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Successful Application of Multiscale Methods in a Real Reservoir Simulator Environment

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Abstract For the past ten years or so, a number of so-called multiscale methods have been developed as an alternative approach to upscaling and to accelerate reservoir simulation. The key idea of all these methods is to construct a set of prolongation operators that map between unknowns associated with cells in a fine grid holding the petrophysical properties of the geological reservoir model and unknowns on a coarser grid used for dynamic simulation. The prolongation operators are computed numerically by solving localized flow problems, much in the same way as for flow-based upscaling methods, and can be used to construct a reduced coarse-scale system of flow equations that describe the macro-scale displacement driven by global forces. Unlike effective parameters, the multiscale basis functions have subscale resolution, which ensures that fine-scale heterogeneity is correctly accounted for in a systematic manner. Among all multiscale formulations discussed in the literature, the multiscale restriction-smoothed basis (MsRSB) method has proved to be particularly promising. This method has been implemented in a commercially available simulator and has three main advantages. First, the input grid and its coarse partition can have general polyhedral geometry and unstructured topology. Secondly, MsRSB is accurate and robust when used as an approximate solver and converges relatively fast when used as an iterative fine-scale solver. Finally, the method is formulated on top of a cell-centered, conservative, finite-volume method and is applicable to any flow model for which one can isolate a pressure equation. We discuss numerical challenges posed by contemporary geomodels and report a number of validation cases showing that the MsRSB method is an efficient, robust, and versatile method for simulating complex models of real reservoirs.

Keywords Multiscale methods · reservoir simulation · review

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1 Introduction

The term 'multiscale method' denotes a variety of methods that all try to model and simulate phenomena that are governed by a wide range of spatial and/or temporal scales. Many engineering problems are characterized by scale separation in the sense that one can easily identify fast, intermediate, and slow processes and/or processes that take place on the macro, meso, and microscale. Multiscale formulations have proved to be particularly efficient for this type of problem by offering a systematic framework for incorporating subscale effects in a manner that is consistent with the physical models governing the behavior on the unresolved scale. Multiphase flow in subsurface rock formations is an example of multiscale processes that do not have apparent scale separation. Modeling processes without scale separation, but with strong media contrasts and orders of magnitude variations in their coefficients, is in many aspects more challenging than modeling problems with scale separation because nonlocal information is needed to compute representative effective properties.

Large (local) variations in the characteristics of sedimentary rock formations are the primary cause of multiscale behavior in reservoir modeling. These variations can be observed on all scales, from the micrometer scale of individual pores and pore throats to the kilometer scale of reservoirs. Naturally, a single model cannot include all pertinent scales. However, state-of-the-art methods for reservoir characterization enable geologists and reservoir engineers to routinely generate complex, high-resolution geological models having grid cells in the range from centimeters to decimeters in the vertical direction and meters to tens of meters in the horizontal direction. This type of resolution is required to accurately account for structural elements like faults, fractures, joints, and deformation bands and stratigraphic characteristics like channels, clinoforms, lobes, shale and mud drapes, etc.

Thanks to a tremendous increase in computational power, which has been accompanied by a similar improvement in computational methods (massive parallelization, multilevel iterative solvers, preconditioning methods, etc.), it is today possible to routinely run reservoir simulations with millions of unknowns (DeBaun et al. 2005; Fjerstad et al. 2007). Direct simulation of high-resolution geological models nevertheless remains a challenge, and contemporary simulators are not yet fully able to utilize all static data and the vast amount of dynamic data that gradually has become available. For many years, there has therefore been a call for a radical change in simulation technology that could offer the computational speedup necessary to enable the oil industry to make a step-change in its modeling processes.

In the following, we will discuss a set of novel two-level solvers designed especially to compute flow and transport of one or more fluid phases in heterogeneous subsurface rock formations. The solvers are called multiscale methods since they originally aimed to solve elliptic problems having variable coefficient with multiscale heterogeneity. When these multiscale methods were first introduced a decade ago, they were said to have the potential to bridge the gap in resolution between models used for characterization and provide the scaling necessary for future model sizes and workflow applications.

The main purpose of the paper is to review the development of multiscale methods and demonstrate that they, after more than a decade of research, now

can handle the full complexity of industry-standard reservoir models. In particular, we outline the state-of-the-art method that is implemented in a commercial simulator, thereby updating the discussion of Kozlova et al. (2015). We also point out numerical challenges encountered during the development and implementation and present a number of complex test cases used to validate the efficiency, robustness, and utility of the multiscale simulator. Computational efficiency is not discussed herein, but our experiments indicate that for representative test cases, the multiscale method may give an order-of-magnitude speedup, which is significant, but less than the theoretical $\mathcal{O}(10^3)$ speedup that was indicated for simple flow problems (Jenny et al. 2003) when these methods first emerged from academia.

2 Review of Major Developments

We start by reviewing the key ideas that paved the road for what is currently considered state-of-the-art multiscale solvers. Somewhat simplified, the evolution from early ideas about multiscale methods and towards deployment in commercial simulators followed two main paths. Developers of multiscale finite-volume methods focused primarily on flow physics and systematic reduction of fine-scale residuals, whereas the development of multiscale mixed finite-element methods focused primarily on geological realism and applicability to general grids. Our discussion is therefore not fully chronological. Instead, we present different hurdles encountered in the research on flow physics and systematic reduction of residuals and in the research focusing on geology and support for general fine and coarse grids. To add context, we also briefly mention methods developed within a more academic context.

2.1 Early development: a robust alternative to upscaling

The standard approach in industry to handle multiscale heterogeneity is to use a homogenization or upscaling method to compute effective properties defined so that the flow equations are satisfied in an averaged sense on a coarser grid (Christie 1996; Renard and De Marsily 1997; Barker and Thibeau 1997; Farmer 2002; Gerritsen and Durlofsky 2005). Despite much research over the past three to four decades, most upscaling methods still only give reliable results for a limited range of flow scenarios. This is in part a result of inaccurate localization assumptions that limit upscaling of static and dynamic properties and in part because of process dependence that reduces robustness of dynamic properties.

The family of multiscale methods discussed herein was first presented in a seminal paper by Hou and Wu (1997), who proposed to use homogeneous solutions of the flow problem posed inside each element as polynomial basis functions. Similar ideas had been introduced a decade earlier by Babuska and Osborn (1983) and Babuska et al. (1994). The multiscale finite element (MsFE) method of Hou and Wu (1997) soon spawned a family of related methods, including the numerical subgrid-upscaling method (Arbogast 2000, 2002; Arbogast and Bryant 2002; Arbogast 2004), the mixed multiscale finite-element (MsMFE) method (Chen and Hou 2003; Aarnes 2004; Aarnes et al. 2005), and the multiscale finite-volume

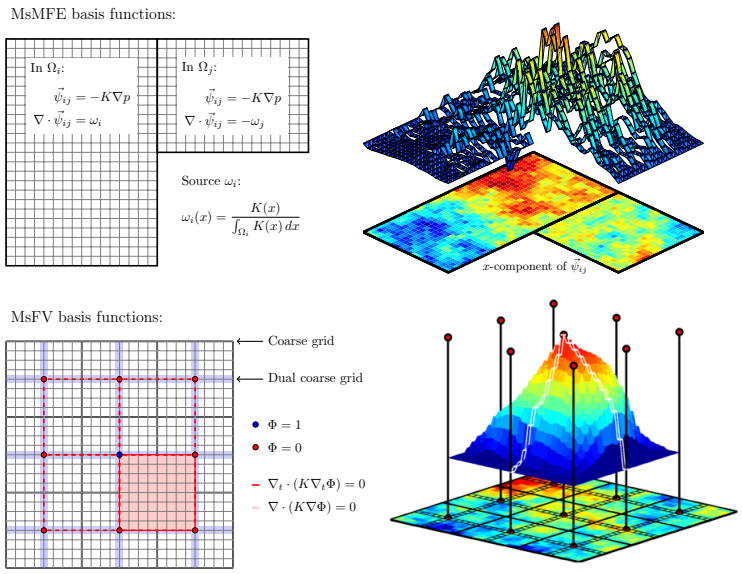

Fig. 1 Definition of basis functions for the MsMFE and the MsFV methods. For MsMFE, one solves a local flow problem inside the union of the two blocks, with no-flow boundary conditions on the perimeter and positive and negative source terms inside the two blocks to drive flow from one block to the other. For MsFV, one sets a unit pressure at the block center and zero pressure at the centers of neighboring blocks, and then solves a reduced flow problem along the dual edges connecting the block centers. Basis functions are then computed by solving flow problems in the regions bounded by the dual edges.

(MsFV) method (Jenny et al. 2003, 2004, 2006). All these methods use numerically computed basis functions satisfying a local flow problem (see Figure 1) to systematically and consistently incorporate the effect of fine-scale heterogeneity into coarse-scale flow equations that describe the macro-scale flow patterns inside the reservoir. Once the flow equations are solved on the coarse grid to account for wells, aquifer support, and so on, the subgrid resolution of the basis functions can be used to prolongate the approximate flow solutions back to define mass-conservative flow fields on the underlying fine grid.

Multiscale methods were initially presented as a more accurate and robust alternative to upscaling methods. The idea was that subresolution and a more natural way of coupling local and global flow effects should improve consistency and reduce the tendency of introducing non-physical coarse-scale properties. The main distinction between an upscaling method and a multiscale method is that whereas the objective of the former is to generate effective properties that can be used to compute approximate coarse-scale solutions, the objective of the latter is compute approximate fine-scale solutions. However, the line of distinction is thin, in particular for so-called local-global upscaling methods (Chen et al. 2003; Chen and Durlofsky 2006; Chen et al. 2009, 2010), or when a suitable fine-scale reconstruction procedure (Gautier et al. 1999; Audigane and Blunt 2004; Babaei and King 2012, 2013) is used much in the same way as in the MsFV method (Jenny et al. 2003, 2004). The interested reader should consult Kippe et al. (2008), who present a comprehensive comparison of multiscale and upscaling methods.

2.2 Adaptivity and computational complexity

The theoretical operational count of a full multiscale solution procedure applied once (i.e., computing basis functions, solving coarse-scale problem, reconstructing fine-scale approximation) is approximately the same as when solving the same problem on the underlying fine grid with an efficient linear solver. So what is the point of using a multiscale method? The primary gain comes from applying these methods to multiphase pressure equations, which are generally time-dependent so that the pressure must be updated multiple times. Multiscale methods are designed to exploit the fact that temporal dependence of the flow equations is moderate compared to the spatial variability and that temporal changes often are localized. By reusing most of the basis functions from one global pressure update to the next, possibly updating a few basis functions locally to account for mobility changes near displacement fronts (Tchelepi et al. 2007), one can obtain qualitatively correct fine-scale solutions at a cost comparable to solving the flow equations on the coarse grid (Kippe et al. 2008). Likewise, since basis functions are defined as localized flow problems, the multiscale formulations offer a natural computational concurrency.

2.3 Adding more flow physics

The multiscale methods introduced above were all designed to compute the solution of a Poisson type elliptic equation with a spatially and strongly variable coefficient without clear scale separation. This meant that all effects not represented in the span of the basis functions could only be resolved on the coarse scale. Early attempts to account more accurately for other physical effects introduced extra basis functions or special correction functions[1] to incorporate gravitational effects (Lunati and Jenny 2006b), compressibility (Lunati and Jenny 2006a; Hajibeygi and Jenny 2009), and wells (Wolfsteiner et al. 2006; Skaflestad and Krogstad 2008; Jenny and Lunati 2009). Formulations were also extended to (simplified versions) of the three-phase, black-oil equations (Lee et al. 2008; Krogstad et al. 2009). Use of correction functions was later superseded by less sophisticated iterative procedures, as will be discussed shortly.

2.4 Numerical instabilities

Extensive numerical tests (see e.g., Kippe et al. (2008)) had shown that multiscale methods are accurate and robust compared with (local) upscaling methods, and in most cases give solutions that are in good agreement with solutions obtained from a fine-scale simulation. On the other hand, the tests also showed that one generally cannot guarantee a low error and that it is relatively simple to define pathological cases making a particular multiscale method loose its accuracy. This is particularly evident for the MsFV method, which tends to produce solutions with

[1] Basis functions prolongate degrees-of-freedom from the coarse to the fine scale and can be thought of as homogeneous solutions to the elliptic operator. Correction functions provide consistent fine-scale description of terms on the right hand side and can be thought of as inhomogeneous or particulate solutions of the elliptic equation.

large unphysical pressure oscillations and highly circular velocity fields for cases with high aspect ratios or channelized media with strong permeability contrasts (Kippe et al. 2008; Lunati and Jenny 2007). This deficiency was further analyzed by Hesse et al. (2008), Nordbotten et al. (2012), and Wang et al. (2016), who explained the inability to provide accurate solutions by inadequate localization assumptions that introduce pronounced lack of monotonicity unless the coarse-scale stencil is modified locally to be closer to the classic two-point scheme.

2.5 Iterative formulation

Significant errors may be introduced at the interfaces between coarse grid blocks as a result of inaccurate boundary conditions that do not necessarily represent the influence from global flow patterns in a good way. To diminish these errors and provide a systematic means of controlling the overall multiscale approximation, the i-MsFV method was proposed by Hajibeygi et al. (2008) and Hajibeygi and Jenny (2009). This iterative method applies line relaxation in each spatial direction to smooth the high-frequency error components and propagate the fine-scale residual from the interfaces and into the coarse blocks. The smoothed solution is used to improve the localization assumption. Then, one updates basis functions, computes a new global coarse-scale approximation, prolongates it back to the fine scale, and so on. To improve efficiency, iterations should be applied adaptively and locally (Hajibeygi and Jenny 2011). The i-MsFV method provided a means for systematic error control and improved accuracy in many cases, both for elliptic and parabolic flow equations. Because the line-relaxation smoother is tied to rectilinear or curvilinear grid geometries, it was later replaced by more general smoothers, which are applicable to non-curvilinear grid geometries.

2.6 Operator formulation and algebraic multiscale methods

Key contributions to the development of multiscale methods were made by Zhou and Tchelepi (2008) and Lunati and Lee (2009), who introduced operator formulations to describe the MsFV method in algebraic form. In this formulation, two operators are constructed: a prolongation operator mapping coarse-scale unknowns to fine-scale unknowns, and a restriction operator mapping fine-scale equations to coarse-scale equations. The prolongation operator $\boldsymbol{P}$ is constructed by assembling the numerically computed basis functions into a matrix, one basis function per column. The specific form of the restriction operator $\boldsymbol{R}$ depends on what coarse-scale discretization one wants: the MsFV method (Jenny et al. 2003) is obtained if $\boldsymbol{R}$ constructs a coarse system by summing all the fine-scale equations inside each coarse block, whereas a MsFE method is obtained if $\boldsymbol{R} = \boldsymbol{P}^T$ (Bonfigli and Jenny 2010). Coarse-scale equations are then derived algebraically by introducing the multiscale prolongation $\boldsymbol{p} \approx \tilde{\boldsymbol{p}} = \boldsymbol{P}\boldsymbol{p}_c$ into the fine-scale problem $\boldsymbol{A}\boldsymbol{p} = \boldsymbol{q}$ and multiplying by the restriction operator from the right,

$$\boldsymbol{A}\boldsymbol{p} = \boldsymbol{q} \qquad \longrightarrow \qquad (\boldsymbol{R}\boldsymbol{A}\boldsymbol{P})\,\boldsymbol{p}_c = \boldsymbol{R}\boldsymbol{q}. \tag{1}$$

To get mass-conservative fluxes on the fine grid, a local flow problem is solved inside each primal grid block with the fluxes derived from $\boldsymbol{p}_c$ imposed as Neumann

boundary conditions (Jenny et al. 2003). This construction applies to both incompressible and compressible flow (Zhou and Tchelepi 2008). Additional insight was provided by Nordbotten and Bjørstad (2008), who showed that the MsFV method without correction functions can be interpreted as a special case of classic non-overlapping domain-decomposition methods (Smith et al. 1996) and then used this interpretation to improve the accuracy of the multiscale approximation.

Based on the operator formulation, Lunati et al. (2011) showed that the multiscale method could be used as preconditioner in a Krylov subspace method like GMRES. Similarly, Zhou and Tchelepi (2012) introduced a two-stage algebraic multiscale solver in which the first global stage is a MsFV or MsFE multiscale solution and the second stage uses a local preconditioner such as additive Schwarz or block incomplete lower-upper factorization (ILU). In another important contribution, Wang et al. (2012, 2014), analyzed the algebraic multiscale solver in more detail and compared the efficiency and accuracy of Galerkin and finite-volume restrictions combined with correction functions, block ILU, and ILU as local preconditioners. Quite interesting, their conclusion is that the best overall performance is obtained by using a Galerkin restriction operator combined with a local ILU(0) preconditioner. This will not give mass-conservative fluxes, but mass conservation can be reintroduced if one finite-volume update is performed at the end. See also (Magri 2015) for a comprehensive overview and analysis of multiscale preconditioning strategies.

The algebraic multiscale framework can, at least in principle, be used in any sequential solution procedure that isolates a pressure equation. Algebraic multiscale solvers have already been applied to polymer flooding with non-Newton fluid rheology (Hilden et al. 2016), discrete and embedded fracture models (Sandve et al. 2013; Shah et al. 2016; Tene et al. 2016a,b), and somewhat simplified (Hajibeygi and Tchelepi 2014) and more comprehensive (Møyner and Tchelepi 2017; Møyner 2016a) compositional models. Work is in progress on extending the method to geomechanics (Castelletto et al. 2016, 2017) and more comprehensive compositional formulations. While promising, none of these capabilities have yet been included in the commercial simulator.

2.7 Geological realism and automated coarsening

Support for realistic geology and complex grids was initially only developed for the MsMFE method. Figure 2 outlines how the capabilities of the MsMFE method were gradually developed towards handling the full complexity of industry-standard stratigraphic grids (corner-point and unstructured Voronoi type grids). As part of this work, the method was included in a streamline simulator during 2006–2008 as the first commercial implementation of the multiscale technology (Natvig et al. 2011), and successfully used to simulate high-resolution models of fracture corridors with multimillion cells.

The MsMFE method is very flexible with respect to grid geometry and topology and can be formulated on any reasonable grid with unstructured topology and general polyhedral cell geometries. Likewise, the only requirement on the coarse partitions is that each grid block consists of a connected set of grid cells. (Reduced accuracy because of grid effects should be expected if the blocks become too irregular). Because of this flexibility, it proved to be relatively simple to develop robust

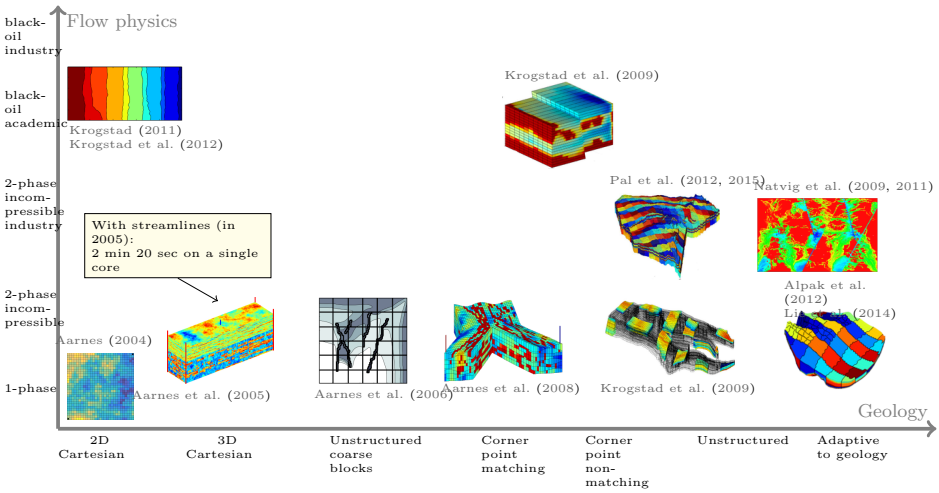

Fig. 2 Developing the MsMFE method towards geological realism.

algorithms for automated grid coarsening. Likewise, the accuracy of the method could in many cases be enhanced by adapting the shape of the grid blocks to the geology and possibly also to preexisting flow solutions. Altogether, the MsMFE method proved to work very well as an approximate solver for incompressible and slightly compressible flow (Aarnes et al. 2005, 2008; Alpak et al. 2012; Pal et al. 2015). Attempts were also made to extend the method to compressible flow by developing various iterative formulations (Krogstad 2011; Krogstad et al. 2012). Unfortunately, this work stranded on the lack of a fully robust mixed formulation of the fine-scale flow equations. Such formulations have been reported in the literature (Bergamashi et al. 1998; Chen 2000), but the resulting linearizations are in our experience not sufficiently robust to handle the many corner cases that tend to arise in models of real assets.

2.8 Forgoing the dual coarse grid for the algebraic multiscale method

Prior to 2011, all work on multiscale finite-volume methods had focused entirely on Cartesian grids in rectangular domains, except for a study of some simple and conceptual fault models consisting of multiple trapezoidal blocks with Cartesian topology (Hajibeygi et al. 2011). The key to construct basis functions for the MsFV method is a wirebasket ordering, in which each cell is categorized as being either a *node*, part of an *edge* (or part of a *face* in 3D), or an *inner* cell. When computing basis functions, a unit or zero pressure is imposed at the nodes, a reduced flow problem is solved in the edge and face cells, and this localizes the flow problems solved in the inner nodes. This process can be formulated as pure algebraic manipulations of the fine-scale discretization matrix, as illustrated in Figure 3.

By utilizing the algebraic operator formulation of the MsFV method, Møyner (2012) and Møyner and Lie (2013, 2014a) were able to formulate and implement the method for unstructured and stratigraphic grids. In principle, this should be

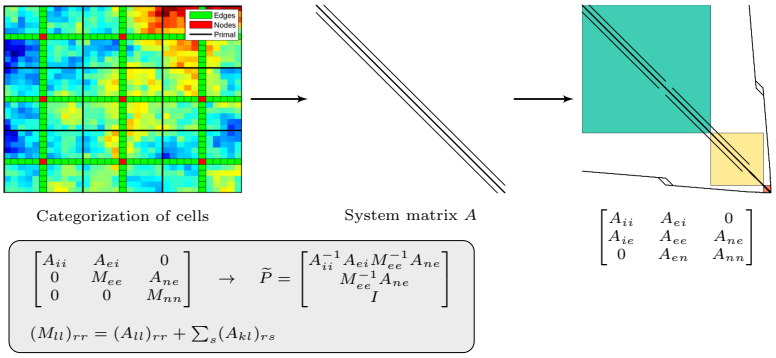

Fig. 3 Computation of the MsFV prolongation operator formulated as an algebraic process. The matrix $\widetilde{P}$ must be permuted back to the normal ordering before it can be used in (1).

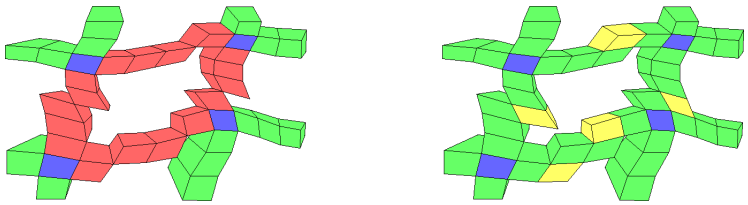

Fig. 4 Connections in wirebasket orderings. Blue cells are either nodes or edges, green cells are part of a connected edge or face, whereas the cells shown in red contain non-contiguous cells that will act as internal barriers for the reduced flow problems. The corresponding edges/faces can be made contiguous by adding the yellow cells so that two-point fluxes can be computed across all internal cell faces.

straightforward, but in practice, it turned out to be difficult to find algorithms for generating admissible primal–dual partitions. The main challenge is that one needs to solve reduced-dimensional problems along the perimeter of the dual blocks to set the boundary conditions that are used to localize the computation of basis functions. This requires a certain connection and regularity of the faces and edges of the dual blocks. That is, cells that are part of edges in the wirebasket ordering (green color in Figure 3) must form a contiguous chain that is connected through cell faces. Cells that are characterized as faces in the wirebasket ordering should be connected to their neighbors the same way. Cells that are only connected through vertices and edges in the fine grid will have no transmissibility between them in the fine-scale discretization, and such connections will thus act as internal no-flow boundaries in the reduced flow problems used to provide localization, see Figure 4.

For simple rectilinear, curvilinear, and triangular grids, the coarsening algorithms proposed in (Møyner 2012; Møyner and Lie 2014a) can be automated and will generally give dual blocks having edges and faces with sufficient connection and regularity. However, sector and field models tend to be much more complicated because of inactive cells, degeneracy in cell geometry, non-matching cell faces, and non-neighboring connections that are used to model pinchouts, erosion, faults, etc. It is therefore very difficult to automate the generation of admissible primal–dual partitions for models of real assets and, in particular, guarantee that the edges and faces of the dual blocks do not contain high-contrast permeability

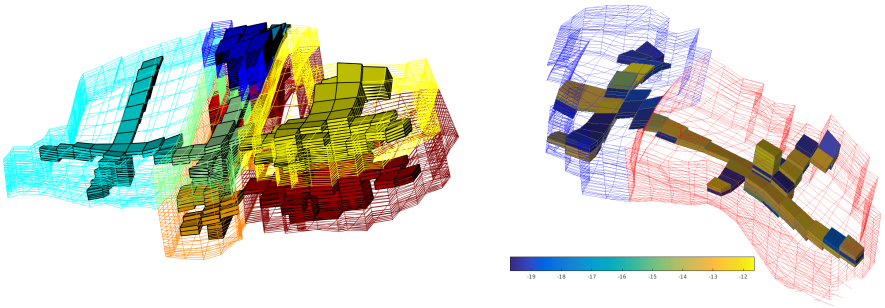

Fig. 5 Parts of a primal–dual partition for the Gullfaks field model (Example 3). The left plot shows one block and its eight face-neighbors. The outline of each coarse block is shown in wireframe with a unique color, whereas cells belonging to the dual edges are shown in the same color. The right plot shows edges inside two coarse blocks, colored by permeability, which here spans seven orders of magnitude. Such extreme contrasts are known to introduce localization errors and severe monotonicity problems.

streaks, see Figure 5, which are known to introduce numerical instabilities. In our experience, the resolution and types of coarse grids one can generate are very limited compared with the MsMFE method and one quickly runs into special cases with degenerate cell geometries and complex local topologies that in the best case can be manually partitioned by the user. With extensive visual support for grid interaction, this becomes a tedious exercise and without, it is virtually impossible for large models. We have also encountered numerous cases that we simply were not able to partition at all.

In an attempt to dispense with the requirement for admissible primal–dual grids, Møyner and Lie (2014b) developed the multiscale two-point flux-approximation method. Borrowing ideas from transmissibility upscaling and the MsMFE method, the authors proposed to first construct a set of elementary solutions, one for each pair of coarse blocks sharing a common face. Then, these elementary functions were multiplied by a set of partition-of-unity functions that each is associated with a single coarse interface and carefully designed so that the resulting basis function will have compact support inside two grid blocks only. This gives a coarse-scale stencil that is (almost) two-point and hence less prone to introduce non-monotonicity in the coarse-scale solutions. Although the resulting method turned out to be flexible, robust, and reasonably accurate, it has not been pursued further, at least not commercially. Instead, the method provided new insight that led to the development of what we today consider as the state-of-the-art multiscale method.

2.9 Basis functions computed by restricted smoothing

Even though the MsFV and the MsMFE methods had both been formulated in an algebraic form, all prolongation operators considered in the literature were derived as basis functions constructed using geometric arguments for the continuous flow equations and then subsequently reinterpreted algebraically. In an algebraic multiscale formulation, however, the main purpose of the prolongation operator is to map a coarse-scale pressure to the fine-scale grid in a non-oscillatory manner and efficiently reduce the low-frequency components of the fine-scale residual in the

same way as the coarse-level stages in a multigrid method. Realizing this, Møyner and Lie (2015, 2016b) proposed to use a simple iterative smoother to construct basis functions algebraically using only a primal coarse grid and a set of support regions that each encompasses a single coarse block. In the multiscale restriction-smoothed basis (MsRSB) method, basis functions are set as constant inside the corresponding coarse blocks and then the local smoother is applied to iterate each basis function so that it gradually becomes consistent with the local differential operator. The iteration is extended with a simple lumping strategy, which ensures that the basis functions do not grow outside their individual support regions and together form a partition of unity. Constructing the support regions is straightforward since we neither need to solve a reduced flow problem nor extrapolate pressure values along their perimeter. As a result, one regains the flexibility of the MsMFE method. Møyner (2014) also shows that acceptable convergence can be obtained using even simpler constant or linear prolongation operators. Prototype implementations of the MsRSB method are available in an open-source framework (Krogstad et al. 2015; Lie 2016).

2.10 Approximation properties and enrichment strategies

The convergence of multiscale methods can be slow for problems with very strong contrasts in the petrophysical parameters. Much research has therefore been devoted to understand and improve approximation properties of multiscale spaces. Efendiev et al. (2011) suggested using eigenvectors of local spectral problems to systematically enrich the initial multiscale space of the MsFE method. Dolean et al. (2014) discuss connections between multiscale and domain-decomposition methods and use a Dirichlet-to-Neumann (DtN) map to obtain an eigenvalue problem that gives a more compact set of eigenvectors. DtN maps were also used to formulate an accurate upscaling method, which in turn inspired an enriched version of the MsMFE method (Lie et al. 2014). Cortinovis and Jenny (2014) proposed a Galerkin-type enrichment technique to add extra coarse-scale degrees of freedom and thereby improve the convergence of the MsFV method for cases with long coherent structures with large media contrasts. Likewise, Manea et al. (2016) introduced an algebraic multiscale solver that systematically enriches the approximation space with local basis functions targeting the largest error components in solution space. The method solves an error equation to identify solution modes missing from the multiscale operator, and repeats the enrichment process iteratively until sufficient convergence is achieved.

The generalized multiscale finite element method (Efendiev et al. 2013; Bush et al. 2014; Chung et al. 2014) is an example of a more general framework that seeks to systematically enrich the coarse approximation space. Computations are divided into an offline and an online stage. In the offline stage, a series of representative snapshots are reduced to a small-dimensional space through spectral decomposition. In the online stage, the offline space is used to generate new basis functions for the particular parameter combinations encountered during the dynamic simulation. Neither of these methods have, however, been developed yet to a stage where they are ready for commercial applications.

Recently, Lie et al. (2017) proposed a new framework in which multiple sets of multiscale prolongation and restriction operators can be combined in a multistage

algorithm to enrich the multiscale approximation space and hence improve accuracy and convergence of (iterative) MsFV-type methods. The framework is flexible and only requires that: (i) each operator is constructed over a non-overlapping coarse partition, (ii) each basis function is restricted to a local support region, and (iii) the basis functions of each prolongation operator form a partition of unity. Although not yet implemented in the commercial simulator, the new method is designed so that it should be straightforward to include.

2.11 Other developments

The literature on numerical methods for elliptic problems offers a great number of methods geared toward solving problems with highly oscillatory coefficients. The multiscale CVD-MPFA method of Parramore et al. (2016) uses a finite-volume formulation and appears to be applicable to a wide variety of structured and unstructured grids. Rigorous error estimates are available both for variational multiscale methods (Hughes et al. 1998; Larson and Målqvist 2007; Juanes and Dub 2008) and for multiscale mortar mixed methods (Arbogast et al. 2007), but only for problems much more simplified than those encountered in industry-grade simulations.

3 Multiscale Formulation Implemented in the Commercial Simulator

In this section, we outline the algebraic multiscale framework as implemented in the research and prototyping branch of a commercial simulator. Several details have already been discussed by Kozlova et al. (2015). We give an updated overview with special emphasis on new features that enable simulation of a much broader spectrum of complex geological models.

3.1 Formulations and solution strategies

The commercial simulator is based on a fully implicit, compositional formulation discretized on general unstructured grids. The simplified fluid description of a black-oil model fits naturally into this framework, with molar densities and mole fractions written in terms of formation-volume factors and solubility ratios, respectively. Cusini et al. (2015) discuss a multiscale preconditioning method for fully implicit discretizations, but this formulation does in our experience not provide the computational speedup desired for commercial applications. Our starting point is therefore a sequential solution strategy, in which the algebraic multiscale method is applied to a pressure equation describing overall mass-balance of the system to reduce its residual below a prescribed tolerance. After the pressure iteration has converged, fine-scale pressures and fluxes are reconstructed locally on each coarse block, and then used to evolve the nonlinear transport equations using a Schwarz overlapping method combined with an implicit upwind scheme on each coarse grid block. More details can be found in (Kozlova et al. 2015; Møyner and Lie 2016a; Watanabe et al. 2016). By adding an outer iteration on the pressure and transport steps, the sequential solver can be made to converge to the solution that would

be computed by a fully implicit discretization (Spillette et al. 1973). To ensure efficient convergence toward a correct solution, it is important that linearization of pressure, flux reconstruction, and transport, are consistent, e.g., as discussed in more detail by Møyner and Lie (2016a). Moreover, in our experience, the way the sequential pressure equation is formulated in the presence of appearing and disappearing phases, phase interfaces, and near wells can greatly influence the performance by reducing the need for outer iterations to correct an inaccurate, first pressure solution.

Improved computational efficiency can be obtained by slacking the tolerance on the outer iteration and the tolerances that determine how far the residual should be reduced and how often basis functions should be recomputed in the multiscale solver. Likewise, computational costs can be reduced by increasing the time step in the sequential formulation beyond that of a fully implicit discretization. Notice, however, that sequential simulators have a lag in the evaluation of the mobility in well models and this may cause large splitting errors for cases with large time steps and abrupt changes in mobility induced by gas in the near-well region. In such cases, the time step needs to be reduced to ensure correct solutions. Generally, it is more difficult to converge the transport equations than the pressure equation, and hence the simulator has the ability to take several substeps in the transport solver for each pressure step. Moreover, work is also in progress to investigate smoother and more robust spatial discretizations (Lee et al. 2015), more efficient and robust nonlinear solvers (Li and Tchelepi 2014; Møyner 2016b; Watanabe et al. 2016), and improved strategies for concurrent computing (Manea et al. 2015; Kozlova et al. 2016).

3.2 The algebraic multiscale solver

As discussed above, the algebraic multiscale framework is very flexible and can be used in several ways to combine various local and global solvers. The commercial simulator initially implemented basis functions based on primal–dual grids as described by Lunati and Lee (2009) with parts of the Jacobian from the nonlinear pressure equation as coefficient matrix for the basis functions (Kozlova et al. 2015). For restriction, the simulator offers the MsFV operator, the MsFE operator, or a combination of the two, i.e., first use MsFE to accelerate convergence and then use MsFV to ensure mass-conservative fluxes. Then, ILU(0) is used as local preconditioner. We are also experimenting with other smoothers in combination with the multiscale preconditioner, but we do not yet have definite results. Recently, we also included the MsRSB method (Møyner and Lie 2016b), which is more robust, at least as accurate, simpler to implement, and applicable to a much larger variety of complex stratigraphic grids than the MsFV method. Figure 6 summarizes the key steps in the resulting solver.

For commercial use, the generation of a coarse partition should be automated and not require any expert knowledge. (Users should be able to benefit from the reduced computational cost of a multiscale solver without having to know its inner workings). As pointed out above, admissible primal–dual partitions are generally cumbersome to construct, if possible at all, for highly heterogeneous models with complex geometry and topology, and general and automated partitioning strategies seem to be far from reality, at least at the moment. A primal coarse grid with

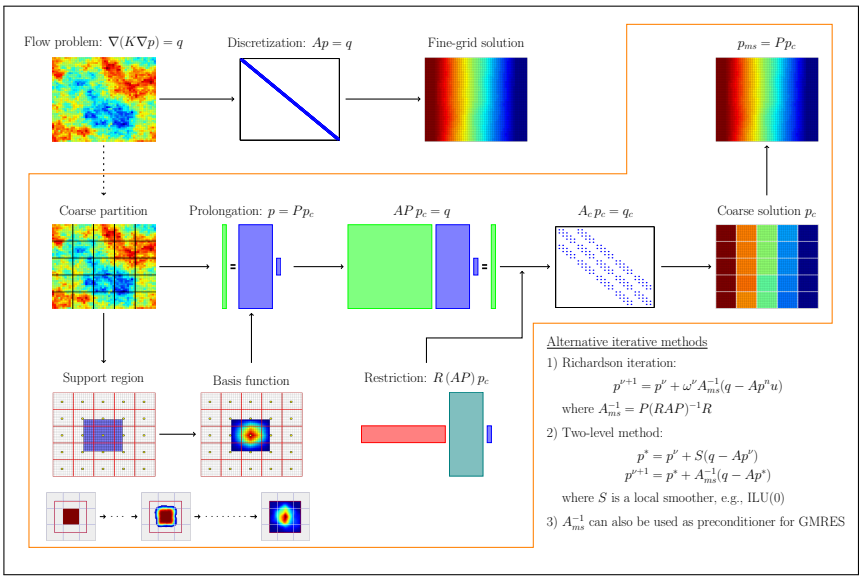

Fig. 6 The algebraic multiscale framework with basis functions generated by restricted smoothing.

support regions that extend between the centers of the neighboring blocks, on the other hand, can easily be generated using one of the many grid partitioning strategies that can be found in the literature. This concept also provides enough flexibility, so that expert users can adapt and fine-tune the coarse partition (e.g., to features in the geological model) to optimize performance. So far, however, we have primarily used rectilinear partitions in index space for simple models, and a graph-portioning software (Karypis and Kumar 1998) for more complex grid models. When used with fine-scale transmissibilities as weights, the latter can generate coarse partitions that adapt to the underlying geology.

4 Examples

In this section, we will present three different models that have been used to verify the accuracy and assess the performance of the multiscale solver. The first model is a 2D conceptual model of fracture corridors. The second model is a benchmark case published by Heriot-Watt University, while the third model is a real reservoir model of a giant oil and gas field from the Norwegian Continental Shelf. Our examples focus on the multiscale solver's ability to compute correct solutions for complex models; computational efficiency is discussed in more detail by Kozlova et al. (2016). The MsRSB prolongation operator is used in all examples.

4.1 Example 1: Fracture corridors

The first model is a simple one-layer model ($60 \times 220 \times 1$ cells), which is a simplification of a model used in (Natvig et al. 2011; Montaron et al. 2007). The main purpose of this model is to illustrate that the multiscale solver computes the

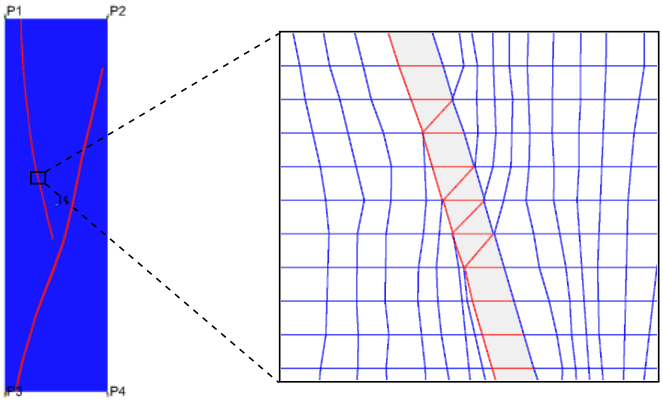

Fig. 7 Permeability, grid, and well positions for the fracture corridor test case. In the color plot, blue color denotes background permeability of 75 md, whereas red color is the 50 darcy fracture corridors.

correct solution for a case with skewed cell geometry and strong media contrasts. The geology is homogeneous except for two explicitly modeled fracture corridors. The permeability is 75 md in most of the layer but 50 darcy in the corridors. A Cartesian background grid has been fitted to the corridors (and other features that have been ignored in the model). The process generated a curvilinear grid with local unstructured features near the fracture corridors, as shown in Figure 7. The model has one injector placed in the middle and four producers located in the corners. Water is injected at a constant rate and the producers are set to operate at constant bottom-hole pressure (BHP). As can be seen in Figure 8 soon after the injection begins, the water moves into the fracture corridors and after approximately 20 days appears close to producers P1 and P3. Water breakthrough is observed in these producers shortly thereafter. Later, P2 is approached by water due to fast flow in the fracture corridor and later by a second front that went via producer P1. The simulation has been performed with both the default fully implicit (FI) solver in the commercial simulator and the multiscale implementation within this simulator (MS). The MS solver used 298 blocks, FE restriction operator, residual tolerance of 10^{-6}, maximum pressure increment of 1.5 psi/step, and an iteration limit of 6. ILU(0) with a tolerance 10^{-3} and an iteration limit of 50 was used as local preconditioner. The transport solver used 536 blocks in the overlapping Schwarz method, a tolerance of 10^{-3}, and 15 as iteration limit. At most three outer iterations were allowed.

It is difficult to see any difference in the water distribution at 21 days and 60 days shown in Figure 8. However, the plot of discrepancy in saturations at 60 days shows minor differences in some locations along the front. Minor differences can also be observed in the water cuts for producers P1 to P4 in Figure 9. Both the FI and the MS solvers are designed to compute solutions to within a prescribed tolerance on the fine-scale residual, but the solutions will generally not be *identical* because of slight differences in interpolations and property avaluations in the two codes. Altogether, we find the match between the multiscale and the fully implicit solvers to be within what is considered as acceptable.

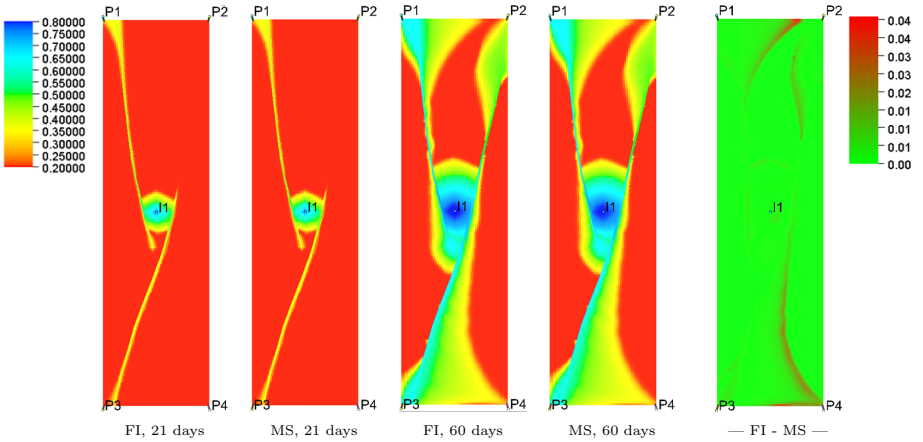

Fig. 8 Water saturation computed by the fully implicit and multiscale simulators. The rightmost plot shows the absolute difference between the solutions at 60 days.

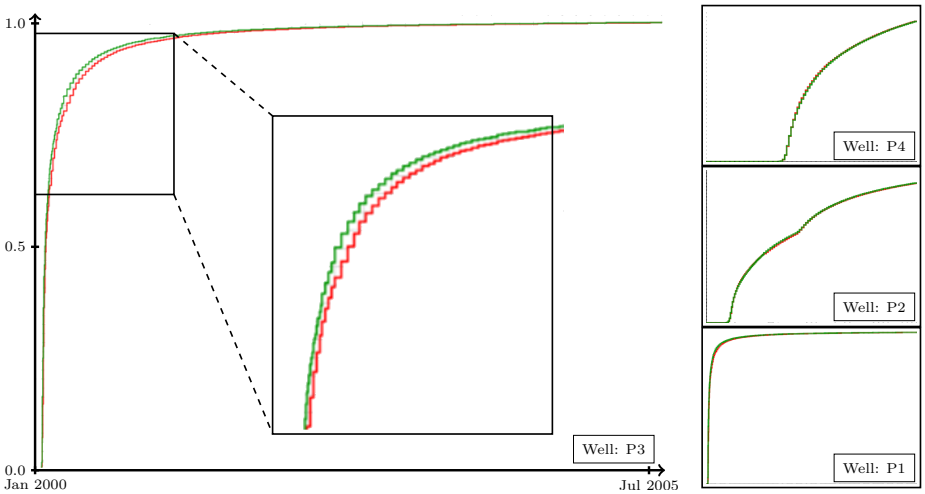

Fig. 9 Water cuts for the fracture corridor model. Producer (P3) with the largest deviation between the FI (green) and MsRSB solution (red lines) is shown to the left and the other three wells to the right.

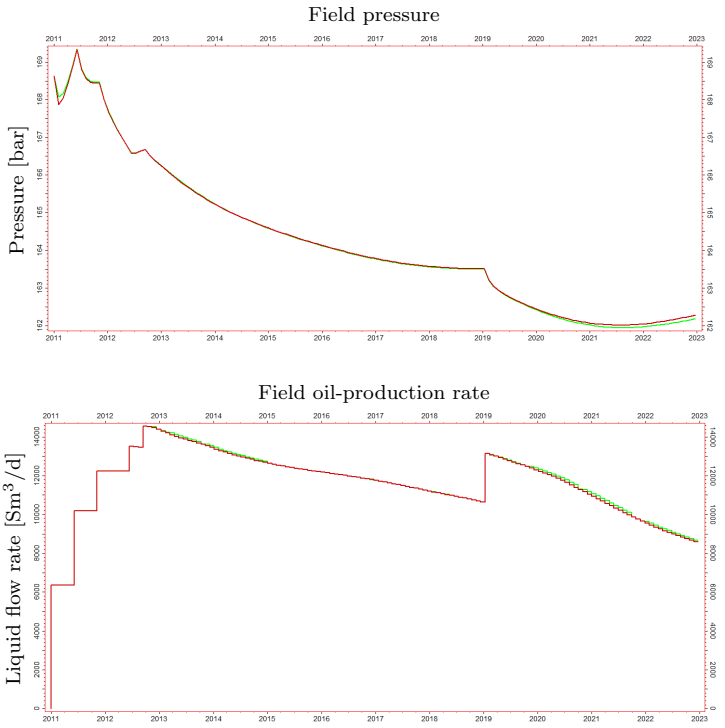

Fig. 10 Average pressure and field oil-production rates for the Watt field computed by the multiscale solver (green lines) and the fully implicit solver (red lines).

4.2 Example 2: The Watt field

The Watt field case (Arnold et al. 2013) is based on a combination of synthetic data and real data from a North Sea oil field and was formulated to study how uncertainty in interpretation is integrated through a reservoir modelling process. Here, we will use a particular realization that was previously used as a benchmark test by Møyner and Lie (2016a). The geological model represents a braided river system, with common facies types including fluvial channel sands, overbank fine sands, and background shales. The model spans a 12.5×2.5 km^2 surface area and has a thickness of approximately 190 m, much of which is below the water contact. We consider a two-phase waterflooding scenario with 15 horizontal production wells located across the central parts of the reservoir and 5 horizontal and 2 vertical injectors around the edges.

The production scenario spans 22 years of oil production with water injected at an increasing rate to maintain the oil production. The simulation is run with both the fully implicit (FI) solver and the sequential fully implicit multiscale (MS) solver. We use default settings for the time-step selection in the fully implicit solver, which means that the time step may be reduced by the simulator. For the multiscale solver, we use a fixed time-step size of 31 days. Figure 10 shows the average reservoir pressure and the field-oil production rate for the two solvers. The solutions follow each other very closely. There are slight discrepancies initially and

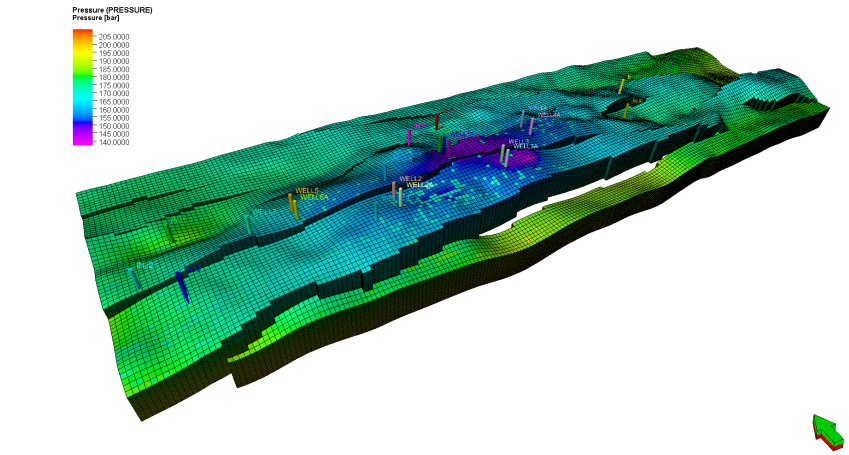

Fig. 11 Field pressure at the end of simulation for the Watt field.

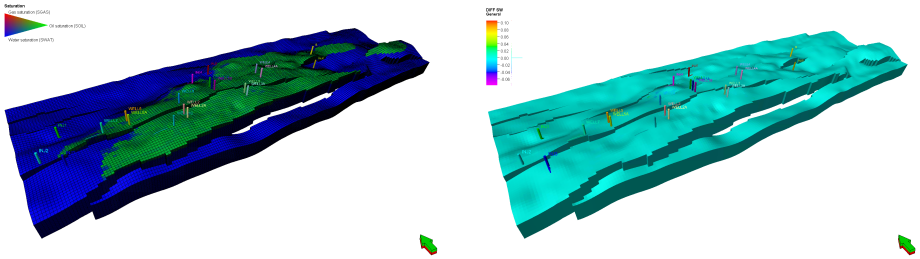

Fig. 12 Saturation at the end of simulation (left) and difference in saturation fields computed by the FI and the MS solver with MsRSB prolongation operator (right).

at the end of the simulation, which are partly due to different time-step lengths and partly due to different criteria to measure convergence. Figure 11 shows the field pressure at the end of the simulation, while Figure 12 shows water saturation and the difference in water saturation between the two solvers. Except for a few outliers, the saturation difference is below 0.04 and close to zero for most cells.

4.3 Example 3: The Gullfaks field

Gullfaks is a mature giant oil and gas field located in the Norwegian sector of the North Sea. Production started in December 1986, reached a peak production of 605 965 bbl/day, and was after that in constant decline until 2001, when the production was stabilized. The main drive mechanism is water injection, but pressure support is also achieved through gas and water-alternating-gas injection some areas of the field. Current recovery rate is 59%, which operator Statoil has an ambition to increase to 62%.

In the following, we will compare the FI and MS simulators on a field model dating back to around the turn of the century. The Gullfaks geology consists of various reservoir zones, including delta sandstones, shallow-marine sandstones, and fluvial-channel and delta-plain formations. The production comes primarily

from Brent sands, i.e., the same type of sedimentary environment as seen in the SPE 10 model, which is used in some form as a test case in almost any paper describing multiscale methods. The reservoir is located in rotated fault blocks in the west and a structural horst in the east, with a highly faulted area in between. This gives a very complex structure consisting of a large number of sloping faults with angles varying from 30 to 80° and throws from 0 and up to 300 m; see (Fossen and Hesthammer 1998) for a discussion of the structural geology. The field model shown in Figure 13 is represented on a $80 \times 100 \times 19$ grid, in which approximately one half of the cells are active. The many faults introduce non-neighboring connection in between 40 and 50% of the active cells. This makes the model very challenging for the original MsFV method. Although it is possible to generate primal–dual partitions (see Figure 5), we have not been able to generate any that are admissible for the computation of basis functions. In the following, we will therefore use MsRSB instead. Møyner and Lie (2016b) have already shown that this method works well on the complex geology of a more refined version of the same model. Assuming single-phase flow and a somewhat contrived well pattern to drive flow through the whole model, they obtained reasonable convergence for simulations with 100 to 500 coarse blocks, and even with an extreme coarsening up to 10 blocks, the MsRSB method managed to reduce the residual eight orders of magnitude in one hundred iterations.

Herein, we will consider the full simulation model using a three-phase fluid description and historic wells. Unfortunately, the field model is not detailed enough to provide accurate well-to-well predictions because of the combination of complex structural architecture and multiple reservoir zones. We will therefore only study field cumulative quantities, which are reported in Figure 14 for both the FI and MS solvers. The maximum difference is observed in cumulative oil production and is less than 3%. Figure 15 shows that discrepancies are located in only a few cells and do not exceed 0.04.

5 Conclusions

After more than a decade of active research, multiscale methods have been developed to a level where they may be ready for commercial application. A large number of tests shows that multiscale methods now can handle the full combination of realistic flow physics and highly heterogeneous geology with complex topologies and rough cell geometries. The most important scientific steps to this end include the development of: (i) an algebraic formulation that interprets the multiscale method in terms of prolongation and restriction operators, (ii) iterative formulations in which multiscale solvers are used as global preconditioners, and (iii) purely algebraic computation of basis functions with support regions derived from a single coarse grid. However, to further improve the multiscale simulators, continued research is needed on sequential formulations, robust spatial discretizations, efficient and robust nonlinear solvers, and improved strategies for concurrent computing. Likewise, we believe that there is a large and untapped potential in the use of multiscale methods as a means to accelerate various reservoir engineering workflows such as uncertainty quantification, history matching, production optimization, etc.

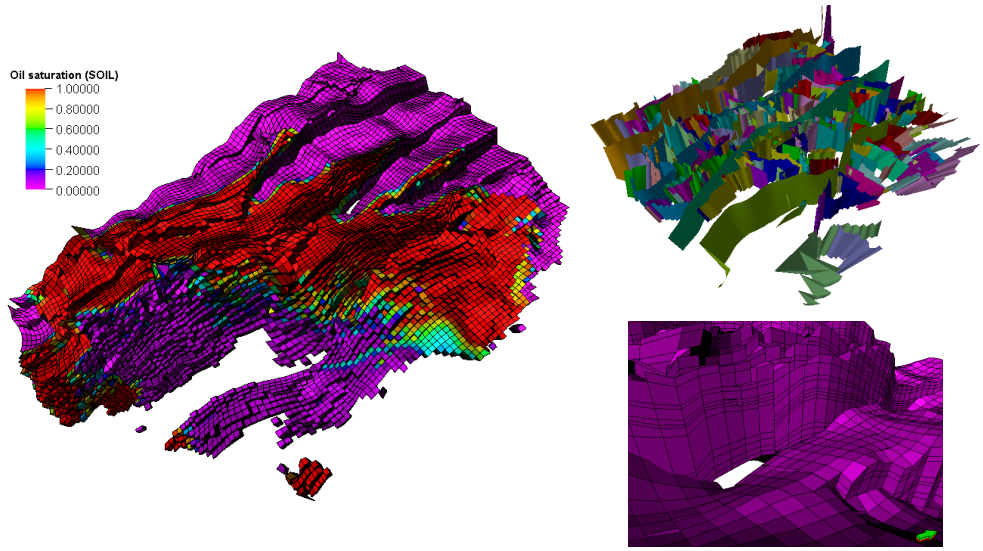

Fig. 13 The Gullfaks model. The plot to the left shows initial saturation, the plot to the upper-right shows the fault network, while the lower-right plot shows a zoom of grid cells near one of the major faults.

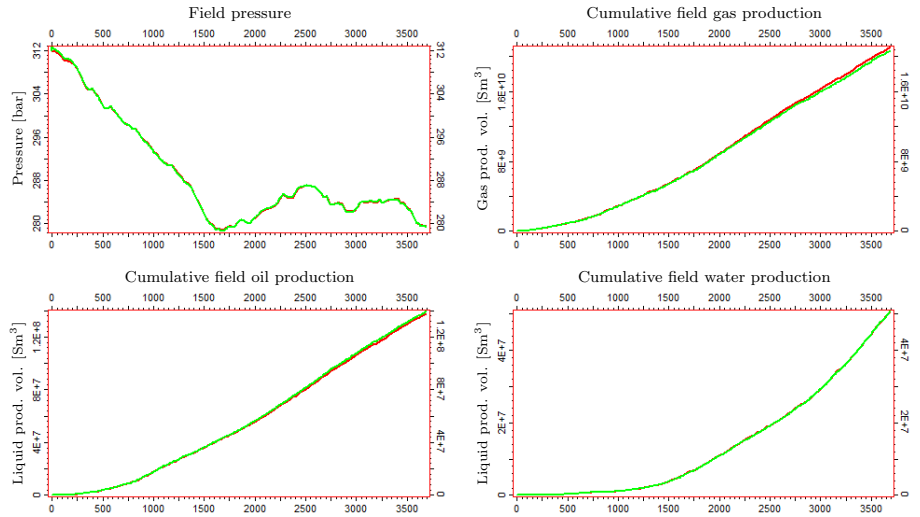

Fig. 14 Field responses for the total simulation period computed by the multiscale solver with MsRSB prolongation operator (red lines) and the fully implicit solver (green lines).

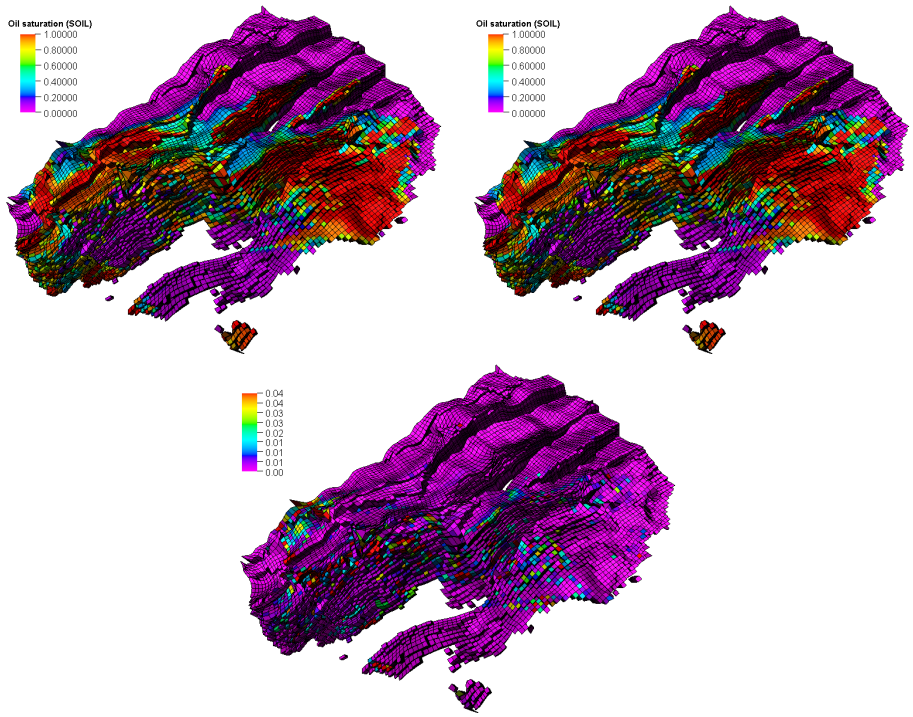

Fig. 15 Oil saturation at the end of simulation computed by the fully implicit solver (upper left) and the multiscale MsRSB solver (upper right). The lower plot shows the discrepancy between the multiscale and fully implicit solutions at the end of simulation.

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