

# Water Values in Future Power Markets

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**Abstract**—In this paper we elaborate on how a hydropower producer’s expected marginal value of water (water value) is affected when considering both sales of energy and reserve capacity in a liberalized market setting. We derive analytical expressions for the water value in this market context, and verify these through numerical computations. We find that the water values are more sensitive to changes in the reservoir level when considering sales of both energy and reserve capacity compared to the energy-only case.

**Index Terms**—Hydroelectric Power Generation, Power Generation Economics, Optimization, Stochastic Processes.

## I. INTRODUCTION

Today’s operational models for long- and medium term hydropower scheduling consider sales of energy as the only opportunity for the producer to earn money. Such scheduling models are important tools for the producers when finding their expected marginal value of water (water values), which in turn is used in the short-term operational scheduling. However, as the producers need to adapt to a future with increasing share of renewable intermittent generation and emphasis on harmonizing various balancing services on European level, so do the scheduling models.

An important research question is then to find how the valuation of water is affected when considering balancing markets. Such multi-market scheduling models have been presented in recent literature, see e.g. [1]–[5]. In this work we broadly define balancing markets to comprise markets for reserve capacity procurement and balancing energy. In theory, all relevant markets should be included when computing the optimal strategy for a hydropower system, but that would be an exhaustive computational task. Keep in mind that the stochastic scheduling models involve uncertainty in inflow and electricity prices and typically apply weekly decision stages over a period of analyses of 3-5 years.

The balancing markets generally allow trading two different products: reserve capacity and balancing energy. It is questionable how the incorporation of additional markets cleared at different time scales involving the same product (energy) would contribute to the strategic scheduling of reservoirs (represented by water values). Thoroughly assessing that impact would call for a model with fine time resolution and many decision stages within the week, similar to those used in short-term multi-market scheduling, see e.g. [6], [7]. In our opinion, incorporating the possibility of selling different products (both

energy and reserve capacity) would potentially have a larger impact on the water values. The work presented here therefore focuses on the two different products energy and reserve capacity in a generic manner, without going into too many details about the different markets in which each of those products are traded.

In the Nordic market, reserve capacity is procured by the TSO’s as several different products. Normally, one separates between primary (FCR), secondary (FRR-A) and tertiary (FRR-M) reserves. Today, each country’s TSO primarily buys the reserve capacity products separately, but there is an aim to further coordinate the provision of reserve capacity [8]. In most European market designs, reserve capacity is primarily procured before clearing the day-ahead market. In the Norwegian case, the TSO (Statnett) buys all three reserve-types the week ahead of operation. This is done to ensure that sufficient reserve capacity is available and that this capacity is not bid into the energy market(s).

In this work we assume that the producer is a risk-neutral price-taker in both the energy and reserve capacity market. In the analyses we have assumed that the capacity should be spinning (rotating) and symmetric (same amount for up- and down-regulation). In line with the current market design, we require the sales of capacity to take place before knowing the energy price. In section II we simplify the decision problem and derive analytical expressions for the water value. Then, in section III we apply a scheduling model on a fictitious hydropower system with attractive reserve capacity prices to verify the analytical expressions and further elaborate on how the water values change. The scheduling model was described in [4], and is based on stochastic dual dynamic programming (SDDP) and uses stochastic dynamic programming (SDP) to represent the price processes [9]–[11].

## II. ANALYTICAL EXPRESSIONS

In this section we present a simplified version of the one-stage decision problem that was defined as a part of the overall stochastic and dynamic optimization problem in [4]. We focus on a single reservoir and assume one time step within the decision stage. The one-stage problem is represented by equations (1)-(5). We analytically evaluate the impact of sales of reserve capacity on the water values.

Assume a system comprising a single hydropower reservoir and a power station with one generator where both energy and reserve capacity can be sold at predefined prices,  $\lambda^E$

Funded by The Research Council of Norway Project No. 228731/E20.

(€/MWh) and  $\lambda^C$  (€/MW/h), respectively, for a period of  $\tau$  hours. The objective function is stated in (1), maximizing revenues from the two markets. The first two terms describe the immediate revenue from the energy and reserve capacity market, respectively. Reserve capacity is sold for the decision period ahead ( $t+1$ ) while energy is sold for the current period ( $t$ ). For simplicity of notation, the period index is omitted when specifying variables and parameters for the current period.

The reservoir volume is denoted  $v$  (in  $\text{Mm}^3$ ), the inflow to the reservoir  $I$ , and a water balance is presented in (2). Water discharge through the station is modeled using one variable  $q_s$  (in  $\text{Mm}^3$ ) per discharge segment  $s \in \mathcal{S}$ . These segments will be used in decreasing order according to their energy equivalent  $\eta_s$ , in  $\text{MWh}/\text{Mm}^3$ , provided that  $\eta_s$  decreases with  $s$ . There is an upper bound on generation  $P^{max}$  (in MW).

The reserve capacity  $C$  sold in the previous period enters the optimization problem for the current period as a fixed requirement. In the formulation we have assumed the reserve capacity to be symmetric (same amount sold for upward and downward regulation). Spinning down-regulation reserve is ensured through (3), and up-regulation through (4). A more detailed description of the methodology and the market context for which this type of model is intended, see [4].

The use of water in the current time period and sales of capacity for the next time period is balanced against the future expected profit  $\alpha_{t+1}$ , which is constrained by linear constraints of type (5), often referred to as *cuts*. These cuts are built iteratively in the overall stochastic and dynamic optimization problem, but for the further discussion we assume that a set of cuts  $\mathcal{K}$  is available when solving the one-stage problem. A cut  $k$  is defined by its cut coefficients  $\tilde{\pi}_k^v$  and  $\tilde{\pi}_k^c$  and right-hand side  $\tilde{\beta}_k$ . Dual values are indicated in parenthesis for each type of constraints (2) - (5).

$$Z_t = \max \left\{ \lambda^E \sum_{s \in \mathcal{S}} \eta_s q_s + \tau \lambda^C c_{t+1} + \alpha_{t+1} \right\} \quad (1)$$

$$v + \sum_{s \in \mathcal{S}} q_s = v_{t-1} + I \quad (\pi^v) \quad (2)$$

$$\frac{1}{\tau} \sum_{s \in \mathcal{S}} \eta_s q_s \geq C \quad (\pi^{c-}) \quad (3)$$

$$\frac{1}{\tau} \sum_{s \in \mathcal{S}} \eta_s q_s \leq P^{max} - C \quad (\pi^{c+}) \quad (4)$$

$$\alpha_{t+1} - \tilde{\pi}_k^v v - \tilde{\pi}_k^c c_{t+1} \leq \tilde{\beta}_k, \forall k \in \mathcal{K} \quad (\pi_k^r) \quad (5)$$

As a further simplification we assume that the reservoir and discharge variables will not hit their maximum or minimum bounds in this time period. The Lagrangian function  $\mathcal{L}$  for the LP problem in (1)-(5) can then be expressed as:

$$\begin{aligned} \mathcal{L}(q_s, c_{t+1}, \alpha_{t+1}, v, \pi^v, \pi^{c-}, \pi^{c+}, \pi^r) = & \\ & \lambda^E \sum_{s \in \mathcal{S}} \eta_s q_s + \tau \lambda^C c_{t+1} + \alpha_{t+1} \\ & + \pi^v \left( v_{t-1} + I - v - \sum_{s \in \mathcal{S}} q_s \right) \\ & + \pi^{c-} \left( C - \frac{1}{\tau} \sum_{s \in \mathcal{S}} \eta_s q_s \right) + \pi^{c+} \left( P^{max} - C - \frac{1}{\tau} \sum_{s \in \mathcal{S}} \eta_s q_s \right) \\ & + \sum_{k \in \mathcal{K}} \pi_k^r \left( \tilde{\beta}_k - \alpha_{t+1} + \tilde{\pi}_k^v v + \tilde{\pi}_k^c c_{t+1} \right) \end{aligned} \quad (6)$$

After differentiating the Lagrangian function with respect to each of the four variables we can obtain the following Kuhn-Tucker conditions for optimality:

$$\frac{\partial \mathcal{L}}{\partial q_s} = \eta_s \left( \lambda^E - \frac{1}{\tau} \pi^{c-} - \frac{1}{\tau} \pi^{c+} \right) - \pi^v = 0 \quad (7)$$

$$\frac{\partial \mathcal{L}}{\partial v} = -\pi^v + \sum_{k \in \mathcal{K}} \pi_k^r \tilde{\pi}_k^v = 0 \quad (8)$$

$$\frac{\partial \mathcal{L}}{\partial \alpha_{t+1}} = 1 - \sum_{k \in \mathcal{K}} \pi_k^r = 0 \quad (9)$$

By joining (7) and (8), we obtain the following relationship:

$$\pi^v = \sum_{k \in \mathcal{K}} \pi_k^r \tilde{\pi}_k^v = \eta_s \left( \lambda^E - \frac{1}{\tau} \pi^{c-} - \frac{1}{\tau} \pi^{c+} \right) \quad (10)$$

Expression (10) tells us that the water value ( $\pi^v$ ) will equal a combination of cut coefficients ( $\tilde{\pi}_k^v$ ) for the binding cuts in  $\mathcal{K}$ . As long as the discharge is not constrained by (3) or (4), the water value will equal the energy price taking the marginal conversion efficiency ( $\eta_s$ ) into account.

If the discharge is constrained by the requirement for down-regulating reserves in (3), then  $\pi^{c-} < 0$  and the water value will be higher than the energy price. Conversely, if discharge is constrained by the up-regulating reserves in (4), then  $\pi^{c+} > 0$  and the water value will be lower than the energy price. From this we can conclude that:

- If the requirement for available down-regulation capacity is binding, it contributes to a higher water value.
- If the requirement for available up-regulation capacity is binding, it contributes to a lower water value.

The conclusions above might seem obvious from a practical point of view. Reserve capacity can in some water courses be sold as a by-product at little additional cost. This is e.g. the case for a hydropower generator running at best efficiency with available capacity for both up- and down regulation. There will however be a threshold after which further sales of capacity more severely impacts system operation, and therefore is associated with a higher cost. This is the case whenever sales of reserve capacity leads to a different optimal solution than would otherwise have been found in the energy-only case. In such cases there is a revenue loss in the energy

market (representing the opportunity cost). If we sell down-regulation capacity and there is an opportunity cost, we use extra water to keep the generators spinning compared to the energy only-case, which in turn increases the marginal value of water. Conversely, if we sell up-regulation capacity and there is an opportunity cost, we use less water to produce energy compared to the energy only-case, which in turn decreases the marginal value of water.

### III. CASE STUDY

In this section we present results for a simple and fictitious test system using a combined SDDP/SDP model described in [4]. The system is kept simple in order to focus on the interpretation of water values.

#### A. Case Description

The system comprises one reservoir and one power station. The reservoir has a storage capacity of 150 Mm<sup>3</sup> and an average annual regulated inflow of 450 Mm<sup>3</sup>. The power station has a maximum generation capacity of 110 MW, following a set of piecewise-linear and gradually decreasing efficiencies, with a best efficiency at 80 MW.

Uncertainty is modeled in inflow and energy prices. Inflow values follows the profile of a historical Norwegian inflow record, and an autoregressive inflow model is fitted to this record. Energy price series were obtained from the EMPS model [12]. Based on these series we generated a price model with two weekly average energy price nodes, see Fig. 1. A transition probability matrix was computed describing the probability of going from a given node in a given week to any of the nodes in the next week. The energy price follows a pre-defined (deterministic) profile within the week, scaled according to the average weekly price.

System operation was computed for a period of 104 weeks, considering 21 sequentially treated time steps within the week. Energy is sold per time step. We let the model sell capacity for the week ahead in blocks, considering 3 blocks per week covering night (00:00-08:00), day (08:00-20:00) and evening (20:00-24:00). We exaggerate the potential for selling capacity in order to provoke significant differences between the two scheduling modes. We allow selling 44 MW of reserve capacity at a fixed price of 60 €/MW. Due to the high reserve capacity price relative to the energy prices in Fig. 1, the model will frequently sell the maximum capacity. At low energy prices, the generation will be at 44 MW, enough to keep the committed reserves spinning. At high energy prices, the station will generate 66 MW leaving 44 MW for upward regulation. There is no minimum power production requirement and start-up cost for the station. Note that the model does not consider activation of reserve capacity.

Simulations were performed in two different modes:

1. E mode (base case). Only allows sales of energy.
2. E+C mode. Allows selling both energy and capacity.

The model was run for 20 main iterations considering 50 samples of inflow and energy price nodes in the forward iterations and 4 "openings" for inflow in the backward iteration. We

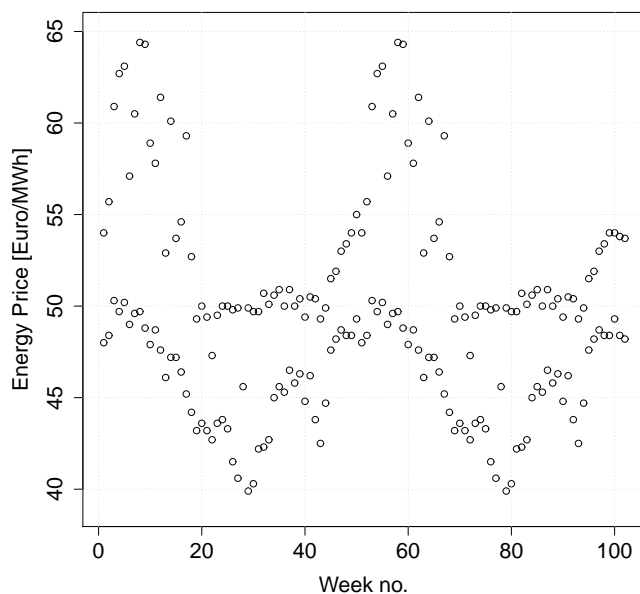


Fig. 1. Energy price nodes representing two probable outcomes of average weekly energy price.

used a set of exogenously created end-value cuts to account for the end-of-horizon value of water and reserve capacity.

The reservoir operation for the E mode is shown in Fig. 2. Operation in the E+C mode is less flexible than in the one-market mode, due to the excessive sales of capacity.

#### B. Water Values

Next we study the water values obtained in the selected weeks 5, 15 and 45, see the vertical lines in Fig. 2. Recall that the evaluation of expected future profit is provided by cuts of type (5). We obtain the resulting water values for a given system state (reservoir volume and reserve capacity sales) by moving these terms to the right-hand side in (5) so that only  $\alpha_{t+1}$  is left as a variable. The cut with the lowest right-hand side value will be binding for that particular state. By repeating the same procedure for many discrete reservoir states we obtain a set of different cuts that are binding. The coefficient of the binding cuts are then treated as the water values for the corresponding reservoir levels.

The water values for weeks 5, 15 and 45 are shown in Fig. 3, Fig. 4 and Fig. 5, respectively. The x-axis range is determined by the minimum and maximum simulated reservoir states for the corresponding week. The E mode trajectories have the largest variation, and are therefore used to define x-axis range.

In week 5, the reservoir is being depleted according to Fig. 2. The water value for the E+C mode is higher than for the E mode, but note that the spread in simulated reservoir levels is still narrow.

In week 15 the reservoir is close to empty, and the water values obtained in the E+C mode are much more sensitive to changes in reservoir level than in the E mode. This is due

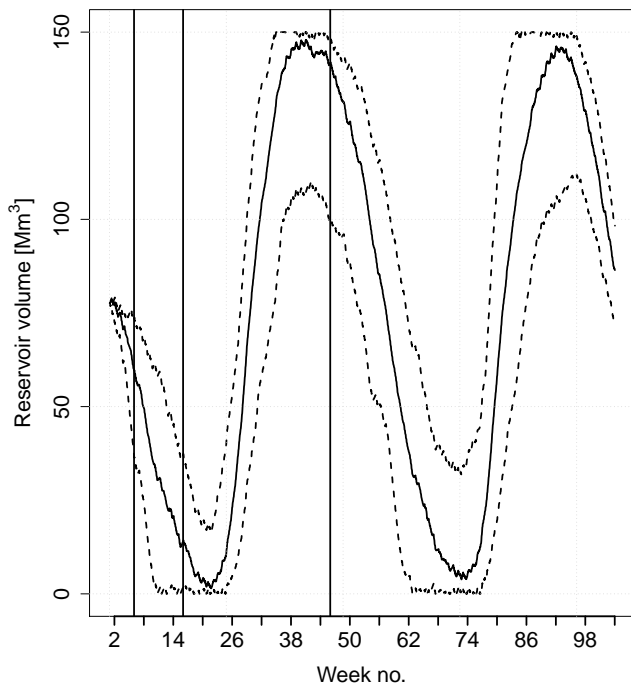


Fig. 2. Simulated reservoir trajectories (max, mean and min) for the E mode, in  $Mm^3$ . The vertical lines indicate weeks for which water values are studied.

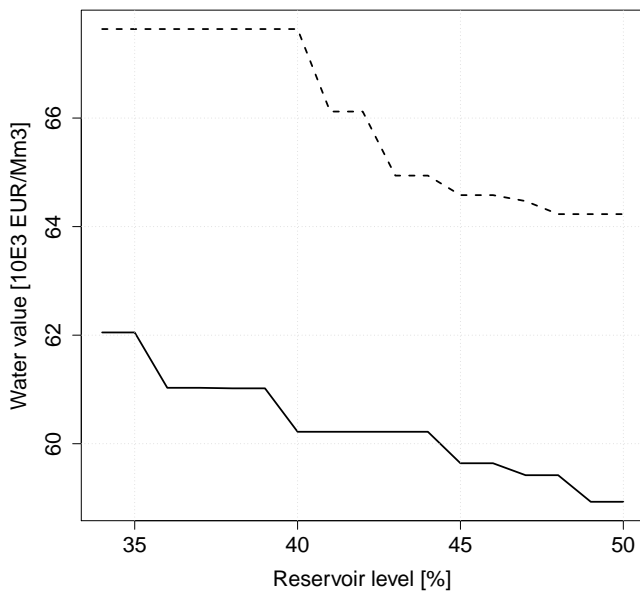


Fig. 3. Water values obtained in the two modes for week 5. E mode values are solid-drawn and E+C values are stapled.

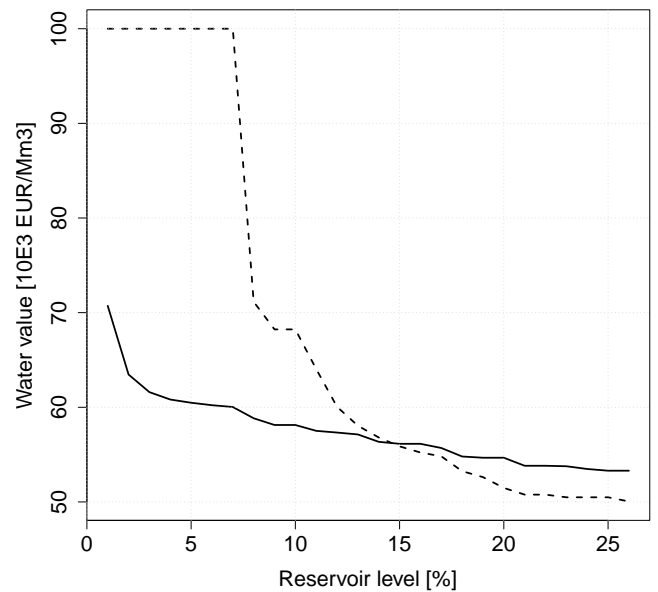


Fig. 4. Water values obtained in the two modes for week 15. E mode values are solid-drawn and E+C values are stapled.

to the limited flexibility when committing a majority of the capacity as reserve. If the reservoir level is high in week 15, the possibility of selling capacity reserves (up regulation) in the filling season will limit the ability to discharge the extra water, and the risk of spillage is therefore high, contributing to lower water values. On the other hand, if the reservoir is low in week 15, the ability to sell capacity reserves (down regulation) will ensure that the station is running, and there is less water left for generating additional energy if prices are favorable. Since the cuts have been generated for simulated reservoir states, the water value curve has finer resolution in the range of simulated reservoir states. This is clearly shown in Fig. 4, where the E+C mode water value is constant ( $100 \text{ k€}/Mm^3$ ) for the lowest reservoir levels (between 0 and 7 % filling).  $100 \text{ k€}/Mm^3$  was set as the cost of buying artificial water to the system, and the binding cut with this coefficient has most likely been generated in an early iteration of the SDP/SDDP model.

In week 45 one enters the season where discharge normally is higher than inflow. Similar to values for week 15, we see from Fig. 5 that the water values obtained in the E+C mode are significantly more sensitive to changes in reservoir than in the E mode. If the E+C mode reservoir level in week 45 is relatively low, the model is left with little flexibility in selling additional energy at favorable prices, and thus a high water value. On the other hand, if the reservoir level gets high enough, the maximum generation constraint of 66 MW (keeping 44 MW in reserve for upward regulation) will be frequently binding, and there is a higher risk of spilling next spring flood, giving lower water values.

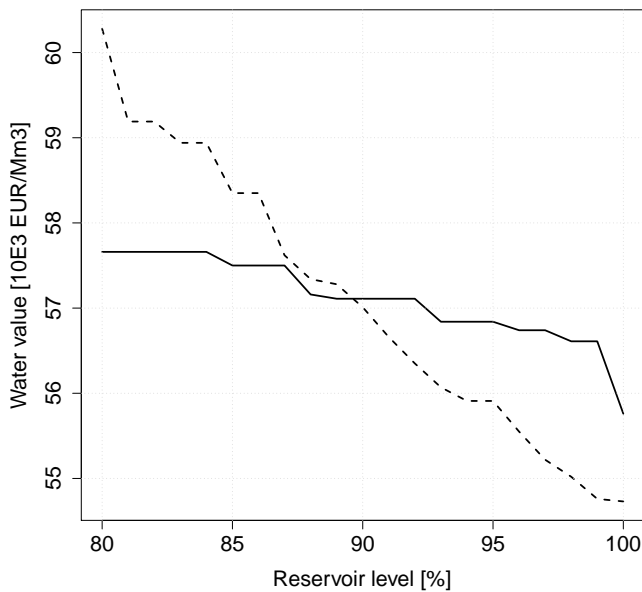


Fig. 5. Water values obtained in the two modes for week 45. E mode values are solid-drawn and E+C values are stapled.

#### IV. CONCLUSIONS

When considering both energy and reserve capacity sales, water values will change. This was elaborated analytically in Section II. When the requirement for available down-regulation capacity is binding, it contributes to an increased water value. Conversely, when the requirement for available up-regulation capacity is binding, it contributes to a reduced water value. These findings were verified in a case study on a fictitious hydropower system in Section III. We introduced a high and constant reserve capacity price to emphasize the impact of an attractive reserve capacity market. From the case study we saw that in cases with low reservoir levels, the down-regulation capacity constraint becomes binding, contributing to an increase in water value (compared to the energy-only case). Conversely, with a high reservoir level and high reserve capacity prices, the up-regulation capacity constraint is binding, contributing to a decrease in water value (compared to the energy-only case).

The case study results indicate that the water values become more sensitive to changes in reservoir levels when considering sales of reserve capacity. Thus, in the future power markets with more intermittent generation and higher demands for reserve capacity, accurate hydropower scheduling becomes even more challenging. More markets will also involve more uncertainty, further complicating the scheduling, but we have not addressed that issue here.

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