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**THE EFFECT OF STICK STIFFNESS OF FRICTION MODELS ON THE BENDING  
BEHAVIOR IN NON-BONDED FLEXIBLE RISERS**

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**ABSTRACT**

*This paper investigates the effect of stick stiffness on the bending behavior in non-bonded flexible risers. The stick stiffness was normally implemented in the friction model for calculating the friction stress between layers in such structures. As the stick stiffness may be too small to achieve the plane-surfaces-remain-plane assumption under low contact pressure in some friction models [1], a new friction model was proposed for maintaining the constant stick stiffness in the present work. Less stick stiffness than that obtained by the plane-surfaces-remain-plane assumption was observed in test data. It was assumed that the stick stiffness reduction is caused by shear deformation of plastic layers. A numerical study on stick stiffness by including the shear deformation effect was carried out and verified against full scale tests with respect to the bending moment-curvature relationship.*

**INTRODUCTION**

The flexible riser concept may be applied for flexible pipes, power cables and umbilicals. All consist of several layers involving different materials and components in the complex cross-section depending on the specific application. In general, flexible pipes are used to transport oil and gas, whereas umbilicals may serve different purposes such as chemical injection, electrical and hydraulic power transmissions/control and monitoring. The umbilical cross-section may therefore include steel tubes,

tensile armors, fluid conduits, electrical cables and fibre optic cables. For the helix elements, the most important stress component with respect to fatigue is the longitudinal stress at critical positions. The longitudinal stress is only governed by axial force and local bending if the friction is not considered. However, ignoring the friction will cause very non-conservative prediction of fatigue life. In many cases, it has been found that friction stress was more dominant than other factors which affected the fatigue life [2–6]. Therefore, accurately modeling of the friction stress behavior becomes important in fatigue design. There are various strategies to take account of friction effect on the fatigue life in such structures. The ideal method may model the complex cross-section by full 3D approaches which will demand in long computing time, however. Therefore, the global response analysis and local stress analysis are normally performed separately. The global results in terms of curvature and tension time history are used as inputs to produce helical elements' stress time history in the local analysis. These local analyses are usually performed at critical positions, i.e., the hang-off, sag, hop, and touch down zone. The local model analysis can be achieved either by the analytical method or by finite element approaches.

One of the finite element approaches is to apply a friction model to calculate friction for modeling individual steel wires in a computer code. The stick stiffness is an important factor which determines the helix element's slip onset and determines the friction stress accuracy. In the present work, it is aimed to develop a new friction model which can calculate the friction

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under constant stick stiffness conditions. The stick stiffness normally considered the contribution from plane-surfaces-remain-plane assumption [1]. As less stick stiffness than that obtained from this assumption was observed in the test data [7], a new method which considered the plastic layers shear deformation was hence developed. The proposed friction model was implemented in a computer code and a numerical study was carried out to verify the friction model against test data.

### Classical Coulomb friction model

An elastic-plastic friction model for predicting friction stress was proposed by Sævik [1], which was implemented in BFLEX [1] for modeling individual steel wires. The criterion of slip onset was the maximum elastic relative displacement which was a fixed stick displacement. The stick stiffness was formulated as:

$$K_{stick} = \frac{\mu f_n}{\Delta_1} \quad (1)$$

where  $\Delta_1$  is the fixed stick displacement,  $f_n$  is the contact pressure, and  $\mu$  is the friction coefficient. In the stress analysis, the stick stiffness was updated due to the dynamic contact pressure. This may result in too small stick stiffness to ensure the plane-surfaces-remain-plane assumption under low contact pressure conditions.

If the variation of contact pressure is within 10%, the stick stiffness is adequate for the plane-surfaces-remain-plane assumption. Otherwise, the assumption can not be achieved.

### Proposed friction model: Smoothed Coulomb friction model

Since the stick stiffness can not be maintained constant under dynamic contact pressure conditions, a new friction model is proposed to address this problem. The most important features are summarized as follows:

- Feature 1: Constant and accurate stick stiffness
- Feature 2: Transition from stick domain to full slip domain

A correct method to determine the stick stiffness of this friction model comes to priority. The stick stiffness is contributed by internal friction and shear deformation of thick plastic layers, which yields:

$$\frac{1}{K} = \frac{1}{K_{friction}} + \frac{1}{K_{shear}} \quad (2)$$

In which  $K$  is the stick stiffness applied in the friction model.  $K_{friction}$  is the stick stiffness contributed by internal friction.

$K_{shear}$  is the stick stiffness contributed by the shear deformation of thick plastic tape. The calculation of  $K_{friction}$  and  $K_{shear}$  will be elaborated in details in following contents.

### Feature 1: Constant stick stiffness

**Equation of  $K_{friction}$**  The flexible riser deforms like a rigid steel pipe where the plane-surfaces-remain-plane in a small global curvature range. This phenomenon is induced by the internal friction mechanism since the relative displacement can be prevented in a small curvature range. When the shear capacity governed by friction, i.e., the maximum static friction, is exceeded with increasing global curvature, the helical elements start to slip against their neighboring layers. A slip model was proposed by Sævik [8] to estimate the  $K_{friction}$ . It is briefly presented as follows.

When the pipe deforms in stick domain where plane surfaces remain plane, the axial force  $Q_1$  in the tendon is expressed as:

$$Q_1 = -EA \cos^2 \alpha R \cos \phi \beta_2 \quad (3)$$

Where  $E$  is Young's modulus of the tendon,  $A$  is the tendon's cross section area.  $\alpha$  is the tendon's lay angle,  $\phi$  is the tendon element angular position in the cross section,  $\beta_2$  is the global curvature about Y axis of the overall cross section center.

The associated shear line force  $q_1$  (force per unit length) along the tendon which fulfills the plane assumption is attained by differentiating the Eq.(3) with respect to the length coordinates  $X^1$  and applying the relationship  $\phi = \frac{\sin \alpha}{R} X^1$ :

$$q_1 = EA \cos^2 \alpha \sin \alpha \sin \phi \beta_2 \quad (4)$$

According to the tendon's position  $\phi$ , the maximum shear line force is obtained at the neutral axis. The shear force capacity is governed by the maximum friction, i.e., the multiplication of friction coefficient and contact line force, which yields  $q_{1c}$ :

$$q_{1c} = \mu (q_3^I + q_3^{I+1}) \quad (5)$$

The relative displacement of the tendon placed at the outermost fiber of the pipe can be obtained by integrating the axial strain associated with Eq.(3) along the tendon within a quarter pitch:

$$u_{1c} = \frac{\mu (q_3^I + q_3^{I+1}) R^2}{EA \sin^2 \alpha} \quad (6)$$

Then the stick stiffness  $K_{friction}$  is obtained by equating the Eq.(5) and Eq.(6):

$$K_{friction} = C_b \frac{EA \sin^2 \alpha}{R^2} \quad (7)$$

Where  $C_b$  is a correction factor that needs to be determined by a sensitivity study. It should fulfill the adequate accuracy of the assumption of plane-surfaces-remain-plane, and compromise with numerical stability issues at the same time.

**Equation of  $K_{shear}$**  When the tensile armor is supported by the thick plastic layers, the shear deformation will occur. In addition to the contribution from the internal friction, shear deformation of plastic tape also contributes the assumption of plane-surfaces-remain-plane. If shear deformation occurs, the assumption will not be achieved before starting of slip. The shear stiffness  $K_{shear}$  is defined in Sævik [9, 10]'s work:

$$K_{shear} = \frac{Gb}{t} = \frac{E_p b}{2(1 + \nu)t} \quad (8)$$

Where  $E_p$  is Young's modulus of the plastic tape,  $G$  is the shear modulus of the plastic tape.  $b$  is the width of the tendon,  $t$  is the thickness of the tendon.  $\nu$  is the poisson ratio. If  $K_{shear}$  goes towards infinite, the assumption of plane-surfaces-remain-plane can be achieved. More details were elaborated in [9, 10].

## Feature 2: Smooth transition from stick domain to full slip domain

A smooth transition from stick domain to full slip domain is designated to improve the numerical performance of the friction model proposed by Sævik [1]. This smooth transition is described by a second order function instead of being linearly increasing as illustrated in FIGURE 1. The friction coefficient in stick domain and in full slip domain were assumed to be equal to 0.15 and 0.20, respectively.

## Brief introduction of full scale tests

A simple sketch of the test set-up is shown in FIGURE 2. The whole length of the 4 inch flexible pipe was 8 meters and was horizontally mounted in a test rig. Two halves of tubes were clamped on the pipe 1.35m away from the right end. This allowed the pipe to be treated as a cantilever structure. Two extensometers were attached at two outermost fibre positions of the pipe as to measure the pipe axial strain at the position 1.19m away from the right end. The curvature was calculated based on the measured axial strain and outer diameter of the pipe by using

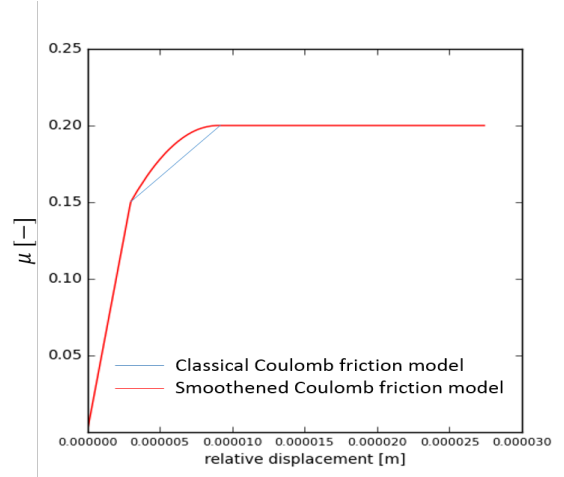


FIGURE 1: Classical Coulomb friction model and Smoothened Coulomb friction model

$\frac{\epsilon_{top} - \epsilon_{bottom}}{D}$ . This test was run in displacement control. The vertical displacement was applied at the right end. The reaction force was recorded by an actuator which was placed at the same load point. Therefore, the global bending moment of the pipe was simply the multiplication of reaction force from actuator and the 1.19m length at the measurement location.

Nine tests were carried out to study the hysteresis behavior due to the internal slip mechanism. In the present work, only the Test No.5 were studied listed in the Table 1.

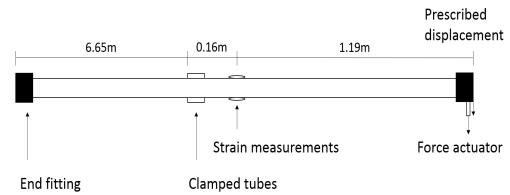


FIGURE 2: Full scale test setup

TABLE 1: Test matrix

Test No.	curvature range [1/m]	Internal pressure [bar]	Frequency [Hz]	Prescribed displacement [mm]
No. 5	1/9	200	0.1	70

## Numerical model

A model of the test specimen was established in BFLEX. The model included the activated length of 1.35m only. The tensile armor's lay angle was  $\pm 38^\circ$  and the pitch length was approximately 0.57m. The friction models' stick stiffness was assumed to be  $1730 [MN/m^2]$  according to the work proposed by Sævik [8]. The pipe cross-section consisted of two cross wound tensile armor layers, four plastic sheath layers including the outer sheath and two anti-wear layers and one pressure barrier. For both the inner and outer tensile armor layer, HS-HEAR353 was employed to model the individual steel wire and 16 steel wires were used to represent the whole tensile armor layer. HCONT453 was applied to simulate the contact surface between these two tensile armor layers. HSHEAR363 was used for the plastic layers and pressure spiral layer. HCONT463 was applied to simulate the contact surface between the cylindrical layer and the tensile armor layer. Other inputs are summarized in Table 2 and the FE model is shown in FIGURES 4 and 3.

The clamped tubes can restrain the rotations of the pipe and axial displacement of the outer sheath. But the tendons can move freely inside. Therefore, springs were applied in the numerical model in order to fulfill the similar boundary conditions as in the test setup. Springs nodes were generated at the same coordinates of the left hand of the pipe. Spring elements were connected between the spring nodes and tendons node. The spring element axial stiffness was calculated as follows:

$$k_s = \frac{EA}{L} \quad (9)$$

Where EA is axial stiffness of the tendon and L is the length of the tendon,  $L = \frac{8-1.35}{\cos\alpha}$ .  $\alpha$  is the lay angle.

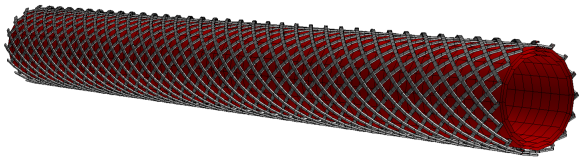


FIGURE 3: Full scale numerical model in a global view

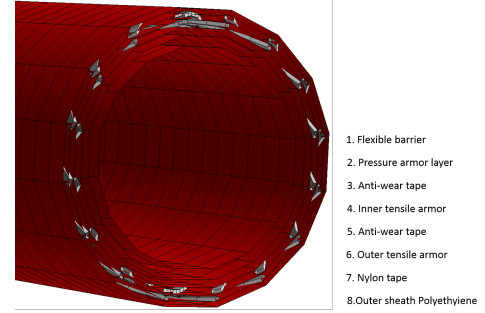


FIGURE 4: Full scale numerical model illustrated by each layer

TABLE 2: Key input parameters of numerical model

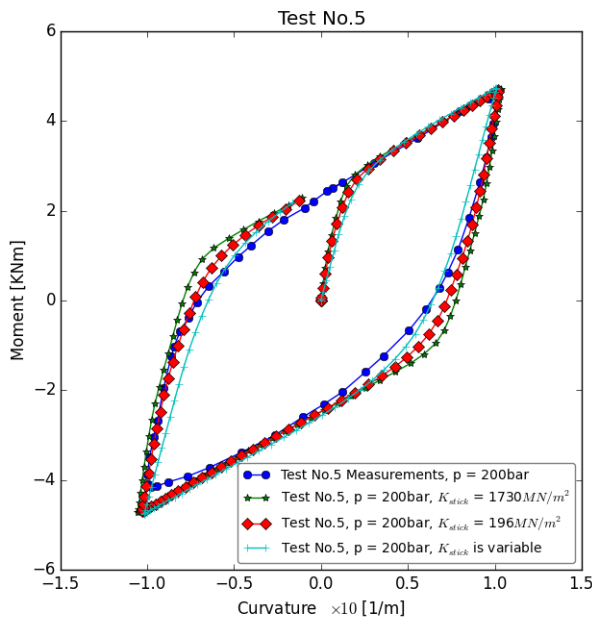
Parameter	Value	Unit
Barrier (Core) mean radius	58.3	mm
Pressure spiral mean radius	64.05	mm
Anti-wear tape 1 mean radius	68.2	mm
Anti-wear tape 2 mean radius	72.2	mm
Inner tensile armor mean radius	70.3	mm
Outer tensile armor mean radius	74.3	mm
Tape (Nylon) mean radius	75.4	mm
Outer sheath mean radius	78.55	mm
Pressure spiral area	35	mm <sup>2</sup>
Inner/outer tensile armor area	10	mm <sup>2</sup>
Inner/outer tensile armor lay angel	-38/38	deg
Number of inner tensile armor	61	
Number of outer tensile armor	65	

## Verification of stick stiffness

A comparison study was carried out for verifying the stick stiffness against full scale test data. Firstly, the stick stiffness was applied based on the plane-surfaces-remain-plane assumption, which was equal to  $1730MN/m^2$  proposed by Sævik [8]. The stick stiffness was further used considering the plastic layers' shear deformation, which was equal to  $196MN/m^2$  based on the parameters of corresponding geometry of steel wire and materials of plastic tape applied in Eq.(2),(7),(8). Thirdly, the Classical Coulomb friction model was applied, where the original stick stiffness was set to  $196MN/m^2$  under contact line force  $f_n = 18757N/m$  induced by the applied static internal pressure 200bar. This resulted in the fixed stick displacement equal to 2E-5m. The contact line force variation during bending was about 4000N/m which resulted in the stick stiffness variation equal to  $42MN/m^2$ .

As illustrated in FIGURE 5, the stick stiffness  $196MN/m^2$

correlated well with that in the test data. However, the stick stiffness based on plane-surfaces-remain-plane assumption is larger than that obtained in test data and the stick stiffness in the classical Coulomb friction model is variable during bending which is smaller than that in the test data. It reveals that the stick stiffness combining the stiffness contributed by internal friction and stiffness contributed by shear deformation of thick plastic layers is correct to be applied in the friction model. In addition, this stick stiffness can be maintained constant which is of great help to stabilize the numerical simulation and predict an accurate friction stress in the alternating stick-slip conditions.



**FIGURE 5:** Correlation between test measurements and BFLEX simulation of Test No.5

### Conclusions

The stick stiffness is an important factor determining the helix element’s slip onset in non-bonded flexible risers. The stiffness observed in the test data was less than that obtained based on the plane-surfaces-remain-plane assumption, which may be caused by the plastic layers’ shear deformation. Therefore, a method to calculate the stick stiffness was proposed, which considered both this assumption and the plastic layers’ shear deformation. As the stick stiffness in some models [1] may be too small to achieve the assumption under dynamic contact pressure, a new friction model was further proposed for maintaining a con-

stant stick stiffness. The friction model was implemented in a computer code and verified against test data. Good agreement was obtained which demonstrated it was correct to predict the stick stiffness considering the plastic layers’ shear deformation. The constant stick stiffness is beneficial to stabilize the numerical simulation and predict an accurate friction stress in the alternating stick-slip conditions.

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