Optimization in offshore supply vessel planning

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This work has been published in Optimization and Engineering.
The final publication is available at Springer via http://dx.doi.org/10.1007/s11081-016-9315-4

Abstract This paper considers the offshore supply vessel (OSV) planning problem, which consists of determining an optimal fleet size and mix of OSVs as well as their weekly routes and schedules for servicing offshore oil and gas installations. The work originates from a project with Statoil, the leading operator on the Norwegian continental shelf. We present both a new arc-flow and a voyage-based model for solving the OSV planning problem. A decision support tool based on the voyage-based model has been used by planners in Statoil, and cost savings from this was estimated to approximately 3 million USD/year. Weather conditions at the Norwegian continental shelf can be harsh; wave heights may limit both an OSV's sailing speed and the time to perform unloading/loading operations at the installations. Hence, we analyze the weather impact on the execution of a schedule and propose robustness approaches to obtain solutions that can better withstand delays due to rough weather. Simulations indicate that such solutions both are more robust and have lower expected costs.

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Key words: Fleet planning, Offshore supply, Routing, Optimization, Simulation

1 Introduction

Norway is among the world's largest oil and gas producing countries, with an oil production of more than two million barrels/day and annual natural gas production of more than 100 000 million m^3. This production takes place at offshore installations on the Norwegian continental shelf. To ensure continuous production, the offshore installations need regular supplies of products from onshore supply depots located along the Norwegian coast. Specialized offshore supply vessels (OSVs) are used for this purpose. Figure 1 shows the relation between the onshore supply depot, OSVs and the offshore installations.

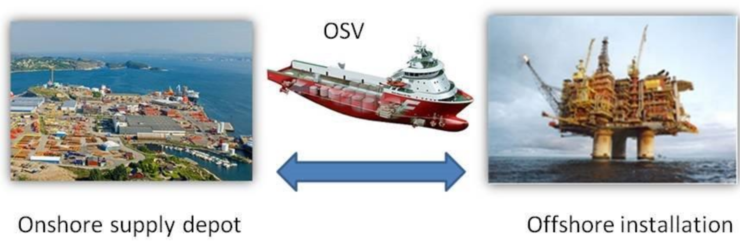

Fig. 1 Relationship between the onshore depot, OSVs and offshore installations.

The OSVs are one of the largest cost drivers in the upstream supply chain of oil and gas installations: Annual time charter cost of a single OSV can be 10-15 million USD. In addition are the costs of operating the OSVs, such as fuel costs.

OSVs are usually time chartered rather than owned by the energy companies. However, the energy companies decide and plan how the OSVs are used. The objective of this paper is to present some main results from a project with Statoil, the largest operator and license holder on the Norwegian continental shelf. The project's aim was to develop mathematical models to support planners obtaining better decisions regarding fleet composition and the weekly schedules. This problem is denoted the *OSV planning problem*, and consist of deciding how many and which types of OSVs to charter in and operate, as well as determining their optimal weekly routes and schedules. Such decisions will typically be valid for a few months depending on when large changes in demand for supplies are observed or expected. Such changes in demand can for example come from new exploration rigs or when an installation goes from drilling operations to production.

In this paper we describe the supply chain in which the OSVs operate, and we propose a new arc-flow model for the OSV planning problem. Its performance is compared with the voyage-based model suggested in [18]. Based on the large costs involved, it is easy to see that proper planning with respect to the fleet size and mix can give significant cost reductions. The environmental aspect from improved planning is also important. Statoil estimates that if the number of OSVs can be reduced by one, the CO_2 emissions can be reduced by about 10 000 tons/year.

The weather conditions at the Norwegian continental shelf can be harsh, especially during the winter season. Wave heights may limit an OSV's sailing speed and its ability to perform unloading/loading operations at the installations. Therefore, we also aim to create robust solutions with high probability to withstand these effects. Hence, we propose robustness approaches and a simulation analysis illustrating the effects on the solutions.

The remainder of this paper is organized as follows: Section 2 describes the supply chain for the distribution of products to offshore installations. A detailed presentation of the problem we consider is given in Section 3. A review of relevant literature is provided in Section 4. In Section 5, two different mathematical models for the problem are proposed, before the computational study on real problems for Statoil is summarized in Section 6. Section 7 discusses the weather impact on the operations. Here, we also present and analyze some approaches to create more robust solutions. Finally, the paper is concluded in Section 8.

2 Supply chain

Planning the supply chain for the distribution of products to the offshore installations is a challenging task. In this section, we give an overview of the different planning activities in the supply chain. This description is based on information from [26]. Figure 2 shows the main activities in the supply chain.

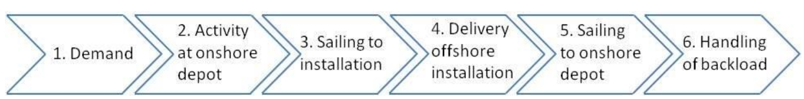

Fig. 2 Supply chain for the distribution of products to offshore installations.

The activities at the offshore installations, such as drilling, well, operations and maintenance, create a continuous demand for different products. These products can be categorized either as deck or bulk cargoes. The deck cargoes will be packed and loaded on the deck of the OSV, while the bulk cargoes are kept in tanks onboard the OSV during transportation. Some of the products may be kept in storage at the onshore supply depots, while others must be ordered from other suppliers when needed. Statoil's general rule is that demand must be registered until 4pm the day before an OSV's departure in order to be loaded on that vessel. In an emergency an offshore installation can use priority for earlier deliveries. There are also significant amounts of backload from the installations to the depot. These volumes represent about 75% of the outgoing product amount.

Government regulated licenses constrain which onshore supply depots that are to supply which offshore installations. All products from external suppliers must be handled at the depots. At the depot, the products are prepared before transportation on an OSV. This includes that appropriate load carriers, such as specialized containers, baskets, frames or tanks, must be selected. The products are packed and secured in load carriers before the load carriers is secured and labeled with the destination installations. Even though there is a weekly plan with routes and schedules, the sailing plan for an individual OSV voyage is usually adjusted before departure based on the actual registered demand and priorities from offshore installations. Before the OSV can depart from the depot, the products must be loaded on the OSV. During stowage planning, there are several issues to consider, such as utilization of the vessel's deck space and bulk tank capacity, reach of the cranes at the offshore installations, and weight and vessel stability.

Rough weather conditions may delay the OSVs' departures from the supply depot. After departure, the OSV continually communicates with both planners at the depot and the offshore installations to be visited. Before an OSV can enter the zone 500 meters around an installation it must receive a permission to do so. After entering this zone the preparation for unloading/loading operations starts at the installation.

The most critical activity in the supply chain is the lifting operation between the OSV and the installation, especially under rough weather conditions. Proper planning in advance is therefore important to reduce the time, and hence the risk, related to this operation. This planning must also consider in which areas of the installation the different products are to be used so that the products can be lifted in a sequence that avoids that the OSV must move between the installation's cranes during the operation. It is also important that the backload has been registered correctly to avoid vessel capacity issues. Bulk products are unloaded/loaded by pumps through tubes. Before the OSV departs from the offshore installation a documentation of what has been unloaded/loaded is exchanged between the installation and the OSV.

After departure from the last offshore installation on the route, the OSV reports about its status, e.g., regarding backload and delays. The OSV sails back to the onshore supply depot, which prepares for its arrival. This preparation includes assigning cranes and a berth to the OSV, and planning to receive the backload. When the OSV berths at the depot it will be unloaded. Bulk tanks will be prepared for the OSV's next voyage, which may involve cleaning of the tanks.

3 Problem description

The OSV planning problem consists of identifying the optimal fleet composition of OSVs that are to service a given set of offshore installations from one common onshore supply depot while at the same time determining the weekly routes and schedules for the OSVs. In this setting a route is a combination of one or more voyages, starting and ending at the supply depot, that an OSV sails during a week. The OSV may visit one or more offshore installations during a voyage. The problem is illustrated in Figure 3 for a problem instance with eight offshore installations. The objective of the OSV planning problem is to minimize the costs while at the same time maintaining a reliable supply service. The costs to be minimized are the OSV time charter costs and the sailing costs of the voyages.

Fig. 3 Example of geographical location of supply depot and offshore installations for an OSV planning problem. The shaded square represents the depot, circles represent the offshore installations.

The onshore supply depot and the offshore installations have *opening hours* during which OSVs can be unloaded/loaded. The depot is open for service during regular Norwegian working hours (0800-1600). Installations are either closed for OSV unloading/loading operations at night (1900-0700) or open for such operations 24 hours. The turnaround time for an OSV at the supply depot is eight hours, which coincides with the opening hours. This implies that an OSV will need to arrive at the depot before 0800 to start on a new voyage the same day. All voyages are assumed to start from the depot at 1600. *Limited capacity at the depot* sets a bound on the number of OSVs that may be prepared for a new voyage on a given day. This number varies during the week due to personnel availability, typically with less capacity at weekends.

Historical data show that deck capacity is the binding capacity resource for the OSVs. This capacity varies from about 600 to 1100 m^2 for the available OSVs. All demands from offshore installations are thus given in m^2 deck capacity. Backloads need to be transported from the offshore installations to the onshore supply depot, but this volume will in almost every case be less than the demand, hence it is assumed that there will be enough capacity to cover the backload. The installations' demands are estimated from historical data as *weekly demands*. The installations need to receive a *given number of visits* during the week, thus the demand for each visit is the weekly demand divided by the number of visits. The demand is not necessarily evenly spread throughout the week; hence it is upscaled by a load factor of 20% to allow for some variations. The total demand for all installations visited on a voyage cannot exceed the *capacity of the OSV* sailing the voyage. The departures from the depot to a given installation need to be *evenly spread* throughout the week, i.e., not in a strict sense, but somewhat spread during the week. It is more important to spread the departures to an installation than the visits as the demand from an installation is reported continuously. Hence, if an installation requires three visits a week and the OSVs visiting the installation leave the depot on three consecutive days, a demand may be called in after the last OSV has left and it will be up to five days until the next departure. This may require that other OSVs must be rerouted or even in some cases that one has to send out a helicopter to meet the demand which will in most cases be very costly. Such situations can to a large extent be avoided with evenly spread departures. Voyages with evenly spread departures for a small instance with two OSVs and seven offshore installations is illustrated in Figure 4.

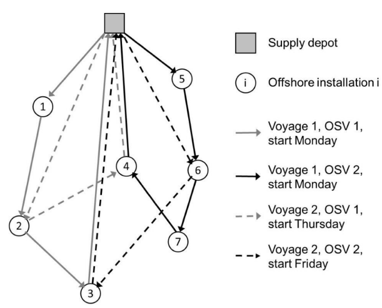

Fig. 4 Example of voyages with evenly spread departures for a problem instance with two OSVs and seven offshore installations.

The duration of a voyage is rounded up to nearest integer number of days, and will vary depending on the number of installations visited, their service times, the sailing distances between them, and the service speed of the OSV sailing the voyage. Too short voyages with few visits are not desired as the OSVs' capacities will not be well exploited. Very long voyages should also be avoided as they involve more uncertainty regarding sailing time. Hence, *minimum* and *maximum duration of voyages* and *number of visits* for each voyage are introduced.

A solution to the OSV planning problem with three OSVs and seven offshore installations (A, B,..., G) is illustrated in Figure 5. The shaded areas

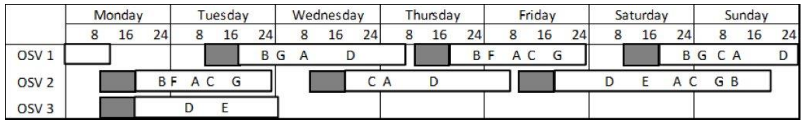

Fig. 5 A solution to the OSV planning problem with three OSVs and seven offshore installations.

represent time spent at the onshore supply depot for preparation of next voyage, and the white boxes represent the voyages sailed by the OSVs. The letters in the white boxes illustrate which installations are visited at what time during the voyages.

4 Literature review

The OSV planning problem can be considered a combined fleet composition and periodic routing problem. It has similarities to the well-studied vehicle routing problem (VRP), e.g., there is one common onshore supply depot where the OSVs' voyages start and end, similar to a depot in a VRP. However, the OSV planning problem is more complex than a traditional VRP. The planning horizon for a VRP is typically one day (a single time period), while the OSV planning problem has a planning horizon of one week (multiple time periods). Other constraints, such as opening hours at the installations and the supply depot, installations that require more than one visit during the planning horizon and OSVs that may sail more than one voyage complicate the OSV planning problem further compared with the VRP. It is also a periodic routing problem where part of the objective is to create a weekly sailing schedule that repeats itself.

There are some papers that consider routing and logistics problems in offshore supply service. [1] discusses the role of OSVs in the offshore logistic chain as these are identified as expensive resources. The focus is on analyzing the design of the OSVs to see how they can be improved to better support operations. Supply operations in the Norwegian Sea are studied in [11] that presents an integer programming model based on a priori generation of voyages. Their model can only be used to solve a simplified version of the OSV planning problem as some complicating aspects such as spread of departures, service capacity constraints for the onshore supply depot and maximum/minimum duration of voyages are omitted. [25] proposes a large neighborhood search heuristic for the periodic OSV planning problem, and [21] presents a simulation model for the problem that can be used to evaluate alternative vessel fleet configuration under stochastic sailing and service times. [16] studies the routing problem arising in the supply service of offshore installations. The problem considered is a single vehicle pickup and delivery problem with capacitated customers where the backload from and limited storage capacity at the installations also are considered. The objective is finding a least-cost voyage starting and ending at the supply depot that visits each installation.

Some studies in the literature have addressed aspects of the OSV planning problem discussed in Section 3. The fleet composition problem considers the strategic decision of determining an optimal fleet of vehicles. Models for the fleet composition problem often include routing decisions as it is necessary to consider the underlying structure of the operational planning problem, see, e.g., [8]. An early reference to this problem is [15] that presents a mathematical formulation for the problem and suggests several heuristic solution procedures. A fleet composition problem from the maritime industry is presented in [10]. [20] presents a literature survey on fleet composition and routing problems in both road-based and maritime transportation. This survey paper discusses industrial aspects and presents basic mathematical models for the problem found in the literature. The authors reviewed in total about 120 scientific papers that could be classified in the category of fleet composition and routing.

VRPs with multiple use of vehicles consider the VRP problem where each vehicle may be assigned to more than one trip during the planning horizon. The problem of assigning a set of trips to a heterogeneous fleet of trucks was studied in [23]. Here, two heuristic methods to solve large sized problems are proposed. [27] proposes a tabu search heuristic for the problem while [5] presents a tabu search heuristic to a more complicated version of the problem. Furthermore, in [6] the authors present a simplified version of their tabu search algorithm to compare it with the one proposed in [27]. [2] presents an exact algorithm for the VRP with time windows and multiple use of vehicles using a branch-and-price approach.

The periodic vehicle routing problem (PVRP) is a generalization of the VRP where vehicle routes are constructed over a planning horizon that can be more than one day, see, e.g., [14]. Decisions to be made are when to visit the customers during the planning horizon (and there can be multiple visits to each customer), assigning visits to vehicles, and optimizing the routes for each vehicle. The OSV planning problem has an extra challenge: A route (voyage) may last more than one day. Therefore it is necessary to ensure that an OSV is not assigned to overlapping voyages. An early study, [4], proposed a heuristic for a PVRP for municipal waste collection. A heuristic algorithm for the PVRP based on an algorithm designed for the VRP (see [12]) can be found in [28]. Some later references are [3] that presents a two-stage heuristic for a real PVRP for the collection of recycling paper containers, and [19] that proposes a heuristic based on variable neighborhood search. A two-phase approach for the tactical problem of deciding dates to visit customers and assigning them to vehicles is presented in [22], leaving the sequencing decisions to be decided on an operational level. Literature on the PVRP where one single route may last more than one day is scarce. Two relevant papers are [10] and [11], where there are also multiple use of vehicles.

5 Mathematical models

This section presents two mathematical model formulations for the OSV planning problem: A new arc-flow model formulation is presented in Section 5.1, followed by a voyage-based model approach in Section 5.2.

5.1 Arc-flow model

In the following we present the arc-flow model for the OSV planning problem. We start by introducing the notations before the general model formulation is presented. Some additional constraints are then introduced.

In the underlying network for the problem, a node represents an offshore installation or the onshore supply depot. Let set $\mathcal{N}$ contain all nodes. Set $\mathcal{V}$ contains all available OSVs that can be time chartered. Set $\mathcal{T}$ contains all time periods (days). The length of the planning horizon is defined as $|\mathcal{T}|$ (seven days/one week).

C_v^{TC} is the weekly time charter cost for OSV v, $v \in \mathcal{V}$. Then, C_{vij}^S is all costs associated with OSV v servicing installation i and then sailing to start the service of installation j, $i, j \in \mathcal{N}$. T_{vij} is the time used by OSV v to service installation i and sail from installation i to j, in hours. Index o represent the onshore supply depot, $o \in \mathcal{N}$. The service time for the onshore supply depot is defined as the turnaround time. S_i is the weekly visit frequency for installation i. T^{MN} and T^{MX} are the minimum and maximum duration of a voyage in days, and R^{MN} and R^{MX} represent the minimum and maximum number of installation visits on a voyage, respectively. D_i is the demand for deck capacity on an OSV for a visit to installation i, and Q_v is the loading capacity of OSV v. H is the length of a time period in hours (24), and B_t is the maximum number of OSVs that can be serviced at the onshore supply depot at day t, $t \in \mathcal{T}$.

The binary variable δ_v equals 1 if OSV v is used, and 0 otherwise. The binary flow variable y_{vijt} equals 1 if OSV v starts on a voyage on day t and sails directly from i to j on that voyage, and 0 otherwise. The support variable z_{vit} equals 1 if OSV v visits node i on a voyage starting on day t, and 0 otherwise. The continuous variable ω_{vjt} is the waiting time variable, and equals the number of hours OSV v, starting on a voyage on day t, waits before start of service at installation j upon arrival at the installation. Finally, variable d_{vt} is the duration of a voyage (in whole days) sailed by OSV v starting on day t.

The arc-flow model for the OSV planning problem can be formulated as follows:

$$min \sum_{v\in\mathcal{V}} C_v^{TC}\delta_v + \sum_{v\in\mathcal{V}}\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}|j\neq i}\sum_{t\in\mathcal{T}} C_{vij}^{S} y_{vijt}, \tag{1}$$

subject to

$$z_{vit} = \sum_{j\in\mathcal{N}|j\neq i} y_{vijt}, \quad v\in\mathcal{V}, i\in\mathcal{N}, t\in\mathcal{T}, \tag{2}$$

$$\sum_{v\in\mathcal{V}}\sum_{t\in\mathcal{T}} z_{vit} \geq S_i, \quad i\in\mathcal{N}\setminus\{o\}, \tag{3}$$

$$\sum_{j\in\mathcal{N}} y_{vjit} - \sum_{j\in\mathcal{N}} y_{vijt} = 0, \quad v\in\mathcal{V}, i\in\mathcal{N}, t\in\mathcal{T}, \tag{4}$$

$$\sum_{j\in\mathcal{N}} y_{vojt} - z_{vit} \geq 0, \quad v\in\mathcal{V}, i\in\mathcal{N}, t\in\mathcal{T}, \tag{5}$$

$$\sum_{j\in\mathcal{N}} y_{vjot} - z_{vit} \geq 0, \quad v\in\mathcal{V}, i\in\mathcal{N}, t\in\mathcal{T}, \tag{6}$$

$$\sum_{j\in\mathcal{N}} y_{vijt} \leq \delta_v, \quad v\in\mathcal{V}, i\in\mathcal{N}, t\in\mathcal{T}, \tag{7}$$

$$d_{vt} = \left\lceil \left(\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}|j\neq i} T_{vij} y_{vijt} + \sum_{j\in\mathcal{N}} \omega_{vjt}\right)\frac{1}{H}\right\rceil, \quad v\in\mathcal{V}, t\in\mathcal{T}, \tag{8}$$

$$T^{MN} z_{vot} \leq d_{vt} \leq T^{MX} z_{vot}, \quad v\in\mathcal{V}, t\in\mathcal{T}, \tag{9}$$

$$R^{MN} z_{vot} \leq \sum_{i\in\mathcal{N}\setminus\{o\}} z_{vit} \leq R^{MX} z_{vot}, \quad v\in\mathcal{V}, t\in\mathcal{T}, \tag{10}$$

$$\sum_{i\in\mathcal{N}\setminus\{o\}} D_i z_{vit} \leq Q_v, \quad v\in\mathcal{V}, t\in\mathcal{T}, \tag{11}$$

$$\sum_{v\in\mathcal{V}} z_{vot} \leq B_t, \quad t\in\mathcal{T}, \tag{12}$$

$$\omega_{vjt} \geq 0, \quad v\in\mathcal{V}, j\in\mathcal{N}, t\in\mathcal{T}, \tag{13}$$

$$\delta_v \in \{0,1\}, \quad v\in\mathcal{V}, \tag{14}$$

$$y_{vijt} \in \{0,1\}, \quad v\in\mathcal{V}, i,j\in\mathcal{N}, t\in\mathcal{T}, \tag{15}$$

$$z_{vit} \in \{0,1\}, \quad v\in\mathcal{V}, i\in\mathcal{N}, t\in\mathcal{T}. \tag{16}$$

The objective function (1) minimizes the time charter costs for the OSVs and the sailing costs for the voyages. Constraints (2) define the support variables, z_{vit}, to be 1 if OSV v sails to installation i on a voyage starting on day t. Constraints (3) ensure that installations get the required number of visits during the planning horizon. Constraints (4) are the flow conservation constraints, and constraints (5)-(6) ensure that voyages start and end at the supply depot. The two constraint sets (4) and (5) will alone ensure that an OSV that sail from the supply depot once also has to sail into the depot once. Hence, constraints (6) are redundant but are kept in the model formulation for consistency. Constraints (7) ensure that there will be a maximum of one departure from the supply depot on a given day for an OSV, and maximum one visit to any given installation during a voyage. The constraints also ensure that if OSV v sails a voyage, δ_v must equal 1. Constraints (8) set the value of the voyage duration variables d_{vt} to the sum of all service, sailing and waiting times rounded up to nearest whole time period (day). Constraints (9) limit the duration of voyages to be within a minimum and maximum number of days, and ensure that d_{vt} is 0 if OSV v does not start on a voyage on day t. Constraints (10) limit the number of installation visits on a voyage to be within a minimum and maximum, constraints (11) are the OSVs' capacity constraints, and constraints (12) are the supply depot capacity constraints limiting the number of OSVs that may be prepared for a voyage starting on day t. The non-negativity requirements for the waiting variables are set by constraints (13), and constraints (14)-(16) set the binary requirements for variables δ_v, y_{vijt} and z_{vit}, respectively.

To avoid OSVs sailing overlapping voyages, these constraints need to be added:

$$\begin{aligned} t \sum_{j\in\mathcal{N}} y_{vojt} + d_{vt} \leq ((t+p) \bmod |\mathcal{T}|) \sum_{j\in\mathcal{N}} y_{voj,((t+1) \bmod |\mathcal{T}|)} \\ + M_t \left(1 - \sum_{j\in\mathcal{N}} y_{voj,((t+p) \bmod |\mathcal{T}|)}\right), \quad p \in \{1,2\}, v \in \mathcal{V}, t \in \mathcal{T}, \end{aligned} \tag{17}$$

For $p = 1$, constraints (17) ensure that if OSV v starts on a voyage on day t, it cannot start on a new voyage on day $t+1$ if the duration of the voyage is more than one day, and for $p = 2$ they ensure that an OSV cannot start on a new voyage on day $t+2$ if the duration of the voyage is more than two days. The expression multiplied by M_t ensures that the constraints are valid if no voyages start on day $t+1$ or $t+2$. The value of M_t is $t + T_{MX}$ as the left hand side of the constraints can never be greater than $t + d_{vt}$ and d_{vt} cannot exceed T_{MX}. The modulus function (mod) is used since the OSV planning problem is a periodic routing problem where the schedule repeat itself every $|\mathcal{T}|$ day. Hence, the modulus function provides that if $t = |\mathcal{T}|$, $t+1 = 1$ and so on.

To prevent subtours and introduce time windows for when installations may be serviced, time variables are added. Let L^E_{iu} and L^L_{iu} be the earliest and latest time for start of service at installation i if start of service takes place on day u, respectively, $u \in \mathcal{T}$. Let variable α_{vitu} equal one if installation i is serviced by OSV v on day u on a voyage starting at day t, and zero otherwise. This variable can only be non-zero if binary variable z_{vit} equals one. The time variable τ_{vit} is the time for start of service at installation i by OSV v on a voyage starting on day t. Then the following constraints can be introduced:

$$y_{vijt}\left(\tau_{vit} + T_{vij} + \omega_{vjt} - \tau_{vjt}\right) = 0, \quad v \in \mathcal{V}, i, j \in \mathcal{N}, t \in \mathcal{T}, \tag{18}$$

$$\begin{aligned} &\sum_{\mu=0}^{2} \alpha_{vit,((u+\mu) \bmod |\mathcal{T}|)} L^{E}_{i,((u+\mu) \bmod |\mathcal{T}|)} \le \tau_{vit} \le \\ &\sum_{\mu=0}^{2} \alpha_{vit,((u+\mu) \bmod |\mathcal{T}|)} L^{L}_{i,((u+\mu) \bmod |\mathcal{T}|)}, \quad v \in \mathcal{V}, i \in \mathcal{N}, t \in \mathcal{T}, u = t, \end{aligned} \tag{19}$$

$$\sum_{\mu=0}^{2} \alpha_{vit,((u+\mu) \bmod |\mathcal{T}|)} = z_{vit}, \quad v \in \mathcal{V}, i \in \mathcal{N}, t \in \mathcal{T}, u = t. \tag{20}$$

Constraints (18) ensure that if an OSV sails from installation i to j on a voyage, the service on installation j cannot start until the OSV has had time to service installation i and sail from installation i to j. These are equality constraints, and waiting before the start of service is allowed by giving the waiting time variable, ω_{vjt}, a value greater than zero. The constraints are nonlinear, but they can easily be linearized following the procedure described in [30], pp. 157-158. The linearized constraints are then expressed as follows:

$$(\tau_{vit} + T_{vij} + \omega_{vjt} - \tau_{vjt}) + M_1 y_{vijt} \ge M_1 \tag{21}$$

$$(\tau_{vit} + T_{vij} + \omega_{vjt} - \tau_{vjt}) + M_2 y_{vijt} \le M_2 \tag{22}$$

The constants M_1 and M_2 are lower and upper bounds on the expression $\tau_{vit} + T_{vij} + \omega_{vjt} - \tau_{vjt}$, respectively.

Constraints (19) are time window constraints, and ensure that the service at an installation starts within the time window on the day the service begins. Constraints (20) ensure that a service at an installation can only start on one specific day if the installation is visited on the voyage.

Constraints (17)-(20) are sufficient if maximum duration of voyages is three days and additional constraints must be added if longer durations are accepted.

To ensure evenly spread departures, we introduce the following sets and parameters: Set $\mathcal{K}$ contains potential weekly installation visit frequencies. Then subset $\mathcal{N}_k \subseteq N$ contains the installations that require k weekly visits. $G_k \in [0, |\mathcal{T}|]$ represent the length of a sub-horizon for installations with visit frequency k, and $\underline{P}_k$ and $\overline{P}_k$ are lower and upper bounds on the number of visits during a sub-horizon of length G_k. Then the following constraints are defined for evenly spread departures:

$$\sum_{v \in \mathcal{V}} z_{vit} \leq 1, \quad i \in \mathcal{N} \setminus \{o\}, t \in \mathcal{T}, \tag{23}$$

$$\underline{P}_k \leq \sum_{v \in \mathcal{V}} \sum_{\mu=0}^{G_k} z_{vi,((t+\mu) \bmod |\mathcal{T}|)} \leq \overline{P}_k, \quad k \in \{2,3,4,5\}, i \in \mathcal{N}_k, t \in \mathcal{T}. \tag{24}$$

Constraints (23) ensure that there will be maximum one voyage starting on day t that is to visit any installation i. They also ensure that there will be no more than one visit to any installation visited on a voyage, and hence make constraints (7) redundant except from the case where $i = o$. These constraints are sufficient for installations that require six or seven visits a week. Constraints (24) ensure that during a given number of consecutive departure days there is a minimum and/or maximum number of visits to an installation with visit frequency k. $\underline{P}_k$ is 0 for visit frequency 2, 1 for visit frequencies 3 and 5, and 2 for visit frequency 4. $\overline{P}_k$ is 1 for visit frequency 2, and set to a high number for all other visit frequencies. The constraints (23)-(24) do not ensure evenly spread departure in a strict sense, but they ensure that the departures are somewhat spread during the week. These constraints were chosen as they were the ones preferred by Statoil.

Constraints (23)-(24) do not ensure that the actual visits to an installation is spread. Hence, two visits to an installation can be very close, or may even be planned at the same time, if an installation is visited late on a voyage starting on one day and early on a voyage starting on the next day. This can be avoided by adding so-called collision constraints. An example of such constraints applied for the voyage-based model can be found in [18]. For the arc-flow formulation, the α_{vitu} variables can be used to express conditions that will avoid planned visits to an installation by two OSVs at the same day.

5.2 Voyage-based model

The OSV planning problem can be reformulated using a voyage-based model in which the variables represent feasible single OSV voyages. The solution determines which OSVs should sail which voyages on what days, and hence the vessel fleet is also decided. A method based on such a model with all feasible voyages generated a priori is proposed in [18] and is summarized in the following. This method is an extension of the one proposed in [7] and [13] for real maritime routing problems. The difference is that in [7] and [13], full ship routes that usually consist of several subsequent voyages are generated a priori, whereas here, the routes will be determined by solving the voyage-based model that chooses and packs voyages for all OSVs. An overview of the method is shown in Figure 6.

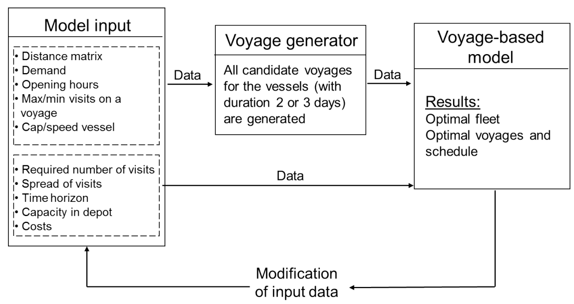

Fig. 6 Voyage-based solution method, schematic overview.

The voyage generator is a program that generates all candidate voyages. These voyages satisfy constraints (4)-(11) and (18)-(20) from Section 5.1. The procedure consists of generating all possible subsets of offshore installations that may be visited during a voyage. Then, for each subset, a traveling salesman problem (TSP) with multiple time windows is solved. Due to the limited sizes of the TSPs they are solved by a full enumeration procedure. For each subset, the voyage with shortest duration is chosen, which in most cases also will be the one with shortest travel distance and least-cost. When the voyage with shortest duration is not the one with shortest travel distance and the one with shortest travel distance is marginally longer, the voyage with the shortest travel distance will be picked. A detailed description of the voyage generation procedure is given in [18]. The set of voyages generated by the voyage generator, $\mathcal{R}$, is input to the voyage-based model.

Let sets $\mathcal{V}$, $\mathcal{N}$, $\mathcal{T}$, $\mathcal{K}$, $\mathcal{N}_k$, parameters G_k, $\underline{P}_k$ and $\overline{P}_k$ and variable δ_v be defined as in Section 5.1. Further, let set $\mathcal{R}_v$ be the set of candidate voyages that OSV v may sail and $\mathcal{L}$ a set containing all possible voyage durations in days (two or three). Then set $\mathcal{R}_{vl}$ contains all candidate voyages of duration l that OSV v may sail, $\mathcal{R}_{vl} \subseteq \mathcal{R}$, $l \in \mathcal{L}$. C_v^{TC} represents the weekly time charter cost for OSV v, C_{vr}^S the sailing cost for OSV v sailing voyage r, $r \in \mathcal{R}$, while D_{vr} is the duration of voyage r sailed by OSV v in days, rounded up to nearest integer. S_i is the required weekly visit frequency to offshore installation i, F_v the number of days OSV v can be in service during a week, B_t the number of OSVs that may be serviced at the onshore supply depot on day t, and the binary parameter A_{vir} is 1 if vessel v visits offshore installation i on voyage r, and 0 otherwise. Finally, we define binary variable x_{vrt} that equals 1 if OSV v sails voyage r on day t, 0 otherwise.

The mathematical formulation for the voyage-based model then becomes:

$$min \sum_{v\in\mathcal{V}} C_v^{TC}\delta_v + \sum_{v\in\mathcal{V}}\sum_{r\in\mathcal{R}_v}\sum_{t\in\mathcal{T}} C_{vr}^{S} x_{vrt}, \tag{25}$$

subject to

$$\sum_{v\in\mathcal{V}}\sum_{r\in\mathcal{R}_v}\sum_{t\in\mathcal{T}} A_{vir}x_{vrt} \geq S_i, \quad i \in \mathcal{N}, \tag{26}$$

$$\sum_{r\in\mathcal{R}_v}\sum_{t\in\mathcal{T}} D_{vr}x_{vrt} - F_v\delta_v \leq 0, \quad v \in \mathcal{V}, \tag{27}$$

$$\sum_{v\in\mathcal{V}}\sum_{r\in\mathcal{R}_v} x_{vrt} \leq B_t, \quad t \in \mathcal{T}, \tag{28}$$

$$\sum_{r\in\mathcal{R}_{vl}} x_{vrt} + \sum_{r\in\mathcal{R}_v}\sum_{\mu=1}^{l-1} x_{vr,((t+\mu) \bmod |\mathcal{T}|)} \leq 1, \quad v \in \mathcal{V}, t \in \mathcal{T}, l \in \mathcal{L} \tag{29}$$

$$\underline{P}_k \leq \sum_{v\in\mathcal{V}}\sum_{r\in\mathcal{R}_v}\sum_{\mu=0}^{G_k} A_{vir}x_{vr,((t+\mu) \bmod |\mathcal{T}|)} \leq \overline{P}_k, \quad k \in K, i \in \mathcal{N}_k, t \in \mathcal{T}, \tag{30}$$

$$\delta_v \in \{0,1\}, \quad v \in \mathcal{V}, \tag{31}$$

$$x_{vrt} \in \{0,1\}, \quad v \in \mathcal{V}, r \in \mathcal{R}_v, t \in \mathcal{T}. \tag{32}$$

The objective function (25) is similar to the objective function for the arc-flow model (1) and minimizes time charter costs and sailing costs. Then constraints (26) like (3) ensure that the installations get their required number of weekly visits, and constraints (27) ensure that an OSV is not in service more days than it is available during a week. These constraints also ensure that δ_v equals one if an OSV is in service. Constraints (28) like (12) limit the number of OSVs that can be serviced at the onshore supply depot on a given weekday, and constraints (29) ensure that an OSV does not start on a new voyage before it has returned to the onshore supply depot after last voyage, as constraints (17). Constraints (30) ensure that the visits to the offshore installations are evenly spread throughout the week, and correspond to (23)-(24). Constraints (31) and (32) set the binary requirements for the δ_v and x_{vrt} variables, respectively.

6 Numerical analysis

The arc-flow model formulation and voyage-based solution method presented in Section 5 have been implemented and tested on problem instances based on a real OSV planning problem faced by Statoil. The problem instances are described in Section 6.1, followed by numerical results in Section 6.2.

6.1 Problem instances

The problem instances are generated from real data provided by Statoil. The data set contains eleven offshore installations that are permanently serviced from Mongstad supply depot, and three floating rigs that at a point in time were also serviced from Mongstad. This is Statoil's largest OSV planning problem.

A total of 22 problem instances were generated. These are the same instances used in [18]. They are numbered after how many offshore installations there are and how many of them that have opening hours (time windows) for when to receive service: Problem instance 3-0 has three installations where none have opening hours, and 14-3 has a total of 14 installations where three have opening hours. The opening hours are equivalent to installations having (multiple) time windows for when to receive service, and they are always between 0700 and 1900 every day.

The problem instances have from three to fourteen installations. The number of weekly required visits for installations varies from one to six, and total number of weekly visits for the problem instances varies from 16 to 59. The installations weekly demands vary from 250 to 960 m^2. The demand for each visit vary from 50 to 334 m^2. The service times at the installations are between 2.25 and 7 hours.

Five OSVs are available for time charter. Their loading capacities vary from 900 to 1090 m^2. Time charter rates are above USD 100 000 per week for all OSVs. The rates vary depending on the OSVs' loading capacities, the OSV with least capacity having the lowest rate. The service speed for all OSVs is 12 knots.

The duration of voyages is two or three days. Minimum number of visits on a voyage is limited by the minimum duration of two days, and maximum number of visits is eight installations. The capacity at the supply depot is three OSVs each day Monday to Saturday, and zero on Sunday when the supply depot is closed.

All results were obtained on a 2.16 GHz Intel Core 2 Duo PC with 2 GB RAM. The arc-flow and voyage-based model formulations were implemented in Xpress-IVE 1.19.00 with Xpress-Mosel 2.4.0 and solved by Xpress-Optimizer 19.00.00. Maximum CPU time when solving the Xpress-MP models was set to 10 000 seconds. The voyage generator was written in C++ using Visual Studio 2005.

6.2 Numerical results

Table 1 shows the results from running the 22 problem instances by the arc-flow and voyage-based models. Reported in the tables are the CPU times in seconds, optimality gap (gap between the solution and best known lower bound), number of vessels used in the solution (#vess) and the total number of voyages they sail (#voy). For the voyage-based model we only report CPU times from running the model in Xpress-MP, and not from the voyage generation procedure. The CPU times for generating voyages varied from 0 seconds for the smallest instance to just over an hour for the largest instance.

Table 1 Results, arc-flow and voyage-based models. Letter *A* and *V* indicate that the numbers are for the arc-flow and voyage-based models, respectively. *O-gap* is the optimality gap reported by Xpress-MP. *Gap* is the percentage gap in objective value between the solutions from the two models, a positive gap means that the objective value of the voyage-based model is lower than that for the arc-flow model.

Prob. inst.	CPU A [s]	CPU V [s]	O-gap A [%]	O-gap V [%]	Gap [%]	#ves A	#ves V	#voy A	#voy V
3-0	1.7	0.0	0.0	0.0	0.0	2	2	6	6
4-0	8906.2	0.1	0.0	0.0	0.0	3	3	6	6
5-0	10 000	1.3	0.3	0.0	0.0	3	3	6	6
6-0	10 000	0.9	0.4	0.0	0.0	3	3	6	6
7-0	5265.5	2.5	0.0	0.0	0.0	3	3	6	6
8-0	10 000	2.3	0.3	0.0	0.0	3	3	6	6
9-0	10 000	36.7	0.0	0.0	0.0	3	3	6	6
10-0	10 000	229.7	0.4	0.0	0.0	3	3	7	7
11-0	10 000	2596.9	1.9	0.0	0.3	3	3	8	8
12-0	10 000	1583.1	24.8	0.0	22.7	4	3	8	8
13-0	10 000	10 000	26.4	13.0	2.0	4	4	9	8
14-0	10 000	10 000	24.5	0.8	2.0	4	4	9	9
5-1	10 000	0.3	0.2	0.0	0.0	3	3	6	6
6-2	10 000	0.6	0.5	0.0	0.0	3	3	6	6
7-2	10 000	1.4	0.1	0.0	0.0	3	3	6	6
8-2	10 000	15.1	0.3	0.0	0.0	3	3	6	6
9-2	10 000	38.7	0.1	0.0	0.0	3	3	6	6
10-3	10 000	253.8	0.7	0.0	0.2	3	3	7	7
11-3	10 000	2004.8	25.2	0.0	23.2	4	3	8	8
12-3	10 000	554.3	26.6	0.0	24.2	4	3	9	8
13-3	10 000	10 000	24.0	0.3	2.7	4	4	9	8
14-3	10 000	10 000	40.2	0.9	20.2	5	4	9	9

The results show that the arc-flow model solves only the smallest problem instances within short CPU times. For most other problem instances, the CPU time limit of 10 000 seconds was reached before an optimal solution was found or proven. We do observe, however, that for many of the problem instances the solutions reported are the optimal ones: The percentage gaps from the solution found from using the voyage-based solution method are small or zero for most of the problem instances with no time windows.

With time windows, we observe that the percentage gaps between the solutions found by the two models are large for some of the large-sized problem instances. The arc-flow model formulation becomes harder to solve when time windows are introduced, while there is no impact on the voyage-based model as these additional constraints only affect the voyage generation procedure. We observe that the problem instances with the highest gaps between solutions from the arc-flow and voyage-based models are the real sized problem instances 11-3 to 14-3. These results indicate that the voyage-based model formulation outperforms the arc-flow model. Still, the arc-flow model has some value: It provides a more precise description of the problem compared to the voyage-based model, in which some of the problem details are hidden in the voyage generation. The arc-flow model's performance can also be improved by adding cuts, e.g., in a branch-and-cut scheme.

We observe that the voyage-based solution method gives optimal solutions within the CPU time limit for all but the largest problem instances, and that the optimality gap is less than 1% for all but one problem instance. For this instance we observe that the gap is still small enough to conclude that it is not possible to reduce the size of the fleet, i.e., the optimal fleet composition is found, which is the main objective for the OSV planning problem.

The problem instances developed represent a good span of the real-life OSV planning problem Statoil needs to solve, and for all practical purposes we obtain good results for all of them with the voyage-based solution method. This indicates that this solution method is well suited for solving the OSV planning problem faced by Statoil.

A decision support tool based on the voyage-based model is implemented and used by planners in Statoil. [18] reports how the model has been extended to deal with a number of additional practical constraints. Examples of such constraints are that some offshore installations require departures on specific days and that the planners prefer voyages with maximum duration of two days except from departures on Fridays and Saturdays, for which they also allow three days voyages. Furthermore, [18] discusses how the visits at the installations can be spread during the week (and not only the departures). The paper also present various what-if analysis for which the model has been used to provide decision support.

7 Weather impact and robust planning

The main objective for the OSV planning problem is to determine the optimal fleet composition. To obtain this objective, voyages and schedules also need to be considered. The sailing schedule should represent a schedule that realistically can be executed. However, the actual sailed schedule will often differ from the planned one as the operational problem involves many uncontrollable factors that can result in rescheduling. In this section we demonstrate how the weather impact may be taken into account leading to more robust planning.

The prevailing weather conditions affect the OSVs' sailing speed and the unloading/loading operations at the offshore installations. This may have huge consequences for the offshore supply service, especially during the winter season in the North Sea. The significant wave height is the critical factor (the average wave height, trough to crest, of the one-third largest waves). Table 2 classifies four weather states based on the significant wave height and shows the reduction in sailing speed (in knots based on a service speed of 12 knots) and percentage increase in service time for unloading/loading operations at offshore installations in each state. Statoil acts in accordance with these states in their supply vessel service. WOW stands for *waiting on weather* and means that OSVs cannot execute offshore unloading/loading operations and will have to wait for better weather conditions. This condition occurs when significant wave heights are above the critical limit of 4.5m.

Table 2 Weather states.

Weather state	Wave height [m]	Sailing speed	Service time
1	< 2.5	0 kn	0%
2	< 2.5,3.5]	0 kn	20%
3	< 3.5,4.5]	- 2 kn	30%
4	> 4.5	- 3 kn	WOW

Other uncertain parameters than weather may also affect the OSV planning problem, but weather is the major one. Hence, we choose to consider weather conditions when creating robust solutions to the OSV planning problem. Figure 5 illustrates a solution where a small decrease in sailing speed or increase in service time may result in delays that will affect later voyages. This is especially critical for the voyages sailed by OSV 1 on Tuesdays and OSV 2 on Wednesdays.

7.1 Robustness approaches

The two models presented in Section 5 find the optimal solution assuming deterministic OSV service speeds and service times at offshore installations. Here we present some robustness approaches that can be incorporated with the voyage-based solution method from Section 5.2 to create robust schedules that can better withstand uncertainties in weather conditions. These are compiled from [17].

7.1.1 Adding slack

We define *slack* as an OSV's idle time after ending a voyage and before the start of preparation for next voyage. Hence, a voyage where an OSV returns to the supply depot at 0430, assuming the OSV needs to start preparation for next voyage at 0800, has slack of 3.5 hours. When voyages are joint in a weekly schedule, a voyage may have more than 24 hours slack if an OSV has one or more idle days before starting on next voyage.

Slack in a schedule gives the OSVs time to catch up when they are delayed. Therefore slack can be used as a robustness approach. Two approaches are identified that introduce and value slack in a schedule. These can easily be incorporated with the voyage-based solution method.

1. Require a given number of slack hours for each voyage: All generated voyages with less slack than required are either discarded (if the duration of the voyage is maximum duration) or are transferred into longer voyages by adding 24 hours slack. Every voyage will then allow for some delays due to reduced sailing speed or increased service time at the offshore installations.
2. Give value to idle days for the OSVs in the schedule: Add a robustness profit to OSVs with idle days. This can for example be to add a robustness profit for each OSV that sails no more than two voyages during the week (which, with maximum duration of voyages of three days means that the OSV will have minimum one idle day).

Figure 5 can be used to illustrate the value of approach two: OSV 3 sails one voyage with duration of two days during the week and has five idle days. OSVs 1 and 2 have no idle days. Hence, a more robust schedule would be if, for example, the voyage sailed by OSV 1 starting on Thursdays was sailed by OSV 3.

The idea of using slack to create robust solutions has been proposed for other planning problems. E.g., to create robust crew schedules for the airline industry, [9] and [29] penalize short ground times when aircrew are changing aircrafts.

7.1.2 An optimization and simulation framework

To create robust and resilient schedules to the OSV planning problem, we propose a solution method that combines optimization and simulation. The method consists of three steps:

Step 1 Generate all candidate voyages the OSVs can sail
Step 2 Simulate each candidate voyage and assign a robustness measure
Step 3 Solve voyage-based model with voyage robustness measures

Steps 1 and 3 are equivalent to the two phases for the voyage-based model. In Step 2 we use statistical weather data to calculate robustness measures for each candidate voyage. These are then used to create a robust weekly schedule for the OSV planning problem by adding a penalty cost in the objective function.

Figure 7 shows a flow chart of the simulation procedure. For each simulation a set of consecutive weather states is drawn from the probability distribution. Four weather states are defined as shown in Table 2. A time interval of six hours is used, which means that each weather state lasts for six hours before weather conditions may change. Each weather state has a start state probability as shown in the last column of Table 3. Next weather state will be dependent only on the current weather state, a random process recognized as a Markov chain, see, e.g., [24]. The probability of moving from one weather state to another, the transition probability matrix, is also provided in Table 3. All probabilities are calculated from historical weather data provided by Statoil for the winter season in the North Sea.

Table 3 Transition probability matrix and start state probabilities

State	1	2	3	4	Start
1	82.5%	16.9%	0.6%	0.0%	22.7%
2	14.0%	60.6%	20.7%	4.7%	27.1%
3	0.5%	23.9%	57.7%	17.9%	28.2%
4	0.0%	0.6%	27.9%	71.5%	22.0%

When weather states are drawn, a voyage is simulated according to the reduction in sailing speed and increase in service times imposed by the weather states. If the voyage cannot be completed within the maximum duration of that voyage, i.e., if the voyage has two day duration and the voyage in the simulation has not returned to the supply depot by the end of day two, the installation with the least demand is removed from the voyage. The procedure

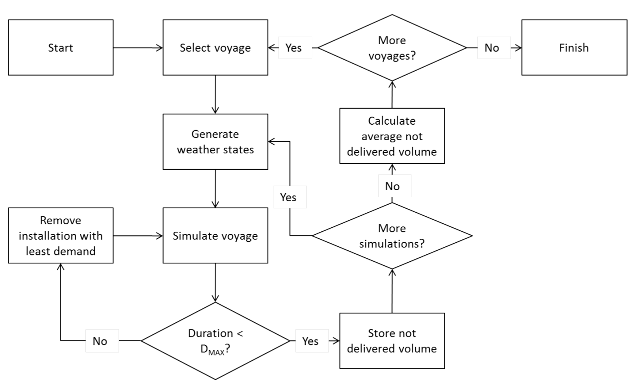

Fig. 7 Flow chart of the simulation procedure.

continues until the voyage can be completed. The sum of the demand from the removed installations, the total demand in m^2 not delivered, is stored and a new simulation executed. The average demand not delivered over all simulations for each candidate voyage is the output from the simulation procedure.

The objective function (25) from Section 5.2 is replaced with:

$$min \sum_{v \in \mathcal{V}} C_v^{TC} \delta_v + \sum_{v \in \mathcal{V}} \sum_{r \in \mathcal{R}_v} \sum_{t \in \mathcal{T}} C_{vr}^{S} x_{vrt} + \sum_{v \in \mathcal{V}} \sum_{r \in \mathcal{R}_v} \sum_{t \in \mathcal{T}} C^P E_{vr} x_{vrt} \quad (33)$$

Here, E_{vr} is the average demand (in m^2) not delivered for voyage r sailed by OSV v, and C^P is the penalty cost for each m^2 not delivered. The penalty cost is estimated based on the real cost of not delivered volume: The volume will have to be delivered by a different OSV at a later time, either as rerouting of an OSV in the fleet or by an additional OSV on short term charter at a higher cost, or by a helicopter at a very high cost. Hence, the penalty cost is calculated as the cost of a three day voyage sailed by an OSV time chartered at a somewhat higher rate divided by the OSV's capacity (in m^2).

To test the weekly fleet schedules obtained with the robustness approaches, a schedule simulation model has been developed. In the simulation model, a sequence of weather states for the whole planning horizon of the schedule is drawn, and for each individual voyage in the schedule the consequences, i.e., the penalty cost for average not delivered volume, is calculated. Idle days for OSVs are added as slack to the voyage sailed prior to the idle day(s), giving the voyage 24 hours (or more) of extra slack.

7.2 Results from applying robustness approaches

To test the robustness approaches, 8 problem instances are generated based on the problem instances from Section 6. As before, there are five OSVs available for time charter. The problem instances have 10 and 13 offshore installations, where 1, 3, 5, or 7 have time windows for service. The total numbers of visits are 43 and 55, and the total weekly demands are 5995 and 7429 for the instances with 10 and 13 offshore installations, respectively. The onshore supply depot has capacity to service three OSVs Monday to Saturday and is closed for service on Sunday.

The voyage and schedule simulation models were both written in C++. Results are all obtained with the same computer setup as in Section 6 with a CPU time limit of 3600 seconds for the voyage-based model.

The problem instances have been solved using five different solution approaches:

Basic Voyage-based model described in Section 5.2

4h	Voyage-based model where each voyage requires minimum 4 hours slack
Max2	Voyage-based model with an extra robustness profit if an OSV sails no more than two voyages
SimSol	Optimization-simulation framework described in Section 7.1.2 with robustness cost for each m^2 not delivered
Combined	Combination of SimSol and Max2, with reduced robustness cost for each m^2 not delivered to 1/3 of the cost used in SimSol

The robustness cost in SimSol and Combined is calculated as described in Section 7.1.2. The robustness profit in Max2 and Combined is set to the average weekly time charter rate divided by 14.

Each candidate voyage is simulated 100 times in the simulation program for problem instances with 10 offshore installations, and 20 times for the instances with 13 offshore installations due to time restrictions. The total CPU times for the simulations were 1-2.5 hours, and 33-50 hours for instances with 10 and 13 offshore installations, respectively.

For a given solution's weekly fleet schedule, we calculated the extra cost for not delivered volume by running the schedule simulation model. Since the schedule simulation model runs simulations on only a small number of voyages the CPU times are low. Hence, we ran 10 000 simulations on each weekly fleet schedule. The total CPU time for 10 000 simulations was up to 90 seconds.

Table 4 Results robustness approaches. Costs are given in NOK/week for the Basic approach and in % relative to the corresponding cost of the Basic solution for all other approaches.

Problem instance		10-1	10-3	10-5	10-7	13-1	13-3	13-5	13-7	Total
Basic	PlanCost	626959	627097	628785	819845	**640273**	**641197**	**645143**	651859	5281158
	ExtraCost	140354	210929	276458	223263	277830	303550	338389	385235	2156008
	Total	767313	838026	905243	1043108	918103	944747	983532	1037094	7437166
4h	PlanCost	100.08	100.24	100.98	100.00	**102.14**	**102.09**	**101.57**	123.07	103.71
	ExtraCost	119.22	73.93	83.77	95.61	89.02	90.47	98.19	63.10	86.53
	Total	103.58	93.62	95.73	99.06	98.17	98.36	100.41	100.79	98.73
Max2	PlanCost	100.00	100.00	100.00	100.00	**100.08**	**99.92**	99.88	**100.00**	99.99
	ExtraCost	129.41	84.99	84.94	86.79	85.52	93.77	85.80	92.50	90.83
	Total	105.38	96.22	95.40	97.17	95.68	97.95	95.03	97.22	97.33
SimSol	PlanCost	103.25	103.48	103.71	101.23	103.00	103.54	102.74	101.97	102.81
	ExtraCost	101.46	79.25	70.28	76.97	81.39	82.46	82.36	83.78	81.33
	Total	102.93	97.38	93.50	96.04	96.46	96.77	95.73	95.21	96.58
Combined	PlanCost	100.27	100.36	101.19	100.41	**100.29**	101.32	100.98	100.87	100.70
	ExtraCost	96.49	89.05	74.08	77.77	91.50	90.19	82.44	96.02	87.13
	Total	99.58	97.51	92.91	95.56	97.63	97.74	94.60	99.07	96.77

Table 4 shows the results. A bold solution cost indicates that the Xpress-MP optimizer did not prove optimality within the CPU limit time of 3600 seconds (gaps from optimal solutions are, however, less than 1% for all instances). The Basic schedule is good for problem instance 10-1 but poor for all other problem instances. The SimSol weekly fleet schedules have the overall best results, 3.42% better on average than the Basic ones. The SimSol weekly fleet schedules beat the Basic ones for all problem instances but 10-1, while the Combined weekly fleet schedules outperform the Basic ones for all problem instances.

In general we see that addressing robustness give added value as the solutions obtained have a higher potential of being successfully executed. The planned cost of the weekly fleet schedule will be somewhat higher than if robustness is not considered, but the simulated extra costs, that represent a better estimate for the real costs, will be less and hence provide a reduction in total cost. However, as the schedules are created by uniting individual voyages in the voyage-based solution method, and we address robustness to individual voyages, a schedule created without considering robustness can potentially perform well with respect to robustness cost, as illustrated by problem instance 10-1.

8 Concluding remarks

This paper considered the offshore supply vessel (OSV) planning problem, which consists of determining an optimal fleet size and mix of OSVs to time charter, as well as their weekly routes and schedules for servicing offshore oil and gas installations from an onshore supply depot. This is a real-life planning problem and the work originates from a research project with Statoil, the leading operator on the Norwegian continental shelf. We have presented both a new arc-flow model and a voyage-based model for solving the OSV planning problem. A computational study showed that the voyage-based model outperforms the arc-flow model on the real planning problems. However, the arc-flow model has some value as it provides a more precise description of the problem compared with the voyage-based model for which some of the problem details are hidden in the voyage generation procedure. A decision support tool based on the voyage-based model has been used by planners at Statoil for the supply service from one of the supply depots along the Norwegian Coast. The use of this tool has enabled a reduction in the number of OSVs while maintaining an efficient and reliable supply service. Cost savings of this reduction was estimated to approximately 3 million USD/year.

The weather conditions at the Norwegian continental shelf can be harsh, especially during the winter season. Wave heights may limit both an OSV's sailing speed and its ability to perform unloading/loading operations at the installations. Hence, we analyzed the weather impact on the execution of a schedule and proposed some robustness approaches to obtain solutions that can better withstand delays due to rough weather conditions. Simulations indicated that such solutions are both more robust and have lower expected costs.

The voyage-based model succeeded in solving Statoil's OSV planning problem. However, it can be observed from the computational results that this method reached its limits with respect to problem sizes. Column generation schemes or heuristic methods should be developed for obtaining solutions to larger problems, like the ones proposed in [25]. Such methods may also be of interest if the scope is extended from considering the OSV planning problem from one onshore supply depot to perform the planning from several depots simultaneously. Today's license regulations restrict the possibility to change the depot from which an installation is serviced, but it can be of interest to evaluate the potential cost reduction by more than one depot sharing a fleet of OSVs. This option results in a larger, more complex problem, and the solution methods presented in this paper would probably no longer suffice.

Acknowledgements

This work has received financial support through the researcher project DOMinant II funded by the Research Council of Norway, and the innovation project MOLO, partly funded by the Research Council of Norway. This support is greatly appreciated.

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