COMPARISON OF WIND AND WAVE EXTREMES IN VERY LONG-TERM CLIMATIC SCALES

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Abstract

The study of very long-term ocean climate is of great interest in a number of different applications. In a climate change perspective, estimations of return values of wind and wave parameters to a future climate are of great importance for risk management and adaptation purposes. However, there are various ways of estimating the required return values, which introduce additional uncertainties in extreme weather and climate variables pertaining to both current and future climates. The different approaches that are considered in the present work include the annual maxima approach, the block maxima approach, and the MENU method which is based on the calculation of return periods of various level values from nonstationary time series data. Furthermore, the effect of different modelling choices within each of the approaches will be explored. Thus, a range of different return value estimates for the different data sets is obtained for a field of datapoints. Long-term datasets for an area in the North Atlantic Ocean are used in the present study, derived for project ExWaCli, comprising of 30 years in the present (historic period) and two sets of 30 years in the future (future projections). The comparison between the results of the various approaches reveals a variability of the return period estimates, and an assessment of this is given. Moreover, it seems that a slight shift towards higher extremes in a future wave climate might be possible based on the particular datasets that have been analysed.

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INTRODUCTION

Extreme value analysis of wave climate parameters is an important part of ocean and coastal engineering where the extreme loads from extreme environmental conditions need to be taken into account.

Several previous studies have discussed the bias and uncertainty of extreme value prediction of metocean parameters, see e.g. [1–3]. Uncertainty of extreme value prediction is divided into statistical uncertainty and modelling uncertainty in [4], which also investigates the impact on structural reliabilities. The uncertainty of design values from extreme value analyses is also addressed in [5] and different methods of extreme value estimation of wind speeds are compared in [6]. Some fundamental problems in extreme value analysis such as uncertainties due to plotting position, the fitting method and due to the fact that the asymptotic conditions are never fulfilled in practice are discussed in [7].

It is noted that there are several sources of uncertainties of future climate projections that are not investigated in this paper. The climate scenario is obviously important, and only two future scenarios are considered in this study. However, studies have demonstrated that the choice of climate model might contribute more to the overall uncertainty of the future wave climate than the climate scenarios themselves [8–12]; see also [13]. Future projections of waves are typically obtained by using wind output from climate models as input to numerical wave models, and also the choice of wave model, the downscaling method and the model resolution will have a big impact on the results. Hence, the uncertainties associated with future wave climate extremes are not restricted to the uncertainties due to the statistical extreme value analysis which is the focus of this paper. Nevertheless, these uncertainties remain important and will also be equally present in the estimation of extremes in historical, present and future climates.

There are large uncertainties associated with extreme value analyses, and the uncertainties generally increase for higher return periods. Ideally, long enough time series should be available to reliably extract return values. In practice, however, return periods much longer than the length of recorded data are needed. Therefore, there is a need to extrapolate in order to obtain estimates of the tail behaviour of the underlying statistical distributions. Intuitively, the further away from the data one has to extrapolate, the larger the uncertainties of the resulting estimates will be. As a rule of thumb, for example, the ISO standard ISO 19901-1 [14] recommends not to use return periods more than a factor of four beyond the length of the data set when deriving return values for design of offshore structures. Hence, for the datasets analysed in this paper, covering a period of 30 years, the longest return periods that should be investigated are 120 years.

There is a number of different approaches to extreme value analysis and return value estimation, which all rely on a set of assumptions. The initial distribution approach fits a statistical model to all the data under the assumption of independent and identically distributed (iid) observations and estimate high return values by extrapolating the fitted distribution to high quantiles corresponding to the desired return periods. However, one fundamental problem with this approach is that it often captures the area close to the mode of the distribution quite well, but gives poor fit to the tail of the distribution.

Some of the classical approaches to extreme value analysis rely on assumptions on the asymptotic behaviour of the extremes as the number of observations approaches infinity. These methods will typically assume that the data are iid, i.e. that the observations are realizations from the same stationary process and can be constructed as independent samples drawn from the same probability distribution. Predicting return periods and the corresponding design values by using data based on annual maxima, essentially overcomes difficulties concerning the validity of iid assumption, as well as the fact that the distribution of the initial population is unknown. In addition, a relatively small number of long-term (say, thirty years) records of wave measurements are usually available, which means that a relatively small population of annual maxima is available.

Two other commonly used approaches to extreme value analysis are the block maxima (BM) approach and the Peaks Over Threshold (POT) approach. An obvious drawback with these approaches is that they are wasteful and only exploits a small subset of all the data available. An introduction to these methods, along with a general introduction to the theory behind extreme value analysis can be found in [15]. Recent applications of the POT approach to analyse the extremes of ocean waves are presented in e.g. [16, 17].

A more recent method for extreme value analysis that allows for the assumptions of independence to be relaxed is proposed in [18], i.e. the Average Conditional Exceedance Rate Method (ACER). A further generalization to the bivariate case has also been presented in [19].

A full Bayesian approach to extreme value modelling is set forth in [20], and a Bayesian hierarchical model is presented in [21] for estimating extremes from climate model output. A review of Bayesian approaches to extreme value methods are given in [22]. An alternative approach to estimate return values of significant wave height, referred to as the (modified) Rice method is proposed in [23].

A further step would be to take into account the dependence structure of the time series by modelling it as, e.g., a stationary stochastic process and calculate its extremes based on the theory of these processes [24]. However, existing wave datasets (measurements) have shown that they can hardly be considered stationary.

The last decades, advanced models for the representation of time series of wind and wave parameters have been proposed; see, e.g. [25–28]. These more sophisticated time series models aim to significantly improve stochastic predictions of extreme values. According to these, except for the stochastic character of the series, the dependence structure and the seasonality exhibited are appropriately modelled. Then, the theory of periodically correlated stochastic processes can be used for the extreme-value predictions.

The nonstationary modelling of wind and wave time series is further combined with a non-Gaussian modelling of the second-order probability structure, to calculate return periods from nonstationary time series. Return period associated with the level value x^* is calculated as the time period in which the MEan Number of Upcrossings of the level x^* becomes equal to unity (MENU method) [29].

In the following, several of the aforementioned extreme-value analysis approaches are applied to wind and wave climatic data for a historical period and for two future climate projections. First, the metocean data will be described and then the various approaches implemented. Then, results from the different extreme value methods will be presented, compared and discussed. Similar analysis for a different wave dataset from other sources have been presented in [30]. In the present paper, both wind and wave data for a large amount of data points (one hundred) have been analysed allowing us: a) to investigate the variability of the extreme estimates of annual maxima methods as well as MENU method, and b) to have a first estimate of the spatial variability of the extreme values for the specific area. The latter will be further investigated in a future work and be compared with different methods such as regional frequency analysis [31].

WIND AND WAVE DATA

For the purpose of this study, three particular datasets have been used. These have been generated by Meteorological Institute of Norway within the ExWaCli project by running the numerical wave model WAM [32] for an area in the North Atlantic with forcings from six CMIP5 climate models listed in [33]. Altogether, the historical and future wave model integrations amount to about 1,000 simulated years, which were combined in one model ensemble, where each model is given equal weight by first averaging the ensemble members for each model. One can find more information about ExWaCli datasets in [33]. See also [34].

The referred area is in the North-eastern Atlantic Ocean, west of British Isles, and the grid, which consists of $10 \times 10 = 100$ datapoints, is shown in Fig. 1. At each datapoint, three-hourly time series of significant wave height and wind speed are available.

The three datasets correspond to a 30-year historical period (1970–1999) and two future periods (2071–2100) consistent with the RCP 4.5 and RCP 8.5 scenarios, respectively. See also [35, 36].

Summary statistics have been derived from the datasets as well as quantiles of the empirical distributions. The results are presented in Tables 1–2. For each statistic, the mean value and the range within the various datapoints is given. In addition, in Figs. 2–3, the same statistics are plotted for all datapoints for the historical period.

An interesting observation is that the variation of the statistics is slightly higher (larger range) in the historical period than in the two future scenarios. Concerning the extremes, the quantiles above 90% are higher in both future scenarios, with few exceptions. Also, in Figs. 2–3, it can be observed that the empirical distributions of significant wave height seem to have a longer tail than

Table 1: Basic statistics and quantiles for H_S (m)

	Historic		RCP 4.5		RCP 8.5	
Stats	Range	Mean	Range	Mean	Range	Mean
Mean	[3.23 - 3.55]	3.44	[3.09 - 3.36]	3.29	[3.11 - 3.35]	3.29
St.Dev.	[1.85 - 1.97]	1.91	[1.79 - 1.89]	1.85	[1.83 - 1.90]	1.88
Min	[0.32 - 0.53]	0.43	[0.25 - 0.38]	0.31	[0.27 - 0.55]	0.41
Max	[15.93-20.80]	18.90	[16.54-19.40]	18.07	[17.33-19.02]	18.32
10%	[1.31 - 1.50]	1.44	[1.27 - 1.43]	1.40	[1.24 - 1.39]	1.36
25%	[1.85 - 2.09]	2.01	[1.78 - 1.99]	1.94	[1.77 - 1.96]	1.92
50%	[2.80 - 3.11]	3.01	[2.69 - 2.93]	2.88	[2.68 - 2.94]	2.87
75%	[4.16 - 4.54]	4.40	[3.96 - 4.27]	4.19	[3.99 - 4.27]	4.20
90%	[5.73 - 6.20]	6.01	[5.41 - 5.82]	5.69	[5.55 - 5.85]	5.76
99%	[9.13 - 9.95]	9.60	[9.16 - 9.74]	9.45	[9.20 - 9.73]	9.56
99.9%	[12.06-13.35]	12.83	[12.75-13.89]	13.31	[12.17-13.39]	13.12
99.99%	[14.65-17.72]	15.88	[15.83-17.24]	16.64	[15.18-17.30]	16.47

Table 2: Basic statistics and quantiles for W_S (m/s)

	Historic		RCP 4.5		RCP 8.5	
Stats	Range	Mean	Range	Mean	Range	Mean
Mean	[9.53 -10.13]	9.76	[9.37 - 9.86]	9.53	[9.40 - 9.90]	9.57
St.Dev.	[4.36 - 4.56]	4.44	[4.27 - 4.46]	4.34	[4.32 - 4.44]	4.36
Min	[0.00 - 0.10]	0.04	[0.00 - 0.10]	0.04	[0.01 - 0.11]	0.04
Max	[27.00-32.33]	29.15	[27.35-33.61]	29.50	[28.02-32.15]	29.93
10%	[4.02 - 4.39]	4.15	[3.90 - 4.32]	4.09	[3.94 - 4.35]	4.12
25%	[6.27 - 6.75]	6.47	[6.16 - 6.59]	6.32	[6.20 - 6.65]	6.38
50%	[9.19 - 9.82]	9.43	[9.03 - 9.50]	9.20	[9.03 - 9.55]	9.22
75%	[12.45-13.17]	12.74	[12.17-12.76]	12.38	[12.20-12.81]	12.41
90%	[15.55-16.36]	15.85	[15.18-15.94]	15.44	[15.32-15.92]	15.51
99%	[20.20-21.30]	20.68	[20.11-21.27]	20.53	[20.48-21.23]	20.74
99.9%	[23.57-24.76]	24.10	[23.65-25.17]	24.36	[23.95-25.28]	24.55
99.99%	[25.74-28.25]	26.79	[26.25-28.39]	27.30	[26.99-29.04]	27.89

in wind speed. This fact will help us later to choose appropriate probability distribution for the wind and wave parameters.

EXTREME VALUE ANALYSIS OF WIND AND WAVE PARAMETERS

In the following, the datasets for significant wave height presented above will be subject to different extreme value analysis methods and it will be investigated how sensitive the results are to the choice of method and different choices within each method. Obviously, the actual results are conditioned on this dataset, but it is believed that it will still give a good indication of the uncertainties in extreme value analysis. The approaches that have been applied are different variations of the annual maxima approach, the GEV approach, and the MENU method.

The annual maxima approach

Let us now assume that, we have a sequence of successive maximum values of wind and wave parameters corresponding to fixed time intervals (e.g., months,

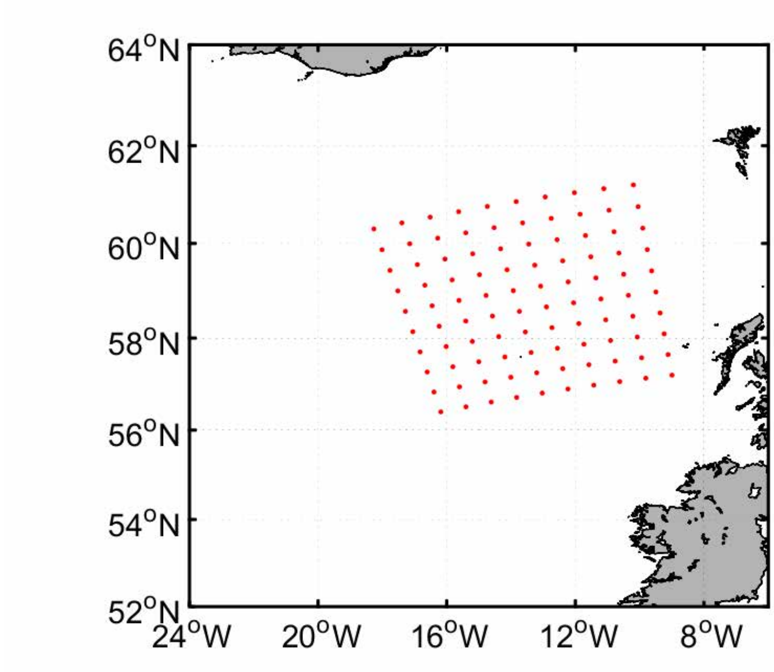

Figure 1: Grid of ExWaCli datapoints.

years, etc). Predicting return periods and the corresponding design values by using data based on annual maxima essentially overcomes all difficulties concerning the initial distribution mentioned above. For, now the distribution of the population of maxima, denoted by $G(x)$, is known as $n \to \infty$ [37], and it is the Generalized Extreme Value (GEV) distribution

$$G(x;\lambda,\delta,k) = \begin{cases} \exp\left\{-\left[1-k\,\dfrac{x-\lambda}{\delta}\right]^{1/k}\right\}, & k \neq 0, \\ \\ \exp\left\{-\exp\left[-\dfrac{x-\lambda}{\delta}\right]\right\}, & k = 0, \end{cases} \tag{1}$$

where $\lambda, \delta > 0$, and $-\infty < k < +\infty$.

The estimation of return periods for high levels x^* is

$$T_R(x^*) = \frac{\Delta\tau}{1-G(x^*)}, \tag{2}$$

where the extreme population distribution $G(x)$ is used instead of the initial population distribution $F(x)$.

The use of $G(x)$ instead of $F(x)$ in equation (2), is justified by the fact that the two distributions $G(x)$ and $F(x)$ are right-tailed equivalent; see, e.g., [37]. The selection of the type of $G(x)$ should be based on the behaviour of $G_{emp}(x)$

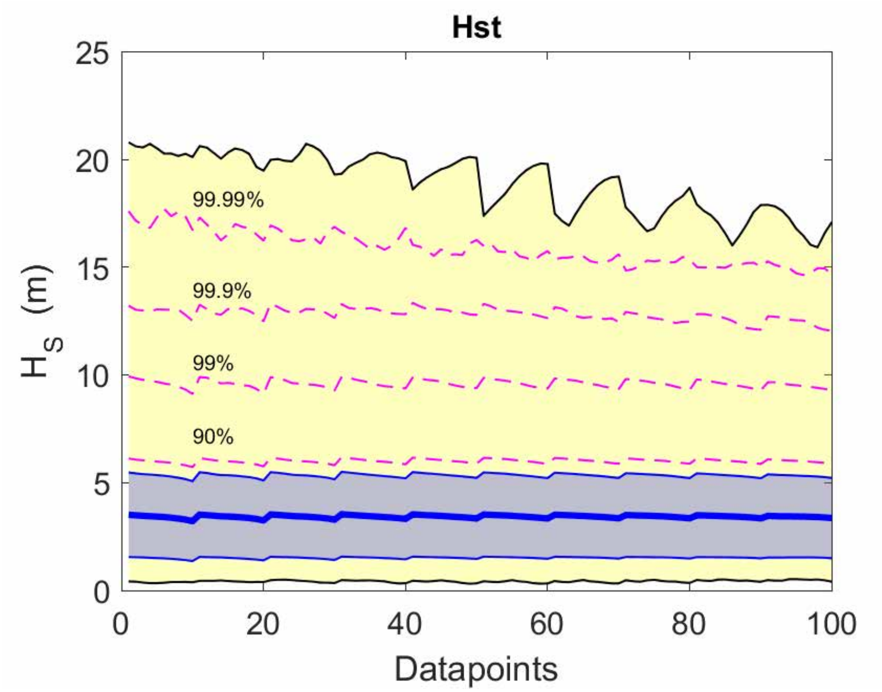

Figure 2: Basic statistics and quantiles for H_S (m)

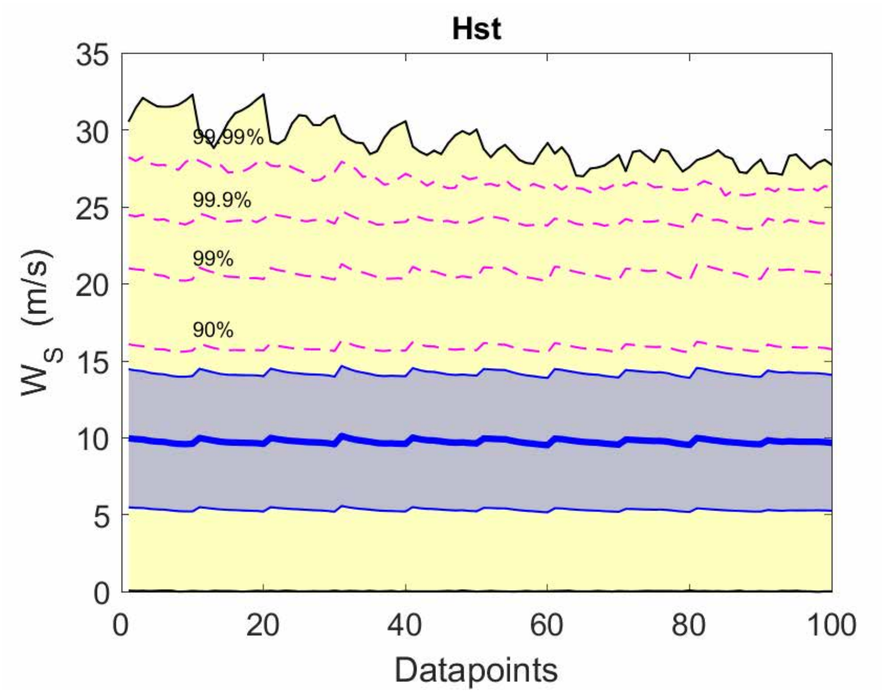

Figure 3: Basic statistics and quantiles for W_S (m/s)

(empirical distribution) at the right tail only. Usually, annual maxima are well modelled by means of the Gumbel distribution, which is a special case of GEV.

There are numerous different approaches for the estimation of the parameters of Gumbel distribution; see, e.g., [37, and references cited therein]. The following four have been selected for the present study:

(i) *The probability paper method.*
In this method, a straight line $y = ax + b$ is fitted by the least-squares method to the data appropriately plotted on Gumbel probability paper. For this, the classical plotting positions $\pi_n = n/(N+1)$ are used. The parameters of the distribution are given by

$$\delta = \frac{1}{a}, \qquad \lambda = -\frac{b}{a}, \tag{3}$$

where a and b are the parameters of the straight line.

(ii) *The probability paper method with different plotting position.*
The method is exactly the same with (i), except that it uses as plotting positions $\pi_n = (n - 0.44)/(N + 1)$.

(iii) *The L-moments method.*
Hosking proposed the concept of L-moments of a random variable [38, 39]. Using L-moments, the parameters of Gumbel distribution are given as follows:

$$\delta = \frac{L_2}{\ln 2}, \qquad \lambda = L_1 - 0.5772\ \frac{L_2}{\ln 2}, \tag{4}$$

where L_1, L_2 are the first two L-moments defined by

$$L_1 = \frac{1}{N}\sum_{i=1}^{N} X_{(i)}, \qquad L_2 = \frac{2}{N}\sum_{i=1}^{N} \frac{i-1}{N-1} X_{(i)} - L_1. \tag{5}$$

(iv) *The method of moments.*
The parameters of the Gumbel distribution are estimated by means of the mean value and the standard deviation of the data, as follows:

$$\delta = s_x\ \frac{\sqrt{6}}{\pi}, \qquad \lambda = m_x + \gamma\ \delta, \tag{6}$$

where m_x and s_x are the sample mean value and the standard deviation, respectively, and γ is the Euler's constant.

In principle, Gumbel's approach forms a sound methodology for predicting long-term extreme values and the corresponding return periods. The most serious problem with this approach is the lack of sufficiently large extreme population data samples that would permit the type of distribution to be safely selected and its parameters to be reliably estimated.

The GEV approach

As it is mentioned in the previous subsection, one can decide to fit instead of Gumbel the general form of GEV model to block of maxima [15]. Block sizes of one year is typical, but different block sizes can be investigated. Furthermore, it is implicitly assumed that the maxima are independent and identically distributed (iid). Within a climate change perspective, this approach needs to assume that the extremes can be considered stationary within each of the time intervals, i.e. that the extremes are stationary during the 30-year reference period and the 30-year projection period. If the effect of any long term trend is small compared to the other variability, this might not be a very unrealistic approximation; see, e.g., [40].

Thus, different GEV-models can be fitted with block sizes corresponding say to six months, one or two years etc. The different block sizes give different subsets of the data for fitting; small block sizes give a larger sample to fit to the model, but the iid assumption becomes less obvious. For example, using the semiannual maxima over the 30 year periods gives 60 samples and assumes that the distributions of wave heights are the same for the first six months of the year as for the last six months. Using the biennial maxima, on the other hand reduces the sample size to 15.

In the present study, annual blocks have been chosen and return periods for them have been calculated by fitting both a GEV and a Gumbel model to the annual maxima.

The MENU approach

The MENU method is based on the MEan Number of Upcrossings of a level value x^*, given by

$$M(x^*;t_1,t_2) = \frac{1}{2}\int_{t_1}^{t_2}\int_{-\infty}^{+\infty} |\dot{x}|\; f_{\tau,\tau}^{X\dot{X}}(x^*,\dot{x})d\dot{x}. \tag{7}$$

The return period $T_R(x^*,t)$ associated with the level x^* and the starting time t is calculated as the unique value T for which

$$M(x^*;t,t+T) = 1. \tag{8}$$

For a detailed description of MENU method; see e.g. [29].

It should be noted that, Eq. (7) is totally independent from the specific modelling adopted for the joint pdf $f_{\tau,\tau}^{X\dot{X}}(s_1,s_2)$.

In the present work, this pdf is obtained by exploiting the following nonstationary time series modelling [25, 28]. Assume that, a stochastic process $X(\tau)$ admits of the representation:

$$X(\tau) = G(\tau) + \sigma(\tau)W(\tau), \tag{9}$$

where $G(\tau)$ and $\sigma(\tau)$ are deterministic time-dependent periodic functions and $W(\tau)$ is a zero-mean stationary stochastic process, which will be called hereafter the residual stochastic process.

Especially, the function $G(\tau)$ is defined as

$$G(\tau) = \overline{X}_{mean} + \mu(\tau), \tag{10}$$

where $\overline{X}_{mean}$ is the overall mean value of the process $X(\tau)$, and $\mu(\tau)$ is the seasonal mean value and $\sigma(\tau)$ is the seasonal standard deviation of the process.

By differentiating Eq. (9), the representation of the derivative $\dot{X}(\tau)$ is obtained:

$$\dot{X}(\tau) = \dot{G}(\tau) + \dot{\sigma}(\tau)W(\tau) + \sigma(\tau)\dot{W}(\tau). \tag{11}$$

The functions $\dot{G}(\tau)$ and $\dot{\sigma}(\tau)$ are the derivatives of $G(\tau)$ and $\sigma(\tau)$ with respect to τ. According to their definition, all these functions $\dot{G}(\tau)$ and $\dot{\sigma}(\tau)$ are periodic.

Eqs (9) and (11) are considered as a linear system (time-dependent transformation) defining $X(\tau)$ and $\dot{X}(\tau)$ by means of $W(\tau)$ and $\dot{W}(\tau)$.

Thus, the calculation of pdf $f^{X\dot{X}}_{\tau,\tau}(s_1, s_2)$ is reduced to the evaluation of the time-invariant bivariate pdf $f^{W\dot{W}}_{\tau,\tau}(v_1, v_2)$, which, in turn, is obtained on the basis of pdf $f^{WW}_{\tau,\tau+\Delta\tau}(u_1, u_2)$. For the modelling of $f^{WW}_{\tau,\tau+\Delta\tau}(u_1, u_2)$, the bivariate Plackett model with given marginals is used. In our study, two different univariate pdfs have been used for the estimation of the univariate pdf of $W(\tau)$: Weibull for the wind speed and Lognormal for the significant wave height.

Concluding, for the application of MENU method, two components are required:

1. Estimations of the seasonal mean value and seasonal standard deviation and their derivatives, and

2. Estimations of the parameters of the joint probability density function of $W(t)$ and $W(t + \tau)$.

NUMERICAL RESULTS

In the sequel, the methods for predicting extreme values presented above have been applied to the datasets of significant wave height and wind speed (100 datapoints) for the three climatic scenarios (one historic and two future).

The first four are variants of the Annual maxima approach using the Gumbel distribution. The fifth applies the GEV distribution again to annual maxima blocks, and the sixth applies the MENU method.

The periods are marked in the tables and figures as “Hst” for Historic, “R45” for RCP 4.5, and “R85” for RCP 8.5.

It should be noted that, the following results are conditioned on the particular dataset analysed and the particular area in the North Atlantic Ocean, and further studies might be needed in order to confirm the findings.

Significant wave height

In Figs. 4–7, the results for the four variants of the Gumbel approach are depicted. In Fig. 8 the results for the GEV distribution and in Fig. 9 the results for MENU. These figures are based on the data of the Historic period. Similar figures were also obtained for the two future scenarios and are omitted due to space limitation.

By inspecting these figures, one can notice the great spatial variability in all methods, which can partially be justified by the fact that each line represents a different geographical location and not exactly the same point. Especially, in the four first methods (Gumbel) the variability of the estimates is very high.

Also, it seems that the variability is increased as we go to higher return periods.

In Table 3, basic statistics (mean value, standard deviation, min and max value) are summarized for one low (20-year) and one high (100-year) return period for the three climatic scenarios. The mean value shows the average behaviour of the 100 points, while the min and max values show the span where the estimates range. This reflects the spatial variability across the geographical area.

According to the results, it seems that the GEV distribution gives on the average the lowest estimates than the other methods,and the Gumbel approach (iv) the highest. In addition, MENU seems to have a more homogeneous spatial behaviour with lower standard deviation and narrower span between the min and the max value.

Another important feature is that the estimates for the future scenarios are a bit higher than the historic one. In addition, the estimates have a narrower range in the future scenarios, showing a more homogeneous spatial behaviour.

Furthermore, in order to examine the spatial variability of the return periods, field plots have been produced for various methods and various return periods. In this way, one can compare the spatial variability of the estimates based on different methods of estimation.

For example, in Figs 10–11, such kind of fields are plotted for the estimates of 100-year return period for the Gumbel approach (i) and the MENU method for the historic (Hst) scenario. There one can observe that the Gumbel estimates are increased in the south-north direction, while the MENU estimates are increased in a southwest-northeast direction. Comparing the two fields, one can infer that greater differences are found in the south than in the north.

Similar figures were also obtained for the two future scenarios and are omitted due to space limitation.

Wind speed

In Figs. 12–15, the results for the four variants of the Gumbel approach are depicted. In Fig. 16 the results for the GEV distribution and in Fig. 17 the results for MENU are shown. These figures are based on the data of the Historic period. Similar figures were also obtained for the two future scenarios and are omitted due to space limitation.

Here the spatial variability is even greater than in wave height. Especially for the future scenarios, it seems that it has been doubled. Again it should be

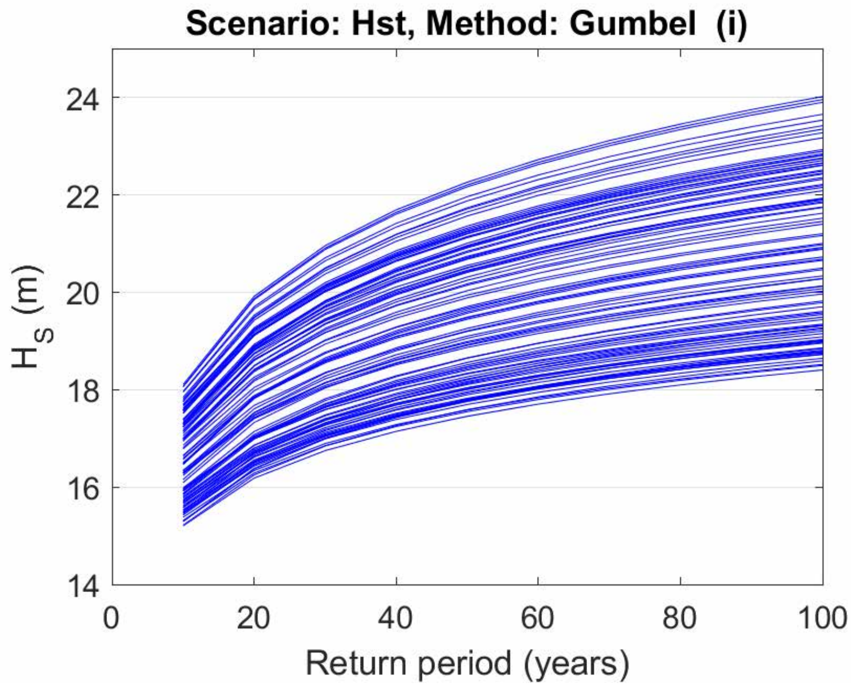

Figure 4: Gumbel (i)

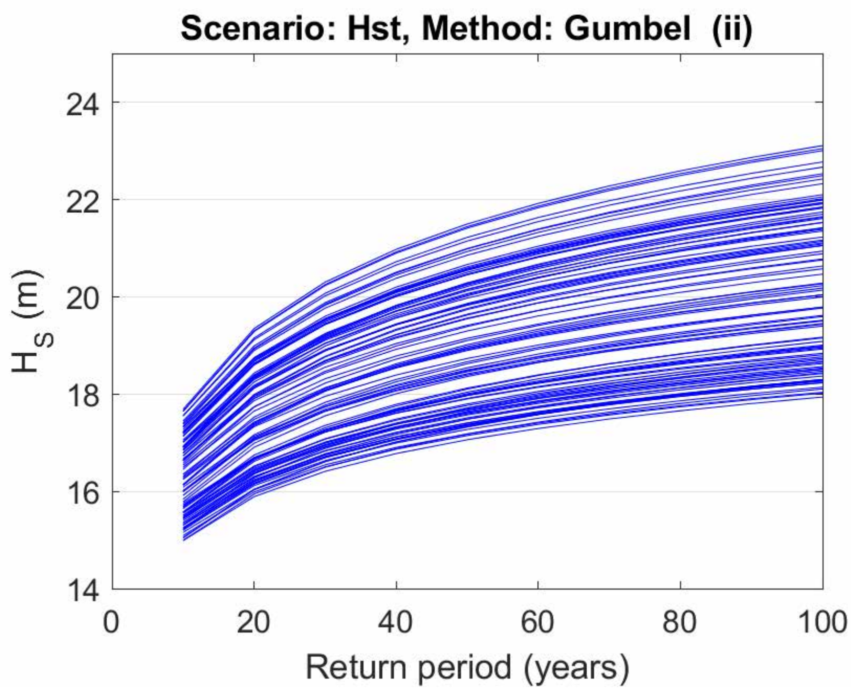

Figure 5: Gumbel (ii)

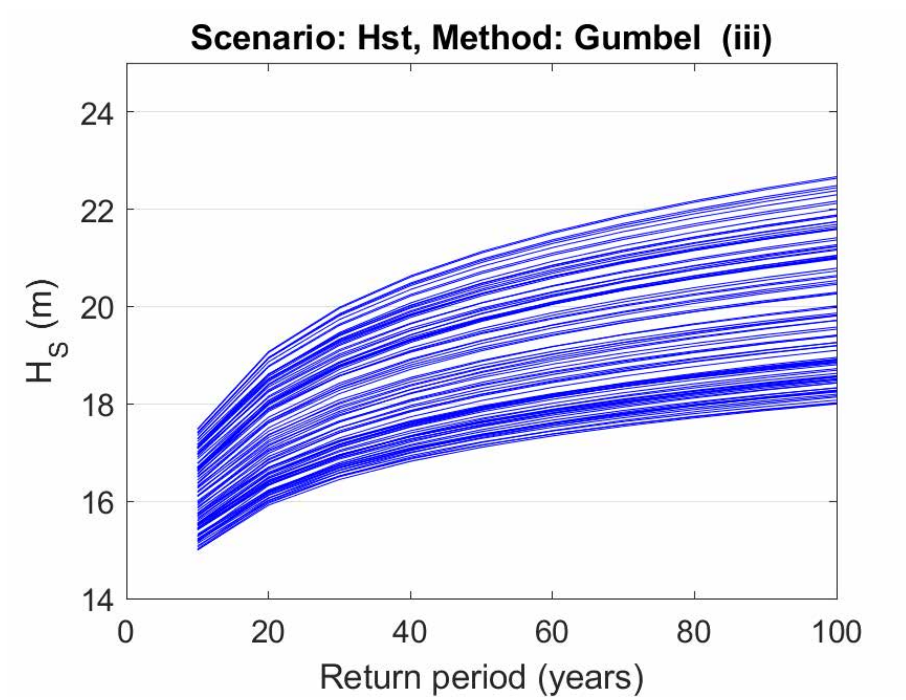

Figure 6: Gumbel (iii)

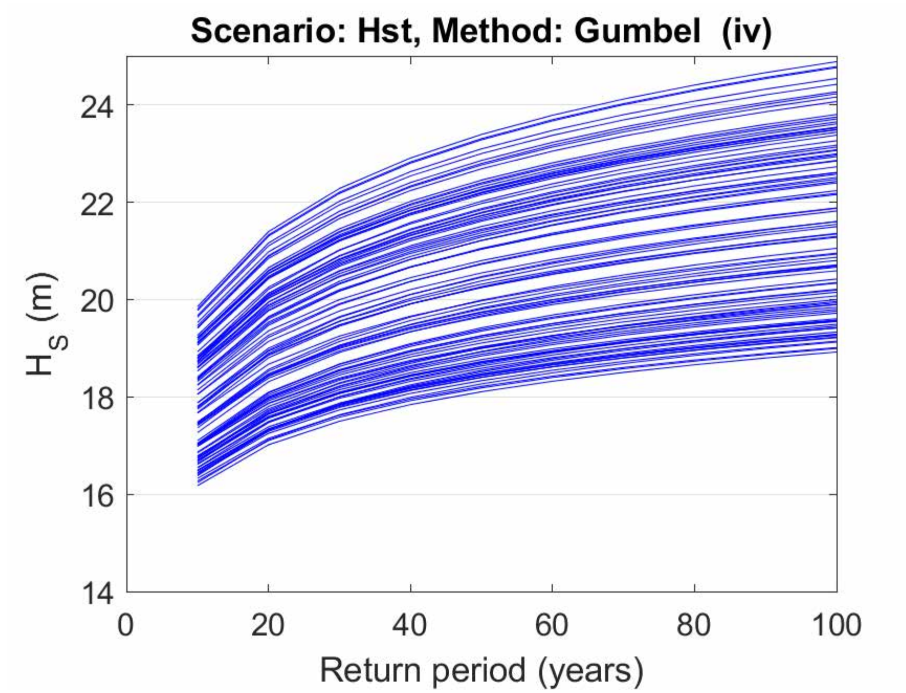

Figure 7: Gumbel (iv)

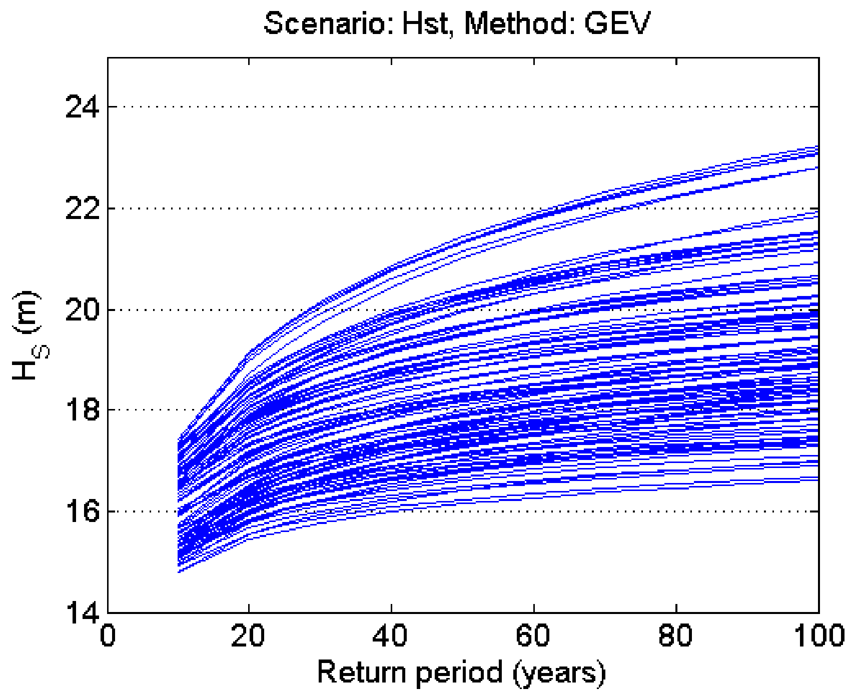

Figure 8: GEV

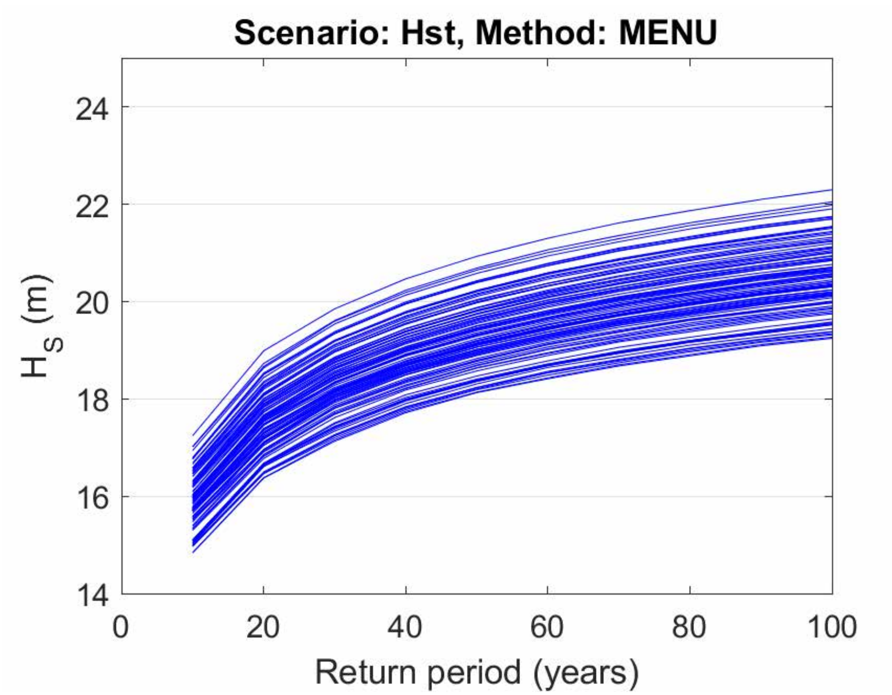

Figure 9: MENU

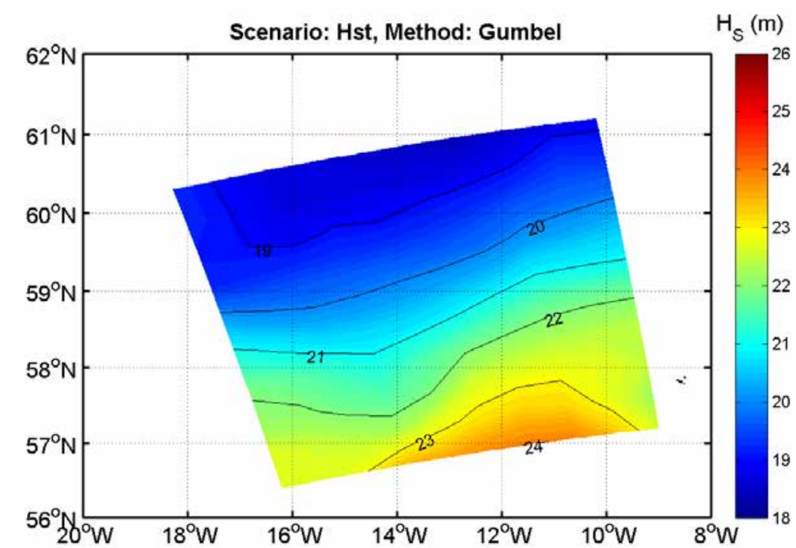

Figure 10: Gumbel field (100 years)

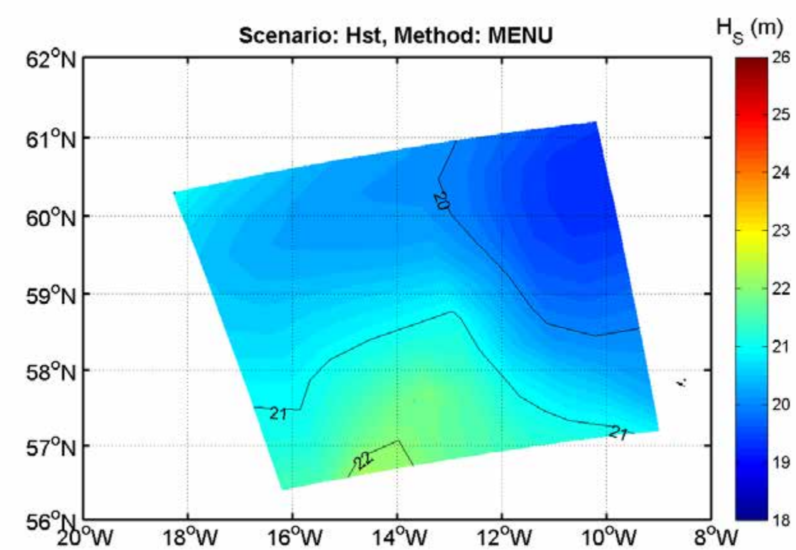

Figure 11: MENU field (100 years)

Table 3: Statistics for H_S-return values. [(1)–(4): Gumbel, (5): GEV, (6): MENU]

Statistic	(1)	(2)	(3)	(4)	(5)	(6)
Hst_20-mean	17.90	17.47	17.34	18.98	17.17	17.50
Hst_20-std	1.10	1.01	0.95	1.29	1.02	0.61
Hst_20-min	16.19	15.90	15.93	17.02	15.43	16.38
Hst_20-max	19.92	19.35	19.08	21.40	19.13	18.99
Hst_100-mean	20.87	20.20	19.99	21.53	19.36	20.56
Hst_100-std	1.65	1.52	1.41	1.76	1.66	0.72
Hst_100-min	18.41	17.95	18.00	18.93	16.61	19.25
Hst_100-max	24.02	23.12	22.67	24.90	23.24	22.30
R45_20-mean	18.49	18.05	18.04	19.70	17.11	17.69
R45_20-std	0.34	0.33	0.31	0.37	0.28	0.57
R45_20-min	17.40	17.02	17.02	18.45	16.01	16.79
R45_20-max	19.07	18.63	18.62	20.34	17.69	18.86
R45_100-mean	21.82	21.13	21.12	22.54	19.00	20.74
R45_100-std	0.44	0.42	0.39	0.46	0.47	0.64
R45_100-min	20.33	19.74	19.76	20.95	18.07	19.75
R45_100-max	22.55	21.83	21.85	23.30	20.43	22.08
R85_20-mean	18.06	17.62	17.53	19.21	17.28	18.68
R85_20-std	0.41	0.40	0.42	0.45	0.35	0.28
R85_20-min	16.55	16.14	16.01	17.62	16.03	18.14
R85_20-max	18.77	18.30	18.22	20.05	17.91	19.26
R85_100-mean	21.20	20.51	20.38	21.89	18.94	21.99
R85_100-std	0.52	0.51	0.54	0.54	0.53	0.33
R85_100-min	19.41	18.75	18.55	20.07	17.30	21.28
R85_100-max	22.23	21.48	21.37	23.01	20.01	22.69

noted that each line represents a different geographical location and not exactly the same point, which partially justifies this.

In Table 4, the same basic statistics (mean value, standard deviation, min and max value) are summarized for the two return periods (20- and 100-years) for the three climatic scenarios.

According to the results, it seems that GEV distribution gives again on the average the lowest estimates than the other methods,and the Gumbel approach (iv) the highest. In addition, MENU seems to have a significantly more homogeneous spatial behaviour with very low standard deviation and narrow span between the min and the max value, especially for the higher return periods.

Furthermore, the estimates for the future scenarios are again a bit higher than the historic one. However, the estimates do not have a narrower range in the future scenarios, but exhibit similar high variability in all three scenarios.

Concerning the spatial variability of the return periods, field plots for the various methods/return periods have been generated. An example is shown in Figs 10–11, where such kind of fields are plotted for the estimates of 100-year return period for the Gumbel approach (i) and the MENU method for the historic (Hst) scenario. There one can observe that the Gumbel estimates have a similar behaviour as in the wave height results, whereas the MENU estimates are increased in a west-east direction (rather than the SW-NE direction in the wave height results). Again, greater differences are found in the south than in the north.

Similar figures were also obtained for the two future scenarios and are omit-

Table 4: Statistics for W_S-return values. [(1)–(4): Gumbel, (5): GEV, (6): MENU]

Statistic	(1)	(2)	(3)	(4)	(5)	(6)
Hst_20-mean	28.82	28.47	28.38	29.73	27.99	29.36
Hst_20-std	0.77	0.71	0.67	0.92	0.82	0.41
Hst_20-min	27.72	27.47	27.36	28.37	26.64	28.63
Hst_20-max	31.03	30.53	30.36	32.32	30.11	30.17
Hst_100-mean	31.36	30.80	30.66	31.88	29.52	31.20
Hst_100-std	1.15	1.04	0.97	1.25	1.59	0.44
Hst_100-min	29.69	29.29	29.14	30.03	27.08	30.36
Hst_100-max	34.51	33.72	33.45	35.30	33.35	32.05
R45_20-mean	29.29	28.93	28.87	30.25	28.99	28.93
R45_20-std	0.90	0.84	0.82	1.05	0.67	0.29
R45_20-min	27.71	27.46	27.44	28.40	27.50	28.31
R45_20-max	31.81	31.28	31.09	33.16	30.43	29.70
R45_100-mean	31.94	31.37	31.28	32.51	30.37	30.75
R45_100-std	1.32	1.22	1.18	1.42	0.92	0.31
R45_100-min	29.60	29.20	29.18	30.01	28.60	30.08
R45_100-max	35.46	34.61	34.32	36.29	32.60	31.54
R85_20-mean	30.19	29.77	29.73	31.32	28.46	29.14
R85_20-std	0.79	0.75	0.73	0.88	0.81	0.28
R85_20-min	28.32	27.99	27.94	29.23	27.02	28.58
R85_20-max	31.75	31.29	31.28	33.11	30.92	29.64
R85_100-mean	33.35	32.68	32.62	34.00	30.05	30.97
R85_100-std	1.09	1.03	1.00	1.13	1.35	0.32
R85_100-min	30.83	30.30	30.28	31.37	27.73	30.28
R85_100-max	35.54	34.72	34.60	36.39	33.85	31.52

ted due to space limitation.

CONCLUDING REMARKS

This paper has revealed that there are large uncertainties associated with the estimation of return values for high return periods in weather and climate data. It seems that GEV approach and MENU method give better results than the Gumbel approach, although it is difficult to single out one method or approach that is best overall. In any case, this is out of scope for the current study, and it should be further investigated by means of longer data records before a final conclusion has to be drawn.

In spite of the large variability of the different return value estimates, there seems to be evidence in the data for a slight increase in the extreme wave heights for both of the future scenarios that have been considered compared to the historical period. Hence, these data indicate that there might be a general trend towards more extreme wave events in a future climate. Obviously, this result is conditioned on the particular dataset analysed and the particular area in the North Atlantic Ocean. Further studies are needed in order to confirm or refute these trends.

Finally, the spatial variability of the extreme values for the specific area have been investigated. Fields of the extreme-value predictions have been produced, revealing that part of the uncertainties discovered can be attributed to the spatial variability of the specific dataset.

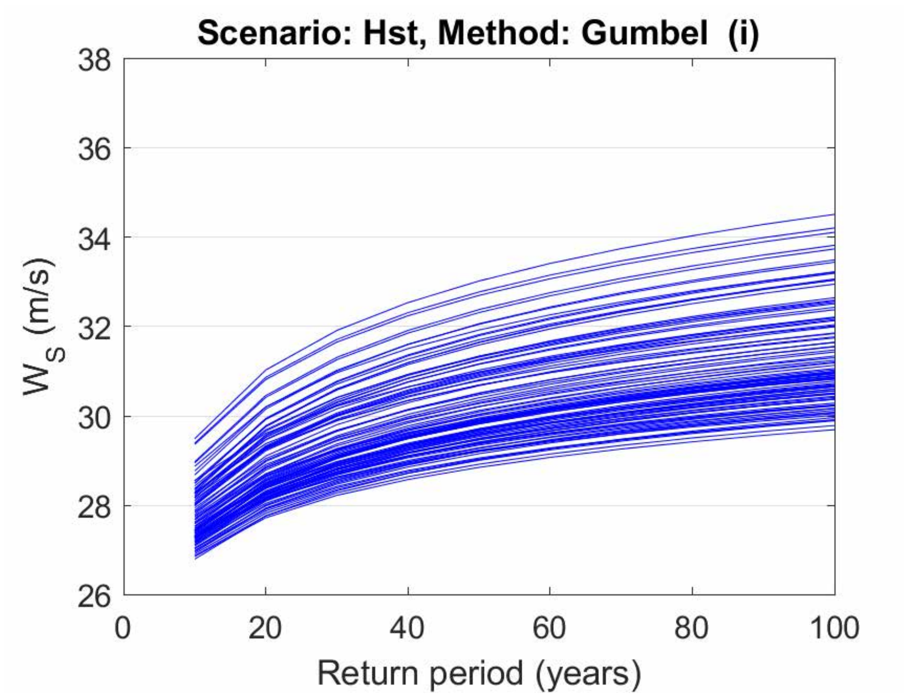

Figure 12: Gumbel (i)

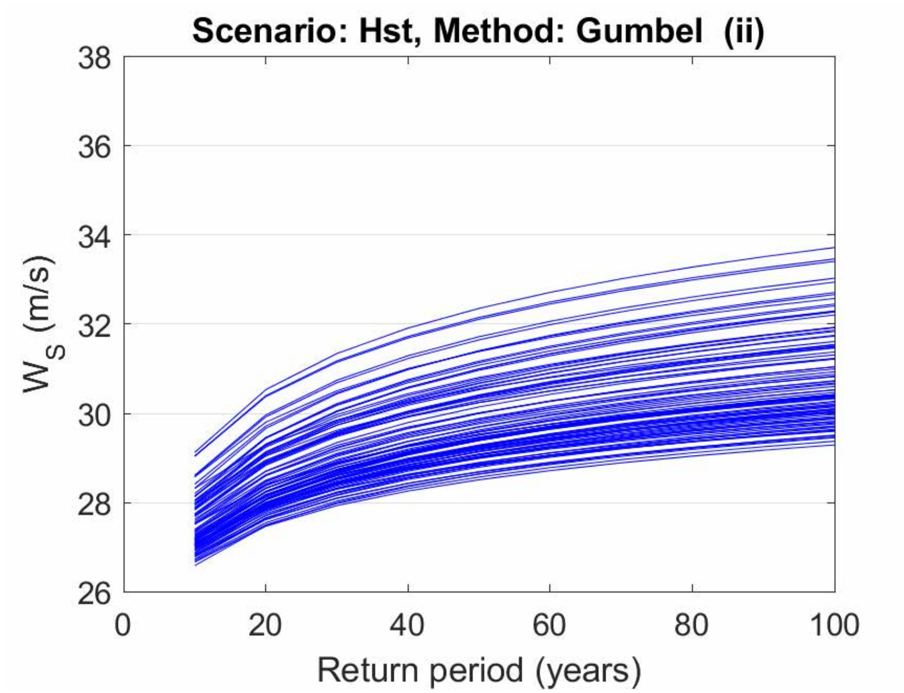

Figure 13: Gumbel (ii)

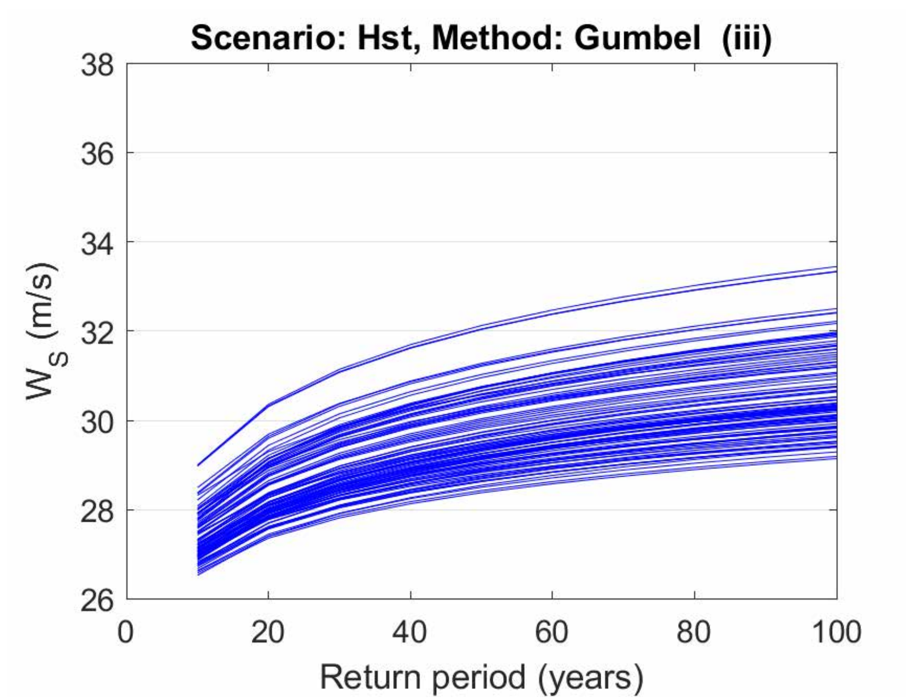

Figure 14: Gumbel (iii)

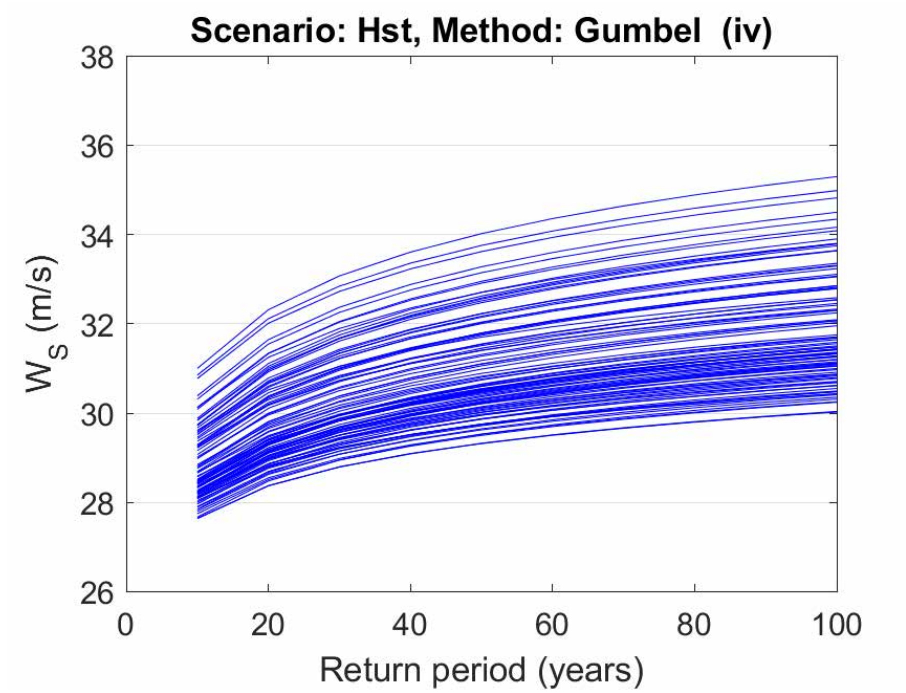

Figure 15: Gumbel (iv)

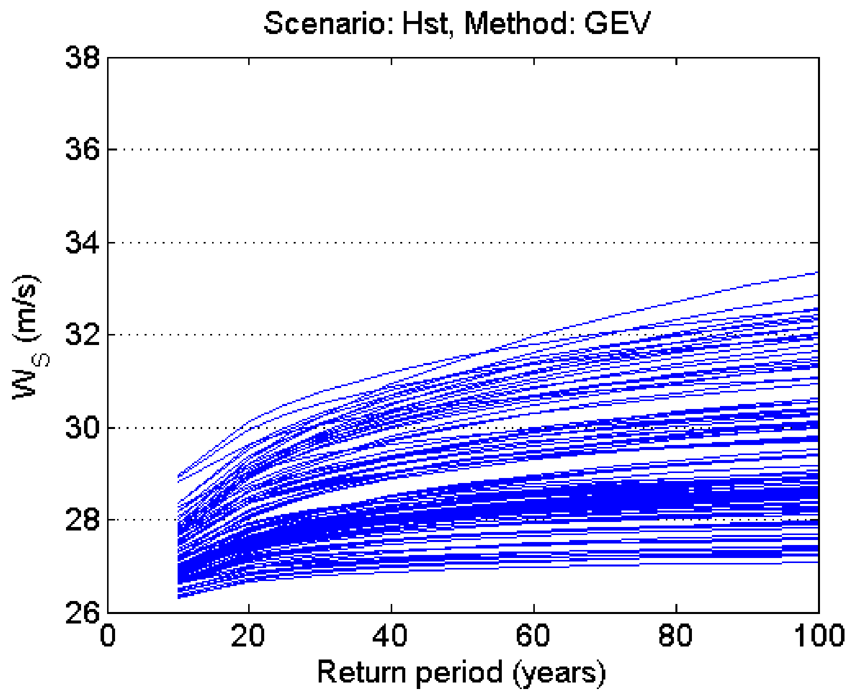

Figure 16: GEV

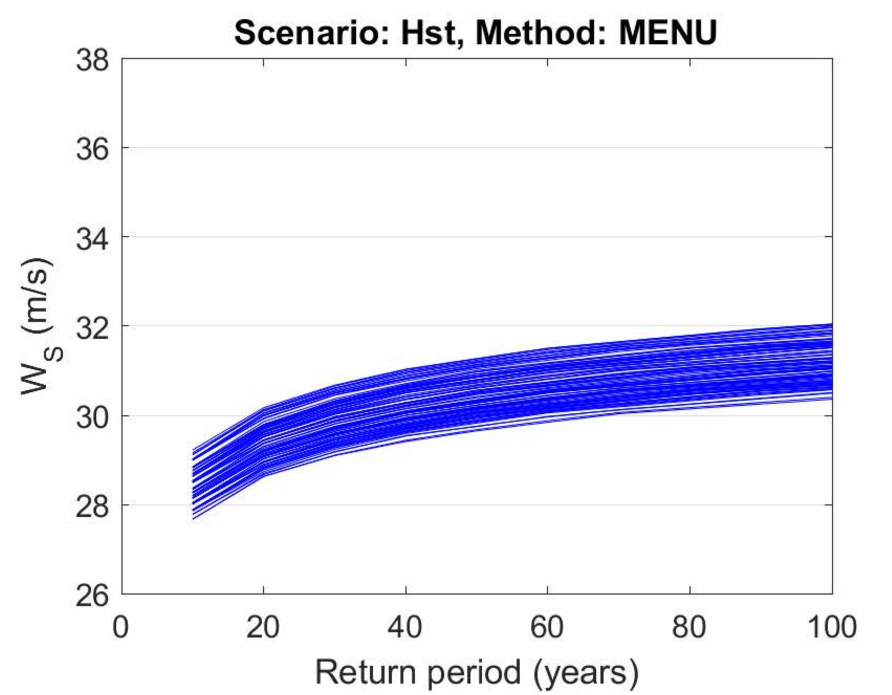

Figure 17: MENU

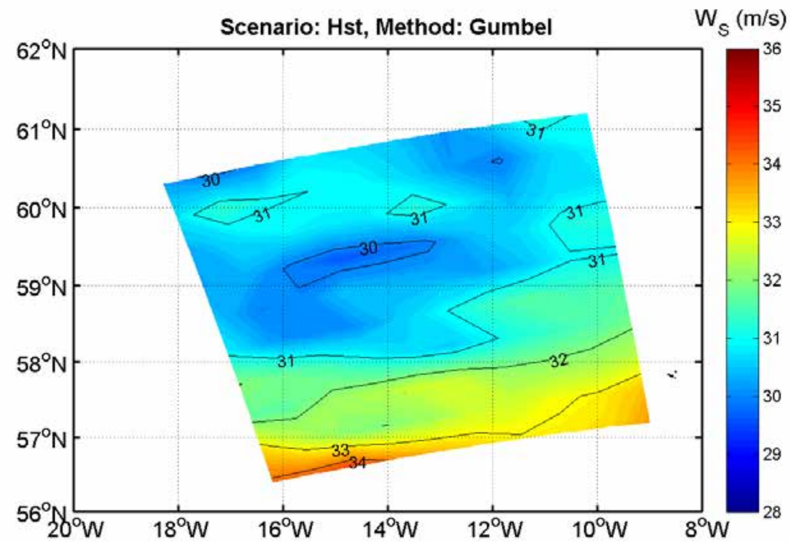

Figure 18: Gumbel field (100 years)

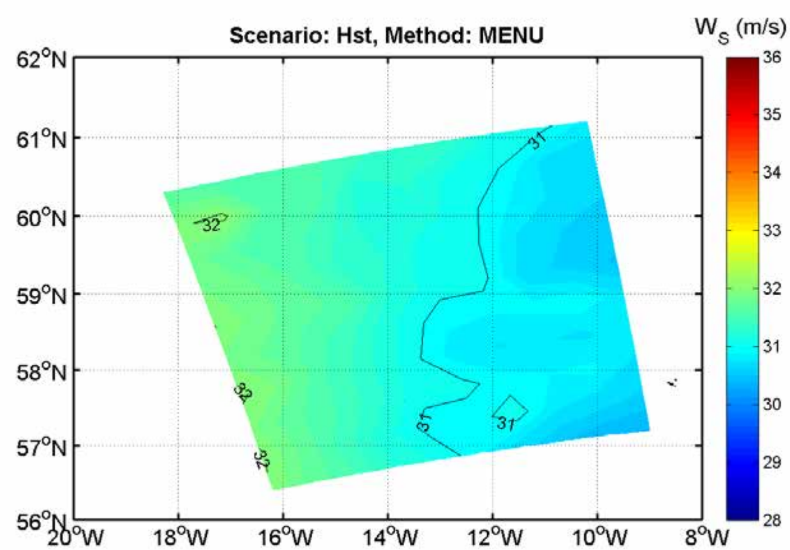

Figure 19: MENU field (100 years)

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