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Optimal design of a regional railway service in Italy

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Abstract
In order to assess the actual ability of planned infrastructure investments to satisfy the required demand of service, railway engineers also need to plan the new offer of train services. In turn, this requires a full re-planning of the line system and of the frequency of each line. In cooperation with the Sales and Network Management department of the Italian infrastructure manager, we developed a model to determine an optimal set of lines and the corresponding train frequencies. The model has been successfully applied to evaluate alternative infrastructure enhancements in the metropolitan rail network of Rome. A set of computational experiments has been carried out, providing interesting insights on the effects of different infrastructural intervention policies.

Keywords
Railway networks, Line planning, Integer programming.

1 Introduction

This work stems from a project carried out with the Sales and Network Management department of the Italian railway infrastructure manager, Rete Ferroviaria Italiana (RFI). In RFI, infrastructure investments are carried out by iterating two planning phases: namely tentative infrastructure design followed by line planning. In general, line planning consists in selecting a set of lines in an infrastructure network and their frequency of operation such that a given travel demand can be routed. Remarkably, both infrastructure investment decisions and line planning are performed ”by hand” in RFI, resulting in a very lengthy and exhausting trial-and-error process, with no guarantee on the quality of the final solutions. Actually, line planning also involves other municipal administrative and political bodies, making the process even more awkward.

We focus on the railway system in the metropolitan area of Rome where RFI has already established an ambitious investment plan, even though some options are still open.

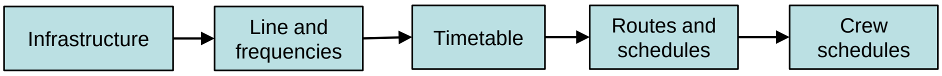

Figure 1: Sequence of planning phases in public transportation.

The plan includes the realization of new stations and tracks, plus the completion of a ring around the city center (*ring closure*). The ring closure is a particularly expensive decision, which is still under consideration. Our major contribution was to support the planners in evaluating investment alternatives, and in particular in analyzing the benefit of the costly ring closure. In particular, we compared three different options. A first option (*plan A*) is the “no-investment” option: Namely, we consider the line planning problem or, simply, the train frequency decision on the existing lines, in order to evaluate the current infrastructure in terms of some passenger-oriented performance indicators. The second option (*plan B*) considers line planning on the new infrastructure, excluding ring closure. Finally, in the last scenario (*plan C*) the ring closure is also taken into account.

1.1 Line Planning and related literature

Line planning is a well studied problem in the literature (see Schöbel (2012); Karbstein (2013) for surveys). In a recent survey paper (Schöbel (2012)), Anita Schöbel observes that the classical planning process in public transportation consists of a number of *consecutive* planning phases, reported in Figure 1. The first phase is infrastructure design, followed by the other phases as we move towards the actual transport operations. In our experience with railway infrastructure planners we quickly found out that things are not so straightforward as it may appear from this picture. For instance, in a recent project with the capacity department of the Norwegian infrastructure manager (*Jernbaneverket*), infrastructure decisions (on single lines[1]) were strongly intertwined with timetabling: indeed, in order to assess if a given railway design suffices to accommodate the expected traffic of trains, one needs to exhibit a suitable conflict-free timetable, see Lamorgese et al (2017). The planning process is in fact a trial-and-error process in which tentative infrastructure and timetabling are repeated until a satisfactory solution is found, see Lamorgese et al (2017). Things get even more complicated when the new infrastructure can change significantly the topology of the original network. In this case, to assess the quality of the new infrastructure it might be necessary to redefine the original lines. Therefore, infrastructure planning and line design converge in a single design problem. In particular, The most common approach (Ceder and Wilson (1986); Schöbel (2012)) in the literature is to select the lines to be activated from a predefined pool. In our case the pool is provided directly by RFI as the new lines are part of the overall plans to be realized. We remark that model presented here can still be used as a building block in a column (line) generation approach even when the pool of lines is not provided by some external source.

The models proposed in the literature typically assume the availability of an O-D ma-

[1] A *line* is defined as a sequence of stations traversed by a train from an origin station to the a destination station.

trix representing the demand on the network, and can be either cost-oriented or passenger oriented, or both (see e.g., Borndoerfer et al (2007); Karbstein (2016)). In the cost-oriented models, the goal is to find a set of lines serving all customers and minimizing the costs for the public transportation company (see e.g., Goosens et al. (2004)). The passenger-oriented approaches aim to minimize the passenger discomfort that can be measured either as the total travel time or as the number of transfers. As an example, in the direct travelers approach by Bussiek et al. (1996) the goal is to maximize the number of direct travelers (i.e. customers that need not change the line to reach their destination), keeping into account the capacity of the network. However, customers who actually need transfers are not considered, so that many transfers could be needed for some of them, and the traveling times are not taken into account. The number of transfers and travelling times are instead directly taken into account in the *change&go* model developed by Schöbel and Scholl (2006) and adopted by several authors, as for instance in Goerigk and Schmidt (2017). In this paper we propose an alternative model with fewer nodes (but potentially more arcs), which allows for a simpler representation of constraints and avoid to resort to big-M coefficients. In our model, passenger needs are expressed by suitable constraints, whereas we try to reduce operational costs by minimizing the total length of the selected lines. We remark that the O-D matrix is not available, so we resorted to use a demand estimate following the approach adopted by RFI's practitioners.

2 Problem description and MILP model

2.1 Notation

The railway network is represented by a directed graph $G = (N, E)$ where the nodes N correspond to the stations (and stops) of the railway, while the set of directed edges E corresponds to the actual tracks existing between the pairs of adjacent stations. We assume that between every pair of adjacent stations $u, v \in V$ we always have two directed edges, namely we have $(u, v) \in E$ and $(v, u) \in E$. Note that even a single track between two stations, say station A and station B, is normally used in both directions to connect A to B and B to A. Correspondingly, we have two directed edges in the graph. A line with terminal stations $s, t \in N$ is simply a pair of directed paths, one from s to t and the other from t to s, going through the same sequence of stations but in reverse order. Next, we are given a set $\mathcal{L}$ of potential lines to be activated. For each line $\ell \in \mathcal{L}$, we let $N^\ell \subseteq N$ be the stations on the line and $N^\ell_{\text{stop}} \subseteq N^\ell$ be the actual stops of ℓ. Also, we let $E^\ell \subseteq E$ be the set of edges on line ℓ. For each line $\ell \in \mathcal{L}$, we also define a set of "logical" arcs between each pair of stations in N^ℓ_{stop}, i.e., there is an arc $(u, v) \in E^\ell_{\text{stop}}$ if u and v are stops on line ℓ: note that E^ℓ_{stop} is not a subset of E^ℓ since the former set includes arcs between non-consecutive stops. Additionally, we indicate by θ^ℓ_{uv} the time necessary to run edge $(u, v) \in E^\ell_{\text{stop}}$ for a train of line ℓ. If s and t are the terminal stations of line ℓ, we let $\theta^\ell = \theta^\ell_{st}$. Finally, we assume here that all trains have the same capacity κ. For a given line, the number of trains running in the peak hour is called *line frequency*.

2.2 Problem statement

Roughly speaking, our problem consists in choosing a subset of lines from $\mathcal{L}$ and decide the hourly frequency of trains on each line so as to satisfy the demand of transportation and meet some service level requirements. Also, we do not want the passengers to travel too long trips and to do too many changes in order to reach their destination. In our experiments we will focus only on one peak hour—however the same model can be used to plan the remaining hours in the planning horizon. In principle, in order to satisfy the travel requirements, we need to know the (hourly) flow of passengers for any possible origin/destination pair $(s, t) \in N$. Unfortunately, this information is not available to the RFI planners (and in general is extremely hard to get). So, the approach followed by RFI is to plan according to the total number d_v of passengers embarking and disembarking at each station $v \in N$. In particular, the policy established at RFI is to evaluate the number of trains through a station v (in the peak hour) sufficient to accommodate d_v passengers, regardless to what happens in the other stations. We call this constraint *demand constraint*. This demand modeling reminds the hose model introduced independently in Duffield et al (1999); Fingerhut et al (1997), and since then widely used in telecommunication networks (see for example Cacchiani et al. (2016); Merakli and Yaman (2017)). In particular, in the hose model, rather than specifying demands for all pairs of nodes (which can be impractical in large networks), each node is assigned an outgoing and incoming traffic bandwidth capacity. This implies optimizing for a set of demands rather than a single traffic matrix, and this makes resource management more flexible to cope with possible changes in the demand realizations. The hose model is a very powerful tool since it relies on cumulative bandwidth capacities, which can be more reliably and easily estimated than individual demand expectations. This is true also for railway networks, since it is extremely hard to have reliable O-D demand matrices, and, even when available, can be highly unstable. On the other hand, a number of passengers expected at a certain station can also be estimated on the basis of the number of citizens of the area.

Next, the restriction on the number of changes is imposed by the so called *one-hop* constraint, which stipulates that a passenger should be able to reach any destination by using at most two trains (i.e. lines). Indeed, this restriction does not limit the possibility for a passenger to choose a different (possibly shorter in terms of travel time) route, rather it only imposes that the line infrastructure is designed so that a steady travel is possible for any possible origin-destination pair.

Finally, we need to limit the maximum distance travelled by any passenger. This is modelled through the *stretch* constraint, which enforces that the duration of a shortest trip between any pair of stations $s, t \in N$ is at most $\alpha \cdot \theta^*_{st}$, where θ^*_{st} is a precomputed benchmark (for instance the expected travel time by car) and $\alpha \geq 1$ is the *stretch parameter*. For each line, the number of trains per hour is our major decision variable and is called *frequency*.

In our model, we assume that (i) each line included in the line plan is active in both directions and with the same frequency; (ii) the stations have infinite capacity; and (iii) an infinite number of trains is available.

2.3 MIP model

The decision concerning line selection, is committed to binary variables x_ℓ, $\ell \in \mathcal{L}$, indicating whether line ℓ is selected ($x_\ell = 1$) or not ($x_\ell = 0$). The goal of the model is also to determine the *frequency* of each line in order to satisfy the travel demand. To this purpose, in our MIP model, we use *integer* variables $f^\ell \in \mathbb{Z}_+$ representing the number of trains traveling on an active line $\ell \in \mathcal{L}$ in the time unit[2]. A detailed description of the objective and constraints and their formulation follows.

The objective function is a measure of the average network load.

$$\min \sum_{\ell \in \mathcal{L}} \theta^{\ell} \, f^{\ell} \tag{1}$$

In (1), the number of trains running in one hour for each line on the railway network is weighted by the running time of the corresponding line. By minimizing such an objective, we aim at minimizing the total distance traveled by trains on the network.

In order to model one-hop constraints (ensuring that it is possible to reach every station $t \in N$ of the network from any other station $s \in N$ with at most one transfer) we make use of the following additional sets of binary variables, each indicating whether or not the corresponding circumstances are met:

- $w_{st} = 1$ if there is at least one active line $\ell \in \mathcal{L}$ that stops in $s \in N$ and $t \in N$ and so it connects the two nodes directly, $w_{st} = 0$ otherwise;
- $z_{sht} = 1$ if two active lines exist, one connecting s to h *and* the other h to t, i.e., this variable indicates whether there is a one-hop connection between stations s and t, $z_{sht} = 0$ otherwise.

The following set of inequalities implement one-hop constraints:

$$w_{st} \leq \sum_{\substack{\ell \in \mathcal{L}\,: \\ s,t \in N^{\ell}_{\text{stop}}}} x_{\ell} \qquad s,t \in N \tag{2}$$

$$z_{sht} \leq w_{sh} \qquad h \in N \setminus \{s,t\} \tag{3}$$

$$z_{sht} \leq w_{ht} \qquad h \in N \setminus \{s,t\} \tag{4}$$

$$w_{st} \geq 1 - \sum_{h \in N} z_{sht} \qquad s,t \in N \tag{5}$$

Constraints (2) ensure that the variable w_{st} is equal to 1 only if there is at least one active line that connects the two nodes $s,t \in N$ and it stops in both of them. Constraints (3) and (4) control that each variable z_{sht} may be equal to 1 only if there are an active line that connects s to h and an active line that connects h to t, so that in station h a transfer to reach the station t starting from the station s is possible. Finally, constraints (5) impose the connectivity for every pair of stations in the network with a direct line or with one-hop.

As we discuss above, stretch constraints put an upper bound on the potential loss, in terms of travel time, of a person choosing to move on a train instead of an alternative (fast) transport. For each origin-destination pair (s,t), $s,t \in N$, we require that among all possible routes for going from s to t along active lines (according to the w-values, see above), there is at least one, that we denote by P^{st}, whose travel time meets the stretch constraint. Therefore, recalling that there is a "logical" arc $(u,v) \in E^{\ell}_{\text{stop}}$ for all stops u and v along line ℓ, we define a binary variable $y^{st}(u,v,\ell)$ for each pair (s,t), line $\ell \in \mathcal{L}$, and logical arc $(u,v) \in E^{\ell}_{\text{stop}}$. For a given pair of stops (s,t), variables $y^{st}(\cdot,\cdot,\cdot)$ identify the route P^{st}: In particular, $y^{st}(u,v,\ell)$ is set equal to 1 if the route P^{st} includes traveling along line $\ell \in \mathcal{L}$ boarding at stop u and getting off at stop v; $y^{st}(u,v,\ell) = 0$ otherwise.

[2] Tipically, number of trains per hour.

In order to impose that the binary variables y^{st} determine a route from s to t, we consider the following *flow constraints*, where $\phi_u^{(s,t)} = 1$, or 0, if $u = s$ or $u \in N \setminus \{s,t\}$, respectively:

$$\sum_{\substack{\ell \in \mathcal{L}: \\ (u,v) \in E^\ell_{\text{stop}}}} y^{st}(u,v,\ell) - \sum_{\substack{\ell \in \mathcal{L}: \\ (v,u) \in E^\ell_{\text{stop}}}} y^{st}(v,u,\ell) = \phi_u^{(s,t)} \quad u \in N \setminus \{t\},\ (s,t) \in N \times N; \tag{6}$$

and we force P^{st} to use only active lines by the following constraints:

$$y^{st}(u,v,\ell) \le x^\ell \qquad\qquad \ell \in \mathcal{L},\ \ (u,v) \in E^\ell_{\text{stop}},\ \ (s,t) \in N \times N. \tag{7}$$

We are now left with computing the travel time of P^{st}, that we denote by τ_{st}, and writing down the stretch constraint. First, we can compute τ_{st}:

$$\tau_{st} = \left(\sum_{\ell \in \mathcal{L}} \sum_{(u,v) \in E^\ell_{\text{stop}}} y^{st}(u,v,\ell)\ (\theta^\ell_{uv} + \lambda) \right) - \lambda, \tag{8}$$

where $\lambda \in \mathbb{R}_+$ is a time-penalty constant that approximates the additional time required by each transfer. Indeed, let $\bar{y}$ be a given route plan, then

$$\sum_{\ell \in \mathcal{L}} \sum_{(u,v) \in E^\ell_{\text{stop}}} \bar{y}^{st}(u,v,\ell) = n^{st}(\bar{y})$$

is the number of lines used by a passenger traveling from s to t and $(n^{st}(\bar{y}) - 1)\lambda$ is the time needed to transfer between subsequent lines. Now, it is possible to write stretch constraints as follows:

$$\tau_{st} \le \alpha\, \theta^*_{st} \qquad (s,t) \in N \times N. \tag{9}$$

Note that constraints (6) do not impose any restriction on the number of transfers of the indicated route, that is, we are only bounding the overall transit time of a possible (in terms of active lines) travel between two stations, disregarding one-hop constraints. Moreover, implementing stretch-constraints requires a large number of variables and inequalities. In practice, we only apply constraints (7) to a "most significant" subset of (s,t) pairs and perform an *ex post* check to verify that the shortest path—with at most one transfer—between every pair of origin-destination stations satisfy such constraint.

The goal of the model is also to determine the *frequency* of each line in order to satisfy the travel demand. To this purpose, in our MIP model, we use *integer* variables $f^\ell \in \mathbb{Z}_+$ representing the number of trains traveling on an active line $\ell \in \mathcal{L}$ in the time unit[3]. Constant $f_{\max}$ is a maximum potential number of trains per time unit (considering a single direction of the line). We also make use of the two additional parameters: capacity c_{uv} of arc $(u,v) \in E$ (i.e., the maximum allowed number of trains travelling on arc (u,v) in a time unit) and capacity κ of a train (maximum number of passengers that may simultaneously

[3]Tipically, number of trains per hour.

travel on a train). The formulation of the constraints is:

$$f^\ell \geq x^\ell \qquad \ell \in \mathcal{L} \tag{10}$$

$$f^\ell \leq f_{\max} x^\ell \qquad \ell \in \mathcal{L} \tag{11}$$

$$\sum_{\substack{\ell \in \mathcal{L}\,:\\ (u,v) \in E^\ell}} f^\ell \leq c_{uv} \qquad (u,v) \in E \tag{12}$$

$$\sum_{\substack{\ell \in \mathcal{L}\,:\\ u \in N^\ell_{\text{stop}}}} 2\kappa f^\ell \geq d_u \qquad u \in N. \tag{13}$$

The meaning of constraints (10)–(12) is obvious: the first constraints (10) ensure that there is at least one train traveling on an active line. Constraints (11) implies that the number of trains traveling along line ℓ is bounded by $f_{\max}$. Constraints (12) implement the capacity satisfaction requirements.

The peculiar expression of the demand satisfaction constraints (13) is motivated by availability of demand data in terms of the number d_u of passengers departing from or arriving to station $u \in N$ (cf. discussion at the beginning of Section 2). For the origin or destination nodes these data represent the actual number of people traveling on the trains. For other stations u, d_u is indeed a lower bound since the passengers already on the train (thus reducing train capacity) are not taken into account. However, in the experiments this model of demand proved to be quite effective in representing actual traffic loads on the network connections. The factor 2 in the constraint is needed in order to keep into account that each line is activated in both directions.

The above described constraints (2–13) define the set of feasible alternatives.

3 The railway network of Rome

The model described in the section above has been tested on the regional railway network of Rome. This network collects the demand not only of people moving in the city center, but also of many passengers coming from neighbouring areas that use the rail public transfer to get to the city, see figure 2.

Current service offers nine lines traveled by regional trains, freight trains, long distance trains and high-speed trains. Along each route there is a "door" station that defines the area of the node and it does not necessarily coincide with the last station of the route. The lines with some basic information (origin and destination stations, off-peak and rush hour frequencies, interchanges) are reported in Table 1. MA and MB indicate the two subway lines of the city. (We remark that subways are not managed by RFI, however they are considered here for the sake of completeness, as they are an important connectivity element of the metropolitan network. As a consequence, in our models, the corresponding lines are always considered as active.)

The railway transportation is the base of public transport due to low pollution rate, reliable travel times, and greater comfort. In particular, in our case it allows the connection with the suburban areas of Rome and the other provinces of the region, ensuring greater accessibility to the city. Furthermore, the railway network improves the mobility of citizens and tourists in the city, connecting to the metro and tram network.

Name	Terminal stations	Frequency min–max	Interchanges
FL1	Fara Sabina–Fiumicino Airport	4 tr/hr.	FL2, FL3, FL5, MA, MB
FL2	Tivoli–Roma Tiburtina	2–4 tr/hr.	FL1, FL3, MB
FL3	Viterbo–Roma Tiburtina	2–4 tr/hr.	FL1, FL2, FL5, MA, MB
FL4	Ciampino–Roma Termini	4 tr/hr.	MA, MB, FL5, FL6, FL7, FL8, LE
FL5	Civitavecchia–Roma Termini	2–3 tr/hr.	FL1, FL3, FL4 FL6, FL7, FL8 LE, MA, MB
FL6	Cassino–Roma Termini	1–4 tr/hr.	FL4, FL5, FL7, FL8, LE, MA, MB
FL7	Latina–Roma Termini	2–4 tr/hr.	FL4, FL5, FL6, FL8, LE, MA, MB
FL8	Nettuno–Roma Termini	1–2 tr/hr.	FL4, FL5, FL6, FL7, MA, MB, LE
LE	Fiumicino Airport–Roma Termini	2 tr/hr.	FL1, FL4, FL5, FL6, FL7, FL8, MA, MB

Table 1: Current regional service offer.

In the current situation, increasing the frequencies of the nine regional lines would increase the number of citizens commuting by train, but in some areas such an enhancement is impossible due to lack of capacity. Indeed, the Italian railway network is one of the safest in Europe. As it is common in railway lines, trains headways are controlled by means of *block sections*: Namely, the track is divided in sections about 1 km long and through suitable signaling devices, each train cannot enter in a section if the preceding train is still occupying it and it must decrease his speed if the preceding train is not at least two section away. Travel times are also affected by the high number of stops of regional trains. Indeed, one of the main problem is the interference between regional and long-distance trains, due to shared infrastructures and lack of passing tracks which is the cause of slowing down and consequent higher travel times. For instance, this situation affects both the connection with Abruzzo and Viterbo on the FL2 and FL3 lines, but also the Leonardo Express between Roma Ostiense and Fiumicino airport. Moreover, there are many crossroads along the tracks and conflicting routes, in particular, at Roma Tuscolana and Roma Ostiense where freight trains and regional trains meet. Finally, some stations, especially head stations like Fiumicino airport, have other infrastructure constraints that significantly impact on the train frequencies.

All these elements limit the capacity of the network; in fact, at most 8 trains per hour are allowed to travel on each track in the same direction. Fortunately, the high-speed (AV) line has separate tracks from regional lines even through also the AV line must keep a low speed in the urban area. In Figure 3, the actual load track is reported, showing the need of an upgrading of the infrastructures and a reorganizations of the services, in order to increase the capacity and to improve the quality of service.

Figure 2: Map of the Lazio region

3.1 Infrastructure improvement plan

Trains are managed by train operators, namely independent companies which are typically separated by the infrastructure manager. However, line design is carried out by the infrastructure manager, also because this may require large investments. The process is complex and may occasionally involve train operators which can point out some needs. At the end of this process, the infrastructure manager has selected a number of lines which are then offered - possibly by a competitive tender - to the train operators.

Within a time-line of five years, RFI has already planned a series of interventions on the infrastructure in order to increase the capacity of the metropolitan and regional area. These projects, combined with a reorganization of the service offer—also relying on our optimization model—aim to improve the quality of service and to further shift the mobility from private to public transport. Beyond a series of investments in technology, the inertial (i.e., already planned) projects, are:

1. Building three new stations: the first one at Fiumicino airport (called “North Fiumicino airport”) corresponding to the building of a new terminal at the airport, the second one, “Pigneto”, in order to allow the change with the new subway line, called metro C, the third one called Villa Senni on the line to Frascati.

2. Reducing block section length in congested areas (as between Roma Ostiense and Roma Tiburtina) and/or doubling the tracks where needed (see for example between Roma Casilina and Ciampino).

3. Enhancing the capacity of some main stations (like Roma Tuscolana).

4. Completing the rail circle line in the north of the city by extending the railway line from Vigna Clara to Roma Smistamento and consequantly to build the Tor di Quinto station.

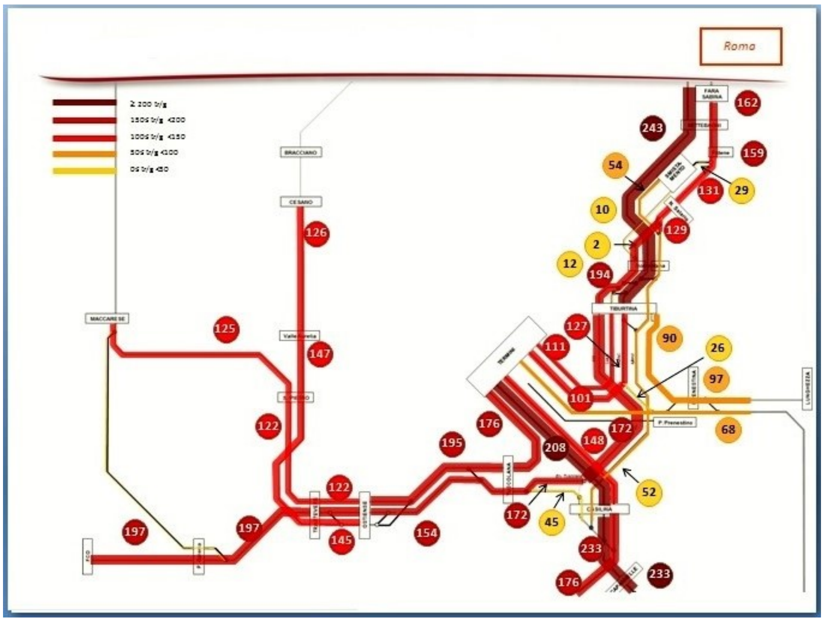

Figure 3: Load of the tracks.

The first three actions have been already scheduled while, due to its major costs, further considerations are required for the fourth operation.

In order to properly evaluate the effects of the changes in the infrastructures, the consequences on the offered service must be evaluated, and this was our contribution. For this reason, we considered three different scenarios for the infrastructure:

Scenario A the actual infrastructure with no interventions (see figure 4). It consists of 25 stations and the line pool proposed by RFI contains 30 lines.

Scenario B the infrastructure obtained after the first three interventions listed above (all except the closure of the ring involving the building of the station of Tor di Quinto). It consists of 27 stations, and the line pool contains 38 lines.

Scenario C the infrastructure of scenario B plus the closure of the ring with the Tor di Quinto station (see figure 5). In this case we have 28 stations and a line pool containing 43 lines.

3.2 Demand data

As already mentioned in section 2, the only available demand data on the Rome railway network, is the number of daily passengers that embark on and disembark at each station of the considered area.

In our case, we refer to the number of passengers embarked on and disembarked at each station of the considered area counted on a certain date of the year. As in the hose model in TLC networks, we use these data to determine an aggregate value for the demand at each node. To scale the data in a time horizon of an hour, we suppose sixteen hours of railway

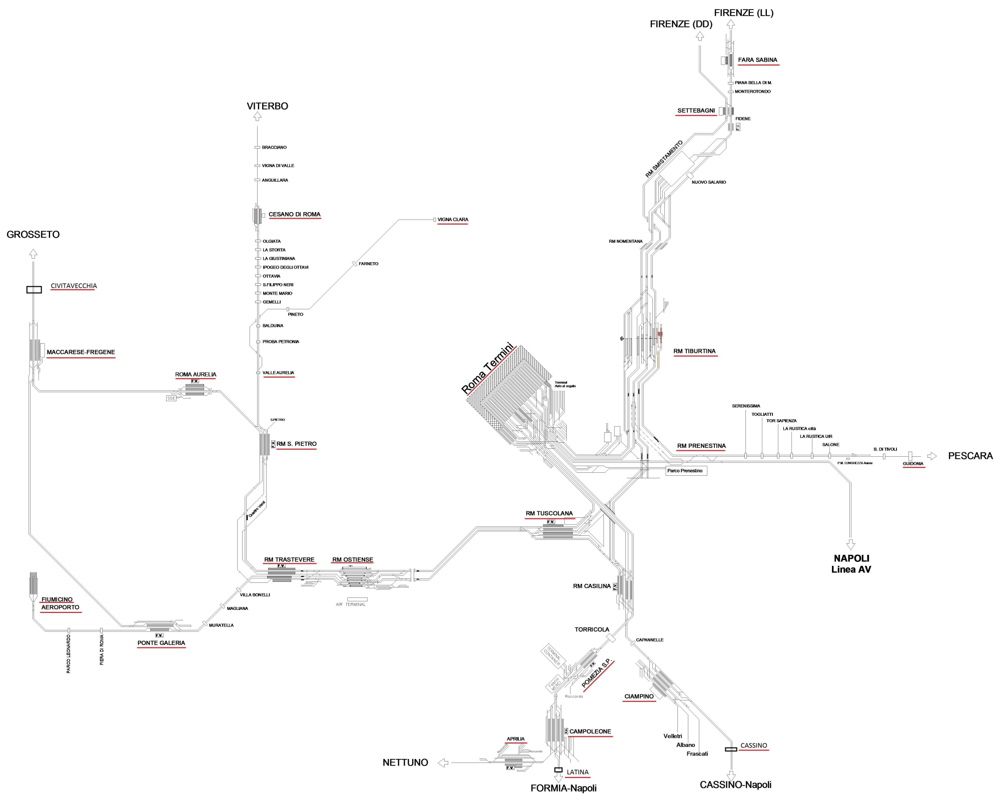

Figure 4: Infrastructure in Scenario A

service, in which six are peak hours and assume that 70 % of the passengers are concentrated in this time window.

The frequency of services on the lines is established considering the demand data during the peak hours, when the network is more congested. Afterwards, once the lines have been settled, train frequencies are determined for the other times of the day.

Capacity. The capacity of each arc is determined on the basis of the number of tracks, knowing that on each track at most 8 train per hour are allowed to travel in both directions. (An exception is the line Ostiense–Tiburtina where this limit is increased up to 12 trains per hour).

4 Results

We tested the model on all the three scenarios and we obtained results that RFI found extremely valuable.

Recall that, since enforcing the stretch constraints implies adding a large number of flow variables $y^{st}(u, v, \ell)$), we relax our model and impose the stretch bound on a limited number of origin destination pairs (in particular, the 26 most significant ones, according to RFI indications). The model described in section 2 has been implemented in AMPL

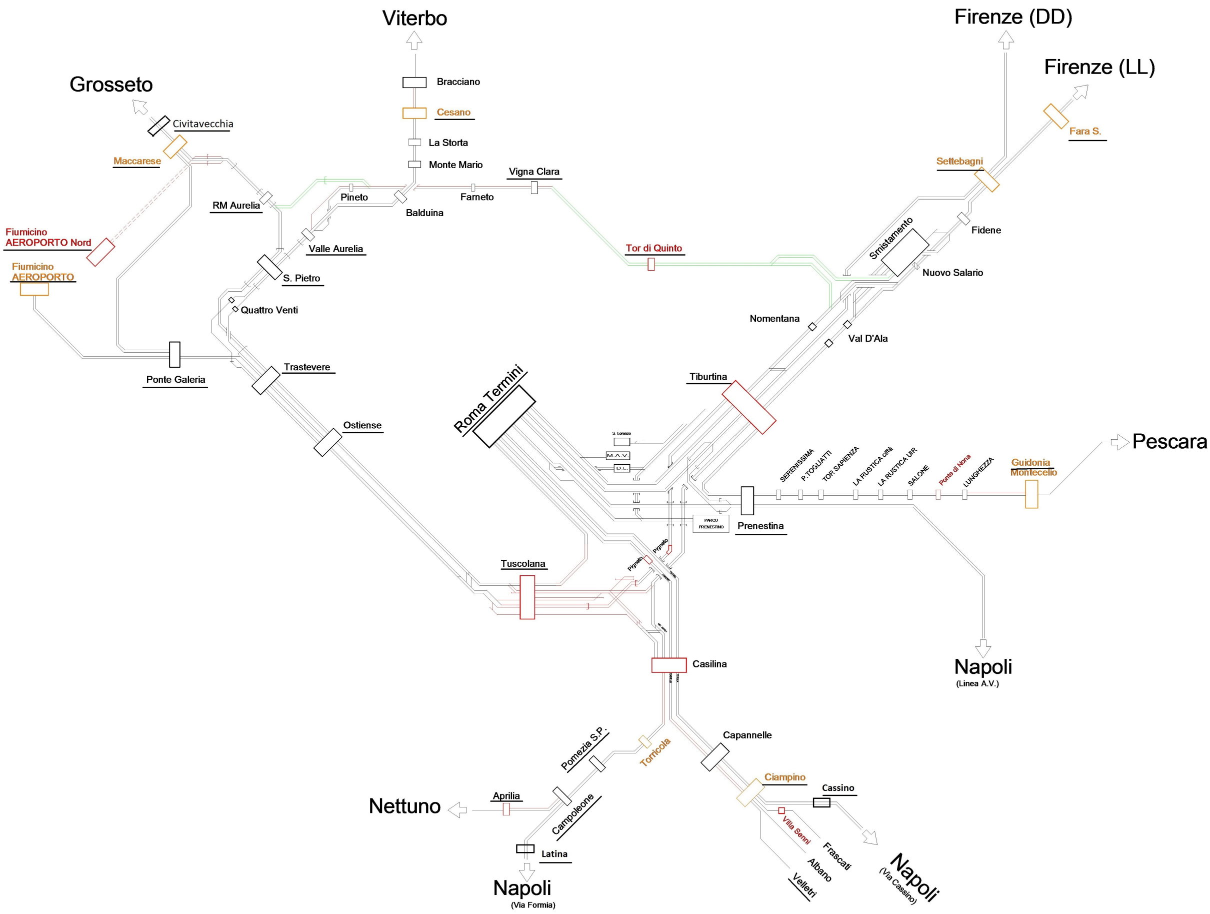

Figure 5: Infrastructure in Scenario C

(see Fourer et al. (2002)) and solved by means of Gurobi (see Gurobi (2016)). In our experiments, we set the following parameters:

1. the capacity of the trains is set to $\kappa = 700$;

2. the time-penalty for each transfer is set to $\lambda = 15$ minutes;

3. the parameters θ^*_{st} used as benchmarks for the stretch constraints is the time needed to go from s to t by car with no traffic as given by Google Maps. We have to point out that traveling by car is the only real viable option as bus services do not cover effectively the whole area considered in this study. Furthermore, these quantities are definitely underestimates of the actual travel times, especially during the rush hours. This in turn corresponds to the fact that α is larger than the actual stretch parameter.

4. $f_{\max}$ is set to 6 trains per hour.

As a first step, we decided to evaluate the current service, that consists in considering Scenario A where we fix as active lines only the ones reported in Table 1.

4.1 Evaluation of the current service

In order to evaluate the service offered at the moment, we considered Scenario A, and fixed the x_ℓ variables by only activating the 9 lines described in section 3. The model does not produce feasible solutions. In particular, one-hop constraints are not satisfied: some origin destination pairs require three transfers. If we relax one-hop constraints, we have that the demand is satisfied even with lower frequencies than in the actual service (which confirms that our demand is a lower estimate). On the other hand, the resulting stretch coefficients turn out to be less or equal than 3, 2, 1.5, and 1, in, respectively, 97.3%, 84.3%, 55.5%, and 26.4% of cases, i.e., possible origin destination pairs. Overall, the actual stretch coefficient is between 3.4 (where the problem is feasible) and 3.3 (where the problem is infeasible).

The conclusion is that the actually offered regional service does not satisfy one-hop constraints, and for some O-D pairs up to three transfers are needed, whereas RFI required this constraint to be satisfied for all the O-D pairs. Moreover, in order to further understand the performance of the actually offered service, in all scenarios, we also evaluated the option of keeping active the current set of lines—by fixing to 1 the corresponding x_ℓ variables—and possibly add new lines so that the one-hop and stretch constraints become satisfied (see Experiments # 4 in Tables 2 and 3, and Experiment #3 in 4).

4.2 Scenario A and B

Given these preliminary results, we ran the model on both Scenario A and Scenario B. In Tables 2 and 3, we summarize the results. In both scenarios, the model does not provide feasible solutions, which is due to bad connections in the north-west area, around Guidonia (Experiments #1, in both Tables 2 and 3). In fact, the consequence is that, in order to connect that area, too many lines cross the central area of the network around Roma Tiburtina, which in turn makes the capacity of the tracks in the area between Fara Sabina and Roma Tiburtina not sufficient.

We tried to relax alternatively one-hop constraints for all pairs involving Guidonia and Prenestina, or to relax the capacity constraint on the tracks between Fara Sabina and Roma Tiburtina. In both cases, the model returns an optimal solution, with the following characteristics:

Scenario A, relaxing the one hop constraint the optimal solution activates 11 lines, for a total number of 16 trains running during the peak hour. Among these ones 7 belong to the set of actual active lines (Experiment #3, Table 2). We also considered the option to fix the variables corresponding to the currently active lines. This option corresponds to keep active the lines actually offered and only add new ones, so that the users that are satisfied by the service do not have to change their habits. In this case, an optimal solution has 12 active lines, for a total number of 17 trains per hour (Experiment #4, Table 2).

Scenario A, relaxing the capacity constraint the optimal solution activates 16 lines and a total number of 23 trains (Experiment #2, Table 2). Among these ones 6 belong to the set of actual lines. In all the feasible simulations, the stretch coefficient can be

#	One-hop	Stretch ($\alpha = 2.8$)	Demand	Capacity	# of lines (current)	# of trains per hr.
1	✓	✓	✓	✓	**no sol**	-
2	✓	✓	✓	(no FS-Tib)	16(6)	23
3	(no Guidonia)	✓	✓	✓	11(7)	16
4	(no Guidonia)	✓	✓	✓	12(**9**)	17

Table 2: Results for Scenario A

#	One-hop	Stretch ($\alpha = 2.8$)	Demand	Capacity	# of lines (current)	# of trains per hr.
1	✓	✓	✓	✓	**no sol**	-
2	✓	✓	✓	(no FS-Tib)	18(6)	24
3	(no Guidonia)	✓	✓	✓	14(7)	17
4	(no Guidonia)	✓	✓	✓	15(**9**)	18

Table 3: Results for Scenario B

reduced up to 2.8.

Scenario B, relaxing one-hop constraints the optimal solution activates 14 lines, for a total number of 17 trains per rush hour. Among these ones 7 belong to the set of actual lines (Experiment #3, Table 3). If we fix the currently active lines, then the solution activates 15 lines with a total number of 18 trains (Experiment #4, Table 3).

Scenario B, relaxing the capacity constraint the optimal solution activates 18 lines, for a total number of 24 trains. Among these ones 6 belong to the set of currently active lines (Experiment #2, Table 3).

In all the considered cases, the stretch coefficient can be reduced up to 2.8. The results are summarized in the two tables (one per each scenario) 2 and 3.

In conclusion, the planned infrastructure intervention (plan B), despite its significant improvements, cannot solve all the critical issues of the actual service. We will show in the next subsection that, on the contrary, the closure of the ring allows to find a feasible solution fixing all the important criticalities.

4.3 Scenario C

In Scenario C, the problem admits a solution, and the quality of the service significantly improves, not only by satisfying one-hop constraints but also by allowing a reduction of the stretch coefficient α, that can be decreased down to 2.5. In Table 4, the results are reported: All the constraints can be satisfied by activating 16 lines (6 of them are already active) with a total number of 21 trains per hour (Experiment #1, Table 4). The stretch coefficient can be shortened to 2.55 (Experiment #2, Table 4). On the other hand, if the currently active lines are fixed, 18 lines are then needed in order to satisfy all the constraints (with stretch coefficient equal to 2.55) with a total number of 21 trains per hour (Experiment #3, Table 4).

#	One-hop	Stretch ($\alpha = 2.8$)	Demand	Capacity	# of lines (current)	# of trains per hr.
1	✓	✓	✓	✓	15(6)	21
2	✓	$\alpha = 2.55$	✓	✓	15(6)	21
3	✓	$\alpha = 2.5$	✓	✓	18(**9**)	21

Table 4: Results for Scenario C

These results imply that the major intervention of the ring closure is justified by the benefits on the whole service, allowing a greater connectivity on the network, without overloading part of the tracks. Of course, the number of trains needed in order to satisfy the actual (unknown) demand may clearly be higher, but since the number of trains in output of the model is lower than the one running on the network now, there is room for increasing the frequencies where needed.

5 Conclusions and future work

In this work, we developed a model to determine a suitable set of lines and the corresponding train frequencies in a railway network. The model proved to be effective in supporting railway's engineers when evaluating alternative infrastructure enhancements in the metropolitan rail network of Rome.

In particular, the model allowed RFI to assess the potential impact of planned and potential infrastructure developments. We remark that, even in the absence of exact or reliable OD-flows, the model is capable of determining other crucial parameters such as, e.g., the connectivity and the stretch parameter. Indeed, the positive results obtained by applying our model were used by RFI in support of the crucial (and expensive) intervention of the closure of the ring. Moreover, due to this first successful experience, RFI decided to apply our model also to the regional railway network of Milan, where more reliable demand data and OD-flows are at hand.

Indeed, infrastructure managers are becoming ever more aware that in order to determine the right mix of new investments, it is necessary to predict the impact of the new infrastructure on current and future demand of transportation. In particular, planning new infrastructure requires the definition of feasible routes and schedules for the trains which ensure that the target demand can be satisfied. In turn, because the overall process is very complex and cannot be handled manually, this can only be achieved by developing suitable optimization models and effective solution algorithms. An example of such two-stage infrastructure planning at the Norwegian infrastructure manager Banenor is described in Lamorgese et al (2017).

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