}{2}\rho|\nabla\phi_1|^2 - \rho\frac{\partial \phi_2}{\partial t} - \rho\nabla(\bar{\phi} - Ux)\cdot\nabla\phi_2 + O\left(U^2\right), \tag{60}$$

where $\bar{\phi}$ is the steady potential which is neglected in this version of VERES3D using the Neumann-Kelvin approximation.

The first order hydrodynamic forces and moments are then obtained by integrating the pressure over the mean position of the surface of the body.

$$H_j = -\iint_S pn_j\,ds, \qquad j = 1,\dots,6. \tag{61}$$

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The force and moments can be divided into two parts as

$$H_j = F_j + G_j, \tag{62}$$

where F_j is the exciting force and moment

$$F_j = \rho e^{i\omega t} \iint_S \left(i\omega(\phi_0 + \phi_7) + \nabla(\bar{\phi} - Ux) \cdot \nabla(\phi_0 + \phi_7) \right) n_j \, ds, \tag{63}$$

and G_j is the force and moment due to the body motions $\eta_1, \dots, \eta_6$

$$G_j = \rho e^{i\omega t} \iint_S \left(i\omega \sum_{k=1}^{6} \phi_k \eta_k + \sum_{k=1}^{6} \nabla(\bar{\phi} - Ux) \cdot \nabla\phi_k \eta_k \right) n_j \, ds$$

$$= \sum_{k=1}^{6} T_{jk} \eta_k e^{i\omega t}. \tag{64}$$

Here, T_{jk} denotes the hydrodynamic force and moment in the jth direction per unit oscillatory displacement in the kth mode.

$$T_{jk} = \rho \iint_S \left(i\omega \phi_k + \nabla(\bar{\phi} - Ux) \cdot \nabla\phi_k \right) n_j \, ds. \tag{65}$$

T_{jk} may be separated into real and imaginary parts and then

$$T_{jk} = \omega^2 A_{jk} - i\omega B_{jk}, \tag{66}$$

where A_{jk} and B_{jk} are the added mass and damping coefficients, respectively.
The added mass and damping coefficients are then obtained as

$$A_{jk} = \frac{1}{\omega^2} \operatorname{Re} T_{jk}, \tag{67}$$

$$B_{jk} = -\frac{1}{\omega} \operatorname{Im} T_{jk}. \tag{68}$$

The potential of the incident waves is given analytically, while the gradients of the radiation and scattering potentials, $\nabla\phi_k, j = 1, \dots, 7$, are obtained by representing the respective potentials by a source distribution over the bodies and free-surface patch.

Under the assumption that the motion responses are linear and harmonic, the six linear coupled differential equations of motion can be written as

$$\sum_{k=1}^{6} \left[(M_{jk} + A_{jk}(\omega))\ddot{\eta}_k + (B_{jk}(\omega) + \hat{B}_{jk})\dot{\eta}_k + (C_{jk} + \hat{C}_{jk})\eta_k \right]$$

$$= F_j(\omega, \beta)e^{i\omega t}, \quad , j = 1, \dots, 6, \tag{69}$$

where

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M_{jk} are the components of the generalized mass matrix.

A_{jk} are the components of the frequency-dependent added mass matrix.

B_{jk} are the components of the frequency-dependent damping matrix.

$\hat{B}_{jk}$ are the components of the external damping matrix.

C_{jk} are the components of the restoring matrix.

$\hat{C}_{jk}$ are the components of the external restoring matrix.

F_j are the complex amplitudes of the wave-exciting forces and moments. The physical forces are given by the real part of $F_j(\omega, \beta)e^{i\omega t}$. F_1, F_2 and F_3 refer to the amplitudes of the surge, sway and heave exciting forces, while F_4, F_5 and F_6 refer to the amplitudes of the roll, pitch and yaw exciting moments, respectively.

η_k are the complex amplitudes of the surge, sway, heave, roll, pitch and yaw motion responses, respectively. The dots stand for time derivatives, so that $\dot{\eta}_k$ and $\ddot{\eta}_k$ are velocity and acceleration terms, respectively.

The mass forces are forces due to the mass of the body and follow directly from Newton's law. Formally, the mass force due to the harmonic motion mode η_k may be written as

$$F_{jk} = -M_{jk}\ddot{\eta}_k, \tag{70}$$

where M_{jk} are the generalized mass coefficients. The generalized mass matrix may be written as

$$
\mathbf{M} = \begin{bmatrix}
m & 0 & 0 & 0 & mz_G & -my_G \\
0 & m & 0 & -mz_G & 0 & mx_G \\
0 & 0 & m & my_G & -mx_G & 0 \\
0 & -mz_G & my_G & I_x & -I_{xy} & -I_{xz} \\
mz_G & 0 & -mx_G & -I_{yx} & I_y & -I_{yz} \\
-my_G & mx_G & 0 & -I_{zx} & -I_{zy} & I_z
\end{bmatrix}. \tag{71}
$$

Here m is the mass of the body, and x_G, y_G, z_G are the coordinates of the centre of gravity of the body. The moments of inertia I_x, I_y and I_z and the products of inertia $I_{xy} = I_{yx}, I_{xz} = I_{zx}$ and $I_{yz} = I_{xz}$ are defined as

$$I_x = \int_V \rho_m(y^2 + z^2)dV, \tag{72}$$

$$I_y = \int_V \rho_m(x^2 + z^2)dV, \tag{73}$$

$$I_z = \int_V \rho_m(x^2 + y^2)dV, \tag{74}$$

$$I_{xy} = \int_V \rho_m\, xy\, dV = \int_V \rho_m\, yx\, dV = I_{yx}, \tag{75}$$

$$I_{xz} = \int_V \rho_m\, xz\, dV = \int_V \rho_m\, zx\, dV = I_{zx}, \tag{76}$$

$$I_{yz} = \int_V \rho_m\, yz\, dV = \int_V \rho_m\, zy\, dV = I_{zy}. \tag{77}$$

When a body is free floating, the restoring forces will follow from hydrostatic and weight considerations. The restoring force coefficients are independent of the velocity potential and wave frequency,

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and depend only on the body geometry and mass distribution. The force and moment components may be written as

$$F_{jk} = -C_{jk}\eta_k, \tag{78}$$

where C_{jk} are the restoring coefficients.

When the coefficients in the equations of motion are known, the equation system (69) may be solved numerically for the complex motion amplitudes $\hat{\eta}_k$ by a direct equation solver after substitution of

$$\eta_k = \hat{\eta}_k \, e^{i\omega t}. \tag{79}$$

The motion transfer functions are then given by the amplitude η_a and phase angle θ defined by

$$\eta_k(t) = \eta_{ka} \cos(\omega t + \theta_k), \quad k = 1, \dots, 6. \tag{80}$$

By expressing the complex amplitude of the incident waves by $-i\zeta_a$, the transfer functions may be referred to the waves by

$$\eta_{ka} = \sqrt{\left[\mathrm{Re}\left(\frac{\hat{\eta}_k}{-i\zeta_a} \right) \right]^2 + \left[\mathrm{Im}\left(\frac{\hat{\eta}_k}{-i\zeta_a} \right) \right]^2}, \tag{81}$$

$$\theta_k = \arctan\left[\frac{\mathrm{Im}\left(\frac{\hat{\eta}_k}{-i\zeta_a} \right)}{\mathrm{Re}\left(\frac{\hat{\eta}_k}{-i\zeta_a} \right)} \right]. \tag{82}$$

The free-surface elevation is obtained as

$$\zeta = -\frac{1}{g}\left(i\omega\varphi + \nabla(\bar{\phi} - Ux) \cdot \nabla\varphi \right). \tag{83}$$

5.6 Calculation of mean second-order forces

5.6.1 Direct pressure integration method

The mean drift forces may be obtained by integrating the pressure over the instantaneous submerged body and then taking the time average. Following a similar analysis as outlined by Ogilvie [1983] for zero current velocity, Zhao and Faltinsen [1989b] derived expressions for calculating the mean drift forces and moment in waves with forward speed. Neglecting the contribution from the second-order potential, we may obtain the mean drift force as

$$\bar{F}_x = \rho \iint\limits_{S_{B_0}} \left\{ \left[\frac{1}{2}\nabla\phi \cdot \nabla\phi + (\eta_T + \eta_R \times \mathbf{r}) \cdot \nabla\left(\frac{\partial\phi}{\partial t} + \nabla\bar{\Phi} \cdot \nabla\phi \right) \right] n_x + (\eta_R \times \mathbf{n}) \cdot \mathbf{i}\left(\frac{\partial\phi}{\partial t} + \nabla\bar{\Phi} \cdot \nabla\phi \right) \right\} ds$$

$$+ \overline{\frac{1}{2}\rho g \oint\limits_{C_B} \zeta_r^2 n_x \, d\ell} + \overline{\rho g \iint\limits_{S_{B_0}} (\eta_R \times \mathbf{n}) \cdot \mathbf{i}\eta_v \, ds} + \overline{\rho g \iint\limits_{S_{B_0}} z\left(-\frac{1}{2}(\eta_5^2 + \eta_6^2)n_x \right) ds}$$

$$+ \overline{\frac{1}{2}\rho \oint\limits_{C_B} \zeta \sum_{k=1}^{6} \eta_k(\beta_k \cdot \nabla)(\mathbf{W} \cdot \mathbf{W}) \, n_x \, d\ell,} \tag{84}$$

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where

$$\eta_T = [\eta_1, \eta_2, \eta_3]^T, \tag{85}$$

$$\eta_R = [\eta_4, \eta_5, \eta_6]^T, \tag{86}$$

and

$$\mathbf{W} = \nabla(\bar{\phi} - Ux). \tag{87}$$

Please note that in the current version of VERES3D, the steady potentials are not calculated, hence the equations shown here can be simplified.

Further, S_{B_0} is the mean wetted body surface. The calculation of the mean drift force requires that second derivatives of the first order potentials are evaluated on the body with forward speed.

Similarly, the transverse and vertical drift forces may be obtained as

$$\bar{F}_y = \rho \iint_{S_{B_0}} \left\{ \left[\frac{1}{2} \nabla\phi \cdot \nabla\phi + (\eta_T + \eta_R \times \mathbf{r}) \cdot \nabla \left(\frac{\partial\phi}{\partial t} + \nabla\bar{\Phi} \cdot \nabla\phi \right) \right] n_y + (\eta_R \times \mathbf{n}) \cdot \mathbf{j} \left(\frac{\partial\phi}{\partial t} + \nabla\bar{\Phi} \cdot \nabla\phi \right) \right\} ds$$

$$+ \frac{1}{2}\rho g \oint_{C_B} \zeta_r^2 n_y \, d\ell + \rho g \iint_{S_{B_0}} (\eta_R \times \mathbf{n}) \cdot \mathbf{j}\eta_v \, ds + \rho g \iint_{S_{B_0}} z\Big(\eta_4\eta_5 n_x + (-\frac{1}{2}(\eta_4^2 + \eta_6^2))n_y\Big) ds$$

$$+ \frac{1}{2}\rho \oint_{C_B} \zeta \sum_{k=1}^{6} \eta_k(\beta_k \cdot \nabla)(\mathbf{W} \cdot \mathbf{W}) \, n_y \, d\ell, \tag{88}$$

$$\bar{F}_z = \rho \iint_{S_{B_0}} \left\{ \left[\frac{1}{2} \nabla\phi \cdot \nabla\phi + (\eta_T + \eta_R \times \mathbf{r}) \cdot \nabla \left(\frac{\partial\phi}{\partial t} + \nabla\bar{\Phi} \cdot \nabla\phi \right) \right] n_z + (\eta_R \times \mathbf{n}) \cdot \mathbf{k} \left(\frac{\partial\phi}{\partial t} + \nabla\bar{\Phi} \cdot \nabla\phi \right) \right\} ds$$

$$+ \frac{1}{2}\rho g \oint_{C_B} \zeta_r^2 n_z \, d\ell + \rho g \iint_{S_{B_0}} (\eta_R \times \mathbf{n}) \cdot \mathbf{k}\eta_v \, ds + \rho g \iint_{S_{B_0}} z\Big(\eta_4\eta_6 n_x + \eta_5\eta_6 n_y + (-\frac{1}{2}(\eta_4^2 + \eta_5^2))n_z\Big) ds$$

$$+ \frac{1}{2}\rho \oint_{C_B} \zeta \sum_{k=1}^{6} \eta_k(\beta_k \cdot \nabla)(\mathbf{W} \cdot \mathbf{W}) \, n_z \, d\ell. \tag{89}$$

The mean drift moment may also be obtained as

$$\overline{dM_z} = \overline{x \, dF_y - y \, dF_x}. \tag{90}$$

5.6.2 Momentum-flux formulation for the mean drift forces

The momentum in the fluid can be expressed in the inertial reference frame as

$$\mathbf{M}(t) = \iiint_{\Omega} \rho\mathbf{V} \, dv, \tag{91}$$

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where Ω is the volume bounded by the instantaneous wetted surface of the body S_B, the instantaneous free-surface S_F, the bottom surface S_b and the control surface S_∞.
The rate of change of momentum can be developed as

$$\frac{d\mathbf{M}}{dt} = -\rho \iint\limits_{S_\Omega} \left(\frac{p}{\rho}\mathbf{n} + \mathbf{V}(V_n - U_n) \right) ds. \tag{92}$$

The surface S_Ω will consist of the surfaces $S_\Omega = S_B + S_F + S_b + S_\infty$ where S_B is the body surface, S_F is the mean free-surface, S_b is the sea bottom and S_∞ is a control surface surrounding the body. On S_F and S_b we will have $U_n = V_n$ and on S_B and S_∞ we will have $U_n = 0$. Further, the pressure is constant on S_B and $p = 0$ on S_F.

$$\frac{d\mathbf{M}}{dt} = -\rho \iint\limits_{S_B} \left(\frac{p}{\rho}\mathbf{n} + \mathbf{V}V_n \right) ds - \rho \iint\limits_{S_b} \left(\frac{p}{\rho}\mathbf{n} \right) ds - \rho \iint\limits_{S_\infty} \left(\frac{p}{\rho}\mathbf{n} + \mathbf{V}V_n \right) ds. \tag{93}$$

As the sea bottom is assumed horizontal, the contribution from S_b to the horizontal forces and moment can be neglected. We may then write

$$\frac{d\mathbf{M}}{dt} = -\rho \iint\limits_{S_B} \left(\frac{p}{\rho}\mathbf{n} + \mathbf{V}V_n \right) ds - \rho \iint\limits_{S_\infty} \left(\frac{p}{\rho}\mathbf{n} + \mathbf{V}V_n \right) ds. \tag{94}$$

Conservation of momentum then gives $\dfrac{\overline{d\mathbf{M}}}{dt} = 0$. The mean horizontal forces may then be obtained as

$$\overline{\mathbf{F}} = \rho \overline{\iint\limits_{S_B} \left(\frac{p}{\rho}\mathbf{n} + \mathbf{V}V_n \right) ds}$$

$$= -\rho \overline{\iint\limits_{S_\infty} \left(\frac{p}{\rho}\mathbf{n} + \mathbf{V}V_n \right) ds}. \tag{95}$$

The control surface S_∞ may be divided into two parts: The contribution from the surface below the undisturbed mean free-surface S_C, and the contribution from the surface above the mean free-surface S_c. Then

$$\overline{\mathbf{F}} = -\rho \overline{\iint\limits_{S_C} \left(\frac{p}{\rho}\mathbf{n} + \mathbf{V}V_n \right) ds} - \rho \overline{\iint\limits_{S_c} \left(\frac{p}{\rho}\mathbf{n} + \mathbf{V}V_n \right) ds}. \tag{96}$$

We will then use that

$$\iint\limits_{S_c} \left(\frac{p}{\rho}\mathbf{n} + \mathbf{V}V_n \right) ds = \oint\limits_{L_C} \int_0^\zeta \left(\frac{p}{\rho}\mathbf{n} + \mathbf{V}V_n \right) dz\, d\ell. \tag{97}$$

After further development, the mean second-order forces may be expressed in terms of the first order potentials only.

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The mean drift forces may then be obtained as

$$
\overline{F_x} = \frac{1}{2}\rho \overline{\iint_{S_C} (\nabla\phi \cdot \nabla\phi)\, n_x\, ds} - \rho \overline{\iint_{S_C} \phi_x\, \phi_n\, ds}
$$

$$
- \rho \overline{\oint_{L_C} \frac{1}{2} g\zeta^2 n_x\, d\ell} - \rho \overline{\oint_{L_C} \left(\bar{\phi}_n \phi_x + \bar{\phi}_x\, \phi_n\right)\zeta\, d\ell} + \rho U \overline{\oint_{L_C} n_x \phi_x\, \zeta\, d\ell}. \tag{98}
$$

Further

$$
\overline{F_y} = \frac{1}{2}\rho \overline{\iint_{S_C} (\nabla\phi \cdot \nabla\phi)\, n_y\, ds} - \rho \overline{\iint_{S_C} \phi_y\, \phi_n\, ds}
$$

$$
- \rho \overline{\oint_{L_C} \frac{1}{2} g\zeta^2 n_y\, d\ell} - \rho \overline{\oint_{L_C} \left(\bar{\phi}_n \phi_y + \bar{\phi}_y\, \phi_n\right)\zeta\, d\ell} + \rho U \overline{\oint_{L_C} n_x \phi_y\, \zeta\, d\ell}. \tag{99}
$$

Similarly, then expression for the moment may be obtained from conservation of angular momentum as

$$
\overline{M_z} = -\rho \overline{\iint_{S_C} \left(\frac{p}{\rho}(xn_y - yn_x) + (\mathbf{r} \times \mathbf{V})V_n\right) ds} - \rho \overline{\iint_{S_c} \left(\frac{p}{\rho}(xn_y - yn_x) + (\mathbf{r} \times \mathbf{V})V_n\right) ds}. \tag{100}
$$

After further developments, this gives

$$
\overline{M_z} = \frac{1}{2}\rho \overline{\iint_{S_C} \left(\nabla\phi \cdot \nabla\phi\right)(xn_y - yn_x)\, ds} - \rho \overline{\iint_{S_C} (x\phi_y - y\phi_x)\phi_n\, ds}
$$

$$
- \rho \overline{\oint_{L_C} \frac{1}{2} g\zeta^2 (xn_y - yn_x)\, d\ell}
$$

$$
- \rho \overline{\oint_{L_C} \left(\left(x\phi_y - y\phi_x\right)\bar{\phi}_n + \left(x\bar{\phi}_y - y\bar{\phi}_x\right)\phi_n\right)\zeta\, d\ell} + \rho \overline{\oint_{L_C} U n_x \left(x\phi_y - y\phi_x\right)\zeta\, d\ell}
$$

$$
- \rho \overline{\oint_{L_C} U y \phi_n \zeta\, d\ell}. \tag{101}
$$

5.6.3 Far-field formulation for mean drift forces for zero speed

The far-field method is based on conservation of momentum. In this case, the control surface has the form of a vertical cylinder far away from the body. Asymptotic formulas can then be derived for radiation and scattering potentials. Integration in the vertical direction of the control surface can be done analytically.

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We may obtain the mean drift force as

$$\bar{F}_x = -\frac{\rho g}{2\pi\nu} \int_0^{2\pi} \left|\Xi(\theta)\right|^2 \cos\theta \, d\theta + \frac{\rho g \zeta_a}{\nu} \cos\beta \operatorname{Re}\left(\Xi(\beta + \pi)\right),$$ (102)

$$\bar{F}_y = -\frac{\rho g}{2\pi\nu} \int_0^{2\pi} \left|\Xi(\theta)\right|^2 \sin\theta \, d\theta + \frac{\rho g \zeta_a}{\nu} \sin\beta \operatorname{Re}\left(\Xi(\beta + \pi)\right),$$ (103)

$$\bar{M}_z = \frac{\rho}{2} \frac{g}{\pi\nu^2} \int_0^{2\pi} \operatorname{Im}\left(\Xi^*(\theta)\Xi'(\theta)\right) d\theta + \rho \frac{g\zeta_a}{\nu^2} \operatorname{Im}\left(\Xi'(\beta + \pi)\right).$$ (104)

The amplitude function is given by

$$\Xi_j(\theta) = -\frac{i}{2}\frac{\omega_0}{g}\nu H_j(\theta).$$ (105)

The derivative of the amplitude function is then obtained as

$$\Xi_j'(\theta) = -i\frac{\omega_0}{2g}\nu H_j'(\theta).$$ (106)

The Kochin functions $H_j(\theta)$ are given by

$$H_j(\theta) = \iint_{S_B} \phi_j \eta_j \frac{\partial\chi(\theta)}{\partial n}\, ds - \iint_{S_B} \chi(\theta)\left(i\omega n_j + M_j\right)\eta_j\, ds, \quad j = 1, \dots, 6,$$ (107)

and

$$H_7(\theta) = \iint_{S_B} \phi_7 \frac{\partial\chi(\theta)}{\partial n}\, ds - \iint_{S_B} \chi(\theta)\frac{\partial\phi_7}{\partial n}\, ds$$

$$= \iint_{S_B} \phi_7 \frac{\partial\chi(\theta)}{\partial n}\, ds + \iint_{S_B} \chi(\theta)\frac{\partial\phi_0}{\partial n}\, ds.$$ (108)

Further, $\chi(\theta)$ is given by

$$\chi(\theta) = e^{\nu(\zeta + i\xi\cos\theta + i\eta\sin\theta)},$$ (109)

$$H_j'(\theta) = \iint_{S_B} \phi_j \eta_j \frac{\partial\chi'(\theta)}{\partial n}\, ds - \iint_{S_B} \chi'(\theta)\left(i\omega n_j + M_j\right)\eta_j\, ds, \quad j = 1, \dots, 6$$ (110)

and

$$H_7'(\theta) = \iint_{S_B} \phi_7 \frac{\partial\chi'(\theta)}{\partial n}\, ds - \iint_{S_B} \chi'(\theta)\frac{\partial\phi_7}{\partial n}\, ds$$

$$= \iint_{S_B} \phi_7 \frac{\partial\chi'(\theta)}{\partial n}\, ds + \iint_{S_B} \chi'(\theta)\frac{\partial\phi_0}{\partial n}\, ds.$$ (111)

Here $\chi'(\theta)$ is given by

$$\chi'(\theta) = -i\nu(\xi\sin\theta - \eta\cos\theta)e^{\nu(\zeta + i\xi\cos\theta + i\eta\sin\theta)}$$

$$= -i\nu(\xi\sin\theta - \eta\cos\theta)\chi(\theta).$$ (112)

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5.7 Removal of irregular frequencies

The methods applied in removing irregular frequencies can be grouped into two categories, Lau and Hearn [1989]:

- Modification of the integral operator.

- Modification of the domain of the integral operator.

The boundary condition for the fictitious inner potential ϕ_j' on S_i may be chosen arbitrarily. We will set

$$\frac{\partial \phi_j'(\mathbf{x})}{\partial n} = 0, \quad \mathbf{x} \in S_i. \tag{113}$$

The resulting boundary value problem for ϕ_j and the fictitious inner problem for ϕ_j' may then be obtained as

$$-4\pi\phi_j(x,y,z) + \iint\limits_{S_B} \phi_j \frac{\partial G}{\partial n}\, ds + \iint\limits_{S_i} \phi_j' \frac{\partial G}{\partial n}\, ds = \iint\limits_{S_B} G \frac{\partial \phi_j}{\partial n}\, ds, \quad j = 1,\dots,7, \tag{114}$$

when (x, y, z) is located on the surface of the body S_B, and

$$4\pi\phi_j'(x,y,z) + \iint\limits_{S_B} \phi_j \frac{\partial G}{\partial n}\, ds + \iint\limits_{S_i} \phi_j' \frac{\partial G}{\partial n}\, ds = \iint\limits_{S_B} G \frac{\partial \phi_j}{\partial n}\, ds, \quad j = 1,\dots,7, \tag{115}$$

when (x, y, z) is located on the surface of the interior free-surface S_i. The two problems are solved simultaneously for ϕ_j on S_B and ϕ_j' on S_i. Then ϕ_j' on S_i is discarded and only ϕ_j is used in the further analysis.

5.8 Including viscous damping

In order to predict the roll motion of a ship it is important to include the contribution from viscous damping due to hull friction, eddy making and bilge keels. In order to account for linear and non-linear viscous damping in roll and pitch, these equations are modified as follows

$$\sum_{k=1}^{6} \left[(M_{4k} + A_{4k}(\omega))\ddot{\eta}_k + (B_{4k}(\omega) + \hat{B}_{4k})\dot{\eta}_k + (C_{4k} + \hat{C}_{4k})\eta_k \right]$$

$$+ B_{44}^{V_1}\dot{\eta}_4 + B_{44}^{V_2}|\dot{\eta}_4|\dot{\eta}_4 = F_4(\omega,\beta)e^{i\omega t}, \tag{116}$$

and

$$\sum_{k=1}^{6} \left[(M_{5k} + A_{5k}(\omega))\ddot{\eta}_k + (B_{5k}(\omega) + \hat{B}_{5k})\dot{\eta}_k + (C_{5k} + \hat{C}_{5k})\eta_k \right]$$

$$+ B_{55}^{V_1}\dot{\eta}_5 + B_{55}^{V_2}|\dot{\eta}_5|\dot{\eta}_5 = F_5(\omega,\beta)e^{i\omega t}. \tag{117}$$

Here the superscripts V_1 and V_2 denote the linear and quadratic viscous damping terms, respectively. The equations are non-linear due to the quadratic terms and must be solved by an iteration technique

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where the non-linear terms are linearised. By using equivalent linearisation a 'linear' term $K\dot{\eta}$ will be introduced. The linearised damping factor K is determined by requiring the work carried out over one oscillation period to be identical for this 'linear' term and the non-linear term. Thus

$$\int_0^{\frac{2\pi}{\omega}} K\dot{\eta}\dot{\eta}\, dt = 4 \int_0^{\frac{\pi}{2\omega}} B^{V_2}\dot{\eta}|\dot{\eta}|\dot{\eta}\, dt. \tag{118}$$

Introducing $\eta = \eta_a \sin \omega t$ yields

$$K = \frac{8\omega}{3\pi} B^{V_2} \eta_a. \tag{119}$$

Since the linearised term is dependent on the motion amplitude, an iterative approach is required in the solution of the motion equations. Strictly, this linearisation is only valid in regular waves. For irregular waves, stochastic linearisation is a better approach. This gives

$$K = \sqrt{\frac{8}{\pi}} B^{V_2} \sigma_{\dot{\eta}}, \tag{120}$$

where $\sigma_{\dot{\eta}}$ is the variance of the roll or pitch velocity. Stochastic linearisation is more conveniently implemented in a post-processor. It should be noted that the transfer functions obtained with equivalent linearisation are strictly valid only for the wave amplitude applied in the linearisation. For a ship, the viscous damping terms may be calculated by VERES and then imported to VERES3D. In a later version it may be directly imported to VERES3D and the solutions for the motions will be obtained by iteration.

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A Output file formats

A.1 WAMIT output files

VERES3D may write output files in the WAMIT format for use with postprocessors compatible with these file formats. The WAMIT file formats are documented in the WAMIT user manual which may be downloaded from WAMIT Inc. (www.wamit.com).

Added resistance can be calculated by multiple methods (direct pressure integration, control surface integration), hence multiple *.2 files can be written:

- If only one method is used, only one *.2 file is written

- If multiple methods are used, two files may be written:

    - $shipCaseName.8$: contains results from the control surface integration.
    - $shipCaseName\_dpi.8$: contains results from the direct pressure integration.
    - $shipCaseName\_ff.8$: contains results from the far-field method.

The wave periods listed in the output files are the global wave periods accounting for current (corresponding to the encounter periods in seakeeping problems).

The files written by VERES3D will use ulen = 1.0.

A.2   VERES output files

A.2.1   Motion transfer functions (*.re1)

This section describes the file that contains data of predicted motions from the VERES program. The format of this file is:

```
'MOTION TRANSFER FUNCTIONS.'
CARDID(1)
CARDID(2)
CARDID(3)
CARDID(4)
CARDID(5)
RHOSW    GRAV
LPP      BREADTH    DRAUGHT
LCG      VCG
NOVEL    NOHEAD     NOFREQ    NDOF
do i=1,NOVEL
   VEL(i)   SINK(i)   TRIM(i)   XMOT(i)   ZMOT(i)
   do j=1,NOHEAD
      HEAD(j)*180.0/3.1415926
      do k=1,NOFREQ
         FREQ(k)
         do l=1,NDOF
            DOF(l)   RETRANS(i,j,k,l)   IMTRANS(i,j,k,l)
         enddo
      enddo
   enddo
enddo
```

Here:

CARDID(1:5)	Vessel-identifying text	Char.	
RHOSW	Density of water	R	$[kg/m^3]$
GRAV	Acceleration of gravity	R	$[m/s^2]$
LPP	Length between the perpendiculars	R	[m]

BREADTH	Vessel breadth	R	[m]
DRAUGHT	Vessel draught	R	[m]
LCG	Longitudinal center of gravity (rel. $L_{pp}/2$)	R	[m]
VCG	Vertical center of gravity (rel. BL)	R	[m]
NOVEL	Number of vessel velocities	I	[-]
NOHEAD	Number of wave headings	I	[-]
NOFREQ	Number of wave frequencies	I	[-]
NDOF	Number of degrees of freedom	I	[-]
VEL(i)	Vessel velocity	R	[m/s]
SINK(i)	Sinkage at a given velocity	R	[m]
TRIM(i)	Trim at a given velocity	R	[deg]
XMOT(i)	Longitudinal pos. of motion coord. system (rel. $L_{pp}/2$)	R	[m]
ZMOT(i)	Vertical pos. of motion coord. system (rel. BL)		
HEAD(j)	Wave heading	R	[rad]
FREQ(k)	Wave frequency	R	[rad/s]
DOF(l)	Degree of freedom	I	[-]
RETRANS(i,j,k,l)	Real part of complex motion transfer function	R	[-]
IMTRANS(i,j,k,l)	Imaginary part of complex motion transfer function	R	[-]

The motion transfer functions are defined as the motion amplitude divided by wave amplitude for all degrees of freedom.

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A.2.2 Added resistance (*.re2)

This section describes the file that contains data of predicted added resistance in waves from the VERES program. The format of this file is:

```
'ADDED RESISTANCE.'
CARDID(1)
CARDID(2)
CARDID(3)
CARDID(4)
CARDID(5)
RHOSW    GRAV
LPP      BREADTH    DRAUGHT
NOVEL    NOHEAD     NOFREQ
do i=1,NOVEL
   do j=1,NOHEAD
      VEL(i)   HEAD(j)*180.0/3.1415926
      do k=1,NOFREQ
         FREQ(k)  ADDEDR(i,j,k)
      enddo
   enddo
enddo
```

Here:

CARDID(1:5)	Vessel identifying text	Char.	
RHOSW	Density of water	R	$[kg/m^3]$
GRAV	Acceleration of gravity	R	$[m/s^2]$
LPP	Length between the perpendiculars	R	[m]
BREADTH	Vessel breadth	R	[m]
DRAUGHT	Vessel draught	R	[m]
NOVEL	Number of vessel velocities	I	[-]
NOHEAD	Number of wave headings	I	[-]
NOFREQ	Number of wave frequencies	I	[-]
VEL(i)	Vessel velocity	R	[m/s]
HEAD(j)	Wave heading	R	[rad]
FREQ(k)	Wave frequency	R	[rad/s]
ADDEDR(i,j,k)	Added resistance in waves	R	[-]

The added resistance is made non-dimensional as:

$$ADDEDR(i,j,k) = \frac{R_{AW}}{\rho g \zeta_a^2 B^2/L} \tag{121}$$

where R_{AW} is the added resistance, ρ is the density of water, g is the acceleration of gravity, ζ_a^2 is the wave amplitude, B is the breadth and L is the length between the perpendiculars.

Added resistance can be calculated by multiple methods (direct pressure integration, control surface integration), hence multiple .re2 files can be written:

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- If only one method is used, only one *.re2 file is written.

- If multiple methods are used, two files may be written:

    - $shipCaseName.re2$: contains results from the control surface integration.

    - $shipCaseName\_dpi.re2$: contains results from the direct pressure integration.

    - $shipCaseName\_ff.re2$: contains results from the far-field method.

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