PAPER ID: 1143
DOI: 10.18462/iir.rankine.2020.1143

Gradient Based Design Optimization of a Radial Inflow Turbine

Brede A. L. Hagen[(a)], Matteo Cavo[(b)], Trond Andresen[(c)], Petter Nekså[(c,a)]

[(a)] Norwegian University of Science and Technology – Department of Energy and Process Engineering
Trondheim, 7034, Norway, brede.hagen@ntnu.no
[(b)] University of Genoa
Genoa, 16126, Italy, matteo.cavo@edu.unige.ito
[(c)] SINTEF Energy Research
Trondheim, 7034, Norway, petter.neksa@sintef.no

ABSTRACT

The expander is one of the key components of an ORC as the cycle efficiency strongly depends on the expander efficiency. This paper presents a method for design optimization of a radial inflow turbine (RIT) using a mean-line model. The novelty of this work lies in the equation-based formulation of the mathematical problem, which enables the use of an efficient gradient based method for optimization. This means that there is no distinction between real decision variables such as specific speed and velocity ratio, and parameters that are unknown a priori such as rotor outlet entropy and velocity. Constraints are imposed to ensure conservation of mass, and to ensure a feasible and consistent design, and the objective is to maximize the total-to-static efficiency. The main results showed an average CPU time less than one second and a success rate of 80% for converging to the global optimum when the independent variables were given random start values. We therefore recommend the proposed method for preliminary RIT-design or to be integrated into an ORC system design model enabling for instance working fluid screening with fluid-dependent expander efficiency.

Keywords: Radial Inflow Turbine; Mean-line model; Optimization; ORC; Preliminary design

1. INTRODUCTION

The Radial inflow turbine (RIT) is a promising expander technology for Organic Rankine Cycles (ORCs). Advantages of the RIT compared to the more common axial turbine is the ease of manufacturing, fewer stages and robustness (Dixon and Hall, 2014).

The traditional ORC analysis involves using a fixed, pre-defined expander efficiency (Song et al., 2016), (Lazzaretto and Manente, 2014). However, this might lead to misleading result because the attainable expander efficiency depends on the expander technology, the operating conditions and the working fluid (Da Lio et al., 2017). Song et al. (2016) performed two separate working fluid screening analyses; One analysis with constant expander efficiency and one analysis using a mean-line RIT model for predicting the expander efficiency. Their results demonstrated that inclusion of the mean-line model significantly changed the result compared to the fixed-efficiency analysis since the RIT efficiency differed by up to 11 percentage point among the considered working fluids.

Predicting expander efficiency based on working fluid properties can be performed in a less computationally expensive way by using a generic efficiency map created by a mean-line model assuming an ideal gas as the working fluid. Lazzaretto and Manente (2014) used efficiency maps from (Perdichizzi and Lozza, 1987) and (Macchi and Perdichizzi, 1981) for predicting the design performance of RITs and axial turbines respectively in their ORC analyses. Although, these maps predict a consistent trend in the attainable expander efficiency with respect to turbine size and pressure ratio, the accuracy with respect to real gas behavior could be questioned because the expansion in an ORC often starts close to the critical pressure of the working fluid. Another limitation of these maps is that they provide the design point efficiency only and are therefore not suitable for preliminary RIT design or off-design analyses.

Several mean-line models for the RIT exist, and the reader is referred to text books on turbine design such as (Moustapha et al., 2003), (Aungier, 2005), or (Dixon and Hall, 2014) for a complete overview of the topic.

Recent literature on RIT mean-line models for ORC application focus accuracy because most existing loss-correlations rely on data for low pressure-ratio RITs. Persky and Sauret (2019) evaluated more than 1.5 million different sets of rotor loss correlations for RITs expanding CO_2 and R143a and identified the set that gave the best agreement with corresponding CFD-simulations. Meroni et al. (2018) calibrated a given set of loss-correlations towards published experimental data on six high pressure ratio RITs with relevance to ORC operating conditions.

Little attention is paid in the open literature to the various options for solving the preliminary RIT design optimization problem. Based on the above-mentioned literature we believe the conventional approach involves treating the mean-line model as a "black box" that takes the real decision variables such as specific speed and velocity ratio as input and solves the remaining unknown parameters to ensure conservation of mass and a consistent design. The optimization is performed by multiple evaluation of the "black box" model; either by manually adjusting the design variables or by using a direct search algorithm for optimizing the design variables. A well-documented example of such an approach is described in (Da Lio et al., 2017).

The intention of this work is to present an alternative and more efficient treatment of the preliminary RIT design optimization problem. The method is inspired by the work by Agromayor and Nord (2019) for design optimization of axial turbines. The method involves optimizing the real decision variables and the unknown parameters simultaneously using a gradient based optimization algorithm. This means for instance that conservation of mass is ensured by imposing equality constraints and that a consistent design only occurs when the optimization algorithm is finished with a feasible result.

2. METHODOLOGY

The proposed method for RIT design optimization consists of a single non-linear constrained optimization problem to be solved by a gradient based optimization algorithm, as is illustrated in Figure 1. Details on the components of this method is outlined in the remainder of this section.

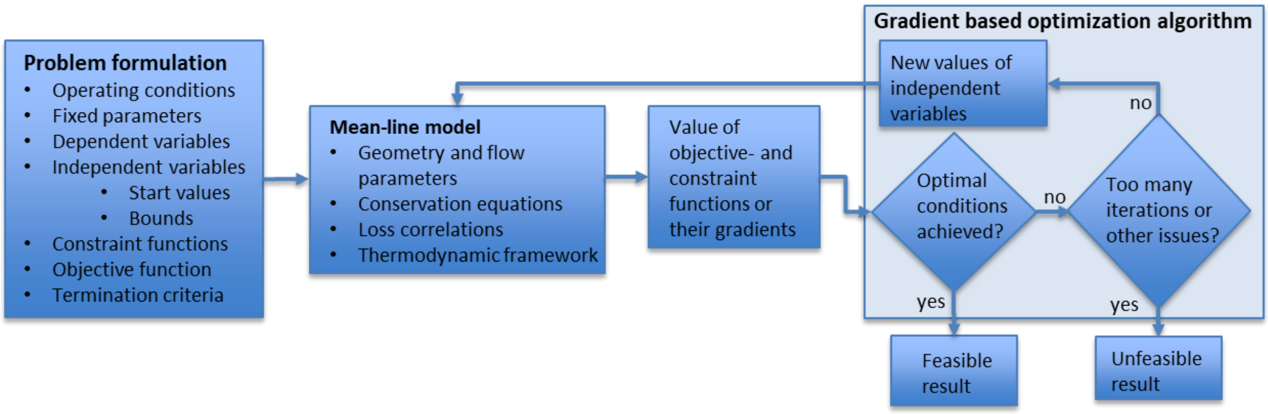
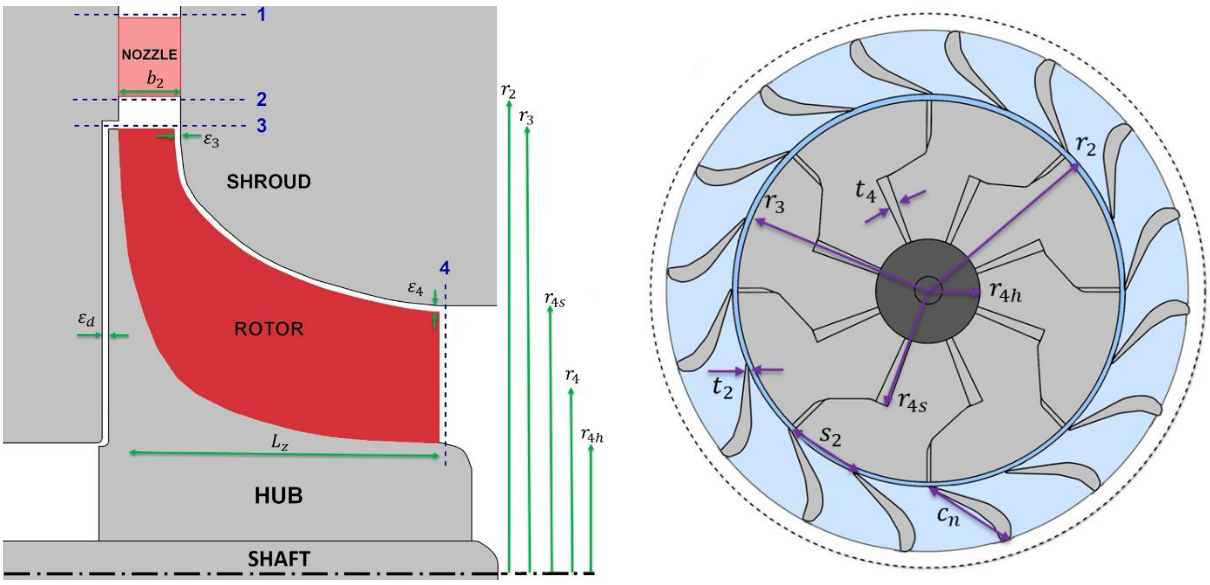


Figure 1: Overview of the proposed method for gradient based RIT design optimization

2.1. Mean-line model

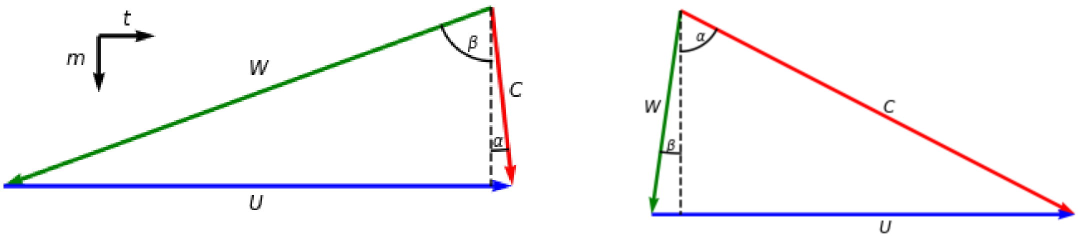
The mean-line model is implemented in the programming language C and consider the nozzle, interspace and the rotor as illustrated in Figure 2. The figure also indicates the involved geometry parameters. The mean-line model also requires knowledge of the flow parameters and thermophysical properties along the mean-line through the turbine. The flow parameters consist of the nozzle outlet velocity (angle and absolute value), and the velocity triangles at rotor inlet and outlet illustrated in Figure 3. Blockage factors are neglected, which means that blade-blockage and boundary layer effects are neglected for mass flow rate calculations. The effective cross-sectional flow areas can thus be computed by Eq. (1).

$$A_i = 2\pi r_i b_2, \quad \text{i=2,3}$$
$$A_4 = \pi\left(r_{4s}^2 - r_{4h}^2\right) \qquad \text{Eq. (1)}$$



a) View of the meridional channel **b) View in the axial direction**

Figure 2: Geometry parameters used in the mean-line model



a) Rotor outlet **b) Rotor inlet**

Figure 3: Flow parameters used in the mean-line model except for the nozzle outlet velocity, which is very similar to the absolute velocity at rotor inlet (no rotational velocity U)

The enthalpy distribution through the turbine is calculated by conservation equations. Nozzle outlet enthalpy is calculated by conservation of energy through the nozzle, Eq. (2)

$$h_2 = h_{01} - \frac{1}{2} C_2^2 \quad \text{Eq. (2)}$$

The rotor inlet enthalpy and velocity are calculated by conservation of angular momentum (Eq. (3)) and energy (Eq. (4)) through the interspace.

$$C_{3t} = \frac{r_2}{r_3} C_{2t} \quad \text{Eq. (3)}$$

$$h_3 = h_{01} - \frac{1}{2}(C_{3m}^2 + C_{3t}^2) = h_{01} - \frac{1}{2} C_3^2 \quad \text{Eq. (4)}$$

The rotor outlet enthalpy is calculated by conservation of rothalpy through the rotor, Eq. (5).

$$h_4 = h_3 + \frac{1}{2}(W_3^2 - U_3^2) - \frac{1}{2}(W_4^2 - U_4^2) \quad \text{Eq. (5)}$$

The thermophysical properties of the working fluid are calculated with Refprop 10.0 (Lemmon et al., 2018).

Many configurations of loss-correlations are possible in a mean-line model because there exist multiple correlations for each loss-mechanism occurring in the RIT. This work employs the set of loss-correlations presented by Meroni et al. (2018) because it was calibrated against experimental data for high pressure ratio

RITs. This set consist of 10 correlations considering loss-mechanisms occurring in the nozzle, interspace and the rotor, including post expansion which activates when the nozzle and/or rotor outlet velocity exceeds speed of sound. Since a diffuser model is not included, it is assumed that all the kinetic energy leaving the rotor is lost. The rotor incidence loss is neglected because it is assumed that the turbine can be designed with zero incidence, as suggested in (Moustapha et al., 2003).

2.2. Problem formulation

The main novelty of the proposed method is the optimization formulation shown in Table 1. The independent variables govern geometry, flow and thermodynamic parameters and finding the optimal value of these is the only mathematical problem to be solved (except explicit calculations and numerical routines within the thermodynamic framework), because the imposed equality constraints ensure conservation of mass and a consistent design. All independent variables are constrained between a lower and an upper bound to ensure a feasible design. In addition, an inequality constraint is imposed to avoid a too large increase in the rotor cross-sectional flow area, as suggested by Da Lio et. al (2017). For numerical reasons it is an advantage to bring all the variables and constraints to the same order of magnitude. Therefore, the spouting velocity $c_0 = (2\Delta h_{is})^{0.5}$ is used as a velocity scale, the angles are unit-converted to radians, and the constraint- and objective functions are written in a non-dimensional form.

Table 1. Formulation of the RIT design optimization problem

Independent variables			
Nozzle outlet velocity	C_2/c_0	$\in$	$[0.1,\ 0.8]$
Nozzle outlet flow angle	α_2	$\in$	$[30°,\ 80°]$
Rotor inlet meridional velocity	C_{3m}/c_0	$\in$	$[0.02,\ 0.4]$
Normalized rotor outlet velocity	W_4/c_0	$\in$	$[0.1,\ 0.9]$
Rotor outlet flow angle	β_4	$\in$	$[-70°, -20°]$
Specific speed	$\omega_s = \omega \dot{V}_{4,is}^{1/2} / \Delta h_{is}^{3/4}$	$\in$	$[0.3, 0.8]$
Velocity ratio	$\nu_s = U_3/c_0 = \omega r_3/c_0$	$\in$	$[0.5, 0.8]$
Rotor radius ratio (shroud/inlet)	r_{4s}/r_3	$\in$	$[0.4,\ 0.7]$
Rotor radius ratio (hub/shroud)	r_{4h}/r_{4s}	$\in$	$[0.4,\ 0.8]$
Blade height to radius ratio	b_2/r_3	$\in$	$[0.04,\ 0.34]$
Outlet entropy[a,b]	s_{out}/s_1	$\in$	$[1.0, s_{ref}/s_1]$
Constraint functions			
Consistent outlet pressure	$1.0 - p(h_4, s_4)/p_4 = 0$		
Consistent outlet enthalpy[a]	$\dfrac{h_{out} - h(p_{out}, s_{in}) - \sum \Delta h_{loss}}{0.5\ W_{out}^2} = 0$		
Conservation of mass[c]	$1.0 - \rho(h, s) W_m A/\dot{m} = 0$		
Maximum rotor area ratio	$2.5 - A_4/A_3 \geq 0$		
Objective function			
Maximize total-to-static efficiency	$\eta_{ts} = \dfrac{h_{01} - h_{04}}{\Delta h_{is}}$		

[a] *Two values (nozzle and rotor)*
[b] *Reference entropy, s_{ref}, is the resulting outlet entropy when $\eta_{ts} = 0.5$*
[c] *Three values (Nozzle outlet, rotor inlet and rotor outlet)*

Some parameters are not suitable for optimization because the total-to-static efficiency is a monotonic function of them, and/or because their values are constrained by factors not included in the model, such as manufacturing limits. Instead, these parameters are either fixed or calculated from the set of independent variables during the optimization. The value of the fixed parameters and formulas for the dependent variables are shown in Table 2, and most of these were suggested default values from Aungier (2005).

2.3. Optimization algorithm and termination criterion

The optimization problem presented in the previous section contains 12 independent variables and 7 non-linear constraints, which is a challenging mathematical problem. However, since the mean-line model is written in an equation-oriented fashion (inner iterations are limited to thermodynamic calculations only) an efficient gradient based method can be applied (Astolfi et al. 2017).

Table 2. Value of fixed parameters and formulas for the dependent variables

Parameter	Symbol	Value	Source
Nozzle blade trailing edge thickness	t_2	$0.012c_n$	(Aungier, 2005)
Rotor blade trailing edge thickness	t_4	$0.02r_4$	(Aungier, 2005)
Rotor axial length	L_z	$1.5(r_{4s} - r_{4h})$	(Aungier, 2005)
Blade tip clearances	ε_i (i=3,4)	$\max(0.4\ mm, 0.02b_i)$	
Disc clearance	ε_d/r_3	0.05	
Nozzle chord to pitch ratio	c_n/s_2	1.33	(Aungier, 2005)
Interspace distance	$r_2 - r_3$	$2b_2 \cos\alpha_2$	(Aungier, 2005)
Number or rotor blades	Z_r	$12 + 0.03(\alpha_2 - 57°)^2$	(Aungier, 2005)
Number of nozzle blades	Z_n	$Z_r + 3$	

The optimization algorithm applied in this work is the Sequential Quadratic Programming (SQP) method, NLPQL (Schittkowski, 1985). Gradients are calculated using a second order central difference approximation for numerical differentiation. The KKT optimal criterion is set to 1.0E-7. The maximum number of iterations are set to 80. This means that NLPQL return an unfeasible result if the KKT optimal criterion have not been met within 80 iterations or other issues occurs, see (Schittkowski, 1985) for details.

3. RESULTS

A design optimization of a RIT for a hypothetical ORC have been performed. The working fluid is propane and the 862-kW heat source enter the heat recovery heat exchanger at 150 °C. The turbine operating conditions are obtained from (Hagen et al., 2020) and showed in Table 3 together with selected result from the optimization.

Table 3. Operating conditions and selected results from the RIT design optimization

Operating conditions		Optimized geometry and performance					
T_{01} [C]	131.57	r_2	43.5 mm	t_2	0.19 mm	v_s	0.62
$\dot{m}$ [kg/s]	1.99	c_n	16.3 mm	t_4	0.84 mm	ω_s	0.41
p_{01} [bar]	46.62	Z_n	22	ε_d	2.11 mm	η_{ts}	0.849
p_4 [bar]	9.58	α_2	72.6°	ω	57142 rpm	Shaft power	142 kW

Loss distribution, rotor geometry and velocity triangles from the design optimization are shown in Figure 4. The single most important loss-mechanism is the rotor clearance loss which accounts for around 4 percentage point reduction in the total-to-static efficiency. The absolute velocity at the nozzle outlet and rotor inlet are supersonic, which resulted in a post expansion loss at the nozzle outlet. The ratio between the rotor shroud and inlet radius (r_{4s}/r_3) was the only independent variable with an active variable bound. This variable went to its upper bound of 0.7, which was suggested by Aungier (2005).

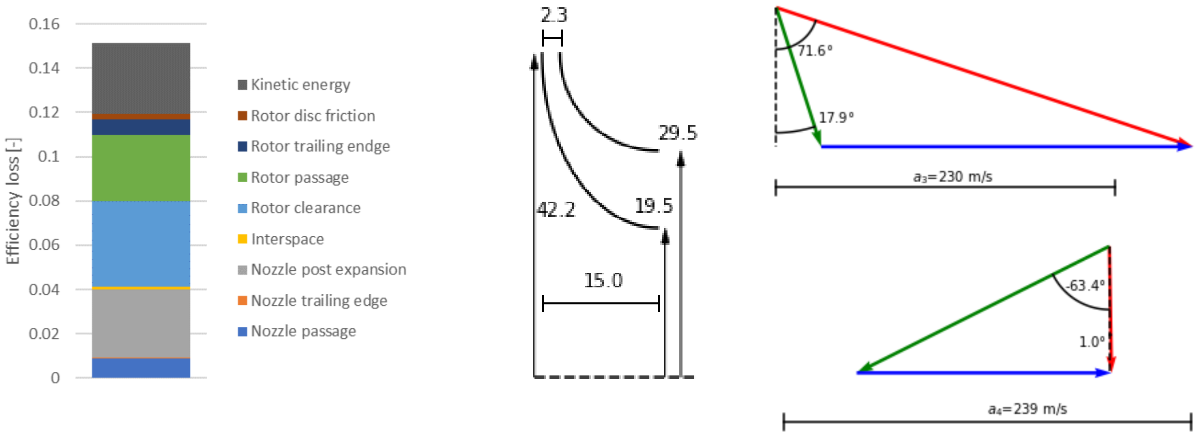
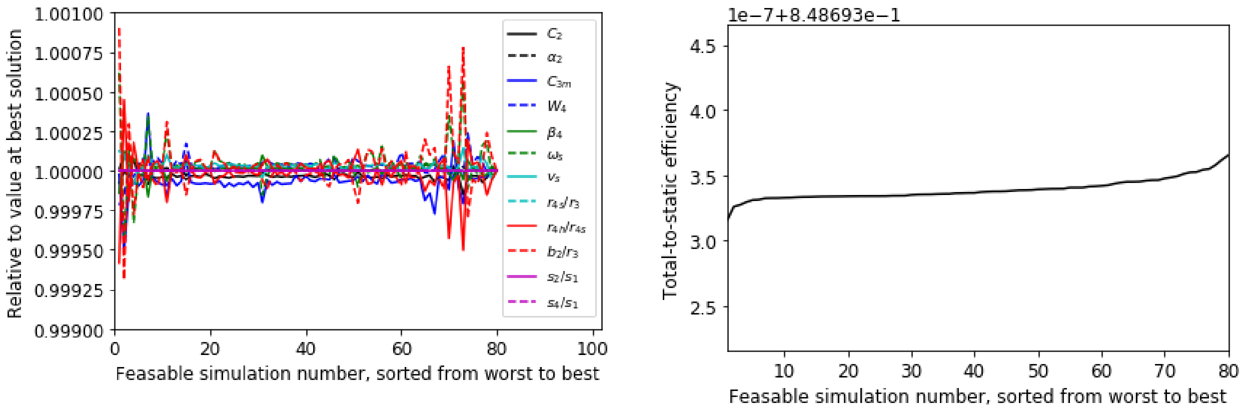


Figure 4: Loss distribution, rotor geometry (values in mm) and velocity triangles for the optimized RIT

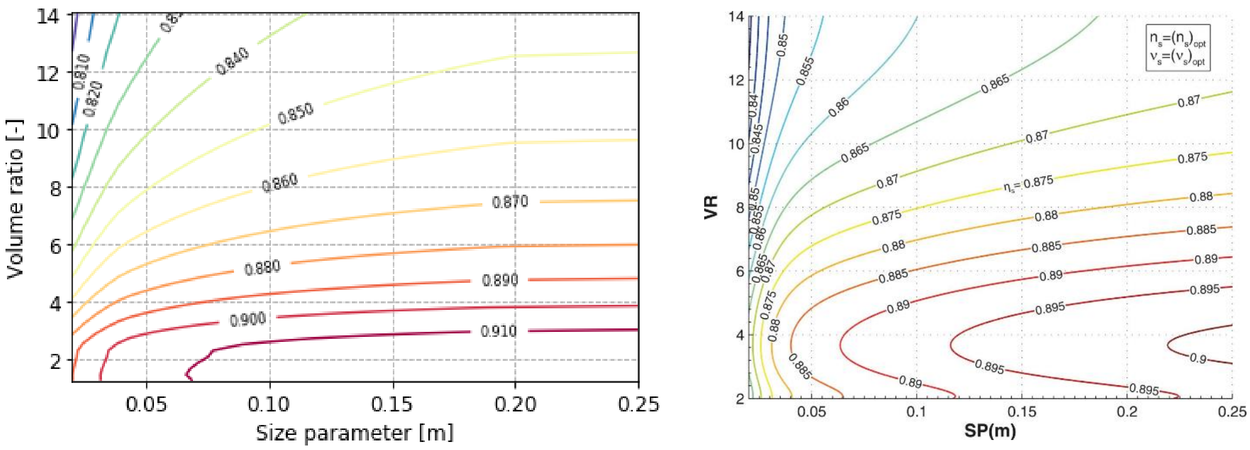
A total of 100 optimizations, each with random start values of the independent variables were carried out to evaluate the risk of converging to local optimum and to assess the robustness of the optimization formulation. The optimized variable values and maximized efficiency from the 80 optimizations that returned a feasible result are shown in Figure 5. The feasible optimization results are quite consistent since all optimized variable values are within ± 0.1 % and the variation in the objective function is smaller than 1E-7. This is a strong indication that all feasible optimizations converged very close to the global optimum, i.e. that the mathematical optimization problem contains none or few local optimums.



a) Independent variables **b) Objective function**

Figure 5: Results from 80 of the 100 optimizations with random start value of the independent variables that converged to a feasible solution. Average CPU time per optimization is 0.83 s

A comparison between maximized RIT efficiency from the presented methodology and results from the open literature has also been performed. Figure 6 shows two RIT performance maps with R245fa as working fluid and an outlet pressure of 1.976 bar. Both maps were generated by performing multiple RIT optimizations with different inlet conditions. Figure 6a shows the maximized total-to-static efficiency using the presented methodology for different size parameters, $SP = \dot{V}_{4s}^{0.5}/\Delta h_s^{0.25}$ and isentropic volume ratios, $VR = \dot{V}_{01}/\dot{V}_{4s}$. Figure 6b shows a similar performance map that are taken from (Da Lio et al., 2017). Both maps predict an increasing efficiency with increasing SP. Otherwise there are some notable differences. Both maps show a VR that maximizes the efficiency (at a constant SP), but the proposed method predict a much lower value of this optimum VR. The proposed method gives larger efficiency for VR less than 3.5-4 and a lower efficiency for larger VRs, i.e. a larger penalty of the attainable total-to-static efficiency for increasing VR.



a) This work **b) Figure from Da Lio et al. (2017)**

Figure 6: Maximized total-to-static efficiency with R245fa as working fluid and outlet pressure of 1.976 bar for different design inlet conditions

4. DISCUSSION

The results show that the presented methodology for RIT design optimization is fast, robust and converges to the global optimum. The main limitation is the uncertainty which is at least as much as the discrepancy between the two performance maps in Figure 6, which is largest for the largest VRs and smallest SPs (up to 4%). One source of uncertainty is the fixed parameters and the dependent variables from Table 2. In particular, the blade tip clearances contribute to uncertainty because they directly affect one of the most important loss mechanisms in the RIT and because their lower bound is constrained by manufacturing difficulty (Dixon and Hall, 2014). We believe that the main source of uncertainty is related to the accuracy of the loss correlations, which is an issue for all mean-line models. Meroni et al. (2018) reports a maximum deviation in efficiency of 2.83% between their calibrated mean-line model and the experimental data used for calibration, and highlights that a larger uncertainty is expected for turbines operating with different characteristics (geometry and operating conditions) than those used for calibration. A logical next step is therefore to validate the applied loss-correlations towards more experimental data. This requires modifications of the optimization formulation from Table 1 in such a way that the mass flow rate and the efficiency is predicted for a given turbine geometry and operating conditions (working fluid, inlet state, and pressure-ratio). This will also enable the calculation of RIT off-design behavior and performance and is left for future work.

5. CONCLUSION

Estimating the expander efficiency considering both fluid properties and operating conditions is a requirement for performing realistic ORC analyses. This paper presents an effective method for preliminary design and performance estimation of a Radial inflow turbine (RIT) using a mean-line model. The novelty of this work is the equation-oriented formulation of the mean-line model that enables use of an efficient gradient based algorithm for optimization. The demonstration of the proposed method showed an average optimization time less than one second and a success rate of 80% of hitting the global optimum when the independent variables were given random start values. We therefore recommend the proposed method for preliminary RIT-design or to be integrated into an ORC system design model for predicting the expander design efficiency. Future work involves extending the presented methodology to also predict RIT performance and behavior for a given geometry and operating conditions. This will enable further experimental validation of the mean-line model and RIT off-design analysis.

ACKNOWLEDGEMENTS

This publication has been funded by HighEFF - Centre for an Energy Efficient and Competitive Industry for the Future, an 8-years' Research Centre under the FME-scheme (Centre for Environment-friendly Energy Research, 257632/E20). The authors gratefully acknowledge the financial support from the Research Council of Norway and user partners of HighEFF.

The authors also greatly acknowledge the discussions with Roberto Agromayor regarding formulation of the Radial Inflow turbine design optimization problem.

NOMENCLATURE

a	Speed of sound (m×s^{-1})	s	Specific entropy; Blade pitch (J×kg^{-1}×K^{-1}); (m)
A	Cross sectional flow area (m^2)	SP	Size parameter (m)
b	Blade height (m)	t	Blade thickness (m)
c	Blade chord length (m)	T	Temperature (K)
c_0	Spouting velocity (m×s^{-1})	U	Rotational velocity (m×s^{-1})
C	Absolute velocity (m×s^{-1})	υ_s	Velocity ratio (-)
h	Specific enthalpy (J×kg^{-1})	VR	Volume ratio (-)
$\dot{m}$	Mass flow (kg×s^{-1})	$\dot{V}$	Volume flow rate (m^3×s^{-1})
p	Pressure (Pa)	W	Relative velocity (m×s^{-1})
r	Radial distance from shaft center (m)	Z	Number of blades (-)
		Greek	
α	Absolute flow angle (rad)	η_{ts}	Total-to-static efficiency (-)
β	Relative flow angle (rad)	ρ	Working fluid density (kg×m^{-3})
ε	Clearance (m)	ω	Rotational speed (rad×s^{-1})
		Subscripts	
0	Total state	m	Meridional component
1-4	Positions along the mean-line, Figure 2a	n	Nozzle
h	Hub	out	Outlet
in	Inlet	s	Specific or shroud
is	Isentropic	t	Tangential component

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