Optimal Hydropower Maintenance Scheduling in Liberalized Markets

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***Abstract*—Maintenance scheduling is an important and complex task in hydropower systems. In a liberalized market, the generation company will schedule maintenance periods to maximize the expected profit. This paper describes a method for hydropower maintenance scheduling suitable for a profit maximizing, price-taking and risk neutral hydropower producer selling energy and reserve capacity to separate markets. The method uses Benders decomposition principle to coordinate the timing of power plant maintenance with the medium-term scheduling of the hydropower system, treating inflow to reservoirs and prices for energy and reserve capacity as stochastic variables.**

The proposed method is applied in a case study for a Norwegian watercourse, and results, in terms of maintenance schedules and computational performance, are presented and discussed.

***Index Terms*—Hydroelectric power generation, Power generation economics, Integer programming, Linear programming, Stochastic processes.**

NOMENCLATURE

A. Index Sets

$\mathcal{H}$	Set of hydropower reservoirs/stations;
$\mathcal{S}_h$	Set of discharge segments for station h;
$\mathcal{K}$	Set of time steps within the week;
$\mathcal{T}$	Set of decision stages;
$\mathcal{T}_R$	Set of stages in maintenance interval;
$\mathcal{N}$	Set of nodes in scenario tree;
$\mathcal{L}^M$	Set of MS Benders cuts;
$\mathcal{L}^H_n$	Set of HPS Benders cuts for node n;
$\mathcal{L}^H_p$	Set of HPS Benders cuts for price cluster p;
$\mathcal{D}_n$	Set of descendant nodes from n, including n;
$\mathcal{D}^S_{n\ell}$	Set of descendant nodes from n sampled in backward iteration ℓ;
$\mathcal{D}^S_{0\ell}$	Set of nodes sampled in backward iteration ℓ;
Ω_h	Set of reservoirs upstream reservoir h.

B. Parameters

$T(n)$	Lookup table giving time stage for node n;
$P(n)$	Lookup table giving price cluster for node n;
$\bar{G}_h$	Max. capacity in station h, in MW;
$\bar{V}_{kh}, \underline{V}_{kh}$	Max./Min. limit for reservoir h, in Mm^3;
$\bar{C}_h$	Max. capacity sales from station h, in MW;
τ_k	Duration of time step k, in hours;
$\tilde{\tau}_k$	Relative duration of time step k, fraction;
ζ^E_k	Energy price scaling factor for time step k;
ζ^C_k	Capacity price scaling factor for time step k;
η_{hs},η_s	Energy equivalent for (station h and) discharge segment s, in MWh/Mm^3;
$\pi_{a(n)}$	Coefficient for reservoir level for node $a(n)$, in €/Mm^3;
$\pi_{nh\ell}$,$\pi_{n\ell}$	Coefficient for reservoir level for node n, (reservoir h) and iteration ℓ, in €/Mm^3;
$\beta_{n\ell}$	Right-hand side for HPS Benders cut at node n and iteration ℓ, in €;
β_ℓ	Right-hand side for MS Benders cut for iteration ℓ, in €;
$\gamma_{nh\ell}$,$\gamma_{n\ell}$,γ_n	Coefficient for maintenance for HPS Benders cuts at node n (reservoir h and iteration ℓ), in €;
$\gamma_{ht\ell}$,$\gamma_{t\ell}$	Coefficient for maintenance for MS Benders cuts (for reservoir h), stage t and iteration ℓ, in €;
$\bar{Q}^D_s$	Max. discharge for segment s, in Mm^3;
$\bar{Q}^B_{kh}$	Max. bypass for plant h in time step k, in Mm^3;
K	Last time step in week;
D	Maintenance duration, weeks;
ϵ	Convergence tolerance, in €;
Θ^M	Optimal solution of MS problem, in €;
Θ^H_n	Optimal solution of HPS subproblem at node n, in €;
$\mathbb{P}_n$	Probability of scenario tree node n;
$\mathbb{P}_{n'\vert n}$	Probability of node n' conditioned on node n.

C. Stochastic Variables

I_n	Inflow at node n, in Mm^3;
I_{nh}	Sum weekly inflow to reservoir h at node n, in Mm^3;
λ_n	Energy price at node n, in €/MWh;
λ^E_p	Weekly average energy price for cluster p, in €/MWh;
λ^C_p	Weekly average reserve capacity for cluster p, in €/MW.

D. Decision Variables

α	Future expected profit for the MS problem, in €;
α_n	Future expected profit for the HPS problem seen from node n, in €;
x_t	Decision on maintenance in week t, binary;
e_{kh}	Generated electricity in time step k for station h, in MWh;
e_n	Generated electricity for node n, in MWh;

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c_{kh}	Allocated capacity in time step k for station h, in MW;
v_{kh}	Volume in time step k for reservoir h, in Mm^3;
v_n	Volume in node n, in Mm^3;
q^D_{khs}	Discharge in time step k through station h at segment s, in Mm^3;
q^D_{sn}	Discharge at segment s in node n, in Mm^3;
q^S_{kh}	Spillage in time step k from reservoir h, in Mm^3;
q^S_n	Spillage in node n, in Mm^3;
q^B_{kh}	Bypass in time step k from reservoir h, in Mm^3;
z_{ht},z_t	Auxiliary variable, continuous;
$\Phi(\ldots)$	End value function, in €.

I. Introduction

A. Background and Motivation

The flexibility and controllability of hydropower play an important role for balancing the unpredictable generation from intermittent generation technologies, such as wind and solar power, in many power systems throughout the world. Consequently, the reliability of hydropower facilities is of great importance for secure system operation, and these facilities should be maintained to provide an adequate level of reliability. Maintenance can be categorized as corrective or preventive. Corrective maintenance is performed after a failure, whereas preventive maintenance is performed at predetermined periods and is intended to reduce the probability of failure, prolong the expected useful lifetime of the system components and ensure continuous operation at acceptable efficiency levels.

This work concerns the scheduling of preventive power plant maintenance in hydropower systems in a liberalized market context. The scheduling consists of finding the optimal timing of maintenance for a predefined set of plants. We study the problem from an economic perspective, assuming a predefined requirement for maintenance within the planning period. In order to minimize the economic impact, it is important to carefully plan when the plants are maintained. The lost revenue from taking a hydropower plant out for maintenance depends on the future market prices and inflow which are uncertain at the time of planning the maintenance. During maintenance the plants will be unavailable, possibly leading to loss of water or profit losses because sales of energy and reserve capacity has to be moved to periods with lower prices. Thus, there is clearly a need to integrate the maintenance scheduling with the process of finding an optimal hydropower dispatch strategy.

B. Literature Review

Both hydropower scheduling [1], [2] and maintenance scheduling [3] are thoroughly addressed in the literature. The early works on scheduling of hydropower reservoirs used the principles of stochastic dynamic programming (SDP), see e.g. [4]. SDP decomposes the multi-stage planning problem into a sequence of single-stage subproblems that can be solved by backward induction. The major drawback of the SDP method is that the state variables (typically reservoir volumes) need to be discretized, so the overall problem size becomes computationally intractable when considering systems with many reservoirs. The stochastic dual dynamic programming (SDDP) introduced in [5] allows solving the scheduling problem without discretizing the state variables, and is therefore computationally tractable for systems with multiple reservoirs. The SDDP algorithm can be classified as a sampling-based variant of multi-stage Benders decomposition method [6], requiring a convex problem formulation. Extensions of the SDDP algorithm have been frequently reported in recent literature, see e.g. [7]–[12].

Several methodological approaches have been presented to approach the general power plant preventive maintenance scheduling problem, see e.g. [3], [13] for an overview. The work in [14] seems like a starting point for the use of mixed integer linear programming (MILP) for optimal maintenance scheduling. In [15] the total operating costs were minimized over an operational planning period of several years while meeting thermal unit maintenance and system constraints. The authors used Benders decomposition, fixing the maintenance schedule in an MILP master problem and obtained cuts by evaluating continuous nonlinear subproblems.

Maintenance coordination schemes between the system operator and the generation companies were proposed in [16], [17] for the purpose of maximizing producer's profits while achieving a sufficient level of reliability. These coordination schemes resemble maintenance scheduling in a competitive market context.

Relatively few works have addressed maintenance scheduling in hydropower-dominated systems. In [18] an integrated approach for maintenance and refurbishment planning in hydropower systems based on SDP and heuristics was presented. In [13] a multi-stage Benders decomposition approach was presented to solve the power plant maintenance scheduling problem considering multiple generation technologies and uncertainty in demand. However, hydropower details such as hydro storages and inflow uncertainties, were not considered. Optimal maintenance scheduling for thermal and hydropower plants under uncertainty was also discussed in [19], but the details of the mathematical model were not presented. In [20] the maintenance scheduling was embedded in a deterministic, continuous and nonlinear hydrothermal scheduling model solved by an evolutionary algorithm.

The maintenance scheduling problem has similarities to the expansion planning problem, in the sense that discrete expansion decisions are coordinated with the system operation. Some approaches integrating expansion and operational planning in a fundamental market modeling context using the SDDP algorithm are presented in [21]–[23].

C. Proposed Method and Contributions

In this paper we present a method for hydropower plant maintenance scheduling (HPMS) by coordinating maintenance scheduling (MS) and detailed hydropower scheduling (HPS) by the use of Benders decomposition [24]. First, the MS problem is solved as an MILP problem to provide a trial maintenance schedule to be considered in the HPS. Subsequently, the HPS problem is evaluated using multi-stage

Benders decomposition, where an outer approximation of a convex future expected profit function is constructed for each time stage by adding Benders cuts. Two different sets of Benders cuts are built iteratively, one to decompose the multi-stage linear HPS problem and one to coordinate the MS and HPS problems.

The presented HPMS method constitutes a novel extension to previous works in two directions. Firstly, we extend the multi-stage Benders decomposition approach presented in [13] by allowing a detailed representation of the hydropower system and the relevant uncertainties in the HPMS problem. Furthermore, we elaborate on how the Benders cuts used in the HPS should be augmented to incorporate information about the maintenance schedule, and thus being valid cuts within the decomposition algorithm. Secondly, the HPS part of the problem is extended using a combined SDP and SDDP algorithm and treating inflow to reservoirs and prices for energy and reserve capacity as stochastic variables. The resulting model is suitable for HPMS in large and complex watercourses.

D. Paper Structure

The paper is outlined as follows. In Section II the basic principles of the HPMS scheduling method are described, using multi-stage Benders decomposition to solve the HPS problem. Subsequently, a more detailed mathematical description of the HPS problem and the combined SDP and SDDP solution strategy is provided in Section III. The proposed method in Section III is applied in a case study in Section IV where the optimal power plant maintenance schedule for a Norwegian watercourse is found. Finally, conclusions are drawn in Section V.

II. Decomposition Principle

The basics of the HPMS method are explained in the following, emphasizing on the decomposition principle. Consider a producer operating a single hydropower plant with an upstream reservoir, aiming to schedule maintenance for the plant within the period $t \in \mathcal{T}_R$. The maintenance scheduling is coordinated with the scheduling of hydropower within the operational planning period $t \in \mathcal{T}$, where $\mathcal{T}_R \subseteq \mathcal{T}$. The objective of the HPMS is to find the economically optimal maintenance schedule while maximizing the expected profit from sales to the energy market. Note that we keep the HPS model formulation simplified in this section to emphasize on the decomposition principle. A more realistic HPS model, suitable for multiple reservoirs and sales of both energy and reserve capacity, will be introduced in Section III. It is assumed that the hydropower producer is a risk neutral price-taker. This is normally considered a valid assumption in mature liberalized markets such as the Nordic, where risk management utilizing financial instruments is typically done separately from the hydropower scheduling. The need for maintenance is treated as a predefined requirement. Moreover, we assume that the cost of maintenance is independent of the time of year, and can thus be disregarded in the scheduling since the contribution is constant.

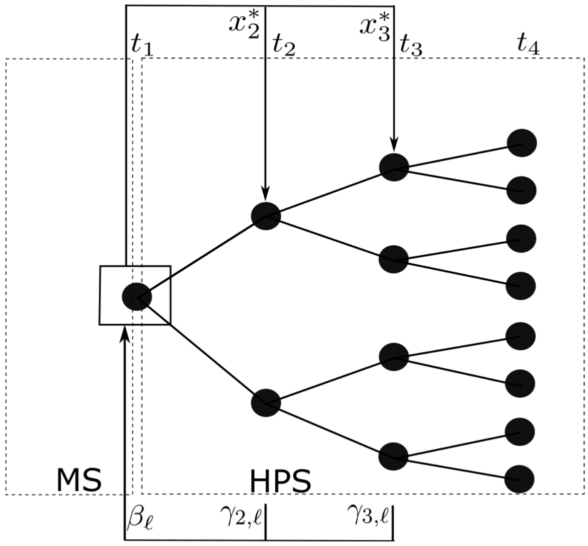

Fig. 1. Illustration of coordination between maintenance scheduling (MS) and hydropower scheduling (HPS). In each iteration ℓ trial schedules x_t^* are sent from the MS to the HPS, and Benders cut coefficients $\gamma_{t\ell}$ and β_ℓ from the HPS to the MS.

Assume that the relevant uncertainties are energy prices and inflow to the reservoir, and that these uncertainties are described in a scenario tree, as illustrated Fig. 1. The scenario tree shown in Fig. 1 comprises 4 decision stages t_1-t_4 and 15 nodes (circles) $n \in \mathcal{N}$. There is one root node at stage t_1 which we index 0, and eight leaf nodes at stage t_4. Except from the root node, each node n has one unique ancestor node $a(n)$ and a set of *all* descendant nodes $\mathcal{D}_n$. We let the set $\mathcal{D}_n$ also include node n itself. The decision stage of node n is denoted $T(n)$. Maintenance is allowed in $\mathcal{T}_R = \{t_2, t_3\}$, and the maintenance decision is made in the root node. All nodes involve decisions on how to operate the hydropower plant, as described in Section II-A. Note that we do not pose strict requirements to the construction of the scenario tree here. The decomposition principle outlined in this Section will be embedded in a hybrid SDP/SDDP approach in Section III, and the use of sampling within this approach is briefly discussed in Sections III and IV.

A. Problem Formulation

The HPMS problem can be formulated as a multi-stage stochastic MILP problem on extensive form as in (1). The objective in (1a) is to maximize the expected revenue from energy sales. For each node, the water balance for the hydro reservoir is formulated in (1b) and the hydropower generation in (1c). In the case of the root node $v_{a(n)}$ equals the initial reservoir volume. Water discharge through the plant is modeled using one variable q_{1sn}^D per discharge segment $s \in \mathcal{S}$. These segments will be used in decreasing order according to their energy equivalent η_s, provided that η_s decreases with s.

The maintenance schedule is represented by binary variables x_t, indicating whether the plant is being maintained (x_t=1) or not (x_t=0) in decision stage t. We require that maintenance should be carried out once over a period of D decision stages in (1d), and that these stages should be consecutive in (1e).

The boundary conditions $x_t = 0; \forall t \leq 0, \forall t > |\mathcal{T}_R|$ must be specified to ensure consecutive maintenance periods. The formulation in (1e) is identical to that in [16] (Eq. 5) and [17] (Eq. 6), but could be formulated differently to serve the same purpose, see e.g. [13]. One could further constrain the maintenance schedule to reflect technical and legislative constraints that represents the problem at hand, as discussed e.g. in [13], [16].

Moreover, one could also allow maintenance on the reservoir, e.g. by forcing the reservoir level below a certain limit for a given period of time. However, such extensions are considered out of scope for this presentation. Note that the maintenance schedule x_t take one common value for all nodes in decision stage t, implying that the decision is taken at the root node in stage t_1. If maintenance is chosen, discharge through the plant is prohibited in (1f).

$$\sum_{n \in \mathcal{N}} \max \left\{ \mathbb{P}_n \lambda_n e_n \right\} \tag{1a}$$

$$v_n + \sum_{s \in \mathcal{S}} q^D_{sn} + q^S_n - v_{a(n)} = I_n \qquad \forall n \in \mathcal{N} \tag{1b}$$

$$e_n - \sum_{s \in \mathcal{S}} \eta_s q^D_{sn} = 0 \qquad \forall n \in \mathcal{N} \tag{1c}$$

$$\sum_{t \in \mathcal{T}_R} x_t = D \tag{1d}$$

$$x_t - x_{t-1} \leq x_{t+D-1} \qquad \forall t \in \mathcal{T}_R \tag{1e}$$

$$0 \leq q^D_{sn} \leq (1 - x_{T(n)}) \bar{Q}^D_s \qquad \forall s \in \mathcal{S}, \forall n \in \mathcal{N} \tag{1f}$$

$$0 \leq v_n \leq \bar{V} \qquad \forall n \in \mathcal{N} \tag{1g}$$

$$e_n, q^S_n \in \mathbb{R}^+ \qquad \forall n \in \mathcal{N} \tag{1h}$$

$$x_t \in \{0, 1\} \qquad \forall t \in \mathcal{T}_R \tag{1i}$$

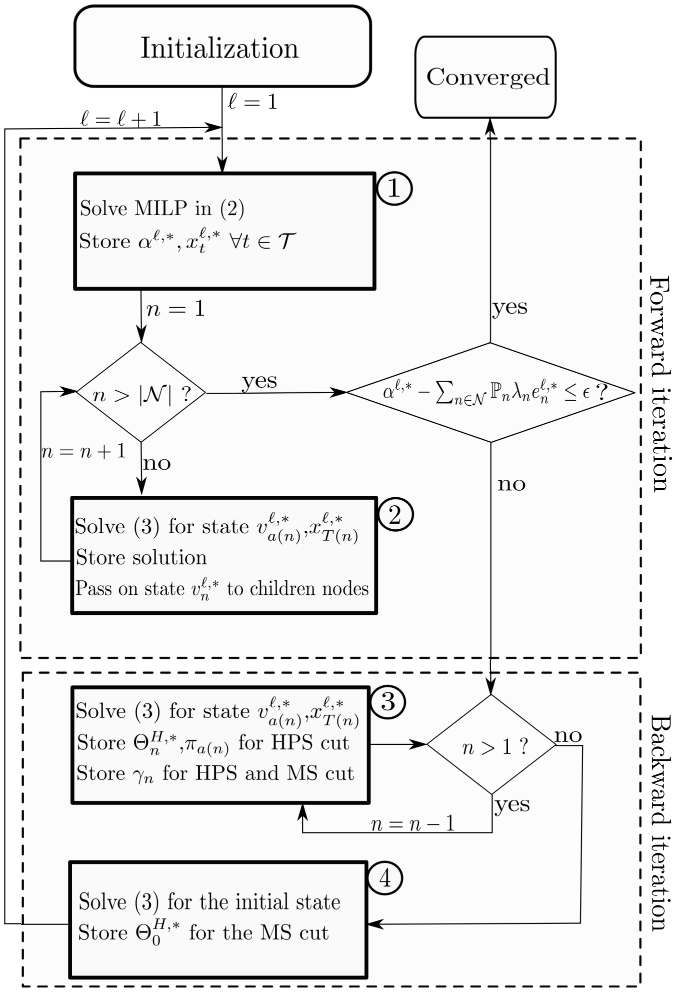

Fig. 2. Flow chart of the decomposition approach to solve the HPMS problem.

B. Problem Decomposition

Although the problem in (1) can be solved directly, practical HPS methods often rely on decomposition approaches to efficiently deal with large scenario trees [25]. We now decompose the HPMS problem in (1) in an MS problem representing the root node binary decisions and an HPS problem representing the continuous variables at all nodes. We apply Benders decomposition principle to coordinate the solution of the MS and HPS problems, as illustrated in Fig. 1. Being an LP problem, the HPS problem itself can be decomposed by node, and efficiently solved by Benders decomposition, as described in [25]–[27].

The MS and HPS problems are coordinated iteratively through forward and backward iterations to arrive at an optimal solution for the HPMS problem, as described in the following, and illustrated in Fig. 2. Note that we have included the iteration counter ℓ in Fig. 2, but we generally omit this counter in the text for notational convenience.

1) Forward Iteration: In a forward iteration, the relaxed MILP MS problem in (2) is solved first, indicated by box ① in Fig. 2. The future expected profit from operating the hydropower system in the period of analyses $\mathcal{T}$ is represented by α in (2a). α is constrained by a set of Benders cuts in (2d) that are added iteratively. We refer to the cuts in (2d) as MS Benders cuts. The cut coefficients $\gamma_{t\ell}$ and β_ℓ are obtained by evaluating the HPS problem, as indicated in Fig. 1, and further described in Section II-B2.

$$\Theta^M = \max \alpha \tag{2a}$$

$$\sum_{t \in \mathcal{T}_R} x_t = D \tag{2b}$$

$$x_t - x_{t-1} \leq x_{t+D-1} \qquad \forall t \in \mathcal{T}_R \tag{2c}$$

$$\alpha - \sum_{t \in \mathcal{T}_R} \gamma_{t\ell} x_t \leq \beta_\ell \qquad \forall \ell \in \mathcal{L}^M \tag{2d}$$

$$\alpha \in \mathbb{R}^+, x_t \in \{0, 1\} \qquad \forall t \in \mathcal{T}_R \tag{2e}$$

From the solution of the MS problem we obtain a trial maintenance schedule x^*_t and the upper bound α^*. The forward iteration is continued by a forward pass visiting all nodes $n \in \mathcal{N}$ in the HPS problem, indicated by box ② in Fig. 2. The trial schedule $x^*_{T(n)}$ enters the nodal HPS subproblems as a parameter. An HPS subproblem at a node n receiving a schedule $x^*_{T(n)}$ is formulated as:

$$\Theta_n^H = \max\left\{\lambda_n e_n + \alpha_n\right\} \tag{3a}$$

$$v_n + \sum_{s\in\mathcal{S}} q_{sn}^D + q_n^S = v_{a(n)} + I_n \qquad \leftarrow \pi_{a(n)} \tag{3b}$$

$$e_n - \sum_{s\in\mathcal{S}} \eta_s q_{sn}^D = 0 \tag{3c}$$

$$z_{T(n)} = x_{T(n)}^* \qquad \leftarrow \gamma_n \tag{3d}$$

$$0 \le q_{sn}^D \le (1 - z_{T(n)})\bar{Q}_s^D, \quad \forall s \in \mathcal{S} \tag{3e}$$

$$\alpha_n - \pi_{n\ell} v_n \le \beta_{n\ell} + \sum_{n'\in\mathcal{D}_n\setminus n} \mathbb{P}_{n'|n}\gamma_{n'\ell}x_{T(n')}^*, \ell \in \mathcal{L}_n^H \tag{3f}$$

$$\alpha_n, e_n, q_n^S, z_{T(n)} \in \mathbb{R}^+ \tag{3g}$$

$$0 \le v_n \le \bar{V} \tag{3h}$$

Dual values extracted from (3) are indicated with an arrow ($\leftarrow$). At each node the immediate profit comes from selling energy e_n at a price λ_n. The immediate profit is balanced against the future expected profit α_n in (3a), constrained by Benders cuts in (3f). We refer to the cuts in (3f) as HPS Benders cuts, and their construction is described in Section II-B2. The sensitivity of the objective to a change in x_t^* is denoted γ_n. In (3d) a continuous variable z_t copies the proposed schedule, and the dual value γ_n can be directly obtained. The variable z_t is then used to define whether discharge through the plant should be allowed in (3e).

Once the forward iteration has completed, convergence can be checked. The solution strategy decomposes the stochastic optimization problem in (1) into a first-stage MILP problem and a multi-stage HPS problem formulated as LP. Assuming that the problem has relatively complete recourse, the decomposition converges within a finite number of iterations at an arbitrary convergence tolerance ϵ, according to [24], [28]. Convergence is obtained when $\alpha^* - \sum_{n\in\mathcal{N}} \mathbb{P}_n\lambda_n e_n^* \le \epsilon$. As long as convergence has not been reached, a new backward iteration is initiated.

2) Backward Iteration: A backward iteration involves traversing the tree from the leaf nodes to the root node and solving the nodal HPS problems, see boxes ③ and ④ in Fig. 2. HPS Benders cuts in (3f) are created for all non-leaf nodes and added to the set $\mathcal{L}_n^H$ used in (3f). Both the initial reservoir level $v_{a(n)}^*$ and the maintenance decisions $x_{T(n')}^*$, $\forall n' \in \mathcal{D}_n$ are state variables and should be represented in the HPS Benders cuts. The incorporation of the root node maintenance decisions in the state space allows HPS Benders cuts that are constructed for different suggested maintenance schedules to be shared at each node. This state space augmentation has similarities to the multi-lag Benders decomposition approach presented in [29].

An HPS Benders cut contribution obtained from the subproblem solved at node n in (3) and passed backward to node $a(n)$ will take the form:

$$\alpha_{a(n)} - \pi_{a(n)}\left(v_{a(n)} - v_{a(n)}^*\right) - \sum_{n'\in\mathcal{D}_n} \mathbb{P}_{n'|n}\gamma_{n'}\left(x_{T(n')} - x_{T(n')}^*\right) \le \Theta_n^{H,*} \tag{4}$$

where $v_{a(n)}$ and $x_{T(n')}$, $\forall n' \in \mathcal{D}_n$ are state variables and the corresponding starred variables are the values set in the forward iteration. After reordering the contribution can be formulated as:

$$\alpha_{a(n)} - \pi_{a(n)}v_{a(n)} - \sum_{n'\in\mathcal{D}_n} \mathbb{P}_{n'|n}\gamma_{n'}x_{T(n')} \le \beta_{a(n)} \tag{5}$$

where $\beta_{a(n)} = \Theta_n^{H,*} - \pi_{a(n)}v_{a(n)}^* - \sum_{n'\in\mathcal{D}_n} \mathbb{P}_{n'|n}\gamma_{n'}x_{T(n')}^*$. The proposed maintenance schedule in the current iteration enters the right-hand side $\beta_{a(n)}$. The HPS Benders cut in (3f) are constructed by averaging the contributions of type (5) for all immediate descendant (or children) nodes. When used later, the values $x_{T(n')}$, $\forall n' \in \mathcal{D}_n$ are known and the term $\sum_{n'\in\mathcal{D}_n} \mathbb{P}_{n'|n}\gamma_{n'\ell}x_{T(n')}$ in (5) can be moved to the right-hand side as in (3f).

The cut coefficients $\gamma_{t\ell}$ and β_ℓ used in the MS Benders cut (2d) are accumulated throughout the backward iteration as in (6) and (7), where $\Theta_0^{H,*}$ is the HPS root-node solution.

$$\gamma_{t\ell} = \sum_{n\in\mathcal{N}:T(n)=t} \mathbb{P}_n\gamma_{n\ell} \qquad \forall t \in \mathcal{T}_R \tag{6}$$

$$\beta_\ell = \Theta_0^{H,*} - \sum_{t\in\mathcal{T}_R} \gamma_{t\ell}x_t^* \tag{7}$$

Note that the value γ_n takes in (3d), strongly depends on the reservoir storage capacity and the expected inflow. In case there is no storage, γ_n will reflect the energy price λ_n, taking the marginal efficiency η_s into account. In the presence of storage capacity, the value of γ_n is reduced compared to the no-storage case according to the expected marginal value of water. That is, the economic loss of maintenance depends on the system's ability to save water across decision stages.

The methodology presented in this section will become computationally intractable for systems with many decision stages and fine-grained representation of uncertainty. In Section III we elaborate on how the decomposition principle can be embedded in a hybrid SDP/SDDP approach, and thus become computationally tractable for large-scale hydropower systems.

III. Hydropower Scheduling Model

In this section we extend the HPS part of the HPMS formulation presented in Section II to realistically deal with the scheduling of larger watercourses. Moreover, we extend the scope by allowing sales of both energy and reserve capacity, assuming that the producer is a price-taker in both markets. We consider inflow, energy price and reserve capacity price as stochastic variables. We assume that the system is confined in a geographical area that can be covered by a single power balance equation (without internal transmission bottlenecks). Thus, the hydropower producer can optimize the operation of a single watercourse indvidually without including other generation or demand obligations that are part of its portfolio. The length of a decision period is one week.

A hybrid SDP/SDDP approach is applied to decompose the overall HPS problem, see e.g. [30], [31] for further details. The price processes are represented in a Markov chain using discrete states (price clusters), and embedded in the SDDP algorithm as in ordinary dynamic programming. The weekly price processes are assumed independent of the inflow. Note that the combined SDP/SDDP algorithm generally requires the stochastic processes being modelled in the SDP part to be independent of those modelled in the SDDP part. In our experience, for a regional system (e.g. a single watercourse within a price zone) it is difficult to find a significant correlation between local inflow changes and the corresponding average spot price change. The HPS Benders cuts can be shared among different system states within a given week and price cluster, according to [32], to efficiently reduce the computational effort.

Since only a sample of the "true" scenario tree is visited in the forward and backward iterations, the construction of HPS and MS Benders cuts differs slightly from what was presented in Section II.

A. HPS Solution Strategy

The HPS solution strategy is briefly described below, emphasizing on the novel parts.

1) Forward Iteration: In the HPMS forward iteration the root node MS problem described in (2) is solved first to obtain a trial maintenance schedule. An upper bound is obtained from the solution of the MS problem. Subsequently, a set of scenarios is sampled for the stochastic variables. Weekly inflows are sampled from a lag-1 autoregressive model, and weekly average energy and reserve capacity prices are sampled based on the conditional transition probabilities in a discrete Markov chain, as described in [10]. For a given node n, the HPS subproblem described in Section III-B is solved. The simulated state at the end of the week is passed forward as an initial state for the next week. The forward simulation gives an updated set of state trajectories and an expected profit for the sampled scenarios, which serves as the lower bound. Convergence can formally be declared when the upper bound is within a predefined confidence interval of the lower bound.

2) Backward Iteration: HPS Benders cuts at the end of the planning horizon $t = |\mathcal{T}|$ can be obtained from a predefined final value function Φ. For $t = |\mathcal{T}| - 1 \cdots 1$ in the backward iteration one loops through each discrete price cluster and each state trajectory obtained from the forward iteration. Starting from the state at the end of week $t-1$, for each realization of stochastic variables one computes the optimal operation for week t by solving (10). From the sensitivities of the objective function to the initial state values, new HPS Benders cuts at the end of week $t-1$ are obtained.

We denote all scenario tree nodes visited in a backward iteration ℓ as $n \in \mathcal{D}^S_{0\ell}$, i.e., sampled descendant nodes seen from the root node. For ease of formulation we include the Markov chain representation of market prices in the scenario tree notation, so that the set $\mathcal{D}^S_{0\ell}$ comprises all price clusters in the discrete Markov chain in addition to the sampled inflows.

The MS Benders cut coefficients $\gamma_{ht\ell}$ and β_ℓ used in (2d) are obtained after one backward iteration as probability weighted averages over all sampled scenarios $n \in \mathcal{D}^S_{0\ell}$.

$$\gamma_{ht\ell} = \sum_{n \in \mathcal{D}^S_{0\ell}: T(n)=t} \mathbb{P}_n \gamma_{nh\ell} \qquad \forall t \in \mathcal{T}_R, \forall h \in \mathcal{H} \tag{8}$$

$$\beta_\ell = \Theta^H_0 - \sum_{t \in \mathcal{T}_R} \gamma_{ht\ell} x^*_{ht} \tag{9}$$

B. HPS Subproblem

The weekly HPS subproblem is formulated as an LP problem described in (10), which is an extension of (3). The formulation is similar to that used in [33].

For a given node n, price cluster $p = P(n)$ and week $t = T(n)$ the realization of weekly inflows I_{nh}, the weekly average energy price λ^E_p, and the weekly average reserve capacity price λ^C_p are known. The amount of energy sold to the spot market and capacity to the reserve market is optimized for the whole water course.

$$\Theta^H_n = \max\left\{\sum_{k \in \mathcal{K}} \sum_{h \in \mathcal{H}} \left(\zeta^E_k \lambda^E_p e_{kh} + \zeta^C_k \lambda^C_p c_{kh}\right) + \alpha_n\right\} \tag{10a}$$

$$v_{kh} + \sum_{s \in \mathcal{S}_h} q^D_{khs} + q^S_{kh} + q^B_{kh} - \tag{10b}$$

$$\sum_{j \in \Omega_h}\left(\sum_{s \in \mathcal{S}_j} q^D_{kjs} + q^S_{kj} + q^B_{kj}\right) = v_{k-1,h} + \tilde{\tau}_k I_{nh} \quad \forall k, h$$

$$e_{kh} - \sum_{s \in \mathcal{S}_h} \eta_{hs} q^D_{khs} = 0 \qquad \forall k, h \tag{10c}$$

$$c_{kh} - \frac{1}{\tau_k} e_{kh} \le 0 \forall k, h \tag{10d}$$

$$c_{kh} + \frac{1}{\tau_k} e_{kh} \le \bar{G}_h \forall k, h \tag{10e}$$

$$v_{kh} - \frac{\tau_k}{\eta_{hS}} c_{kh} \ge \underline{V}_{kh} \forall k, h \tag{10f}$$

$$\alpha_n - \sum_{h \in \mathcal{H}} \pi_{nh\ell} v_{kh} \tag{10g}$$

$$\le \beta_{n\ell} + \sum_{n' \in \mathcal{D}^S_{n\ell} \setminus n} \mathbb{P}_{n'|n} \gamma_{n'h\ell} x^*_{T(n')}, k = K, \ell \in \mathcal{L}^H_p$$

$$z_{ht} = x^*_{ht} \quad \leftarrow \gamma_{nh} \tag{10h}$$

$$0 \le q^D_{khs} \le (1 - z_{ht}) \bar{Q}^D_{khs} \quad \forall k, h \quad \forall s \in \mathcal{S}_h \tag{10i}$$

$$\underline{V}_{kh} \le v_{kh} \le \bar{V}_{kh} \qquad \forall k, h \tag{10j}$$

$$0 \le q^B_{kh} \le \bar{Q}^B_{kh} \qquad \forall k, h \tag{10k}$$

$$0 \le c_{kh} \le \bar{C}_h \qquad \forall k, h \tag{10l}$$

$$\alpha_n, e_{kh}, q^S_{kh}, z_{ht} \in \mathbb{R}^+ \tag{10m}$$

The objective (10a) is to maximize the profit from both markets, subject to constraints (10b-10m). Energy and reserve capacity prices corresponding to a specific time step k within the week are found by scaling the weekly average values by predefined expected profiles.

The water balance equation for a specific reservoir h and time step k is formulated in (10b).

Electricity generation e_{kh} in (10c) and capacity allocation c_{kh} in (10d)-(10e) are determined per plant. The generation in (10c) can be seen as a simplified representation of the generally nonconvex hydropower production function. See e.g. [34], [35] for more detailed representations.

Allocated capacity should be spinning and symmetric. The spinning requirement is taken care of in (10d), ensuring that a plant cannot offer more reserve capacity than what is already spinning. Eqn. (10e) ensures that the generation capacity sold in the two markets does not exceed the plant's installed capacity. Eqn. (10f) ensures that there is enough water in the reservoir to deliver up-regulation reserves at the lowest efficiency η_{hS} for the entire time period in question.

The profit obtained for the current week is balanced against the future expected profit α_n. This variable is constrained by HPS Benders cuts in (10g). The HPS Benders cuts should relate to all state variables, i.e., decision and stochastic variables that define the system state passed on to the subsequent week. Although not explicitly represented here, it should be noted that inflow is also a state variable due to the time coupling in the autoregressive inflow model. As inflow is not a decision variable, its contribution to the cut will enter the right-hand side in (10g), as described in [31]. The HPS Benders cuts are built and stored for each price cluster in a set $\mathcal{L}_p^H$ in each backward iteration of the algorithm. In (10g) the cut coefficients $\gamma_{n'h\ell}$ obtained from (10h) are averaged over all descendant nodes, e.g. combinations of descendant price clusters and sampled inflow nodes. The set $\mathcal{D}_{n\ell}^S$ contains all descendant nodes seen from n that are visited in a particular backward iteration ℓ.

It is important that the HPS subproblem has a feasible solution for all possible states, and thereby ensure that the HPMS problem has relatively complete recourse. During a maintenance period for a given station, upstream reservoirs may experience a constrained release capacity. In the HPS subpoblem described above, the spillage variable q_{kh}^S has no defined upper bound and will therefore serve as a slack variable in case both the discharge q_{khs}^D and bypass q_{kh}^B reach their upper bounds in (10i) and (10k), respectively.

IV. Case Study

A. Case Description

A computer model was established implementing the HPMS algorithm described in Section II using the combined SDP/SDDP algorithm described in Section III to solve the HPS problem. The model was tested on a Norwegian watercourse comprising 7 hydropower reservoirs with corresponding power plants, with a total capacity of 986 MW. An illustration of the topology and specification of some technical characteristics is provided in Fig. 3. For each reservoir shown in the figure the average annual inflow and storage capacity are stated, both in Mm^3. Each power plant is identified with a number and its installed capacity in MW.

A scheduling horizon of 2 years was applied with weekly decision stages. Each week was divided into 21 time steps allowing for energy and reserve capacity sales. Each plant is allowed to commit a maximum of 10 % of its installed capacity to the reserve market.

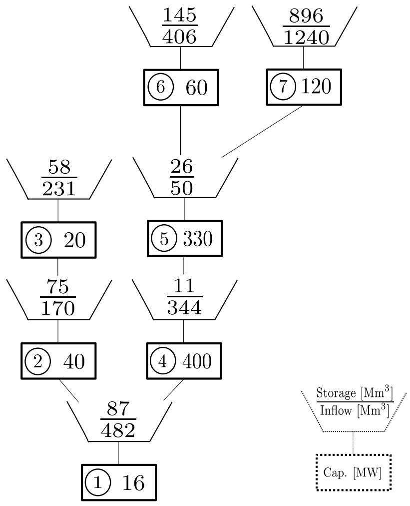

Fig. 3. Watercourse topology and technical data.

Conceptually, one could run each case with a scheduling horizon long enough to make the impact of end-valuation of reservoirs negligible. In order to obtain acceptable computation times, we adopted an approximate and practical approach. A set of cuts of type (10g) was used to ensure that state variables at the end of the scheduling horizon were valuated. These cuts were obtained as a result of a few initial model runs for case A with a time horizon of four years, and the same set of cuts was used for all simulated cases.

Energy and reserve capacity price scenarios were obtained from the EMPS model, which is a fundamental hydrothermal market model [36]. The EMPS model was run on a system description of the Nordic power system, using 80 historical inflow years, and with reserve capacity constraints per price zone. We extracted 80 energy and reserve capacity price scenarios from the simulation, and from these scenarios a discrete price model comprising 6 price nodes per stage (3 energy and 2 capacity price clusters) was identified by following the approach described in [10]. The lag-1 autoregressive inflow model was fitted using a single inflow series comprising 80 historical years, and the model error was sampled from a normal distribution.

A total of 200 scenarios of inflow and prices were resampled in each forward iteration, and 12 discrete inflow error terms were sampled at each stage in the backward iterations in the SDDP-part of the algorithm. A total of $200 \times 104 = 20800$ HPS subproblems are solved in the forward iteration and $200 \times 12 \times 3 \times 104 = 748800$ in the backward iteration when considering 3 price clusters. In general, increasing the number of scenarios in the forward iteration will improve the convergence properties at the cost of higher computational effort in each iteration.

The model was implemented in C++, using the Gurobi 7.5 library [37] for solution of both the MILP and the LP problems. All tests were carried out on an Intel Core i7-4940MX processor with 3.30 GHz and 32 GB RAM.

We ran 4 cases A-D, see Table I separating between whether maintenance scheduling was considered and whether sales of reserve capacity was allowed. A maintenance period of 3 con-

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secutive weeks was required, letting $\mathcal{T}_R$ comprise weeks 15-45. In all cases the simultaneous scheduling of maintenance in two plants are considered, which to our knowledge represents a realistic two-year case study. The MS problem comprises 1 continuous and 62 binary variables, and is constrained by 64 linear constraints without counting the MS Benders cuts. The HPS subproblem comprises 1247 continuous variables and 1078 linear constraints without counting the HPS Benders cuts. The total computation times are indicated in Table I. Although not exploited in this work, the algorithm is well suited for parallel processing, see e.g. [38].

TABLE I
SIMULATED CASES.

Case	Maint. Included	Plants	Markets	Iter.	Time [hours]
A	✓	4,7	Energy + Capacity	340	168
B	✓	4,7	Energy	323	81
C	-	-	Energy + Capacity	40	20
D	✓	6,7	Energy + Capacity	520	263

B. Results

Both cases A and B concludes on co-scheduling the maintenance of plants 4 and 7. In case A the optimal maintenance period for both plants are in weeks 42-44, whereas the optimal schedule is weeks 29-31 for both plants in case B. The higher reserve capacity prices in the summer period makes it profitable to postpone maintenance to the autumn in case A. The serial connection between plants 4 and 7 explains why it is profitable to co-schedule maintenance for these two plants. A three-week maintenance period at plant 4 leads to loss of water due to the large inflow and upstream discharge capacity compared to the reservoir size. Thus, it is wise to avoid discharging from plant 7 during the maintenance period of plant 4. When testing other plant combinations, e.g. in case D for plants 6 and 7 which are arranged parallel, co-scheduling of maintenance is no longer optimal.

The convergence characteristic of the algorithm is shown in Fig. 4, comparing the upper bounds for all cases for the first 100 iterations. The lower bound for case B is also included to indicate that the cost gap gradually closes as the iteration number increases. From a theoretical point of view, the algorithm should converge within a pre-defined confidence interval, as discussed in [39]. However, through initial testing on a small-scale system, we found that this convergence criterion was sensitive to the specific case and the number of scenarios, inflow error terms and price clusters used. Thus, we used the stabilization of the upper bound as the convergence criterion. As discussed in [40], the stabilization of the lower (minimization problem) bound indicates that further runs of the algorithm do not significantly improve the constructed policy, and may serve as a practical convergence criterion. The number of iterations required for convergence are shown in fifth column in Table I. As expected, case C (with no maintenance requirement) converges in significantly less iterations than the other cases. The storage dynamics and system complexity slow down the convergence process when

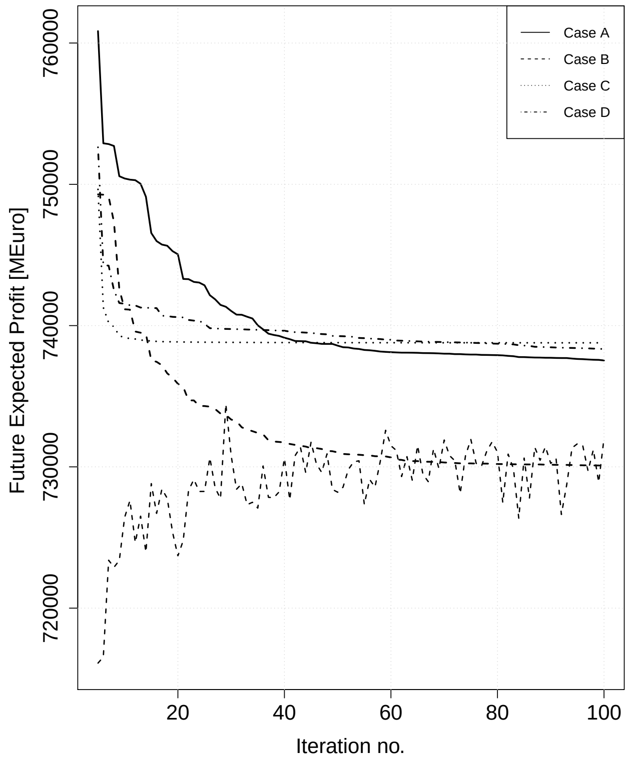

Fig. 4. Convergence characteristics. Upper bounds for all cases and lower bound for case B, considering the 100 first iterations.

maintenance is considered. A stable maintenance schedule over several iterations is needed to ensure that the HPS Benders cuts are built around that schedule.

In cases A, B and D we have allowed the model to explore 29 maintenance starting weeks (weeks 15-43) for each of the two plants considered, giving a total of 841 possible combinations. Fig. 5 shows the models suggestions for the first maintenance week for case B in all iterations, before stabilizing at the optimal solution (starting week 29 for both plants). The figure shows that a modest subset of all combinations are tested before arriving at the optimal solution, and many of these are only visited once. Moreover, Fig. 5 shows that the diagonal elements are visited more frequently, i.e. the different timings for co-scheduling of maintenance are explored. In case D maintenance is required for two reservoirs connected in parallel, and we observed that a wider search was carried out for that case, avoiding co-scheduling. Thus, convergence was slower in case D than in cases A and B, as indicated in Table I.

C. Discussion

It is possible to improve the computational performance of the model by some straightforward adjustments. Typically, the MS problem defined in (2) can be further constrained by incorporating the expert knowledge and specific requirements from the producer. Moreover, to avoid starting from scratch, one can run the case to convergence (with no or fixed

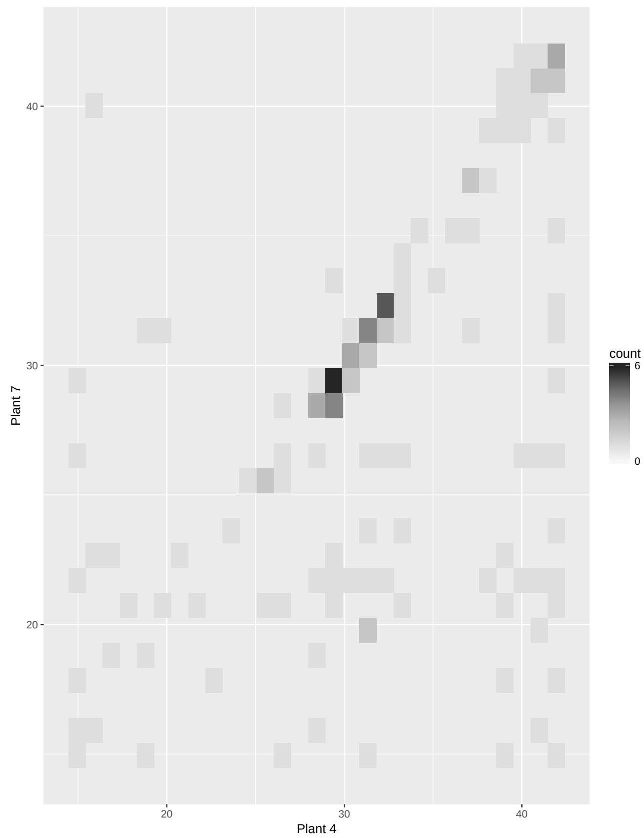

Fig. 5. Suggested first week of maintenance for plants 4 and 7 in case B counted over all iterations.

maintenance plans) and use the resulting HPS Benders cuts as a starting point for the algorithm. We also believe that computational improvements can be seen by allowing multiple iterations within the HPS problem for the same maintenance schedule before creating a MPS Benders cut, similar to the reasoning in [21]. Finally, it should be noted that we did not conduct sensitivity analysis on the number of scenarios sampled in the forward iteration in the presented case study, and that adjusting this number may improve computational performance.

V. Conclusions

A new method suitable for solving the hydropower maintenance scheduling problem for a profit maximizing and price-taking producer considering both the markets for energy and reserve capacity was presented. The method uses Benders decomposition principle to coordinate the timing of power plant maintenance with the medium-term scheduling of the hydropower system.

Traditionally, maintenance and hydropower scheduling are treated as separate or at least loosely coupled tasks, primarily due to the computational complexity. The proposed method was applied in a case study of a Norwegian watercourse, demonstrating that an optimal solution to the hydropower maintenance scheduling problem can be found by coordinating the maintenance and hydropower scheduling tasks. Although the convergence rate is significantly lower than for the hydropower scheduling problem, the results indicate that the proposed method is capable of efficiently exploring the search space of possible maintenance periods for multiple plants.

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