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EXECUTIVE SUMMARY

This report is the public deliverable from work package 7 (WP7) of the GARPUR project. Since 2013, the GARPUR project has been designing and developing new probabilistic reliability management approaches and criteria for transmission system planning and operation. As part of this work, the practical use of these reliability management concepts is being evaluated. The objective of WP7 has been to develop the GARPUR Quantification Platform (GQP) – a prototype software platform to allow quantitative comparison of different reliability management approaches and criteria via numerical simulation.

This report provides an overview of the features of such a quantification platform for comparison of reliability criteria for the operation of the transmission system. Therefore, a consistent framework is developed to bring together calculation modules of a state-of-the-art security constrained optimal power flow (SCOPF), a contingency probability calculation module, an interface to read CIM files (Common Information Model) and a module for representing reliability management processes. Furthermore, algebraic deterministic-equivalent mathematical models for reliability management in a day-ahead and real-time context are derived, implemented in AMPL, and explored computationally.

The first part of the report focuses on the development of the framework to support the comparison of reliability criteria, using a prototype quantification platform developed throughout the GARPUR project. In the second part, gaps between this prototype version and a version suitable for industrial use are identified, and a set of recommendations is developed for how to evolve into an industry-grade reliability management tool.

The GARPUR reliability management proposal is based on the general notion of the GARPUR reliability management approach and criterion (RMAC) developed in the work package in the project responsible for the development of new reliability criteria (WP2). In the first part of the present report, the concept of optimal power flow (OPF) is discussed, after which it is extended to security-constrained optimal power flow, which is then further extended with a modified version of the GARPUR RMAC ingredients, as per the scope of pilot testing. Afterwards, approximations and simplifications are developed, to deal with the computational challenges pertaining to this specific variant & implementation of the GARPUR RMAC[1]. This SCOPF is then used as a basis for simulation studies of day-ahead and real-time reliability management.

Next, two case studies are developed, for which a number of reliability criteria are compared. As the probabilities of N-k contingencies are usually orders of magnitude smaller than N-1, the associated risk of these are also very small and as such they do not incentivise to take more costly preventive actions. The benefit of using a probabilistic approach is more significant if probabilities of different contingencies significantly differ from each other. It should be emphasized that the tests carried out using the current implementation of the GARPUR RMAC-QP only address the system operation (i.e. short-term) aspects of reliability management. Furthermore, even if the results for the N-1 approach and the GARPUR RMAC-QP differ only in a limited way for most hours of the year, the cumulative benefits of the GARPUR RMAC-QP over a longer horizon could be significant.

[1] Henceforth denoted as RMAC-QP to avoid confusion.

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The inclusion of topological actions in the SCOPF increase the computation time significantly. In order to include topological actions in the SCOPF, better convexification techniques than the DC power flow approach need to be used in order to achieve feasibility of the nonlinear AC problem.

The computational challenges at hand are significant. As noted, a full GARPUR RMAC (AC) SCOPF implementation ticks a number of boxes which each have notable complexity:

1. large-scale due to the nature of the stochastic optimization model
2. with binary variables required in the modelling of the contingency discarding and the indicator variables for the corrective actions (and others),
3. with nonconvex constraints, due to the (AC) power flow physics.

These problems can be classified as large-scale mixed-integer optimal power flow problems. It is recommended that methods specific to such problems are to be studied in new research projects. We refer the reader to [GARPUR,2016c] for a discussion of a proof-of-concept algorithmic solution approach to the AC-SCOPF implementation of the GARPUR RMAC.

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1 TERMS AND DEFINITIONS

1.1 Abbreviations

AC	Alternating current
ACER	Agency for the Cooperation of Energy Regulators
AGC	Automatic generation control
AMPL	A Mathematical Programming Language
API	Application programming interface
B&B	Branch-and-bound
BFM	Branch Flow Model
BIM	Bus Injection Model
CIM	Common information model (here: for power systems)
CORESO	COoRdination of Electricity System Operators
CQCP	Convex quadratically-constrained programming
DA-RMAC	Day-ahead RMAC
DACF	Day-ahead congestion file
DC	Direct current
EMS	Energy management system
ENS	Energy not served
ENTSO-E	European network of transmission system operators for electricity
FACTS	Flexible Alternating Current Transmission System
GARPUR	Generally accepted reliability principle with uncertainty modelling and through probabilistic risk assessment
GQP	GARPUR quantification platform
H2020	Horizon 2020 EU research and innovation programme
HVDC	High-voltage direct current
IA	Innovation Action
IP	Interior-point
LPAC	Linear Programming AC
Max	maximize / maximum
MI	Mixed integer
Min	minimize / minimum
MIP	Mixed integer programming
MILP	Mixed integer linear programming
MISOCP	Mixed integer second-order cone programming
MICP	Mixed integer convex programming
MPEC	Mathematical programming with equilibrium constraints
MTTR	Mean time to repair

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NCQCP	Nonconvex quadratically-constrained programming
NLP	Nonlinear (here: ≈ nonconvex) programming
LP	Linear programming
OLTC	On-load tap changing transformer
OPF	Optimal power flow
PNS	Power not served
PST	Phase-shifting transformer
QP	Quadratic programming
RBTS	Roy Billinton test system
ref	Reference
RIA	Research and Innovation Action
RMAC-QP	Reliability management approach and criterion, quantification platform variant
RT-RMAC-QP	Real-time RMAC-QP
RTE	Réseau de Transport d'Électricité (TSO of France)
SCADA	System control and data acquisition
SCOPF	Security-constrained optimal power flow
SDP	Semidefinite programming
SOC	Second-order cone
SOCP	Second-order cone programming
s.t.	Subject to
ST	Short-term post-contingency (stage)
SVC	Static var compensator
TF	Transformer
TSC(NET)	Transmission system operator security cooperation
TSO	Transmission system operator
VOLL	Value of lost load
WP	Work Package

1.2 Symbols

It is noted that symbols (and equations) are given assuming SI units. Nevertheless, quantities through-out the document may be provided in more appropriate engineering units (e.g. kWh instead of J). Furthermore, there may be a nondimensionalization step (e.g. per unit conversion) before the problem is passed to the numerical solver.

P	Active power (W)
Q	Reactive power (var)
S	Apparent / complex power (VA)
U	Voltage magnitude (V)

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θ	Voltage angle (rad)
α	Line state (0/1)
i	Unit state (0/1)
φ	PST phase shift (rad)
ρ	Transformer voltage magnitude ratio (-)
y	Admittance (Siemens)
g	Conductance (Siemens)
b	Susceptance (Siemens)
z	Impedance (Ohm)
r	Resistance (Ohm)
x	Reactance (Ohm)
K	Cost (€)
π	Probability (-)
λ	Failure rate (1/s)
τ	Time (s)
$\mathcal{L}$	Set of lines (-)
$\mathcal{N}$	Set of nodes (-)
$\mathcal{U}$	Set of units (loads + generators) (-)
$\mathcal{C}$	Set of contingencies (-)
$\mathcal{T}$	Set of tuples describing line–node connectivity (uniquely) (-)
$\mathcal{G}$	Set of tuples describing unit-node connectivity (-)
l	Index for lines (-)
i	Index for nodes (-)
j	Second index for nodes (-)
u	Index for units (-)
c	Index for contingencies (-)
$[\quad] \vee [\quad]$	Model disjunction (-)
$\vee$	Logical or
$\wedge$	Logical and
$\{a, b\}$	Set of only a *and* b
$[a, b]$	Set of range from a *to* b, including a and b
(a, b)	Coordinate or tuple a, b
$\mathcal{A} \times \mathcal{B}$	Cartesian product of sets
$\lvert \mathcal{A} \rvert$	Cardinality (amount of elements) of set

1.3 Typography

$\color{red}{a}$	Generic parameter – known value (normal font, red)
$\color{blue}{a}$	Sizing parameter – known value (normal font, blue)
a	Variable – to be optimized (normal font, black)
$\color{red}{\mathcal{A}}$	Known set (calligraphic, red)
$\mathcal{A}$	Set – to be composed by optimization (calligraphic, black)

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1.4 Definitions

Contingency	A contingency is the unexpected failure or outage of a system component, such as a generator, transmission line, circuit breaker, switch, or other electrical element. A contingency may also include multiple components, which are related by situations leading to simultaneous component outages [ENTSO-E, 2004].
Corrective operation	In the real-time context, corrective operation concerns the application of post-contingency actions, in the aftermath of specific contingencies [GARPUR, 2016c].
Operational planning	Operational planning is the group of reliability management activities linked to system optimization occurring ahead of real-time operation, within the short-term and mid-term horizons.
Outage	An outage is the state of a component or system when it is not available to properly perform its intended function due to some event directly associated with that component or system [IEEE, 1997].
Power flow model	Set of equations describing the physics of power flow, considered only in steady-state throughout this work, derived from Kirchhoff's circuit laws and the conservation of energy. These equations can be used as derived or approximated.
Preventive operation	In the real-time context, preventive operation concerns the potential application of pre-contingency actions to achieve security and improve the ability to withstand the possible effects of potential contingencies. In the short-term context, preventive operation concerns the application of actions that apply to any realisation of the short-term uncertainty [GARPUR, 2016c].
Real-time horizon	The real-time horizon (system operation) in GARPUR focuses on the observed system state, i.e., it covers monitoring, control of the power system, and actions based on observed system state. Control covers corrective actions and activating manual preventive (planned) actions.
Reliability management	Power system reliability management means to take a sequence of decisions under uncertainty. It aims at meeting a reliability criterion, while minimising the socio-economic costs of doing so [GARPUR, 2015a].
Residual probability	The aggregate probability of all discarded and/or not explicitly modelled events in a reliability assessment.
Residual risk	The aggregate risk of all discarded and/or not explicitly modelled events, as defined [GARPUR, 2016c].
Socio-economic surplus	The sum of surplus or utility of all stakeholders, including external costs and benefits (e.g. environmental costs) as defined in [GARPUR, 2016a].
Trajectory	A sequence of events affecting the state of the transmission system, such as contingencies, system response, and corrective control, over multiple operational periods.

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2 INTRODUCTION

This report is the public deliverable from work package 7 (WP7) of the GARPUR project. Since its initiation in 2013, the GARPUR project has been designing and developing new, probabilistic reliability criteria and approaches with the aim of maximizing social welfare. As part of this work, the practical use of these reliability management concepts is being evaluated. The objective of WP7 has been to develop the GARPUR Quantification Platform (GQP) – a prototype software platform to allow quantitative comparison of different reliability criteria via numerical simulation of the reliability management process.

'N-1' is the term used for the conventional power system reliability criterion currently used in the reliability management of power systems. As described in previous work in the GARPUR project [GARPUR, 2014a], [GARPUR, 2014b], transmission system operators (TSOs) have different practical implementations of the N-1 criterion. However, broadly speaking, the conventional reliability criterion is a deterministic criterion in which contingencies involving a single system component (N-1) are considered in the reliability management process.

Throughout the GARPUR project, the GARPUR Reliability Management Approach and Criterion (RMAC) has been developed as a general mathematical formalization of reliability management, formulated for the GQP as a Security-Constrained Optimal Power Flow (SCOPF) problem. The main ingredients of the GARPUR RMAC are described in [GARPUR, 2016c] and reproduced in Appendix 12.1. In the context of a reliability management through SCOPF tools, the main ingredients of the GARPUR RMAC with respect to conventional reliability criteria are the following:

- moving to a probabilistic and risk-based approach, i.e., taking into account probabilities of contingencies and using a compound socio-economic objective function blending the costs of TSO preventive and corrective actions with a monetization of the risk of service interruptions;
- covering in an adaptive way a variable set of contingencies well approximating the incurred risk of service interruptions, rather than a fixed set of N-1 contingencies;
- ;
- relaxing infeasible post-contingency trajectories, while considering the corresponding risk increase;
- including the risk of failure of corrective actions, which may cause infeasible post-contingency trajectories;
- limiting the probability of occurrence of an infeasible trajectory.

The GARPUR quantification platform is used to represent combinations of the ingredients listed above. In this way, new reliability criteria including different aspects of the GARPUR RMAC can be compared with each other and with the conventional N-1 criterion.

This report provides a detailed presentation of the SCOPF that is formulated for and implemented in the GQP. Using this formulation and a set of case studies, the report gives a broader comparison of different reliability criteria. It furthermore discusses the use of the GARPUR Quantification Platform and the lessons learned. Finally, recommendations are given on how to evolve the GQP into a tool suitable for industrial use (i.e. an industry grade tool). To provide the necessary context, the remainder of this chapter will introduce the concept of an SCOPF and discuss the purpose of the GQP.

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2.1 Security-constrained optimal power flow

Security-constrained optimal power flow is a concept underlying reliability management for future power systems in real-time operation and day-ahead planning. Alsaç and Stott pioneered the concept of SCOPF in 1973 [Alsaç,1973], and SCOPF has been a topic of research in recent EU projects such as Pegase and iTesla [PegaseD3.1], [iTesla,2015]. Recent developments are discussed in [Capitanescu,2015], [Capitanescu,2011].

Optimal power flow (OPF) refers to a class of optimization problems subject to the physical power flow model of a power grid. The power flow model can be an exact (AC) or can be based on a valid approximation. For a real-life system, the AC optimal power flow problem is a nonconvex non-linear program, and its computational tractability may present considerable challenges.

Contingency-constrained OPF or security-constrained OPF refers to a class of OPF problems that minimize the cost or risk of the operation of a power system over a set of contingencies. A contingency is the unexpected failure or outage of one or multiple system component. In other words, contingencies are the unexpected unavailability of elements of this system such as generators, lines, PSTs, TFs, etc. A conventional SCOPF, minimizing only preventive costs represents the conventional N-1 reliability criterion in which the probability of the contingencies is not explicitly considered. Even a conventional SCOPF may be computationally very demanding for a realistic system and including a considerable number of contingencies.

A SCOPF can be viewed more generally as a stochastic programming problem over a set of contingency scenarios. Such stochastic problems can be considered with and without recourse. Without recourse, the objective of the SCOPF is to decide on an operational unit dispatch and grid configuration that can withstand any individual contingency within the set of contingencies. With recourse (two-stage decision-making), it is also allowed to take corrective actions following a realization. This may make the overall system cost (including the corrective risk) lower, as the risk related to a contingency does not need to hedged against exclusively preventively.

In terms of its applications, SCOPF may become an important tool for TSOs for both operational and planning purposes [Capitanescu,2011]. Some TSOs have reported on their experience with AC SCOPF tools used for near real-life reliability management [López,2015], but on the whole, few TSOs currently seem to have implemented AC SCOPF [GARPUR,2016d]. A survey of some commercially available SCOPF tools is presented in [Sperstad,2016]. In the Pegase and iTesla projects, SCOPF has been a topic of research; the main contributions and conclusions of these projects are listed in Table 1. It can be highlighted that the treatment of binary (or integer) variables presents one of the major outstanding research challenges for application of SCOPF to practical problems.

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Table 1: Summary of lessons learned in the context of SCOPF in Pegase and iTesla

Pegase	iTesla
• Generally, AC OPF with binary variables is hard to tackle computationally • Rounding of small discrete steps (e.g. taps) • Approaches based on MPEC (math programming with equilibrium constraints) were experimented with, but scalability remained challenging	• Risk-based SCOPF is superior to only-preventive-cost SCOPF • Experiments with topological actions using convex relaxations formulations • Start from high-quality candidate topologies • Sequential fixing of variables: PST → generation → shunts

2.2 GARPUR Quantification Platform

The GARPUR Quantification Platform (GQP) allows comparison of different reliability management approaches via numerical simulations of their application in different contexts. This approach allows to appraise the socio-economic impact if new reliability management strategies were used instead of the current N-1 approaches. The prototype version of the platform is designed to cover day-ahead and real-time operations.

In GARPUR, the purpose of the GQP primarily is to evaluate and compare different reliability management approaches and criteria. The SCOPF implemented as part of the GQP thus serves to represent and emulate parts of the reliability management process for this purpose. However, one could also envision the GQP SCOPF be used as a tool for the purpose of providing decision support for operational reliability control. It is noted that the quantification platform is much broader in scope than a SCOPF tool: the steps of the reliability management process prior to those represented by the SCOPF are also considered as part of this quantification approach. For instance, pre-processing of input data and post-processing of results, deriving of indicators, modelling of contingencies, etc.

In GARPUR, the GQP has been designed as a general-purpose platform for evaluating different reliability criteria. At the same time, it has throughout the project also been developed more specifically with its application to specific pilot tests in mind. In work package 8 (WP8) of GARPUR pilot tests of the new proposed reliability criteria are performed, by French transmission system operator RTE, using the GQP and focusing on a part of the French control zone. Reliability criteria are compared and aim to establish the robustness of the results of the GQP.

2.3 Scope and structure

This report aims to:

- provide an overview of features of a quantification platform for the comparison of reliability criteria, including detailed specifications for the implementation of:
    - a state-of-the-art SCOPF;
    - contingency probability calculation;
    - reliability management approaches covering multiple decision stages;
- derive algebraic and deterministic-equivalent mathematical models of these features;
- develop case studies to demonstrate the developed functionality;

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- propose recommendations to further evolve the GQP implementation to an industry-grade level.

However, it does not aim to:
- be a complete specification of the implementation of the GQP;
- encompass all theoretical aspects of reliability management; the reader is referred to GARPUR deliverables published on the website [GARPUR,2017].

The aim of the derivation of the mathematical models is to encompass the requirements of:
- conventional N-1 SCOPF;
  - at least supporting the 'DC' OPF formulation;
- real-time RMAC-QP ingredients as adapted in work package 6 of GARPUR ('System operation');
- day-ahead RMAC-QP ingredients of WP6;
- RTE GQP-based pilot test of WP8.

Finally, this report is organized as follows:
- chapter 1 lists abbreviations, terms and symbols;
- chapter 2 introduces the topics of study of this report and delineates the scope;
- chapter 3 sets up a SCOPF framework for reliability criteria;
- chapter 4 analyses tractability of this framework, and develops a number of approximations to trade off accuracy and calculation requirements;
- chapter 5 discusses how this SCOPF framework can be applied in the context of day-ahead and real-time reliability management;
- chapter 6 describes the implementation aspects of the previously developed approaches;
- chapter 7 compares and analyses a variety of criteria, using academic test cases;
- chapter 8 suggests recommendations for how to evolve to an industry-grade reliability management tool from the basis described in this deliverable;
- chapter 9 provides the general conclusions;
- chapter 10 provides a list of references;
- chapters 11 and 12 are the appendices, providing supplementary background to the reader.

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3 RELIABILITY CRITERIA IN A SCOPF FRAMEWORK

This chapter aims to provide insight into how to fit the GARPUR RMAC ingredients together, while building upon the ideas underlying SCOPF and multi-stage stochastic programming, and the original work of implementing the RMAC ingredients in SCOPF formulations for real-time operation and short-term operational planning discussed in [GARPUR,2016c, Karangelos,2016, Karangelos,2017]. Therefore, an extensive symbol list is developed, which is later re-used in the context of the definition of the reliability criteria as represented in the SCOPF. However, the aim is not to develop a complete implementation guide for the SCOPF or to list all equations involved exhaustively, deriving them from first principles.

Throughout this chapter, the complexity is gradually increased. First, the concept of optimal power flow (OPF) is discussed in Section 3.2, after which it is extended to security-constrained optimal power flow in Section 3.3, which is then further extended with the GARPUR RMAC-QP ingredients specific to the SCOPF in Section 3.4. In the next chapter (Chapter 4), approximations and simplifications are developed, to deal with the computational challenges at hand.

3.1 Preliminaries to the GQP SCOPF framework

Throughout this chapter, it will be assumed that a 'conventional' SCOPF has the following features:

- AC optimal power flow formulation
    - Support for parallel lines
- Variety of loads / generators
    - Multiple per node
    - Marginal costs used to calculate dispatch costs
- Two stages: preventive and corrective
    - A single preventive stage
    - Each contingency corresponds to a corrective stage
- Minimization of preventive costs as objective
    - No cost related to corrective actions
- Voltage-regulation capable sources such as generators, SVCs, Statcoms
    - PQ/PV bus definition
- Ability to define participation of generators & PSTs in actions in preventive and corrective stages separately

The detailed aspects of (the implementation of) these features are left to the reader and the literature [Pegase D3.1], [Pegase D3.2], [GARPUR, 2016c].
Historically, SCOPF implementations have considered limited subsets of the following features:

- Objective function considering corrective control costs and risk of service interruptions [Capitanescu,2015b], [Karangelos,2016]
- Risk of failure of corrective actions [Karangelos,2013]
- Short-term post-contingency system stage [Capitanescu,2015b]
- Contingency relaxation, when contingencies are too difficult or expensive to secure against and need to be discarded. [Karangelos,2016]
- Discrete actions:
    - Line & breaker switching [Henneaux,2016], [Kocuk, 2015]
    - PST actions [Guha, 2015]

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- Generation start-up and shutdown [Fernandez,2017]
- Indicator variables for actions, used in discrete cost models for operator actions
    - E.g. to support PST activation, switching, generation start-up & shutdown

These aspects will be the focus of the derivations made in this document.

3.2 Conventional OPF

This section gives a mathematical description of a conventional AC OPF, with switching, voltage magnitude tap changing and phase shifting, on the basis of which the GARPUR extensions will be detailed.

3.2.1 Grid element model

The symbols in the remainder of the document are derived on the basis of the line model depicted in Figure 1. The line parameters are summarized in Table 20 in Appendix I. The used line model is a unified extended branch model based on [Andersson,2004]. It uses a symmetrical pi-section representation including lossless switches and ideal complex valued transformers (to model phase angle shift and voltage magnitude ratios).

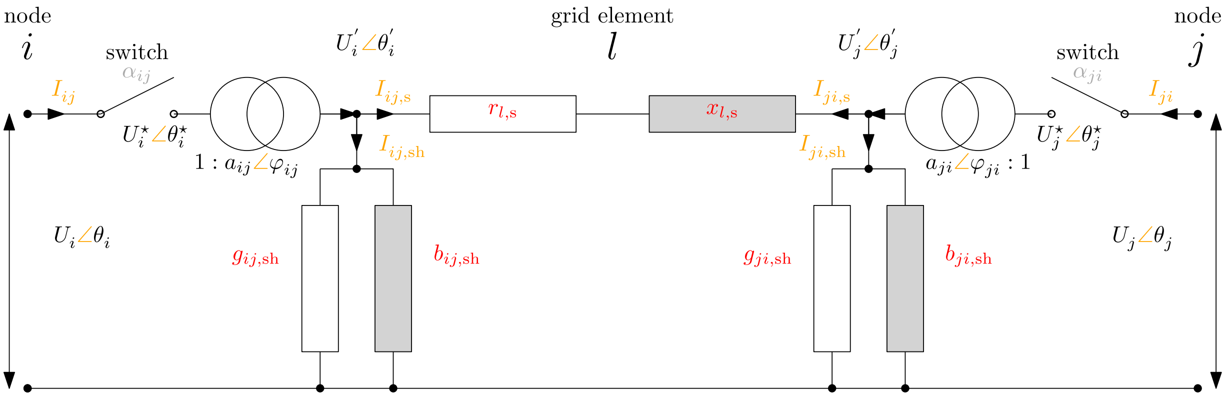

Figure 1: Extended line model

The OPF formulation is provided in Appendix I including the representation of units, lines, switches and PSTs. The sets and indices lines, units, stages and contingencies are defined in Table 2 and are used in these formulations and will be referenced in the upcoming chapters.

Table 2: Overview of sets and indices

Entity	Symbols
Lines	$l \in \mathcal{L}$
Nodes	$i, j \in \mathcal{N}$
Units	$u \in \mathcal{U}$
Contingencies	$c \in \mathcal{C}$
Grid topology (unique, original)	$lij = (l, i, j) \in \mathcal{T} \subset \mathcal{L}\times\mathcal{N}\times\mathcal{N}$
Grid topology (reversed)	$\mathcal{T}^{\text{reversed}} = \{(l, j, i) \mid \forall(l, i, j) \in \mathcal{T}\}$
Grid topology bidirectional	$\mathcal{T}^{\text{bidir.}} = \mathcal{T}^{\text{rev.}} \cup \mathcal{T}$
Unit connection	$ui = (u, i) \in \mathcal{G} \subset \mathcal{U}\times\mathcal{N}$
Stages	$\{'', 'ST', 'corr'\}$

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The preventive stage is not indicated explicitly in the variable symbols.

3.3 SCOPF

3.3.1 Overall formulation

The overall structure of the SCOPF problem in the framework of the GQP can be summarized as follows:

input parameters	• Parameters to initialize grid elements and topology, loads and generators • Parameters indicating participation in preventive and corrective actions, and AGC
minimize	• Total risk = Preventive cost + corrective risk + blackout risk
decision variables	• Line, PST, switch power flow and state + state change
	• PST shift + shift change
	• Load and generation dispatch and state + dispatch change + state change
	• Dispatch, redispatch, switching and shifting costs
subject to	• OPF and unit model replicated for each stage and contingency: preventive – short-term post-corrective – corrective
	• Nodal balance: power flow – load – generation – bus shunt
	• Power flow equations
	• Load and generation flexibility model
post	• Assess approximation error if simplifications of the model were performed

A visualization of this SCOPF framework is given in Figure 2.

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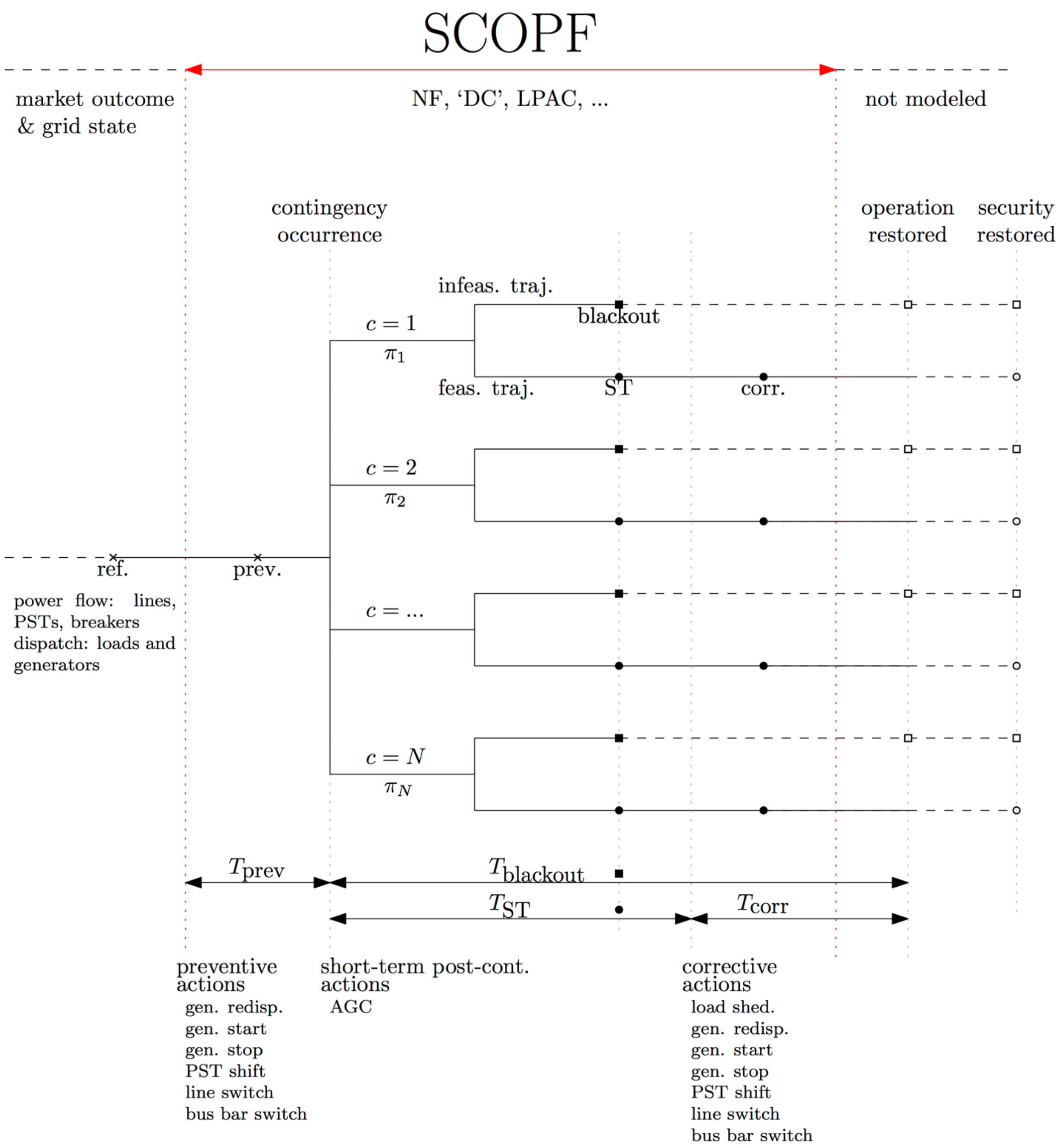

Figure 2: Structure of the GQP SCOPF framework

3.3.2 Decision-making stages

The SCOPF framework is developed to encompass *both* day-ahead and real-time reliability management. Therefore, the stages are abstracted as follows:

- The reference stage describes an expected situation of the system under consideration. E.g. this is the post market-clearing dispatch and DACF.
- The preventive stage is identical to that of the conventional preventive SCOPF. The outcomes of the preventive stage are the set-points that are planned to be implemented.

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- The short-term post-contingency stage is a simulation of what happens in the preventive stage after the occurrence of the contingency, but following automatic actions. Here AGC is considered as an automatic action. This stage has power flow limits which are less strict than other stages, and therefore the system operation in the longer term may not be entirely secure. It is stressed that there is no freedom (slack) in the actions (decisions) in this stage, i.e. the actions are fully defined by the preventive set points, as they are automatic.
- The corrective stage simulates how to return the system to secure operation after the short-term post-contingency stage. This stage has a unique set of operator decisions for each contingency scenario.
- The SCOPF is solved for each time step (hour) considered, and each time step is treated independently (no time-coupling constraints)

One of the main challenges is balancing the preventive and corrective decisions, as the preventive costs are committed costs, whereas the corrective costs are only known following a realization.

3.3.3 Contingencies

3.3.3.1 Definition of contingency sets

Contingencies are considered as realizations of a set of unavailabilities of both units, i.e. loads and generators, and lines, i.e. lines, switches, TFs and PSTs. Not all contingencies are explicitly considered in a SCOPF, due to the enormous size of the contingency set of all possible contingencies $\mathcal{C}$. This contingency set is composed of the expected no-contingency state ($\mathcal{C}^{\mathrm{N-0}}$) together with the N-1 ($\mathcal{C}^{\mathrm{N-1}}$) and all N-k contingencies:

$$\mathcal{C} = \mathcal{C}^{\mathrm{N-0}} \cup \mathcal{C}^{\mathrm{N-1}} \cup \mathcal{C}^{\mathrm{N-2}} \cup \ldots \cup \mathcal{C}^{\mathrm{N-N}}.$$

To keep the problem size manageable, a subset of contingencies is selected $\mathcal{C}^{\mathrm{selected}}$ which implies that other contingencies are immediately discarded:

$$\mathcal{C} = \mathcal{C}^{\mathrm{selected}} \cup \mathcal{C}^{\mathrm{not\ selected}}.$$

This selection can be made across the N-k categories. A selection of contingencies $\mathcal{C}^{\mathrm{prob}}(\pi^{\mathrm{min}}, \mathcal{C})$ out of the set $\mathcal{C}$ with a probability cut-off π^{min} could be described as:

$$\mathcal{C}^{\mathrm{prob}}(\pi^{\mathrm{min}}, \mathcal{C}) = \{c \in \mathcal{C} \mid \pi_c \geq \pi^{\mathrm{min}}\}.$$

The overall set of all these selected contingencies is $\mathcal{C}^{\mathrm{selected}}$, which can be secured against or not:

$$\mathcal{C}^{\mathrm{selected}} = \mathcal{C}^{\mathrm{secure}} \cup \mathcal{C}^{\mathrm{insecure}}.$$

However, this process of choosing which contingencies to secure against is part of the contingency discarding that is performed by the GARPUR approach.

The set of secured contingencies itself can be further divided into a set of purely preventively secured contingencies ($\mathcal{C}^{\mathrm{prev}}$) and a set of contingencies which is preventively-correctively secured ($\mathcal{C}^{\mathrm{prev-corr}}$):

$$\mathcal{C}^{\mathrm{secure}} = \mathcal{C}^{\mathrm{prev}} \cup \mathcal{C}^{\mathrm{prev-corr}}.$$

Overall the contingency set structure is therefore:

$$\mathcal{C} = \underbrace{\mathcal{C}^{\mathrm{prev}} \cup \mathcal{C}^{\mathrm{prev-corr}}}_{\mathcal{C}^{\mathrm{secure}}} \cup \underbrace{\mathcal{C}^{\mathrm{insecure}} \cup \mathcal{C}^{\mathrm{not\ selected}}}_{\mathcal{C}^{\mathrm{discarded}}}$$

In a conventional N-1 SCOPF, all the selected contingencies $\mathcal{C}^{\mathrm{selected}} = \mathcal{C}^{\mathrm{N-0}} \cup \mathcal{C}^{\mathrm{N-1}}$ are also secured against, and therefore $\mathcal{C}^{\mathrm{insecure}} = \{\}$.

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3.3.4 Stage variables

To clarify the implementation and simplify the reading of this document, separate symbols are defined for the parameters and variables in the stages discussed (Table 3). The relationship between the stage variable for units and lines are provided in Appendix I.

Table 3: Definition of SCOPF symbols across stages

		Reference	Δ	Preventive	Δ	Short-term post-contingency	Δ	Corrective
Units	Active dispatch	P_u^{ref}	P_u^{Δ}	P_u	$P_{u,c}^{\text{ST}\Delta}$	$P_{u,c}^{\text{ST}}$	$P_{u,c}^{\Delta}$	$P_{u,c}^{\text{corr}}$
	Reactive dispatch	Q_u^{ref}	Q_u^{Δ}	Q_u	$Q_{u,c}^{\text{ST}\Delta}$	$Q_{u,c}^{\text{ST}}$	$Q_{u,c}^{\Delta}$	$Q_{u,c}^{\text{corr}}$
	State	i_u^{ref}	i_u^{Δ}	i_u	$i_{u,c}^{\text{ST}\Delta}$	$i_{u,c}^{\text{ST}}$	$i_{u,c}^{\Delta}$	$i_{u,c}^{\text{corr}}$
	Dispatch cost	K_u^{ref}		K_u^{disp}				$K_{u,c}^{\text{disp}}$
	Redispatch cost		K_u^{redisp}				$K_{u,c}^{\text{redisp}}$	
	Start/stop cost		$K_u^{\text{startstop}}$				$K_{u,c}^{\text{startstop}}$	
Line	State	α_l^{ref}	α_l^{Δ}	α_l		$\alpha_{l,c}^{\text{ST}}$	$\alpha_{l,c}^{\Delta}$	$\alpha_{l,c}^{\text{corr}}$
	Phase shift	φ_l^{ref}	φ_l^{Δ}	φ_l		$\varphi_{l,c}^{\text{ST}}$	$\varphi_{l,c}^{\Delta}$	$\varphi_{l,c}^{\text{corr}}$
	Switch cost		K_l^{switch}				$K_{l,c}^{\text{switch}}$	
	Shift cost		K_l^{shift}				$K_{l,c}^{\text{shift}}$	
Line	Flow	P_{lij}^{ref}		P_{lij}		$P_{lij,c}^{ST}$		$P_{lij,c}^{\text{corr}}$
		Q_{lij}^{ref}		Q_{lij}		$Q_{lij,c}^{ST}$		$Q_{lij,c}^{\text{corr}}$
	Rating	S_{lij}^{rated}		S_{lij}^{rated}		$S_{lij}^{\text{rated,ST}}$		S_{lij}^{rated}
		I_{lij}^{rated}		I_{lij}^{rated}		$I_{lij}^{\text{rated,ST}}$		I_{lij}^{rated}
Unit		U_i^{ref}		U_i		$U_{i,c}^{\text{ST}}$		$U_{i,c}^{\text{corr}}$
	Rating	S_u^{rated}		S_u^{rated}		S_u^{rated}		S_u^{rated}
Node	Rating	U_i^{rated}		U_i^{rated}		U_i^{rated}		U_i^{rated}
		$U_i^{\max}$		$U_i^{\max}$		$U_i^{\max,\text{ST}}$		$U_i^{\max}$
		$U_i^{\min}$		$U_i^{\min}$		$U_i^{\min,\text{ST}}$		$U_i^{\min}$
Contingency	Discarding	-		-		β_c		β_c
Cost			K^{prev}			K_c^{ST}		K_c^{corr}
Duration				T		T^{ST}		T^{corr}

3.4 GARPUR's SCOPF extensions

A 'big M' formulation corresponding to the disjunctive formulation is developed. Unless otherwise noted, the unit and power flow models are identical for each stage and contingency.

3.4.1 Unit dispatch

The active and reactive power dispatch of loads and generators are P_u, Q_u and the rated power is S_u^{rated}.

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A generator's or load's state is either on or off ($i_u \in \{0,1\}$). If the unit is on, it operates between its minimum and maximum operational active and reactive power limits ($P_u^{\max}, P_u^{\min}, Q_u^{\max}, Q_u^{\min}$). If a unit needs to start up or shut down, a cost is assigned.

3.4.2 Switching actions

Topological actions are modelled using lossless switches as a part of the line model (Figure 1). The switch state variable is a binary variable $i_l \in \{0,1\}$. The cost of operation switches can be incorporated. Using a cost for operating switches often improve calculation speed. As TSOs consider switching actions usually as free, in practice this cost should be very small compared to e.g. generation redispatch cost.

3.4.3 PST actions

The PST actions are only possible when the line is in operation ($\alpha_l = 1$). The effective PST shift $\varphi_{l,ij}$ remains between $\varphi_{l,ij}^{\min}$ and $\varphi_{l,ij}^{\max}$. A binary indicator variable μ_l is used to signal the activation of PST actions. Using a cost for PST actions often improves calculation speed in combination with binary indicator variables. As TSOs consider PST actions usually as free, in practice this cost should be very small compared to e.g. generation redispatch cost.

3.4.4 AGC actions

AGC actions are used to model the short-term post-contingency stage. AGC actions account for the power generation lost due to a contingency in re-establishing the short-term post-contingency power balance, e.g. through frequency control mechanisms. For each contingency, the active and reactive power immediately following the contingency c are $P_c^{\text{ST,lost}}, Q_c^{\text{ST,lost}}$. Proportionality factors are used to determine the participation of operating generators in the AGC action based on their actual generation value.

3.4.5 Indicator variables for corrective actions

Binary indicator variables are defined to signal the corrective actions of units, lines and PSTs. These variables are then further used in the modelling of the failure of corrective actions.

3.4.6 Contingency discarding

The indicator variable for the inclusion or discarding of a contingency is β_c. If a contingency is included, the power flow model of this contingency must be satisfiable. Conversely, if the power flow trajectory post-contingency and correctively is infeasible, the contingency must be discarded to maintain an overall feasible SCOPF problem.

$$\begin{bmatrix} \text{Power flow model for } c \\ \text{Flow and voltage bounds} \\ \beta_c = 0 \text{ (included)} \end{bmatrix} \vee \begin{bmatrix} \text{No power flow model for } c \\ \text{No bounds} \\ \beta_c = 1 \text{ (discarded)} \end{bmatrix} \beta_c \le \beta^{\max}.$$

Contingency discarding can be disabled using a parameter $\beta^{\max} \in \{0,1\}$, resulting in a conventional SCOPF constrained by the set of preselected contingencies: $\beta_c \le \beta^{\max}$.

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3.4.7 Optimized contingency sets

The set of contingencies not-secured therefore is $\mathcal{C}^{\text{insecure}} = \{c \in \mathcal{C}^{\text{selected}} | \beta_c = 1\}$.
If contingency discarding is not allowed, a conventional preventive-corrective SCOPF is solved and therefore $\mathcal{C}^{\text{insecure}} = \{\}$. The set of purely preventively secured contingencies therefore is:

$$\mathcal{C}^{\text{prev}} = \left\{ c \in \mathcal{C}^{\text{selected}} \middle| \underbrace{\beta_c = 0}_{\substack{\text{contingency} \\ \text{secured}}} \wedge \sum_{u \text{ in } \mathcal{U}} \underbrace{\nu_{u,c}^{\text{corr}} = 0}_{\substack{\text{no corr.} \\ \text{unit action}}} \wedge \sum_{l \text{ in } \mathcal{L}} \underbrace{\nu_{l,c}^{\text{corr}} = 0}_{\substack{\text{no corr.} \\ \text{line action}}} \right\},$$

namely, the contingencies which are not discarded, and for which no corrective line or unit actions are taken. The contingencies preventively-correctively secured are now derived as:

$$\mathcal{C}^{\text{prev-corr}} = \mathcal{C}^{\text{selected}} \backslash (\mathcal{C}^{\text{prev}} \cup \mathcal{C}^{\text{insecure}}) \,.$$

3.4.8 Failure of corrective actions

The probability of failure of corrective actions π_c^{fail} is conditional on the occurrence of an action and can be estimated based on historic failure probability data.

3.4.9 Per-contingency probability of blackout

The probability of blackout is either determined by the failure of corrective actions, or by the discarding of a contingency:

$$\begin{bmatrix} \pi_c^{\text{blackout}} = \pi_c \\ \nu_{l,c}^{\text{corr}} = \nu_{u,c}^{\text{corr}} = 0 \text{ (no corrective actions)} \\ \beta_c = 1 \text{ (included)} \end{bmatrix} \vee \begin{bmatrix} \pi_c^{\text{blackout}} = \pi_c \cdot \pi_c^{\text{fail, unstable}} \cdot \pi_c^{\text{fail}} \\ \nu_{l,c}^{\text{corr}}, \nu_{u,c}^{\text{corr}} \geq 0 \text{ (corrective actions)} \\ \beta_c = 0 \text{ (discarded)} \end{bmatrix}.$$

In general, the failure of corrective actions does not need to lead to a blackout, but may often lead to an insecure state. Actions may exist to return to a secure operational state. Therefore, there is a probability that the failure of corrective actions actually leads to a blackout $\pi_c^{\text{fail, unstable}}$. In the worst case, $\pi_c^{\text{fail, unstable}} = 1,$ which assumes that every failure of corrective actions leads to a blackout state.

3.4.10 Reliability target

In the GQP SCOPF a specific version of the GARPUR RMAC reliability target was implemented, expressing an upper bound on the probability to experience “blackout” as defined above:

$$\sum_{c \in C} \pi_c^{\text{blackout}} \leq \pi^{\text{blackout,max}}.$$

The setting $\sigma^{\text{rel.target}} \in \{0,1\}$ can be used to remove this constraint:

$$\sigma^{\text{rel.target}} \cdot \sum_{c \in C} \pi_c^{\text{blackout}} \leq \pi^{\text{blackout,max}}.$$

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3.4.11 Cost and risk components

The total risk K^{total}, i.e. the objective, is composed of three components, namely, preventive cost, corrective risk and blackout risk, which can each be independently considered using parameters $k^{\text{prev}}, k^{\text{corr}}, k^{\text{blackout}} \in \{0,1\}$:

$$\min K^{\text{total}}$$

$$K^{\text{total}} = k^{\text{prev}} \cdot K^{\text{prev}} + k^{\text{corr}} \cdot \sum_{c\in C} \pi_c \cdot K_c^{\text{corr}} + k^{\text{blackout}} \cdot \sum_{c\in C} K_c^{\text{blackout}}.$$

The preventive and corrective cost terms are designed identically:

$$K^{\text{prev}} = \sum_{u\in\mathcal{U}} \underbrace{k^{\text{redisp}} \cdot K_u^{\text{redisp}} + k^{\text{disp}} \cdot K_u^{\text{disp}} + k^{\text{startstop}} \cdot K_u^{\text{startstop}}}_{\text{unit costs}} + \sum_{l\in\mathcal{L}} \underbrace{k^{\text{PST}} \cdot K_l^{\text{PST}} + k^{\text{switch}} \cdot K_l^{\text{switch}}}_{\text{line costs}},$$

$$K_c^{\text{corr}} = \sum_{u\in\mathcal{U}} \underbrace{k^{\text{redisp}} \cdot K_{u,c}^{\text{redisp}} + k^{\text{disp}} \cdot K_{u,c}^{\text{disp}} + k^{\text{startstop}} \cdot K_{u,c}^{\text{startstop}}}_{\text{unit costs}} + \sum_{l\in\mathcal{L}} \underbrace{k^{\text{PST}} \cdot K_{l,c}^{\text{PST}} + k^{\text{switch}} \cdot K_{l,c}^{\text{switch}}}_{\text{line costs}}.$$

The parameters $k^{\text{redisp}}, k^{\text{disp}}, k^{\text{startstop}}, k^{\text{PST}}, k^{\text{switch}} \in \{0,1\}$ are used to indicate the inclusion/exclusion of the following cost components: redispatch cost (includes load shedding as well), dispatch cost, start-up/shutdown cost, PST shifting cost and line switching cost. Finally, the blackout risk depends on the system's total VOLL and the probability of the blackout outcome $K_c^{\text{blackout}} = \pi_c^{\text{blackout}} \cdot \mathbb{E}(\text{VOLL})$. The system's expected VOLL is considered as a parameter in this approach.

3.4.12 Acceptability constraints

To find a better balance between preventive costs and corrective risks, SCOPF tools have commonly included constraints to limit corrective load shedding through limits on the energy not served (ENS) and power not served (PNS) [GARPUR,2016d].

3.4.13 Parameterized criteria

A mapping of the previously-introduced parameters to reliability criteria in the SCOPF is provided in Table 4. The GQP SCOPF is capable of representing an N-1 reliability criteria as well as the GARPUR RMAC-QP through different values of these parameters. Criteria in-between N-1 and the full RMAC-QP are easily parameterized. Furthermore, to allow for validation w.r.t MATPOWER, a mode is parameterized to obtain a OPF (not security-constrained). In this mode, only the N-0 scenario is used (hence probability 1) and the true dispatch cost is considered instead of the *re*dispatch cost.

Table 4: Overview of parameterized reliability criteria

	N-1 DA	'Full' RMAC DA	MATPOWER validation
$\mathcal{C}^{\text{selected}}$	$\mathcal{C}^{\text{N-0}} \cup \mathcal{C}^{\text{N-1}}$	$\mathcal{C}^{\text{N-0}} \cup \mathcal{C}^{\text{N-1}} \cup \mathcal{C}^{\text{N-2}}$	$\mathcal{C}^{\text{N-0}}$
π_c	$\frac{1}{\lvert\mathcal{C}^{\text{selected}}\rvert}$	Original contingency probability data	(1)

k^{prev}	1	1	1
k^{corr}	0	1	0
k^{blackout}	0	1	0
k^{redisp}	1	1	0
k^{disp}	0	0	1
$k^{\mathrm{startstop}}$	1	1	0
k^{PST}	1	1	0
k^{switch}	1	1	0
$\sigma^{\mathrm{rel.target}}$	0	1	0
β^{max}	0	1	0
$\pi_c^{\mathrm{fail,\ unstable}}$	1	1	0

3.5 Summary

A mathematical framework is presented to bridge conventional SCOPF with formulations of the GARPUR RMAC-QP ingredients, based on [Karangelos,2016, Garpur,2016c]. The parameterization of this framework is illustrated and a day-ahead and real-time N-1 criterion and a GARPUR RMAC are defined.

State-of-the-art algebraic modelling toolboxes (e.g. AMPL [AMPL,2017], GAMS, Pyomo [Hart, 2011]) allow for a relatively straight-forward implementation of these equations. However, it is assumed that a (symbolic) presolver is available to remove redundant variables and constraints, and to perform substitution of variables. If not available, the formulation may lead to less-than-optimal model building and solving times. To deal with this, the constraint sets can be derived more concretely for specific model components, e.g. differentiate again between loads and generators, flexible or not, etc.

Notice that with respect to the GARPUR RMAC implementations of SCOPF for real-time operation discussed in [Karangelos,2016, Garpur,2016c], this document presents a specific SCOPF implementation to fit the scope of pilot testing. Likewise, the day-ahead SCOPF implementation presented here differs from the implementation presented in [Karangelos,2017] in the sense that it is specifically adapted for the pilot testing context.
In an optimization context, the developed SCOPF framework can be described as:

- a three-stage stochastic programming problem reformulated as a deterministic equivalent, subject to the possible power system contingencies as realizations of uncertainty vectors of the availability of lines and units;
- with nonconvex constraints due to the AC power flow equations;
- with disconnected search space due to the binary variables required to model, amongst others, contingency discarding and topological actions;
- therefore overall being a large-scale MINLP problem.

As the computational tractability of such problems is limited, different trade-offs between tractability and accuracy are analysed in the next chapter.

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4 SCOPF TRACTABILITY: APPROXIMATION AND ACCURACY

In this chapter, the computational tractability of the developed mathematical model is analysed first. To support the application of such models to real-life case studies, approaches are developed to deal with the computational challenge. The focus lies on dealing with the combination of the nonconvex power flow equations and the use of binary variables in an optimization context. Different approximations to the original AC power flow formulations can be used in the context of SCOPF, each having their own trade-off in terms of accuracy and tractability.

4.1 Mathematical model complexity

Since the development of interior-point methods it has been understood that the true distinction between easy-to-solve and hard-to-solve problems does not align with linear programming (LP) versus nonlinear programming (NLP[2]) problems but with convex versus nonconvex optimization. In theory and in practice, large convex problems can be solved reliably (convergence guaranteed), quickly (polynomial-time) and to global optimality. Such problem classes include quadratic programming (QP), convex quadractically-constrained programming (CQCP) and second-order cone programming (SOCP). With nonconvex (smooth) optimization, one largely has to choose between solving problems quickly but locally optimal or globally optimal but slowly. With the development of practical semidefinite programming (SDP) methods, efforts have been made to leverage their expressive power to find strong SDP approximations to nonconvex problems. A hierarchy of continuous optimization complexity classes can be developed as follows:

$$\underbrace{\text{LP} \subset \text{QP} \subset \text{CQCP}^3 \subset \text{SOCP} \subset \text{SDP}}_{\text{convex} \sim \text{tractable}} \subset \underbrace{\text{NCQCP} \subset \text{NLP}}_{\text{nonconvex}}$$

Relaxations, by a process of only *removing* equations from the feasible set of an original problem, provide strong quality assurances on the solution of both problems:

- if the original problem is feasible, the relaxed problem is feasible;
- if the relaxed problem is infeasible, the original problem is infeasible;
- the optimum of the relaxed problem will be a lower bound (in case of minimization) for the optimum of the original problem.

Other approximation strategies, e.g. linearization (that cannot be shown to be a relaxation), do *not* provide such guarantees. Nevertheless, the underlying idea of these approximations is that they are sufficiently accurate, but for parameters and decision variables limited to certain ranges.

4.2 Complexity of GARPUR SCOPF

4.2.1 Power flow equations

The power flow equations (static, balanced) are challenging to include directly in a SCOPF, due to the large-scale nature of SCOPF and the nonconvexity of the power flow equations [PegaseD3.1]. Depending on the application, specific approximations of the power flow equations can be used [Taylor,2015]. Historically, the linear 'DC' OPF formulation was proven popular in the context of approximating the nonconvex power flow equations in transmission systems. More recently, new

[2] Including nonconvex quadratically-constrained programming (NCQCP)

[3] QP is included in the CQCP through the epigraph transformation

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approaches have been developed, using (nonlinear) convex relaxation techniques [Taylor,2015]. An overview of power flow formulations for OPF problems is given in Table 5.

Table 5: A variety of power flow formulations

	Paper	Complexity	Technique	Relaxation?
AC OPF		Nonconvex NLP		-
SDP 'moment'	[Josz, 2015]	SDP	[Lasserre,2001]	Yes
SDP	[Bai, 2008]	SDP	[Shor,1987]	Yes
SOCP BIM	[Jabr, 2006]	SOCP (+ LP through [Ben–Tal,1999] polyhedral relax.)	[Kim,2003]	Yes
SOCP BFM	[Baran,1989]	SOCP (+ LP through [Ben–Tal,1999] polyhedral relax.)		Yes
QC	[Coffrin,2016a]	SOCP (+ LP through [Ben–Tal,1999] polyhedral relax.)	Convex hull, McCormick	Yes
'DC'		LP	Linearization	No
LPAC	[Coffrin,2014a]	LP	Convex hull, linearization	No
Network Flow	[Coffrin,2016]	LP		Yes or No ([Coffrin,2016])
Copper plate	[Coffrin,2016]	LP		Yes or No ([Coffrin,2016])

A great deal of interest has recently been shown in SDP (and SOCP) power flow formulations [Coffrin,2015], [Madani,2016] which may return the (proven) global optimum of the original nonconvex AC OPF problem. In a radial test case, the SDP formulation also automatically reduces to an SOCP formulation, which can even be proven to be exact under mild conditions [Taylor,2015]. Nevertheless, in general, convex relaxation formulations are not exact. However, SOCP models are tractable (with similar performance as local NLP solvers for polar OPF). SDP solvers are not as performant as SOCP ones, and the application of SDP power flow models for SCOPF seems out of reach for larger optimization models at the moment of writing.

Next to these methods, a hybrid convex relaxation–linearization formulation, named the linear programming AC (LPAC) approximation was proposed [Coffrin,2014a].

It is furthermore noted that a lot of research has gone towards scalable methods to incorporate switching in these new formulations [Bestuzheva,2016], [Coffrin,2014b], [Kocuk, 2015] in a mathematically rigorous way.

4.2.2 Binary variables

The number of binary variables in the GARPUR SCOPF depends on the modelling detail considered and the settings chosen. Sources of binary variables in the model are listed in Table 6.

Table 6: Overview of binary variables in the GARPUR SCOPF

Symbol	Amount	Presolve amount	Meaning	Removal option?
β_c	$\|C\|$	$\|C\|$	Contingency discarding	Don't use contingency discarding
$\nu_{l,c}^{\mathrm{corr}}$	$\|L\| \cdot \|C\|$	$\|L\| \cdot \|C\|$	Indicator variable for line corrective action	Don't use failure of corrective actions model

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$\nu_{u,c}^{\text{corr}}$	$\lvert U\rvert \cdot \lvert C\rvert$	$\lvert U\rvert \cdot \lvert C\rvert$	Indicator variable for unit corrective action	Don't use failure of corrective actions model
μ_{l},	$\lvert L\rvert$	$\lvert L\rvert$	Indicator variable for PST action preventive	Don't use PST action costs
$\mu_{l,c}^{\text{corr}}$	$\lvert L\rvert \cdot \lvert C\rvert$	$\lvert L\rvert \cdot \lvert C\rvert$	Indicator variable for PST action corrective	Don't use PST action costs
α_{l}	$\lvert L\rvert$	$\lvert L\rvert$	Line switch variable (open/closed) preventive	Don't allow line switching
i_{u}	$\lvert U\rvert$	$\lvert U\rvert$	Unit state variable (on/off) preventive	Don't allow unit start-up/shutdown
$\alpha_{l,c}^{\text{ST}}$	$\lvert L\rvert \cdot \lvert C\rvert$	0	Line switch variable (open/closed) short-term post-contingency	Automatic presolve, for $C_{l,c} = 1$: $\alpha_{l,c}^{\text{ST}} = \alpha_l$, $C_{l,c} = 0$: $\alpha_{l,c}^{\text{ST}} = 0$
$i_{l,c}^{\text{ST}}$	$\lvert L\rvert \cdot \lvert C\rvert$	0	Unit state variable (on/off) short-term post-contingency	Automatic presolve, for $C_{l,c} = 1$: $\alpha_{l,c}^{\text{ST}} = \alpha_l$, $C_{l,c} = 0$: $\alpha_{l,c}^{\text{ST}} = 0$
$\alpha_{l,c}^{\text{corr}}$	$\lvert L\rvert \cdot \lvert C\rvert$	$\lvert L\rvert \cdot \lvert C\rvert$	Line switch variable (open/closed) corrective	Don't allow line switching
$i_{u,c}^{\text{corr}}$	$\lvert U\rvert \cdot \lvert C\rvert$	$\lvert U\rvert \cdot \lvert C\rvert$	Unit state variable (on/off) corrective	Don't allow unit start-up/shutdown
i_{u}^{start}	$\lvert U\rvert$	$\lvert U\rvert$	Indicator variable for unit start-up preventive	Don't allow unit start-up/shutdown
i_{u}^{stop}	$\lvert U\rvert$	$\lvert U\rvert$	Indicator variable for unit shutdown preventive	Don't allow unit start-up/shutdown
$i_{u,c}^{\text{start,ST}}$	$\lvert U\rvert \cdot \lvert C\rvert$	0	Indicator variable for unit start-up short-term post-contingency	Automatic presolve, for $C_{u,c} = 1$: $i_{u,c}^{\text{start,ST}} = 0$
$i_{u,c}^{\text{stop,ST}}$	$\lvert U\rvert \cdot \lvert C\rvert$	0	Indicator variable for unit shutdown short-term post-contingency	Automatic presolve, for $C_{u,c} = 1$: $i_{u,c}^{\text{stop,ST}} = 0$
$i_{u,c}^{\text{start}}$	$\lvert U\rvert \cdot \lvert C\rvert$	$\lvert U\rvert \cdot \lvert C\rvert$	Indicator variable for unit start-up corrective	Don't allow unit start-up/shutdown
$i_{u,c}^{\text{stop}}$	$\lvert U\rvert \cdot \lvert C\rvert$	$\lvert U\rvert \cdot \lvert C\rvert$	Indicator variable for unit shutdown corrective	Don't allow unit start-up/shutdown

The basic relationships between the RMAC ingredients and the use of binary variables are:

- contingency discarding requires 1 binary variable per contingency;
- failure of corrective actions requires 1 variable per contingency per line and per unit.

Other SCOPF features also depend on binary variables:

- a discrete cost model of PST actions requires one variable per PST preventively, and one variable per PST per contingency correctively;
- line switching actions require one variable per switchable line preventively, and one variable per switchable line per contingency correctively;
- unit state actions require one variable per unit preventively, and one variable per unit per contingency correctively;
- considering start-up and shutdown separately, triples the number of binary variables for each unit (generator).

4.2.3 Circular constraints

The number of circular constraints ($\sim x_1^2 + x_2^2 \leq x_3^2, x_3 \geq 0$) depends on the number of flow limits in the power flow model. There are three distinct categories of SOC constraints discussed in this document:

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- 1 apparent power flow limit per unit preventively, and 1 per unit per contingency, both short-term and correctively (due to ST AGC);
- 2 line apparent power flow limit per line preventively, and 2 per line per contingency, both short-term and correctively.

4.2.4 True conic constraints

The SOC formulation includes 1 rotated cone constraint ($\sim x_1^2 + x_2^2 \leq x_3 \cdot x_4, x_3 \geq 0, x_4 \geq 0$) per line preventively, and 1 further constraint per line per contingency both short-term and correctively.

Furthermore, current limits are also enforced through true conic equations:

- 2 line current limit per line preventively, and 2 per line per contingency, both short-term and correctively;

Commonly, only the apparent power flow limits are used. The SCOPF includes an option to consider the apparent power and current limits constraints separately.

4.3 Solution approaches

Mathematical optimization solvers have been developed for specific problem types. Solution approaches to the GARPUR RMAC in the context of probabilistic SCOPF have been developed and discussed in [GARPUR,2016c], [Karangelos,2016], [Karangelos,2017]. These are not rediscussed below.

4.3.1 Convex solvers

Convex solvers typically have three possible end states:

- feasible: global solution found;
- certificate of infeasibility;
- numerical problems.

CPLEX, Gurobi, Xpress and MOSEK are commonly used solvers for convex problems.

4.3.2 Local NLP solvers

When applied to nonconvex problems, local solvers can have the following end states:

- feasible, solution found is locally optimal;
- infeasible;
- numerical problems;
- convergence to infeasible solution;
- lack of convergence.

If the problem is known to be convex, any locally optimal solution is also globally optimal. Otherwise, a certificate of global optimality is not expected. KNITRO and Ipopt are examples of NLP solvers

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4.3.3 Global (MI)NLP solvers

Global / spatial branch & bound (B&B) algorithms can be used to solve nonconvex problems to global optimality. However, such approaches are not scalable enough to consider in the context of SCOPF. Baron is an example of a solver in this category.

4.3.4 Branch and bound schemes for mixed-integer convex problems

In a B&B scheme, the mixed integer programming (MIP) gap is an indicator for the quality of the best integer solution found so far. Reducing the MIP gap to below a certain threshold can take a lot of time, even for small problems. In such a scheme, one can choose to end the calculation under one of the two following criteria:

- MIP gap below target value;
- calculation budget exceeded.

The MIP gap measures the difference in optimality (objective value) between the continuous relaxation of the original problem and the so-far-found best integer solution. There are four outcomes of a B&B scheme:

- (close-to-) global solution found when MIP gap is small enough;
- local integer solution found when MIP gap remains too large;
- infeasible if continuous relaxation is infeasible;
- infeasible, no integer solution found, but continuous relaxation is feasible.

Only in the first two cases the solution returned can be used (feasible), but comparing results when the MIP gap remains too large can be tricky, due to the lack of guarantees on the optimality. CPLEX, Gurobi, Xpress and MOSEK are commonly used solvers for mixed-integer convex quadratic (SOC) problems.

4.3.5 Summary

An overview of commonly-used solvers for the complexity classes discussed throughout this report is given in Table 7.

Table 7: Illustration of common solver packages for different complexity classes

	Convex					
	LP	SOCP	SDP	Nonconvex	Integer	License
Gurobi, CPLEX[4], XPRESS	✓	✓	×	×	✓	Commercial
MOSEK[5]	✓	✓	✓	×	✓	Commercial
KNITRO	✓	✓	×	✓ (local)	✓	Commercial
Ipopt	✓	✓	×	✓(local)	×	Open
GLPK	✓	×	×	×	✓	Open
ECOS(_BB)	✓	✓	×	×	(✓)	Open
Baron	✓	✓	×	✓ (global)	✓	Commercial

[4] CPLEX supports nonconvex QP.

[5] MOSEK does not support MISDP, just continuous SDP.

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4.4 Power flow approximations

A number of common OPF formulations and their properties relevant to a SCOPF with binary variables are summarized in Table 8. These approximate power flow formulations are described and derived in the following to illustrate how different formulations can fit in the SCOPF framework introduced in Chapter 3.

The ‘DC’ is a commonly used approximation, suitable when modelling highly inductive & meshed transmission grids. It is lossless and is based on a linearization of the voltage angles w.r.t. the active power flow. As it is linear and has few variables and equations, it is highly tractable. However, it is inexact due to the underlying approximations, but in practice the results correlate significantly with the true AC OPF solutions.

LPAC is a more advanced linearization approach applied to the power flow equations that linearizes the model around an AC power flow feasible starting point [Coffrin,2014a]. The formulation contains a graph for both active and reactive power and uses a polyhedral convex hull to approximate the cosine function of the underlying polar AC power flow formulation.

Recently, convex relaxations to the power flow equations have received a great deal of interest. Due to the relaxation process, certain guarantees can be derived. In general, the convex relaxation formulations obtain either the global optimum of the underlying problem or a truly relaxed[6] solution with a lower minimum (in case of minimization). An overview of a larger number of convex relaxation formulations is given in [Geth,2017].

Table 8: Properties of power flow formulations for SCOPF

		Lossless?	Variables	Can handle Integers?	Tractability	Feasibility	Optimality
DC	LP	Yes	P, θ	+	+++	correlated	correlated
LPAC	LP	No	P, Q, θ, U	+	++	strongly correlated	strongly correlated
Convex BFM (DistFlow)	SOCP	No	P, Q, U^2, I^2	+	++	exact (including cert. of infeasibility) or over-relaxed	global or over-relaxed
AC OPF	NLP local	No	P, Q, U, θ	---	+	local heuristic	local
AC OPF	NLP global	No	P, Q, U, θ	+	---	exact	global

As the ‘DC’ and LPAC power flow approximations are inherently approximate due to the *linearization* of the power flow equations, there are discrepancies between the results from the linearized model and the AC power flow check. Therefore, the amount of infeasible trajectories in the linearized model may differ from the true amount, which means the risk estimate is only an approximation and contingencies may have been discarded improperly.

[6] The relaxed problem isn’t necessarily feasible w.r.t. the original problem

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In a post-solve AC check, the real risk of the solution can be quantified, but this solution is not necessarily optimal with respect to the underlying problem. The only way to avoid suboptimality is to implement the original (nonconvex) power flow equations in the SCOPF.

Due to the fact that (binary) indicator variables are used to indicate the activation of corrective actions, the overall problem, which also includes the power flow equations, is a nonconvex MINLP (even without topological actions). Improving the computational tractability is a significant challenge.

Finally, transient power flow stability is not explicitly considered, even though it impacts the feasibility of a number of actions, e.g. topological actions.

In Appendix I, section 11.5 three power flow formulations are discussed, as derived for the SCOPF framework within the GQP.

4.4.1 Comparison of tractability

The aim of the derivation of the previous approximate power flow formulations was to illustrate how different formulations can fit in the framework developed. More formulations exist, and are compatible as long as they can be mapped to the underlying AC OPF model.

To illustrate the effectiveness of the GARPUR RMAC, considering binary variables is crucial.

- MILP and MISOCP offer the best scalability;
  - This maps to ‘DC’ OPF, LPAC, SOC BFM OPF formulations
- MINLP (nonconvex) is not scalable.
  - This maps to the AC OPF formulation with binary variables

In chapter 7, computational tests will be performed for different power flow formulations. An overview of the settings and their associated complexity is given in Table 9.

Table 9: Power flow settings and complexity

Setting	Meaning	Power flow equations	Power flow bounds
$\sigma_{\text{DC}} \in \{0,1\}$	‘DC’ formulation	Linear	linear
$\sigma_{\text{LPAC}} \in \{0,1\}$	LPAC formulation	Linear	SOC
$\sigma_{\text{BFM}} \in \{0,1\}$	BFM formulation	SOC	SOC
$\sigma_{\text{AC}} \in \{0,1\}$	Polar AC formulation	Nonlinear (polar)	SOC

Only one formulation is used at the same time ($\sigma_{\text{DC}} + \sigma_{\text{AC}} + \sigma_{\text{BFM}} + \sigma_{\text{LPAC}} = 1$).

4.5 Model simplifications

As indicated in Table 10, certain aspects of the mathematical model can be removed through a number of settings.

Table 10: SCOPF model settings

Setting	Application
$\sigma_{\text{STstage}} \in \{0,1\}$	0: two-stage SCOPF, no ST stage, no AGC actions;

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	1: three-stage SCOPF, AGC actions in ST stage
$\sigma_{\text{Sbounds}} \in \{\text{box}, \text{circular}\}$	box: $\begin{cases} -S_{lij}^{\text{rated}} \leq P_{lij} \leq S_{lij}^{\text{rated}} \\ -S_{lij}^{\text{rated}} \leq Q_{lij} \leq S_{lij}^{\text{rated}} \end{cases}$ circular: $(P_{lij})^2 + (Q_{lij})^2 \leq (S_{lij}^{\text{rated}})^2$
$\sigma_{\text{startstop}} \in \{0,1\}$	0: no start-up/shutdown constraints and binary variables 1: start-up/shutdown model included
$\sigma_{\text{ENSPNS}} \in \{0,1\}$	0: no ENS-PNS constraints / limits 1: ENS-PNS constraints /limits are enforced

Relationship between settings and model complexity is illustrated in Table 11.

Table 11: Impact of settings on SCOPF complexity

Setting	Impact
σ_{STstage}	Removes all variables and constraints of the ST-post-contingency stage
$\sigma_{\text{Sbounds}} = \text{box}$	All flow constraints are linear
$\sigma_{\text{Sbounds}} = \text{circular}$	All flow constraints are SOC
$\sigma_{\text{startstop}} = 0$	Binary start-stop variables are removed, start-up/shutdown costs are set to 0
$\sigma_{\text{ENSPNS}} = 0$	All ENS-PNS constraints / limits are removed (all linear)

4.6 Summary

The SCOPF is subject to inaccuracy coming from a number of approximations:

- Approximations of the power flow model
  - Power flow equations:
    - linear approximation, and convex relaxation.
  - PST model approximation:
    - continuous relaxation of discrete steps.
    - fixed impedance whereas impedance depends on phase angle in reality.
  - Steady-state power flow, assuming balanced phasors
- Approximations in modelling the reliability management processes w.r.t. real-world operations
  - No restoration of security after corrective actions, i.e. after the corrective action the reliability criterion is not necessarily satisfied.
  - No black-out avoidance strategy after
    - failure of corrective actions;
    - contingency discarding.
  - AGC / frequency control model in the post-contingency stage is approximate.

The trade-offs between accuracy of the model formulations and the tractability of the resulting problem was discussed and analysed. A number of power flow formulations are implemented in a SCOPF framework to allow for numerical experiments.

The research leading to these results has received funding from the European Union Seventh Framework Programme under Grant Agreement No 608540.

5 SIMULATING RELIABILITY MANAGEMENT USING SCOPF

5.1 Introduction

In this chapter, it is illustrated how a SCOPF calculation core fits into the simulation processes of reliability management in a combined day-ahead and real-time context. Special attention is given to the approach used in the RTE pilot. Note that the time steps are evaluated independently without coupling constraints.

Three different modes are developed:

- three-step N-1;
- two-step RMAC-QP;
- three-step RMAC-QP.

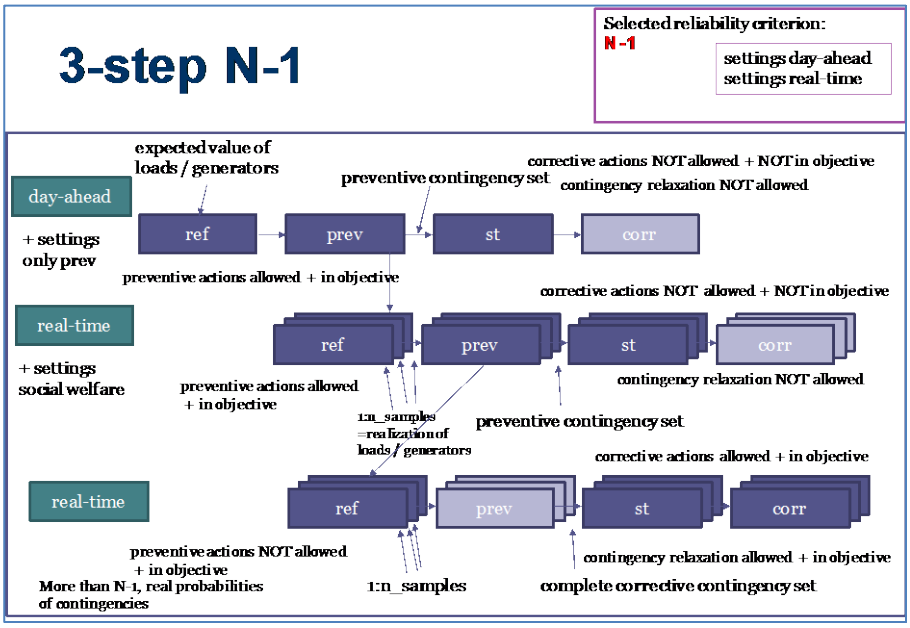

Figure 3: Three-step day-ahead and real-time N-1 reliability assessment

5.2 GARPUR day-ahead and real-time RMAC-QP

For performing the RTE pilot test, the GQP has been set up to perform in a three-step configuration. The reason behind that is to mimic the decision-making process of the TSO as well as possible, especially when assessing the N-1 security criterion. In general, the optimization problem has two time domains (steps), namely the day-ahead step and the real-time step. During the day-ahead step, the optimization problem is performed for one expected value of generation and demand, whereas in the real-time step, many different realisations of generation and demand are investigated. Besides the two time domains, also two different contingency sets are used in the comparison, namely a smaller of set of contingencies which are preventively secured by the TSO and a more extensive set of contingencies which are correctively secured. Depending on the security criteria chosen, different settings are used for the GQP.

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5.2.1 Three-step N-1 configuration

As mentioned in the previous chapters, the SCOPF of the GQP consists of a reference stage, a preventive stage, a short-term post-contingency stage and a corrective stage which are interlinked with each other (see Figure 3). In the three-step N-1 configuration, the day-ahead SCOPF is calculated on the basis of all N-1 contingencies of the preventive contingency list. As it aims to mimic a “classical” N-1 analysis, no corrective actions (topological actions, generation dispatch, PST tap changes and load shedding) are allowed in the corrective stage of the SCOPF. That means all contingencies need to be secured preventively, using topological actions, generation dispatch and PST tap changes, as preventively no load shedding may be performed.

When the day-ahead preventive SCOPF is performed (see Figure 3, upper part), the obtained optimal values are passed to the real-time problem as reference values. Nevertheless, because different samples regarding generation and demand are analysed in the real-time problem, a real-time deviation (difference between day-ahead prediction and real-time value) is added to the optimal generation and demand values obtained in the day-ahead step.

In real time, at first, the same problem is solved as in the day-ahead (with the exception of the load and generation error of the samples), to find the optimal real-time preventive actions for all samples. (See Figure 3, middle part). Eventually, the risk of the contingencies in the corrective contingency set is assessed on a contingency per contingency basis in the real time corrective step. (See Figure 3, lower part). The optimal values obtained in the real-time preventive step are passed as reference values for the real-time corrective step. In this step, the settings for the SCOPF are configured such that no preventive actions are allowed. In this step, also the probabilistic RMAC-QP objective and constraints are used, in order to determine the corrective risk as well as the risk of blackouts, in case there are insufficient corrective actions available, or due to risk of failure of corrective actions, as discussed in the previous sections.

5.2.2 Two-step RMAC configuration

To be able to quantify the RMAC-QP in the same fashion as the N-1 criterion, the two-step RMAC-QP configuration consists of the day-ahead and the real-time stage. In the day-ahead stage the preventive contingency list is used (see Figure 4, upper part). The SCOPF is performed using the RMAC-QP settings including the corrective risk and probabilistic contingency discarding.

As in the case of the three-step N-1 configuration, the result of the preventive stage of the day-ahead step is passed as the reference value for the real-time step (See Figure 4, lower part). The results of the corrective stage of the problem are not further used, but they are needed to quantify the overall risk. Similarly, the real-time error of generation and demand is added to the optimal day-ahead preventive dispatch values. In real time, the full functionality of the SCOPF is used, including the corrective risk and probabilistic contingency discarding. The SCOPF is performed in one step over the more extensive corrective contingency list.

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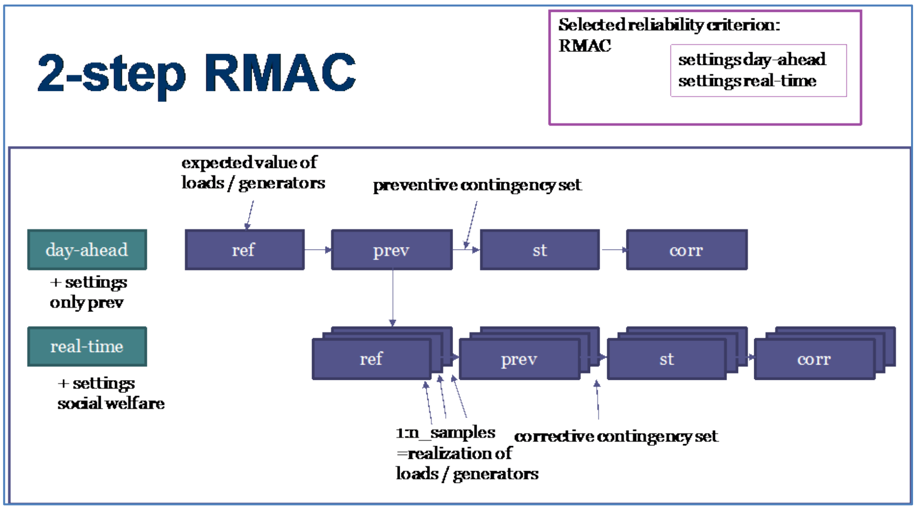

Figure 4: Two-step day-ahead and real-time GARPUR RMAC-QP

5.2.3 Three-step RMAC-QP configuration

The three-step RMAC-QP configuration is implemented in the GQP because it is somewhat more intuitive for the TSO as it is used in the same fashion as the N-1 configuration. It is important to mention at this point that the three-step configuration does not allow the use of the full potential of the RMAC-QP, as a trade-off between the preventive and corrective contingency list is still used outside the SCOPF. To be more specific, by using two different contingency sets in the real-time problem, and by making the decisions of the corrective sets dependent on the results of the real-time preventive step, means that the optimal trade-off between the preventive and corrective actions in real time cannot be found. Therefore, the three-step RMAC-QP configuration results in a lower economic performance.

The working principle of the three-step RMAC-QP configuration is very similar to the three-step N-1 configuration (see Figure 5). In the day-ahead problem the corrective risk and probabilistic contingency discarding are used over the preventive set of contingencies (see Figure 5 upper part). The preventive results of the day-ahead problem are passed to the real time preventive problem, where the SCOPF is performed using the corrective risk and probabilistic contingency discarding over the preventive contingency set, for all different generation and load samples. (See Figure 5 middle part.)

Similar to the real-time corrective step in the N-1 approach, no preventive actions are allowed in the real-time corrective step of the RMAC-QP, which again uses corrective risk and probabilistic contingency discarding to assess the risk of the corrective contingency set. (See Figure 5 lower part.)

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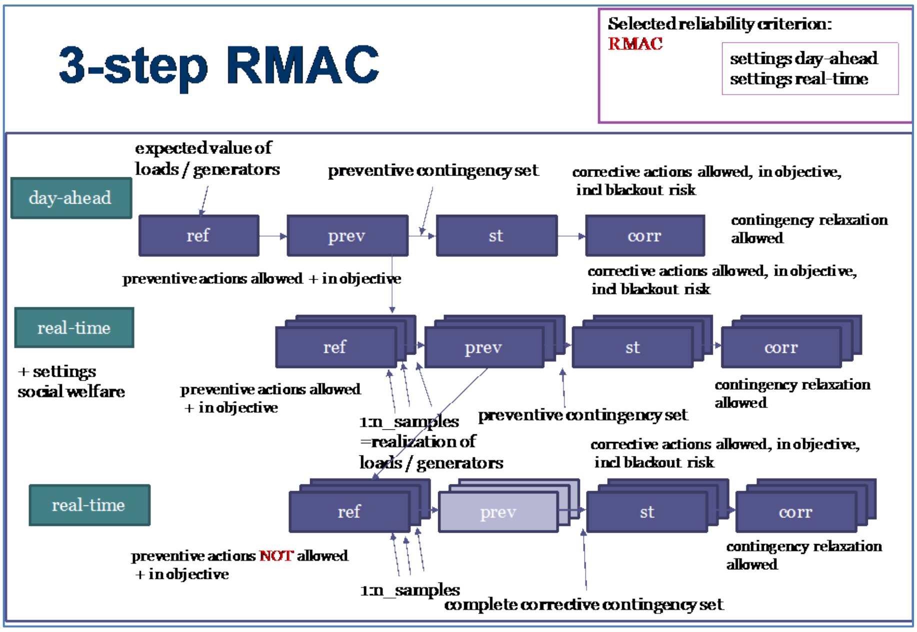

Figure 5: Three-step day-ahead and real-time GARPUR RMAC-QP

5.3 Summary

Different options to incorporate the parameterized GARPUR SCOPF in the simulation of day-ahead and real-time N-1 and GARPUR RMAC-QP reliability management are developed and analysed. Table 12 summarizes the simulation settings used for comparison of the criteria.

Table 12: Overview of parameterized reliability criteria for the different steps

	N-1			RMAC-QP		
	DA	RT prep	RT corr	DA	RT prev	RT corr
$\mathcal{C}^{\text{selected}}$	$\mathcal{C}^{\text{N}-0} \cup \mathcal{C}^{\text{N}-1}$	$\mathcal{C}^{\text{N}-0} \cup \mathcal{C}^{\text{N}-1}$	$\mathcal{C}^{\text{N}-0} \cup \mathcal{C}^{\text{N}-1} \cup \mathcal{C}^{\text{N}-2}$	$\mathcal{C}^{\text{N}-0} \cup \mathcal{C}^{\text{N}-1}$	$\mathcal{C}^{\text{N}-0} \cup \mathcal{C}^{\text{N}-1}$	$\mathcal{C}^{\text{N}-0} \cup \mathcal{C}^{\text{N}-1} \cup \mathcal{C}^{\text{N}-2}$
π_c	$\frac{1}{\lvert\mathcal{C}^{\text{selected}}\rvert}$	$\frac{1}{\lvert\mathcal{C}^{\text{selected}}\rvert}$	Orig. data	Orig. data	Orig. data	Orig. data
k^{prev}	1	1	0	1	1	0
k^{corr}	0	0	1	1	1	1
k^{blackout}	0	0	1	1	1	1
k^{redisp}	1	1	1	1	1	1
k^{disp}	0	0	0	0	0	0
$k^{\text{startstop}}$	1	1	1	1	1	1
k^{PST}	1	1	1	1	1	1
k^{switch}	1	1	1	1	1	1
$\sigma^{\text{rel.target}}$	0	0	1	1	1	1
β^{max}	0	0	1	1	1	1
$\pi_c^{\text{fail, unstable}}$	1	1	1	1	1	1

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6 IMPLEMENTATION OF SCOPF FRAMEWORK FOR THE GARPUR QUANTIFICATION PLATFORM

Basic functional requirements are:

- ability to read in CIM data;
- ability to read in other text-based reliability data;
- setting up contingencies;
- evaluation of socio-economic indicators to allow for comparison of criteria on this basis;
- simulation of reliability management using SCOPF calculation core with GARPUR ingredients.

Throughout this chapter, first the high-level architecture is described, and second the design of the modules.

6.1 Architecture

The general-purpose programming for the GQP has been performed in MATLAB. Partners in the project access the GQP through a remote-desktop connection (RDC). From MATLAB, interfaces were set up with:

- MATPOWER, a research-oriented power flow toolbox [Zimmerman,2011];
- a CIM parser in Python [PyCIM,2016];
- a CIM to MATPOWER case struct converter [CIM2MAT,2016];
- AMPL, a toolbox for algebraic modelling of optimization problems [AMPL,2017]
- CPLEX, a MISOCP solver [CPLEX,2016].

In addition to the CIM data input, a part of the required data was read from custom spreadsheet files. An overview of the general architecture is shown in Figure 6.

In a model file in AMPL, the parameterized mathematical models of the SCOPF were developed. AMPL then interfaces with the actual solvers, e.g. CPLEX [CPLEX,2016] and Ipopt [Wächter,2006].

6.2 Computation hardware

The KU Leuven server for the GQP has following specifications:

Table 13: KU Leuven Server Specifications

Processor	2.6+ GHz, 6 processors, (Intel Xeon Processor E5-2690 v3)
Memory	32 GB
OS	Windows Server 2012R2
CPU Cache	30 MB

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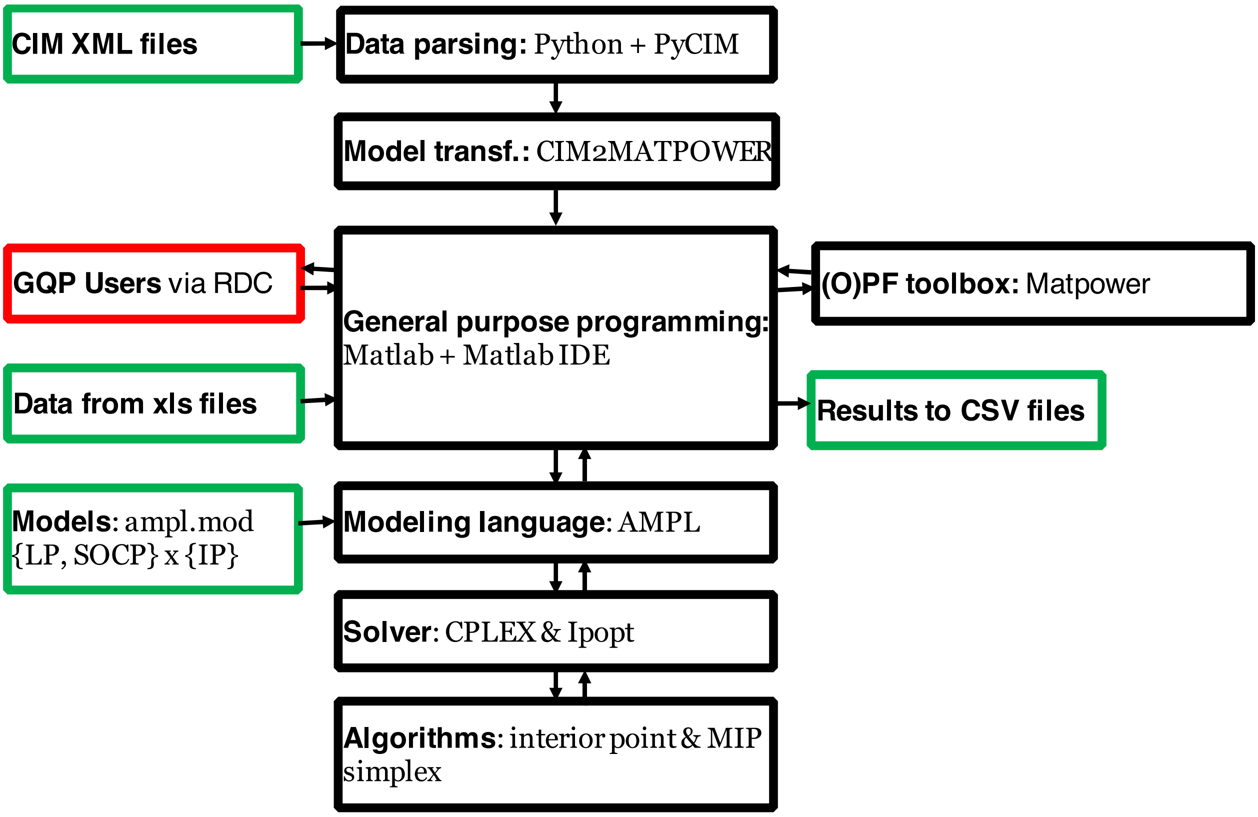

Figure 6: Overview of interactions of data and programs in the GQP

6.3 Design

6.3.1 Contingencies module

This module's task is to calculate the probability of a contingency, using the information available on failure rates and MTTR of lines and generators. A contingency is defined by the unavailability of one or multiple system components. The probabilities calculated depend on the weather conditions and use the historical data collected. For example, the probability of failure and mean-time-to-repair of certain lines may change significantly depending on the specific weather situation. If the data describing a contingency reflect this information, the reliability management approach can incorporate this into the decision-making.

6.3.2 Reliability management module

In this module, the processes for DA and RT preventive and corrective reliability management are programmed. For each DA and RT problem, the quantitative simulation module is being run – first considering a subset of contingencies, and finally over a larger set of contingencies to see if feasible corrective actions exist. This module encompasses the processes described in chapter 5.

6.3.3 Quantitative simulation module

The quantitative simulation module simulates reliability management using the SCOPF calculation core. This SCOPF calculation core is composed out of 6 entities, as visualized in Figure 7.

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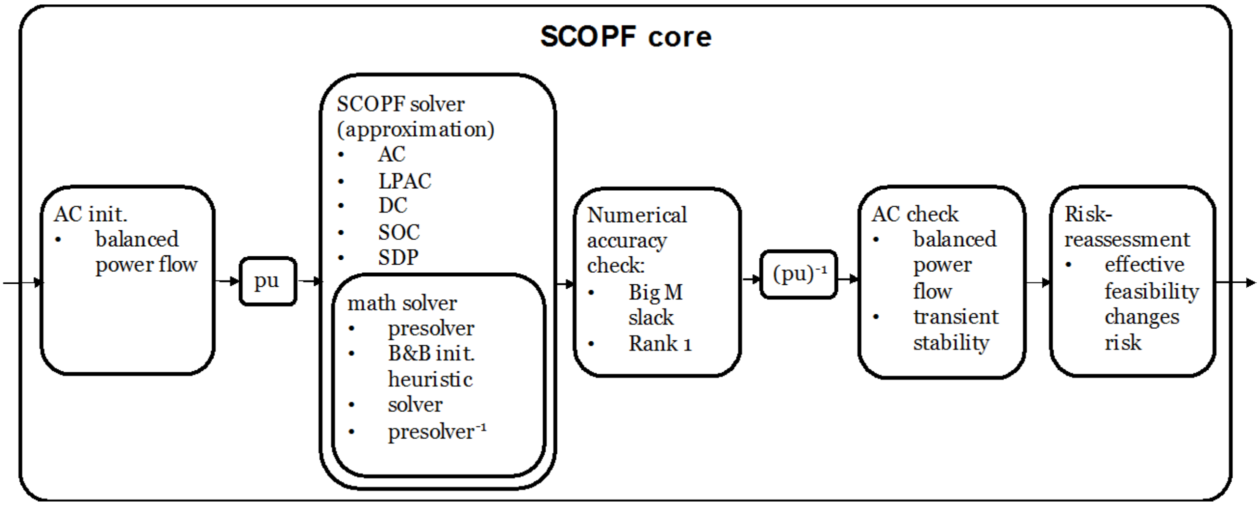

Figure 7: Structure of SCOPF core as part of the quantitative simulation module

The scope of the submodules is described in the subsections below.

6.3.3.1 Per unit conversion, and its own reverse

Numerical solvers operate on numerical values, not physical quantities. Therefore, nondimensionalization of the physical quantities is performed. For numerical reasons, variables and parameters should also be scaled 'well' in the optimization problem. One can accomplish this by performing the 'per unit conversion', which performs a nondimensionalization as well as a scaling of the variables. After solving the SCOPF, and validating its accuracy, this process is reversed.

6.3.3.2 AC Initialization

In specific cases, it may make sense to derive candidate solutions for the SCOPF. For instance, certain linearized power flow formulations, e.g. warm-start LPAC [Coffrin,2014a], use voltage magnitude estimates to improve the problem accuracy.

6.3.3.3 Parameterized SCOPF

The N-1 and GARPUR RMAC-QP SCOPF can be evaluated for a number of different settings:

- Power flow representation: network flow, 'DC' OPF, LPAC, AC OPF, …
- Start-up/shutdown constraints, …

Depending on the problem formulation complexity, a specific mathematical solver is called. For MISOCP and MILP models, CPLEX is called; or the AC formulation, IPOPT is called.

6.3.3.4 Numerical accuracy check

The quality of the (approximate) SCOPF results is assessed. For example, in the big M 'DC' OPF, it is assumed that all the voltage angles in the system are in the range $[-2\pi, 2\pi]$. Maximum, minimum and absolute value operators were also replaced with convex equivalents, and therefore it must be checked post-solve that there is no slack on the approximation.

Furthermore, for certain SOCP and SDP formulations, a criterion for global optimality is rank-1-ness of a certain variable matrix.

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6.3.3.5 Power flow accuracy check

The AC check is performed by a power flow tool. Here, MATPOWER [Zimmerman,2011] is used. As an input, a power flow solver takes:

- a grid topology, described by set T;
- parameters and states for all lines, switches, transformers and phase-shifters $z_{l,s}, y_{lij,\text{sh}}, y_{lji,\text{sh}}, \rho_{l,ji}^{\text{ref}}, \varphi_{l,ij}, \alpha_l$;
- net load and generation aggregated by node $P_i^{\text{unit}} + jQ_i^{\text{unit}}$.

The power flow solver then returns the voltages and complex power flows throughout this grid:

$$\left(\underbrace{[\tilde{U}_i \angle \tilde{\theta}_i]_{i\in N}}_{\text{node voltages}}, \underbrace{[\tilde{P}_{lij} + j\tilde{Q}_{lij}]_{lij\in T}}_{\text{line flows}}\right)$$
$$= \Phi_{\text{AC}}\left(\underbrace{[P_i^{\text{unit}} + jQ_i^{\text{unit}}]_{i\in N}}_{\text{unit power by node}}, \underbrace{T}_{\text{topology}}, \left[\underbrace{z_{l,s}, y_{lij,\text{sh}}, y_{lji,\text{sh}}, \rho_{l,ji}^{\text{ref}}}_{\text{line parameters}}, \underbrace{\varphi_{l,ij}, \alpha_l}_{\text{line state}}\right]_{lij\in T}\right)$$

Then the AC feasibility is assessed as:

$$f_{\text{AC}}\left(\underbrace{[\tilde{U}_i \angle \tilde{\theta}_i]_{i\in N}}_{\text{node voltages}}, \underbrace{[\tilde{P}_{lij} + j\tilde{Q}_{lij}]_{lij\in T}}_{\text{line flows}}, \underbrace{\left[S_{lij}^{\text{rated}}, I_{lij}^{\text{rated}}\right]_{lij\in T}}_{\text{line ratings}}, \underbrace{[U_i^{\min}, U_i^{\max}]_{i\in N}}_{\text{node limits}}\right) = 1$$
$$\Leftrightarrow \begin{cases} \forall i \in \mathcal{N}: U_i^{\min} \le U_i \le U_i^{\max} \\ \forall lij \in \mathcal{T}: \left(P_{lij}\right)^2 + \left(Q_{lij}\right)^2 \le \left(S_{lij}^{\text{rated}}\right)^2 \\ \forall lij \in \mathcal{T}: \left(P_{lij}\right)^2 + \left(Q_{lij}\right)^2 \le \left(I_{lij}^{\text{rated}}\right)^2 (U_i)^2 \end{cases},$$

otherwise $f_{\text{AC}} = 0$.[7]

If solved as an AC OPF, by definition, applying a post-solve AC power flow is idempotent:

$$\left([\tilde{U}_i \angle \tilde{\theta}_i]_{i\in N}, [\tilde{P}_{lij} + j\tilde{Q}_{lij}]_{lij\in T}\right) = \left([U_i \angle \theta_i]_{i\in N}, [P_{lij} + jQ_{lij}]_{lij\in T}\right)$$

However, in case an inexact power flow formulation was used, the values of these power flows and voltages may differ from the values calculated in the approximate SCOPF. Then, by observing the actual feasibility of the AC power flow, certain contingencies that were classified as secured by the SCOPF may actually be insecure w.r.t. AC power flow. This is then taken into account into the risk re-assessment.

Note that the power flow and the feasibility re-assessment are performed for each stage and each contingency separately.

More advanced AC feasibility checks can be developed. For example, proposed switching actions can be assessed based on their transient stability.

6.3.3.6 Risk re-assessment post-SCOPF

This module may also perform a risk re-assessment after the SCOPF has solved an inherently approximate model. The risks, need to be reconsidered on effective feasibility following the AC check. This may mean that certain contingencies are actually insecure, and should effectively be considered as discarded. Then, the risk is to be re-assessed.

[7] In the impleme[ntation, a small numerical tolerance is used.

The research leading to these results has received funding from the European Union Seventh Framework Programme under Grant Agreement No 608540.

6.3.4 Transmission system modelling module

This module sets up the model of the grid under consideration. Data is read from an XML serialization of CIM through the Pica package [PyCIM,2016]. In the RTE pilot, the CIM'14 version was used.

6.3.5 Socio-economic evaluation module

In this module, socio-economic indicators, developed in D3.1 [GARPUR,2016], were implemented. Some indicators are calculated as part of the SCOPF. The value-of-lost-load (VOLL) can be diversified: Not only do different groups of customers have different needs, but the system operator may also choose to contract large consumers to have preferential disconnection.

6.3.6 External systems module

The GQP was designed to be modular to be able to serve a number of different applications covering different parts of the scope of the GARPUR project. This includes the modelling of exogenous parameters (given by what is referred to as "external systems") and associated uncertainties, in case this is not included as part of the input data to the GQP. Although not utilized in the GQP version used for the RTE pilot test and the case studies reported in this deliverable, external systems sub-modules for renewable generation, load and market clearing were implemented in the general-purpose version of the GQP.

6.3.6.1 Renewable generation

The renewable generation sub-module generates time series of day-ahead forecasts of wind power generation. It is based on CorWind, an advanced tool able to simulate temporal and spatially correlated wind power generation patterns developed at DTU Wind Energy Department [Sørensen,2010]. The tool also simulates forecast errors, reproducing the forecast error characteristics observed in measured data. This can be used to provide multiple realizations of the uncertainty in wind power generation to the market clearing sub-module.

6.3.6.2 Load

A neural-network based load forecast model was implemented to forecast hourly day-ahead loads given temperature forecasts, holiday information and historical loads. The output of the load sub-module is day-ahead load forecast in form of a single time-series of forecasted value for each bus. When required, the model can be run to give a number of realizations of the uncertainties in the load to the market clearing sub-module.

6.3.6.3 Market clearing

The main purpose of the market clearing sub-module is to provide generation and load snapshots (i.e. operating states) to the GQP in the absence of real (e.g. day-ahead or historic) market clearing data. It is an adapted version of PowerGAMA [PowerGAMA,2017; Svendsen,2016], a lightweight open-source Python package for power system grid and market analyses developed by SINTEF Energy Research. Based on optimal unit dispatch, it outputs generation, load, power prices and producer and consumer surplus for each bus for each of the time steps within the time horizon that is considered. The market clearing sub-module is especially relevant for modelling future market and grid scenarios and could in principle also be replaced by more sophisticated market models.

The research leading to these results has received funding from the European Union Seventh Framework Programme under Grant Agreement No 608540.

7 COMPARISON OF CRITERIA FOR A SET OF CASE STUDIES

In this chapter, numerical experiments are performed for a number of reliability criteria. Two case studies are used to illustrate the simulation features. The goal is to illustrate that reliability criteria can be compared in a neutral and consistent manner, to further the development of future industry-grade tools to perform reliability management.

7.1 Default settings

The default calculation settings of AMPL, CPLEX and MATPOWER are summarized in Table 14.

Table 14: Calculation settings

AMPL presolve accuracy (-)	1E-8
CPLEX feasibility and optimality (-)	1E-6
CPLEX MIP gap (-)	1E-4
CPLEX timeout (s)	3600
MATPOWER accuracy (-)	1E-8

Furthermore, the default SCOPF model settings are listed in Table 15.

Table 15: Model settings

$\sigma_{\mathrm{DC}} = 1$	'DC' OPF formulation
$\sigma_{\mathrm{STstage}} = 1$	Removal of short-term post-contingency stage
$\sigma_{\mathrm{Sbounds}} = \mathrm{box}$	Apparent power flow limits as box constraints
$\sigma_{\mathrm{startstop}} = 1$	Inclusion of start-up and shut-down constraints
$\sigma_{\mathrm{ENSPNS}} = 1$	Inclusion of ENS and PNS constraints and limits
$\sigma^{\mathrm{rel.target}} = 1$	Inclusion of reliability target
$\pi_c^{\mathrm{fail,\,unstable}} = 1$	Inclusion of corrective actions failure (always leads to blackout)

7.2 IEEE Reliability test system

In this section, numerical results are generated for the IEEE Reliability Test System, based on [Grigg,1999]. All simulations are performed using setting as indicated in section 7.1, unless specified differently.

7.2.1 Case study details

The single-area version of the IEEE Reliability Test System is used. The reliability data, grid data, generation and demand data is taken from [Grigg,1999]. For the base case calculations 3 generators on of the original system have been converted to wind generators with a total generation of 78.4 MW which corresponds to 2.7% of the total generation. The positions of the wind generators are indicated in Figure 8. In further analysis, more conventional generation will be replaced the wind power in order to analyze the effect of available reserves on the total risk. The short-term thermal limits of the lines have been set to twice of the long-term thermal limit. The tests on the IEEE reliability test system are carried out using the SCOPF in a single step, as opposed to the test on the RTE system, which compose of a day-ahead and real-time step.

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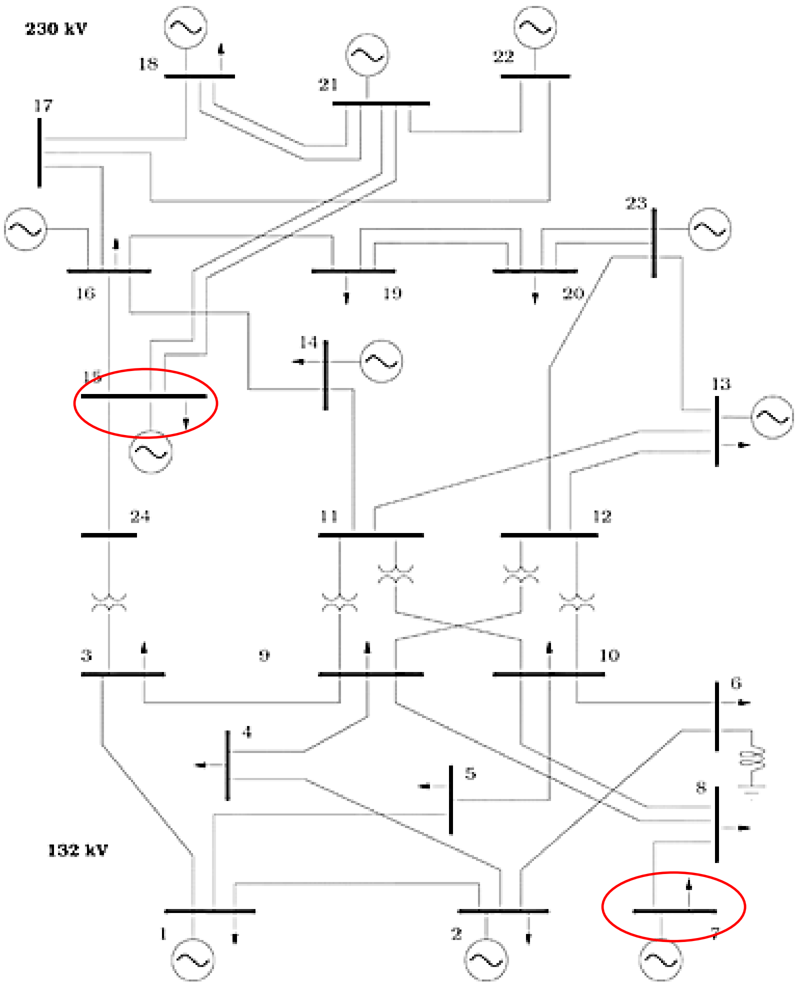

Figure 8: Overview of IEEE RTS system: topology, grid element types and generator connectivity (wind generator locations indicated in red)

7.2.2 Base case comparison N-1 vs RMAC-QP

For a base case comparison between the conventional N-1 approach and the GARPUR RMAC-QP a set of contingencies is used consisting of all branch contingencies up to N-2. For the N-1 approach, the SCOPF is performed first for all N-1 contingencies, where no corrective actions may be used and no contingency discarding is allowed. Thus, only preventive actions are taken to secure the system against all N-1 contingencies. In a second step, the preventive actions have been implemented, and the corrective and blackout risk of the remaining N-2 are assessed using the RMAC-QP formulation with probabilistic contingency discarding.

In the RMAC-QP approach, the SCOPF is performed using the probabilistic contingency discarding and the preventive and corrective stage are optimized in one go, allowing to find best balance between preventive and corrective risk.

For the base case comparison, some further assumptions are made:

- line switching actions are deactivated;
- PST switching actions are deactivated;
- the failure rates for all corrective actions are set to 0.

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Figure 9 shows the cost of preventive actions and the corrective and blackout risk for both the N-1 and the RMAC-QP approach. As can be seen in this figure, the preventive costs for the RMAC-QP are comparable, where the corrective and blackout risk are three orders of magnitude smaller. This stems from the fact that the calculated contingency probabilities are very low (in the order of 1E-5), and the corrective risk for some of the N-1 contingencies becomes very low.

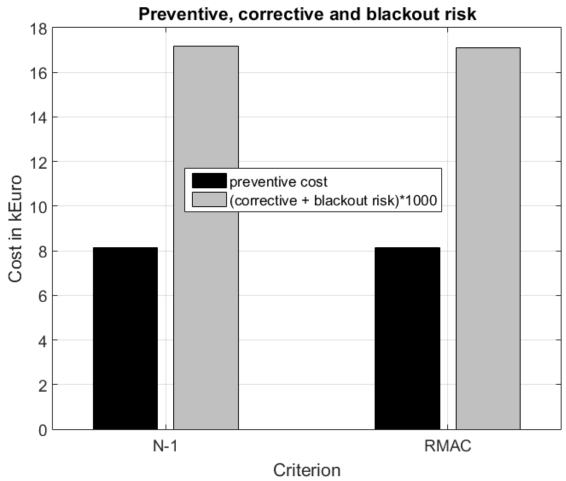

Figure 9: Cost of preventive actions and corrective and blackout risk. (Corrective and blackout risk are multiplied by 1000 as they are 3 orders of magnitude smaller than the preventive costs in the base case.)

In general, the probability of an N-2 contingency is several orders of magnitude smaller than an N-1 contingency. As such, if the contingency probabilities are low, the RMAC-QP approach tends to secure only N-1 contingencies preventively, resulting in the total risk. Another reason therefore is, that all N-1 contingency probabilities are in the same order of magnitude, such that the RMAC-QP approach does not shift them to the corrective problem. With more nuanced probability data, where contingency probabilities can have large differences, it can be expected that the RMAC-QP approach selects a different preventive contingency set than the N-1 approach.

Figure 10 shows how the total risk for the N-1 approach and the probabilistic RMAC-QP evolve, if the failure rates of the branches are increased. For this analysis, all contingency probabilities have been multiplied by 10 and 100 respectively. In these cases, we can clearly see that the GARPUR RMAC-QP approach prefers to use more expensive preventive actions than the N-1 approach in order to decrease the corrective and blackout risk. As a general conclusion we can say that the N-1 approach is too conservative if contingency probabilities are low, and not conservative enough if contingency probabilities are high.

The research leading to these results has received funding from the European Union Seventh Framework Programme under Grant Agreement No 608540.

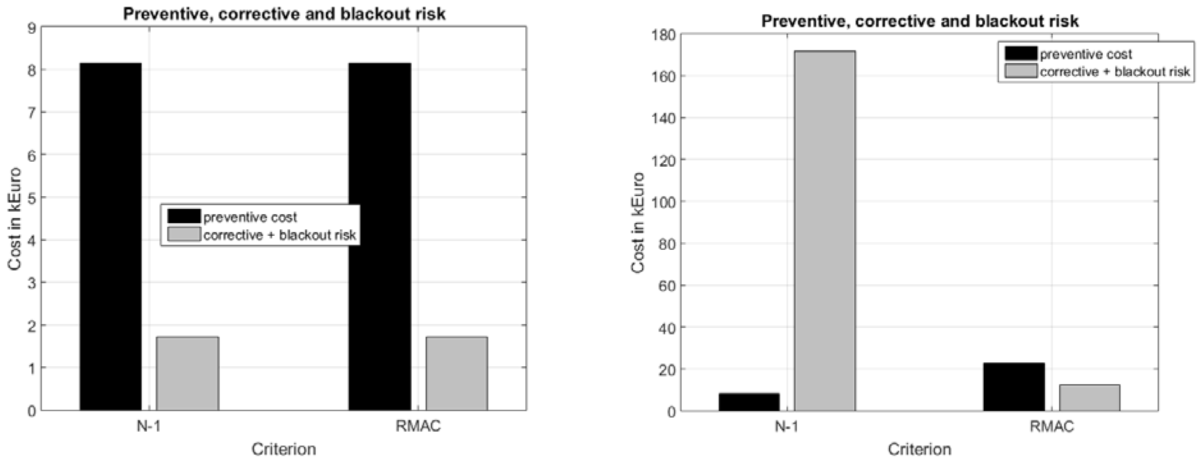

Figure 10: Sensitivity of cost of preventive actions and corrective and blackout risk to component failure rates 10 times (left) and 100 times higher (right) than in the base case

7.2.3 Effect of PST switching actions

Two PSTs have been added between buses 4 and 9 and 9 and 11 respectively. A PST switching action cost of 0.5 € per switching action is used. Figure 11 shows the total risk of the case study including PSTs. The analysis performed using equipment failure rates ten times higher than in the base case. The figure shows that the difference in the total risk between the N-1 and the RMAC-QP approach are very similar. We can see again that the RMAC-QP option uses more expensive preventive actions to reduce the corrective and blackout risk, resulting in total costs which are 6% lower. Compared to the case without PST actions (Figure 10, left), we can see that very similar values are obtained for both the N-1 and the RMAC-QP.

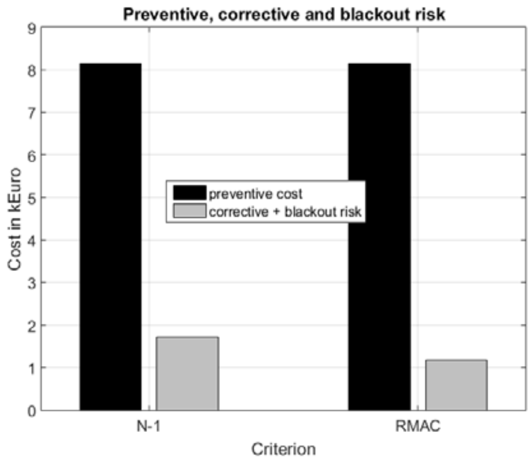

Figure 11: Preventive, corrective and blackout risk if PST switching is allowed

7.2.4 The effect of transmission switching

To analyse the effect of transmission line switching on the total risk K^{total} (preventive cost + corrective risk + blackout risk) and the calculation time, a certain number of lines have been defined as switchable. Figure 12 depicts the calculation time and the obtained objective function value of the problem in dependence of a variable number of switchable lines. The x-axis value refers to the number line, e.g. a ‘3’ means that the first three lines are defined as switchable, according to the numbering provided in Appendix 12.2. On the right-hand side, the N-1 approach is depicted. We can see that the objective function value does not change for fewer than 13 lines. We can also see that the computation time does not necessarily increase with the number of switchable lines. This stems also from the fact that in the N-1 approach only 38 contingencies are used in the preventive stage, and the corrective and blackout risk are assessed on a contingency per contingency base, limiting the calculation time.

In the RMAC-QP approach (Figure 12, right) we can see a similar behaviour as in the case of the N-1 approach namely that the calculation time varies greatly without a specific pattern and the objective value is constant for 13 switchable lines. However, the computation time is ~ 3 times higher than in the N-1 approach, due to fact that all N-1 and N-2 line contingencies are considered in the optimization problem. Another observation is that after the number of switchable lines is increased to 14 (i.e. to a value for which an effect on the objective function takes place), the optimization stopped after the time limit of 24 h without finding a feasible integer solution.

This means that the set of switchable lines needs to be selected carefully, especially for the RMAC-QP approach, as it can have a significant effect on the economic performance as depicted in the N-1 case as well as on the computational efficiency.

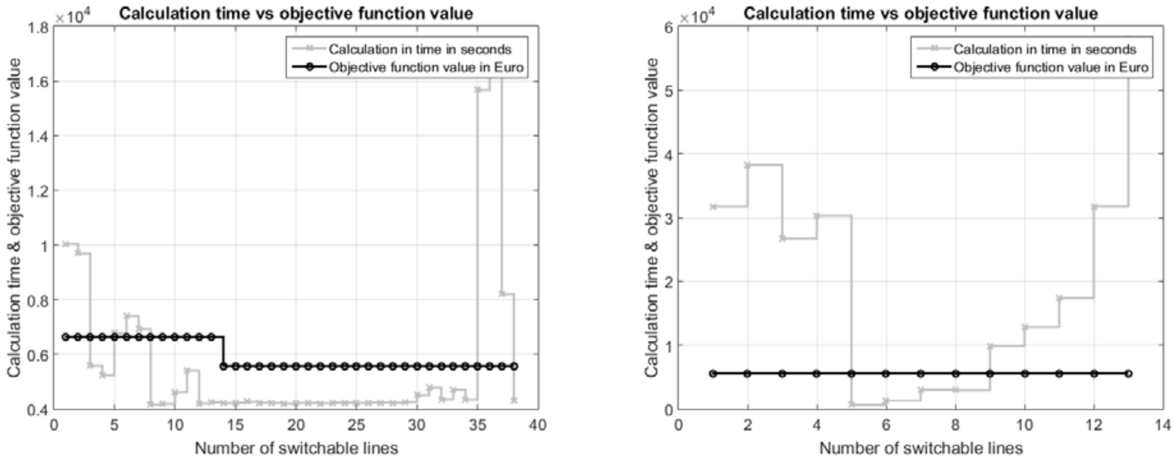

Figure 12: Effect of line switching on calculation time and total risk value K^{total} (left N-1 approach, right RMAC-QP approach)

Using switching actions in the implemented SCOPF can introduce feasibility problems with respect to the AC check, as the feasibility of the topological change for a linearized system does not guarantee feasibility of the non – linear system. Figure 13 shows the feasibility behaviour of the AC check in dependence of the number of switching actions for both, the N-1 and the RMAC-QP approach. The left hand figure shows clearly that the feasibility of the AC check is drastically reduced, if switching actions are performed (see Figure 12, left). In the RMAC-QP approach, where no switching actions were performed, we can see a consistent feasibility behaviour. Figure 13 also shows that once the solution is feasible, the voltage magnitude and power flow limits are respected mostly also during the AC check.

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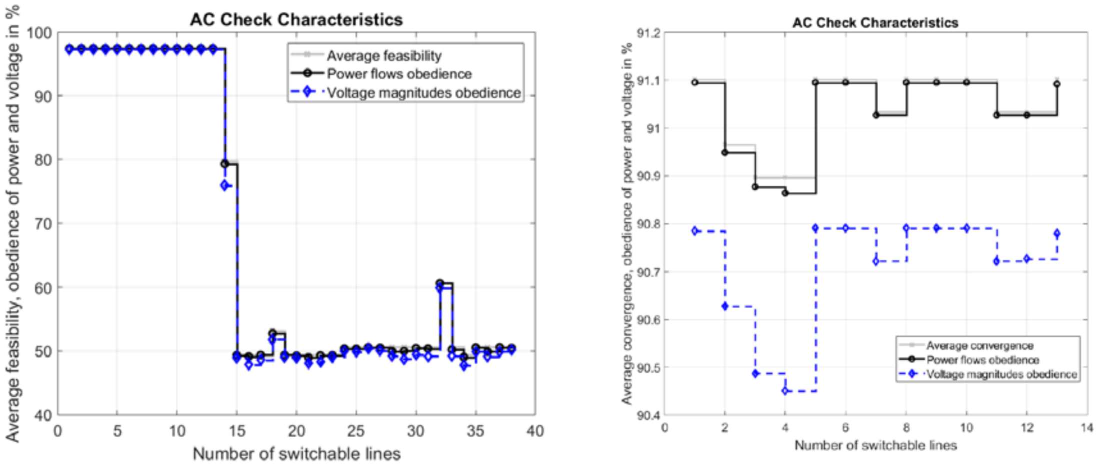

Figure 13: Feasibility of AC check in dependence of topological changes for the N-1 (left) and RMAC-QP approach (right)

7.2.5 Effect of the failure probability of corrective actions

The failure rate of corrective actions has been varied in order to analyse the sensitivity of the objective value on this parameter. As Figure 14 shows, the obtained result is very robust with respect to these failure rates. The left and right hand side figures show the expected risk if the failure rate of corrective actions is set to 1 % and 5 % respectively. We can see that there is almost no difference in the total obtained risk, as also shown in Table 16.

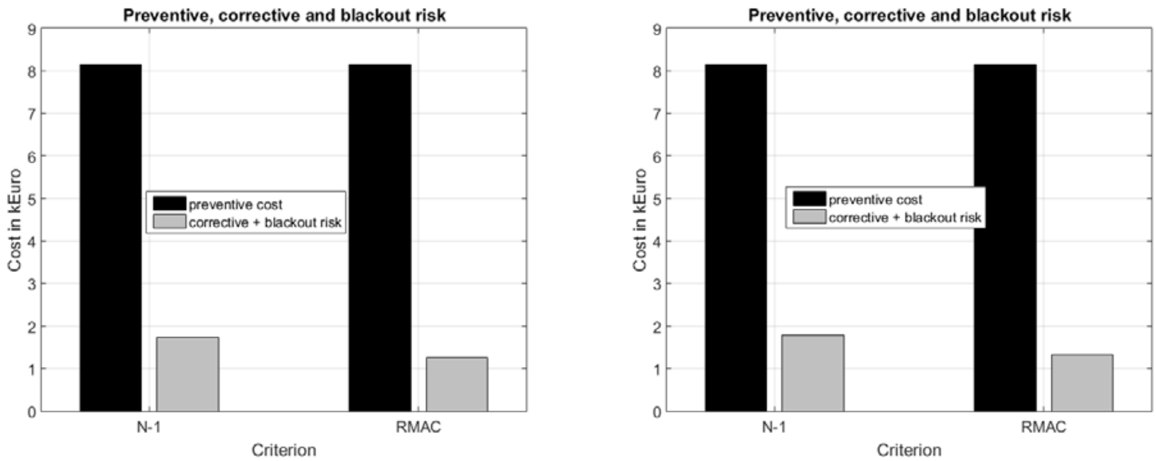

Figure 14: Sensitivity of preventive cost, and corrective and blackout risk, to the failure rates of corrective actions (left hand side failure rate of 1 %, right hand side 5 %)

Table 16: Dependence of total risk value on the failure rate of corrective actions

Failure rate of corrective actions	N-1 risk K^{total} (k€)	RMAC risk K^{total} (k€)
0%	9871.4	9338.9
1%	9885.3	9417.9
5%	9939.8	9494.1

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7.2.6 Effect of the Value of Lost Load

Different values of lost load have been used in order to see the sensitivity of the total risk on the value of lost load. Three different values, 26 000 €/MWh, 5000 €/MWh and 1000 €/MWh have been used for this purpose. Figure 15 shows how the total risk consisting of preventive action costs and corrective and blackout risk behave for these values for the N-1 and the RMAC-QP approach respectively. We can see that for lower values of lost load of 1000 €/MWh, which come closer to actual generation costs within the system, the RMAC-QP approach avoids preventive actions and prefers to take a higher corrective risk, which still results in a better socio-economic performance overall as shown in Table 17.

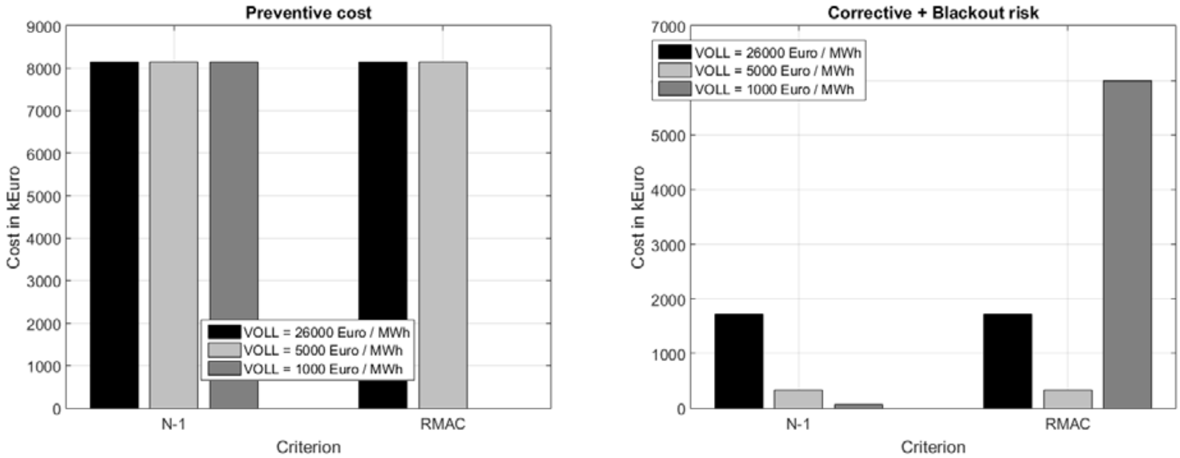

Figure 15: Sensitivity of the total risk consisting of preventive cost (left) and corrective and blackout risk (right) to the value of lost load

Table 17: Preventive cost, corrective risk and blackout risk for different values of lost load

	N-1			RMAC-QP		
VOLL	Preventive cost in € K^{prev}	Corrective + Blackout risk in € $\sum_{c\in C}\pi_c \cdot K_c^{\text{corr}} + \sum_{c\in C} K_c^{\text{blackout}}$	Total risk in € K^{total}	Preventive cost in € K^{prev}	Corrective + Blackout risk in € $\sum_{c\in C}\pi_c \cdot K_c^{\text{corr}}$	Total risk in € K^{total}
26 000 €/MWh	8155	1716	**9871**	8155	1716	**9871**
5000 €/MWh	8155	330	**8485**	8155	330	**8485**
1000 €/MWh	8155	66	**8221**	0	6001	**6001**

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7.3 RTE Tavel-Realtor corridor

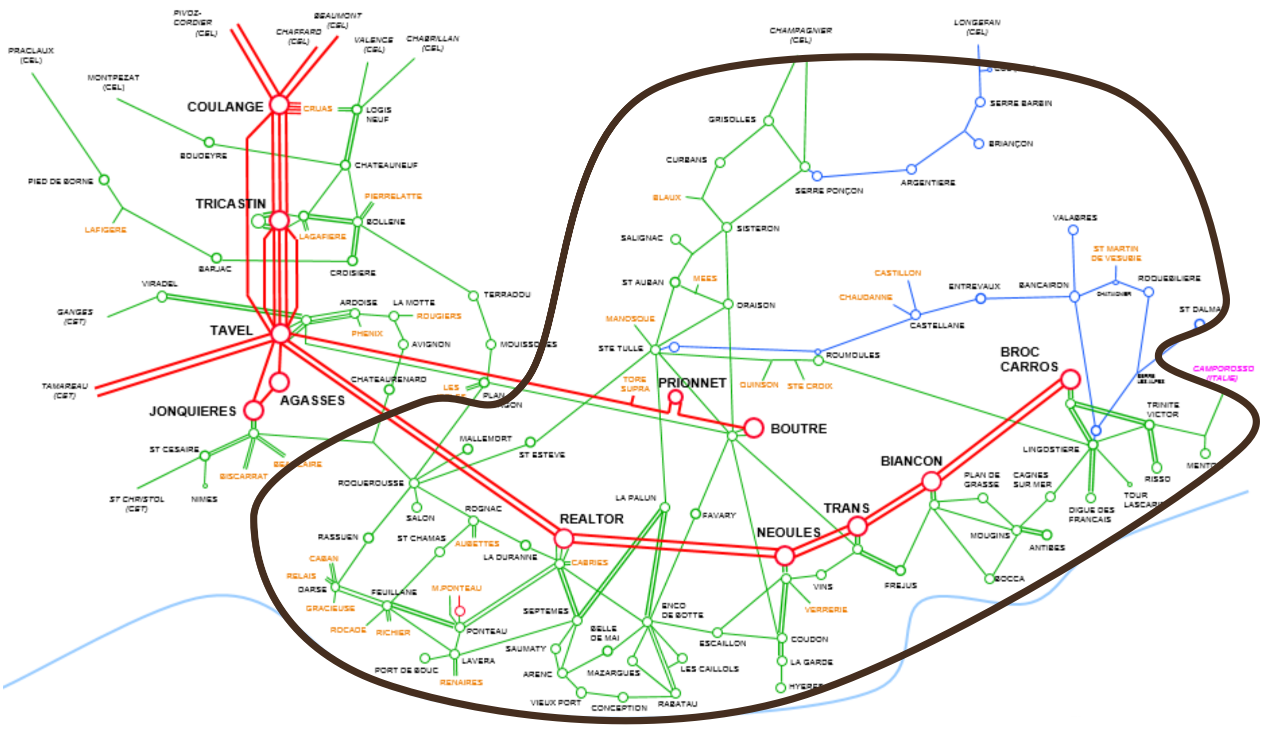

Figure 16: Overview of Tavel-Realtor corridor of the transmission network in the South of France

The GQP is applied to the southeast France region. The specifications of the test case are illustrated in Table 18.This case study is also used in the RTE pilot test performed as part of work package 8 of the GARPUR project. The test region has following characteristics:

- there are some generation units available in this zone, which however are not sufficient to fully supply the local load in the highlighted area;
- four corridors, plus the Italian cross-border phase-shifting transformer (PST), are the main axes which can supply electricity to the load to highlighted area;
- the renewable energy sources, mainly photovoltaic, are significant and represent approximately an amount of 30% of the average load.

The test aims to seek a good balance between the flows on the different corridors through combinations of actions, such as topology, PST, and generation redispatch. The cross-border PST between France and Italy can be an efficient means to alleviate some constraints in the French grid, but changing its tap requires inter-TSO coordination.

Table 18: Specifications of the RTE Pilot test

Entity	Set	Set cardinality	Contingencies	Preventive participation	Corrective participation	Samples
Nodes	$\mathcal{I}$	450	-	-	-	-
Idealized switches	-	207	-	67	67	-
Lines	$\mathcal{L}$	641	86	-	-	-

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PSTs	-	6	-	2	2	-
Switchable lines	-	192	-	192	192	-
Units	$\mathcal{U}$	786	9	86	342	-
Generators	-	202	9	86	13	10
Loads	-	584	-	-	329	10

The N-1 and GARPUR RMAC-QP approaches are compared using the three-step approach as described within Section 5. All simulations are performed using setting as indicated in section 7.1, unless specified differently.

Twelve different snapshots (time steps, i.e. hours) have been analysed using two different real-time load/generation samples per snapshot. The starting point in the day-ahead step has balanced load and generation, such that no preventive dispatch needs to be performed to compensate for a possible imbalance.

Figure 17 shows the total cost of preventive actions, and corrective and blackout risk. We can see that there is no difference between the approaches. The reason for this is that the costs of real time preventive actions caused by the necessary load balancing actions by far overweigh the corrective and blackout risk, as in real time different load / generation samples are used. In the three-step approach the same contingency set is considered during the real-time preventive stage in both approaches, and the corrective contingency set is assessed on a contingency by contingency case. As such the RMAC-QP approach cannot use its full potential to find the trade-off between preventive and corrective actions as they are decoupled for the real-time problem.

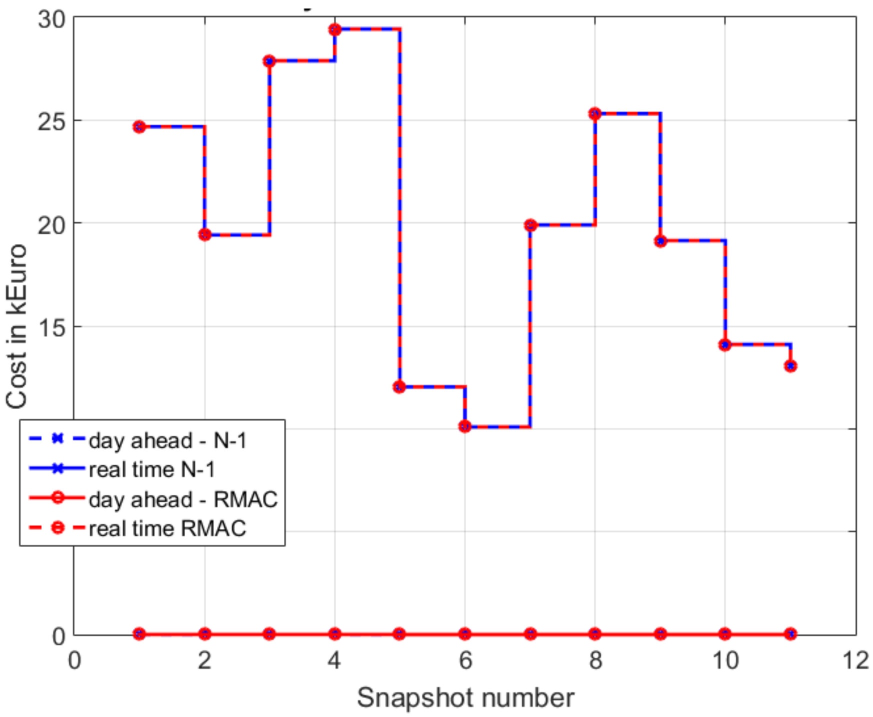

Figure 17: Comparison of the total risk (preventive cost + corrective and blackout risk) for the three-step N-1 and three-step RMAC-QP approach on the RTE test case

To have a better comparison, the N-1 approach is compared with the two-step RMAC-QP approach as described in section 5. The comparison is performed for one snapshot (i.e. one hour), as the

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computation time for the two-step RMAC-QP approach is significantly higher (Figure 18). The analysed case study foresees a number of possible topological actions, which results in a higher computation time (see previous section). As a result, very often the calculation time limit is reached before the global optimum solution is found. For this test, at the day-ahead step, the load and generation is assumed to be balanced.

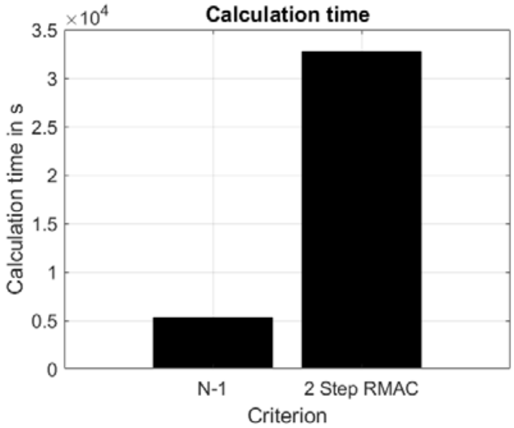

Figure 18: Comparison of calculation time for the three-step N-1 vs two-step RMAC-QP approach

Figure 19 shows a case where the global optimum result is found for the two-step RMAC-QP approach. The figure depicts the total risk of the N-1 approach and the two-step RMAC-QP approach respectively. The total risk values look very similar, however the total risk value of the two-step RMAC-QP approach is 2.1% lower than the N-1 approach. The reason for this is that the considered N-1 contingency set in both cases is the same, and the N-k contingencies have a much lower probability of occurring. For higher N-k contingency probabilities, it can be expected that the two-step RMAC-QP approach uses costlier real time preventive actions in order to avoid a larger real time corrective and blackout risk.

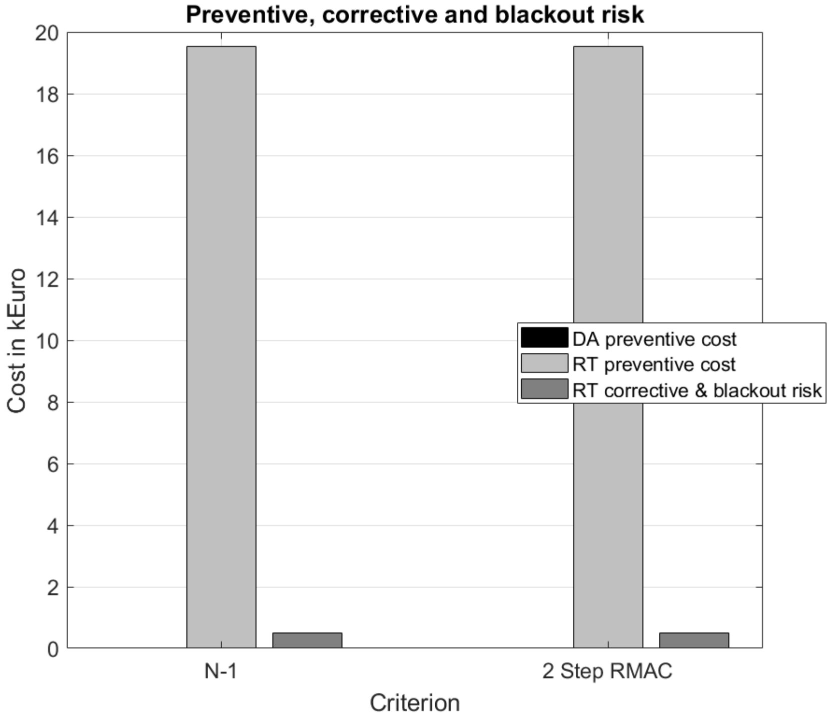

Figure 19: Comparison of total risk: N-1 approach vs. two-step RMAC-QP approach

7.4 Conclusions

From the performed test cases a number of conclusions can be drawn.

- The choice of the preventive and corrective actions and the associated risk depends on several factors such as contingency probabilities, the used acceptability limits and the probability of the failure of corrective actions. As such, the difference between the risk values obtained for the N-1 and the RMAC-QP approach can be both higher or lower. For instance, Figure 9 and Figure 10 illustrate the dependence of the total risk on the component failure rates and thus the contingency probabilities. We see that for small contingency probabilities the total risk for the N-1 and the RMAC-QP approach can be similar, whereas for higher contingency probabilities the corrective risk can be shifted to the preventive problem, e.g. using more expensive day-ahead dispatch to alleviate expensive re-dispatch in case a contingency happens.
- PST shift actions can be used both preventively and correctively, and offer more flexibility and have limited impact on the computational tractability.
- The inclusion of topological actions in the SCOPF increases the computation time significantly. In order to include topological actions in the SCOPF, better convexification techniques than the DC power flow approach need to be used in order to achieve feasibility of the nonlinear AC problem.
- In case acceptability constraints are not very tight, the impact on the risk of increasing failure rates of corrective actions is rather limited. However, with tight acceptability constraints, increasing failure rate of corrective actions can increase the occurrence of contingency discarding.

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- The selection of the preventive contingency set for the 2 and 3 step RMAC-QP approach heavily influences the associated risk value and the feasibility behaviour of the considered problem.
- To achieve even better reliability management, next to the uncertainty stemming from the unavailability of lines and generators, the uncertainty related to intermittent renewable generation (PV, wind) output should also be considered in the day-ahead preventive problem.

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8 RECOMMENDATIONS FOR AN INDUSTRY-GRADE QUANTIFICATION PLATFORM

The term “industry-grade” or “industrial-grade” is used to describe a software tool to contrast it from a “research-grade” prototype. The Oxford Dictionary defines industrial-grade as denoting that “a type or quality suitable, intended, or necessary for industrial use”. In the context of the GQP, we understand “industrial use” to mean the operational use by non-academic stakeholders such as TSOs and regulators. Using the definitions of Technology Readiness Level (TRL) used by the European Commission, this could correspond to TRL 7 (“system prototype demonstration in operational environment”). That a software tool is industry-grade does not necessarily mean that it is commercially available, but its applicability for the industry should be equivalent in terms of robustness, user friendliness and availability.

The industrial version of the GQP should be considered as a tool for quantitatively evaluating and comparing different reliability management approaches and criteria developed within GARPUR. As the implemented GQP prototype emulates system operation according to the GARPUR RMAC-QP through its evaluation, one could eventually envision a tool inspired by the GQP that is used to support actual system operation decisions. Using the GQP to compare different RMACs can support deciding the operational policies the TSO should adopt, which is also a reliability management decision.

This chapter suggests recommendations for the evolution towards an industry-grade quantification platform based on the lessons learnt during the development of the GARPUR prototype version of such a platform. Furthermore, most attention is given to system operation (rather than asset management and system development) because this is the context considered in the current version of the GQP. First, possible extensions of the scope and context of the current version of the GQP is discussed in Section 8.1 and specific potential long-term improvements are listed 8.2. Section 8.3 outlines the role of different stakeholders.

8.1 Identified improvement paths

A number of specific improvements of the GQP as a reliability management tool were identified during the development and testing of the GQP. Improvements can be described along different axes, and here three categories have been chosen:

- computational tractability;
- modelling detail and accuracy;
- interfaces and I/O.

The following subsections describe the identified improvement paths within the current scope of the GQP along these three axes.

8.1.1 Computational tractability

Computational tractability of the day-ahead and real-time reliability management process currently implemented in the GQP remains challenging. The main issues and potential improvements identified are:

- The interaction between day-ahead and real-time steps can be parallelized in conjunction with the sequential approach for the time steps.

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- Currently, a common computational approach is taken for both the DA and RT GARPUR RMAC. More customized approaches can be developed.
- The implementation can be optimized further (i.e. using the same model, but making it easier to solve) through:
    - Fine-tuning of specific formulations
    - Providing a high-quality initial solution (heuristic) for the B&B algorithm
    - Developing custom B&B (or branch-and-cut) algorithms
        - E.g. branching priorities or cut generation
        - These algorithms can be implemented on top of an existing solver through callbacks.
- The underlying algorithms (IP, B&B, decomposition techniques) can be improved, i.e. using better solvers for the same model.
    - Such an undertaking may be better suited for the operations research community rather than the power systems community.
- One could experiment with and develop decomposition approaches (using a similar/identical model, but making it faster to solve):
    - Decomposing the model by scenario (contingency, sample) or by stage
        - E.g. Benders decomposition, nested Benders decomposition, stochastic (dual) dynamic programming, approximate dynamic programming
- The available computation power should be increased
    - Use faster hardware
    - Using more computation nodes for parallel computation
        - Access computational power on demand in the cloud or high performance computing centre
- One could develop better heuristics and prove their quality (using a new model or algorithm that hopefully is faster and still accurate):
    - Develop algorithms to perform contingency discarding outside of the SCOPF
    - Develop advanced heuristics for topological actions
        - The expert knowledge of operators should be integrated in such heuristics.

8.1.2 Modelling detail and accuracy

The main requirements of the current implementation of the GARPUR RMAC in the GQP prototype were to support the evaluation of academic test cases and to support the RTE pilot test. The modelling features were developed to support these cases, but a broader perspective is to be considered. Identified possibilities for further improving the level of detail and the accuracy include:

- Full AC GARPUR SCOPF formulation
- Add detailed models for all power system components:
    - Modelling of automatic tap-changing in response to contingencies
    - Modelling of demand-response and storage
    - Modelling of HVDC and FACTS equipment
    - Modelling nonlinear PST impedance
    - Modelling voltage dependence of loads (ZIP model)
    - Modelling of islanding due to corrective actions
    - Modelling of short-circuit power, specifically in case of topological actions
- Extend to include multi-period aspects of reliability management optimization problems:

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- Including more stages, sequential implementation of actions, and energy storage.

- How to order multiple ‘simultaneous’ actions for operator. If multiple grid elements/units are involved in a single corrective action, it may make sense to consider the order of implementing the control signals explicitly.

- Relationship between weather conditions and the probability of failure and MTTR
- Develop accurate proxy for transient stability to include in quasi-static models such as the SCOPF in GQP
  - E.g. to improve the selection of topological actions in the model
- Add more options to achieve uncertainty modelling:
  - Support for robust optimization to capture specific realizations of uncertainty
  - Chance-constrained programming techniques to handle load and generation uncertainty (e.g. convex formulations of single-sided and double-side chance constraints)
- Returning to *secure* state after occurrence of contingency which is correctively secured
- Different power flow formulations for different stages
  - Trade-off between accuracy and computational requirements, e.g. focus the computational resources on the preventive stage and use convex relaxation model for the corrective stage
  - Or an use accurate model (e.g. [Kocuk,2015]) for preventive decisions, in combination with approximate model for short-term post-contingency and corrective stages.

8.1.3 Interfaces and I/O

Finally, a GARPUR RMAC implementation can be used in a number of settings, interacting with different users and different systems. Depending on the application, interfaces for humans (operators) and machines (computer systems) need to be developed. This is particularly important if the implementation is to be used as an operational decision support tool for reliability control.

8.1.3.1 Human operators

- Risk should be quantify and presented in a way that is meaningful to an operator, i.e. it should provide useful decision support to the decision maker.
  - Visualization of weather-dependence of the risk
  - Vizualization of critical failure paths
- Interactive visualization, exploration and comparison of results
  - Advanced approaches for multidimensional visualization
- Guidelines for operator interfaces should be developed for decision support tools based on the GARPUR RMAC, considering interpretation and psychology of decision making under time pressure
  - There could be lessons to be learnt from other industries, cf. e.g. the analogy to autopilots in airplanes.

8.1.3.2 Other

- APIs to SCADA/EMS, supported by the vendors
  - Potentially use an API in the EMS to allow a reliability management tool to use the system response model implemented within the EMS

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- Broader support for data import and export (CIM15, CIM16, PSSE, ...)
    - Relevant data formats include e.g. CIM15, CIM16, and PSS/E.
    - The extended set of reliability data required for the GARPUR RMAC could be integrated as part of the CIM format.
    - The recommendations of D8.4 of the iTesla [iTesla,2015b] project regarding exchange of contingencies and reliability criteria (e.g. using reliability criteria definitions developed as parameterizations) between TSOs should be followed up.
- Interface for scripting the set-up of studies and test routines
- A database of standard test cases should be available

8.2 Long-term evolution of scope of GQP

In its current implementation, the GQP prototype is built around a SCOPF calculation core, however, the scope of this core can be expanded. A number of directions in which it can be expanded are listed below.

- Security-constrained market-clearing (energy, reserves, ancillary)
    - In the current version, market clearing and reliability management are considered sequentially. However, eventually market clearing and reliability management could be performed simultaneously, for example in day-ahead
- Reliability management contexts with longer time horizons (mid- and long-term planning)
    - Asset management and system development
- Multi-TSO coordination of reliability management
- Not separating state estimation and real-time reliability management
    - Today, in operation, state estimation and reliability management are performed sequentially. Nevertheless, measurement uncertainty can be considered explicitly in the reliability management approach.
- Uncertainty can be dealt with in multiple ways:
    - Robustness against data uncertainty as well as against erroneous data
    - Hedging risk against continuous uncertainty sources, e.g. variability and correlation in renewable (wind and PV) generation
    - Dealing with uncertainty of the behaviour of market players
    - Improved uncertainty modelling, i.e. in representing distributions/correlations behind contingencies.

8.3 Stakeholder groups for potential GQP exploitation

The objective of the use of a reliability management tool varies with the interests of the stakeholder. A number of categories of stakeholders for the GARPUR quantification platform and their potential interests are identified and summarized in Table 19.

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Table 19: Identification of stakeholder groups with interest in reliability management tools

Group		Focus
TSO (operations)	Inside consortium	Convince management to use more advanced reliability management approaches Experiment with applying probabilistic approaches and/or improved reliability management tool operationally (for decision support)
	Outside consortium	Experiment with probabilistic approaches, to improve understanding of concepts and their benefits
TSO (planning)		Advocate for research and development of reliability management tool for long-term applications
DSOs		Start to adapt more advanced and probabilistic reliability management approaches
Coordination entities	ENTSO-E	Consider probabilistic approaches in adapting and developing network codes and guidelines for cross-border reliability management Develop planning guidelines based on probabilistic reliability assessment and evaluate these guidelines through simulation
	TSCNET, CORESO	Perform coordination of cross-border reliability management
Regulators	ACER	Develop incentives for system operators to improve reliability management. Investigate cross-border reliability implications
	National regulators	Develop incentives for system operators to improve reliability management.
Consulting firms	e.g.Tractebel, Ecofys, 3E, Technofi, …	Adopting tool for carrying out grid reliability studies for other stakeholders
Academia & research institutes		Acquire knowledge of implementation of ingredients of the GARPUR RMAC in GQP Continue development, in particular on approaches for visualization and presenting results to users
Power system software vendors	e.g. Digsilent, Neplan, GE, Siemens PTI, ABB…	Continue research and development, improving modelling detail Developing improved computational approaches

The interests listed above primarily relate to the GQP applied as a tool for evaluating reliability management approaches. Today, TSOs, coordination entities, and regulators are considered the most relevant stakeholders and the first-line potential users of GQP. Since a prototype version of a reliability management tool as the GQP requires a strong competence background and significant training to use for real cases, it is advantageous if this use can be supported by the research community. A more industry-grade version, in which it is less demanding to set up cases and interpret results, could more easily be adapted by other potential users, including consulting firms and TSOs that are not already familiar with the underlying concepts. The stakeholders listed above could also have interests in the application of a reliability management tool without being a potential user. For instance, power system software developers could be interested in adopting elements of the implementation in the GQP in their own SCOPF software tools.

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9 CONCLUSIONS

The tests carried out using the current implementation of the GARPUR GQP only address the system operation (i.e. short-term) aspects of reliability management. In other words, in the work reported in the present deliverable, potential uses of the GARPUR GQP as concerns the benefits of using a probabilistic RMAC in the context of asset management and system development are not considered. This therefore remains for future work. However, complementary results on the potential benefits of the GARPUR approach to all these three reliability management contexts have been developed in other work packages of the GARPUR project.

The GQP makes risks explicit through a SCOPF implementation. It quantifies and shows the break-down of the socioeconomic components (preventive costs, corrective risk and blackout risk) of different solutions. Such explicit quantification is also useful when considering conventional N-k criteria. The benefit of using a probabilistic approach is more significant if probabilities of different contingencies significantly differ from each other.

For the application of data in probabilistic reliability management, TSOs will have to gain experience collecting this data and safeguarding the quality. Data quality and data formats are to be standardized to be able to exchange them efficiently. CIM has made significant steps, but further extensions towards probabilistic data are required.

The consequences of contingencies need to be modelled in higher detail. E.g. corrective topological actions can lead to unwanted system transients which are not assessed with the current version of the GQP, whereas the assumption that failure of a corrective action would result in a blackout is overly conservative. In reality this will not always be the case, as other back-up actions can be taken in case of failure of corrective actions.

The computational challenges at hand are significant. As noted, a full AC SCOPF implementation of the GARPUR RMAC ticks a number of boxes which each have notable complexity:

1. large-scale due to the nature of the stochastic optimization model,
2. with binary variables required in the modelling of the contingency discarding and the indicator variables for the corrective actions (and others),
3. with nonconvex constraints due to the (AC) power flow physics.

These problems can be classified as large-scale mixed-integer optimal power flow problems. Solution methods specific to such problems are to be researched in future projects. We refer the reader to [Garpur,2016c] for a discussion of a proof-of-concept algorithmic solution approach to the AC-SCOPF implementation of the GARPUR RMAC.

The full potential of the GARPUR approach is not realized when decomposing the reliability management into a three-step problem (day-ahead, real-time preventive and real-time corrective). However, solving it as a two-step problem (day-ahead and real-time) results in much higher computational requirements.

Even if the total risk value between the N-1 approach and the GARPUR RMAC differs only in a limited way for most hours of the year, the cumulative benefits over a longer horizon could be significant. However, this is to be assessed in future tests.

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Given the GQP is the first tool of its kind, its validation is particularly difficult. To do so a strong collaboration between the developers and the operators is required. Adding a graphical interface would be a logical next step. As already mentioned, effective visualization of the results is of utmost importance.

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11 APPENDIX I

11.1 Line parameters of the SCOPF model

Table 20: Overview of line parameters

Symbol	Meaning (physical unit)
$y_{l,s}$	Series admittance (Siemens)
$g_{l,s}$	Series conductance (Siemens)
$b_{l,s}$	Series susceptance (Siemens)
$z_{l,s}$	Series impedance (Ohm)
$r_{l,s}$	Series resistance (Ohm)
$x_{l,s}$	Series reactance (Ohm)
$g_{lij,\mathrm{sh}}$	Shunt conductance at the i side of the line (Siemens)
$b_{lij,\mathrm{sh}}$	Shunt susceptance at the i side of the line (Siemens)
$g_{lji,\mathrm{sh}}$	Shunt conductance at the j side of the line (Siemens)
$b_{lji,\mathrm{sh}}$	Shunt susceptance at the j side of the line (Siemens)
$\varphi_{l,ij}^{\mathrm{ref}}$	Reference phase shift at the i side of the line (rad)
$\varphi_{l,ij}^{\mathrm{min}}$	Minimum phase shift at the i side of the line (rad)
$\varphi_{l,ij}^{\mathrm{max}}$	Maximum phase shift at the i side of the line (rad)
$\varphi_{l,ji}^{\mathrm{ref}}$	Reference phase shift at the j side of the line (rad)
$\varphi_{l,ji}^{\mathrm{min}}$	Minimum phase shift at the j side of the line (rad)
$\varphi_{l,ji}^{\mathrm{max}}$	Maximum phase shift at the j side of the line (rad)
$\rho_{l,ij}^{\mathrm{ref}}$	Reference voltage magnitude ratio at the i side of the line (-)
$\rho_{l,ji}^{\mathrm{ref}}$	Reference voltage magnitude ratio at the j side of the line (-)
α_l^{ref}	Reference line state (0/1)
c_l^{switch}	Switching action cost (€)
c_l^{PST}	PST shifting action cost (€)
τ_l	Mean time to repair after failure (s)
λ_l	Failure rate (1/s)
π_l^{fail}	Probability of failure of a corrective line action (-)
S_{lij}^{rated}	Apparent power flow rating (long-term) (VA)
I_{lij}^{rated}	Current rating (long-term) (A)
$S_{lij}^{\mathrm{rated,ST}}$	Apparent power flow rating (short-term) (VA)
$I_{lij}^{\mathrm{rated,ST}}$	Current rating (short-term) (A)
$\varphi_{l,ij}^{\mathrm{rated}}$	Rated phase shift at the i side of the line (rad)
$\varphi_{l,ji}^{\mathrm{rated}}$	Rated phase shift at the j side of the line (rad)

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11.2 OPF Formulation

11.2.1 AC Power flow

The nonconvex power flow equations can be derived as:

$$P_{lij} = (g_{l,s} + g_{lij,\mathrm{sh}}) \cdot (U'_{lij})^2 - g_{l,s} U'_{lij} U'_{lji} \cos(\theta'_{lij} - \theta'_{lji}) - b_{l,s} U'_{lij} U'_{lji} \sin(\theta'_{lij} - \theta'_{lji})$$

$$Q_{lij} = -(b_{l,s} + b_{lij,\mathrm{sh}}) \cdot (U'_{lij})^2 + b_{l,s} U'_{lij} U'_{lji} \cos(\theta'_{lij} - \theta'_{lji}) - g_{l,s} U'_{lij} U'_{lji} \sin(\theta'_{lij} - \theta'_{lji})$$

These equations are nonconvex and represent one of the major hurdles in the tractability of power system optimization. The above formulation, being the common 'polar' formulation is commonly used in AC OPF solvers, in combination with a NLP interior-point algorithm.

The power flow through a line can be bound by the apparent power rating or current rating

$$(P_{lij})^2 + (Q_{lij})^2 \le (S_{lij}^{\mathrm{rated}})^2,$$

$$\frac{(P_{lij})^2 + (Q_{lij})^2}{(U_i)^2} \le (I_{lij}^{\mathrm{rated}})^2.$$

11.2.2 Line aggregation by node

The power flow through the lines connected to a certain node i is defined as follows:

$$P_i^{\mathrm{lines}} = \sum_{lij \in \mathcal{T}^{\mathrm{bidir.}}} P_{lij},$$

$$Q_i^{\mathrm{lines}} = \sum_{lij \in \mathcal{T}^{\mathrm{bidir.}}} Q_{lij}.$$

Note that these variables can be bounded by:

$$-\sum_{lij \in \mathcal{T}} S_{lij}^{\mathrm{rated}} \le P_i^{\mathrm{lines}} \le \sum_{lij \in \mathcal{T}} S_{lij}^{\mathrm{rated}},$$

$$-\sum_{lij \in \mathcal{T}} S_{lij}^{\mathrm{rated}} \le Q_i^{\mathrm{lines}} \le \sum_{lij \in \mathcal{T}} S_{lij}^{\mathrm{rated}}.$$

11.2.3 Unit power generation and consumption

Loads and generators are considered as active and reactive power sinks and sources. A load can be considered as a negative generator. In this document, both loads and generators are referred to as 'units'. The parameters needed to initialize unit models are listed in Table 21.

Table 21: Overview of unit parameters

Symbol	Meaning
P_u^{ref}	Reference active power dispatch (W)
P_u^{min}	Minimum active power during operation (W)
P_u^{max}	Minimum active power during operation (W)
Q_u^{ref}	Reference reactive power dispatch (vary)
Q_u^{min}	Minimum reactive power during operation (var)
Q_u^{max}	Minimum reactive power during operation (var)
c_u	(Re)dispatch cost (€/J)

The research leading to these results has received funding from the European Union Seventh Framework Programme under Grant Agreement No 608540.

c_u^{quad}	(Re)dispatch cost (€/J^2)
c_u^{start}	Start-up cost (€/-)
c_u^{stop}	Shutdown cost (€/-)
k_u	Participation factor in automatic generation control (W/W)
τ_u	Mean time to repair after failure (s)
λ_u	Failure rate (1/s)
S_u^{rated}	Apparent power rating (VA)

The power rating of a unit can be derived as a circular constraint

$$(P_u)^2 + (Q_u)^2 \leq \left(S_u^{\text{rated}}\right)^2$$

Or as box constraints:

$$-S_u^{\text{rated}} \leq P_u \leq S_u^{\text{rated}}$$
$$-S_u^{\text{rated}} \leq Q_u \leq S_u^{\text{rated}}$$

Further extensions can be developed, e.g. field and armature apparent power limits for synchronous generators. Furthermore, reactive power can be used to regulate the grid voltage, thereby adding a voltage equality constraint in the node the unit is connected to.

The state of the unit i_u involves a disjunction:

$$\begin{bmatrix} P_u^{\min} \leq P_u \leq P_u^{\max} \\ Q_u^{\min} \leq Q_u \leq Q_u^{\max} \\ i_u = 1 \text{ (on)} \end{bmatrix} \vee \begin{bmatrix} P_u = 0 \\ Q_u = 0 \\ i_u = 0 \text{ (off)} \end{bmatrix}$$

This notation means that either the unit is in the on-state, in which the power dispatch is flexible between a minimum and a maximum, or (symbol: ∨) the unit is in the off-state, in which its power output is zero.

11.2.4 Unit aggregation by node

The units can also be aggregated based on the node they are connected to:

$$P_i^{\text{units}} = \sum_{ui \in G} P_u$$
$$Q_i^{\text{units}} = \sum_{ui \in G} Q_u$$

Note that, similar to the lines, bounds can be derived on these variables as follows:

$$\sum_{ui \in G(i)} S_u^{\text{rated}} \leq P_i^{\text{units}} \leq \sum_{ui \in G(i)} S_u^{\text{rated}},$$
$$\sum_{ui \in G(i)} S_u^{\text{rated}} \leq Q_i^{\text{units}} \leq \sum_{ui \in G(i)} S_u^{\text{rated}}.$$

11.2.5 Bus shunts

Shunts $g_i, b_i$are assumed to be aggregated by bus:

$$P_i^{\text{shunt}} = g_i \cdot (U_i)^2,$$
$$Q_i^{\text{shunt}} = -b_i \cdot (U_i)^2.$$

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11.2.6 Node balance

Finally, at each grid node the balance between power transfer through the lines connected to it, the bus shunts and the local loads and generators must be satisfied:

$$P_i^{\text{lines}} = P_i^{\text{units}} + P_i^{\text{shunt}},$$
$$Q_i^{\text{lines}} = Q_i^{\text{units}} + Q_i^{\text{shunt}}.$$

11.2.7 Node voltages

Node voltages U_i must be within the long-term limits $U_i^{\text{min}}, U_i^{\text{max}}$:

$$U_i^{\text{min}} \le U_i \le U_i^{\text{max}}.$$

In the short-term post-contingency stage, the short-term ratings $U_i^{\text{ST,min}}, U_i^{\text{ST,max}}$ are considered instead:

$$U_i^{\text{ST,min}} \le U_i \le U_i^{\text{ST,max}}.$$

where $U_i^{\text{max}} \le U_i^{\text{ST,max}}, U_i^{\text{min}} \ge U_i^{\text{ST,min}}$

11.2.8 Switches

A disjunctive model of a switch is:

$$\begin{bmatrix} U_i = U_{lij}^* \\ \theta_i = \theta_{lij}^* \\ \alpha_l = 1 \text{ (closed)} \end{bmatrix} \vee \begin{bmatrix} U_{lij}^* = 0 \\ \theta_{lij}^* = 0 \\ \alpha_l = 0 \text{ (open)} \end{bmatrix}$$

11.2.9 Transformer

Only fixed transformer taps are considered:

$$\rho_{lij}^{\text{ref}} = \frac{U'_{lij}}{U_{lij}^*}$$

11.2.10 PST

Only if a PST is in operation, it can perform a phase shift:

$$\begin{bmatrix} \varphi_{l,ij}^{\text{min}} \le \varphi_{l,ij} \le \varphi_{l,ij}^{\text{max}} \\ \alpha_l = 1 \text{ (closed)} \end{bmatrix} \vee \begin{bmatrix} \varphi_{l,ij} = 0 \\ \alpha_l = 0 \text{ (open)} \end{bmatrix}$$

11.3 SCOPF Formulation

11.3.1 Definition of contingency sets and parameters

The symbols used to define specific contingency sets are listed in Table 22

Table 22: Definition of specific contingency sets

Symbol	Meaning
$\mathcal{C}$	Set of all possible contingencies
$\mathcal{C}^{\text{prev}}$	Subset of purely preventively secured contingencies
$\mathcal{C}^{\text{prev-corr}}$	Subset of preventive-correctively secured contingencies

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$\mathcal{C}^{\text{secure}}$	Subset of secured contingencies
$\mathcal{C}^{\text{insecure}}$	Subset of contingencies not explicitly secured
$\mathcal{C}^{\text{not selected}}$	Subset of contingencies not selected for securing
$\mathcal{C}^{\text{discarded}}$	Subset of discarded contingencies
$\mathcal{C}^{\text{prob}}(\pi^{\text{min}}, \mathcal{C})$	Subset of contingencies from a set $\mathcal{C}$ that have a probability of at least π^{min}
$\mathcal{C}^{\text{N}-0}$	No-contingency case
$\mathcal{C}^{\text{N}-\text{k}}$	Subset of N-k contingencies

11.3.1.1 Contingency parameters

Parameters $C_{l,c}, C_{u,c}$ describe the availability (1) / unavailability (0) of lines and units respectively under each contingency c. Multiple lines and units can be unavailable at the same time. For each specific contingency, depending on the type, e.g. N-1, N-2, N-k, a specific energy-not-served $E_c^{\text{not-served,max}}$ and power-not-served $P_c^{\text{not-served,max}}$ limit can be defined. E.g. a higher ENS and PNS may be acceptable for more severe contingencies. Contingency parameters are defined in Table 23.

Table 23: Overview of contingency parameters

Symbol	Meaning
N_c^{lines}	Set of lines available during contingency c (-)
F_c^{lines}	Set of lines unavailable during contingency c (-)
N_c^{units}	Set of units available during contingency c (-)
F_c^{units}	Set of units unavailable during contingency c (-)
$C_{u,c}$	Unit u unavailable during contingency c if $C_{u,c} = 0$, otherwise $C_{u,c} = 1$
$C_{l,c}$	Line l unavailable during contingency c if $C_{l,c} = 0$, otherwise $C_{l,c} = 1$
$P_c^{\text{not-served,max}}$	Maximum power-not-served during contingency c (W)
$E_c^{\text{not-served,max}}$	Maximum energy-not-served during contingency c (J)
$k_{u,c}^*$	AGC constant of unit u during contingency c (W/W)
π_c	Probability of contingency c (-)
τ_c^{MTTR}	Mean-time-to-repair of contingency c (s)

Now, either the τ_c^{MTTR} is known for each contingency, or it is estimated using the MTTR for individual line τ_l or load/generator failures τ_u. For instance, assuming the MTTR depends on the element with the highest MTTR:

$$\tau_c^{\text{MTTR}} = \max\left(\max_{u\in U}\left(\left(1 - C_{u,c}\right) \cdot \tau_u\right), \max_{l\in L}\left(\left(1 - C_{l,c}\right) \cdot \tau_l\right)\right).$$

The individual line or unit contingency probabilities are calculated as

$$\pi_u(\Delta T) = 1 - e^{-\lambda_u \cdot \Delta T}$$

$$\pi_l(\Delta T) = 1 - e^{-\lambda_l \cdot \Delta T}$$

where ΔT is the time horizon and e is the natural logarithm. Note that these parameters can be considered as a function of exogenous parameters, e.g. the weather. The probability of simultaneous failure of multiple elements during a contingency can now be calculated as:

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$$\pi_c = \underbrace{\prod_{u\in F_c^{\text{units}}} \pi_u \prod_{u\in N_c^{\text{units}}} (1-\pi_u)}_{\text{probability of unit state realization}} \underbrace{\prod_{l\in F_c^{\text{lines}}} \pi_l \prod_{l\in N_c^{\text{lines}}} (1-\pi_l)}_{\text{probability of line state realization}}$$

11.3.2 Equipment variables in different stages

11.3.2.1 Unit state

The unit's preventive state equals that of the reference stage unless a start-up or shutdown occurs.

$$i_u = i_u^{\text{ref}} + i_u^{\Delta}$$

The short-term post-contingency stage after the occurrence of the contingency:

$$i_{u,c}^{\text{ST}} = C_{u,c} \cdot i_u$$
$$i_{u,c}^{\text{corr}} = i_{u,c}^{\text{ST}} + i_{u,c}^{\Delta}$$

A corrective start-up/shutdown is disabled for the unit which was involved in the contingency:

$$-C_{u,c} \le i_{u,c}^{\Delta} \le C_{u,c}$$

11.3.2.2 Unit active power

The active power dispatch is defined analogously. Furthermore, certain generators can participate in automatic generation control actions $P_{u,c}^{\text{ST}\Delta} \neq 0$ during the short-term post-contingency stage to maintain the power balance in the system.

$$P_u = P_u^{\text{ref}} + P_u^{\Delta}$$
$$P_{u,c}^{\text{ST}} = C_{u,c} \cdot P_u + P_{u,c}^{\text{ST}\Delta}$$
$$P_{u,c}^{\text{corr}} = P_{u,c}^{\text{ST}} + P_{u,c}^{\Delta}$$

Corrective redispatch is disabled for the unit which underwent the contingency

$$-C_{u,c} \le P_{u,c}^{\Delta} \le C_{u,c}$$
$$-C_{u,c} \le P_{u,c}^{\text{ST}\Delta} \le C_{u,c}$$

11.3.2.3 Unit reactive power

The reactive power dispatch is defined analogously.

$$Q_u = Q_u^{\text{ref}} + Q_u^{\Delta}$$
$$Q_{u,c}^{\text{ST}} = C_{u,c} \cdot Q_u + Q_{u,c}^{\text{ST}\Delta}$$
$$Q_{u,c}^{\text{corr}} = Q_{u,c}^{\text{ST}} + Q_{u,c}^{\Delta}$$
$$-C_{u,c} \le Q_{u,c}^{\Delta} \le C_{u,c}$$
$$-C_{u,c} \le Q_{u,c}^{\text{ST}\Delta} \le C_{u,c}$$

11.3.2.4 Line state

The line state is defined analogously:

$$\alpha_l = \alpha_l^{\text{ref}} + \alpha_l^{\Delta}$$
$$\alpha_{l,c}^{\text{ST}} = C_{l,c} \cdot \alpha_l$$
$$\alpha_{l,c}^{\text{corr}} = \alpha_{l,c}^{\text{ST}} + \alpha_{l,c}^{\Delta}$$
$$-C_{l,c} \le \alpha_{l,c}^{\Delta} \le C_{l,c}$$

11.3.2.5 Line phase shift

The line shift is defined analogously:

$$\varphi_{l,ij} = \varphi_{l,ij}^{\text{ref}} + \varphi_{l,ij}^{\Delta}$$
$$\varphi_{l,ij,c}^{\text{ST}} = C_{l,c} \cdot \varphi_{l,ij}$$

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$$\varphi^{\mathrm{corr}}_{l,ij,c} = \varphi^{\mathrm{ST}}_{l,ij,c} + \varphi^{\Delta}_{l,ij,c}$$
$$-C_{l,c} \leq \varphi^{\Delta}_{l,ij,c} \leq C_{l,c}$$

11.4 GARPUR SCOPF Extensions

11.4.1 Unit dispatch

The power rating of a load or generator is considered as:
$$-S^{\mathrm{rated}}_u \leq P_u \leq S^{\mathrm{rated}}_u,$$
$$-S^{\mathrm{rated}}_u \leq Q_u \leq S^{\mathrm{rated}}_u.$$
A generator's or load's state is either on or off:
$$i_u \in \{0,1\}.$$
If the unit is on, it should operate between its minimum and maximum operational limits
$$P^{\mathrm{min}}_u \cdot i_u \leq P_u \leq P^{\mathrm{max}}_u \cdot i_u,$$
$$Q^{\mathrm{min}}_u \cdot i_u \leq Q_u \leq Q^{\mathrm{max}}_u \cdot i_u.$$
A change in state reflects either a start-up or shutdown:
$$i^{\Delta}_u = i^{\mathrm{start}}_u - i^{\mathrm{stop}}_u, i^{\mathrm{start}}_u \in \{0,1\}, i^{\mathrm{stop}}_u \in \{0,1\}.$$
It cannot be in shutdown and start-up simultaneously:
$$i^{\mathrm{start}}_u + i^{\mathrm{stop}}_u \leq 1.$$
These variables allow to define the cost of start-up and shutdown:
$$K^{\mathrm{startstop}}_u = c^{\mathrm{start}}_u i^{\mathrm{start}}_u + c^{\mathrm{stop}}_u i^{\mathrm{stop}}_u.$$
Either the quadratic polynomial dispatch costs can be considered:
$$K^{\mathrm{disp}}_u = c_u P_u T + c^{\mathrm{quad}}_u (P_u T)^2.$$
Which are convexified in the implementation, but is met in equality in the optimum, as the costs are minimized:
$$\Rightarrow^{\mathrm{convex}} K^{\mathrm{disp}}_u \geq c_u P_u T + c^{\mathrm{quad}}_u (P_u T)^2.$$
Alternatively, the redispatch can be considered:
$$K^{\mathrm{redisp}}_u = c_u |P^{\Delta}_u T|.$$
Which can be convexified as:
$$K^{\mathrm{redisp}}_u = c_u |P^{\Delta}_u T| \Rightarrow^{\mathrm{convex}} K^{\mathrm{redisp}}_u \geq c_u |P^{\Delta}_u T| \Leftrightarrow \begin{cases} K^{\mathrm{redisp}}_u \geq c_u P^{\Delta}_u T \\ K^{\mathrm{redisp}}_u \geq -c_u P^{\Delta}_u T \end{cases}.$$
Big M integer-linear representations of maximum, minimum and absolute value, which do not introduce slack, could be used as well. However, it is not expected that there is slack on these operators, as the costs are being minimized. Furthermore, this convex relaxation formulation results in a problem with fewer binary variables, and therefore higher tractability.

11.4.2 Switching actions

Complex power flow through a line satisfies the following loss model for a pi-section:
$$P_{lij} + P_{lji} = P^{\mathrm{loss}}_l = P^{\mathrm{loss}}_{lij,\mathrm{sh}} + P^{\mathrm{loss}}_{l,s} + P^{\mathrm{loss}}_{lji,\mathrm{sh}},$$
$$Q_{lij} + Q_{lji} = Q^{\mathrm{loss}}_l = Q^{\mathrm{loss}}_{lij,\mathrm{sh}} + Q^{\mathrm{loss}}_{l,s} + Q^{\mathrm{loss}}_{lji,\mathrm{sh}},$$
The state of a switch is either on or off:
$$i_l \in \{0,1\}.$$
The circular bounds can be used to force the power flow through the switch to zero (SOC):

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$$\left(P_{lij}\right)^2 + \left(Q_{lij}\right)^2 \leq \left(S_{lij}^{\mathrm{rated}}\right)^2 \cdot \alpha_l.$$

Alternatively, box constraints can be used:

$$-S_{lij}^{\mathrm{rated}} \cdot \alpha_l \leq P_{lij} \leq S_{lij}^{\mathrm{rated}} \cdot \alpha_l,$$
$$-S_{lij}^{\mathrm{rated}} \cdot \alpha_l \leq Q_{lij} \leq S_{lij}^{\mathrm{rated}} \cdot \alpha_l.$$

Switching costs are calculated as:

$$K_l^{\mathrm{switch}} = c_l^{\mathrm{switch}} \left|\alpha_l^{\Delta}\right|.$$

And reformulated as

$$\Rightarrow^{\mathrm{convex}} \begin{cases} K_l^{\mathrm{switch}} \geq c_l^{\mathrm{switch}} \alpha_l^{\Delta} \\ K_l^{\mathrm{switch}} \geq -c_l^{\mathrm{switch}} \alpha_l^{\Delta} \end{cases}.$$

11.4.3 PST actions

The PST actions are only possible when the line is in operation ($\alpha_l = 1$)

$$\varphi_{l,ij}^{\min} \cdot \alpha_l \leq \varphi_{l,ij} \leq \varphi_{l,ij}^{\max} \cdot \alpha_l,$$
$$\theta'_{l,ij} = \theta_{l,ij}^{*} + \varphi_{l,ij}.$$

The clause 'if $\varphi_{l,ij}^{\Delta} \neq 0$ then $\mu_l = 1$' is implemented using binary variable μ_l, which indicates the use of a PST action:

$$\mu_l, \mu_{l,c}^{\mathrm{corr}} \in \{0,1\},$$
$$2 \cdot \varphi_{l,ij}^{\mathrm{rated}} \cdot \mu_l \leq \varphi_{l,ij}^{\Delta} \leq 2 \cdot \varphi_{l,ij}^{\mathrm{rated}} \cdot \mu_l.$$
$$2 \cdot \varphi_{l,ij}^{\mathrm{rated}} \cdot \mu_{l,c}^{\mathrm{corr}} \leq \varphi_{l,ij,c}^{\Delta} \leq 2 \cdot \varphi_{l,ij}^{\mathrm{rated}} \cdot \mu_{l,c}^{\mathrm{corr}}.$$

The reverse, if $\varphi_{l,ij}^{\Delta} = 0$ then $\mu_l = 0$, is not guaranteed. However, this can be avoided by using a cost for PST actions, which often also improves calculation speed. As TSOs consider PST actions usually as free, in practice this cost should be very small compared to e.g. generation redispatch cost.

11.4.4 AGC actions

11.4.4.1 Power lost due to the contingency occurrence

AGC actions account for the power generation lost due to a contingency in re-establishing the short-term post-contingency power balance, e.g. through frequency control mechanisms. For each contingency, the complex power immediately following the contingency c, i.e. $P_c^{\mathrm{ST,lost}}, Q_c^{\mathrm{ST,lost}}$, is calculated as:

$$P_c^{\mathrm{ST,lost}} = \sum_{u \in U} P_u - P_u \cdot C_{u,c},$$
$$Q_c^{\mathrm{ST,lost}} = \sum_{u \in U} Q_u - Q_u \cdot C_{u,c}.$$

11.4.4.2 Proportionality factors for each contingency

Each generator is associated with a nonnegative constant k_u.

$$k_u \geq 0$$

In each problem, at least one generator must have a strictly positive constant, or:

$$\sum_{u \in U} k_u > 0.$$

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Then, for each contingency, a new, normalized constant is derived, which takes into account the occurrence of generator contingencies:

$$k^*_{u,c} = \frac{k_u \cdot C_{u,c}}{\sum_{u \in U} k_u \cdot C_{u,c}}.$$

Now, the generator constants for each contingency are guaranteed to be in the range of [0,1]:

$$0 \leq k^*_{u,c} \leq 1.$$

Finally, for each individual contingency, the generator constants add up to 1:

$$\sum_{u \in U} k^*_{u,c} = 1.$$

11.4.4.3 Short-term post-contingency power balance

Each generator's AGC action $P^{\mathrm{ST}\Delta}_{u,c}$ is proportional to the constant $k^*_{u,c}$.

$$P^{\mathrm{ST}\Delta}_{u,c} = k^*_{u,c} \cdot P^{\mathrm{ST}\Delta}_{c}$$
$$Q^{\mathrm{ST}\Delta}_{u,c} = k^*_{u,c} \cdot Q^{\mathrm{ST}\Delta}_{c}$$

In general, the power lost due to the contingency is similar to the power replaced due to the AGC actions, however they are not identical due to the impact on the grid losses:

$$P^{\mathrm{ST}\Delta}_{c} \approx P^{\mathrm{ST,lost}}_{c},$$
$$Q^{\mathrm{ST}\Delta}_{c} \approx Q^{\mathrm{ST,lost}}_{c}.$$

Nevertheless, in lossless power flow formulations such as 'DC' OPF, the power lost always equals the power replaced.

11.4.5 Indicator variables for corrective actions

11.4.5.1 Indicator variables for lines

If a line switches correctively, or a PST shifts correctively, the indicator variable $\nu^{\mathrm{corr}}_{l,c}$ is set to a unity value.

$$\nu^{\mathrm{corr}}_{l,c} \in \{0,1\}$$

The clause 'if $\mu^{\mathrm{corr}}_{l,c} = 1$ then $\nu^{\mathrm{corr}}_{l,c} = 1$' is implemented as:

$$\mu^{\mathrm{corr}}_{l,c} \leq \nu^{\mathrm{corr}}_{l,c}.$$

The clause 'if $\alpha^{\Delta}_{l,c} \neq 0$ then $\nu^{\mathrm{corr}}_{l,c} = 1$' is formulated as:

$$\alpha^{\Delta}_{l,c} \leq \nu^{\mathrm{corr}}_{l,c}, \qquad -\alpha^{\Delta}_{l,c} \leq \nu^{\mathrm{corr}}_{l,c}.$$

11.4.5.2 Indicator variables for units

If a unit starts or stops, or is redispatched, the variable $\nu^{\mathrm{corr}}_{u,c}$ is forced to a unity value.

$$\nu^{\mathrm{corr}}_{u,c} \in \{0,1\}$$

The clause 'if $i^{\Delta}_{u,c} \neq 0$ then $\nu^{\mathrm{corr}}_{u,c} = 1$' is implemented as:

$$i^{\Delta}_{u,c} \leq \nu^{\mathrm{corr}}_{u,c}, \qquad -i^{\Delta}_{u,c} \leq \nu^{\mathrm{corr}}_{u,c}.$$

The clause 'if $P^{\Delta}_{u,c} \neq 0$ or $Q^{\Delta}_{u,c} \neq 0$ then $\nu^{\mathrm{corr}}_{u,c} = 1$' is implemented as:

$$-2 \cdot S^{\mathrm{rated}}_{u} \cdot \nu^{\mathrm{corr}}_{u,c} \leq P^{\Delta}_{u,c} \leq 2 \cdot S^{\mathrm{rated}}_{u} \cdot \nu^{\mathrm{corr}}_{u,c},$$
$$-2 \cdot S^{\mathrm{rated}}_{u} \cdot \nu^{\mathrm{corr}}_{u,c} \leq Q^{\Delta}_{u,c} \leq 2 \cdot S^{\mathrm{rated}}_{u} \cdot \nu^{\mathrm{corr}}_{u,c}.$$

11.4.6 Contingency discarding

The indicator variable for the inclusion or discarding of a contingency is β_c. If a contingency is included, the power flow model of this contingency must be satisfiable. Conversely, if the power

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flow trajectory post-contingency and correctively is infeasible, the contingency must be discarded to maintain an overall feasible SCOPF problem.

$$\begin{bmatrix} \text{Power flow model for } c \\ \text{Flow and voltage bounds} \\ \beta_c = 0 \text{ (included)} \end{bmatrix} \vee \begin{bmatrix} \text{No power flow model for } c \\ \text{No bounds} \\ \beta_c = 1 \text{ (discarded)} \end{bmatrix}$$

Through a big M and a binary variable, nodal the power balance can be relaxed:

$$P_{i,c}^{\text{units}} + P_{i,c}^{\text{shunt}} - P_{i,c}^{\text{lines}} \leq M_i \cdot \beta_c$$
$$Q_{i,c}^{\text{units}} + Q_{i,c}^{\text{shunt}} - Q_{i,c}^{\text{lines}} \leq M_i \cdot \beta_c$$
$$P_{i,c}^{\text{units}} + P_{i,c}^{\text{shunt}} - P_{i,c}^{\text{lines}} \geq -M_i \cdot \beta_c$$
$$Q_{i,c}^{\text{units}} + Q_{i,c}^{\text{shunt}} - Q_{i,c}^{\text{lines}} \geq -M_i \cdot \beta_c$$

The power flow model is thereby invalidated/deactivated, where the M_i can be derived as:

$$M_i = \max\left(\sum_{lij \in T(i)} S_{lij}^{\text{rated}}, \sum_{ui \in G(i)} S_u^{\text{rated}} \right).$$

The voltage and power flow bounds can be relaxed depending on β_c as:

$$U_i^{\text{ST,min}} - \beta_c \cdot U_i^{\text{rated}} \leq U_{i,c}^{\text{ST}} \leq U_i^{\text{ST,max}} + \beta_c \cdot U_i^{\text{rated}},$$
$$U_i^{\text{min}} - \beta_c \cdot U_i^{\text{rated}} \leq U_{i,c}^{\text{corr}} \leq U_i^{\text{max}} + \beta_c \cdot U_i^{\text{rated}},$$
$$\left(P_{lij,c}^{\text{ST}}\right)^2 + \left(Q_{lij,c}^{\text{ST}}\right)^2 \leq \left(S_{lij}^{\text{rated,ST}}\right)^2 \cdot \alpha_{l,c}^{\text{ST}} + M \cdot \beta_c \cdot \left(S_{lij}^{\text{rated,ST}}\right)^2,$$
$$\left(P_{lij,c}\right)^2 + \left(Q_{lij,c}\right)^2 \leq \left(S_{lij}^{\text{rated}}\right)^2 \cdot \alpha_{l,c} + M \cdot \beta_c \cdot \left(S_{lij}^{\text{rated}}\right)^2.$$

Corrective actions and contingency discarding should not be performed simultaneously:

$$\beta_c \cdot \nu_{l,c}^{\text{corr}} = 0, \qquad \beta_c \cdot \nu_{u,c}^{\text{corr}} = 0.$$

As both β_c and $\nu_{l,c}^{\text{corr}}$ / $\nu_{u,c}^{\text{corr}}$ are defined as binary variables, this can be linearly formulated as:

$$\beta_c + \nu_{l,c}^{\text{corr}} \leq 1, \qquad \beta_c + \nu_{u,c}^{\text{corr}} \leq 1.$$

Contingency discarding can be disabled using a parameter $\beta^{\text{max}} \in \{0,1\}$, resulting in a conventional SCOPF constrained by the set of preselected contingencies:

$$\beta_c \leq \beta^{\text{max}}.$$

11.4.7 Optimized contingency sets

The set of contingencies not-secured therefore is:

$$\mathcal{C}^{\text{insecure}} = \left\{ c \in \mathcal{C}^{\text{selected}} \middle| \beta_c = 1 \right\}.$$

If $\beta^{\text{max}} = 0$, a conventional preventive-corrective SCOPF is solved and therefore $\mathcal{C}^{\text{insecure}} = \{\}$. The set of purely preventively secured contingencies therefore is:

$$\mathcal{C}^{\text{prev}} = \left\{ c \in \mathcal{C}^{\text{selected}} \middle| \underbrace{\beta_c = 0}_{\substack{\text{contingency}\\ \text{secured}}} \wedge \underbrace{\sum_{u \text{ in } \mathcal{U}} \nu_{u,c}^{\text{corr}} = 0}_{\substack{\text{no corr.}\\ \text{unit action}}} \wedge \underbrace{\sum_{l \text{ in } \mathcal{L}} \nu_{l,c}^{\text{corr}} = 0}_{\substack{\text{no corr.}\\ \text{line action}}} \right\},$$

namely, the contingencies which are not relaxed, and for which no corrective line or unit actions are taken. The contingencies preventively-correctively secured are now derived as

$$\mathcal{C}^{\text{prev-corr}} = \mathcal{C}^{\text{selected}} \setminus \left(\mathcal{C}^{\text{prev}} \cup \mathcal{C}^{\text{insecure}} \right).$$

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11.4.8 Residual probability

The residual probability π^{res} is now calculated as:

$$\pi^{\text{res}} = \sum_{c \in \mathcal{C}} \pi_c \beta_c.$$

11.4.9 Failure of corrective actions

The probability of failure of corrective actions π_c^{fail} is conditional on the occurrence of an action and can be estimated based on historic failure probability data:

$$\pi_c^{\text{fail}} = \underbrace{\sum_{l \in L} \pi_l^{\text{fail}} \cdot \nu_{l,c}^{\text{corr}}}_{\text{line-related failure}} + \underbrace{\sum_{u \in U} \pi_u^{\text{fail}} \cdot \nu_{u,c}^{\text{corr}}}_{\text{unit-related failure}}.$$

Note that this approximation may lead to $\pi_c^{\text{fail}} > 1$ when a large number of corrective actions are needed to find a feasible operational state.

11.4.10 Per-contingency probability of blackout

The probability of blackout is either determined by the failure of corrective actions, or by the discarding of a contingency:

$$\begin{bmatrix} \pi_c^{\text{blackout}} = \pi_c \\ \nu_{l,c}^{\text{corr}} = \nu_{u,c}^{\text{corr}} = 0 \text{ (no corrective actions)} \\ \beta_c = 1 \text{ (included)} \end{bmatrix} \vee \begin{bmatrix} \pi_c^{\text{blackout}} = \pi_c \cdot \pi_c^{\text{fail, unstable}} \cdot \pi_c^{\text{fail}} \\ \nu_{l,c}^{\text{corr}}, \nu_{u,c}^{\text{corr}} \geq 0 \text{ (corrective actions)} \\ \beta_c = 0 \text{ (discarded)} \end{bmatrix}.$$

Which is implemented as:

$$\pi_c^{\text{blackout}} = \pi_c \cdot \left(\beta_c + \pi_c^{\text{fail, unstable}} \cdot \pi_c^{\text{fail}}\right).$$

In general, the failure of corrective actions does not need to lead to a blackout, but may often lead to an insecure state. Actions may exist to return to a secure operational state. Therefore, there is a probability that the failure of corrective actions actually leads to a blackout $\pi_c^{\text{fail, unstable}}$. In the worst case, $\pi_c^{\text{fail, unstable}} = 1$, which assumes that every failure of corrective actions leads to a blackout state. Note that $\beta_c \cdot \pi_c^{\text{fail}} = 0$, due to the disabling of corrective actions when performing contingency discarding.

11.4.11 Reliability target

In the GQP SCOPF a specific version of the GARPUR RMAC reliability target was implemented, expressing an upper bound on the probability to experience "blackout" as defined above:

$$\sum_{c \in C} \pi_c^{\text{blackout}} \leq \pi^{\text{blackout,max}}.$$

The setting $\sigma^{\text{rel.target}} \in \{0,1\}$ can be used to remove this constraint:

$$\sigma^{\text{rel.target}} \cdot \sum_{c \in C} \pi_c^{\text{blackout}} \leq \pi^{\text{blackout,max}}.$$

The research leading to these results has received funding from the European Union
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11.4.12 Cost and risk components

The total risk K^{total}, i.e. the objective, is composed of three components, namely, preventive cost, corrective risk and blackout risk, which can each be independently considered using parameters $k^{\text{prev}}, k^{\text{corr}}, k^{\text{blackout}} \in \{0,1\}$:

$$\min K^{\text{total}}$$

$$K^{\text{total}} = k^{\text{prev}} \cdot K^{\text{prev}} + k^{\text{corr}} \cdot \sum_{c\in C} \pi_c \cdot K_c^{\text{corr}} + k^{\text{blackout}} \cdot \sum_{c\in C} K_c^{\text{blackout}}.$$

The preventive and corrective cost terms are designed identically:

$$K^{\text{prev}} = \sum_{u\in\mathcal{U}} \underbrace{k^{\text{redisp}} \cdot K_u^{\text{redisp}} + k^{\text{disp}} \cdot K_u^{\text{disp}} + k^{\text{startstop}} \cdot K_u^{\text{startstop}}}_{\text{unit costs}}$$
$$+ \sum_{l\in\mathcal{L}} \underbrace{k^{\text{PST}} \cdot K_l^{\text{PST}} + k^{\text{switch}} \cdot K_l^{\text{switch}}}_{\text{line costs}},$$
$$K_c^{\text{corr}} = \sum_{u\in\mathcal{U}} \underbrace{k^{\text{redisp}} \cdot K_{u,c}^{\text{redisp}} + k^{\text{disp}} \cdot K_{u,c}^{\text{disp}} + k^{\text{startstop}} \cdot K_{u,c}^{\text{startstop}}}_{\text{unit costs}}$$
$$+ \sum_{l\in\mathcal{L}} \underbrace{k^{\text{PST}} \cdot K_{l,c}^{\text{PST}} + k^{\text{switch}} \cdot K_{l,c}^{\text{switch}}}_{\text{line costs}}.$$

The parameters $k^{\text{redisp}}, k^{\text{disp}}, k^{\text{startstop}}, k^{\text{PST}}, k^{\text{switch}} \in \{0,1\}$ are used to indicate the inclusion/exclusion of the following cost components: redispatch cost (includes load shedding as well), dispatch cost, start-up/shutdown cost, PST shifting cost and line switching cost. Finally, the blackout risk depends on the system's total VOLL and the probability of the blackout outcome $K_c^{\text{blackout}} = \pi_c^{\text{blackout}} \cdot \mathbb{E}(\text{VOLL})$. The system's expected VOLL is considered as a parameter in this approach.

11.4.13 Acceptability constraints

To find a better balance between preventive costs and corrective risks, SCOPF tools have commonly included constraints to limit corrective load shedding through limits on the energy not served (ENS) and power not served (PNS) [GARPUR,2016d].

The following acceptability constraints and limits have been defined specifically for the GQP implementation. The PNS is calculated for the loads only, and is bounded for each contingency by $P_c^{\text{not-served,max}}$:

$$P_c^{\text{not-served}} = \sum_{u\in\text{loads}} P_{u,c}^{\Delta},$$
$$P_c^{\text{not-served}} \leq P_c^{\text{not-served,max}}.$$

A probabilistic limit $P^{\text{not-served,max}}$ is enforced as:

$$P^{\text{not-served}} = \sum_{c\in C} \pi_c \cdot P_c^{\text{not-served}}$$
$$P^{\text{not-served}} \leq P^{\text{not-served,max}}$$

The equivalent for ENS uses the mean time to repair (MTTR) of the specific contingency:

$$E_c^{\text{not-served}} = \sum_{u\in\text{loads}} P_{u,c}^{\Delta} \cdot \tau_c^{\text{MTTR}},$$
$$E_c^{\text{not-served}} \leq E_c^{\text{not-served,max}}.$$

The equivalent probabilistic limit $P^{\text{not-served,max}}$ is enforced as:

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$$E^{\text{not-served}} = \sum_{c \in C} \pi_c \cdot E_c^{\text{not-served}}$$
$$E^{\text{not-served}} \leq E^{\text{not-served,max}}$$

11.5 Power flow formulations supported by the GQP

11.5.1 DC OPF

The assumptions underlying the DC OPF approximation are: $U^{*\text{ref}}_{lij} \approx 1$ pu, $b_{l,s} \approx -\frac{1}{x_{l,s}}$, $\sin(\theta'_{lij} - \theta'_{lji}) \approx (\theta'_{lij} - \theta'_{lji})$. Under these assumptions, the active power flow equation becomes:

$$P_{lij} = \underbrace{(g_{lij,\text{sh}}) \cdot \left(U^{*\text{ref}}_{lij}\rho^{\text{ref}}_{lij}\right)^2 \cdot \alpha_l}_{\text{shunt}} - \underbrace{b_{l,s}\left(U^{*\text{ref}}_{lij}\rho^{\text{ref}}_{lij}\right)\left(U^{*\text{ref}}_{lji}\rho^{\text{ref}}_{lij}\right)(\theta'_{lij} - \theta'_{lji})}_{\text{series}}$$

The reactive power flow equations are dropped from the equation set, although they can also be included through a network flow formulation.

11.5.2 LPAC OPF

In LPAC, the power flow equations are linearized around an operating point U^{ref}_i.

$$U'^{\text{ref}}_{lij} = U^{\text{ref}}_i \cdot \rho^{\text{ref}}_{lij}$$
$$U'_{lij} \approx U'^{\text{ref}}_{lij} + U'^{\Delta}{}_{lij}$$

Then the power flow equations are developed, using the initial value for the voltage magnitude U'^{ref}_{lij}, linearizing the sine function around $\theta'_{lij} - \theta'_{lji} \approx 0$ and deriving a convex hull for the cosine function in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$:

$$P_{lij} = (g_{l,s} + g_{lij,sh}) \cdot \left(U'^{\text{ref}}_{lij}\right)^2 - g_{l,s}U'^{\text{ref}}_{lij}U'^{\text{ref}}_{lji}c'_{lij} - b_{l,s}U'^{\text{ref}}_{lij}U'^{\text{ref}}_{lji}(\theta'_{lij} - \theta'_{lji})$$
$$Q_{lij} = -(b_{l,s} + b_{lij,sh}) \cdot \left(U'^{\text{ref}}_{lij}\right)^2 + b_{l,s}U'^{\text{ref}}_{lij}U'^{\text{ref}}_{lji}c'_{lij} - g_{l,s}U'^{\text{ref}}_{lij}U'^{\text{ref}}_{lji}(\theta'_{lij} - \theta'_{lji}) + Q^{\Delta}_{lij}$$
$$c'_{lij} \in \text{convexhull}\left(\cos(\theta'_{lij} - \theta'_{lji})\right)$$

The reactive power flow is linearized:

$$Q^{\Delta}_{lij} = -2 \cdot (b_{l,s} + b_{lij,sh})U'^{\text{ref}}_{lij}U'^{\Delta}{}_{lij} + b_{l,s} \cdot \left(U'^{\text{ref}}_{lij}U'^{\Delta}{}_{lji} + U'^{\text{ref}}_{lji}U'^{\Delta}{}_{lij}\right)$$

Again, this linearization relates the reactive power flow to the voltage magnitude. A polyhedral convex hull formulation is used as in [Coffrin,2014a]. The resulting equation set is linear.

11.5.3 SOC Branch Flow Model OPF

First, a number of new variables are defined to represent the squared magnitudes of the originally defined voltage and current variables:

$$\left(U'_{lij}\right)^2 \to u'_{lij} \geq 0$$
$$\left(U^*_{lij}\right)^2 \to u^*_{lij} \geq 0$$
$$(U_i)^2 \to u_i \geq 0$$
$$\left(|I_{lij,s}|\right)^2 \to i'_l \geq 0$$

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The power flow equations can then be written as:

$$P_{lij} + P_{lji} = g_{lij,\mathrm{sh}} u'_{lij} + r_{l,s} i'_{l} + g_{lji,\mathrm{sh}} u'_{lji}$$
$$Q_{lij} + Q_{lji} = -b_{lij,\mathrm{sh}} u'_{lij} + x_{l,s} i'_{l} - b_{lji,\mathrm{sh}} u'_{lji}$$

The series current and complex power flow should satisfy:

$$\left(P_{lij} - g_{lij,\mathrm{sh}} u'_{lij}\right)^2 + \left(Q_{lij} + b_{lij,\mathrm{sh}} u'_{lij}\right)^2 = u'_{lij} \cdot i'_{l}$$

This equation is relaxed as:

$$\left(P_{lij} - g_{lij,\mathrm{sh}} u'_{lij}\right)^2 + \left(Q_{lij} + b_{lij,\mathrm{sh}} u'_{lij}\right)^2 \leq u'_{lij} \cdot i'_{l}$$

This inequality describes a rotated second-order cone.

An off-set voltage tap is modelled as:

$$\left(\rho_{lij}^{\mathrm{ref}}\right)^2 u_{lij}^{*} = u'_{lij}$$

The switch model is:

$$-U_i^{\max}(1 - \alpha_l) \leq u_i - u_{lij}^{*} \leq U_i^{\max}(1 - \alpha_l)$$
$$u_{lij}^{*} \leq \alpha_l (U_i^{\max})^2$$

And finally, Ohm's law becomes:

$$u'_{lij} - u'_{lji} = -2\left(r_{l,s}\left(P_{lij} - g_{lij,\mathrm{sh}} u'_{lij}\right) + x_{l,s}\left(Q_{lij} + b_{lij,\mathrm{sh}} u'_{lij}\right)\right) + \left(\left(r_{l,s}\right)^2 + \left(x_{l,s}\right)^2\right) i'_{l}$$

Furthermore, current limits can be enforced as:

$$\left(P_{lij}\right)^2 + \left(Q_{lij}\right)^2 \leq \left(I_{lij}^{\mathrm{rated}}\right)^2 u_i$$

This equation set is therefore mixed-integer second-order cone.

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12 APPENDIX II

12.1 GARPUR RMAC Ingredients

The theoretical model for reliability management approaches and criteria (RMACs) proposed by the GARPUR project is composed of 4 main ingredients [excerpt from [GARPUR,2016c]]:

1. A Socio-Economic Objective Function (SEOF) to be minimized (accounting for costs and benefits resulting from TSO reliability management decisions).
2. A Risk-averse Reliability Target (RaRT) aiming to ensure that the decisions considered as acceptable indeed lead with high enough probability to an acceptable system behaviour.
3. A Discarding Principle allowing one to avoid detailed computations over the generally intractable space of exogenous uncertainties, by specifying how to neglect large portions of this uncertainty space, both in the context of reliability assessment and reliability control.
4. A Relaxation Principle prescribing how the reliability management problem should be progressively relaxed, whenever no feasible decision can be found according to the previous three components. It basically indicates that the level of approximation tolerated by the discarding principle should be relaxed as little as possible in order to enable the determination of a decision compliant with the reliability target.

12.2 Data for the IEEE RTS Test System in Matpower format

12.2.1 Bus data

```
%   bus_i   type    Pd  Qd  Gs  Bs  area    Vm  Va  baseKV  zone    Vmax
Vmin
mpc.bus = [
    1   2   108.0   22.0    0.0     0.0     1       1.04723     -6.75543    138.0
1       1.05000     0.95000;
    2   2   97.0    20.0    0.0     0.0     1       1.04721     -6.83108    138.0
1       1.05000     0.95000;
    3   1   180.0   37.0    0.0     0.0     1       1.01378     -6.18747    138.0
1       1.05000     0.95000;
    4   1   74.0    15.0    0.0     0.0     1       1.01625     -9.57433    138.0
1       1.05000     0.95000;
    5   1   71.0    14.0    0.0     0.0     1       1.03543     -9.73755    138.0
1       1.05000     0.95000;
    6   1   136.0   28.0    0.0     -100.0  2       1.03241     -12.31055   138.0
1       1.05000     0.95000;
    7   2   125.0   25.0    0.0     0.0     2       1.03600     -9.87656    138.0
1       1.05000     0.95000;
    8   1   171.0   35.0    0.0     0.0     2       1.00896     -12.55316   138.0
1       1.05000     0.95000;
    9   1   175.0   36.0    0.0     0.0     1       1.02393     -7.81824    138.0
1       1.05000     0.95000;
    10  1   195.0   40.0    0.0     0.0     2       1.05000     -9.67832    138.0
1       1.05000     0.95000;
    11  1   0.0     0.0     0.0     0.0     3       1.02263     -2.61309    230.0
1       1.05000     0.95000;
    12  1   0.0     0.0     0.0     0.0     3       1.01727     -1.87607    230.0
1       1.05000     0.95000;
    13  3   265.0   54.0    0.0     0.0     3       1.03348     -0.00000    230.0
1       1.05000     0.95000;
    14  2   194.0   39.0    0.0     0.0     3       1.03987      1.01244    230.0
1       1.05000     0.95000;
```

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```
     15   2   317.0   64.0    0.0     0.0     4      1.04106      9.32436  230.0
1       1.05000     0.95000;
     16   2   100.0   20.0    0.0     0.0     4      1.04428      8.54258  230.0
1       1.05000     0.95000;
     17   1   0.0     0.0     0.0     0.0     4      1.04756     12.89163  230.0
1       1.05000     0.95000;
     18   2   333.0   68.0    0.0     0.0     4      1.05000     14.24704  230.0
1       1.05000     0.95000;
     19   1   181.0   37.0    0.0     0.0     3      1.03895      7.49316  230.0
1       1.05000     0.95000;
     20   1   128.0   26.0    0.0     0.0     3      1.04397      8.45230  230.0
1       1.05000     0.95000;
     21   2   0.0     0.0     0.0     0.0     4      1.05000     15.02296  230.0
1       1.05000     0.95000;
     22   2   0.0     0.0     0.0     0.0     4      1.05000     20.69119  230.0
1       1.05000     0.95000;
     23   2   0.0     0.0     0.0     0.0     3      1.05000      9.68377  230.0
1       1.05000     0.95000;
     24   1   0.0     0.0     0.0     0.0     4      1.00585      3.66828  230.0
1       1.05000     0.95000;
];
```

12.2.2 Generation Data

```
%    bus Pg   Qg   Qmax    Qmin    Vg  mBase    status  Pmax    Pmin    Pc1 Pc2
Qc1min  Qc1max  Qc2min  Qc2max  ramp_agc     ramp_10 ramp_30 ramp_q  apf
mpc.gen = [
     1    16.0    4.862   10.0    0.0     1.04723     100.0   1   20.0    16.0
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
     1    16.0    4.862   10.0    0.0     1.04723     100.0   1   20.0    16.0
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
     1    76.0    -1.576  30.0    -25.0   1.04723     100.0   1   76.0    15.2
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
     1    76.0    -1.576  30.0    -25.0   1.04723     100.0   1   76.0    15.2
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
     2    16.0    4.814   10.0    0.0     1.04721     100.0   1   20.0    16.0
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
     2    16.0    4.814   10.0    0.0     1.04721     100.0   1   20.0    16.0
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
     2    76.0    -2.923  30.0    -25.0   1.04721     100.0   1   76.0    15.2
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
     2    76.0    -2.923  30.0    -25.0   1.04721     100.0   1   76.0    15.2
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
     7    70.343  16.438  60.0    0.0     1.036   100.0   1   100.0   25.0
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
     7    70.343  16.438  60.0    0.0     1.036   100.0   1   100.0   25.0
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
```

The research leading to these results has received funding from the European Union
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```
    7    70.343  16.438  60.0    0.0     1.036   100.0   1   100.0   25.0
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
    13   78.579  32.592  80.0    0.0     1.03348     100.0   1   197.0   69.0
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
    13   78.579  32.592  80.0    0.0     1.03348     100.0   1   197.0   69.0
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
    13   78.579  32.592  80.0    0.0     1.03348     100.0   1   197.0   69.0
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
    14   0.0     114.908     200.0   -50.0   1.03987     100.0   1   0.0
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0     0.0;
    15   2.4     6.0     6.0     0.0     1.04106     100.0   1   12.0    2.4
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
    15   2.4     6.0     6.0     0.0     1.04106     100.0   1   12.0    2.4
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
    15   2.4     6.0     6.0     0.0     1.04106     100.0   1   12.0    2.4
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
    15   2.4     6.0     6.0     0.0     1.04106     100.0   1   12.0    2.4
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
    15   2.4     6.0     6.0     0.0     1.04106     100.0   1   12.0    2.4
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
    15   155.0   80.0    80.0    -50.0   1.04106     100.0   1   155.0   54.3
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
    16   155.0   80.0    80.0    -50.0   1.04428     100.0   1   155.0   54.3
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
    18   400.0   72.898  200.0   -50.0   1.05    100.0   1   400.0   100.0
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
    21   400.0   -7.458  200.0   -50.0   1.05    100.0   1   400.0   100.0
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
    22   50.0    -6.413  16.0    -10.0   1.05    100.0   1   50.0    10.0
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
    22   50.0    -6.413  16.0    -10.0   1.05    100.0   1   50.0    10.0
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
    22   50.0    -6.413  16.0    -10.0   1.05    100.0   1   50.0    10.0
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
    22   50.0    -6.413  16.0    -10.0   1.05    100.0   1   50.0    10.0
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
    22   50.0    -6.413  16.0    -10.0   1.05    100.0   1   50.0    10.0
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
```

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```
    22   50.0    -6.413  16.0    -10.0   1.05    100.0   1   50.0    10.0
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
    23   155.0   2.548   80.0    -50.0   1.05    100.0   1   155.0   54.3
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
    23   155.0   2.548   80.0    -50.0   1.05    100.0   1   155.0   54.3
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
    23   350.0   40.548  150.0   -25.0   1.05    100.0   1   350.0   140.0
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
0.0;
];
```

12.2.3 Generation cost data

```
%   2   startup shutdown    n   c(n-1)  ... c0
mpc.gencost = [
    2    1500.0  0.0     3     0.000000  130.000000  400.684900;
    2    1500.0  0.0     3     0.000000  130.000000  400.684900;
    2    1500.0  0.0     3     0.014142   16.081100  212.307600;
    2    1500.0  0.0     3     0.014142   16.081100  212.307600;
    2    1500.0  0.0     3     0.000000  130.000000  400.684900;
    2    1500.0  0.0     3     0.000000  130.000000  400.684900;
    2    1500.0  0.0     3     0.014142   16.081100  212.307600;
    2    1500.0  0.0     3     0.014142   16.081100  212.307600;
    2    1500.0  0.0     3     0.052672   43.661500  781.521000;
    2    1500.0  0.0     3     0.052672   43.661500  781.521000;
    2    1500.0  0.0     3     0.052672   43.661500  781.521000;
    2    1500.0  0.0     3     0.007170   48.580400  832.757500;
    2    1500.0  0.0     3     0.007170   48.580400  832.757500;
    2    1500.0  0.0     3     0.007170   48.580400  832.757500;
    2    1500.0  0.0     3     0.000000    0.000000    0.000000;
    2    1500.0  0.0     3     0.328412   56.564000   86.385200;
    2    1500.0  0.0     3     0.328412   56.564000   86.385200;
    2    1500.0  0.0     3     0.328412   56.564000   86.385200;
    2    1500.0  0.0     3     0.328412   56.564000   86.385200;
    2    1500.0  0.0     3     0.328412   56.564000   86.385200;
    2    1500.0  0.0     3     0.008342   12.388300  382.239100;
    2    1500.0  0.0     3     0.008342   12.388300  382.239100;
    2    1500.0  0.0     3     0.000213    4.423100  395.374900;
    2    1500.0  0.0     3     0.000213    4.423100  395.374900;
    2    1500.0  0.0     3     0.000000    0.001000    0.001000;
    2    1500.0  0.0     3     0.000000    0.001000    0.001000;
    2    1500.0  0.0     3     0.000000    0.001000    0.001000;
    2    1500.0  0.0     3     0.000000    0.001000    0.001000;
    2    1500.0  0.0     3     0.000000    0.001000    0.001000;
    2    1500.0  0.0     3     0.000000    0.001000    0.001000;
    2    1500.0  0.0     3     0.008342   12.388300  382.239100;
    2    1500.0  0.0     3     0.008342   12.388300  382.239100;
    2    1500.0  0.0     3     0.004895   11.849500  665.109400;
];
```

12.2.4 Branch data

```
%   fbus    tbus    r   x   b   rateA   rateB   rateC   ratio   angle   status
angmin  angmax
```

```
mpc.branch = [
    1     2    0.0026   0.0139   0.4611   175.0    193.0    200.0    0.0      0.0       1
-30.0    30.0;
    1     3    0.0546   0.2112   0.0572   175.0    208.0    220.0    0.0      0.0       1
-30.0    30.0;
    1     5    0.0218   0.0845   0.0229   175.0    208.0    220.0    0.0      0.0       1
-30.0    30.0;
    2     4    0.0328   0.1267   0.0343   175.0    208.0    220.0    0.0      0.0       1
-30.0    30.0;
    2     6    0.0497   0.192    0.052    175.0    208.0    220.0    0.0      0.0       1
-30.0    30.0;
    3     9    0.0308   0.119    0.0322   175.0    208.0    220.0    0.0      0.0       1
-30.0    30.0;
    3     24   0.0023   0.0839   0.0      400.0    510.0    600.0    1.03     0.0       1
-30.0    30.0;
    4     9    0.0268   0.1037   0.0281   175.0    208.0    220.0    0.0      0.0       1
-30.0    30.0;
    5     10   0.0228   0.0883   0.0239   175.0    208.0    220.0    0.0      0.0       1
-30.0    30.0;
    6     10   0.0139   0.0605   2.459    175.0    193.0    200.0    0.0      0.0       1
-30.0    30.0;
    7     8    0.0159   0.0614   0.0166   175.0    208.0    220.0    0.0      0.0       1
-30.0    30.0;
    8     9    0.0427   0.1651   0.0447   175.0    208.0    220.0    0.0      0.0       1
-30.0    30.0;
    8     10   0.0427   0.1651   0.0447   175.0    208.0    220.0    0.0      0.0       1
-30.0    30.0;
    9     11   0.0023   0.0839   0.0      400.0    510.0    600.0    1.03     0.0       1
-30.0    30.0;
    9     12   0.0023   0.0839   0.0      400.0    510.0    600.0    1.03     0.0       1
-30.0    30.0;
    10    11   0.0023   0.0839   0.0      400.0    510.0    600.0    1.02     0.0       1
-30.0    30.0;
    10    12   0.0023   0.0839   0.0      400.0    510.0    600.0    1.02     0.0       1
-30.0    30.0;
    11    13   0.0061   0.0476   0.0999   500.0    600.0    625.0    0.0      0.0       1
-30.0    30.0;
    11    14   0.0054   0.0418   0.0879   500.0    600.0    625.0    0.0      0.0       1
-30.0    30.0;
    12    13   0.0061   0.0476   0.0999   500.0    600.0    625.0    0.0      0.0       1
-30.0    30.0;
    12    23   0.0124   0.0966   0.203    500.0    600.0    625.0    0.0      0.0       1
-30.0    30.0;
    13    23   0.0111   0.0865   0.1818   500.0    600.0    625.0    0.0      0.0       1
-30.0    30.0;
    14    16   0.005    0.0389   0.0818   500.0    600.0    625.0    0.0      0.0       1
-30.0    30.0;
    15    16   0.0022   0.0173   0.0364   500.0    600.0    625.0    0.0      0.0       1
-30.0    30.0;
    15    21   0.0063   0.049    0.103    500.0    600.0    625.0    0.0      0.0       1
-30.0    30.0;
    15    21   0.0063   0.049    0.103    500.0    600.0    625.0    0.0      0.0       1
-30.0    30.0;
    15    24   0.0067   0.0519   0.1091   500.0    600.0    625.0    0.0      0.0       1
-30.0    30.0;
    16    17   0.0033   0.0259   0.0545   500.0    600.0    625.0    0.0      0.0       1
-30.0    30.0;
    16    19   0.003    0.0231   0.0485   500.0    600.0    625.0    0.0      0.0       1
-30.0    30.0;
```

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```
    17   18  0.0018  0.0144  0.0303  500.0   600.0   625.0   0.0     0.0     1
-30.0   30.0;
    17   22  0.0135  0.1053  0.2212  500.0   600.0   625.0   0.0     0.0     1
-30.0   30.0;
    18   21  0.0033  0.0259  0.0545  500.0   600.0   625.0   0.0     0.0     1
-30.0   30.0;
    18   21  0.0033  0.0259  0.0545  500.0   600.0   625.0   0.0     0.0     1
-30.0   30.0;
    19   20  0.0051  0.0396  0.0833  500.0   600.0   625.0   0.0     0.0     1
-30.0   30.0;
    19   20  0.0051  0.0396  0.0833  500.0   600.0   625.0   0.0     0.0     1
-30.0   30.0;
    20   23  0.0028  0.0216  0.0455  500.0   600.0   625.0   0.0     0.0     1
-30.0   30.0;
    20   23  0.0028  0.0216  0.0455  500.0   600.0   625.0   0.0     0.0     1
-30.0   30.0;
    21   22  0.0087  0.0678  0.1424  500.0   600.0   625.0   0.0     0.0     1
-30.0   30.0;
];
```

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