Transmission Line Length Estimation based on Electrical Parameters

Til Kristian Vrana
SINTEF Energi, Trondheim, Norway

Iver Bakken Sperstad
SINTEF Energi, Trondheim, Norway

Abstract—Some grid data sets contain only electrical parameters, but not the individual transmission line lengths. The failure rates of transmission lines are often estimated based on aggregated statistics given in failure rate per unit length of transmission line. These failure rates are key input parameters to power system reliability assessments. When length data are not available, such data need to be estimated to carry out reliability assessments. A simple and practical method is proposed for estimating transmission line lengths from the electrical parameters inductance and capacitance (which normally are available in grid data sets). The validity of the method is corroborated by testing it on a grid data set of the Danish transmission system that contains length information. The average error was 3.7 % for calculating the length of overhead lines and 27.3 % for estimating the length of all transmission lines (overhead, cable and combinations of these). The value of the method is illustrated by applying it in a case study on the reliability assessment of a part of the Norwegian transmission system, where length information was not available.

I. Introduction

Unavailability of suitable data is traditionally an important barrier to the practical application of probabilistic methods for power system reliability assessment [1]. This barrier has also been highlighted in a recent survey of European Transmission System Operators [2]. Data availability is also a substantial challenge for the research community and academia, as it inhibits the testing of newly developed methods beyond the standard reliability test systems available in the research literature. Furthermore, the need for more detailed and higher-quality reliability data increases as more probabilistic reliability management approaches are being adopted. Their benefits are greater if failure probability data for different components significantly differ from each other, in contrast to assuming e.g. that all lines have the same failure rate [3] [4].

This article specifically considers the failure rates of transmission lines and addresses one particular challenge related to such reliability data, namely how to estimate the lengths of transmission lines; when only aggregated reliability statistics is available, length estimates are needed for estimating the failure rates of individual transmission lines. Although this may seem as somewhat banal problem, it is a real and common practical challenge for case studies on real systems where limited data is available to the analyst, researcher or student.

Failure rate statistics are usually given on an aggregated form as the expected number of failures per year per unit length of the transmission line. Uncertainties in the failure rate for individual transmission lines are due to a number of factors, e.g. weather conditions [1] [5], but actually having an estimate of the length of the transmission line is a first step in reducing these uncertainties. In lack of data on the length of the lines, they may in principle be estimated by finding e.g. the line resistance per unit length and comparing with grid data, but this requires data on the conductor material and cross section. Similarly, using geographical information requires either Geographical Information System (GIS) data that are consistent with the grid data or time-consuming manual inspections of maps.

A practical and easily implemented method for estimating transmission line lengths has been developed. This method can be useful when detailed data sets are inaccessible or non-existing. The proposed method only requires electrical data of the transmission line and thus serve as a convenient alternative to the estimation approaches mentioned above.

The rest of the article is organised as follows: Section II introduces basic theoretical background and derives the relevant equations. Section III explains a practical method to calculate overhead line lengths. Section IV demonstrates the use of the method and validates it using a real data set. Section V proposes a generalised transmission line length estimation method that is not limited to overhead lines. Section VI briefly describes the application of the method in a reliability assessment case study. This case study (with an incomplete data set) originally motivated the development of the method. Section VII concludes the article.

II. Overhead Line Length Calculation

In this section, the theoretical basis of length calculation is introduced. The relation between the electrical data and the line geometry is demonstrated. An equation for overhead transmission line length calculation is derived. This will in turn form the basis for general transmission line length estimation.

A. Characteristic Impedance

Based on inductance L_j and capacitance C_j, when neglecting losses, the characteristic impedance of overhead transmission line j can be calculated, based on Equation 6.10 in [6]:

$$Z_j^0 = \sqrt{\frac{L_j}{C_j}} \tag{1}$$

The vacuum wave impedance is given in Equation (2):

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 376.730\,\Omega \tag{2}$$

Based on Equation (1) and Equation (2), the relative characteristic impedance α_j^{imp} of overhead transmission line j can be calculated:

$$\alpha_j^{\mathrm{imp}} = \frac{Z_j^0}{Z_0} \tag{3}$$

B. Geometry

Radius r_j denotes the radius of a conductor of overhead transmission line j. In case of a conductor bundle (not a single conductor) it represents the geometric mean radius of the conductor bundle. This means that it represents the radius of a solid non-bundled conductor that would have similar electromagnetic field properties as the real conductor bundle. Distance d_j of overhead transmission line j denotes the geometric mean distance between the conductors/conductor bundles.

Based on the radius r_j and the distance d_j, an overhead transmission line geometry parameter α_j^{geo} is introduced in Equation (4):

$$\alpha_j^{\mathrm{geo}} = \frac{1}{2\pi} \ln\left(\frac{d_j}{r_j}\right) \tag{4}$$

The inductance L_j of overhead transmission line j of length l_j in a medium with permeability μ is given by Equation (5), which is based on Equation (4) and Equation 6.1 in [6]:

$$L_j = l_j \mu \frac{1}{2\pi} \ln\left(\frac{d_j}{r_j}\right) = l_j \mu \alpha_j^{\mathrm{geo}} \tag{5}$$

The capacitance C_j of overhead transmission line j of length l_j in a medium with permittivity ϵ is given by Equation (6), which is based on Equation (4) and Equation 6.3 in [6]:

$$C_j = l_j \epsilon \frac{2\pi}{\ln\left(\frac{d_j}{r_j}\right)} = l_j \frac{\epsilon}{\alpha_j^{\mathrm{geo}}} \tag{6}$$

C. Relation between Characteristic Impedance and Geometry

Inserting Equation (5) and Equation (6) in Equation (1), the characteristic impedance Z_j^0 of line j can be expressed as:

$$Z_j^0 = \sqrt{\frac{l\mu\alpha_j^{\mathrm{geo}}}{l\epsilon/\alpha_j^{\mathrm{geo}}}} = \alpha_j^{\mathrm{geo}}\sqrt{\frac{\mu}{\epsilon}} = \alpha_j^{\mathrm{geo}}\sqrt{\frac{\mu_0}{\epsilon_0}}\sqrt{\frac{\mu_r}{\epsilon_r}} \tag{7}$$

Equation (7) can be simplified using Equation (2):

$$Z_j^0 = \alpha_j^{\mathrm{geo}} Z_0 \sqrt{\frac{\mu_r}{\epsilon_r}} \tag{8}$$

The conductors of overhead lines are surrounded by air, yielding:

$$\mu_r \approx 1 \quad \epsilon_r \approx 1 \quad \Rightarrow \quad \sqrt{\frac{\mu_r}{\epsilon_r}} \approx 1 \tag{9}$$

Applying the simplification of Equation (9) on Equation (8) yields:

$$Z_j^0 = \alpha_j^{\mathrm{geo}} Z_0 \tag{10}$$

Combining Equation (3) and Equation (10) yields:

$$\alpha_j^{\mathrm{geo}} = \alpha_j^{\mathrm{imp}} \tag{11}$$

Equation (11) shows that the geometry of an overhead line directly determines the characteristic impedance. It is therefore possible to retrieve information on the geometry from the electrical data found in a grid data set.

D. Length Calculation

Given capacitance C_j and inductance L_j, the electromagnetic time constant T_j of overhead line j is defined as:

$$T_j = \sqrt{L_j C_j} \tag{12}$$

Inserting Equation (5) and Equation (6) and applying the simplification of Equation (9) yields:

$$T_j = \sqrt{l_j \mu_0 \alpha_j^{\mathrm{geo}} \frac{l_j \epsilon_0}{\alpha_j^{\mathrm{geo}}}} = l_j \sqrt{\mu_0 \epsilon_0} \tag{13}$$

The speed of light in vacuum is:

$$c_0 = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 299.792 \, \frac{\mathrm{m}}{\mathrm{\mu s}} \tag{14}$$

Combining Equation (13) and Equation (14) yields an expression for the overhead line length:

$$l_j = c_0 T_j \tag{15}$$

III. PRACTICAL APPLICATION METHOD

A practical overhead line length calculation method for application on real data sets is described in this section.

A. Nomenclature

H	set of transmission line entries in a data set
I	set of transmission lines ($I \subset H$)
J	set of overhead lines ($J \subset I$)
K	set of other transmission lines ($K \subset I$) (cables, aggregate combination of cable and overhead line sections, compensated lines, etc.)

B. Data Quality Check

The length calculation formula proposed in Subsection II-D needs both the inductance L and the capacitance C of an transmission line. Equation (16) specifies which entries h of a set of transmission line entries H in a grid data set qualify to be considered in the set of transmission lines I:

$$I = \{h \in H \mid L_h > 0 \wedge C_h > 0\} \tag{16}$$

If the data set does not contain the necessary information for entry h, it is not considered a transmission line, and the length calculation cannot be applied for that entry. This is often the case for very short connections within substations, where capacitance is neglected ($C_h = 0$).

C. Overhead Line Identification

The user manual of the Network Information System (NIS) [7] contains data for $|H| = 78$ generic overhead line types between 66 kV and 400 kV. All entries of the data set are complete ($I = H$). The parameter α_i^{imp} is calculated for all $i \in I$ with Equation (3). The α_i^{imp}-values of all lines were in the range $0.70 \leq \alpha_i^{\mathrm{imp}} \leq 1.08$, with a mean value of $\overline{\alpha}_i^{\mathrm{imp}} = 0.91$. Thus, the relative characteristic impedance α_i^{imp} is within a quite narrow range for all 78 overhead line configurations.

Cables ($k \in K$) have a significantly different geometry compared to overhead lines ($\alpha_k^{\mathrm{geo}} < \alpha_j^{\mathrm{geo}}$), as the spacing between the conductors is much smaller. The relation between conductor geometry and characteristic impedance is also different for cables, as the space between the conductors in not filled with homogeneous air ($\alpha_k^{\mathrm{imp}} < \alpha_k^{\mathrm{geo}}$). This leads to cables ($\alpha_k^{\mathrm{imp}} \approx 0.1$) having much lower impedance values than overhead lines ($\alpha_j^{\mathrm{imp}} \approx 1.0$). The impedance difference between cables and overhead lines ($\alpha_k^{\mathrm{imp}} << \alpha_j^{\mathrm{imp}}$) is used to identify overhead lines J in a set of transmission lines I, as given in Equation (17):

$$J = \left\{ i \in I \mid \alpha_i^{\mathrm{imp}} \geq 0.7 \right\}$$
$$K = \left\{ i \in I \mid \alpha_i^{\mathrm{imp}} < 0.7 \right\} \tag{17}$$

The threshold value is based on the overhead line type with the lowest impedance, as mentioned above. It should be noted that K does not represent only cables, but also other transmission assets (e.g. aggregations of cable and overhead line). For the data set in the user manual of the NIS, which only contains overhead lines, this yields $J = I$.

Equation (17) is of course not a very precise criterion to identify overhead lines. A resonant circuit can be tuned to realise $\alpha_i^{\mathrm{imp}} \geq 0.7$ without being an overhead line. Thick conductor bundles can lead to $\alpha_i^{\mathrm{imp}} < 0.7$, even though they are overhead lines. Also double-circuit lines can fall below the threshold, leading to false identification. The criterion in Equation (17) can only give an indication, which should work for the majority of transmission lines in a grid data set, but it will not be correct for all.

D. Overhead Line Length Calculation

For those transmission line entries $j \in J$, which were identified as overhead lines using Equation (17), the time constant T_j can be calculated using Equation (12). Based on this time constant, the length can be calculated using Equation (15).

It should be noted that for K, Equation (15) cannot be expected to deliver meaningful results.

IV. Validation

A methodology for validation of Equation (15) is defined and applied using generic line type data and real grid data.

A. Validation Methodology

l_j^{calc} denotes the length of overhead transmission line j calculated with Equation (15). l_j^{ref} denotes the length of overhead transmission line j as specified in the grid data set. The measures for deviation and error used in this study are based on [8], where a justification can be found. The logarithmic length calculation deviation is defined as:

$$D_j = \log_2 \left(\frac{l_j^{\mathrm{calc}}}{l_j^{\mathrm{ref}}} \right) \tag{18}$$

The following quality criterion for a valid length calculation is defined and applied here:

$$|D_j| \leq 0.5 \quad \Leftrightarrow \quad \frac{\sqrt{2}}{2} \leq \frac{l_j^{\mathrm{calc}}}{l_j^{\mathrm{ref}}} \leq \sqrt{2} \tag{19}$$

The overall root mean square (RMS) error for line length calculation of a set of lines is:

$$E_J = \sqrt{\frac{1}{|J|} \sum_j^J (D_j)^2} \tag{20}$$

B. Validation based on Generic Line Type Data

The developed method has been validated using the data from the user manual of the NIS [7] (Subsection III-C). The length of all 78 overhead line types has been calculated with Equation (15). The calculation had an RMS error of only $E_J = 2.84\,\%$. This accuracy is very high as the error is within a similar range as the rounding error of the input data.

However, the generic overhead line type data have likely been calculated based on similar equations as the calculation method is based on, possibly causing the achieved good calculation accuracy. Therefore, the validation methodology is applied using real grid data in the following subsection.

C. Validation based on Danish Grid Data

The transmission system in Danmark has been used as a test case for validation. The Danish data contain a set of transmission line entries H with $|H| = 358$. Of these 358 entries, $|I| = 304$ fulfill the criterion in Equation (16), meaning the necessary data are complete for these entries.

The α_i^{imp}-values have been calculated for I, and this characteristic of the transmission lines is shown in Figure 1. The green area covers J, and the red area covers K, as defined in Equation (17).

The transmission system in Danmark is a special case, since it contains an unusually large share of transmission lines that are (at least partly) cables. Considering Equation (17), this yields $J = 99$ and $K = 205$. For comparison, the α_i^{imp}-characteristic is shown for Norway in Figure 5 in Section VI.

The length of the 99 overhead lines has been calculated with Equation (15). The calculation had a RMS error of only $E_J = 3.70\,\%$. This calculation accuracy indicates that Equation (17) and Equation (15) are valid.
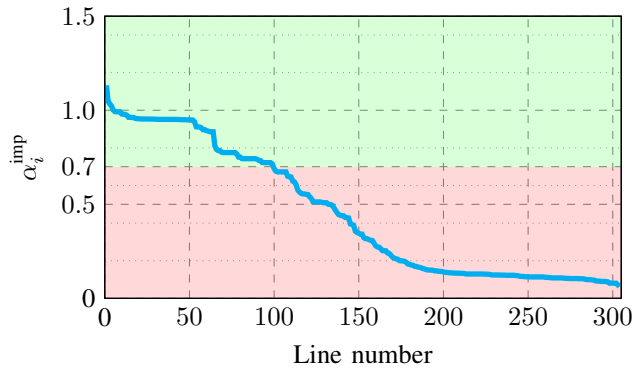
Fig. 1: Impedance characteristic of the Danish data set

The calculation deviations D_j of J are now plotted in relation to their α_j^{imp}-value, shown in Figure 2. The red horizontal lines display the limits of a valid length calculation (Equation (19)).
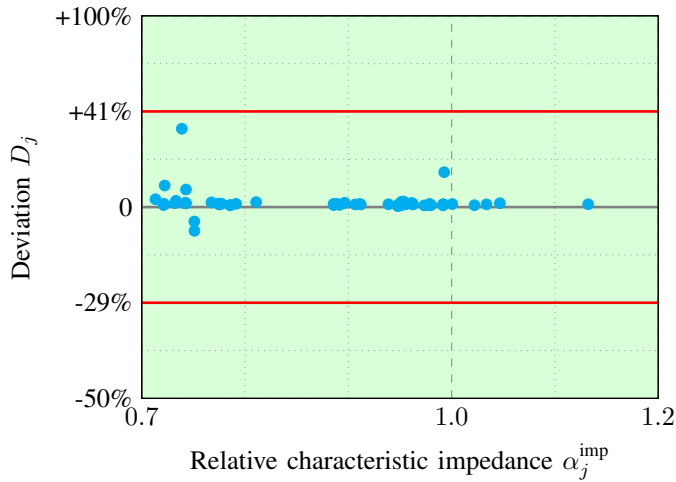

Fig. 2: Length calculation deviations for overhead lines

Figure 2 shows that the quality criterion in Equation (19) is fulfilled for all 99 calculated lengths. There is a large percentage of very good estimates $D_j \approx \pm 1\,\%$ and a few not so good estimates. The RMS value is dominated by one badly calculated length with $\alpha_j^{\mathrm{imp}} = 0.74$ and $D_j = 0.41(+33\,\%)$ in the upper left corner. This α_j^{imp}-value is just above the cut-of limit between J and K, as given in Equation (17). This badly calculated length stems likely from an aggregated combination of overhead line and cable (where Equation (15) does not deliver meaningful results), which falsely has been identified as overhead line. This shows the limits to the validity of Equation (17).

V. EMPIRICAL LENGTH ESTIMATION FOR ALL TRANSMISSION LINES

As mentioned before, Equation (15) is only applicable to overhead lines. It has not been possible to find a similarly simple and general formula for cables, due to their more complex internal structure (including shield and armour).

Although the cable length therefore can not be calculated, an approximate method for estimating the length of all transmission lines (not only overhead lines) would be useful, and such an estimation method is proposed in this section.

A. Length Calculation applied on All Transmission Lines

When length calculation is performed for all transmission line entries I instead of J in the Danish data set, the average error is $E_I = 142.87\,\%$. This shows that calculation accuracy is highly dependent on the α_i^{imp}-values.

In Figure 3, the calculation deviations D_i of I are plotted in relation to their α_i^{imp}-value, similarly as in Figure 2.
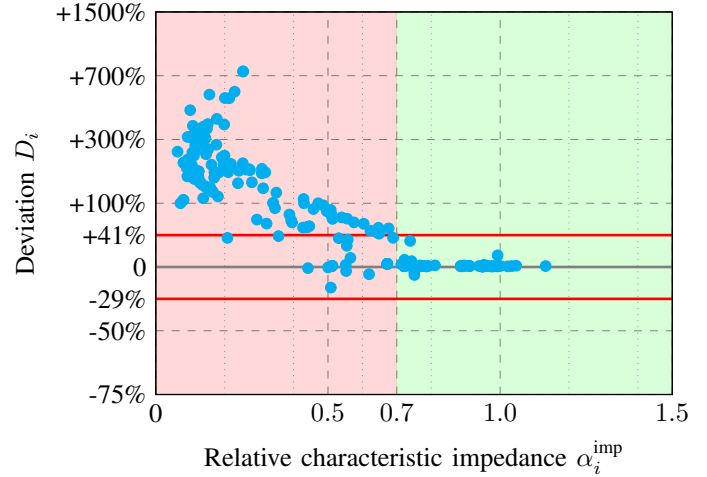

Fig. 3: Length calculation deviations for all transmission lines

Figure 3 clearly shows how the estimation accuracy, which is good for the overhead lines (green area), deteriorates as one also considers other transmission lines (red area).

B. Introduction of a Correction Term

Figure 3 clearly shows large but systematic errors for all transmission lines with low α_i^{imp}. As this error is clearly a function of α_i^{imp}, an empirical correction term γ, which also is a function of α_i^{imp}, can be introduced to cope with this systematic error. This empirical correction term can be used create a generalised version of Equation (15) that can be applied for all transmission lines I:

$$l_i^{\mathrm{est}} = \frac{l_i^{\mathrm{calc}}}{\gamma(\alpha_i^{\mathrm{imp}})} = \frac{c_0 T_i}{\gamma(\alpha_i^{\mathrm{imp}})} \tag{21}$$

A possible form of such a correction term $\gamma(\alpha_i^{\mathrm{imp}})$ is proposed in Equation (22):

$$\gamma(\alpha_i^{\mathrm{imp}}) = \gamma_1 + \gamma_2 e^{-\gamma_3 \alpha_i^{\mathrm{imp}}} \tag{22}$$

The values of the three parameters of the correction term that were found to give the best estimation accuracy for the Danish data set are given in Table I.

If the values for these three parameters are determined using a different grid data set than the Danish one used here, the

TABLE I: Correction term parameters

Parameter	Value
γ_1	0.85
γ_2	4.0
γ_3	3.6

parameters will likely be slightly different. However, as cable and overhead line types do not differ dramatically between different countries, it is likely that the parameters will be in a similar range. This implies that the values in Table I (based on the Danish data) are also likely to be applicable for other grids as well.

C. Application of Line Length Estimation

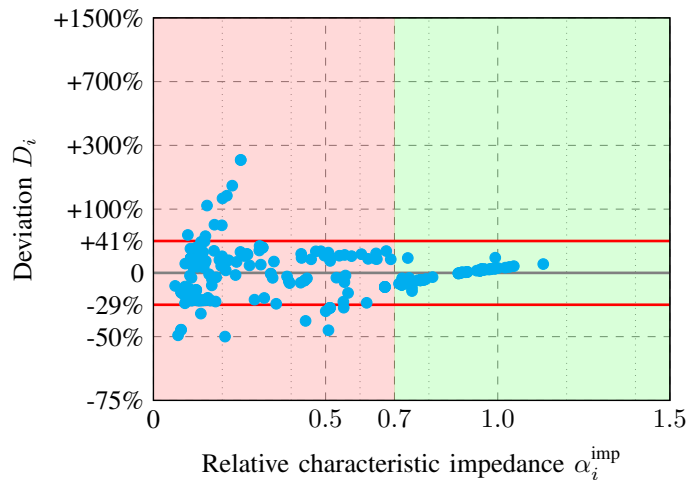The estimation deviations obtained with Equation (21) are displayed in Figure 4.

Fig. 4: Length estimation deviations for all transmission lines

The results obtained with Equation (15) and Equation (21) are compared in Table II.

TABLE II: Results for the Danish grid

Set	Number	Error	Equation (15)	Equation (21)
I	304	E_I	142.87 %	27.32 %
J	99	E_J	3.70 %	7.34 %
K	205	E_K	194.55 %	33.64 %

As expected, the empirical estimation method (Equation (21)) is less accurate for overhead lines than the calculation method (Equation (15)), and it is not supported by physical equations. However, it delivers significantly better results for other transmission lines, and it can thus be applied more generally. The empirical method might be a useful tool, when a quick estimation of the length of all transmission lines is needed, and special attention to cables (as needed when using Equation (15)) is not practicable.

VI. APPLICATION

The development of the above methods was motivated by a reliability assessment case study considering a part of the Norwegian transmission system. The application of the line length calculation formula and estimation method described above are illustrated in this section.

A. Data Quality Check

This data set included a set of transmission line entries H with a total of $|H| = 1170$ entries. The reactances were supplied, so reactances have been converted to inductances.

A majority of the entries in H have complete data, fulfilling Equation (16), leading to the set of transmission lines I with $|I| = 1009$. The others were mostly very short connections between different busses within a transformer station, lacking capacitance specification.

B. Overhead Line Identification

A majority of the transmission lines in I fulfill the criterion in Equation (17), leading to the set of overhead lines J with a total of $|J| = 807$ overhead lines. The characteristic of the α_i^{imp}-values in the Norwegian grid is displayed in Figure 5
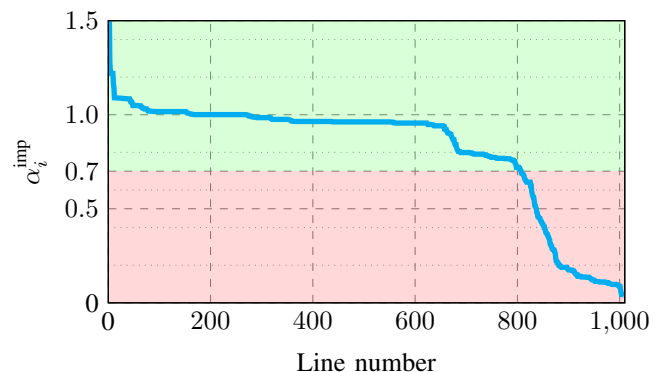

Fig. 5: Impedance characteristic of the Norwegian data set

C. Length Calculation and Estimation

The developed methods have been used to determine the line lengths for the Norwegian data set. The length has been calculated for J based on Equation (15) and estimated for I based on Equation (21) Both results are displayed in Figure 6.

The results appear reasonable, but cannot be validated, as no length data is available in the Norwegian data set. The longest lines appear very long on European standards (length up to 309 km). However, the Norwegian data set also includes aggregated representations of other parts of the Nordic power system (e.g Sweden), which can explain these lengths.
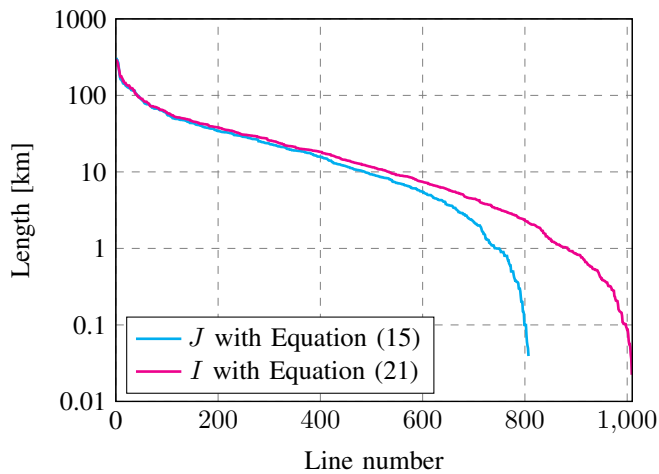
Fig. 6: Determined line lengths for the Norwegian data set

D. Application to Reliability Assessment

The objective of the reliability assessment case study was to estimate the expected annual interruption costs for a particular region of Norway with a subset $I' \subset I$ of the transmission lines ($|I'| = 89$). For this subset I', l_i was determined using Equation (21).

The failure rate λ of the individual transmission lines is crucial input for reliability assessment. This is true for basic analytical approaches, as described in [9], as well as for more advanced methods. This failure rate is calculated with Equation (23), where λ_i' is the expected number of failures per year per km of transmission line. In this case study, λ_i' is taken from Norwegian failure statistics [10]:

$$\lambda_i = \lambda_i' l_i \qquad (23)$$

These failure rate data are used to calculate the expected interruption costs for the region. For simplicity, contingencies involving other network elements (e.g. generators, transformers) or multiple transmission lines are not considered in the reliability assessment. As this article focuses on the quality and value of the input data for reliability assessment rather than the reliability assessment as such, the reader is simply referred to [11] for information about the reliability assessment methodology, and more information about the case is forthcoming in [12].

To illustrate the impact of uncertainties in the length estimates on the reliability assessment, and hence the value of obtaining more accurate length estimates, a simple sensitivity analysis is carried out. As a reference, one considers the interruption cost estimate obtained as described above. Comparing this with the presumably less accurate assumption that all transmission lines have the same length (here optimistically chosen to coincide with the average length as estimated by Equation (21)), one underestimates the interruption costs by 24 %. However, one should keep in mind that this sensitivity is case dependent, as it depends on which contingencies are most critical for a given case.

VII. CONCLUSION

A practical method has been proposed for estimating the length of transmission lines from electrical data. This has been done for the purpose of estimating failure rates for power system reliability assessments, but it can serve many other purposes. The method includes procedures for checking the quality of grid data sets and characterising transmission lines in terms of the geometry they are likely to represent. This may help analysts pinpoint transmission lines whose length estimates could be checked manually to further reduce the uncertainties in failure rate assumptions. The validity of the method has been explored by testing it on real data sets.

For future work, it would be interesting to test the method on other real data sets to better assess the limits of its validity. Also the threshold in Equation (17) can possibly be improved if more data sets are considered. It would also be interesting and relevant to test possible improvements by considering more information, e.g. the line's resistance. This could contribute to improving the identification of overhead lines (Equation (17)) or the determination of cable lengths.

ACKNOWLEDGEMENT

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