# Nonconvex Environmental Constraints in Hydropower Scheduling

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***Abstract*—Environmental constraints in hydropower systems serve to ensure sustainable use of water resources. Through accurate treatment in hydropower scheduling, one seeks to respect such constraints in the planning phase while optimizing the utilization of hydropower. However, many environmental constraints introduce state-dependencies and even nonconvexities to the scheduling problem, making them challenging to capture. This paper describes how the recently developed stochastic dual dynamic integer programming (SDDiP) method can incorporate nonconvex environmental constraints in the medium- and long-term scheduling of a hydropower system in a liberalized market context. A mathematical model is presented and tested in a multi-reservoir case study, emphasizing on the improvements observed when accurately modelling a particular type of nonconvex environmental constraint.**

***Index Terms*—Hydroelectric power generation, Power generation economics, Linear programming, Integer linear programming, Stochastic processes.**

## I. Introduction

Hydropower is a dominant generation technology in the Nordic power system, e.g., accounting for 143 TWh/year or 96% of the total power production in Norway in 2017 [1]. In the future, the Nordic power system will have tighter connections with Europe and an increasing proportion of intermittent renewable generation from, for example, wind and solar power. Rapid and unpredictable fluctuations in intermittent generation will offer new possibilities for controllable generation, such as regulated hydropower, to be able to respond to these fluctuations. Flexible and fast-responding power plants able to produce at demand peaks will therefore see a higher profit potential.

Operational planning (or scheduling) models have been widely used by Nordic hydropower producers for several decades. Although these have been developed along different methodological tracks, the stochastic dual dynamic programming (SDDP) algorithm introduced in [2], is currently the state-of-the-art method for medium-term hydropower scheduling in the Nordic market. In particular, the extended algorithm to incorporate uncertainty in power price presented in [3]–[5] has become popular. The SDDP algorithm allows for optimization of hydropower schedules provided a detailed and complex system description and uncertainties in e.g. inflow and power price. The SDDP algorithm is based on linear programming (LP) and requires a convex model formulation. A recent reformulation of the SDDP algorithm, known as stochastic dual dynamic integer programming (SDDiP), has proven convergence also in the nonconvex case. SDDiP was first presented in [6], and has later been applied to hydropower scheduling in [7]. This work compares the strategies obtained from the SDDiP algorithm and its linear approximation (akin to SDDP) when applied to a scheduling problem with nonconvex environmental constraints.

In symphony with the ongoing power market changes, the physical and environmental requirements associated with hydropower operation are changing, e.g. through proposed revisions of hydropower concessions conditions and the implementation of EU Water Framework Directive. The directive strives to ensure sustainable use of water resources, balancing the multiple uses such as hydropower, agriculture and recreation [8]. Consequently, hydropower producers need to both adjust their operational schedules according to the new price patterns seen in the market and at the same time relate to new operational constraints. In this context, the producers need scheduling models that represent physical and environmental constraints in a precise and consistent manner.

*Environmental constraints* in water resources systems come in many flavors, depending physical and legislative conditions. The technical literature tend to emphasize on ecologically acceptable flows, in terms of magnitude and rate of change [9]–[12]. The impact of minimum flows and maximum ramping rates certainly limits the flexibility of the hydro system, but from a modelling point of view these constraints fit well into algorithms relying on a convex model formulation, such as SDDP. Other environmental constraints involve state-dependencies which are not easily treated in the SDDP algorithm, see e.g. [13]. The main complicating factor is the nonconvexities associated with such constraints.

In this paper we focus on constraints on maximum discharge from hydropower reservoirs and their dependencies on reservoir level. This constraint type is enforced in some Norwegian reservoirs and is believed to have significant impact on the hydropower scheduling for a number of watercourses in the future. We elaborate on how to accurately model this type of state-dependent and nonconvex constraint within the SDDiP algorithm, and compare the exact solution with a linear approximation. The different formulations are tested in multi-reservoir case study, and compared in terms of economic

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indicators and reservoir operation. The novel contributions of this paper lie in the modelling of the nonconvex environmental constraints within the SDDiP algorithm and in the assessment of the performance of the proposed model when applied to a realistic case study.

## II. Problem Formulation

In a liberalized electricity market a risk-neutral hydropower producer's primary objective is to find an operating strategy that maximizes the expected profit for the entire planning period while respecting all relevant constraints. This decision problem can be formulated as a multi-stage stochastic optimization problem, and the expectation is to be taken over the stochastic variables, typically inflow and power price. We assume that the probability distributions of the stochastic variables can be discretized, and that the problem can be decomposed into weekly decision stages.

For the Nordic electricity market, we assume that there is a sufficient number of players such that a price-taker assumption is reasonable. Moreover, we assume that the system is confined within a single price zone where potential grid bottlenecks can be disregarded. This assumption can be justified in the Nordic market context, where the scheduling mainly provides decision support for the day-ahead bidding process, and intra-zone grid bottlenecks are handled through redispatch by the transmission system operator after day-ahead market clearing. Consequently, the hydropower producer can optimize the operation of a single watercourse individually without including other generation or demand obligations that are part of its portfolio.

We define a scheduling problem in (1), comprising state variables $x_t$ and stage variables $y_t$ for each decision stage $t$. The state variables are typically the reservoir volumes. There may also be other states in $x_t$, such as inflows, but these will not be emphasized here. The stage variables represent the operational decisions to be made in a given stage, typically concerning water releases, overflows and transactions with the electricity market. For the scheduling problem in (1), we want to find an operating strategy that maximizes the expected profit in (1a) and accounts for the end-of-horizon valuation of stored water in $\Phi(x_T)$. The expectation in (1a) is taken over the stochastic parameter $\xi_t$, representing the inflow. For simplicity we will treat price as a deterministic exogenous variable in this work, see e.g. [3], [14] or [15] for treatment of stochastic price within the SDDP algorithm.

$$\max_{(x_1,y_1),\ldots,(x_T,y_T)} \mathbb{E}\left\{ \sum_{t=1}^{T} f_t(x_t,y_t) + \Phi(x_T) \right\} \tag{1a}$$

$$\text{s.t.} \quad Wx_t + Hx_{t-1} + Gy_t = h(\xi_t) \tag{1b}$$

$$By_t = 0 \tag{1c}$$

$$(x_t,y_t) \in X_t \tag{1d}$$

$$\forall t \in \{1,2,\ldots,T\}$$

The constraints are indicated in (1b)-(1d), where the initial state vector $x_0$ is given and $W$, $H$, $G$, and $B$, are matrices of suitable dimensions, and where $X_t$ is a non-empty mixed-integer polyhedral set. The right-hand-side parameter vector $h(\xi_t)$ is dependent on the random data vector $\xi_t$ whose distribution is known, and where $\xi_t$ are the realizations.

For efficient solution of problems like (1) the use of decomposition techniques are often crucial [16]. The problem in (1) can be decomposed into stage-wise nested decision problems of type:

$$\mathcal{Q}_t(x_{t-1}) = \max_{x_t,y_t} f_t(x_t,y_t) + \alpha_t(x_t) \tag{2a}$$

$$\text{s.t.} \quad (x_t,y_t) \in X_t(x_{t-1},\xi_t) \tag{2b}$$

In Section II-A we elaborate on the stage-wise problem in (2) and define the environmental constraints. The overall solution strategy for solving (1) is outlined in Section III.

### A. Basic Stage Problem

In the following we describe the basic scheduling problem of a given decision period $t$ and inflow scenario, with a typical length of one week. The stage and state variables comprised in vectors $x_t$ and $y_t$ in (1) are now specified. As a simplification, but without loss of generality, we only consider the reservoir volume as a state variable in this work. A thorough treatment of inflow as a state variable in SDDiP and SDDP, can e.g. be found in [7] and [17], respectively.

The problem in (3) is an LP problem, but will be extended with nonconvex environmental constraints and will be cast as a mixed integer linear programming (MIP) problem in Section II-B. We omit the scenario index for brevity.

$$\mathcal{Q}_t(v_{t-1}) = \max\Big(p_t e_t + \alpha_t\Big) \tag{3a}$$

$$v_{ht} + q^D_{ht} + q^O_{ht} + q^B_{ht} - \sum_{j\in\Omega_h}\left(q^D_{jt} + q^O_{jt} + q^B_{jt}\right) = v_{h,t-1} + I_{ht} \qquad \forall h \tag{3b}$$

$$q^D_{ht} = \sum_{k\in\mathcal{K}_h} q^S_{hkt} \qquad \forall h \tag{3c}$$

$$e_t - \sum_{h\in\mathcal{H}}\sum_{k\in\mathcal{K}_h} \eta_{hk} q^S_{hkt} = 0 \tag{3d}$$

$$\alpha_t - \sum_{h\in\mathcal{H}} \pi_{htc} v_{ht} \le \beta_{tc} \qquad \forall c \tag{3e}$$

$$\underline{V}_{ht} \le v_{ht} \le \bar{V}_{ht} \qquad \forall h \tag{3f}$$

$$\underline{Q}^D_{ht} \le q^D_{ht} \le \bar{Q}^D_{ht} \qquad \forall h \tag{3g}$$

$$\underline{Q}^B_{ht} \le q^B_{ht} \le \bar{Q}^B_{ht} \qquad \forall h \tag{3h}$$

$$e_t, q^O_{ht} \in \mathbb{R}^+ \tag{3i}$$

The objective (3a) is to maximize the profit from selling energy $e_t$ to the market in week $t$ at a price $p_t$. This 'here-and-now' profit is balanced against the future expected profit (FEP)

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function. The FEP function is represented by the variable $\alpha_t$ which is constrained by a set $c \in \mathcal{C}$ Benders cuts in (3e) with cut coefficients $\pi_{htc}$ and intercepts $\beta_{tc}$.

The state vector $v_{t-1}$ in (3a) comprises the initial reservoir volumes for all reservoirs $h \in \mathcal{H}$. The water balance equation for a specific reservoir $h$ is formulated in (3b). The reservoir volume $v_{ht}$ at the end of the week is a function of the reservoir at the beginning of the week $v_{h,t-1}$, the inflow $I_{ht}$, the water spillage $q^O_{ht}$ and release (bypass $q^B_{ht}$ and the discharge through the station $q^D_{ht}$) to downstream modules, and the spillages and releases from upstream modules in $\Omega_h$.

The generation in (3d) can be seen as a simplified representation of the generally nonconvex hydropower production function, which is well suited for many Norwegian hydropower stations with rather small variations in relative head. Water discharge through the station is modeled using one variable $q^S_{hkt}$ per discharge segment in (3d). These segments will be uploaded in ascending order provided that the efficiency $\eta_{hk}$ decreases with $k$.

### B. Environmental Constraints

As mentioned in the introduction, environmental constraints comes in many flavors. Time-dependent boundaries on reservoir (3f), discharge (3g) and bypass (3h) variables is one example. Constraints on ramping on water releases is another. These constraints have been addressed previously in the literature, and fits well within the LP formulation in (3) and do not challenge the convexity requirement of the SDDP algorithm.

In the following we will emphasize on state-dependent maximum discharge constraints. This type of constraint is often enforced to limit release and thereby ensure a secure refilling of the reservoirs prior to and during the summer season to protect the landscape and satisfy recreational interests. If the reservoir is below a certain threshold surface level, the operator is not allowed to release water for generation, only to secure that more critical constraints downstream are met. Since the threshold surface level corresponds to a given reservoir volume $V^{lim}_{ht}$, the constraint can be formulated as:

$$\underline{Q}^D_{ht} \leq q^D_{ht} \leq \bar{Q}^D_{ht} \qquad \texttt{if} \qquad \frac{v_{ht} + v_{h,t-1}}{2} \geq V^{lim}_{ht} \tag{4a}$$

$$q^D_{ht} = \underline{Q}^D_{ht} \qquad \texttt{if} \qquad \frac{v_{ht} + v_{h,t-1}}{2} \leq V^{lim}_{ht} \tag{4b}$$

The average reservoir volume for the week is used to indicate if the constraint (4b) should be activated. The binary nature of the constraint (4) causes a significant discontinuity in the scheduling problem that is not easily approximated in LP formulations. Constraint (4) can be modelled by introducing a binary variables $\gamma_h$, so that $\forall h \in \mathcal{H}$:

$$\underline{Q}^D_{ht} \leq q^D_{ht} \leq \gamma_h \left(\bar{Q}^D_{ht} - \underline{Q}^D_{ht}\right) + \underline{Q}^D_{ht} \tag{5a}$$

$$\frac{v_{ht} + v_{h,t-1}}{2} \geq \gamma_h V^{lim}_{ht} \tag{5b}$$

$$\gamma_h \in \{0,1\} \tag{5c}$$

To incorporate the modelling of state-dependent release constraints, we add (5) to (3), and remove (3g). Due to the binary variable $\gamma_h$ the problem is now cast as an MIP problem.

Note that in some hydropower systems, the *physical* dependency of maximum discharge on the reservoir level is significant, but does not have the binary nature of the environmental constraints presented here. In such cases, linear approximations can often provide satisfactory results.

## III. Solution Method

Although the MIP problem formulated in the previous section could be solved directly using classical stochastic dynamic programming such as in [18], this type of algorithm has limits in dealing with the state space in multi-reservoir systems. The recently developed SDDiP algorithm [6] is believed to have a higher potential for handling realistic multi-reservoir hydropower systems than SDDP, and is outlined in the following.

### A. The SDDiP algorithm

As the scheduling problem described in the previous section contains binary stage variables, the FEP function (represented by $\alpha$) is nonconvex with respect to the state variables. To solve the nonconvex scheduling problem comprising (3) and (5) as specified in Sections II-A and II-B, we apply the SDDiP method [6]. The key concept of the method is to represent state variables as binary variables, knowing that any function of binary variables can be represented as a convex polyhedral function. In our case, the reservoir volumes, which are the continuous variables $v_{ht}$ in (3), are the state variables and can be represented by a binary approximation in (6).

$$v_{ht} = \sum_{n \in \mathcal{N}_h} 2^{n-1} \epsilon \lambda_{hn} = C_{hn} \lambda_{hn} \tag{6a}$$

$$\lambda_{hn} \in \{0,1\} \tag{6b}$$

$$|\mathcal{N}_h| = \lfloor \log_2 \left(\frac{\bar{V}_{ht}}{\epsilon}\right) + 1 \rfloor \tag{6c}$$

where $\epsilon \in (0,1)$ is the approximation precision (which we will set equal for all reservoirs), and $|\mathcal{N}_h|$ is the number of binary variables needed to approximate $v_{ht}$ to the precision $\epsilon$. As an example, the approximation of a reservoir volume variable with $\bar{V}_{ht}$ = 200 Mm$^3$ to a precision of 0.01 Mm$^3$ requires 15 binary variables. With this binary approximation, we can substitute $v_{ht}$ as expressed in (6a) in (3b), (3e), (3f) and (5b).

The second important reformulation used in the SDDiP method is to generate local copies of the state variables. This reformulation ensures that one is able to generate cuts that accurately approximate the FEP function. The DP formulation of the problem with binary state variables becomes:

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$$\mathcal{Q}_t(\lambda_{t-1}) = \max \Big( p_t e_t + \alpha_t \Big) \tag{7a}$$

$$\text{s.t.} \quad \text{(3) and (5)} \tag{7b}$$

$$z_{hnt} = \lambda_{hn,t-1} \qquad \forall h, n \tag{7c}$$

$$z_{hnt} \in [0, 1] \qquad \forall h, n \tag{7d}$$

$$\lambda_{hnt} \in \{0, 1\} \qquad \forall h, n \tag{7e}$$

Where (7c) are the local copies of the state variables, connecting the previous state solution, $\lambda_{hn,t-1}$, and the copy variables $z_{hnt}$ for all reservoirs $h \in \mathcal{H}$ and binary variables $n \in \mathcal{N}_h$.

The SDDiP algorithm is outlined in below. In the forward iteration, the MIP problem defined in (7) is solved. In the backward iteration different relaxations of (7) are solved, depending on the type of cuts to be computed.

Apply binary expansion on reservoir variables
Set $\lambda_0$, $i \leftarrow 1$, $\mathrm{UB} = +\infty$ and $\mathrm{LB} = -\infty$,
**while** $i < i^{\max}$ **or** some other stopping criteria **do**
  Sample $S$ scenarios
  /* Forward Iteration */
  **for** s=1,...,S **do**
    **for** t=1,...,T **do**
      Solve $\mathcal{Q}^i_{ts}(\lambda^i_{t-1,s})$
      Collect solution
    **end for**
    $\mathrm{lb}_s \leftarrow \sum_{t=1,\ldots,T} \mathcal{Q}^i_{ts}$
  **end for**
  /* Compute lower bound */
  $\mu \leftarrow \frac{1}{S}\sum_{s=1}^{S} \mathrm{lb_s}$ and $\sigma^2 \leftarrow \frac{1}{S-1}\sum_{s=1}^{S}(\mathrm{lb^s} - \mu)^2$
  $\mathrm{LB} \leftarrow \mu + \mathrm{z}_\alpha \frac{\sigma}{\sqrt{\mathrm{S}}}$
  /* Backward Iteration */
  **for** t=T,..., 2 **do**
    **for** s=1,..., S **do**
      **for** b=1,..., B **do**
        Solve a suitable relaxation of $\mathcal{Q}^i_{tsb}(\lambda^i_{t-1,s})$
        Collect cut coefficients and parameters
      **end for**
      Create desired cuts as described in Sec. III-B
    **end for**
  **end for**
  /* Compute upper bound */
  $\mathrm{UB} \leftarrow \mathcal{Q}^{\mathrm{i}}_1(\lambda_0, \xi_0)$
  $i \leftarrow i + 1$
**end while**

### B. Cut Types

Cuts to be constructed in the backward iteration will take the form:

$$\alpha_t - \sum_{h\in\mathcal{H}} \sum_{n\in\mathcal{N}_h} \pi_{hntc} C_{hn} \lambda_{hnt} \leq \beta_{tc} \tag{8}$$

A brief description of the 3 cut types used in the SDDiP algorithm in the case study in Section IV is provided below, please see [6], [7] for a more thorough description.

The *Benders cuts* are constructed by solving the LP relaxation of (7), giving an optimal value $\mathcal{Q}^{LP}_t$. The coefficients $\pi_{hntc}$ are computed as the dual values of (7c), and the right-hand side $\beta_{tc} = \mathcal{Q}^{LP}_t - \sum_{h\in\mathcal{H}}\sum_{n\in\mathcal{N}_h} \pi_{hnt}\lambda_{hn,t-1}$. Note that this approach will lead to the same solution as one would get solving the MIP in (3) and (5) in the SDDP forward iteration and its LP relaxation in the backward iteration.

The *Lagrangian cuts* are constructed by solving the Lagrangian dual of (7), relaxing the copy constraint (7c). The Lagrangian multiplier is obtained by

$$\bar{\pi}^i_t = \operatorname*{argmin}_{\bar{\pi}_t} \Big\{ \mathcal{L}^i_t(\bar{\pi}_t) + \sum_{h\in\mathcal{H}} \sum_{n\in\mathcal{N}_h} \bar{\pi}_{hnt}\lambda_{hn,t-1} \Big\}, \tag{9}$$

where $\mathcal{L}^i_t$ is defined as:

$$\mathcal{L}^i_t(\bar{\pi}_t) = \max \Big( p_t e_t + \alpha_t - \sum_{h\in\mathcal{H}} \sum_{n\in\mathcal{N}_h} \bar{\pi}_{hnt} z_{hnt} \Big) \tag{10a}$$

$$\text{s.t.} \quad \text{(3), (5), (7d) and (7e)} \tag{10b}$$

Equation (9) is solved repeatedly, with the aim to gradually improving the vector of Lagrangian multipliers $\bar{\pi}_t$ to provide tight Lagrangian cuts. The multipliers are updated taking steps with the subgradient method [19]. The coefficients $\pi_{hntc}$ are found as the Lagrangian multipliers, and the right-hand side $\beta_{tc}$ as $\mathcal{L}^i_t$. Interesting related work on Lagrangian relaxation within the SDDP algorithm has been published in [20], [21].

The *Strengthened Benders cuts* are constructed by first solving the LP relaxation of problem (7). Subsequently, problem (10) is solved with the Lagrangian multiplier vector $\bar{\pi}_t$ equal to the optimal LP dual solutions with respect to constraints (7c). From the optimal objective of the latter problem we obtain the right-hand side $\mathcal{L}^i_t(\bar{\pi}^i_t)$, which together with $\bar{\pi}_t$ is used to construct the Strengthened Benders cut of type (8).

## IV. Case Study

### A. Case Description

A computer model was established implementing the algorithm described in Section III-A using the different cut types described in Section III-B. The model was used to obtain schedules for a Norwegian watercourse comprising 3 hydropower reservoirs with corresponding power plants, and with a total capacity of 202 MW. An illustration of the topology and specification of some technical characteristics is provided in Fig. 1. For each reservoir shown in the figure the average annual inflow and storage capacity are stated, both in $\mathrm{Mm}^3$. Each power plant is identified with a number and its installed capacity in MW. An environmental constraint is attached to reservoir 2, stating that no water should be discharged from the reservoir in between weeks 18 to 35 if the reservoir volume is lower than 140 $\mathrm{Mm}^3$.

A scheduling horizon of 1 year was applied with weekly decision stages. A set of cuts of type (8) was used to ensure that state variables at the end of the scheduling horizon were valuated. These cuts were obtained by model calibration, but could also be provided as a boundary condition from a long-term scheduling model.

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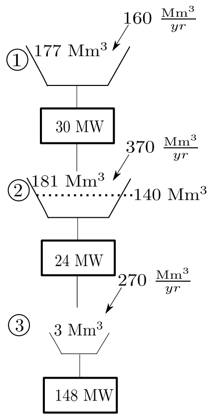


Fig. 1. Watercourse topology and technical data.

A relatively coarse representation of uncertainty and time discretization was applied in order to focus on the modelling of environmental constraints within acceptable computation times. An inflow model was fitted using a single inflow series comprising 80 historical years, and the model error was sampled from a normal distribution. A total of 50 inflow scenarios were re-sampled in each forward iteration, and 6 discrete inflow error terms were used at each stage in the backward iterations of the algorithm. We ran the model with 3 different price scenarios, following the weekly average NordPool system prices for 2006 and 2015 as well as the average system price for the years 2005-2018. The price scenarios are referred to as *high* (2006), *average* (average for 2005-2018) and *low* (2015).

The model was implemented in `Julia`, using the `JuMP` package [22] and CPLEX 12.9 solver [23] for solution of both the MIP and the LP problems. All tests were carried out on an Intel Core i7-4940MX processor with 3.30 GHz and 32 GB RAM.

The system in Fig. 1 was optimized using the 3 different cut-types described in Section III-B and with the 3 different price scenarios. The resulting 9 cases are listed in Table I. The system operation for each case was simulated in a final simulation using a fixed and separately sampled set of 1000 inflow scenarios. The resulting expected profits are given in Table I. The improved economic performance with exact modelling is most pronounced in the high-price case. The high summer prices in this case provides a clear incentive to keep water available for summer production, rewarding the accurate modelling of the environmental constraint in reservoir 2. Recall that the environmental constraints associated with reservoir 2 are always met in the forward simulation, but may be approximated in the backward iteration, depending on the type of cuts being used. For the average-price case, the 0.09 M€ (or 0.16 %) increase in expected profit when using Lagrangian compared to Benders cuts may seem modest, but one should keep in mind that the producer will often hunt for such marginal improvements in a competitive market.

The choice of cut type significantly impacts the computation time. With the use of Benders cuts, the majority of computation time is spent solving the MIP problems in the forward simulation, resulting in a run time in the range 2-3 hours. The run-time approximately increased with a factor of 5 when using Strengthened Benders cuts compared to using Benders cuts, due to the need for solving one MIP problem per problem instance in the backward iteration. The use of Lagrangian cuts further increases computation time, strongly depending on the number of iterations when searching for improved multipliers in (9). Note that parallel processing was not applied in this case study for simplicity.

TABLE I
SIMULATED CASES AND EXPECTED PROFITS [M€].

| | **Price Scenario** | | |
|---|---|---|---|
| **Cut Type** | **High Price** | **Average Price** | **Low Price** |
| Benders | 65.73 | 56.74 | 34.59 |
| Strengthened Benders | 65.91 | 56.81 | 34.62 |
| Lagrangian | 65.96 | 56.83 | 34.63 |

## B. Results

The convergence characteristic of the algorithm is shown in Fig. 2, comparing the upper bounds obtained using the 3 different cut types in the high-price scenario. We used a maximum number of 30 iterations in all presented cases. The lower bound obtained when using the Lagrangian cuts is also included to indicate that the cost gap gradually closes as the iteration number increases. From Fig. 2 it can be observed that the cost gap using Lagrangian cuts is slowly decreasing, but has not closed after 30 iterations. Further improving the cost gap turned out to be difficult and time-consuming, which we believe is due to the inefficient search for improved Lagrangian multipliers (using the subgradient method). The significant difference in upper bound when using the Strengthened Benders cuts compared to the Benders cuts indicate that the former is a substantial improvement compared to the approximation made when linearizing the nonconvex constraints in (5). This finding is in-line with the conclusions in [7], although this case study is clearly different from the one in [7].

Operation of reservoirs 1 and 2 when using the high-price scenario is presented in Fig. 3, comparing the use of Strengthened Benders and Benders cuts. The threshold volume of 140 $Mm^3$ for reservoir 2 is indicated with the stapled horizontal line in Fig. 3. The figures show that the use of Strengthened Benders cuts leads to more released water from reservoir 1 and less from reservoir 2 prior to the price spike in week 34. The difference in operation obtained when using Strengthened Benders and Lagrangian cuts were less pronounced than between Strengthened Benders and Benders and is therefore not emphasized here.

# V. CONCLUSIONS

We presented a hydropower scheduling model based on SDDiP for exact treatment of reservoir-level dependent maximum discharge, which can be classified as a nonconvex

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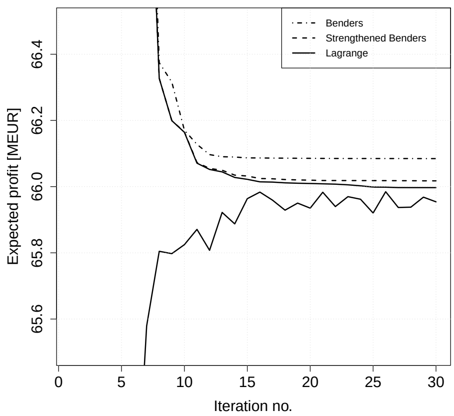


Fig. 2. Convergence characteristics for the high-price cases. Upper bounds for all cut types and lower bound for the case with Lagrangian cuts.

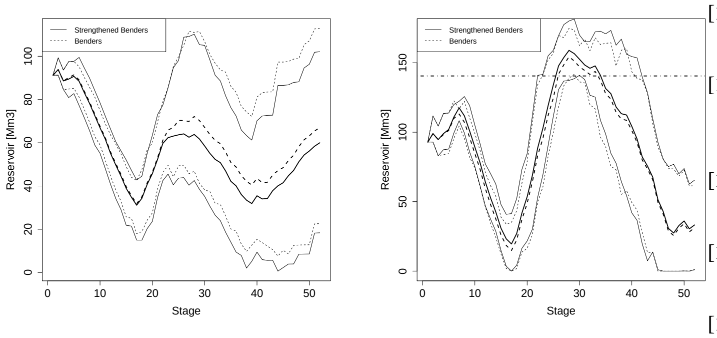


Fig. 3. Simulated operation of reservoirs 1 (left) and 2 (right), shown as mean (emphasized) and 0 and 100 percentiles. The solid-drawn and stapled lines are obtained using Strengthened Benders and Benders cuts, respectively. The reservoir threshold impacting maximum discharge from reservoir 2 is indicated with the stapled, horizontal line.

environmental constraint. The model was tested in a multi-reservoir case study, and its performance in terms of economic indicators and reservoir operation was presented. The model relies on several simplifying assumptions, a few of which should be challenged to arrive at more robust results. In particular we believe that adding uncertainty to the exogenous power price, using finer time resolution, and extending the scheduling horizon would impact the results.

The case study results show that there is a potential for improving the scheduling by accurately treating this type of environmental constraints compared to a traditional linear approximation. In particular we found that the use of Strengthened Benders cuts seems to capture a majority of the improvement potential. Further work on the algorithm could focus on improving the search for multipliers for the Lagrangian cuts and combining cut types to improve both the computational performance and the convergence.

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