EXPERIMENTAL AND NUMERICAL STUDY ON THE BEHAVIOUR OF PVC AND HDPE IN BIAXIAL TENSION

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Abstract: This paper presents and discusses a set of biaxial tests on two different polymer materials; PVC and HDPE. The biaxial tests are used to evaluate a hyperelastic-viscoplastic constitutive model involving the pressure-dependent Raghava yield criterion. The parameters in the model are calibrated from uniaxial tension and compression tests. Subsequently, the capability of the model is explored through nonlinear finite element analyses of the biaxial tests. The analyses are carried out using the material model implemented in LS-DYNA. A comparison of the force-displacement curves and the displacement fields from the simulations with those from the experiments shows that the model captures the main features of the behaviour observed in the tests.

Keywords: Polymer, LS-DYNA, biaxial tension, constitutive modelling, Raghava yield function

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1. Introduction

Together with the rise in use of polymers in industrial and structural applications, the request for numerical models for these materials has increased. Much effort is put into the development of material models for applications in the finite element method. To ensure that these models are reliable we need to investigate thoroughly how the materials behave under different loading conditions. It is well-known that most thermoplastics are sensitive to pressure. This is usually taken into account in material models for polymers. In particular for thin-walled structures, tensile biaxial load cases occur in several practical applications, and it is therefore important that the yield function represents this stress state properly. Within this context, it is relevant to evaluate the capability of the material model to describe large deformations in a biaxial loading mode.

Some investigations on the mechanical response of different polymers in biaxial deformation have already been reported (Adams et al., 2000; Buckley et al., 1996; Chandran and Jabarin, 1993a, b, c; Chevalier and Marco, 2007; Sweeney and Ward, 1995; Zeng et al., 2010). The studies on biaxial deformation found in the literature often concern manufacturing conditions involving high strain rates and high temperatures or the behaviour of polymer films. Paying attention to validation of material models, Chevalier and co-workers (2001; 2002) have shown that by using a multiaxial testing machine, a charge-coupled device (CCD) camera and digital image correlation (DIC) software, biaxial displacement and strain fields from such tests can be evaluated for rubber-like materials. By assuming incompressibility they derived the stress evolution in the test specimen during deformation and compared this with the stress calculated by different rubber material models.

The deformation of thermoplastic polymers commonly involves large elastic and plastic deformations. Their mechanical response is often sensitive to strain rate and temperature. Polymers are often regarded as pressure sensitive materials; a higher yield strength in compression than in tension is commonly observed. Another feature is that the volume changes during plastic deformation (Delhaye et al., 2010; Delhaye et al., 2011; Grytten et al., 2009; Mohanraj et al., 2006; Moura et al., 2010). Moreover, some polymers have a stress softening behaviour after the yield limit, while others experience monotonic hardening (G'Sell et al., 1992; Moura et al., 2010). These are some characteristics a material model for thermoplastics should allow for. Based on the original idea of Haward and Thackray (1968), Polanco-Loria et al. (2010) presented a model separating the response in two parts describing the intermolecular resistance and the molecular network resistance. The constitutive model includes the pressure dependent Raghava yield criterion (Raghava et al., 1973; Raghava and Caddell, 1973).

The purpose of this work is to investigate how the constitutive model proposed by Polanco-Loria et al. (2010), employing Raghava's yield criterion calibrated from uniaxial tension and compression tests, predicts the mechanical response in biaxial tension. Both experimental tests and numerical simulations have been performed. As the thermoplastic materials in this study dilate during plastic deformation, it is not possible to find the stresses from the experiments without measuring the volume changes. This was not achieved in this work as only one CCD-camera was used. However, in-plane strain fields from the deformation are found using DIC. They are subsequently compared with the strain fields from finite element simulations. In addition the global force-displacement relationships for the biaxial experimental tests are compared with those from the analysis. Two different thermoplastics are addressed; an amorphous PVC (polyvinyl chloride) and a semi-crystalline HDPE (high-density polyethylene).

This paper presents first, in Section 2, the setups of uniaxial tension and compression tests for the calibration as well as the biaxial tests for the validation performed on the two materials. Thereafter, experimental results are provided in Section 3. Next, Section 4 describes the material model and gives a short description on how the material parameters were calibrated from the uniaxial tests. Section 5 presents the results from finite element analysis utilizing the constitutive model. The paper is rounded off with discussion and conclusions in the last section.

2. Experiments

2.1 Materials

Both materials PVC and HDPE were bought as 5 mm thick extruded plates, delivered as regular off-the-shelve products from the German producer SIMONA. All test specimens presented in this paper are cut from these plates. The PVC is an amorphous thermoplastic. Scanning electron micrography of the material, coupled with spectroscopy, has revealed that it contains some calcium carbonate ($CaCO_3$) particles. The volume fraction of particles is around 20% (Ognedal et al., 2012). According to the technical data sheet provided by the producer (SIMONA) the PVC a material with high rigidity and increased impact strength. The HDPE is a semi-crystalline thermoplastic referred by the producer (SIMONA) to be very tough. Both materials are the same as were presented by Moura and co-workers (2010), but they took all specimens from plates of 10 mm thickness.

2.2 Uniaxial test setup

Uniaxial tension and compression tests were performed on both materials in order to collect experimental data as input for calibration of the material model. Test specimens were cut out from larger plates both parallel and normal to the extrusion direction. The tension and compression tests were performed at nominal strain rate 10^{-3} s^{-1} in a servo-hydraulic Dartec machine with a 20 kN load cell connected to an Instron controller. Test specimens with the standard "dog bone" shape were used for the tensile tests. The length of the specimens' parallel section was 33 mm, and the width and thickness were 12 and 5 mm, respectively. Prior to all tension tests a speckled pattern was applied to the surface. A digital camera monitored the displacements in the pattern during the test in order to capture the local deformation. Strain fields were then computed using the DIC software 7D (Vacher et al., 1999). For the compression tests cylindrical test specimens with a diameter and a height of 5 mm were used. Their size was restricted by the thickness of the extruded plates. With focus at the outer edge of the test specimens, images were taken regularly during the compression test. After testing the diameter of the cross section as well as the height of the cylinder were measured on the images.

2.3 Biaxial test setup

The cross-shaped test specimens, see Figure 1, were also cut out from the extruded PVC and HDPE plates. At the centre of the samples, the thickness was reduced to control the location of the initial strain localization. All experiments were performed in the Astree triaxial testing machine (Chevalier et al., 2001; Marco et al., 2002) at LMT-Cachan. Two of the three axes of this machine were employed using displacement controlled loading. The software LabView was employed for computer control of the test and data acquisition. Each test specimen was mounted in the testing machine with the extrusion direction parallel to the horizontal *x*-axis and the transverse direction parallel to the vertical *y*-axis. In order to obtain different states of biaxial loading, three biaxial load cases with different ratios were investigated for each material, see Table 1. The extension ratio $\rho = v_y / v_x$, i.e. the ratio between the cross-head velocities v_x and v_y in the *x*- and *y*-directions, respectively, was equal to ¼, ½ and 1. To obtain this, v_y varied between the different tests while v_x was fixed. During each test, however, v_x and v_y were constant. The displacement in the two directions started and stopped simultaneously. As a special case, one of the biaxial samples was tested in uniaxial tension for both materials, applying a servo-hydraulic MTS testing machine. For these tests the two transverse arms of the specimen were unconstrained and free to move in the *y*-direction.

All tests were carried out at room temperature. According to Table 1, the strain rate was set relatively low, ensuring that plastic dissipation of the samples did not cause any large increase of temperature. Further, an inspection of the clamped areas after the tests revealed no signs of sliding in the fixtures.

In a similar way as for the uniaxial tension tests, all cross-shaped specimens were painted with a speckled pattern before testing, facilitating post-test analysis with the DIC program 7D (Vacher et al., 1999) to find the Green strain fields E_{xx}, E_{yy} and E_{xy} at the surface of the biaxial test samples. Corresponding Green strain fields were taken out from the simulations for comparison.

3. Experimental results

3.1 Uniaxial tests

Tension and compression tests were done with specimens cut both parallel and normal to the extrusion direction. Isotropic transverse deformation, i.e. equal strains in the width and thickness direction, was assumed when calculating the true stress both in tension and in compression. This has earlier been shown to be a good approximation for these materials (Moura et al., 2010).

All tension specimens exhibited necking after some deformation. Stress-strain curves, see Figure 2, were established from the results for the cross section where necking started. This section was easily found using the DIC software. Towards the end of the tests, the increase in deformation from one image to the next in this cross section was small since the neck had propagated through the specimen. The curves have been smoothened before plotted in Figure

2. The HDPE test specimen did not reach the fracture level with the applied setup. The test was aborted when the speckled pattern was so distorted that it was difficult to get accurate measurements of the strain field.

For the compression tests the true stresses were calculated using the data from force and diameter measurements. The longitudinal strains were assumed to be uniform over the specimen, and were calculated from the change of specimen height. Bulging, or "barrelling", started after some deformation of the compression test coupons, probably due to friction between the steel platen and the test coupon's surface. The onset of barrelling is marked with circles in Figure 2.

For the PVC specimens the maximum load registered was about 6 % higher in the extrusion direction than in the transverse in-plane direction. Hardly any difference was observed in HDPE. The constitutive model (Polanco-Loria et al., 2010) assumes, however, isotropic material behaviour, and the properties in the extrusion direction were applied during the calibration procedure, see Section 4.2. In both tension and compression the PVC softens after reaching a local maximum stress, see Figure 2. This is not the case for HDPE.

3.2 Biaxial tests

Figure 3 a)-d) shows the force-displacement curves for PVC for the four different extension ratios defined in Table 1. Results from the finite element simulations are plotted in the same figure and will be discussed later. With the exception of the specimen loaded in the *x*-direction only, see Figure 3 a), results from both directions *x* and *y* are included. All PVC load curves show a rather linear behaviour up to maximum load, corresponding to the onset of yielding. It is interesting to note that the yielding starts at the same force level in the tests loaded in uniaxial and the equibiaxial tension, while the two tests with extension ratios $\rho = ½$ and $\rho = ¼$ reach higher force levels at yielding. After the point of maximum load and onset of necking there is a softening-effect observed as a load drop before cold-drawing and failure. For all tests, an X-shaped neck developed in the centre region of the specimen. This led to extensive whitening of the PVC in this region, as seen from the post-test image in Figure 4 a). The X-shaped neck appeared at an earlier stage and it was more pronounced for the equibiaxial test than for the other tests. When the cross section thickness decreases, less material is left to restrict the deformation, and a softer response is observed. This might explain why the magnitude of the load drop seems to increase slightly with the extension ratio ρ , see Figure 3. The failure pattern of the specimen with $\rho =$ ½ is also shown in Figure 4 a). The other biaxial PVC specimens ruptured in a similar manner.

Figure 5 a)-d) shows force-displacement curves from HDPE together with results from simulations that again are to be discussed later. Also the HDPE samples experienced a load drop after reaching the maximum force. In all four tests, extensive drawing, induced by necking at the moment of reaching maximum force, made the centre region very thin. The thinning in the X-shaped neck in the centre of the test specimens was more pronounced for the HDPE specimens than for the PVC specimens. For the uniaxial specimen, buckling was observed at this location; as the left and the right end of the specimen moved apart in the *x*-direction, the upper and lower free ends were forced to approach each other. The drawing of the centre region in the three biaxial samples resulted in creation of holes. Due to the ductile behaviour of the material, these holes continued to grow, without causing cracking and global failure of the test specimens, until the tests were aborted. The first appearances of the holes are marked with circles in Figure 5. An example of such a hole is shown in Figure 4 b). The picture shows the specimen with $\rho =$ ½ at the stage when the test was aborted.

4. Material model

4.1 Outline of the constitutive model

The hyperelastic-viscoplastic material model presented by Polanco-Loria et al. (2010) consists of two parts, Part A and Part B, coupled in parallel. An outline of the model is shown in Figure 6. The main kinematic variable in the model is the deformation gradient $\mathbf{F}$. It is the same for Part A and Part B, i.e. $\mathbf{F} = \mathbf{F}_A = \mathbf{F}_B$. The total Cauchy stress $\boldsymbol{\sigma}$ is taken as the sum of the stress contributions from the two parts, viz. $\boldsymbol{\sigma} = \boldsymbol{\sigma}_A + \boldsymbol{\sigma}_B$.

Part A describes a hyperelastic-viscoplastic response due to intermolecular resistance. A multiplicative split $\mathbf{F}_A = \mathbf{F}_A^e \cdot \mathbf{F}_A^p$ is used to decompose the deformation gradient of Part A into elastic and plastic parts. The plastic part, $\mathbf{F}_A^p$, defines an intermediate configuration, invariant to the rigid body rotations of the current configuration. The evolution of the intermediate configuration is defined by the differential equation $\dot{\mathbf{F}}_A^p = \bar{\mathbf{L}}_A^p \cdot \mathbf{F}_A^p$, where $\bar{\mathbf{L}}_A^p$ is the plastic velocity gradient.

A Neo-Hookean model is used to allow for large elastic deformations

$$\boldsymbol{\tau}_A = \lambda \ln J_A^e \,\mathbf{I} + \mu[\mathbf{B}_A^e - \mathbf{I}] \tag{1}$$

where $\boldsymbol{\tau}_A = J_A^e \boldsymbol{\sigma}_A$ is the Kirchhoff stress, $J_A^e = \det \mathbf{F}_A^e$ is the elastic part of the Jacobian, $\mathbf{I}$ is the second-order unit tensor, and $\mathbf{B}_A^e = \mathbf{F}_A^e \cdot (\mathbf{F}_A^e)^T$ is the elastic left Cauchy-Green deformation tensor. The Lamé constants λ and μ are used to define the elastic response, which can also be expressed by Young's modulus E and Poisson's ratio ν.

The viscoplastic contribution of Part A is computed on the intermediate configuration, applying the Mandel stress tensor $\bar{\boldsymbol{\Sigma}}_A$. The relationships between the Kirchhoff and Mandel stress tensors read $\boldsymbol{\tau}_A = (\mathbf{F}_A^e)^{-T} \cdot \bar{\boldsymbol{\Sigma}}_A \cdot (\mathbf{F}_A^e)^T$ and $\bar{\boldsymbol{\Sigma}}_A = (\mathbf{F}_A^e)^T \cdot \boldsymbol{\tau}_A \cdot (\mathbf{F}_A^e)^{-T}$. Note that the Mandel stress tensor is symmetric due to the assumed isotropy of the material. The yield criterion is formulated as $f_A = \bar{\sigma}_A - \sigma_T - R = 0$. The Raghava equivalent stress $\bar{\sigma}_A$ is used to express pressure dependency (Raghava et al., 1973)

$$\bar{\sigma}_A = \frac{(\alpha-1)I_{1A} + \sqrt{(\alpha-1)^2 I_{1A}^2 + 12\alpha J_{2A}}}{2\alpha} \tag{2}$$

where $I_{1A} = \operatorname{tr} \bar{\boldsymbol{\Sigma}}_A$ and $J_{2A} = \frac{1}{2} \bar{\boldsymbol{\Sigma}}_A^{dev} : \bar{\boldsymbol{\Sigma}}_A^{dev}$ are invariants of respectively the Mandel stress tensor and the deviatoric part of $\bar{\boldsymbol{\Sigma}}_A$. The parameter $\alpha = \sigma_C / \sigma_T$ represents the ratio between the yield stresses in compression and tension. These two stress data provide sufficient information to define the shape of the yield surface. Setting $\alpha = 1$, we get the von Mises' yield surface as a special case of the Raghava function. Further, the isotropic strain hardening or softening R of Part A, see Figure 6 b), is a function of the accumulated plastic strain $\bar{\varepsilon}_A^p$, and it is controlled by the saturation stress σ_S and the hardening/softening parameter H, viz.

$$R\left(\bar{\varepsilon}_A^p\right) = \left(\sigma_S - \sigma_T\right)\left[1 - \exp\left(-H\bar{\varepsilon}_A^p\right)\right] \quad (2)$$

A non-associated viscoplastic flow rule is assumed to define the plastic velocity gradient on the intermediate configuration as

$$\bar{\mathbf{L}}_A^p = \dot{\bar{\varepsilon}}_A^p \frac{\partial g_A}{\partial \bar{\mathbf{\Sigma}}_A} \quad (3)$$

where the plastic potential g_A is defined in the form

$$g_A = \frac{(\beta - 1)I_{1A} + \sqrt{(\beta-1)^2 I_{1A}^2 + 12\beta J_{2A}}}{2\beta} \geq 0 \quad (4)$$

Here, β is the plastic dilation parameter, determining the increase of volume during plastic flow.

The equivalent plastic strain rate $\dot{\bar{\varepsilon}}_A^p$ of Equation (3) is defined by the constitutive relation

$$\dot{\bar{\varepsilon}}_A^p = \begin{cases} 0 & \text{if } f_A \leq 0 \\ \dot{\varepsilon}_{0A}\left\{\exp\left[\dfrac{1}{C}\left(\dfrac{\bar{\sigma}_A}{\sigma_T + R} - 1\right)\right] - 1\right\} & \text{if } f_A > 0 \end{cases} \quad (5)$$

In this expression, two rate-sensitivity parameters, $\dot{\varepsilon}_0$ and C, are introduced.

Part B of the material model describes a hyperelastic entropic resistance originally proposed by Arruda and Boyce (1993)

$$\boldsymbol{\sigma}_B = \frac{C_R}{3J_B}\frac{\bar{\lambda}_L}{\bar{\lambda}} L^{-1}\left(\frac{\bar{\lambda}}{\bar{\lambda}_L}\right)(\mathbf{B}_B^* - \bar{\lambda}^2\mathbf{I}) \quad (6)$$

where C_R is the initial elastic modulus of Part B, $\bar{\lambda}_L$ is the locking stretch, L^{-1} is the inverse Langevin function, and $J_B = \det \mathbf{F}_B$ is the Jacobian (recall that $\mathbf{F} = \mathbf{F}_B$). The average total stretch ratio $\bar{\lambda}$ is calculated as

$$\bar{\lambda} = \sqrt{\frac{1}{3}\mathrm{tr}\left(\mathbf{B}_B^*\right)} \tag{7}$$

where $\mathbf{B}_B^* = \mathbf{F}_B^* \cdot (\mathbf{F}_B^*)^{\mathrm{T}}$ is the distortional left Cauchy-Green deformation tensor, and $\mathbf{F}_B^* = J_B^{-1/3}\,\mathbf{F}_B$ denotes the distortional part of $\mathbf{F}_B$.

The model involves 11 coefficients to be determined from uniaxial tension and compression tests. For further details the reader is referred to Polanco-Loria et al. (2010). Working for brick elements, the model is implemented as a user-defined material model in LS-DYNA (2007). Neither thermal effects nor a fracture criterion is incorporated in the model.

4.2 Calibration of the material model

Applying results from the uniaxial tension and compression tests the coefficients of the material model were identified. Starting with Part A, Young's modulus E for both materials was defined as the initial slope of the stress-strain curve. Poisson's ratio was determined from the initial relationship between transverse and longitudinal strains. Further, the yield stress σ_T , was taken as the first local maximum of the stress-strain curve for PVC, adjusted by subtracting the contribution of Part B. The HDPE material did not show any obvious maximum point. Therefore, an offset strain of 0.2 % was used to define the yield stress for this material. After determining the yield stress in compression in similar manners for both materials, the pressure sensitivity parameter α could be found. This is the parameter controlling the shape of the yield surface, and it is determined as the ratio of the yield stresses in compression and tension at equal strain rates. The other parameters were found according to Hovden (2010). All coefficients are summarized in Table 2. Alternatively, an inverse modelling procedure for parameter identification could also be applied as proposed by Polanco-Loria et al. (2012).

As shown in Figure 2, the uniaxial tensile and compression tests revealed that PVC exhibited higher yield stress in compression than in tension, while yielding in HDPE is hardly pressure sensitive. The pressure dependency for PVC is taken into account by introducing $\alpha = 1.3$ in the Raghava function, while α is set to 1.0 for the HDPE material, see Table 2. For HDPE the Raghava yield criterion is then reduced to the von Mises yield criterion without pressure sensitivity.

Considering plane stress conditions with $\sigma_z = 0$, the Raghava yield criterion in Equation (2) can be expressed as

$$R_x^2 + R_y^2 - R_x R_y + (\alpha - 1)(R_x + R_y) = \alpha \tag{8}$$

where the normalizing stresses are defined as $R_x = \sigma_x / \sigma_T$ and $R_y = \sigma_y / \sigma_T$ (Raghava et al., 1973). It is assumed that the coordinate axes are aligned with the principal axes of the stress tensor. The Raghava function is plotted for both materials in Figure 7, addressing the principal as well as the invariant stress space. It can clearly be seen that PVC requires a higher stress to reach the yield limit in compression than in tension. This is not the case for HDPE where the function gives a von Mises surface, symmetric about the origin in Figure 7 a). From Table 2 it can be found that the value of the plastic dilation parameter β is quite close to the corresponding value of the pressure sensitivity parameter α. This means that for both materials the plastic potential function has a shape rather similar to the yield function, as depicted in Figure 7, again with a clear difference between the two materials. During plastic deformation, this will lead to different responses for PVC and HDPE because the gradients of the potential functions do not have the same directions in stress space.

5. Numerical simulations

5.1 Simulations of uniaxial tension tests

In order to check the capability of the model to describe the response of a simple structure, the uniaxial tension specimen was modelled in LS-DYNA, see Figure 8 a). The left and the right ends were modelled as rigid bodies. The model has 5 elements over the thickness, 12 elements in the width direction and 52 elements along the gauge part. The material parameters in Table 2 were applied in these numerical simulations of the tension test. Figure 9 compares force – displacement curves from the experiments and analyses. It shows that the material model is capable of predicting the main features of the response for both materials.

5.2 Simulation of biaxial tension tests

Two meshes with three and five elements through the thickness were considered for the finite element simulations of equibiaxial loading of cross shaped biaxial test specimens. The two meshes gave similar results as seen in Figure 10. To save computational time, the mesh with three elements trough the thickness was used in the further simulations. The employed mesh is shown in Figure 8 b). Smaller elements were applied in the areas where large deformations were expected. In total, 12660 elements were used to model the deformable part of the sample. The four clamping areas of the specimen were idealized as a rigid material. Velocities corresponding to those from the experiments were applied to the rigid parts. The specimens were modelled with the constitutive model presented in Section 4.1 and the material parameters of Table 2.

In Figure 3 and Figure 5, results from simulations are plotted together with results from the experiments. The numerical and experimental results are further compared in Figure 11 and Figure 12 for PVC and HDPE, respectively, where sub-figures a) and b) in turn represent the laboratory tests and the numerical predictions. Figure 11 and Figure 12 express the effect of biaxial loading mode on the force-displacement curve, or in other words, how the response in the *x*-direction is affected by a change of the deformation in the *y*-direction.

It can be seen both from Figure 3 and from Figure 11 that the finite element model underestimates the force for the cross-shaped test specimen of PVC loaded in uniaxial tension. For the biaxial loading cases, the maximum force predicted from the simulation is somewhat higher than the experimental results. The model captures what is seen from the experiments, namely that the maximum force is largest when the extension ratio ρ is equal to ½ or ¼. This is in accordance with the shape of the yield surface employed for this material, see Figure 7 a). Clearly, a higher value of the *x*-direction stress can be obtained in uniaxial tension (corresponding to the horizontal axis) than for equibiaxial tension (corresponding to the dashed line in the figure). The ultimate rupture of the specimens was not captured in the simulation because no failure criterion was employed in the numerical simulation. Figure 12

a) shows that the force level reached in the uniaxial test for HDPE is about the same as in the equibiaxial test. This is captured well by the numerical model, see Figure 12 b). Also for HDPE the intermediate extension ratios result in a larger maximum force in the *x*-direction in the tests as well as in the numerical analyses.

As in the experiments, the large deformations in the finite element model lead to strain localisation and necking of the X-shaped centre region of all specimens. The Green strains E_{xx}, E_{yy} and E_{xy} in the centre region of the PVC specimen loaded in equibiaxial tension are plotted in Figure 13, which adresses the deformation state at 5 mm global displacement. In this figure, the experimentally obtained strain fields found with the use of digital image correlation are plotted on the left hand side, as sub-figures a), c) and e), while the sub-figures to the right, b), d) and f), present the corresponding strain fields from the finite element analysis. Similar experimental and numerical strain fields are plotted in Figure 14 for HDPE. It is seen that in both cases, a good agreement is obtained between the experimentally and numerically obtained strain fields.

As the strain fields are inhomogeneous, only the centre point is chosen to show how the extension ratio ρ affects the evolution of the strains with increasing deformation. The strain components E_{xx} and E_{yy} at this location are plotted in Figure 15 for PVC and in Figure 16 for HDPE, again comparing experimental data with numerical predictions. The shear strains at are small at this location, and are therefore not plotted in these figures. The large deformations at the centre point caused distortion of the speckle pattern sprayed on the specimens for both materials, so it was not possible to follow the strains towards the end of the experiments. Therefore, the plotting of the curves is aborted when the DIC software was unable to determine the strain.

During the uniaxial tests of the cross shaped test specimen the left and right parts are pulled away from each other in the *x*-axis direction, causing the upper and lower free parts to move closer in the *y* direction. This Poisson effect is clearly visible in the centre region of the specimen. By examination of the strain curves for the uniaxial test in Figure 15 and Figure 16 it can be seen that E_{yy} is negative, indicating compression, for both materials. This is observed in the experimental test as well as in the numerical simulation. With respect to the strains in the biaxial specimens, it can be seen that the higher the extension ratio ρ, the larger the strain component E_{yy}.

6. Discussion and conclusions

The two materials tested in this paper, PVC and HDPE, show different mechanical responses. This was seen already in Figure 2, displaying the stress-strain curves for uniaxial tension and compression. A material model (Polanco-Loria et al., 2010) with parameters calibrated from these curves was employed to predict the mechanical response of the same materials in biaxial tension. PVC was modelled with α = 1.3 in the Raghava yield function to incorporate pressure sensitivity of the yield surface. HDPE was modelled with α = 1.0, which corresponds to the von Mises yield criterion. Opposed to HDPE, PVC shows stress softening after reaching the yield stress. This might be related to physical ageing of the material (Meijer and Govaert, 2005) or to debonding of the $CaCO_3$-particles (Ognedal et al., 2012).

From Figure 15 and Figure 16 it can be seen that the Green strains in the centre points of the specimens found from the experiments and the numerical simulations are comparable. These plots also show that the model is able to predict the earlier localisation of strains for PVC compared with HDPE. This can be seen from the sudden increase of strain in the centre point. Also the strain fields plotted in Figure 13 and Figure 14 confirm this observation. The strain fields are plotted at the same global deformation level, yet localisation can be seen in the strain fields for PVC but not in those for HDPE. These figures are plotted at 5 mm deformation in both *x*- and *y*-direction, which is after the load maximum for PVC but before the load maximum for HDPE, see Figure 3 and Figure 5. The drawing of the centre region of the HDPE specimens makes the specimens very thin here. In the experiments, this thinning leads to creation of holes in the three biaxial tests on HDPE. This hole-growth is not included in the model. However, the finite elements in the simulation in this region are extremely thin at this stage, so the global numerical response is still quite similar to the one observed in the experiments for some time after the hole initiation.

By examination of the yield surface used for modelling of PVC, see Figure 7, it would be expected that the peak load in uniaxial tension is higher than the one in equibiaxial tension.

Due to the transverse compression at the centre of specimen subjected to uniaxial loading, however, the stress state in this specimen is not uniaxial. Thus, the location on the yield surface corresponding to this stress state is not on the *x*-axis of Figure 7, but slightly below. Figure 15 shows that at the centre of the specimen, the ratio E_{xx} / E_{yy} at 10 mm deformation can roughly be estimated to –6.3 and –5.9 for the experiments and simulations, respectively. This also suggests that the corresponding stress state is somewhere within the fourth quadrant of Figure 7a). Some difference in onset of yielding in experiments and simulations can be observed for this specimen. Another yield criterion, for instance a high-exponent yield function that accounts for the dependence of the third stress invariant, might be able to predict the onset of yielding in a better way.

The numerical analyses of the biaxial tests slightly overestimate the force level for PVC, while for HDPE the force level is somewhat lower in the simulations than in the tests. From Figure 11 it can be seen that also after the onset of yielding in PVC there is some mismatch between experiments and simulations, especially for the uniaxially loaded specimen. In the simulations, the load drop is less pronounced than in the experiments. This might be related to the plastic potential g_A. If the plastic dilation is overestimated, the load drop will be too small.

Compression could be observed also at the centre of the HDPE specimen tested in uniaxial tension, see Figure 16. It is seen that the ratio E_{xx} / E_{yy} at 10 mm deformation is around –3.8 for both experiment and simulation. After applying some additional deformation, buckling could also be observed at the centre of the specimen. Due to the comprehensive thinning of this section, the load-bearing capacity might be reduced so much that the global response of the specimen is not affected by the transverse compression in the same manner as in the case of PVC. Modelling yielding in this particular HDPE with a von Mises criterion seems to be in reasonable accordance with our experimental results.

Due to the geometry of the cross-shaped test specimens described in this paper, the evolution of the strain fields in the specimens is not directly linked to the extension ratio ρ. An example of this is the biaxial test specimens in uniaxial tension producing compression in the centre region. Another aspect to be aware of is that the strain fields are not at all uniform. Therefore, such tests alone are not very well suited for investigation of mechanisms of biaxial deformation. They should rather be employed for validation of material models. Still, in combination with numerical analysis, the results from biaxial experiments could verify whether some of the assumptions made during the formulation of such a model are realistic. Because of the variation in strain field observed in these tests, the tests could also be subject for determination of material properties through inverse modelling.

In conclusion, it was found that the constitutive model proposed by Polanco-Loria et al. (2010) was capable of describing the response of PVC and HDPE in biaxial tension with good accuracy. The model has rather few parameters. They can be determined from uniaxial tension and compression tests at various strain rates, facilitating an industrial use of the model. Therefore, it is believed to be a good candidate for large-scale simulations of polymeric components.

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Table 1. Biaxial test programme.

Material	Extension ratio ρ	v_x [mm/s]	v_y [mm/s]	Initial strain rate [s^{-1}]	Sequence
PVC	–	0.05		$4.1 \cdot 10^{-4}$	Uniaxial
PVC	¼	0.035	0.0086	$2.3 \cdot 10^{-4}$	Biaxial
PVC	½	0.035	0.017	$2.3 \cdot 10^{-4}$	Biaxial
PVC	1	0.035	0.035	$2.3 \cdot 10^{-4}$	Equibiaxial
HDPE	–	0.05		$4.1 \cdot 10^{-4}$	Uniaxial
HDPE	¼	0.045	0.011	$3.7 \cdot 10^{-4}$	Biaxial
HDPE	½	0.045	0.022	$3.7 \cdot 10^{-4}$	Biaxial
HDPE	1	0.045	0.045	$3.7 \cdot 10^{-4}$	Equibiaxial

Table 2. Material parameters for PVC and HDPE.

	E [MPa]	ν	$\dot{\varepsilon}_0$ [s^{-1}]	C	σ_T [MPa]	C_R [MPa]	$\bar{\lambda}_L$	α	β	σ_S [MPa]	H
PVC	1800	0.30	0.0010	0.0700	47.3	4.4	1.87	1.3	1.27	38	16
HDPE	450	0.40	0.00045	0.108	12.1	1.2	3.00	1.0	1.04	19	24

Table and figure captions

Table 1. Biaxial test programme.

Table 2. Material parameters for PVC and HDPE.

Figure 1. Sketch of the biaxial test specimen including some relevant measures (in mm).

Figure 2. Cauchy stress – logarithmic strain curves from uniaxial tension and compression tests of PVC and HDPE.

Figure 3. Force-displacement curves for PVC: a) uniaxial test on biaxial sample, and the biaxial tests: b) $\rho = 1/4$, c) $\rho = 1/2$, and d) $\rho = 1$. Force in *x*-direction is plotted against displacement in *x*-direction, and force in *y*-direction is plotted against displacement in *y*-direction.

Figure 4. Failure of biaxial tensile test specimen with extension ratio $\rho = ½$ made of a) PVC, and b) HDPE.

Figure 5. Force-displacement curves for HDPE: a) uniaxial test on biaxial sample, and the biaxial tests: b) $\rho = 1/4$, c) $\rho = 1/2$, and d) $\rho = 1$. Force in *x*-direction is plotted against displacement in *x*-direction, and force in *y*-direction is plotted against displacement in *y*-direction.

Figure 6. a) Rheological representation of the constitutive model with inter-molecular (A) and network (B) contributions, and b) Stress contributions from Parts A and B.

Figure 7. Raghava yield surfaces for PVC and HDPE in plane stress: a) principal stress space, and b) stress space defined by invariants.

Figure 8. The finite element mesh of a) the uniaxial tensile specimen, and b) the biaxial specimen. The samples are not drawn in the same scale.

Figure 9. Force-displacement curves from experimental tests and numerical simulations of the dogbone-shaped specimen loaded in tension.

Figure 10. Results from numerical simulations with meshes with three and five elements through thickness.

Figure 11. Force-displacement curves in *x*-direction for biaxial sample of PVC loaded at various extension ratios: a) experimental test results, and b) finite element simulations.

Figure 12. Force-displacement curves in *x*-direction for biaxial sample of HDPE loaded at various extension ratios: a) experimental test results, and b) finite element simulations.

Figure 13 Green strain fields after 5 mm displacement for the PVC specimen subjected to equibiaxial tension: a) E_{xx} from the experiment, b) E_{xx} from the numerical analysis, c) E_{yy}

from the experiment, d) E_{yy} from the numerical analysis, e) E_{xy} from the experiment, and f) E_{xy} from the numerical analysis. Note that the shear strains fields are plotted with a colour bar different from the other strain fields.

Figure 14. Green strain fields after 5 mm displacement in for the HDPE specimen subjected equibiaxial tension: a) E_{xx} from the experiment, b) E_{xx} from the numerical analysis, c) E_{yy} from the experiment, d) E_{yy} from the numerical analysis, e) E_{xy} from the experiment, and f) E_{xy} from the numerical analysis. Note that the shear strains fields are plotted with a colour bar different from the other strain fields.

Figure 15. Green strains at the centre point of the biaxial tension specimens of PVC as function of displacement in *x*-direction: a) E_{xx} from the experiment, b) E_{xx} from the numerical analysis, c) E_{yy} from the experiment, and d) E_{yy} from the numerical analysis.

Figure 16. Green strains at the centre point of the biaxial tension specimens of HDPE as function of displacement in *x*-direction: a) E_{xx} from the experiment, b) E_{xx} from the numerical analysis, c) E_{yy} from the experiment, and d) E_{yy} from the numerical analysis.