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Market Driven Hydro-Thermal Scheduling

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RESULT (summary)

Modelling of target freemarket power system performance is expected to become increasingly relevant, as deregulation of power sectors spread internationally and associated market processes mature:

- Such modelling may supply information on reference or 'benchmark' system performance, in discussing or evaluating practical implementations and aspects of nonperfect behaviour within power market clearing and coordination.
- It may provide for insight into power trade capabilities and limitations between differently designed systems, when new regimes of operation opens up for freemarket trade between them.
- It may give ideas and clues as to how essential societal concerns that today are more or less external to the market clearing process, could be internalized in that process. – And thus provide for realism in the very concept of modelling of freemarket power system performance.

The report presents a concept and an associated computational scheme for approximate modelling of market clearing, unit commitment and unit dispatch of an ideally performing deregulated hydrothermal power system exposed to various component- and systems related constraints. Criterion of performance and relevance of concept and methodology is illustrated via four small (up to 9-bus and 9-generator) example analyses, that in part allow for comparison of results with those from an alternative solution method.

Relative to traditional methods of analysis, the present optimization scheme includes capabilities/features not observed in available literature. E.g.:

- Clearing of distributed electrical power markets and scheduling of distributed power production hour-by-hour over the week, while considering main aspects of cost of start and spinning reserve together with hydraulic and electrical constraints, and constraints pertaining to accumulated (e.g. water/fuel/emission-related) quantities over the horizon of analysis.
- Solving of formal part of the optimization task by a nonlinear (Newtonian) process that internalizes all concerns of inequality constraints.

KEYWORDS

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	Power market clearing	Deregulation
	Hydro-Thermal Scheduling	Unit Committment

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SINTEF Energy Research

MARKET DRIVEN HYDRO-THERMAL SCHEDULING

REPORT SUMMARY

SCOPE AND MOTIVATION

The scope is twofold:

- Firstly, to present a concept and an associated computational scheme for simulating market clearing, unit committment and unit dispatch of an ideally performing deregulated hydro-thermal power system exposed to various component- and systems related constraints.

- Secondly, to demonstrate criterion of performance and practical relevance of concept and methodology, via example analyses that in part allow for comparison of results with those from an alternative solution method.

Modelling of target freemarket power system performance is expected to become increasingly relevant, as deregulation of power sectors spread internationally and associated market processes mature. E.g.:

Such modelling may supply information on reference or 'benchmark' system perfomance, in discussing or evaluating practical implementations and aspects of nonperfect behaviour within power market clearing and coordination.

It may provide for insight into power trade capabilities and limitations between differently designed systems, when new regimes of operation opens up for freemarket trade between them. -In case of e.g. the Norwegian power system, this may imply information on target volume and utilization of power trade agreements vis-a-vis external partners.Local, regional and interregional constraints that are included , will hereby contribute in setting premises for this utilization.

It may give ideas and clues as to how essential societal concerns that today are more or less external to the market clearing process, could be internalized in that process. – And thus provide for realism in the very concept of modelling of target freemarket power system performance.

'NEW' vs. 'TRADITIONAL' PROBLEM FORMULATION

The above stated scope implies internalizing proper consideration of essential societal concerns in the process of simulating geographically dispersed power market clearing and unit scheduling. Among such concerns: The enhancement of free competition whereever relevant, securing overall efficiency of resource utilization, observing quality of power supply requirements, accounting for environmental concerns.

Internalizing the stated societal concerns in the analysis, leads to what the report outlines as *Collaborative Scheduling*. Such scheduling may involve different tasks, depending on the time horizon at hand. Illustrations:

Market Driven Optimal Power Flow is Collaborative Scheduling within the hour. This task is dealt with in a separate SINTEF report.

Market Driven Optimal Power Flow is in principle an extension of *Optimal power Flow (OPF)* in that not only price insensitive- but also price sensitive demand, may be present at respective buses of the system.

In the *Market Driven Optimal Power Flow* as well as *Optimal Power Flow*, hydro production is either exogenously specified, or endogenously given via valuation of water. The latter valuation comes readily as a byproduct from what this report defines as Market Driven Hydro-Thermal Scheduling. See next.

Market Driven Hydro-Thermal Scheduling is Collaborative Scheduling within the week. It is in effect the interconnection and integrated solution of the previous *Market Driven Optimal Power Flow*, over a sequence of (say) 168 hours, observing time dependent aspects such as start/stop of units, ramping, volume constraints on water/fuel/emissions, and price sensitivity of electrical demand. Market Driven Hydro-Thermal Scheduling is the main theme of this report.

Market Driven Hydro-Thermal Scheduling is in principle a generalization of Unit Committment (UC) in that price sensitivity of loads, constraints on power transmission, constraints on water flow, and constraints specifically associated with integrals or volumes over time, are added features to account for.

'NEW' vs. 'TRADITIONAL' PROBLEM SOLUTION

Problem complexity and magnitude, together with the fact that many power supply systems are dominated by either thermal or hydro generation, give rise to the observation that traditional power scheduling processes can be categorized within two main methodologies:

> *The 'Unit Commitment methodology'*, where focus is on the precise treatment of costs and constraints associated with start/stop of production units, - and where e.g. extensive systems related simplifications are made to afford such dealing with the committment aspects. This methodology is usually applied when power generation is purely thermal, or predominantly so.
>
> In solving this problem, a frequent approach is to interlink continous and discrete type mathematics in iterative processes based on a combination of formal and intuitive type logic for updating of variables. Very often Lagrange Relaxation is a chosen mathematical decomposition tool to afford such a solution process.
>
> *The 'Hydro production planning methodology'*, where focus is on capability to model the often complex constraint picture associated with hydro generation and power transmission, and where the aspects of start/stop and spinning reserve are ignored, to allow for a manageable 'real variable problem'. This methodology is usually applied when scheduling of hydro generation is the major task.
>
> In solving this nonlinear,real variable, and multi-constraint problem, iterative processes are required, and they may be based on direct nonlinear problem formulation, stepwise linearization, or a combination of both. With the advent of special/efficient algorithms within the realms of linear programming (LP) and linear decomposition, the stepwise linearization approach would seem to have the edge here.-

The scope of *Market Driven Hydro-Thermal Scheduling* is in principle to model main aspects of power production, power transmission and power market clearing, on an equal foot, so as to retain a balanced consideration of all aspects that go into the continous matching of electrical power supply and demand. In concrete terms, the modelling should – whenever appropriate- take into account the main cost characteristics of start and operation of units, ramping constraints, other local constraints associated with thermal as well as hydro generation, power transmission constraints, relevant premises for distributed market clearing, reserve constraints and constraints that relate to accumulated quantities over the horizon of analysis

In trying to deal with this formidable, nonlinear, mixed integer,multiconstraint task,a strategy of iteratively using only continous mathematics is being researched.This means that also the cost of start of units and the constraints of system spinning reserve are handled via real variables only:

The cost of start is dealt with by a real variable $\delta_{t(t+1)}$ in the range 0.0-1.0, for each generator unit. The variable describes the 'part of the start process' that –in a continous frame of reference - accrues from time interval t to interval (t+1). The corresponding 'incremental' cost of start is defined by the product $\delta_{t(t+1)}\cdot$Startcost, and added to the criterion of performance as a cost element. For each 'startinvolved' generator unit, Startcost is a variable to adapt so that the product (Startcost$\cdot\sum\delta$) equals the actual cost of start. The cost of start of a unit is generally assumed constant (which means that the startcost's dependency of the temperature status of the unit is ignored).

The spinning reserve constraints implies knowledge of maximum production capacity of each unit that is up and running in any hour. This capacity figure is approximated by applying continous but strongly nonlinear ∫-shaped functions, to model maximum production unit output as function of actual unit output.

Practical solution to the task of optimally matching power supply and demand over the week, is found via processes of optimization on two main levels of analysis: On level I) mathematical methods that involve only continous variables, are applied to generate an initial (and most likely) overcommitted committment plan. Level II) comprises a decommittment process that aims at targeting *in* on a close to optimal final plan for matching of electrical supply and demand over the week:

Level I analysis: Generation of initial (overcommitted) committment plan

The analysis comprises two distinct steps: Evaluation of the best possible *continous variable* production schedule for the week, followed by the definition of an initial (and most likely) overcommitted committment plan based on the established *continous variable* solution:

- *The best continous variable solution for matching of power supply and demand*

  This theoretical solution accounts in an approximate and iteratively determined way, for all the effects listed above in characterizing the scope of *Market driven Hydro-Thermal Scheduling.*

  At start of the iterative process nominal startcosts and best estimates of 'specific cost descriptions' of all production units are applied.The latter description allows for production in the theoretical range zero to maximum output for respective units. Within this range, specific production cost (NOK/MWh$_{el}$) is defined as a linear function of P_{gen} and chosen close to constant. At the end of the optimization process the 'specific cost description' of respective generator units, should be consistent with the actual specific production cost figure for respective units. By allowing for production also over the theoretical range from zero output to actual minimum production $P_{g(min)}$, a systematic handling of the tradeoff between cost of start and operational transactions (such as cost of fuel, cost of various operational constraints, income from spotsale) may be brought into the total market clearing context. Details on generator modelling are given in App.1.2 . The further analysis task is as follows:

  1) *Solve the formal problem* with current costs of start and current specific cost descriptions of generator units. Solution of this (often) large-scale, nonlinear, continous variable problem is afforded by a version of Newton's method. Equality constraints are dealt with by Lagrange Multipliers, and inequality constraints are all internalized in the formal process via logarithmic or inverse barrier functions.

  2) *Check for feasibility* with respect to handling of cost of start and efficiency description of generators. If ok, go to next task which is definition of an initial committment plan. If the cost of start for a given 'startinvolved' generator is not properly set, its formal Startcost is adjusted as indicated above. In a corresponding way the efficiency modelling of a generator may have to be modified, if current modelling is not consistent with current production level of the unit. After implementing proper adjustments, return to 1).

- *The initial (overcommitted) committment plan*

  The established formal solution based on continous variables only, forms the basis for defining an initial committment plan. Decision logic in relation to each generator unit:

  If $P_g \geq$ (say) $0.5\cdot P_{gmin}$:Unit is tentatively online in considered hour

  If $P_g <$ (say) $0.5\cdot P_{gmin}$:Unit is offline, unless required to sustain the power balance.

Level II analysis: The decommittment process
On this level each committed generating units is described by its correct/ 'instantaneous' efficiency curve valid for the actual output range from $P_{g(min)}$ to $P_{g(max)}$. For this initial committment schedule the associated optimal unit dispatch is found from optimizing the matching of power supply and demand over the week, considering all the constraints that are commented on earlier. Solution to this task is again afforded by Newton's method.

The objective of the ensuing analysis is to approach on final committment plan and associated optimal power market clearing, by investigating – for each prospective hour – which unit(s) (if any) should be disconnected in that hour. For each prospective hour to consider, decision analysis is based on the following problem formulation:

A generator unit that a) produces at P_{min} or close to this limit (say less than $1.1 \cdot P_{min}$), and b) is not required to be online for feasibility of supply reasons,- is in principle to be considered a candidate for disconnection. Disconnection of an 'eligible' unit is made, if the unit-related money saved by disconnecting that unit, exceeds the systems related costs incurred by alleviating the loss of that same unit.

By including the effect of cost of start when relevant, the decision logic will also identify which generator units (if any) should not at all have been committed over the characteristic cycle of time at hand. Implementation of the decision logic results in principle in a stepwise process, since incremental cost information from current solution, is being used as part of decision basis. The decommittment logic is dealt with in App. 1.3.

FOUR EXAMPLE ANALYSES

The background for these analyses is two-fold: To illustrate problem formulation and solution within the scope and methodology of *Market Driven Hydro-Thermal Scheduling*, and to compare results with those obtainable from another solution method. An existing program labeled *Market Driven Unit Commitment* (based on Dijkstra's Shortest Path Algorithm), is available to the latter end. This program – as most other schemes for Unit Commitment – limits the power transmission network to one single bus. To allow for comparison of results, the three first example cases are therefore restricted to being single bus cases. The fourth analysis exemplifies the general multibus case, as it addresses the task of geographically distributed market clearing.

'Conventional' Unit Commitment. Spinning reserve constraints neglected. The system comprises 7 thermal production units; 5 conventional coalfired units of different size and efficiency, 1 gasfired combined cycle unit, and 1 'light' gasturbine unit. Rated capacity of smallest and biggest unit is 50 and 200 MW, respectively. The load is specified for 168 hours, with slightly lower load profile during the weekend than during workdays. There are two load peaks during the day; one major at noon, and one evening peak.

The cost of starting thermal production units is assumed independent of the temperature status of the unit. Ramping constraints are not included in this case. Requirements to up- and downtimes of units are neglected (and generally presumed taken care of by the criterion of operation, and by hydro units that may have the capability of providing for 'finetuning' of the power market balance).

As operational boundary conditions it is specified that production units 1 and 2 are online all the time and unit 3 is online in hour –1 and hour 168.

Scope of analysis: To decide on unit committment and unit dispatch so that specified demand over the week is covered at minimum cost, taking into consideration stated boundary conditions, - but disregarding spinning reserve constraints as given by the premises.

Solution is found after two optimization runs on 'Level I analysis' (to generate the initial committment plan), and two optimization runs on 'Level II analysis' (to finalize the decommittment process). The solution's cost criterion is 0.002% above true minimum value found from the reference (shortest path) analysis.

Of interest to note solutionwise, relative to the ensuing analysis which observes spinning reserve constraints: The gas turbine is committed for a few hours every workday, and actually loaded up, whereas a bigger/old coalfired unit (no. 5) remains offline throughout the week. Thus – when ignoring spinning reserve constraints – it is here found less expensive to suffer the heavy operational cost of the gas turbine, than the consequences of starting the coalfired unit- even if the latter unit from a running cost point of view, is quite competitive.

'Conventional' Unit Commitment. Spinning reserve constraints observed. The case is identical to the previous one, apart from the spinning reserve constraints, which are now introduced. In this latter respect it is required that any dispatched set of production units, shall have the capability of increasing power output 10% beyond scheduled sum production.This holds for every hour of the period of analysis.

Scope of analysis: To decide on unit committment and unit dispatch so that specified demand over the week is covered at minimum cost, taking into account all given constraints including those relating to spinning reserve.

Solution is found after two optimization runs on 'Level I analysis' (to generate the initial committment plan), and two runs on 'Level II analysis' (to finalize the decommittment process). The solution's cost criterion is 0.03% above true minimum value found from the reference analysis. (The cost is furthermore 1.1% above the cost of previous analysis, which ignored spinning reserve requirements)

Although almost identical in terms of criterion value, the two solution methods produce results that differ slightly in terms of workday utilization of the gasfired combiplant and the gas turbine: The reference solution committs the gas turbine twice each workday, while the *Market Driven Hydro-Thermal Scheduling* committs the turbine once,but then retains the combiplant online in more hours to also cover the evening peak.

Solutionwise it is of interest to observe that –when spinning reserve requirements are included- the gas turbine is committed two hours each workday only to run at minimum output. I.e.: The gas turbine's primary function is now to supply spinning reserve,while the bigger/old coalfired unit (5) is called upon from Monday through Friday to contribute to covering the load.

'Conventional' Unit Commitment including hydro production. Spinning reserve constraints observed. The system comprises the same 7 thermal units as in the previous two example studies. In addition 2 hydro units of capacity 150 and 70MW, respectively, are included.

As operational boundary conditions it is specified that thermal units 1 and 2 are online all time and unit 3 is online in hours -1 and 168. For the two hydro units, available gross energy volumes (i.e. natural energy volumes) for the week are specified. (For comparison of result purposes, the available volumes are set equal to the volumes that are registered from the reference 'Dijkstra-solution' - which is based on (arbitrarely) choosing some individual valuation of stored gross energy upstream of the two hydro units.)

With respect to spinning reserve, it is again required that any dispatched set of production units shall have the capability of increasing power output 10% beyond scheduled sum production.

Scope of analysis: To decide on unit committment and unit dispatch for 7 thermal and 2 hydro production units, so that specified demand over the week is covered at minimum cost – taking into account all given constraints, including those relating to spinning reserve and availability of water.

Solution is found after three optimization runs on 'Level I analysis' (to generate the initial committment plan), and four runs on 'Level II analysis' (to finalize the decommittment process). The solution's cost criterion is 0.4% above true minimum value found from the reference analysis.

Market Driven Hydro-Thermal Scheduling The power system layout is stylized: 9 different regions are defined, and the 7 thermal and 2 hydro production units of the previous study provide for the supply of power to this system. Each generator is feeding power into its regional area, where there is a local market to clear in consistency with the hour-by-hour clearing of the rest of the system. Each region is via lossy transmission connected to the central system bus where main contractual (i.e. 'firm'/price-insensitive) demand is located. Transmission losses are described in terms of B-coefficients. The cost of starting thermal production units is assumed independent of the temperature status of the unit. Requirements to up- and downtimes of units are neglected. Ramping constraints are not included in this example.

As operational boundary conditions it is specified that thermal units 1 and 2 are online all the time. Unit 3 is online in hour -1 and hour 168. For the two hydro units, available gross energy volumes for the week are specified. Most of the energy presumed available for one of the hydro units (unit no 9), stems from nonregulated inflow – hence it is specified that this unit should be up and running in all hours of the week.

Power demand is forecasted in terms of contractual (i.e. price insensitive) demand as well as spot type (i.e. price sensitive) demand:

> *Hourly contractual demand* is assumed to have the same daily cycle Monday through Friday.The weekend days have their own demand profiles.
>
> *Spot power demand for given hour and given region* is described in terms of a linear incremental price vs. volume curve. Market description differ from day- to nighttime, and from workday to weekend.

With respect to spinning reserve, it is required that any dispatched set of production units shall have the capability of increasing power output 15.4% beyond scheduled sum production.

Scope of analysis: To decide on hour-by-hour clearing of geographically dispersed power markets over the week, together with commitment and dispatch of production units, so that the *expected sum of consumer- and producer surplus* over the stated period is maximized – taking into account all given constraints, including those relating to spinning reserve and availability of water power.

Solution is found after three optimization runs on 'Level I analysis' (to generate the initial committment plan), and two runs on 'Level II analysis' (to finalize the decommittment process). There is no alternative or reference solution available to this multibus case. Points of interest to notice from the solution:

> Not only the contractual demand, but also some (although strongly time-variable) volume of spot power is supplied in all hours of the week. The background here being (as specified via data input) a forecasted/registered willingness among potential buyers, to pay relatively high prices in respective local spot power markets.

The coalfired units 1 2 3 4 participate in all hours of the week. The oldest/most inefficient coalfired unit 5 is brought online at start of hour 7 on Monday and remains connected until hour 24 on Friday. During workdays, the gasfired combi unit is up and running at minimum output from t=10 to t=17, thus contributing to supplying spinning reserve in that period of daytime. Hydro unit 8 is producing at daytime each day of the week, while hydro unit 9 -as specified- is connected in all hours.

The incremental cost associated with the system spinning reserve constraint, is taking on values above zero in 9 daytime hours on workdays, implying that the matching of power supply and demand in these hours is constrained by the reserve requirement. 6 hours during the weekend days, are only marginally influenced in this way.

In consistency with foregoing incremental cost observations, it is also noticed that the cleared incremental power cost (associated with the system power balance) is forced high during daytime hours when the system reserve constraint is felt as strongest. –

The previous observation illustrates that a process of market clearing which internalizes the constraints of spinning reserve, may yield prices in the marketplaces that in a consistent and effective/optimal way reflects these concerns.- Other concerns of similar common/societal interest, may be internalized in the same way. Ill.:Assume that weekly emission to air of some material from certain plants/units, has to be constrained. This would imply volume constraints associated with one or more thermal units, in the same way as above illustrated for hydro units. If the emission-related constraint(s) is/ (are) activated, the market will respond and observe this concern in the best possible way from the point of view of the agreed-upon criterion of market performance. (Remark: In the present scheme of analysis both 'fuel' price and volume constraint are per definition 'attached' to each and every production unit. In the example studies, neither emission- nor fuel deficiency-aspects are chosen as themes for special consideration. Therefore, volume constraints on thermal units are set (arbitrarely high) beyond reach, so as not to become binding in the solutions)

SUMMARY OBSERVATIONS

Market Driven Hydro-Thermal Scheduling as well as e.g. *Unit Committment*, involve in principle operational evaluation of all prospective configurations of production units within a sequence of discrete time intervals, and subsequently including them in an optimization process over the agreed-upon period if analysis. Unfortunately, such combinatorial problems soon suffer the curse of dimensionality when the number of (here) generating units increases. Ill.: With 10 such units, the no of prospective combinations for a given hour is 2^{10} =1024, - with 20 units the number is 2^{20} =1048576. Thus, it is readily observed that the ambition of applying solution techniques that evaluate the one and only global optimal solution sequence over time, will limit analysis to power systems comprising less than (say) 10-15 production units.

For practical handling of the stated tasks, one has to resort to iterative solution processes that limit their scope to approaching and identifying not *the* optimal solution sequence, but rather one out of the many that belong to the region of solution space where the best ones reside:

Analysis schemes based on the *Unit Committment methodology* are very effective in terms of properly handling aspects related to start and stop of large number of production units connected to one or a very few system buses. The methodology would seem to have inherent difficulties, if called upon to also cope with arbitrary hydraulic constraints, distributed market clearing, power transmission constraints, and multiple constraints relating to accumulated quantities over the horizon of analysis.

Analysis schemes based on the *Hydro production planning methodology* are very powerful for dealing with the multiconstraint and large scale 'real variable problems' that are often formulated in scheduling of complex systems comprising hydro generation, power transmission, and distributed power supply. The methodology would normally entail iterative processes based on linear programming, where the aspects of start/stop of units and system spinning reserve are not internalized. This *linearization* approach of the hydro production planning methodology would seem to have inherent difficulties, if called upon to also deal with spinning reserve constraints in the optimization; then it is likely that also integer variables have to be applied, and that could seriously hamper applicability of the linearization scheme. - But what could be the prospects of a *nonlinear* approach here ?

The present report focuses on the just stated *nonlinear* approach of the *Hydro production planning methodology*, for investigating the prospects of extending its scope to also handle – in an approximate way – the concerns of spinning reserve as well as cost of start. The main 'trick' being – in relation to the previous linear concept – to adapt strongly *nonlinear* functions in modelling of system spinning reserve constraints.

The Market Driven Hydro-Thermal Scheduling – which can be termed a generalization of *Unit Committment* – is formulated in approximate terms, and solved via processes of optimization on two levels of analysis; one where Newtonian optimization is applied to only continous variables to generate an initial (and most likely) overcommitted commitment plan, and one that –via a process of stepwise decommittment- aims at targeting in on a close to optimal final plan for matching of electrical supply and demand over the week. The Newtonian optimization process has been tested for systems comprising up to 30 production units and 30 interconnected regions where local market clearing takes place. Convergence properties seem promising. Further testing and development is required before deciding on the merit and robustness of this approach of analysis, for large-scale handling of the task defined as *Market Driven Hydro-Thermal Scheduling.*

Relative to traditional methods of analysis, the present optimization scheme includes capabilities /features not observed in available litterature. E.g.:

Clearing of electrical power markets and scheduling of production hour-by-hour over the week, while also considering the main aspects of cost of start and spinning reserve.

Handling –in the stated market clearing and scheduling context- also of hydraulic and electrical constraints and constraints that relate to accumulated quantities over the horizon of analysis

Solving of inner/formal optimization task by a nonlinear process that internalizes all concerns of inequality constraints.

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1 SCOPE AND OVERVIEW OF WORK

Governments worldwide are endorsing deregulation of power sectors on the premise that free-market performance in all main respects shall prove beneficial to the general consumer. Can ideal or target performance be expected in practice?

EFI together with numerous other research undertakings are concerned with modelling of future power system performance.- And the question raised above is met with in this specific form: How should we today most realistically model the future behaviour of a hydro-thermal power system in deregulated mode?

The answer is not readily given. This report will not discuss the question of most realistic modelling per se, but focus on the modelling and simulation of ideal or target system performance. The scope of the report is twofold:

- Firstly, to present a concept and an associated computational scheme for simulating market clearing, unit committment and unit dispatch of an ideally performing deregulated hydro-thermal power system exposed to various component- and systems related constraints. (1.1)

- Secondly, to demonstrate criterion of performance and practical relevance of concept and methodology, via example analyses that in part allow for comparison of results with those from an alternative solution method.

Modelling of target freemarket power system peformance is expected to become increasingly relevant, as deregulation of power sectors spread internationally and associated market processes mature. E.g.:

Such modelling may supply information on reference or 'benchmark' system perfomance, in discussing or evaluating practical implementations and aspects of nonperfect behaviour within power market clearing and coordination.

It may provide for insight into power trade capabilities and limitations between differently designed systems, when new regimes of operation opens up for freemarket trade between them. -In case of e.g. the Norwegian power system, this may imply information on target volume and utilization of power trade agreements vis-a-vis external partners.Local, regional and interregional constraints that are included , will hereby contribute in setting premises for this utilization.

It may give ideas and clues as to how essential societal concerns that today are more or less external to the market clearing process, could be internalized in that process. – And thus provide for realism in the very concept of modelling of target freemarket power system performance.

The substance of the report is organized in four chapters, the main content of which is summarized in the following:

Competitive Market vs. Central Planning
Demonstration of conditions *for* and rationality *in* assuming equality between Maximum Societal Welfare Behaviour and the aggregate behaviour resulting from individual decisions within a presumed perfect free-market setting.

Collaborative Scheduling
In terms of system simulation analyses, scope (1.1) implies internalizing proper consideration of competition , overall resource utilization, environmental concerns as well as security constraints, in the process of simulating market clearing and unit scheduling.

Internalizing the stated societal concerns in the analysis leads to what the chapter outlines as Collaborative Scheduling. Such scheduling may involve different time horizons, depending on the analysis task at hand. Illustrations:

- Collaborative Scheduling within the hour ; this analysis is denoted Market Driven Optimal Power Flow, and is planned dealt with in a separate SINTEF report.
- Collaborative Scheduling over an horizon of (typically) one week; this analysis is labeled Market Driven Hydro-Thermal Scheduling, and is the main theme of this report. See the following text.

Market Driven Hydro-Thermal Scheduling
The Market Driven Hydro-Thermal Scheduling – which can be termed a generalization of *Unit Commitment* – is formulated in approximate terms, and solved via Newtonian processes of optimization on two main levels of analysis: On the first level mathematical methods that involve only contionous variables, are applied to generate an initial (overcommitted) committment plan. The second level comprises a decommittment process that aims at targeting *in* on a close to optimal final plan for matching of electrical supply and demand over the week.

Relative to traditional methods of analysis, the present optimization scheme includes capabilities /features not observed in available litterature. E.g.:

Clearing of electrical power markets and scheduling of production hour-by-hour over the week, while also considering the main aspects of cost of start and spinning reserve.

Handling in this context also of hydraulic and electrical constraints and constraints that relate to accumulated quantities over the horizon of analysis

Solving of formal optimization task by a mathematical process that internalizes all concerns of inequality constraints

Example analyses
Four cases of increasing complexity are dealt with to 1) demonstrate problem formulation and solution, and 2) compare results with those obtained from other solution method:

- 'Conventional' Unit Committment, spinning reserve constraints neglected.

- 'Conventional' Unit Committment, spinning reserve constraints observed

- 'Conventional' Unit Committment including hydro production, spinning reserve constraints observed

- Market Driven Hydro-Thermal Scheduling, spinning reserve constraints observed

Available programs for Unit Committment generally presumes single bus representation of the electrical network.To allow for comparison of the results with those from an alternative computational scheme, single bus representation is resorted to in the three first example cases .The fourth example is inherently a multibus problem formulation, hence there is no alternative solution to compare with in this case.

2 COMPETITIVE MARKET vs. CENTRAL PLANNING

The scpoe of this chapter is two-fold; 1) to be a reminder on the content of 'general freemarket behaviour', 'ideal freemarket behaviour' and 'maximum societal welfare behaviour', and 2) to point to the main conditions for equality between the two latter concepts.

2.1 Free-market behaviour/ Individual decisions

In principle there are two categories of participants in the marketplace; GENERATORS, each one of which is concerned with own Producer Surplus (PS) , and CONSUMERS, each of which similarly is concerned with own Consumer Surplus (CS) .

For the purpose of formal outline , the following nomenclature is applied:

PS : Producer Surplus (NOK/time interval)
CS : Consumer Surplus (NOK/time interval)

p : power price in the market place (NOK/MWh)

P_i : Production from GENERATOR 'i' (MW)
C_i : Cost of Production, GENERATOR 'i' (NOK/time interval)

Q_j : Quantity taken by CONSUMER 'j' (MW)
U_j : Utility of power taken by CONSUMER 'j' (NOK/time interval)

The GENERATOR in the marketplace

The GENERATOR wishes to maximize his own producer surplus PS. Thus for GENERATOR 'i' the objective of operation can be stated as follows [1] :

$$\max_{P_i} \quad \{ PS_i = p \cdot P_i - C_i \} \tag{2.1}$$

Taking the derivative with respect to the GENERATOR's decision variable P_i, we get the operational optimality condition for this market participant:

$$\underline{p + P_i \cdot (\partial p / \partial P_i) = dC_i/dP_i} \tag{2.2}$$

Eqn. (2.2) describes the operational condition to observe by GENERATOR 'i' , in the general freemarket case.

The second left-side term of (2.2) describes an influence on the market price of the decision made by GENERATOR 'i' . The term reflects in principle the effect of market power exerted by the considered market participant .If this type of effect is generally nil, we have the classical 'price taker' condition , characterizing the ideal freemarket situation where production from a unit that is up and running, is set so that marginal cost of production is equal to current power price at the production site.

The CONSUMER in the marketplace

The CONSUMER wishes to maximize his own consumer surplus CS. Thus for CONSUMER 'j' the objective of operation can be stated as follows [1]:

$$\max_{Q_j} \{ CS_j = U_j - p \cdot Q_j \} \qquad (2.3)$$

Taking the derivative with respect to the CONSUMER's decision variable Qj , we get the operational optimality condition for this market participant:

$$\underline{p + Q_j \cdot (\partial p / \partial Q_j) = dU_j / dQ_j} \qquad (2.4)$$

Eqn. (2.4) describes the operational condition to optimally observe by CONSUMER 'j' , in the general free-market case.

The second left-side term of (2.4) displays an influence on the market price of the decision made by CONSUMER 'j'. The term reflects in principle the effect of market power exerted by the considered market participant. If this type of effect is generally absent, we have the ideal price taker condition commented on above.

Summary observations

Summary formalism on free-market behaviour is presented in Figure 2.1. The ideal price taker condition which presumes that all market power sensitivity terms are nil, is characterized in the box to the right in the figure.

The ideal (price taker) free-market situation presumes in principle a large no. of comparably sized participants both on the generator- and consumer side. We see from the figure that the ideally cleared power market is characterized by the conditions

$$\underline{p = dC_i / dP_i = dU_j / dQ_j} \qquad (2.5)$$

which says that the market price is where short-term marginal production cost equals short-term marginal willingness to pay. - And this price is also the one reached, if maximum societal welfare is the agreed-upon objective of power system operation. See next section 2.2.

2.2 Maximum societal welfare behaviour / 'The Invisible Hand' decides

The perfect market/system coordinator (i.e. 'The Invisible Hand') who - by definition - has total insight into all local as well as global aspects related to power production, transmission and market clearing , decides on market clearing and production scheduling so as to in principle maximize an agreed-upon common welfare goal for the society.

On behalf of all GENERATORS and CONSUMERS, the common objective would be to maximize the sum of all Producer and Consumer Surpluses (PCS) :

$$\underset{(P,Q)}{\text{Max}} \quad \{ \text{PCS} = \sum_{x}^{Nc} U_x - \sum_{y}^{Np} C_y \} \qquad (2.6)$$

which leads to the previously defined ideal free-market condition (2.5) , - a condition that most probably should characterize target power market behaviour from the point of view of the authorities in charge of deregulation of power sectors.

The concepts and tools presented in this report are in principle of the genre required by 'The Invisible Hand ' , to sustain maximum societal welfare behaviour, - or equivalently - , ideal freemarket power system performance.

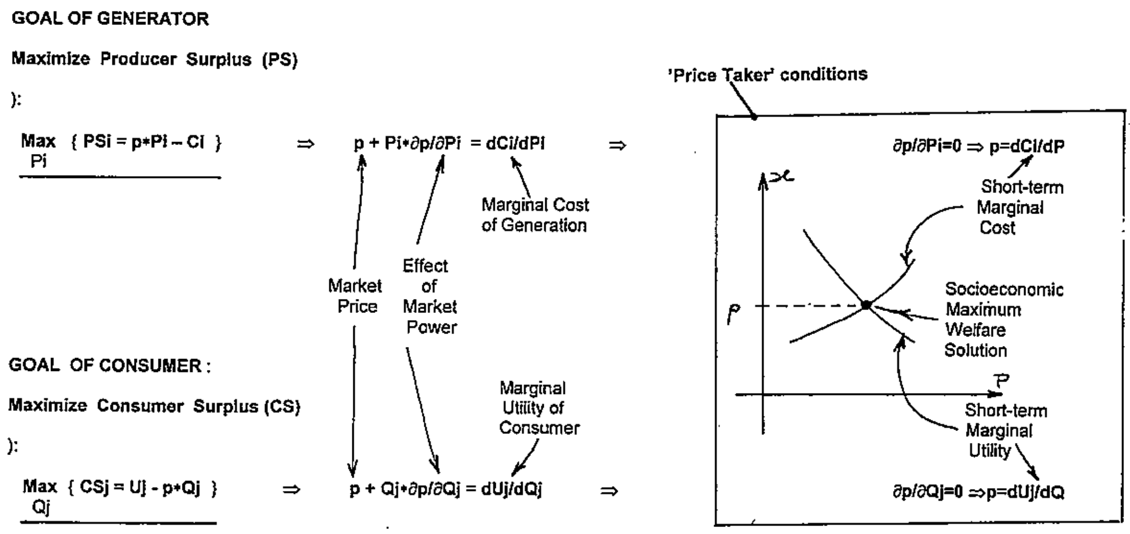


Figure 2.1 Summary formalism on individual free-market behaviour :
- Objective of the GENERATOR (producer)
- Objective of the CONSUMER
- The ideal Price Taker conditions

3 COLLABORATIVE SCHEDULING

The simulation-directed concept of Collaborative Scheduling aims at modelling target behaviour of the deregulated hydro-thermal power system.

Modelling of target behaviour implies internalizing proper consideration of competition, overall resource utilization,environmental concerns as well as security constraints, in the process of simulating market clearing an unit scheduling.

The task of Collaborative Scheduling can in general terms be stated in this way:

For a market clearing period of representative duration and relevant time resolution: To evaluate the bus-by-bus matching of electrical power supply and demand that contributes to maximizing system-wide expected sum of PRODUCER- and CONSUMER SURPLUS, due consideration given to all relevant global and local operational constraints.	(3.1)

Task of Collaborative Scheduling

For given bus and given time interval of the day, the classical Marshallian supply-demand cross of Figure 3.1 illustrates the content of the terms PRODUCER SURPLUS, CONSUMER SURPLUS, and their sum, - which contributes to the criterion to be maximized. In economic litterature this sum is also termed Social Welfare or an element of it [1].

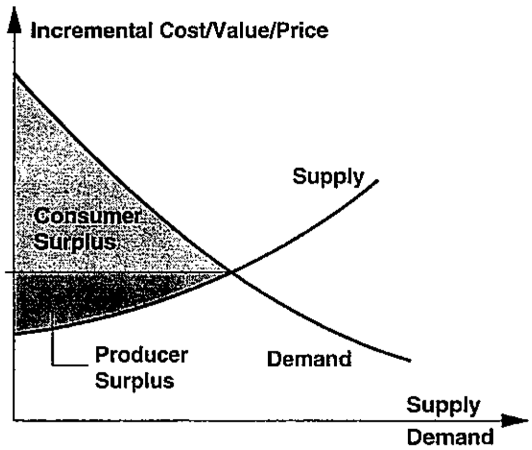


Figure 3:1 : Illustration of concepts of Producer- and Consumer Surplus

The Figure gives power supply and demand along the x-axis, and incremental cost, value and price in vertical direction. The crossing point defines market price as well as volume traded.

The task of (3.1) may involve different time horizons, depending on the analysis scope at hand. Main illustrations:

Collaborative Scheduling within the hour

This scheduling is denoted Market Driven Optimal Power Flow, ând differs from the ordinary Optimal Power Flow in that power demand has become an additional decision variable in the process of optimization.

Practical solution is afforded by Newton's method , where all inequality constraints are internalized in the mathematical process via logarithmic or inverse barrier functions.

Convergence properties and speed of solution seem very satisfactory. Ill, : A 51-bus system that includes 31 generators, 38 local spotmarkets and 12 transformers with variable turns ratio, is solved in about 5s on a 130Mz laptop PC. With fixed transformer settings , solution time reduces to ca. 2s.

Collaborative Scheduling over the week *)

This task is labeled Market Driven Hydro-Thermal Scheduling, and involves in effect interconnecting the former hourly market clearing over a sequence of (say) 168 hours, observing time dependent aspects such as;

- start/stop of units
- volume constraints on water,fuel, emissions
- price-sensitivity of demand

Formulating, solving and exemplifying of this problem is the main theme of the present report. See next two chapters.

The large-scale task of Market Driven Hydro-Thermal Scheduling, represents a formidable computational challenge even if – as is the case in this report – stochasticity of processes is defined out, or treated in simplified manner.

In solving scheduling problems of similar nature, a frequent approach is to interlink continous and discrete type mathematics in iterative processes based on a combination of formal and intuitive type logic. This is e.g. the case in commonly used methods for solving the unit committment problem.

As part of a project aimed at investigating alternative schemes for solving new and challenging problems, a strategy of iteratively using only continous mathematics is being researched for dealing with the task of Market Driven Hydro-Thermal Scheduling.

The Newton optimization method is again used. Since start/stop cannot be dealt with correctly by continous mathematics alone, simplified modelling of this aspect is introduced in the iterative process leading to the final collaborative integer solution. Simplifications are made in view of the capability of reservoir hydro units to provide for 'tuning' of the power market balance.- In brief the method could be labeled ' Newton's Method with Integer Finalizing' .

*) Although the principal horizon of collaborative scheduling is one week, it may at time be required to look (say) 7+1 days ahead, in order to secure proper boundary conditions for the analysis. Ill.: In a given situation a thermal unit of considerable startcost may have to be committed on Monday morning and retained online at least until after peaktime on Friday. To decide on whether the unit should be de-committed on Friday or kept online also during the weekend, it may be significant for the optimization process to 'see' the market conditions coming up on the ensuing Monday.

4 MARKET DRIVEN HYDRO-THERMAL SCHEDULING

This chapter reports on the methodological part of the development work.

Following practical problem specification in section 4.1, section 4.2 deals with mathematical formulation and solution of the stated problem.

4.1 Problem formulation , practical level

Consistent with the idealized scope of (1.1) and correspondingly also (3.1), practical problem formulation can be summarized as follows:

> GIVEN an electrical production and transport apparatus for matching of geographically dispersed power demand over a period of 168 (or say 168+24) hours.
>
> Power demand at a given bus may comprise price-indifferent load (as e.g. a contractual delivery), and/or demand that depends on the spot-price to be cleared at the bus at hand. (4.1)
>
> DETERMINE for respective system buses, the matching of power supply and demand over time, that contributes to maximizing systemwide sum of producer- and consumer surplus, due consideration given to all relevant global and local constraints

MARKET DRIVEN HYDRO-THERMAL SCHEDULING
Problem formulation, practical level

A user oriented simulation tool for solving of (4.1) , should permit the system analyst to define any desired layout of the system. To provide full such flexibilty is however not deemed relevant at this demonstration stage of development.

Figure 4.1 illustrates the power system structure presumed in the present research version of the model.The system comprises a number of (up to 100) buses or regional areas at/in each of which there may be a thermal or hydro generator as well as local spot market to clear in consistency with the hour-by-hour clearing of the rest of the system. Each region is via lossy transmission connected to the central system where main contractual or price-insensitive demand is located.

In ensuing sections , (4.1) is formalized in appropriate detail, and expressed in mathematical terms suited for a direct, non-linear optimization process.

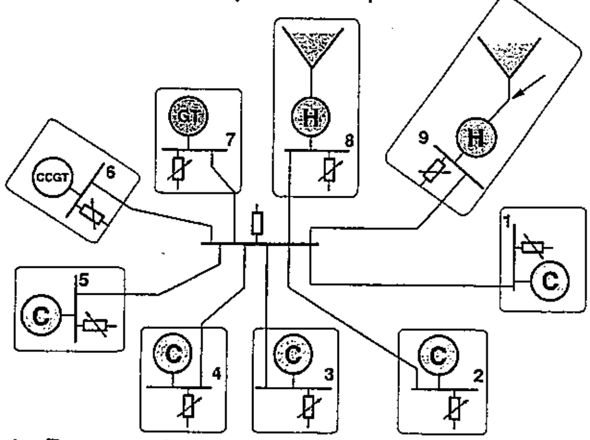


Figure 4.1 : Presumed system structure in modelling scheme

4.2 Problem formulation , mathematical level

4.2.1 The generic problem

The verbal formulation (4.1) can be equivalenced by the following compact, formal description:

$$\max_{\underline{x}} \{ F(\underline{x}) \} \qquad \text{a)}$$

$$\text{s/t;} \qquad (4.2)$$

$$L_i(\underline{x}) = 0 \qquad i = 1,2,\ldots.I \qquad \text{b)}$$

$$H_j(\underline{x}) \geq Hlim_j \qquad j = 1,2,\ldots J \qquad \text{c)}$$

Compact/generic description of verbal formulation (4.1)

Here:

$\underline{x}$ = n-dimensional vector comprising the n problem variables that are to be determined. $\underline{x}$ will in principle include both real and integer type variables.

$F(\underline{x})$ = nonlinear criterion to be maximized; i.e. the systemwide sum of producer- and consumer surplus over a relevant period of time – here defined to be 168h.

$L_i(\underline{x})$ = the 'i'-th equality constraint. Alltogether 'I' such linear and nonlinear constraints are to be fulfilled , to secure feasible power market clearing over the week

$H_j(\underline{x})$ = left side of 'j'-th inequality constraint. To secure valid solution , alltogether 'J' such linear and nonlinear constraints have to be fulfilled. ((4.2c) also cover inequalities of the type 'less or equal' , as multiplication of these inequalities by –1, brings them on the form stated above)

$Hlim_j$ = specified limt for 'j'-th inequality constraint

The number of variables and constraints will increase about proportional to problem size, and attain considerable magnitude for most prospective real-life systems. Ill. : A hydro-thermal power system that includes 7 thermal production units, 23 hydro units, and 30 local power markets to clear hour-by-hour over a week, will (in the current scheme of analysis) incur about 35000 problem variables, 10000 equalities and 20000 inequalities.

4.2.2 The concrete / mathematical problem

CRITERION f($\underline{X}$) AND ASSOCIATED CAPACITY CONSTRAINTS

Electrical power will in part have status as price insensitive (e.g. contractual deliveries), in part price sensitive.

A load defined as price insensitive, should in principle be fully covered regardless of cleared price at the delivery point. Thus, from a mathematical viewpoint, this delivery is exogenously given, and could readily be accounted for in terms of constraint formulation.

However, power system deficiencies or contingencies may at times cause part of contractual load to be curtailed to sustain system security constraints. To generally retain feasibility of mathematical solution, part of, or all of contractual demand is here formally treated as price sensitive, with prices set high to reflect the special/agreed-upon inconveniance or cost, of having outage of contractual or firm power delivery.

Valuating power deliveries in the light of foregoing considerations, the systemwide sum of producer- and consumer surplus over 168 hours- together with associated capacity constraints, can be expressed as given by (4.3) – (4.8). The special constraints to validate the formulation of cost of start in (4.6), are developed in a separate section below.

$$\underline{F(x) = \text{Utility} - \text{Fuelcost} - \text{Startcost}} \qquad [\text{NOK/week}] \qquad 4.3)$$

Where;

F($\underline{x}$) = Systemwide sum of producer- and consumer surplus over (typically) 168h [(NOK/week]

$$\text{Utility} = \sum_{i=1}^{ng} \sum_{t=1}^{nt} [\, \alpha(i,0,t) + \alpha(i,1,t)\cdot Ps(i,t) - \alpha(i,2,t)\cdot Ps(i,t)^2 \,] \qquad (4.4)$$

$$\text{Fuelcost} = \sum_{t=1}^{nt} \sum_{j=1}^{ng} [\, \gamma(j,1,t) + \gamma(j,2,t)\cdot Pg(j,t) + \gamma(j,3,t)\cdot Pg(j,t)^2 \,] \qquad 4.5)$$

$$\text{Startcost} = \sum_{j=1}^{ng} \sum_{t=1}^{nt} \text{Cstart}(j,t)\cdot \text{Delta}(j,t) \qquad (4.6)$$

$$Psmin(i,t) \le Ps(i,t) \le Psmax(i,t) \qquad i=1,2,\ldots ns \qquad t=1,2,\ldots nt \qquad (4.7)$$

$$Pgmin(j,t) \le Pg(j,t) \le Pgmax(j,t) \qquad j=1,2,\ldots ng \qquad t=1,2,\ldots nt \qquad (4.8)$$

Criterion F($\underline{x}$) and associated capacity constraints

The terminology of (4.3)-(4.8) is as follows:

Utility : Total consumer utility over the horizon of analysis of 168h. [KR/week]

Ps(i,t) : Price-sensitive power delivery at regional bus 'i', time interval t. [MWh/h]

Psmax(i,t) : Max. Price-sensitive demand at regional bus 'i', time interval t. [MWh/h]

Psmin(i,t) : Min. price-sensitive demand at bus 'i',time interval t. (Normally, Psmin will be zero, but Psmin>0 can be encountered- e.g. when power is bought for pumping and the pump has a minimum power consumption)

α(i,0,t) : First coefficient of polynomial that describes the utility of electrical power as function of delivered volume. Load located at bus 'i', time interval t.

α(i,1,t) : Second coefficient of above utility polynomial.

α(i,2,t) : Third coefficient of above utility polynomial. For further outline on market modelling, see Appendix 1.1

Fuelcost : Total running cost of generation over the horizon of analysis. [NOK/week] For reasons of economy, scarcity of resourse and/or emission limitations,fuel price as well as volume constraints are presumed specified for every generator of the system. Illustrations:

If the fuel price is set to reflect expected market cost or market value, and the volume constraint is set very high, (i.e. beyond reach) the latter constraint will not be binding and we have the traditional 'thermal' case, where fuel cost and max/min limits on production are decisive for output from synchronized generators.

If the fuel price is zero or very low, and the volume constraint is set within reach,the latter constraint will most likely be activated, and thus decisive for operation. This may typically be the operational case for many reservoir hydro production units.

If the price is set marketwise realistic, and the volume constraint is set within reach, only the solution of the market clearing process will reveal which constraint is binding; the fuel price, or the volume limitation on resource use. This could exemplify the operational case for a coal-fired power plant , where environmental concerns incur a limit on e.g. weekly burned volume of coal. In this situation, the cost of other production alternatives together with the market's willingness to pay, will be decisive factors in determining which constraint is binding.

Pg(j,t) : Generator production at regional bus 'j', time interval t. [MWh/h]

Pgmax(j,t) : Max. generator production at regional bus 'j', time interval t. [MWh/h]

Pgmin(j,t) : Min. generator production at bus 'j', time interval t. [MWh/h]

γ(j,1,t) : First coefficient of polynomial that describes running cost as function of power production at regional bus 'j', time interval t.

γ(j,2,t) : Second coefficient of above running cost polynomial

γ(j,3,t) : Third coefficient of above polynomial. For further details on running cost modelling , see App. 1.2

Startcost : Total cost of start over the horizon of analysis. [NOK/week]

Cstart(j,t) : Cost of start of unit 'j' , if brought online at beginning of hour t. [NOK/start]

Delta(j,t) : Pu output increase from unit 'j' from (t-1) to t. See details in the following.

CONSTRAINTS ASSOCIATED WITH COST OF START

The cost of starting a thermal generator unit can generally be approximated by the following equation:

$$S = S_0 \cdot (1 - a \cdot e^{-Tdown/To}) \qquad (4.9)$$

Where;

S = Cost of starting considered unit [NOK]
S_0= Cost of start of cold unit [NOK]
a = pu cost coefficient
T_{down} = Downtime of considered unit [h]
T_0 = Boiler cool-down time constant [h]

To internalize the time-dependency of cost of start in the optimization is a complex matter that (per today) cannot be implemented unless major system simplifications are made. To illustrate; it can be included in UNIT COMMITTMENT analyses, provided the power transmission network is reduced to one or a very few buses, and volume constaints (on e.g. water,fuel,emissions) are absent or only very few.

The cost of start of units expressed by (4.6), neglects the time-dependency of S. This is deemed appropriate in view of

1) the need for giving priority to other features such as modelling of power transmission, local market places and volume constraints,

2) the capability of reservoir hydro units to provide for 'final tuning' of the power market balance, and thus opening up for harvesting (part of) the benefit that may accrue from optimally observing time-dependency of startcosts.

The constraints that are found relevant for governing the variation of the continous variables Delta(i,t) of (4.6), are summarized in (4.10) – (4.13). To demonstrate the relevance of this modelling, the operation of an arbitrary generator unit over two consecutive time intervalst t and (t+1) – and next over the period of analysis - , is discussed in the following:

$$pg(j,t-1)^{\mu} - pg(j,t)^{\mu} + Delta(j,t) = \delta D(j,t) \qquad t=1,2,\ldots(nt+1) \quad j=1,2,\ldots ng \qquad (4.10)$$

$$\delta D(j,t) \geq 0.0 \qquad t=1,2,\ldots(nt+1) \quad j=1,2,\ldots ng \qquad (4.11)$$

$$Delta(j,t) \geq 0.0 \qquad t=1,2,\ldots(nt+1) \quad j=1,2,\ldots ng \qquad (4.12)$$

$$0 < \mu \leq 1.0 \qquad (4.13)$$

where;

pg(j,t) : Pu generator production at regional bus 'j', time interval t
Delta(j,t) : Pu output increase from unit 'j' from (t-1) to t
$\delta D(j,t)$: Pu Dummy variable associated with unit 'j' from (t-1) to t
μ : constant less or equal to 1.0 (and greater than 0)

Constraints associated with modelling of cost of start

A continous variable $\delta t(t+1)$ in the range 0.0-1.0 is introduced to describe the 'part of the start process' that – in a continous frame of reference - accrues from time interval t to interval (t+1). The corresponding incremental cost of start is defined by the product [$\delta t(t+1)$*S] , and added to the criterion of performance as a cost element. To govern the variation of $\delta t(t+1)$ over the period of analysis, eqns. (4.14)-(4.15) are introduced:

$$pt - pt(t+1) + \delta t(t+1) \geq 0 \qquad t=0,1,2,\ldots nt \qquad (4.14)$$

$$\delta t(t+1) \geq 0 \qquad t=0,1,2,\ldots nt \qquad (4.15)$$

where;

pt = Pt/Ptmax=pu production in time interval t
p(t+1) = pu production in time interval (t+1)
$\delta t(t+1)$ = continous variable, range 0.0-1.0

(4.15) is included to prevent the product $\delta t(t+1) \cdot S$ from becoming an artificial income, as it will , if δ is allowed to turn negative. Main consequences to observe from (4.14) - (4.15):

If pt=0.0 and p(t+1)=1.0 , corresponding to startup of unit and subsequent operation at full output, $\delta t(t+1)$ will have to equal 1.0 in order to make (4.14) valid. Full start cost 1.0·C will then contribute to the criterion,- which is correct.

If pt>p(t+1), any value of $\delta t(t+1) \geq 0.0$ will make (4.14) valid. The criterion itself will see to that the value $\delta t(t+1)=0.0$ is chosen, - leading to zero start cost. This is correct as no start of unit is implemented.

If pt=0.0 and p(t+1)= (say) 0.65, $\delta t(t+1)$ will take on the value 0.65 to secure feasibility of (4.14). Criterionwise, an amount 0.65·S will be added to the cost. If later in that production cycle, p increases further so that the accumulated δ-values amount to 1.0, full start cost is correctly included by way of incrementally adding up to full cost of start.

If- in general- the accumulated value of δ over the characteristic cycle of production for the unit,is above 0.0 but less than 1.0, the solution found is infeasible with respect to handling of the cost of start of the unit. Feasibility can then be attained via an iterative solution process as follows:

1) Solution of the formal problem with current value of cost of start of unit. The first time this means nominal cost S.

2) Check of feasibility with respect to handling of cost of start. If ok: exit. If not ok, a new formal cost of start is defined so that the product ($\Sigma\delta$)$\cdot S_{(new)}$=S. Then return to 1).

The process is terminated when acceptable consistency of solution is attained. For a given generator, three characteristic 'exit situations' can in general occur with respect to accumulated deltas over the relevant cycle of load- or production output variation:

Accumulated deltas=0.0, implying that the unit is offline, or is operated at constant or diminishing output.

Accumulated deltas =1.0, implying start of unit and loading it up to nominal output in the course of its characteristic production cycle.

Accumulated deltas<1.0, and greater than zero, implying start of unit, but loading it up to less than nominal capacity. In the course of the iterative solution process, a 'new' cost of start $S_{(new)}$ is found such that accumulated deltas over the production cycle multiplied by the 'new' startcost, equals actual cost of start S.

Eqn. (4.14) defines a linear relationship between increment of pu power output (p) and associated pu increment (δ) of cost of start. This relationship is illustrated by the straight line in the diagram of Figure 4.2. A nonlinearity can be introduced in such a way that a considerable part of the cost of start is incurred even for small 'trigging' increments of pu power output. This will in principle tend to fit with practice, where full cost of start is suffered once production- however small- is initiated from the unit.

Replacing the term pu power production p in (4.14) by the modified pu expression p^{μ}, where μ is a constant less or equal to 1.0 (but greater than zero), a prospective nonlinear effect can be achieved. Figure 4.2 illustrates how the modified term p^{μ} varies as function of p, for μ =1.0, 0.7 and 0.1, respectively. We see e.g. that for μ = 0.1, ca. 80% of the cost of start would be incurred by 10% increase of power output from zero initial value. The nonlinearity caused by applying the form of p^{μ}, is - for reasons of further research - introduced into the main scheme of analysis.

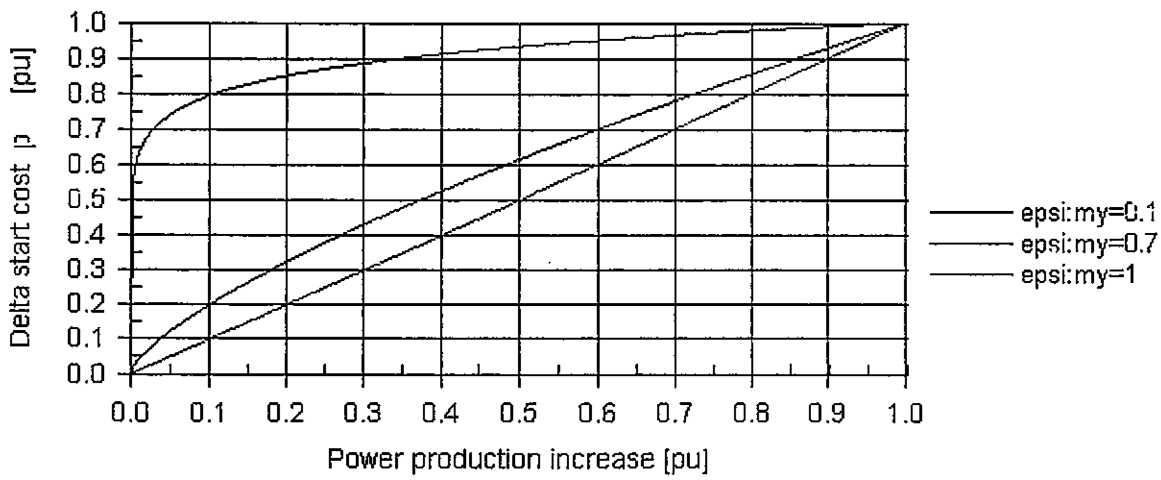


Figure 4.2 : Illustration of nonlinear pu form p^{μ} to 'trigger' large part of startcost for small increase of pu power output p

Eqns. (4.16)-(4.18) shows the set of constraints that replaces (4.14)-(4.15), when introducing the exponent μ .

$$p_t^{\mu} - p_{(t+1)}^{\mu} + \delta_{t(t+1)} \geq 0.0 \qquad t=0,1,2,\ldots nt \qquad (4.16)$$

$$\delta_{t(t+1)} \geq 0.0 \qquad t=0,1,2,\ldots nt \qquad (4.17)$$

$$0 < \mu \leq 1.0 \qquad (4.18)$$

Each inequality of (4.16) interlinks a multitude of variables. From the point of view of internalizing handling of inequalities in the mathematical process, it is found efficient to deal only with single-variable inequalities. (4.16) can be transformed into a system of nt single-variable inequalities and nt equalities, by introducing nt dummy variables δ_D ('D' for 'Dummy'). δ_D is set equal to the left side of (4.16).

Introducing the new dummy variables, we get the following system of constraints to ensure that the cost of start in strategic/reasonable way is brought into a trade-off position vis-a-vis fuel related cost and the cost of security:

$$p_t^{\mu} - p_{(t+1)}^{\mu} + \delta_{t(t+1)} = \delta_{Dt} \qquad t=0,1,2,\ldots nt \tag{4.19}$$

$$\delta_{Dt} \geq 0.0 \qquad t=0,1,2,\ldots nt \tag{4.20}$$

$$\delta_{t(t+1)} \geq 0.0 \qquad t=0,1,2,\ldots nt \tag{4.17}$$

$$0 < \mu \leq 1.0 \tag{4.18}$$

(4.17) – (4.20) are the sought constraints for one , un-indexed generator. We easily extend this description to cover ng generators by adopting the notation already introduced, see (4.3)-(4.8) with ensuing nomenclature comments : Final, extended constraint description emanating from above equations (4.17)-(4.20) , are already set forth as (4.10)-(4.13).

ELECTRICAL PROCESS CONSTRAINTS ('Load flow' constraints)

On a 'high ambition level' simulation of market clearing over the week, the detailed modelling scope of Marked Driven Optimal Power Flow, - documented in TR A4872, - would provide an appropriate basis for establishing the hour-by-hour electrical process constraints to observe, over the defined period of analysis.

In the present research version of the market clearing model , it is not deemed important neither to include full AC-presicion in the electrical network modelling, nor having full flexibility in data input definition of layout of the power system. On this background a power system structure as shown on page 8, is presumed here. Figure 4.2 shows the nomenclature applied for an arbitrary region 'j'.

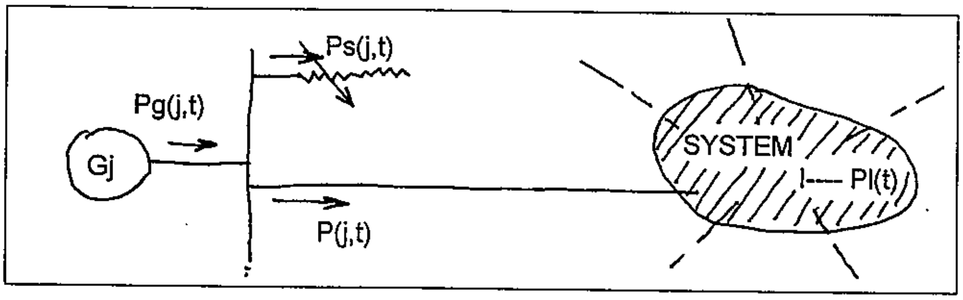


Figure 4.2 : Nomenclature associated with arbitrary region 'j'

From Figure 4.1 – 4.2 we see that the hourly system constraints will comprise ng regional power balances and nt global balance:

Local power balances:

$$P(j,t)+Ps(j,t)-Pg(j,t)=0 \qquad t=1,2,\ldots nt \qquad j=1,2,\ldots ng \qquad (4.21)$$

Global power balances:

$$Pl(t)+\sum_{i=1}^{ng}\sum_{j=1}^{ng}P(i,t)\cdot B(i,j,t)\cdot P(j,t) - \sum_{j=1}^{ng}P(j,t)=0 \qquad t=1,2,\ldots nt \qquad (4.22)$$

Where new variables/parameters are;

P(j,t) : Power injected into the transmission line connected to region 'j' ,time interval t. [MWh/h]

Pl(t) : Price-indifferent (e.g. contractual) delivery at central system bus [MWh/h]

B(i,j,t) : Loss coefficient pertaining to the power transmission system. For each time interval a set of such coefficients describes transmission losses as function of injected power $\underline{P}$ in that time interval. See further outline below.

Pg(j,t) : Generator production,region 'j',time interval t [MWh/h]

Ps(j,t) : Price sensitive power delivery,region 'j',time interval t [MWh/h]

Electrical process constraint comprising
(ng·nt) regional and nt global power balances

Further comment on B-coefficients:

Theory and experience show that total transmission losses during a characteristic period of the day (e.g. 'peak winter workday' ,or 'nighttime midsummer') can be well approximated by a quadratic function of input active powers to the transmission system. For a system of N inputs, this function can in compact matrix form be expressed as:

$$\text{Losses} = \underline{P}^t \cdot \underline{B} \cdot \underline{P} \qquad [\text{MWh/h}] \qquad (4.23)$$

Where;

$\underline{P}$ = Vector comprising the N power inputs. [MWh/h]

$\underline{P}^t$ = Tranpose of $\underline{P}$

$\underline{B}$ = N*N symmetric coefficient matrix comprising $M=0.5\cdot(N^2-N)+N=0.5\cdot N\cdot(N+1)$ independent elements. Ill.: Example no 4 of Chapter 5.5 comprises 9 buses or regions. This gives rise to M=0.5(81-9)+9=45 different B-coefficients.

(4.23) is a compact matrix formulation of the double summation contained in (4.22)

Assume for illustration purposes that the system of Figure 4.1 comprises 3 regions. Transmission losses in a given hour could then be described by a set of loss coefficients in this way:

$$\text{Losses} = \begin{bmatrix} P_1 & P_2 & P_3 \end{bmatrix} \cdot \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{13} \\ B_{31} & B_{32} & B_{13} \end{bmatrix} \cdot \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} \qquad (4.24)$$

$$\begin{aligned} = B_{11}\cdot P_1^2 &+ 2\cdot B_{12}\cdot P_1\cdot P_2 + 2\cdot B_{13}\cdot P_1\cdot P_3 \\ &+ B_{22}\cdot P_2^2 + 2\cdot B_{23}\cdot P_2\cdot P_3 \\ &+ B_{33}\cdot P_3^2 \end{aligned} \qquad (4.25)$$

The form illustrated by (4.25) also gives a key for determining the loss coefficients: For given power input values ($\underline{P}$) , we see that (4.25) is a linear equation in terms of the loss coefficients. Thus , if we perform M separate load flow analyses based on representative variation of the power incections $\underline{P}$, we can register the losses from each case and establish M linear, independent equations for determination of M unknown loss coefficients.

HYDRAULIC PROCESS CONSTRAINTS

Constraints relating to flow of water may take on similar or more complex forms, compared to those describing flow of electricity.

Hydraulic constraints are not included in the present modelling scheme. They will add to the size of a given system problem, but pose per se little modelling problems as a powerful nonlinear optimization method is being applied to solve the problem at hand.

VOLUME CONSTRAINTS

The system comprises ng generator units of thermal and/or hydroelectric type. To provide maximum flexiblity in choosing operational premises for the production units, volume constraints are formulated for each of them. The constraints can apply either to produced electrical energy , or to the use of primary resource. For a thermal unit, primary energy is considered to be the MWh thermal energy content of the fuel. For a hydroelectric unit , primary energy is here defined as the natural energy content of stored water (referred to some expected or reference weekly head profile).

The main practical premises for specifying volume constraints are summarized on p. 11. A general formulation of the constraints is set forth as follows:

$$\mathrm{Wmin}(i) \le \sum_{t=1}^{nt} [C(i,0,t)+C(i,1,t)\cdot Pg(i,t)+C(i,2,t)\cdot Pg(i,t)^2] \le \mathrm{Wmax}(i) \qquad i=1,2,\ldots ng \qquad (4.26)$$

The coefficients C( ) for a given generator take on different interpretation, depending on where the constraint applies in the production chain :

> If the volume constraint relates to produced electrical energy for generator 'i', C(i,0,t)=C(i,2,t)=0 , and C(i,1,t)=1.0
>
> If the volume constraint relates to use of primary resource, the coefficients C( ) are equal to the γ-coefficients divided by the resource cost. See eqns. (4.5).

(4.26) displays a situation (similar to (4.16)) in that each inequality interlinks a multitude of problem variables. For reasons already explained, new dummy vaiables are introduced to replace (4.26) by single-variable inequalities.In specific tems, (4.26) is transformed into a system of ng single-variable inequalities and ng equalities, by introducing ng dummy variables PgD ('D' for 'Dummy'). With reference to (4.26), PgD(i) is defined equal to the left side of the equation, - meaning that PgD(i) equals actual accumulated production or resource use , for generator 'i' , over the chosen market clearing horizon.

Introducing the new variables as stated, we get the following system of equations for effective handling of of volume constraints on generation:

$$\sum_{t=1}^{nt} \sum_{k=0}^{2} C(i,k,t)\cdot Pg(i,t)^k = PgD(i) \qquad i=1,2,\ldots ng \qquad (4.27)$$

$$\mathrm{Wmin}(i) \le PgD(i) \le \mathrm{Wmax}(i) \qquad i=1,2,\ldots ng \qquad (4.28)$$

New variables/parameters;

PgD(i) : Accumulated production/resource use over the chosen market clearing horizon, generator 'i'

Wmax(i),Wmin(i) : Weekly wolume limits not to be violated, generator 'i'.

C(i,k,t) : Volume constraint coefficients for unit 'i', time interval t. k=0,1,2 . See (4.26)

Volume constraints on generation

SPINNING RESERVE CONSTRAINTS

To handle unforeseeable events (such as e.g. forced outages) as well as foreseeable (such as e.g. morning load gradients within the hour), it is required that any dispatched set of production units be able to cover some specified increase in total output, beyond the (average hourly) value actually cleared in the coordination process.

In a given hour t , the reserve constraint to observe can be expressed in this way:

$$Kr(t)\cdot\sum_{i=1}^{ng} Pg(i,t) \le \sum_{i=1}^{ng} Pgmax(i,t)\cdot Z(i,t) \qquad (4.29)$$

Where;

Kr(t) : Pu reserve factor. If Kr(t) = (say) 1.12, eqn. (4.29) says that the considered set of units should be capable of increasing production 12% beyond total production cleared (as average hourly value) in the market solution process

Pgmax(i,t) : Max. capacity of unit 'i' , hour t [MWh/h]

Z(i,t) : Integer variable which is 1 if unit 'i' is synchronized in hour t, and 0 if not synchronized.

The formal optimization process is based on applying only continous variables (and thus coping with the discrete/integer aspect of scheduling and market clearing in an 'outer loop' of the analysis). Therefore, Z( ) has to be replaced by a continous approximation in the formal part of the analysis. The following approximation to Z( ) is introduced (using the formal shape of a 2-timelag steprespons of a dynamical system, as basic conceptual model):

$$z(i,t) = 1+(b/(a-b))\cdot e^{-(a\cdot\psi(i,t))} - (a/(a-b))\cdot e^{-(b\cdot\psi(i,t))} \qquad (4.30)$$

where (a,b) are chosen 'time constants' and $\psi(i,t)=(Pg(i,t)/Pgmax(i,t))^{\varsigma}$. ς is an exponent >1.0 chosen so as to contribute to giving z(i,t) the desired ʃ-shape. a and b should be greater than (say) 10.0 . We note from (4.30) and the foregoing comments on (a,b,ς):

- If Pg(i,t)=0 , z(i,t) is also 0. This fits with 'reference' description Z( ).

- If Pg(i,t)=Pgmax(i,t), z(i,t)≈1.0 . This also fits with Z( ).

- If Pg(i,t) is greater than zero and less than Pgmax(i,t), z(i,t) will take on a value in the range 0.0-1.0. By choosing the parameters (a,b,ς) appropriately in view also of the value of Pgmin(i,t) , it may be relevant to let z( ) replace Z() for practical analyses.

The example studies of this report are based on the following set of parameters: a=20.0, b=30.0, ς=1.5. z(i,t) then take on the following concrete form;

$$z(i,t)= 1+2\cdot e^{-30\cdot\varsigma} - 3\cdot e^{-20\cdot\varsigma} \qquad (4.30a)$$

Figure 4.3 illustrates the z-function of (4.30a). We note that 90% of full capacity is 'made available' at 0.3 pu output from the production unit. For the set of thermal generators of this study, Pgmin is in the range 0.2-0.5.

Some further discussion of the implications of applying z( ) instead of the correct description Z( ) in the formal optimization process, is delayed until the final constraint equations (4.33) - (4.34) to handle the reserve aspect, have been establishd.

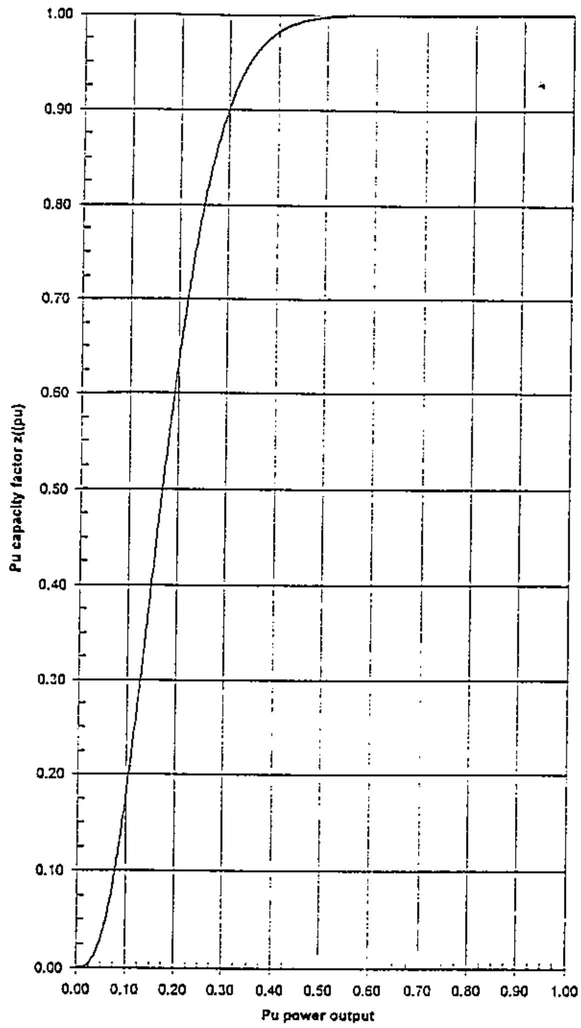


Figure 4.3 : Continous variable z(i,t) to replace integer variable Z(i,t), for generator 'i', hour t

Introducing (4.30) into (4.29) , and extending the description to cover all time intervals ,we get the following reserve constraints to observe , - now solely in terms of continous variables:

$$\sum_{i=1}^{ng} Pg(i,t) \leq \sum_{i=1}^{ng} [CC(i,t) \cdot z(i,t)] \qquad t=1,2,\ldots nt \qquad (4.31)$$

Where:

$$CC(i,t) = Pgmax(i,t)/Kr(t) \qquad (4.31a)$$

z(i,t) = f(P(i,t))= pu continous variable of range 0.0-1.0, to approximate per unit rated capacity of unit 'i' in time interval t. z(i,t) is proposed modelled by (4.30), which takes on the form (4.30a) for chosen set of parameter values (a,b,ς).

(4.31) displays a situation (similar to (4.16)) in that each inequality interlinks a multitude of problem variables. For reasons already explained, new dummy variables are intoduced to replace (4.31) by single-variable inequalities:

First we rearrange (4.31) by moving the right-side term over to the left side, and get:

$$\sum_{i=1}^{ng} [\, Pg(i,t) - CC(i,t) \cdot z(i,t)] \leq 0 \qquad t=1,2,\ldots nt \qquad (4.32)$$

Then we set the left side equal to a new dummy variable PDD(t). In this way we transform (4.32) to an equivalent description comprising nt equalities and nt single variable inequalities. Thus, we finally get the following set of constraints to ensure robust operation of the power system:

$$\sum_{i=1}^{ng} [\, Pg(i,t) - CC(i,t) \cdot z(i,t) \,] = PDD(t) \qquad t=1,2,\ldots nt \qquad (4.33)$$

$$PDD(t) \leq 0 \qquad t=1,2,\ldots nt \qquad (4.34)$$

Where;

Pg(i,t) : Generator production at regional bus 'i', time interval t

CC(i,t) : Coefficient, see (4.31a)

ε(i,t) : " " " " "

PDD(t) : Reserve beyond required (after shift of sign)

Spinning reserve constraints

Remark re. implications of applying z( ) instead of Z( ) :
To each and every equality constraint formulated in the model, the solution will yield an associated Lagrange multiplier reflecting the cost of fulfilling that constraint.- So also will a Lagrange multiplier be associated with the reserve constraint equation (4.33) for each time interval. In the following we discuss qualitatively the solution for an arbitrary chosen time interval:

Assume that (4.34) is registered as non-binding, i.e. PDDt is found less than zero. This implies surplus of reserve in the considered hour, and hence no extra cost associated with providing for reserve; the aboundancy of reserve is so to speak a free by-product of the economic clearing of the market.The associated 'Reserve Lambda' will be zero, signalling this free surplus of reserve. Assume now (for the sake of discussion) that the solution yields a gas turbine up and running in the chosen hour. We then know that not reserve consideration, but the economics of operating the unit has been decisive for loading it up. This kind of insight is to be utilized when preparing premises from one iteration to the next in the process of reaching a final integer solution. See Chapter 5

Assume next that (4.34) is binding, i.e. PDDt is found equal to zero. This implies scarcity of reserve in the considered hour, now signalled by a value greater than 0 for the associated 'Reserve Lambda'. Assume again that the solution yields a gas turbine up and running in the chosen hour – and output is (say) 1.4 MW, which is far below minimum permissible production of (say) 10MW.We now know from the solution that the gas turbine is brought in to supply reserve, and the quantity provided is given by the gasturbine's function z(i,t). The insight commented on here together with use of additional signals that are discussed in Chapter 5, provide a basis for deciding on whether the gas turbine should be offline or online (with production at least 10MW) in the considered hour.

RAMPING CONSTRAINTS

Ramp rate limits restrict the change of generator output from one time period to the next. The practical limits may be set by e.g. physical limitations (as in case of thermal production units), or by regulatory agencies (as in case of some hydro production units).

The ramping constraint is expressed and discussed in Appendix 1.2. The constraint may apply both to increase and decrease of output from generating units.

For thermal units the constraint will normally apply to regimes of increase of production. For many such units the rate limit is in the range 2 – 4% of nominal power output per minute. For these units the ramping constraint will be non-binding, if a time resolution of e.g. 1 hour is chosen,- since power output may reach any desired value within nominal range in less than one hour.

Ramping constraints that limit the rate of increase of production, imply mathematical variables that are already available in terms of Delta(i,j). See eqns. (4.10) – (4.13). To handle 'thermal' ramping, it only remains to individualize the limitation of Delta(i,j), which presently is set to default value 1.0 for all units in all time intervals. (The default value is the proper value to apply when the only function of Delta(i,j) is monitoring/controlling the cost of starts)

For hydro units ramping constraints may in the most complex cases apply 'both ways'. To cover also limits on the rate of reduction of power production, new sets of variable similar to Delta(i,j) and $\delta D(i,t)$ will have to be defined.

In the present scheme of analysis time resolution is chosen to be one hour. This causes ramping to most likely be a phenomenon of less prominence to our defined market clearing simulation task. Ramping constraints are therefore at present disregarded.

SUMMARY PROBLEM FORMULATION

In the course of present chapter 4.4.2, mathematical expressions have been established to describe the task of modelling of Market Driven Hydro-Thermal Scheduling. Final descriptions have been placed in boxes, the aggregate of which provides for modelling of target power system performance.

For overview purposes final descriptions are compiled and presented in Figure 4.4. In consistency with the generic scheme of (4.2), the concrete model comprises three main challenges to be dealt with jointly:

Maximizing the objective of operation, which is the systemwide sum of producer- and consumer surplus over an horizon of (say)168h. The objective comprises up to second order polynomials in power production- and power demand variables, and lends itself well for Newtonian optimization.

Handling of equality constraints that are introduced to deal with cost of start- , electrical process- , generation volume- , and system reserve restrictions. Many of these constraints are complex in the sense that they are nonlinear and interlinks multiple variables. Equality constraints- however complex – are effectively handled via Lagrange Multipliers in the context of Newtonian optimisation .

Handling of inequality constraints that are introduced to limit the operating range of explicitly as well as implcitly defined variables. All inequality constraints applied in the optimization process are simple in the sense that each of them is linear and relate only to a single variable. Given this constraint characteristic, they are conveniantly dealt with by inverse or logarithmic barrier functions.

$$\text{MAX} \;\{ F(\underline{x}) = \text{Utility} - \text{Fuelcost} - \text{Startcost} \}$$ [NOK/week]

Where;

$F(\underline{x})$ = Systemwide sum of producer- and consumer surplus over (typically) 168h [(NOK/week]

$$\text{Utility} = \sum_{i=1}^{ng} \sum_{t=1}^{nt} [\, \alpha(i,0,t) + \alpha(i,1,t)\cdot Ps(i,t) - \alpha(i,2,t)\cdot Ps(i,t)^2 \,]$$

$$\text{Fuelcost} = \sum_{t=1}^{nt} \sum_{j=1}^{ng} [\, \gamma(j,1,t) + \gamma(j,2,t)\cdot Pg(j,t) + \gamma(j,3,t)\cdot Pg(j,t)^2 \,]$$

$$\text{Startcost} = \sum_{j=1}^{ng} \sum_{t=1}^{nt} \text{Cstart}(j,t)\cdot \text{Delta}(j,t)$$

Criterion F(x) and associated capacity constraints

s/t:

$Psmin(i,t) \le Ps(i,t) \le Psmax(i,t)$ i=1,2,...ns   t=1,2,...nt

$Pgmin(j,t) \le Pg(j,t) \le Pgmax(j,t)$ j=1,2,...ng   t=1,2,...nt

$Pg(j,t-1)^{\mu} - pg(j,t)^{\mu} + Delta(j,t) = \delta D(j,t)$ t=1,2,...(nt+1)   j=1,2,...ng

$\delta D(j,t) \ge 0.0$ t=1,2,...(nt+1)   j=1,2,...ng

$Delta(j,t) \ge 0.0$ t=1,2,...(nt+1)   j=1,2,...ng

$0 < \mu \le 1.0$

Constraints associated with modelling of cost of start

Local power balances:

$P(j,t)+Ps(j,t)-Pg(j,t)=0$ t=1,2,...nt   j=1,2,...ng

Global power balances:

$Pl(t) + \sum_{i=1}^{ng} \sum_{j=1}^{ng} P(i,t)\cdot B(i,j,t)\cdot P(j,t) - \sum_{j=1}^{ng} P(j,t) = 0$ t=1,2,...nt

Electrical process constraints

$\sum_{t=1}^{nt} \sum_{k=0}^{2} C(i,k,t)\cdot Pg(i,t)^k = PgD(i)$ i =1,2,...ng

$Wmin(i) \le PgD(i) \le Wmax(i)$ i =1,2,...ng

Volume constraints on generation

$\sum_{i=1}^{ng} [\, Pg(i,t) - CC(i,t)\cdot z(i,j) \,] = PDD(t)$ t=1,2,...nt

$PDD(t) \le 0$ t=1,2,...nt

Spinning reserve constraints

Figure 4.4 Summary problem formulation comprising sets of complex equality constraints and simple inequality constraints

4.3 Problem solution

4.3.1 Solution strategy

Practical solution to the task of optimally matching power supply and demand over the week, is found via processes of optimization on two main levels of analysis: On level I) mathematical methods that involve only continous variables, are applied to generate an initial (and most likely) overcommitted committment plan. Level II) comprises a decommittment process that aims at targeting *in* on a close to optimal final plan for matching of electrical supply and demand over the week:

Level I analysis: Generation of initial (overcommitted) committment plan

The analysis comprises two distinct steps: Evaluation of the best possible *continous variable* production schedule for the week, followed by the definition of an initial (and most likely) overcommitted committment plan based on the established *continous variable* solution:

- *The best continous variable solution for matching of power supply and demand*

This theoretical solution accounts in an approximate and iteratively determined way, for all the effects listed above in characterizing the scope of *Market driven Hydro-Thermal Scheduling.*

At start of the iterative process nominal startcosts and best estimates of 'specific cost descriptions' of all production units are applied.The latter description allows for production in the theoretical range zero to maximum output for respective units. Within this range, specific production cost is defined as a linear function of P_{gen} and chosen close to constant. At the end of the optimization process the 'specific cost description' of respective generator units, should be consistent with the actual specific production cost figure for respective units. By allowing for production also over the theoretical range from minimum permissible production and down to zero output, a systematic handling of the tradeoff between cost of start and cost related to fuel and/or volume constraints, may be brought into the total market clearing context. Details on generator modelling are given in App. 1.2 . The further analysis task is as follows:

1) *Solve the formal problem* with current costs of start and current specific cost descriptions of generator units. Solution of this (often) large-scale, nonlinear, contionous variable problem is afforded by a version of Newton's method. Equality constraints are dealt with by Lagrange Multipliers, and inequality constraints are all internalized in the formal process via logarithmic or inverse barrier functions.

2) *Check for feasibility* with respect to handling of cost of start and efficiency description of generators. If ok, go to next task which is definition of an initial committment plan. If the cost of start for a given 'startinvolved' generator is not properly set, its formal Startcost is adjusted as indicated above. In a corresponding way the efficiency modelling of a generator may have to be modified, if current modelling is not consistent with current production level of the unit. After implementing proper adjustments, return to 1).

- *The initial (overcommitted) committment plan*

The established formal solution based on continous variables only, forms the basis for defining an initial committment plan. Decision logic in relation to each generator unit:

If $P_g \geq$ (say) $0.5 \cdot P_{gmin}$:Unit is tentatively online in considered hour
If $P_g <$ (say) $0.5 \cdot P_{gmin}$:Unit is offline, unless required to sustain the power balance.

Level II analysis: The decommittment process

On this level each committed generating units is described by its correct/ 'instantaneous' efficiency curve valid for the actual output range from $P_{g(min)}$ to $P_{g(max)}$. For this initial committment schedule the associated optimal unit dispatch is found from optimizing the matching of power supply and demand over the week, considering all the constraints that are commented on earlier. Solution to this task is again afforded by Newton's method.

The objective of the ensuing analysis is to approach on final committment plan and associated optimal power market clearing, by investigating – for each prospective hour – which unit(s) (if any) should be disconnected in that hour. For each prospective hour to consider, decision analysis is based on the following problem formulation:

A generator unit that a) produces at P_{min} or close to this limit (say less than $1.1 \cdot P_{min}$), and b) is not required to be online for feasibility of supply reasons,- is in principle to be considered a candidate for disconnection. Disconnection of an 'eligible' unit is made, if the unit-related money saved by disconnecting that unit, exceeds the systems related costs incurred by alleviating the loss of that same unit.

By including the effect of cost of start when relevant, the decision logic will also identify which generator units (if any) should not at all have been committed over the characteristic cycle of time at hand. Implementation of the decision logic results in principle in a stepwise process, since incremental cost information from current solution, is being used as part of decision basis. The decommittment logic is dealt with in App. 1.3.

4.3.2 The formal solution process of 'Level I'

OPTIMIZATION METHOD
The method applied may be termed Newton's method. The concept is very simple, but it allows for strong convergence properties as second derivative information is effectively used in the search for optimum. Formal basis for the method is briefly outlined in the following:

Given an unconstrained objective function of n real (or continous) variables

$$f(x_1,x_2,x_3,\ldots x_n) = f(\underline{x}) \tag{4.35}$$

The column vector $\underline{x}$ is the point with coordinates $(x_1,x_2,x_3,\ldots.x_n)$ in n-dimensional Euclidean space. A vector x^* that minimizes $f(\underline{x})$ is sought. The minimum may be a local minimum, or it may (hopefully) be the global minimum.

The gradient of $f(\underline{x})$, i.e. the vector with components $(\partial f/\partial x_1, \partial f/\partial x_2, \ldots.. \partial f/\partial x_n)$ is denoted by $\nabla f(\underline{x})$ (or sometimes $g(\underline{x})$).

The Hessian matrix of $f(\underline{x})$ is denoted by $G(\underline{x})$, and is the symmetric n n matrix with elements

$$Gij = \partial^2 f/(\partial x_i \partial x_j) \tag{4.36}$$

Assuming certain differentiability properties of the objective function , $f(\underline{x})$ may in general be modelled in the vicinity of a chosen point x_0, by a Taylor series expansion :

$$f(\underline{x}) = f(\underline{x}_0) + (\underline{x}-\underline{x}_0)^t \cdot \nabla f(\underline{x}_0) + (1/2!) \cdot (\underline{x}-\underline{x}_0)^t \cdot G(\underline{x}_0) \cdot (\underline{x}-\underline{x}_0) + \ldots\ldots.. \tag{4.37}$$

Starting from xo and the series expansion that comprises an infinite number of terms, a new and reduced objective function value can in principle be attained, by properly utilizing derivative information from the right side of (4.37), in moving to a new solution point .

In practise the Taylor series has to be truncated, and we here hypothesize that the three first terms of the right side of (4.37), be adequate for describing $f(\underline{x})$ in practical vicinity of $\underline{x}^*$. Accordingly, we define a new truncated function $\varphi(\underline{x})$ in this way:

$$\varphi(\underline{x})=f(\underline{x}_0)+(\underline{x}-\underline{x}_0)^t \cdot \nabla f(\underline{x}_0)+(1/2!) \cdot (\underline{x}-\underline{x}_0)^t \cdot G(\underline{x}_0) \cdot (\underline{x}-\underline{x}_0) \tag{4.38}$$

It flows from the foregoing that a process that minimizes $\varphi(\underline{x})$, will yield a reasonable approximation to the solution point $\underline{x}^*$ that minimizes $f(\underline{x})$.

Provided $G(\underline{x})$ is positive definite within the solution space of interest, minimum of $\varphi(\underline{x})$ is found by utilizing the necessary extremum condition $\nabla\varphi(\underline{x})=0$: Taking the derivative of (4.38), we get the following system of linear equations ;

$$\nabla f(\underline{x}_0) + G(\underline{x}_0)\cdot(\underline{x}-\underline{x}_0) = 0 \qquad (4.39)$$

which solved with respect to $\underline{x}$, gives

$$\underline{x} = \underline{x}_0 - G^{-1}(\underline{x}_0)\cdot\nabla f(\underline{x}_0) \qquad (4.40)$$

The concept of Newton optimization is readily grasped from (4.40) : From a given starting point $\underline{x}_0$, an improved solution vector $\underline{x}$ is computed from (4.40). If $\underline{x}$ is sufficiently close to $\underline{x}^*$, the solution is found. If exit condition is not fulfilled, (4.40) is solved again using the latest computed point $\underline{x}$ as current starting point $\underline{x}_0$, and so on .

Practical large-scale Newton optimization cannot be based on algorithm (4.40), - however conceptually attractive the algorithm may be: Matrix inversion is in this case an unnecessary cumbersome process that most probably (both from the point of view of computer execution time and memory use) will prohibit solution of commercial-sized optimization tasks.

For effective optimization we shall instead focus on the direct solution of (4.39), observing that $G(\underline{x}_0)$ inherently is a very sparse matrix. Introducing $\Delta\underline{x}=(\underline{x}-\underline{x}_0)$, and rearranging (4.39) , we get the following systems of equations to replace (4.39) :

$$\mathbf{G(\underline{x}_0)\cdot\Delta\underline{x} = -\nabla f(\underline{x}_0)} \qquad (4.41)$$

$$\mathbf{\underline{x} = \underline{x}_0 + \Delta\underline{x}} \qquad (4.42)$$

Basic Newton optimization algorithm

Algorithms (4.41)-(4.42) represent the nucleus of the inner/formal optimization process of the present scheme of analysis. The basic solution process is as follows:

1) Compute non-zero elements of Hessian $G(\underline{x}_0)$, and elements of gradient $\nabla f(\underline{x}_0)$. Apply sparse matrix technique and solve (4.41) for increments $\Delta\underline{x}$ of the problem variables.

   The sparsity of G( ) can be illustrated by an example: In section 4.2.1 the number of variables and constraints were indicated for a hydro-thermal power system comprising 7 thermal production units, 23 hydro units, and 30 local power markets to clear hour-by-hour over a week. The no. of nonzero elements of G() in this case is about 70000, whereas total number of elements of the upper (or lower) triangular part of G( ) is 35000*(35000+1)*0.5 = 612517500. Thus, only 70000*100/612517500 = 0.01% of the elements of G( ) have a nonzero value.

2) Compute new point $\underline{x}$ from (4.42), and check for exit from formal optimization process. If exit conditions not fulfilled, return to 1) with current point $\underline{x}$ to apply as $\underline{x}_0$ in (4.41).

Two conditions are presently required fulfilled before exit from computation is allowed: each elements of the gradient vector must be sufficiently close to nil, and so also the improvement of the criterion value from one iteration to the next.

HANDLING OF EQUALITY CONSTRAINTS

Equality constraints can be handled in different ways: E.g. via substitution processes, penalty functions, or Lagrange multipliers. The latter method is in many cases the most efficient, provided a sufficiently convergence-strong scheme of minimization (or maximization) is used. (Applying Lagrange multiplires implies finding a stationary point that identifies a saddle point type solution, and this is inherently more demanding convergence-wise, than e.g. merely locating a functional minimum)

Present scheme of analysis applies the method of Lagrange multipliers, - the concept of which now is introductory explored via formally solving a minimization problem comprising two variables and one equality constraint:

$$\min_{(x_1,x_2)} \{ f = f(x_1,x_2) \} \qquad (4.43)$$

s/t:

$$l(x_1,x_2) = 0 \qquad (4.44)$$

At optimum the differential of function f( ) as well as function l( ) must be zero:

$$df = (\partial f/\partial x_1)\cdot dx_1 + (\partial f/\partial x_2)\cdot dx_2 = dx_1\cdot[\,\partial f/\partial x_1 + (\partial f/\partial x_2)\cdot(dx_2/dx_1)\,] = 0 \qquad (4.45)$$

$$dl = (\partial l/\partial x_1)\cdot dx_1 + (\partial l/\partial x_2)\cdot dx_2 = 0 \qquad (4.46)$$

(4.46) is solved with respect to the quotient (dx_2/dx_1) :

$$dx_2/dx_1 = -(\partial l/\partial x_1)/(\partial l/\partial x_2) \qquad (4.47)$$

and this quotient is inserted into the right side equation (4.45), giving;

$$\partial f/\partial x_1 + [\,-(\partial l/\partial x_1)/(\partial l/\partial x_2)\,]\cdot(\partial f/\partial x_2) = 0 \qquad (4.48)$$

):

$$\partial f/\partial x_1 + [\,-(\partial f/\partial x_2)/(\partial l/\partial x_2)\,]\cdot(\partial l/\partial x_1) = 0 \qquad (4.49)$$

The expression in square bracket of (4.49), is by definition set equal to a new variable λ :

$$\lambda = -(\partial f/\partial x_2)/(\partial l/\partial x_2) \qquad (4.50)$$

Equation (4.49) with lambda introduced in it , together with (4.50) rewritten, and the originally specified restriction (4.44) , - yield the following three user oriented equations for solving of the originally stated problem (4.43)-(4.44):

$$\boldsymbol{\partial f/\partial x_1 + \lambda\cdot\partial l/\partial x_1 = 0} \qquad (4.51)$$

$$\boldsymbol{\partial f/\partial x_2 + \lambda\cdot\partial l/\partial x_2 = 0} \qquad (4.52)$$

$$\boldsymbol{l(x_1,x_2) = 0} \qquad (4.53)$$

We notice the following fundamental features of (4.51)-(4.53), relative to initial problem formulation (4.43)-(4.44) : The original problem comprising 2 variables plus 1 restriction, has been reformulated to a restrictionfree problem comprising 3 equations for solving 2+1=3 variables.

The third variable being the lambda. Introducing the concept of a Lagrangian function $\Im(\,)$, we may write (4.51)-(4.53) in the following compact way:

$$\underset{(x_1,x_2,\lambda)}{\text{Opt}} \; \{ \Im(x_1,x_2,\lambda) = f(x_1,x_2) + \lambda\cdot l(x_1,x_2) \} \qquad (4.54)$$

The necessary condition for a stationary point of the (unconstrained) Lagrangian function $\Im$, is that the gradient of $\Im(\,)$ is zero: Taking the partial derivatives of (4.54), we arrive at the equations (4.51)-(4.53), - which demonstrates the validity of the established, compact problem formulation (4.54). (In (4.54) the abbreviation 'Opt' (for 'Optimize') is used instead of 'Min' (for 'Minimize'), since the task now is to find a stationary point in n-dimensional Euclidean space characterized by a non-definite Hessian $G(\underline{x})$, rather than a functional minimum conditioned by a positive definite Hessian).

Equation (4.54) which relates to 2 variables and 1 equality constraint, can easily be extended to cover the general case of n variables and k equality constraints:Each such constraint brings in 'its own' multiplier term additively into the Lagrangian function, extending (4.54) to the following general and conveniantly applicable form:

$$\underset{(\underline{x},\underline{\lambda})}{\textbf{Opt}} \; \{ \Im(\underline{x},\underline{\lambda}) = f(\underline{x}) + \sum_{j=1}^{k}(\lambda_j \cdot l_j(\underline{x})) \} \qquad (4.55)$$

where;

$\Im(\underline{x},\underline{\lambda})$ = defined Lagrangian function
$f(\underline{x})$ = objective function to be minimized in view of constraints
$lj(\underline{x})$ = equality constraint 'j'. j= 1,2,...k
$\underline{x}$ = problem vaiables $x_1, x_2, x_3, \ldots x_n$
λ_j = Lagrange multiplier associated with equality constraint 'j'

Equality constraints handled via Lagrange multipliers. Algorithmic basis.

An original problem comprising n variables and k equality constraints, is transformed into a restrictionfree equivalent problem comprising (n+k) variables.

HANDLING OF INEQUALITY CONSTRAINTS

All inequalities dealt with in the following are linear and each of them relate to only one problem variable. We can restrict ourself to this case, since complex inequalities are eliminated by the feature of introducing appropriate dummy variables. This is dealt with in Section 4.2.2.

Simple inequality constraints can be handled in different ways: E.g. via heuristics, transformation of variables, or barrier functions.

Treating inequalities via heuristics means surveilling the 'inequality picture' during the solution process, and interfering appropriately when constraints change from nonbinding to binding status and/or vice versa. The goodness of this solution strategy which is widely applied within electrical power system analysis , depends strongly on the quality of the expert type logic built into the scheme of optimization. From a mathematical point of view the method is in principle less attractive, since there is not a strict/formal process toward optimality of solution.

A process of variable transformation can in principle internalize the concern of inequalities. To illustrate: Defining a new variable u via the transformation $x=x(min)+[x(max) - x(min)]\cdot \sin(u)^2$, will automatically secure feasible values of x (in the range x(min) to x(max)) , regardless of the value of u . Own experience seem to disclose that such transformations are prone to have detrimental effects on convergence – particularly in large-scale applications.

Present scheme of analysis applies interior barrier function methods. By 'interior' is understood that feasibility is always retained during the process of solution. Two alternative such schemes have been implemented; one based on inverse- and one on logarithmic modelling. They both work well, and are further outlined in the following.

The general and formal problem met with in our case, can at the outset be stated compactly by (4.56):

$$\min_{\underline{x}} \{ f(\underline{x}) \} \qquad (4.56)$$

s/t:

$$xj \geq xj(min) \qquad (j=1,2,\ldots\ldots m)$$

The basic idea of the barrier concept is to arrive at the solution of (4.56) by carrying through a restrictionfree minimization of a fictitious function

$$\varphi = f(\underline{x}) + P(\underline{x}) \qquad (4.57)$$

$P(\underline{x})$ is a penalty term so defined that minimum of (4.57) during an iterative solution process, converges toward the minimum implied by (4.56).To highlight the barrier concept, we start by delving into a small example problem that is handled first by inverse- and next by logarithmic barrier logic:

The concrete problem:

$$\min_{x} \{ f(x)=x \} \qquad (4.58)$$

s/t:

$$x \geq 3$$

The solution is apparently $f(x^*)=3$, for $x^*=3$

Applying the inverse modelling approach we define the penalty term $P = r/(x-3)$, where $x\geq 3$ and $r>0$ We then arrive at the following φ-function (4.57) :

$$\varphi(x,r) = x + r/(x-3) \tag{4.59}$$

which is shown in Figure 4.5 for 5 alternative values of parameter r.

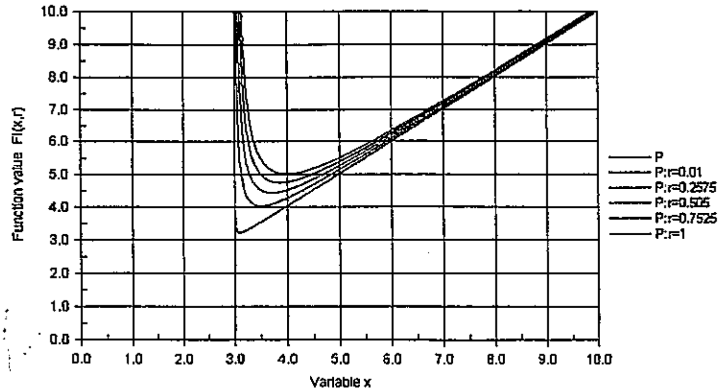


Figure 4.5 : Inverse barrier function $\varphi = x+r/(x-3)$, for 5 different values of r in the range 0.01-1.00

We see from the figure that as r becomes smaller, the minimization of the restrictionfree barrier function (4.59), predicts a consistently better estimate of the solution x* implied by the stated problem (4.58).

The validity of the concept rests on the premise that the penalty term P=r/(x-3) will vanish as r reduces to nil, even if the denominator in that process approaches zero value (as in present case with x*=3) . Our simple example allows us – on a formal basis -- to pursue the discussion of approaching limits : Minimizing (4.59) while treating r as a constant, we find the conditions:

$$\partial\varphi/\partial x = 1 - r/(x-3)^2 = 0 \qquad \text{and} \qquad \partial^2\varphi/\partial x^2 = 2\cdot r/(x-3)^3$$

From the left equation we find the solutions $x^*=3 \pm \sqrt{r}$. Using the sign of the second derivative, it is seen that plus sign provides for the sought minimum of $\varphi(x,r)$. We notice that x is within feasible range for all values r, and attains optimal value for r=0. This confirms the validity of the presumed penalty term of (4.59).

Fiacco and McCormick [3] have done fundamental research on this methodology which is also denoted *Sequential Unconstrained Minimization Technique ('SUMT')*, and have proved its general validity. On this basis we can extend our specific/intuitive function (4.59) to handle any set of inequalities in optimization:

Handling of inequalities is generally afforded via solving of the following restrictionfree inverse barrier problem:

$$\min_{\substack{\underline{x} \\ r\to 0}} \left\{ \varphi(\underline{x},r) = f(\underline{x}) + r\cdot\sum_{i=1}^{m} 1/(x_i - x_i(\lim)) \right\} \tag{4.60}$$

Inequality constraints handled via inverse barrier function.

An original problem comprising n variables and m inequalities, is transformed into a restrictionfree n-variable problem to be solved sequentially with diminishing values of penalty factor r

(4.60) is valid both for inequalities of the type 'greater or equal' (which was the type leading to formulation (4.60)), and inequalities of the type 'less or equal':

For inequalities of type 'x>x(min)': (4.60) is valid with x(lim)=x(min) in the denominator of the penalty term .

For inequalities of type 'x<x(max)' : Adapt to the premises for (4.60) by multiplying the inequalities by −1 , getting (-x) > (-x(max)). Thus, (4.60) is valid when setting x(lim)=x(max), and shifting sign of the denominator of the penalty term.

Applying the logarithmic modelling approach we define the penalty term P= -r·ln(x-3), where x>3. We then arrive at the following φ-function (4.57) :

$$\varphi(x,r) = x - r\cdot\ln(x-3) \qquad (4.61)$$

(Comment in paranthesis to the form of the penalty term P : For values of (x-3) less than 1 and moving towards zero, ln(x-3) is negative and increases toward infinity in (negative) value. The negative sign chosen for P, is introduced to compensate for this negative sign of ln(x-3). To secure valid performance of the penalty term in practical applications, the argument to ln has to be scaled to a value Z , where $0<Z\leq 1$)

The φ-function (4.61) is shown in Figure 4.6 for 5 alternative values of parameter r.

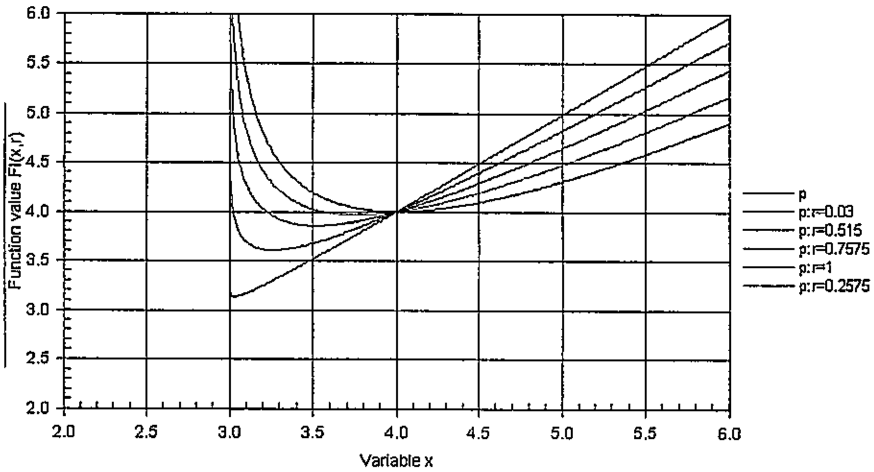


Figure 4.6 :Logarithmic barrier function φ = x - r·ln(x-3), for 5 different values of r in the range 0.03 – 1.00

We see again that as r becomes smaller, the minimization of the restrictionfree barrier function (4.61) predicts a steadily better estimate of the solution x* implied by the stated problem (4.58) We also (vaguely) perceive that the logarithmic strategy may provide a more efficient process toward optimum of x , than the inverse barrier strategy will do.-

Our simple example again permits us to evaluate the formal solution in terms of r : Minimizing (4.61) with respect to x :

$$\partial\varphi/\partial x = 1 - r/(x-3) = 0\,, \quad \text{giving} \quad x = 3+r$$

We notice that x is within feasible range for all permissible values of r , and attains optimal value for r=0. This confirms the validity of the presumed logarithmic penalty term of (4.61).

The interior logarithmic barrier methodology was (most probably) first suggested by the Norwegian economist and Nobel Laureate Ragnar Frisch [5] , and its general applicability has since been well established.

In summary, handling of inequalities is generally afforded via solving of the following restrictionfree logarithmic barrier problem:

$$\underset{\substack{\underline{x} \\ (r \to 0)}}{\text{Min}} \; \{ \; \varphi(\underline{x},r) = f(\underline{x}) - r \cdot \sum_{i=1}^{m} \ln(D_i) \; \} \qquad (4.62)$$

where;

$D_i = x_i(pu) - x_{i(min)}(pu)$, if the inequality is $x_i \geq x_{i(min)}$
$D_i = x_{i(max)}(pu) - x_i(pu)$, if the inequality is $x_i \leq x_{i(max)}$

The subscript '(pu)' indicates scaling of variable

Inequality constraints handled via logarithmic barrier function

An original problem comprising n variables and m simple inequalities, is transformed into a restrictiongfree n-variable problem to be solved sequentially with decrementing values of scalar penalty factor r

CONSTRAINT-FREE SOLUTION PROCESS

From the foregoing mathematical outline it is evidenced that minimization of a problem comprising n variables , k equalities and an arbitrary number of of inequalities , can be afforded sequentially in terms of a restrictionfree problem that comprises (n+k+v) variables. v is the number of 'complex' inequalities , i.e. inequalities that are nonlinear and/or involve more than one variable. (The v 'complex' inequalities are eliminated by defining v new 'dummy' variables, see e.g. p. 14.)

Given that all complex inequalities have been eliminated, the total restrictionfree minimization problem to be solved sequentially by Newton's method, can be summarized as follows, when *the inverse barrier strategy* is chosen for dealing with the inequalities:

$$\underset{\substack{(\underline{x},\underline{\lambda},r) \\ (r \to 0)}}{\text{Opt}} \; \{ \; \varphi(\underline{x},\underline{\lambda},r) = f(\underline{x}) + \sum_{j=1}^{(k+v)} (\lambda_j \cdot l_j(\underline{x})) + r \cdot \sum_{i=1}^{m} (1/D_i) \; \} \qquad (4.63)$$

where;

$\varphi(\underline{x},\underline{\lambda},r)$: 'Extended Lagrange function' to handle equality- as well as inequality constraints
$\underline{x}$: problem variables $x_1, x_2, x_3, \ldots x_{(n+v)}$
r : scalar factor to be decremented in sequential process
$f(\underline{x})$: given objective function
λ_j : Lagrange multiplier associated with equality constraint 'j'
$l_j(x)$: equality constraint 'j' . j=1,2,3,....(k+v)
m : number of inequalities
D_i : $(x_i - x_{i(min)})$, if the inequality is $x_i \geq x_{i(min)}$
D_i : $(x_{i(max)} - x_i)$, if the inequality is $x_i \leq x_{i(max)}$

Scheme for restrictionfree solution of the complete inner/formal optimization problem. Sequential process. Inequalities handled via inverse barrier strategy

If – alternatively – the logarithmic barrier strategy is applied for dealing with inequality constraints , the scheme of (4.63) will be replaced by (4.64) :

$$\underset{\substack{(\underline{x},\underline{\lambda},r)\\(r\to 0)}}{\textbf{Opt}} \left\{ \varphi(\underline{x},\underline{\lambda},r) = f(\underline{x}) + \sum_{j=1}^{(k+v)} (\lambda_j \cdot l_j(\underline{x})) - r \cdot \sum_{i=1}^{m} \ln(D_i) \right\} \qquad (4.64)$$

where;

D_i is defined in connection with equation (4,62), and the rest of the quantities with (4.63)

Scheme for restrictionfree solution of the complete inner/ formal optimization problem. Inequalities handled via logarithmic barrier strategy.

The complete inner/formal solution process based on (4.63) or (4.64) , can be summarized as follows :

1. **Initialization.** Set r and vectors $\underline{x}$ and $\underline{\lambda}$ to initial values

2. **Decrement r.** E.g. by setting new r = current r divided by Kr (= (say)10)

3. **Problem solution for given r.** Gradient $\nabla\varphi(\underline{x}_0,\underline{\lambda},r)$ and Hessian $G(\underline{x}_0,\underline{\lambda},r)$ is computed from (4.63) or (4.64) . New and improved solution vector found by applying Newton's algorithm (4.41) - (4.42). See p.xx for details on this algorithmic process.

4. **Test for exit.** Is penalty term (associated with inequalities) below specified bound ? If 'yes' , the solution is reached. If 'no' , go to 2.

Basis for evaluating gradient $\nabla\varphi(\underline{x}_0,\underline{\lambda},r)$ and Hessian $G(\underline{x}_0,\underline{\lambda},r)$, is the 'Extended Lagrange function ' $\varphi(\underline{x},\underline{\lambda},r)$ of (4.63) or (4.64) .

The concrete algorithmic content of $\varphi(\underline{x},\underline{\lambda},r)$ is illustrated in the following, choosing (arbitrarely) (4.63) as basis for optimization. I.e. we choose in the following demonstration to deal with all inequality constraints via inverse barrier formulation.

We introduce earlier developed expressions for criterion $f(\underline{x})$, equality constraints $l_j(\underline{x})$ and inequalty terms D_i , into (4.63) and get :

$$\varphi(\underline{x},\underline{\lambda},r) = \sum_{i=1}^{ng}\sum_{t=1}^{nt}[\ \alpha(i,0,t)+\alpha(i,1,t)\cdot Ps(i,t)-\alpha(i,2,t)\cdot Ps(i,t)^2\ ] \qquad \text{(Utility)}$$

$$-\sum_{j=1}^{ng}\sum_{t=1}^{nt}[\ \gamma(j,1,t)+\gamma(j,2,t)\cdot Pg(j,t)+\gamma(j,3,t)\cdot Pg(j,t)^2\ ] \qquad \text{(Fuelcost)}$$

$$-\sum_{j=1}^{ng}\sum_{t=1}^{nt} Cstart(j,t)\cdot Delta(j,t) \qquad \text{(Startcost)}$$

$$+\sum_{j=1}^{ng}\sum_{t=1}^{(nt+1)}\lambda_\delta(j,t)\cdot[\ pg(j,t-1)^\mu - pg(j,t)^\mu + Delta(j,t) - \delta D(j,t)\ ] \qquad \text{('start-constraints.')}$$

$$+\sum_{t=1}^{nt}\sum_{j=1}^{ng}\lambda(j,t)\cdot[\ P(j,t)+Ps(j,t)-Pg(j,t)\ ] \qquad \text{(local power balances)}$$

$$+\sum_{t=1}^{nt}\lambda_{syst}(t)\cdot[\ Pl(t)-P(j,t)+\sum_{i=1}^{ng}\sum_{j=1}^{ng}P(i,t)\cdot B(i,j,t)\cdot P(j,t)\ ] \qquad \text{(global power balances)}$$

$$+\sum_{j=1}^{ng}\lambda_w(j)\cdot[\sum_{t=1}^{nt}\sum_{k=0}^{2}(C(j,k,t)\cdot Pg(j,t)^k) - PgD(j)\ ] \qquad \text{(volume constraints)}$$

$$+\sum_{t=1}^{nt}\lambda_{reserv}(t)\cdot[\ \sum_{i=1}^{ng}[Pg(i,t)-CC(i,t)\cdot z(i,j)] - PDD(t)\ ] \qquad \text{(security constraints)}$$

(4.65)

$$+\sum_{t=1}^{nt}\sum_{j=1}^{ng} r\cdot[\ 1/(Pgmax(j,t)-Pg(j,t))+1/(Pg(j,t)-Pgmin(j,t))\ ] \qquad \text{(generator. capacity constraints)}$$

$$+\sum_{t=1}^{nt}\sum_{j=1}^{ng} r\cdot[\ 1/(Psmax(j,t)-Ps(j,t))+1/(Ps(j,t)-Psmax(j,t))\ ] \qquad \text{(spot capacity constraints)}$$

$$+\sum_{i=1}^{ng} r\cdot[\ 1/(Wmax(i)-PgD(i)) + 1/(PgD(i)-Wmin(i))\ ] \qquad \text{(volume constraints on generation)}$$

$$+\sum_{j=1}^{ng}\sum_{t=1}^{(nt+1)} r\cdot[\ 1/(Deltamax - Delta(j,t)) + 1/(Delta(j,t)-Deltamin)\ ] \qquad \text{(Delta-constraints, start aspect)}$$

$$+\sum_{j=1}^{ng}\sum_{t=1}^{(nt+1)} r\cdot[\ 1/(\ \delta Dmax-\delta D(j,t)) + 1/(\ \delta D(j,t)-\delta Dmin)\ ] \qquad \text{(constraints on dummy variables,start aspect)}$$

$$+\sum_{t=1}^{nt} r\cdot[\ 1/(PDDmax-PDD(t)) + 1/(PDD(t)-PDDmin))\ ] \qquad \text{(constraints on dummu variables, reserve aspect)}$$

' Extended Lagrange Function ' $\varphi(\underline{x},\underline{\lambda},r)$ **as given by (4.63)**

$\varphi(\underline{x},\underline{\lambda},r)$ is basis for computing gradient $\nabla\varphi(\underline{x},\underline{\lambda},r)$ and Hessian $G(\underline{x},\underline{\lambda},r)$.
The inverse barrier function strategy chosen for dealing with inequality constraints

PROBLEM VARIABLES AND THEIR MEMORY ALLOCATION

The x-vector of $\varphi(\underline{x},\underline{\lambda},r)$ will comprise the following variables , with size of variable field in parenthesis ;

Pg(i,t) : Generator production (ng·nt)
Ps(i,t) : Price-sensitive power delivery (ng·nt)
Delta(i,t) : Variable to describe start cost (ng·(nt+1))
$\delta_D(i,t)$: Dummy variable associated with start of generators (ng·(nt+1))
P(i,t) : Power injected into the transmission system at bus/region (ng·nt)
PgD(i) : Dummy variable associated with volume constraints (ng)
PDD(t) : Dummy variable associated with security constraints (nt)

The λ*-vector of* $\varphi(\underline{x},\underline{\lambda},r)$ will correspondingly comprise the following;

$\lambda(i,t)$: Lambdas associated with local/regional power balances (ng·nt)
$\lambda_{syst}(t)$: " " " global power balances (nt)
$\lambda_\delta(i,t)$: " " " start constraints (ng·(nt+1))
$\lambda_w(i)$: " " " volume constraints (ng)
$\lambda_{reserv}(t)$: " " " reserve constraints (nt)

ALGORITHMIC HANDLING OF INEQUALITY CONSTRAINTS

It is worth noting that very little additional logic and programming is required to include consideration of inequaliry constraints in the optimization , when this is done by the use of interior barrier function logic. The 'inequality contributions' to Nabla ($\nabla\varphi$) and Hessian (G) are found from the first and second derivative respectively, of the Extended Lagrange function :

Assume that both upper and lower limit is to be taken into account for a given variable:

The consequence to Nabla vector ($\nabla\varphi(\underline{x},\underline{\lambda},r)$) is then merely to add to the corresponding variable's vector element, the quantity;

$r\cdot[\ 1/(xmax\text{-}x)^2 - 1(x\text{-}xmin)^2]$ if inverse barrier strategy is chosen, or

$r\cdot[\ 1/(xmax\text{-}x) - 1/(x\text{-}xmin)\ ]$ if logarithmic barrier strategy is chosen.

The consequence to Hessian matrix ($G(\underline{x},\underline{\lambda},r)$) will be to add to the variable's diagonal element, the quantity;

$2\cdot r\cdot[\ 1/(xmax\text{-}x)^3 + 1/(x\text{-}xmin)^3\ ]$ if inverse barrier strategy is chosen, or

$r\cdot[\ 1/(xmax\text{-}x)^2 + 1/(x\text{-}xmin)^2\ ]$ if logarithmic barrier strategy is chosen.

A major reason for using a *logarithmic* barrier strategy in preference for an *inverse* such strategy, would seem to be demonstrated by the exponents of the expressions above: *The lower exponents for the logarithmic algorithms* will secure slower approach toward infinity for the bracketed terms when a variable approaches its limit, - and thus - in relative terms - enhance the required progress toward zero of the resulting product form of the penalty term.

5 EXAMPLE ANALYSES

5.1 Overview

Four cases are dealt with in Chapter 5. The background is two-fold : 1) To demonstrate problem formulation and solution, and 2) to compare results with those obtained from other solution method.

An existing program labeled Market Driven Unit Committment is available to this end [6]. As most computational schemes for Unit Committment, also this program limits the power transmission network to one single bus. To allow for comparison of results, the three first example cases are therefore restricted to being single bus cases. The fourth example is inherently the general multibus case, as it addresses the task of geographically distributed market clearing.

The four example cases dealt with in ensuing separate chapters , are the following :

'Conventional' Unit Committment. Spinning reserve constraints neglected. The system comprises 7 thermal production units to cover a specified load profile over 168 hours.

'Conventional' Unit Committment. Spinning reserve constraints observed. The system is identical to the previous one, but the scheduling task is extended to also consider reserve constraints.

'Conventional' Unit Committment including hydro production. Spinning reserve constraints observed. The previous production system is extended by two hydro production units for which available water volumes are specified for the coming week.

Market driven Hydro-thermal Scheduling. The previous hydro-thermal production scheduling case is extended to include a power tramsmission system and multiple power markets to clear. Spinning reserve constraints are observed.

5.2 'Conventional' Unit Committment. Spinning reserve constraints neglected

5.2.1 Problem formulation

The system comprises 7 thermal production units that are described in overview terms in Table 4.1. For further details on generator- and generator model data, see App. 1.2

Table 4.1 Main data for thermal production units

No	System "unit"	Production range [MW]	Efficiency at Pmax [pu]	Remarks
1	Coal-fired unit. New/very efficient	50 - 200	0,43	Startcost equiv. to P=Pmax in 3,5 h
2	Coal-fired unit. High efficiency	60 - 170	0,40	Startcost: T(equiv)=3.5 h
3	Coal-fired unit. Medium efficient	40 - 120	0,37	T(equiv)=4,0 h
4	Coal-fired unit. Old/inefficient	50 - 150	0,33	T(equiv)=4,0 h
5	Coal-fired unit. Very old	30 - 100	0,30	T(equiv)=4,5 h
6	Gas-fired unit. New combi-type	30 - 60	0,50	T(equiv)=0,6 h
7	Light gas turbine	10 - 50	0,32	T(equiv)=0,1 h

The load is specified for all 168 hours of the week. Each day Monday through Friday is assumed loadwise the same, but different from Saturday and Sunday both days of which are assumed equal. There are two load peaks during the day; one major at noon,and one evening peak. See data input formats in Appendix 1.4 for detailed load specification.
Special point re. modelling of loads: If the load of a given hour is exogenously specified, and total production capacity of the system is less than this load, there will be no feasible solution to the problem- and hence no convergence of the inner/formal optimization process. To secure feasibility of solution (also) in such cases, the 'last' part (here 210 MWh/h) of firm power demand in all hours, is formally treated as price-sensitive with worth of power set far above variable cost of production.Demand will then be covered whenever possible, and curtailed in peri-

ods when demand exceeds total generation capacity. Thus; instead of 'no convergence', we get a feasible solution showing what curtailment has been necessary, in order to retain valid system performance.

The cost of starting thermal production units is assumed independent of the temperature status of the unit. Ramping constraints are not included in this case. Requirements to up- and downtimes of units are neglected (and presumed taken care of by the criterion of operation).

As operational boundary conditions it is specified that production units 1 and 2 are online all the time, unit 3 is online in hour –1 and hour 168.

The task of operation in this case is to decide on unit committment and unit dispatch so that specified demand is covered at minimum cost, taking into consideration previously stated boundary conditions, but disregarding spinning reserve constraints.

5.2.2 Problem solution

Solution is found after two optimization runs on 'Level I analysis' (to generate the initial committment plan), and two optimization runs on 'Level II analysis' (to finalize the decommittment process). The solution's cost criterion is 0.002% above true minimum value found from the reference (shortest path) analysis. App. 1.4 gives further details on the solution process.

Main characteristics of the solution are presented in Figure 5.1- 5.4: Fig. 5.1 gives overview mathematical data. Fig. 5.2 shows how hourly production is distributed among the generators over the week: Fig. 5.2b) also represents Wednesday andThursday. Fig 5.2c) shows Friday, and 5.2d) displays production on Saturday as well as Sunday,- as the two days solutionwise turn out equal. Fig. 5.3 shows how the spinning reserve is distributed on Monday, and Fig. 5.4 displays summary results on the market solution for Monday. Comments by column to the latter results:

- System lambda is hourly incremental power cost as defined by the system power balance constraint.
- Required (MW) reserve is 'empty' since no reserve is specified in this example.
- Actual (MW) reserve is the difference between the production capacity of current combination of units and current sum production from the units.
- Power reserve lambda is in this case (per definition) zero, as no security constraints are formulated.
- The sum of the two last columns constitutes in this case specified load: As commented on earlier, we have for reasons of convergence defined 210MWh/h of the firm load as price-sensitive demand.

Of interest to note solutionwise, relative to the ensuing analysis which observes spinning reserve constraints: The gas turbine is committed for a few hours every workday, and actually loaded up, whereas the bigger/old coalfired unit (no. 5) remains offline throughout the week. Thus – when ignoring spinning reserve constraints – it is here found less expensive to suffer the heavy operational cost of the gas turbine, than the consequences of starting the coalfired unit- even if the latter unit from a running cost point of view, is quite competitive.

```
Minimum found:          f(x)=     -23384.25

Production cost(KKr)      :        8976.96
Spot market income(KKr)   :       32281.21
-------------------------------------------
Minimised criterion(Kkr):         -23384.25

No of production units          :          7
No of hydro units               :          0
No of local power markets       :          7
No of time intervals            :        168
Duration of time intervals (h)  :          1

No of variables                          :     8771
No of inequalities                       :     4893
No of equalities                         :     2702
No of non-zero elements in Hessian       :    16842
No of iterations in r-loop               :        8
Solution time (s)                        :     10.6
```

Figure 5.1 Overview mathematical info on case 'Conventional UC. Spinning reserve neglected'

MW output from resp. plants 1 to 7

t		1	2	3	4	5	6	7
1	< 1>	200.0	100.0	40.0	0.0	0.0	0.0	0.0
2	< 2>	200.0	80.0	40.0	0.0	0.0	0.0	0.0
3	< 3>	200.0	70.0	40.0	0.0	0.0	0.0	0.0
4	< 4>	200.0	70.0	40.0	0.0	0.0	0.0	0.0
5	< 5>	200.0	[illegible]	40.0	0.0	0.0	0.0	0.0
6	< 6>	200.0	170.0	80.0	0.0	0.0	0.0	0.0
7	< 7>	200.0	170.0	91.4	98.6	0.0	0.0	0.0
8	< 8>	200.0	170.0	120.0	130.0	0.0	30.0	0.0
9	< 9>	200.0	170.0	120.0	147.8	0.0	32.2	0.0
10	<10>	200.0	170.0	120.0	150.0	0.0	50.0	0.0
11	<11>	200.0	170.0	120.0	150.0	0.0	60.0	20.0
12	<12>	200.0	170.0	120.0	150.0	0.0	60.0	50.0
13	<13>	200.0	170.0	120.0	150.0	0.0	60.0	50.0
14	<14>	200.0	170.0	120.0	150.0	0.0	60.0	0.0
15	<15>	200.0	170.0	120.0	150.0	0.0	50.0	0.0
16	<16>	200.0	170.0	120.0	150.0	0.0	40.0	0.0
17	<17>	200.0	170.0	120.0	110.0	0.0	30.0	0.0
18	<18>	200.0	170.0	120.0	120.0	0.0	30.0	0.0
19	<19>	200.0	170.0	120.0	140.0	0.0	30.0	0.0
20	<20>	200.0	170.0	120.0	150.0	0.0	50.0	0.0
21	<21>	200.0	170.0	120.0	150.0	0.0	50.0	0.0
22	<22>	200.0	170.0	120.0	147.8	0.0	32.2	0.0
23	<23>	200.0	170.0	113.7	106.3	0.0	0.0	0.0
24	<24>	200.0	64.1	40.0	75.9	0.0	0.0	0.0

a) Generator production Monday

t		1	2	3	4	5	6	7
25	< 1>	170.3	60.0	40.0	69.7	0.0	0.0	0.0
26	< 2>	151.0	60.0	40.0	69.0	0.0	0.0	0.0
27	< 3>	141.4	60.0	40.0	68.6	0.0	0.0	0.0
28	< 4>	141.4	60.0	40.0	68.6	0.0	0.0	0.0
29	< 5>	199.3	[illegible]	40.0	70.7	0.0	0.0	0.0
30	< 6>	200.0	132.2	40.0	77.8	0.0	0.0	0.0
31	< 7>	200.0	170.0	91.4	98.6	0.0	0.0	0.0
32	< 8>	200.0	170.0	120.0	130.0	0.0	30.0	0.0
33	< 9>	200.0	170.0	120.0	147.8	0.0	32.2	0.0
34	<10>	200.0	170.0	120.0	150.0	0.0	50.0	0.0
35	<11>	200.0	170.0	120.0	150.0	0.0	60.0	20.0
36	<12>	200.0	170.0	120.0	150.0	0.0	60.0	50.0
37	<13>	200.0	170.0	120.0	150.0	0.0	60.0	50.0
38	<14>	200.0	170.0	120.0	150.0	0.0	60.0	0.0
39	<15>	200.0	170.0	120.0	150.0	0.0	50.0	0.0
40	<16>	200.0	170.0	120.0	150.0	0.0	40.0	0.0
41	<17>	200.0	170.0	120.0	110.0	0.0	30.0	0.0
42	<18>	200.0	170.0	120.0	120.0	0.0	30.0	0.0
43	<19>	200.0	170.0	120.0	140.0	0.0	30.0	0.0
44	<20>	200.0	170.0	120.0	150.0	0.0	50.0	0.0
45	<21>	200.0	170.0	120.0	150.0	0.0	50.0	0.0
46	<22>	200.0	170.0	120.0	147.8	0.0	32.2	0.0
47	<23>	200.0	170.0	113.7	106.3	0.0	0.0	0.0
48	<24>	200.0	64.1	40.0	75.9	0.0	0.0	0.0

b) Generator production Tuesday

t		1	2	3	4	5	6	7
97	< 1>	170.3	60.0	40.0	69.7	0.0	0.0	0.0
98	< 2>	151.0	60.0	40.0	69.0	0.0	0.0	0.0
99	< 3>	141.4	60.0	40.0	68.6	0.0	0.0	0.0
100	< 4>	141.4	60.0	40.0	68.6	0.0	0.0	0.0
101	< 5>	199.3	60.0	40.0	70.7	0.0	0.0	0.0
102	< 6>	200.0	132.2	40.0	77.8	0.0	0.0	0.0
103	< 7>	200.0	170.0	91.4	98.6	0.0	0.0	0.0
104	< 8>	200.0	170.0	120.0	130.0	0.0	30.0	0.0
105	< 9>	200.0	170.0	120.0	147.8	0.0	32.2	0.0
106	<10>	200.0	170.0	120.0	150.0	0.0	50.0	0.0
107	<11>	200.0	170.0	120.0	150.0	0.0	60.0	20.0
108	<12>	200.0	170.0	120.0	150.0	0.0	60.0	50.0
109	<13>	200.0	170.0	120.0	150.0	0.0	60.0	50.0
110	<14>	200.0	170.0	120.0	150.0	0.0	60.0	0.0
111	<15>	200.0	170.0	120.0	150.0	0.0	50.0	0.0
112	<16>	200.0	170.0	120.0	150.0	0.0	40.0	0.0
113	<17>	200.0	170.0	120.0	110.0	0.0	30.0	0.0
114	<18>	200.0	170.0	120.0	120.0	0.0	30.0	0.0
115	<19>	200.0	170.0	120.0	140.0	0.0	30.0	0.0
116	<20>	200.0	170.0	120.0	150.0	0.0	50.0	0.0
117	<21>	200.0	170.0	120.0	150.0	0.0	50.0	0.0
118	<22>	200.0	170.0	120.0	147.8	0.0	32.2	0.0
119	<23>	200.0	170.0	113.7	106.3	0.0	0.0	0.0
120	<24>	200.0	64.1	40.0	75.9	0.0	0.0	0.0

c) Generator production Friday

t		1	2	3	4	5	6	7
121	< 1>	170.3	60.0	40.0	69.7	0.0	0.0	0.0
122	< 2>	151.0	60.0	40.0	69.0	0.0	0.0	0.0
123	< 3>	141.4	60.0	40.0	68.6	0.0	0.0	0.0
124	< 4>	141.4	60.0	40.0	68.6	0.0	0.0	0.0
125	< 5>	199.3	60.0	40.0	70.7	0.0	0.0	0.0
126	< 6>	200.0	132.2	40.0	77.8	0.0	0.0	0.0
127	< 7>	200.0	170.0	83.9	96.1	0.0	0.0	0.0
128	< 8>	200.0	170.0	120.0	120.0	0.0	0.0	0.0
129	< 9>	200.0	170.0	120.0	130.0	0.0	0.0	0.0
130	<10>	200.0	170.0	120.0	140.0	0.0	0.0	0.0
131	<11>	200.0	170.0	120.0	150.0	0.0	0.0	0.0
132	<12>	200.0	170.0	120.0	145.0	0.0	0.0	10.0
133	<13>	200.0	170.0	120.0	140.0	0.0	0.0	0.0
134	<14>	200.0	170.0	106.3	103.7	0.0	0.0	0.0
135	<15>	200.0	170.0	98.8	101.2	0.0	0.0	0.0
136	<16>	200.0	170.0	61.6	88.4	0.0	0.0	0.0
137	<17>	200.0	170.0	61.6	88.4	0.0	0.0	0.0
138	<18>	200.0	170.0	91.4	98.6	0.0	0.0	0.0
139	<19>	200.0	170.0	98.8	101.2	0.0	0.0	0.0
140	<20>	200.0	170.0	106.3	103.7	0.0	0.0	0.0
141	<21>	200.0	170.0	83.9	96.1	0.0	0.0	0.0
142	<22>	200.0	161.4	40.0	78.6	0.0	0.0	0.0
143	<23>	200.0	103.0	40.0	77.0	0.0	0.0	0.0
144	<24>	200.0	64.1	40.0	75.9	0.0	0.0	0.0

d) Generator production Saturday

Figure 5.2 Summary print of generator production in first example analysis : 'Conventional Unit Committment. Spinning reserve constraints neglected'.

MW reserve from resp. units 1 to 7

t		1	2	3	4	5	6	7
1	< 1>	0.0	70.0	80.0	0.0	0.0	0.0	0.0
2	< 2>	0.0	90.0	80.0	0.0	0.0	0.0	0.0
3	< 3>	0.0	100.0	80.0	0.0	0.0	0.0	0.0
4	< 4>	0.0	100.0	80.0	0.0	0.0	0.0	0.0
5	< 5>	0.0	40.0	80.0	0.0	0.0	0.0	0.0
6	< 6>	0.0	0.0	40.0	0.0	0.0	0.0	0.0
7	< 7>	0.0	0.0	28.6	51.4	0.0	0.0	0.0
8	< 8>	0.0	0.0	0.0	20.0	0.0	30.0	0.0
9	< 9>	0.0	0.0	0.0	2.2	3.0	27.8	0.0
10	<10>	0.0	0.0	0.0	0.0	0.0	10.0	0.0
11	<11>	0.0	0.0	0.0	0.0	0.0	0.0	30.0
12	<12>	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	<13>	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	<14>	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	<15>	0.0	0.0	0.0	0.0	0.0	10.0	0.0
16	<16>	0.0	0.0	0.0	0.0	0.0	20.0	0.0
17	<17>	0.0	0.0	0.0	40.0	0.0	30.0	0.0
18	<18>	0.0	0.0	0.0	30.0	0.0	30.0	0.0
19	<19>	0.0	0.0	0.0	10.0	0.0	30.0	0.0
20	<20>	0.0	0.0	0.0	0.0	0.0	10.0	0.0
21	<21>	0.0	0.0	0.0	0.0	0.0	10.0	0.0
22	<22>	0.0	0.0	0.0	2.2	0.0	27.8	0.0
23	<23>	0.0	0.0	6.3	43.7	0.0	0.0	0.0
24	<24>	0.0	105.9	80.0	74.1	0.0	0.0	0.0

Figure 5.3 Distribution of spinning reserve on Monday. First example analysis: 'Conventional Unit Committment. Spinning reserve constraints neglected'.

t		System Lambda <Kr/Mwh>	Reserve<Mw> Required	Actual	Power reserve Lambda<Kr/Mw & h>	Deliveries<MW> Firm	Flexible
1	< 1>	84.8	0.0	150.0	0.0	130.0	210.0
2	< 2>	84.5	0.0	170.0	0.0	110.0	210.0
3	< 3>	84.4	0.0	180.0	0.0	100.0	210.0
4	< 4>	84.4	0.0	180.0	0.0	100.0	210.0
5	< 5>	85.2	0.0	120.0	0.0	160.0	210.0
6	< 6>	93.4	0.0	40.0	0.0	240.0	210.0
7	< 7>	95.3	0.0	80.0	0.0	350.0	210.0
8	< 8>	110.4	0.0	50.0	0.0	440.0	210.0
9	< 9>	110.9	0.0	30.0	0.0	460.0	210.0
10	<10>	123.1	0.0	10.0	0.0	480.0	210.0
11	<11>	185.9	0.0	30.0	0.0	510.0	210.0
12	<12>	309.1	0.0	0.0	0.0	540.0	210.0
13	<13>	310.0	0.0	0.0	0.0	540.0	210.0
14	<14>	219.0	0.0	0.0	0.0	490.0	210.0
15	<15>	123.1	0.0	10.0	0.0	480.0	210.0
16	<16>	120.8	0.0	20.0	0.0	470.0	210.0
17	<17>	100.7	0.0	70.0	0.0	420.0	210.0
18	<18>	105.6	0.0	60.0	0.0	430.0	210.0
19	<19>	115.2	0.0	40.0	0.0	450.0	210.0
20	<20>	123.1	0.0	10.0	0.0	480.0	210.0
21	<21>	123.1	0.0	10.0	0.0	480.0	210.0
22	<22>	118.9	0.0	30.0	0.0	460.0	210.0
23	<23>	99.0	0.0	50.0	0.0	380.0	210.0
24	<24>	84.3	0.0	260.0	0.0	170.0	210.0

Figure 5.4 Summary results on the market solution for Monday. First example analysis: 'Conventional Unit Committment. Spinning reserve constraints neglected'.

5.3 'Conventional' Unit Committment. Spinning reserve constraints observed

5.3.1 Problem formulation

The system comprises the same 7 thermal production units as in previous chapter 5.2. See overview in Table 4.1. All other premises are likewise the same, apart from the spinning reserve constraints, which are now introduced.

With respect to spinning reserve it is required that any dispatched set of production units shall have the capability of increasing power output 10% beyond scheduled sum production. This holds for every hour of the period of analysis.

The task of operation in this case is to decide on unit committment and unit dispatch so that specified demand over the week is covered at minimum cost, taking into account all given constraints - including those relating to spinning reserve.

5.3.2 Problem solution

Solution is found after two optimization runs on 'Level I analysis' (to generate the initial committment plan), and two runs on 'Level II analysis' (to finalize the decommittment process). The solution's cost criterion is 0.03% above true minimum value found from the reference analysis. (The cost is furthermore 1.1% above the cost of previous analysis, which ignored spinning reserve requirements). App.1.5 gives further details on the solution process.

Although almost identical in terms of criterion value, the two solution methods produce results that differ slightly in terms of workday utilization of the gasfired combiplant and the gas turbine: The reference solution committs the gas turbine twice each workday, while the *Market Driven Hydro-Thermal Scheduling* committs the turbine once,but then retains the combiplant online in more hours to also cover the evening peak.

Main characteristics of the solution are presented in Figure 5.5- 5.8: Fig. 5.5 gives overview mathematical data. Fig. 5.6 shows how hourly production is distributed among the generators over the week: Fig. 5.6b) also represents Wednesday andThursday. Fig 5.6c) shows Friday, and 5.6d) displays production on Saturday as well as Sunday,- as the two days solutionwise turn out equal. Fig. 5.7 shows how the spinning reserve is distributed on Monday, and Fig. 5.8 displays summary results on the market solution for Monday.

Solutionwise it is of interest to observe that –when spinning reserve requirements are included- the gas turbine is committed two hours each workday only to run at minimum output. I.e.: The gas turbine's primary function is now to supply spinning reserve,while the bigger/old coalfired unit (5) is called upon from Monday through Friday to contribute to covering the load.

```
  Minimum found:        f(x)=      -23264.68

  Production cost(KKr)     :        9016.44
  Spot market income(KKr)  :       32281.12
 ------------------------------------------
  Minimised criterion(Kkr):       -23264.68

No of production units          :          7
No of hydro units               :          0
no of local power markets       :          7
No of time intervals            :        168
Duration of time intervals (h)  :          1

  No of variables                          :        8771
  No of inequalities                       :        4893
  No of equalities                         :        2702
  No of non-zero elements in Hessian       :       16842
  No of iterations in r-loop               :           9
  Solution time (s)                        :         9.8
```

Figure 5.5 Overview mathematical info on the second example analysis: 'Conventional Unit Committment. Spinning reserve constraints observed'.

MW output from resp. plants 1 to 7

t		1	2	3	4	5	6	7
1	< 1>	200.0	100.0	40.0	0.0	0.0	0.0	0.0
2	< 2>	200.0	80.0	40.0	0.0	0.0	0.0	0.0
3	< 3>	200.0	70.0	40.0	0.0	0.0	0.0	0.0
4	< 4>	200.0	70.0	40.0	0.0	0.0	0.0	0.0
5	< 5>	200.0	130.0	40.0	0.0	0.0	0.0	0.0
6	< 6>	200.0	132.2	40.0	77.8	0.0	0.0	0.0
7	< 7>	200.0	170.0	91.4	98.6	0.0	0.0	0.0
8	< 8>	200.0	170.0	120.0	122.3	37.7	0.0	0.0
9	< 9>	200.0	170.0	120.0	130.8	49.2	0.0	0.0
10	<10>	200.0	170.0	120.0	126.5	43.5	30.0	0.0
11	<11>	200.0	170.0	120.0	139.2	60.8	30.0	0.0
12	<12>	200.0	170.0	120.0	147.2	71.7	31.1	10.0
13	<13>	200.0	170.0	120.0	147.2	71.7	31.1	10.0
14	<14>	200.0	170.0	120.0	130.8	49.2	30.0	0.0
15	<15>	200.0	170.0	120.0	126.5	43.5	30.0	0.0
16	<16>	200.0	170.0	120.0	122.3	37.7	30.0	0.0
17	<17>	200.0	170.0	98.8	101.2	30.0	30.0	0.0
18	<18>	200.0	170.0	106.3	103.7	30.0	30.0	0.0
19	<19>	200.0	170.0	120.0	110.0	30.0	30.0	0.0
20	<20>	200.0	170.0	120.0	126.5	43.5	30.0	0.0
21	<21>	200.0	170.0	120.0	126.5	43.5	30.0	0.0
22	<22>	200.0	170.0	120.0	130.8	49.2	0.0	0.0
23	<23>	200.0	170.0	91.4	98.6	30.0	0.0	0.0
24	<24>	180.0	60.0	40.0	70.0	30.0	0.0	0.0

a) Generator production Monday

t		1	2	3	4	5	6	7
25	< 1>	141.4	60.0	40.0	68.6	30.0	0.0	0.0
26	< 2>	122.1	60.0	40.0	67.9	30.0	0.0	0.0
27	< 3>	112.4	60.0	40.0	67.6	30.0	0.0	0.0
28	< 4>	112.4	60.0	40.0	67.6	30.0	0.0	0.0
29	< 5>	170.3	60.0	40.0	69.7	30.0	0.0	0.0
30	< 6>	200.0	103.0	40.0	77.0	30.0	0.0	0.0
31	< 7>	200.0	170.0	69.0	91.0	30.0	0.0	0.0
32	< 8>	200.0	170.0	120.0	122.3	37.7	0.0	0.0
33	< 9>	200.0	170.0	120.0	130.8	49.2	0.0	0.0
34	<10>	200.0	170.0	120.0	126.5	43.5	30.0	0.0
35	<11>	200.0	170.0	120.0	139.2	60.8	30.0	0.0
36	<12>	200.0	170.0	120.0	147.2	71.7	31.1	10.0
37	<13>	200.0	170.0	120.0	147.2	71.7	31.1	10.0
38	<14>	200.0	170.0	120.0	130.8	49.2	30.0	0.0
39	<15>	200.0	170.0	120.0	126.5	43.5	30.0	0.0
40	<16>	200.0	170.0	120.0	122.3	37.7	30.0	0.0
41	<17>	200.0	170.0	98.8	101.2	30.0	30.0	0.0
42	<18>	200.0	170.0	106.3	103.7	30.0	30.0	0.0
43	<19>	200.0	170.0	120.0	110.0	30.0	30.0	0.0
44	<20>	200.0	170.0	120.0	126.5	43.5	30.0	0.0
45	<21>	200.0	170.0	120.0	126.5	43.5	30.0	0.0
46	<22>	200.0	170.0	120.0	130.8	49.2	0.0	0.0
47	<23>	200.0	170.0	91.4	98.6	30.0	0.0	0.0
48	<24>	180.0	60.0	40.0	70.0	30.0	0.0	0.0

b) Generator production Tuesday

t		1	2	3	4	5	6	7
97	< 1>	141.4	60.0	40.0	68.6	30.0	0.0	0.0
98	< 2>	122.1	60.0	40.0	67.9	30.0	0.0	0.0
99	< 3>	112.4	60.0	40.0	67.6	30.0	0.0	0.0
100	< 4>	112.4	60.0	40.0	67.6	30.0	0.0	0.0
101	< 5>	170.3	60.0	40.0	69.7	30.0	0.0	0.0
102	< 6>	200.0	103.0	40.0	77.0	30.0	0.0	0.0
103	< 7>	200.0	170.0	69.0	91.0	30.0	0.0	0.0
104	< 8>	200.0	170.0	120.0	122.3	37.7	0.0	0.0
105	< 9>	200.0	170.0	120.0	130.8	49.2	0.0	0.0
106	<10>	200.0	170.0	120.0	126.5	43.5	30.0	0.0
107	<11>	200.0	170.0	120.0	139.2	60.8	30.0	0.0
108	<12>	200.0	170.0	120.0	147.2	71.7	31.1	10.0
109	<13>	200.0	170.0	120.0	147.2	71.7	31.1	10.0
110	<14>	200.0	170.0	120.0	130.8	49.2	30.0	0.0
111	<15>	200.0	170.0	120.0	126.5	43.5	30.0	0.0
112	<16>	200.0	170.0	120.0	122.3	37.7	30.0	0.0
113	<17>	200.0	170.0	98.8	101.2	30.0	30.0	0.0
114	<18>	200.0	170.0	106.3	103.7	30.0	30.0	0.0
115	<19>	200.0	170.0	120.0	110.0	30.0	30.0	0.0
116	<20>	200.0	170.0	120.0	126.5	43.5	30.0	0.0
117	<21>	200.0	170.0	120.0	126.5	43.5	30.0	0.0
118	<22>	200.0	170.0	120.0	130.8	49.2	0.0	0.0
119	<23>	200.0	170.0	91.4	98.6	30.0	0.0	0.0
120	<24>	200.0	64.1	40.0	75.9	0.0	0.0	0.0

c) Generator production Friday

t		1	2	3	4	5	6	7
121	< 1>	170.3	60.0	40.0	69.7	0.0	0.0	0.0
122	< 2>	151.0	60.0	40.0	69.0	0.0	0.0	0.0
123	< 3>	141.4	60.0	40.0	68.6	0.0	0.0	0.0
124	< 4>	141.4	60.0	40.0	68.6	0.0	0.0	0.0
125	< 5>	199.3	60.0	40.0	70.7	0.0	0.0	0.0
126	< 6>	200.0	132.2	40.0	77.8	0.0	0.0	0.0
127	< 7>	200.0	170.0	83.9	96.1	0.0	0.0	0.0
128	< 8>	200.0	170.0	106.3	103.7	0.0	30.0	0.0
129	< 9>	200.0	170.0	113.7	106.3	0.0	30.0	0.0
130	<10>	200.0	170.0	120.0	110.0	0.0	30.0	0.0
131	<11>	200.0	170.0	120.0	110.0	0.0	30.0	10.0
132	<12>	200.0	170.0	120.0	115.0	0.0	30.0	10.0
133	<13>	200.0	170.0	120.0	110.0	0.0	30.0	0.0
134	<14>	200.0	170.0	106.3	103.7	0.0	0.0	0.0
135	<15>	200.0	170.0	98.8	101.2	0.0	0.0	0.0
136	<16>	200.0	170.0	61.6	88.4	0.0	0.0	0.0
137	<17>	200.0	170.0	61.6	88.4	0.0	0.0	0.0
138	<18>	200.0	170.0	91.4	98.6	0.0	0.0	0.0
139	<19>	200.0	170.0	98.8	101.2	0.0	0.0	0.0
140	<20>	200.0	170.0	106.3	103.7	0.0	0.0	0.0
141	<21>	200.0	170.0	83.9	96.1	0.0	0.0	0.0
142	<22>	200.0	161.4	40.0	78.6	0.0	0.0	0.0
143	<23>	200.0	103.0	40.0	77.0	0.0	0.0	0.0
144	<24>	200.0	64.1	40.0	75.9	0.0	0.0	0.0

d) Generator production Saturday

Figure 5.6 Summary print of generator production in second example analysis : 'Conventional Unit Committment. Spinning reserve constraints observed'.

MW reserve from resp. units 1 to 7

t		1	2	3	4	5	6	7
1	< 1>	0.0	70.0	80.0	0.0	0.0	0.0	0.0
2	< 2>	0.0	90.0	80.0	0.0	0.0	0.0	0.0
3	< 3>	0.0	100.0	80.0	0.0	0.0	0.0	0.0
4	< 4>	0.0	100.0	80.0	0.0	0.0	0.0	0.0
5	< 5>	0.0	40.0	80.0	0.0	0.0	0.0	0.0
6	< 6>	0.0	37.8	80.0	72.2	0.0	0.0	0.0
7	< 7>	0.0	0.0	28.6	51.4	0.0	0.0	0.0
8	< 8>	0.0	0.0	0.0	27.7	62.3	0.0	0.0
9	< 9>	0.0	0.0	0.0	19.2	50.8	0.0	0.0
10	<10>	0.0	0.0	0.0	23.5	56.5	30.0	0.0
11	<11>	0.0	0.0	0.0	10.8	39.2	30.0	0.0
12	<12>	0.0	0.0	0.0	2.8	28.3	28.9	40.0
13	<13>	0.0	0.0	0.0	2.8	28.3	28.9	40.0
14	<14>	0.0	0.0	0.0	19.2	50.8	30.0	0.0
15	<15>	0.0	0.0	0.0	23.5	56.5	30.0	0.0
16	<16>	0.0	0.0	0.0	27.7	62.3	30.0	0.0
17	<17>	0.0	0.0	21.2	48.8	70.0	30.0	0.0
18	<18>	0.0	0.0	13.7	46.3	70.0	30.0	0.0
19	<19>	0.0	0.0	0.0	40.0	70.0	30.0	0.0
20	<20>	0.0	0.0	0.0	23.5	56.5	30.0	0.0
21	<21>	0.0	0.0	0.0	23.5	56.5	30.0	0.0
22	<22>	0.0	0.0	0.0	19.2	50.8	0.0	0.0
23	<23>	0.0	0.0	28.6	51.4	70.0	0.0	0.0
24	<24>	20.0	110.0	80.0	80.0	70.0	0.0	0.0

Figure 5.7 Distribution of spinning reserve on Monday. Second example analysis: 'Conventional Unit Committment. Spinning reserve constraints observed'.

t	System Lambda (Kr/Mwh)	Reserve(Mw) Required	Reserve(Mw) Actual	Power reserve Lambda(Kr/Mw & h)	Deliveries(MW) Firm	Deliveries(MW) Flexible
1 < 1>	84.8	34.0	150.0	0.0	130.0	210.0
2 < 2>	84.5	32.0	170.0	0.0	110.0	210.0
3 < 3>	84.4	31.0	180.0	0.0	100.0	210.0
4 < 4>	84.4	31.0	180.0	0.0	100.0	210.0
5 < 5>	85.2	37.0	120.0	0.0	160.0	210.0
6 < 6>	85.2	45.0	190.0	0.0	240.0	210.0
7 < 7>	95.3	56.0	80.0	0.0	350.0	210.0
8 < 8>	106.7	65.0	90.0	0.0	440.0	210.0
9 < 9>	110.7	67.0	70.0	0.0	460.0	210.0
10 <10>	108.7	69.0	110.0	0.0	480.0	210.0
11 <11>	114.8	72.0	80.0	0.0	510.0	210.0
12 <12>	118.6	75.0	100.0	0.0	540.0	210.0
13 <13>	118.6	75.0	100.0	0.0	540.0	210.0
14 <14>	110.7	70.0	100.0	0.0	490.0	210.0
15 <15>	108.7	69.0	110.0	0.0	480.0	210.0
16 <16>	106.7	68.0	120.0	0.0	470.0	210.0
17 <17>	96.5	63.0	170.0	0.0	420.0	210.0
18 <18>	97.7	64.0	160.0	0.0	430.0	210.0
19 <19>	100.7	66.0	140.0	0.0	450.0	210.0
20 <20>	108.7	69.0	110.0	0.0	480.0	210.0
21 <21>	108.7	69.0	110.0	0.0	480.0	210.0
22 <22>	110.7	67.0	70.0	0.0	460.0	210.0
23 <23>	95.3	59.0	150.0	0.0	380.0	210.0
24 <24>	81.5	38.0	360.0	0.0	170.0	210.0

Figure 5.8 Summary results on the market solution for Monday. Second example analysis: 'Conventional Unit Committment. Spinning reserve constraints observed'.

5.4 'Conventional Unit Committment including hydro production. Spinning reserve constraints observed

5.4.1 Problem formulation

The system comprises the the same 7 thermal units as in previous two chapters. See overview in Table 4.1. In addition, two hydro units are now included; one of output range 50-150 MWh/h, the other of range 20-70 MWh/h. Detailed description of all units - both thermal and hydro - is given in App.1.2.

As operational boundary conditions it is specified that thermal units 1 and 2 are online all time and unit 3 is online in hours -1 and 168. For the two hydro units, available gross energy volumes (i.e. natural energy volumes) for the week are specified at 15.2 and 4.3GWh$_n$, respectively. (For comparison of result purposes, the available volumes are set equal to the volumes that are registered from the reference 'Dijkstra-solution' - which is based on (arbitrarely) choosing some individual valuation of stored gross energy upstream of the two hydro units.)

With respect to spinning reserve, it is again required that any dispatched set of production units shall have the capability of increasing power output 10% beyond scheduled sum production. This holds for every hour of the period of analysis which is one week.

Scope of analysis: To decide on unit committment and unit dispatch for 7 thermal and 2 hydro production units, so that specified demand over the week is covered at minimum cost – taking into account all given constraints, including those relating to spinning reserve and availability of water.

5.4.2 Problem solution

Solution is found after three optimization runs on 'Level I analysis' (to generate the initial committment plan), and four runs on 'Level II analysis' (to finalize the decommittment process). The solution's cost criterion is 0.4% above true minimum value found from the reference analysis. App.1.6 gives further details on the solution process.

Main characteristics of the solution are presented in Figure 5.9- 5.12: Fig. 5.9 gives overview mathematical data. Fig. 5.10 shows how hourly production is distributed among the generators over the week: Fig. 5.10b) also represents Wednesday andThursday. Fig 5.10c) shows Friday, and 5.10d) displays production on Saturday as well as Sunday, as the two days solutionwise turn out equal. Fig. 5.11 shows how the spinning reserve is distributed on Monday, and Fig. 5.12 displays summary results on the market solution for Monday.

Minimum found: f(x)=	-28139.38
Production cost(KKr) :	7014.54
Spot market income(KKr) :	35153.92
Minimised criterion(KKr):	-28139.38
No of production units :	9
No of hydro units :	2
no of local power markets :	9
No of time intervals :	168
Duration of time intervals (h) :	1
No of variables :	11133
No of inequalities :	6243
No of equalities :	3378
No of non-zero elements in Hessian :	21558
No of iterations in r-loop :	5
Solution time (s) :	13.6

Figure 5.9 Overview mathematical info on the third example analysis: 'Conventional Unit Committment including hydro production. Spinning reserve constraints observed'.

MW output from resp. plants 1 to 9

t		1	2	3	4	5	6	7	8	9
1	< 1>	200.0	100.0	40.0	0.0	0.0	0.0	0.0	0.0	0.0
2	< 2>	200.0	80.0	40.0	0.0	0.0	0.0	0.0	0.0	0.0
3	< 3>	200.0	70.0	40.0	0.0	0.0	0.0	0.0	0.0	0.0
4	< 4>	200.0	70.0	40.0	0.0	0.0	0.0	0.0	0.0	0.0
5	< 5>	200.0	130.0	40.0	0.0	0.0	0.0	0.0	0.0	0.0
6	< 6>	200.0	157.8	40.0	0.0	0.0	0.0	0.0	0.0	52.1
7	< 7>	200.0	170.0	85.0	0.0	0.0	0.0	0.0	105.0	0.0
8	< 8>	200.0	170.0	120.0	0.0	41.0	0.0	0.0	119.0	0.0
9	< 9>	200.0	170.0	120.0	0.0	55.8	0.0	0.0	124.2	0.0
10	<10>	200.0	170.0	118.5	0.0	30.0	0.0	0.0	110.7	60.8
11	<11>	200.0	170.0	120.0	0.0	43.8	0.0	0.0	119.9	66.3
12	<12>	200.0	170.0	120.0	0.0	56.5	0.0	10.0	124.5	69.0
13	<13>	200.0	170.0	120.0	0.0	56.5	0.0	10.0	124.5	69.0
14	<14>	200.0	170.0	120.0	0.0	31.1	0.0	0.0	115.4	63.6
15	<15>	200.0	170.0	118.5	0.0	30.0	0.0	0.0	110.7	60.8
16	<16>	200.0	170.0	110.7	0.0	30.0	0.0	0.0	109.4	60.0
17	<17>	200.0	170.0	119.2	0.0	30.0	0.0	0.0	110.8	0.0
18	<18>	200.0	170.0	120.0	0.0	33.7	0.0	0.0	116.3	0.0
19	<19>	200.0	170.0	120.0	0.0	48.4	0.0	0.0	121.6	0.0
20	<20>	200.0	170.0	118.5	0.0	30.0	0.0	0.0	110.7	60.8
21	<21>	200.0	170.0	118.5	0.0	30.0	0.0	0.0	110.7	60.8
22	<22>	200.0	170.0	120.0	0.0	55.8	0.0	0.0	124.2	0.0
23	<23>	200.0	170.0	85.0	0.0	30.0	0.0	0.0	105.0	0.0
24	<24>	200.0	110.0	40.0	0.0	30.0	0.0	0.0	0.0	0.0

a) Generator production Monday

t		1	2	3	4	5	6	7	8	9
25	< 1>	200.0	70.0	40.0	0.0	30.0	0.0	0.0	0.0	0.0
26	< 2>	190.0	60.0	40.0	0.0	30.0	0.0	0.0	0.0	0.0
27	< 3>	180.0	60.0	40.0	0.0	30.0	0.0	0.0	0.0	0.0
28	< 4>	180.0	60.0	40.0	0.0	30.0	0.0	0.0	0.0	0.0
29	< 5>	200.0	100.0	40.0	0.0	30.0	0.0	0.0	0.0	0.0
30	< 6>	200.0	170.0	50.0	0.0	30.0	0.0	0.0	0.0	0.0
31	< 7>	200.0	170.0	59.3	0.0	30.0	0.0	0.0	100.7	0.0
32	< 8>	200.0	170.0	120.0	0.0	41.0	0.0	0.0	119.0	0.0
33	< 9>	200.0	170.0	120.0	0.0	55.8	0.0	0.0	124.2	0.0
34	<10>	200.0	170.0	118.5	0.0	30.0	0.0	0.0	110.7	60.8
35	<11>	200.0	170.0	120.0	0.0	43.8	0.0	0.0	119.9	66.3
36	<12>	200.0	170.0	120.0	0.0	56.5	0.0	10.0	124.5	69.0
37	<13>	200.0	170.0	120.0	0.0	56.5	0.0	10.0	124.5	69.0
38	<14>	200.0	170.0	120.0	0.0	31.1	0.0	0.0	115.4	63.6
39	<15>	200.0	170.0	118.5	0.0	30.0	0.0	0.0	110.7	60.8
40	<16>	200.0	170.0	110.7	0.0	30.0	0.0	0.0	109.4	60.0
41	<17>	200.0	170.0	119.2	0.0	30.0	0.0	0.0	110.8	0.0
42	<18>	200.0	170.0	120.0	0.0	33.7	0.0	0.0	116.3	0.0
43	<19>	200.0	170.0	120.0	0.0	48.4	0.0	0.0	121.6	0.0
44	<20>	200.0	170.0	118.5	0.0	30.0	0.0	0.0	110.7	60.8
45	<21>	200.0	170.0	118.5	0.0	30.0	0.0	0.0	110.7	60.8
46	<22>	200.0	170.0	120.0	0.0	55.8	0.0	0.0	124.2	0.0
47	<23>	200.0	170.0	85.0	0.0	30.0	0.0	0.0	105.0	0.0
48	<24>	200.0	110.0	40.0	0.0	30.0	0.0	0.0	0.0	0.0

b) Generator production Tuesday

t		1	2	3	4	5	6	7	8	9
97	< 1>	200.0	70.0	40.0	0.0	30.0	0.0	0.0	0.0	0.0
98	< 2>	190.0	60.0	40.0	0.0	30.0	0.0	0.0	0.0	0.0
99	< 3>	180.0	60.0	40.0	0.0	30.0	0.0	0.0	0.0	0.0
100	< 4>	180.0	60.0	40.0	0.0	30.0	0.0	0.0	0.0	0.0
101	< 5>	200.0	100.0	40.0	0.0	30.0	0.0	0.0	0.0	0.0
102	< 6>	200.0	170.0	50.0	0.0	30.0	0.0	0.0	0.0	0.0
103	< 7>	200.0	170.0	59.3	0.0	30.0	0.0	0.0	100.7	0.0
104	< 8>	200.0	170.0	120.0	0.0	41.0	0.0	0.0	119.0	0.0
105	< 9>	200.0	170.0	120.0	0.0	55.8	0.0	0.0	124.2	0.0
106	<10>	200.0	170.0	118.5	0.0	30.0	0.0	0.0	110.7	60.8
107	<11>	200.0	170.0	120.0	0.0	43.8	0.0	0.0	119.9	66.3
108	<12>	200.0	170.0	120.0	0.0	56.5	0.0	10.0	124.5	69.0
109	<13>	200.0	170.0	120.0	0.0	56.5	0.0	10.0	124.5	69.0
110	<14>	200.0	170.0	120.0	0.0	31.1	0.0	0.0	115.4	63.6
111	<15>	200.0	170.0	118.5	0.0	30.0	0.0	0.0	110.7	60.8
112	<16>	200.0	170.0	110.7	0.0	30.0	0.0	0.0	109.4	60.0
113	<17>	200.0	170.0	119.2	0.0	30.0	0.0	0.0	110.8	0.0
114	<18>	200.0	170.0	120.0	0.0	33.7	0.0	0.0	116.3	0.0
115	<19>	200.0	170.0	120.0	0.0	48.4	0.0	0.0	121.6	0.0
116	<20>	200.0	170.0	118.5	0.0	30.0	0.0	0.0	110.7	60.8
117	<21>	200.0	170.0	118.5	0.0	30.0	0.0	0.0	110.7	60.8
118	<22>	200.0	170.0	120.0	0.0	55.8	0.0	0.0	124.2	0.0
119	<23>	200.0	170.0	63.4	0.0	0.0	0.0	0.0	101.4	55.2
120	<24>	200.0	140.0	40.0	0.0	0.0	0.0	0.0	0.0	0.0

c) Generator production Friday

t		1	2	3	4	5	6	7	8	9
121	< 1>	200.0	100.0	40.0	0.0	0.0	0.0	0.0	0.0	0.0
122	< 2>	200.0	80.0	40.0	0.0	0.0	0.0	0.0	0.0	0.0
123	< 3>	200.0	70.0	40.0	0.0	0.0	0.0	0.0	0.0	0.0
124	< 4>	200.0	70.0	40.0	0.0	0.0	0.0	0.0	0.0	0.0
125	< 5>	200.0	130.0	40.0	0.0	0.0	0.0	0.0	0.0	0.0
126	< 6>	200.0	157.8	40.0	0.0	0.0	0.0	0.0	0.0	52.1
127	< 7>	200.0	170.0	76.4	0.0	0.0	0.0	0.0	103.6	0.0
128	< 8>	200.0	170.0	79.1	0.0	0.0	0.0	0.0	104.0	56.8
129	< 9>	200.0	170.0	87.0	0.0	0.0	0.0	0.0	105.4	57.6
130	<10>	200.0	170.0	94.9	0.0	0.0	0.0	0.0	106.7	58.4
131	<11>	200.0	170.0	102.8	0.0	0.0	0.0	0.0	108.0	59.2
132	<12>	200.0	170.0	106.7	0.0	0.0	0.0	0.0	108.7	59.6
133	<13>	200.0	170.0	94.9	0.0	0.0	0.0	0.0	106.7	58.4
134	<14>	200.0	170.0	102.1	0.0	0.0	0.0	0.0	107.9	0.0
135	<15>	200.0	170.0	93.5	0.0	0.0	0.0	0.0	106.5	0.0
136	<16>	200.0	170.0	50.8	0.0	0.0	0.0	0.0	99.2	0.0
137	<17>	200.0	170.0	50.8	0.0	0.0	0.0	0.0	99.2	0.0
138	<18>	200.0	170.0	85.0	0.0	0.0	0.0	0.0	105.0	0.0
139	<19>	200.0	170.0	93.5	0.0	0.0	0.0	0.0	106.5	0.0
140	<20>	200.0	170.0	102.1	0.0	0.0	0.0	0.0	107.9	0.0
141	<21>	200.0	170.0	76.4	0.0	0.0	0.0	0.0	103.6	0.0
142	<22>	200.0	144.0	40.0	0.0	0.0	0.0	0.0	96.0	0.0
143	<23>	200.0	170.0	50.0	0.0	0.0	0.0	0.0	0.0	0.0
144	<24>	200.0	140.0	40.0	0.0	0.0	0.0	0.0	0.0	0.0

d) Generator production Saturday

Figure 5.10 Summary print of generator production in third example analysis : 'Conventional Unit Committment including hydro production. Spinning reserve constraints observed'.

MW reserve from resp. units 1 to 9

t		1	2	3	4	5	6	7	8	9
1	< 1>	0.0	70.0	80.0	0.0	0.0	0.0	0.0	0.0	0.0
2	< 2>	0.0	90.0	80.0	0.0	0.0	0.0	0.0	0.0	0.0
3	< 3>	0.0	100.0	80.0	0.0	0.0	0.0	0.0	0.0	0.0
4	< 4>	0.0	100.0	80.0	0.0	0.0	0.0	0.0	0.0	0.0
5	< 5>	0.0	40.0	80.0	0.0	0.0	0.0	0.0	0.0	0.0
6	< 6>	0.0	12.2	80.0	0.0	0.0	0.0	0.0	0.0	17.9
7	< 7>	0.0	0.0	35.0	0.0	0.0	0.0	0.0	45.0	0.0
8	< 8>	0.0	0.0	0.0	0.0	59.0	0.0	0.0	31.0	0.0
9	< 9>	0.0	0.0	0.0	0.0	44.2	0.0	0.0	25.8	0.0
10	<10>	0.0	0.0	1.5	0.0	70.0	0.0	0.0	39.3	9.2
11	<11>	0.0	0.0	0.0	0.0	56.2	0.0	0.0	30.1	3.7
12	<12>	0.0	0.0	0.0	0.0	43.5	0.0	40.0	25.5	1.0
13	<13>	0.0	0.0	0.0	0.0	43.5	0.0	40.0	25.5	1.0
14	<14>	0.0	0.0	0.0	0.0	68.9	0.0	0.0	34.6	6.4
15	<15>	0.0	0.0	1.5	0.0	70.0	0.0	0.0	39.3	9.2
16	<16>	0.0	0.0	9.3	0.0	70.0	0.0	0.0	40.6	10.0
17	<17>	0.0	0.0	0.8	0.0	70.0	0.0	0.0	39.2	0.0
18	<18>	0.0	0.0	0.0	0.0	66.3	0.0	0.0	33.7	0.0
19	<19>	0.0	0.0	0.0	0.0	51.6	0.0	0.0	28.4	0.0
20	<20>	0.0	0.0	1.5	0.0	70.0	0.0	0.0	39.3	9.2
21	<21>	0.0	0.0	1.5	0.0	70.0	0.0	0.0	39.3	9.2
22	<22>	0.0	0.0	0.0	0.0	44.2	0.0	0.0	25.8	0.0
23	<23>	0.0	0.0	35.0	0.0	70.0	0.0	0.0	45.0	0.0
24	<24>	0.0	60.0	80.0	0.0	70.0	0.0	0.0	0.0	0.0

Figure 5.11 Distribution of spinning reserve on Monday. Third example analysis: 'Conventional Unit Committment including hydro production. Spinning reserve constraints observed'.

t		System Lambda (Kr/Mwh)	Reserve(Mw) Required	Actual	Power reserve Lambda(Kr/Mw & h)	Deliveries(MW) Firm	Flexibl
1	< 1>	84.8	34.0	150.0	0.0	70.0	270.0
2	< 2>	84.5	32.0	170.0	0.0	50.0	270.0
3	< 3>	84.4	31.0	180.0	0.0	40.0	270.0
4	< 4>	84.4	31.0	180.0	0.0	40.0	270.0
5	< 5>	85.2	37.0	120.0	0.0	100.0	270.0
6	< 6>	85.6	45.0	110.0	0.0	180.0	270.0
7	< 7>	94.2	56.0	80.0	0.0	290.0	270.0
8	< 8>	107.9	65.0	90.0	0.0	380.0	270.0
9	< 9>	113.0	67.0	70.0	0.0	400.0	270.0
10	<10>	99.8	69.0	120.0	0.0	420.0	270.0
11	<11>	108.8	72.0	90.0	0.0	450.0	270.0
12	<12>	113.3	75.0	110.0	0.0	480.0	270.0
13	<13>	113.3	75.0	110.0	0.0	480.0	270.0
14	<14>	104.3	70.0	110.0	0.0	430.0	270.0
15	<15>	99.8	69.0	120.0	0.0	420.0	270.0
16	<16>	98.5	68.0	130.0	0.0	410.0	270.0
17	<17>	99.9	63.0	110.0	0.0	360.0	270.0
18	<18>	105.3	64.0	100.0	0.0	370.0	270.0
19	<19>	110.4	66.0	80.0	0.0	390.0	270.0
20	<20>	99.8	69.0	120.0	0.0	420.0	270.0
21	<21>	99.8	69.0	120.0	0.0	420.0	270.0
22	<22>	113.0	67.0	70.0	0.0	400.0	270.0
23	<23>	94.2	59.0	150.0	0.0	320.0	270.0
24	<24>	84.9	38.0	210.0	0.0	110.0	270.0

Figure 5.12 Summary results on the market solution for Monday. Third example analysis: 'Conventional Unit Committment including hydro production. Spinning reserve constraints observed'.

5.5 Market Driven Hydro-Thermal Scheduling, spinning reserve constraints observed

5.5.1 Problem formulation

The system comprises the the same 7 thermal units and 2 hydro units as is previous Chapter 5.4. Detailed description of all production units is given in App.1.2. As operational boundary conditions it is specified that thermal unit 1 and 2 are online all the time. Unit 3 is online in hour –1 and hour 168.

For hydro unit 8 (of capacity 150MW) a volume of natural energy of 12.73 $GWh_{natural}$ is specified available for the considered market clearing period of 168 hours. For hydro unit 9 (of capacity 70MW) the corresponding volume is 7.49 $GWh_{natural}$. Most of the energy presumed available for unit 9 stems from nonregulated inflow,- hence it is specified that unit 9 should be up and running in all hours of the planning period.

The power system layout is stylized, as shown in Figure 4.1 : Each generator is feeding power into its regional area, where there is a local market to clear in consistency with hour-by-hour clearing of the rest of the system. Each region is via lossy transmission connected to the central system bus where main contractual (or 'firm') demand is located. Transmission losses are described in terms of B-coefficients, the formal content of which is outlined via eqns. (4.23)-(4.25). Actual parameter values for the example are defined in App. 1.7.

Power demand is described in terms of price-insensitive as well as price-sensitive demand :

> *Contractual demand* is assumed to have the same daily cycle Monday through Friday. Range of variation: 330-820 MWh/h. Saturday and Sunday have each separate contractual load profiles with range of variation 330 -685 , and 330-695 MWh/h, respectively. The 'last' 100MW of contractual load is modelled as high-priced spot demand, the pricing of which is further outlined below where spot power demand is dealt with. The main price-insensitive part of contractual demand is given in App.1.7. The rationale for splitting of contractual load in a price-insensitive and a price-sensitive part, is outlined at the start of Chapter 4.2.2.
>
> *Spot power demand for given hour and location* is presumed described in terms of a linear incremental price vs. volume curve. For modelling details, see App.1.1. Table A1.1 of that appendix shows the forecast of price-sensitive power demand by region, that is used in the current example. Table A1.2 displays the formal high-priced '100-MW market' that represents the 'last'/balancing part of contractual delivery.

With respect to spinning reserve it is required that any dispatched set of production units shall have the capability of increasing power output 15.4% beyond scheduled sum production. This holds for every hour of the period of analysis which is one week.

The task of operation in this case is to decide on hour-by-hour clearing of geographically dispersed power markets over the week, together with committment and dispatch of production units , so that the EXPECTED SUM OF CONSUMER- AND PRODUCER SURPLUS over the stated period is maximized – taking into account all given constraints, including those relating to spinning reserve and availability of water power. See Figure 3.1 and Chapter 4.1 as a reminder of basic premises of analysis.-

5.5.2 Problem solution

Solution is found after three optimization runs on 'Level I analysis' (to generate the initial committment plan), and two runs on 'Level II analysis' (to finalize the decommittment process). There is no alternative or reference solution available to this multibus case. App.1.7 gives further details on the solution process.

Main characteristics of the solution are presented in Figure 5.13- 5.18: Fig. 5.13 gives overview mathematical data. Fig. 5.14 shows how hourly production is distributed among the generators over the week: Fig. 5.14b) also represents Wednesday andThursday. Fig 5.14c) shows Friday, and 5.14d) displays production on Saturday as well as Sunday, as the two days solution-wise turn out equal. Fig. 5.15 shows how the spinning reserve is distributed on Monday, Fig. 5.16 displays summary results on the market solution for Monday, and Fig. 5.17 shows the detailed results from Monday's clearing of one of the local markets,- here chosen to be the one at region '8'.

```
  Minimum found:       f(x)=     -14801.00

  Production cost(KKr)     :      9860.07
  Spot market income(KKr) :     24661.07
 ______________________________________
  Minimised criterion(Kkr):    -14801.00

No of production units         :           9
No of hydro units              :           2
no of local power markets      :           9
No of time intervals           :         168
Duration of time intervals (h) :           1

  No of variables                       :       11133
  No of inequalities                    :        6243
  No of equalities                      :        3378
  No of non-zero elements in Hessian    :       23070
  No of iterations in r-loop            :           5
  Solution time (s)                     :        15.5
```

Figure 5.13 Overview mathematical info on the fourth example analysis: 'Market Driven Hydro-Thermal Scheduling'.

5.5.3 Discussion of sample results

Unit committment and dispatch (Figure 5.14)
Unit 1 and 2 are conventional coalfired units of high efficiency; they appear by and large fully loaded in all hours of the week.

Units 3 and 4 are older conventional coalfired units of less efficiency. They contribute decisively each day with production adapted to the demand profile.

Unit 5 is an old and inefficient unit that is brought online at start of hour 7 on Monday and remains connected until hour 24 on Friday.

Unit 6 is a 60MW gasfired 'combitype' production unit that is brought to participate with minimum output each workday during hours 10 to 17. Thus the main function of unit 6 is to supply peaktime spinning reserve.

Unit 7 is a light gasfired gas turbine of 50MW that remains offline all week.

Unit 8 and 9 are hydro units that provide for peaking power each day of the week. Unit 8 comes up from zero each morning, while unit 9 is connected all the time – as specified in advance.

Power market clearing (See 'Monday snapshot' Figure 5.16)
'On top' of the contractual power market there is a spotmarket to clear on a regional basis from hour to hour over the week. Prospective spotprice varies in the range 160 NOK/MWh to 60 NOK/MWh, with highest valuation during workday peak hours and lowest valuation at night-time in the weekend. (There is also - for reasons of convergence - a formal/high-priced spot market segment of capacity 100 MWh/h in each hour of the week, as part of contractual delivery). See Table A1.1 – A1.2 of Appendix 1.1

A general observation is that contractual power is delivered in full in all hours, see two last columns of figure 5.16. (Total contractual delivery in a given hour is the sum of 'Firm delivery' of second last column and the 'first' (i.e. highest priced) 100MWh/h of 'Flexible' (or spot-) delivery of last column). Another overview observation is that some volume of price-sensitive demand (beyond the 100MWh/h representing firm load) is also covered throughout the week; we see e.g. that lowest such delivery is in sum for all regions (162.3-100) =62.3 MWh/h in peak hour 12 on workdays, and attains a maximum sum of (326.6-100) =226.6 MWh/h in hour 3 and 4 on workdays. Some further specific comments to respective columns of the figure:

'System Lambda' is the cleared incremental power cost associated with the system power balance, see eqn. (4.22). *We notice that the incremental power cost is forced high during daytime hours when the system reserve constraint is being activated.* See further on this below.

'Required reserve' is given by the premise that any configuration of production units up and running, shall be capable of increasing sum output (here) 15.4% beyond scheduled sum production. Returning again to hour 12 on workdays, it is seen from Figure 5.14a) that sum production is P_{Gsum}=(200.0+170.0+120.0+142.9+67.6+30.0+112.7+40.6)=883.8MWh/h. Required reserve is then 883.8·0.154=136.1, which fits with the figure displayed in Figure 5.16.

'Actual reserve' is given as the difference between rated output from the configuration of generators running, and sum scheduled production from that same configuration. 'Actual reserve' is in this case equal to 'Required reserve', implying that the spinning reserve constraint is active in hour 12.

'Power reserve lambda' is the incremental power cost associated with the spinning reserve constraint. It is noticed from Figure 5.16 that this multiplier is taking on values greater than zero, in 9 hours on workdays, implying that the matching of power supply and demand in these hours is constrained by the reserve requirement.

'Deliveries' to the far right in Figure 5.16, comprises two result columns related to the covering of demand:

'Firm' delivery is exogenously specified contractual load.

'Flexible' delivery is total price sensitive delivery cleared in respective hours. As pointed to above, the figure is (here) the sum of two main contributions: A special quantity of 100MWh/h demand that is the 'last' part of firm or contractual load, and the sum of 'ordinary' price sensitive regional demand as described in Table A1.1 of App.1.1

Figure 5.17 shows details of the market clearing within a given region of the system. Region 8 is chosen. This is one of the 'hydro regions' , - and also the region in which the formal/high-priced 100MWh/h spot market (arbitrarely) has been placed. Specific comments to the figure:

On top is given the implied incremental worth of natural- or gross- stored energy up-streams of hydro unit 8: If the volume constraint is relaxed 1 MWh of gross energy, its beneficial consequences to the system is seen to be NOK 127.7.

Three columns relate to regional generation; the first gives hourly production, the next two cite upper and lower limits when the unit is connected to the system.

The three next columns relate to the local spot market; the first of them gives hourly cleared volume, the next two give low and high limit on the demand. In consistency with previous discussion, it is to notice that the formal 100MWh/h spotmarket that equivalences part of firm load, is 'filled up' in all hours.

The last three columns present different incremental cost figures pertaining to the considered region:

> The first is the cost associated with fulfilling the regional hourly power balance, see enqn. (4.21). As transmission losses in this case are relatively small and transmission constraints are nonbinding, this lambda value will differ little from the one associated with the system power balance. See lambda values in Fig. 5.16.
>
> The second last column of Figure 5.17 contains the incremental price defined by current delivery in the regional power spot market,- which here is the formal market segment representing the 'last' 100MWh/h of firm load. As this market segment is fully 'filled up',the shown lambda equals the specified worth of the last delivered MWh of firm power. See Table A1.2 of App.1.1 .
>
> The last column comprises regional incremental production cost. In e.g. hour 12 this cost is 117.5 NOK/MWh (for P_8=112.7 MWh/h). This value is far below the corresponding regional power balance cost of 150.6 NOK/MWh. The feature of lifted regional and global power balance cost relative to the level of physical incremental production costs, is due to the spinning reserve constraint: To limit the coverage of price sensitive demand in order to observe the reserve requirement, the market clearing process is brought to 'believe' that incremental power cost has attained some high level. For region 8 the cost difference (see Figure 5.17) is (150.6-117.5) = 33.1 NOK/MWh, which is in agreement with the incremental system cost of 33.1 NOK/MWh, associated with observing the security constraint in hour 12. See Figure 5.16.

Figure 5.18 gives on top a summary on power supply and delivery for the week. The next three columns show accumulated *resource use*, *production*, and *available resource volume* for each generator of the system:

For the thermal units (1 to 7) *resource use* is the thermal energy content of the consumed fuel. For these units the *available resource volume* is set (arbitrarely) high so as not to risk that the volume constraint on fuel becomes binding in the example studies. See further comments on the resource constraint aspect in the next to final part of the Summary Report.

For the hydro units (8 and 9) *resource use* is gross energy content of the turbined water at given head. For these units we note that the *available resource volumes* have been set so as to become binding in the solution.

MW output from resp. plants 1 to 9

t		1	2	3	4	5	6	7	8	9
1	< 1>	200.0	170.0	77.1	93.6	0.0	0.0	0.0	0.0	32.3
2	< 2>	200.0	170.0	69.8	91.1	0.0	0.0	0.0	0.0	31.8
3	< 3>	200.0	170.0	66.1	89.8	0.0	0.0	0.0	0.0	31.6
4	< 4>	200.0	170.0	66.1	89.8	0.0	0.0	0.0	0.0	31.6
5	< 5>	200.0	170.0	88.0	97.5	0.0	0.0	0.0	0.0	32.9
6	< 6>	200.0	170.0	107.1	104.1	0.0	0.0	0.0	0.0	34.1
7	< 7>	200.0	170.0	120.0	121.7	38.0	0.0	0.0	0.0	37.0
8	< 8>	200.0	170.0	120.0	121.2	37.4	0.0	0.0	103.1	36.9
9	< 9>	200.0	170.0	120.0	124.1	41.4	0.0	0.0	104.4	37.4
10	<10>	200.0	170.0	120.0	143.1	67.6	30.0	0.0	112.6	40.6
11	<11>	200.0	170.0	120.0	143.0	67.6	30.0	0.0	112.7	40.6
12	<12>	200.0	170.0	120.0	142.9	67.6	30.0	0.0	112.7	40.6
13	<13>	200.0	170.0	120.0	143.0	67.6	30.0	0.0	112.7	40.6
14	<14>	200.0	170.0	120.0	143.1	67.6	30.0	0.0	112.6	40.6
15	<15>	200.0	170.0	120.0	143.1	67.6	30.0	0.0	112.6	40.6
16	<16>	200.0	170.0	120.0	143.1	67.6	30.0	0.0	112.6	40.6
17	<17>	200.0	170.0	120.0	143.2	67.6	30.0	0.0	112.5	40.6
18	<18>	200.0	170.0	120.0	118.2	33.4	0.0	0.0	101.8	36.4
19	<19>	200.0	170.0	120.0	121.2	37.4	0.0	0.0	103.1	36.9
20	<20>	200.0	170.0	120.0	125.6	43.4	0.0	0.0	105.0	37.7
21	<21>	200.0	170.0	120.0	125.6	43.4	0.0	0.0	105.0	37.7
22	<22>	200.0	170.0	120.0	122.6	39.4	0.0	0.0	103.7	37.2
23	<23>	200.0	170.0	120.0	126.3	44.4	0.0	0.0	0.0	37.8
24	<24>	200.0	170.0	80.8	94.9	30.0	0.0	0.0	0.0	32.5

a) Generator production Monday

t		1	2	3	4	5	6	7	8	9
25	< 1>	200.0	170.0	66.2	89.8	30.0	0.0	0.0	0.0	31.6
26	< 2>	200.0	170.0	58.8	87.3	30.0	0.0	0.0	0.0	31.2
27	< 3>	200.0	170.0	55.2	86.0	30.0	0.0	0.0	0.0	31.0
28	< 4>	200.0	170.0	55.2	86.0	30.0	0.0	0.0	0.0	31.0
29	< 5>	200.0	170.0	77.1	93.7	30.0	0.0	0.0	0.0	32.3
30	< 6>	200.0	170.0	106.4	103.9	30.0	0.0	0.0	0.0	34.0
31	< 7>	200.0	170.0	120.0	121.6	38.0	0.0	0.0	0.0	37.0
32	< 8>	200.0	170.0	120.0	121.2	37.4	0.0	0.0	103.1	36.9
33	< 9>	200.0	170.0	120.0	124.1	41.4	0.0	0.0	104.4	37.4
34	<10>	200.0	170.0	120.0	143.1	67.6	30.0	0.0	112.6	40.6
35	<11>	200.0	170.0	120.0	143.0	67.6	30.0	0.0	112.7	40.6
36	<12>	200.0	170.0	120.0	142.9	67.6	30.0	0.0	112.7	40.6
37	<13>	200.0	170.0	120.0	143.0	67.6	30.0	0.0	112.7	40.6
38	<14>	200.0	170.0	120.0	143.1	67.6	30.0	0.0	112.6	40.6
39	<15>	200.0	170.0	120.0	143.1	67.6	30.0	0.0	112.6	40.6
40	<16>	200.0	170.0	120.0	143.1	67.6	30.0	0.0	112.6	40.6
41	<17>	200.0	170.0	120.0	143.2	67.6	30.0	0.0	112.5	40.6
42	<18>	200.0	170.0	120.0	118.2	33.4	0.0	0.0	101.8	36.4
43	<19>	200.0	170.0	120.0	121.2	37.4	0.0	0.0	103.1	36.9
44	<20>	200.0	170.0	120.0	125.6	43.4	0.0	0.0	105.0	37.7
45	<21>	200.0	170.0	120.0	125.6	43.4	0.0	0.0	105.0	37.7
46	<22>	200.0	170.0	120.0	122.6	39.4	0.0	0.0	103.7	37.2
47	<23>	200.0	170.0	120.0	126.3	44.4	0.0	0.0	0.0	37.8
48	<24>	200.0	170.0	80.8	94.9	30.0	0.0	0.0	0.0	32.5

b) Generator production Tuesday

t		1	2	3	4	5	6	7	8	9
97	< 1>	200.0	170.0	66.2	89.8	30.0	0.0	0.0	0.0	31.6
98	< 2>	200.0	170.0	58.8	87.3	30.0	0.0	0.0	0.0	31.2
99	< 3>	200.0	170.0	55.2	86.0	30.0	0.0	0.0	0.0	31.0
100	< 4>	200.0	170.0	55.2	86.0	30.0	0.0	0.0	0.0	31.0
101	< 5>	200.0	170.0	77.1	93.7	30.0	0.0	0.0	0.0	32.3
102	< 6>	200.0	170.0	106.4	103.9	30.0	0.0	0.0	0.0	34.0
103	< 7>	200.0	170.0	120.0	121.6	38.0	0.0	0.0	0.0	37.0
104	< 8>	200.0	170.0	120.0	121.2	37.4	0.0	0.0	103.1	36.9
105	< 9>	200.0	170.0	120.0	124.1	41.4	0.0	0.0	104.4	37.4
106	<10>	200.0	170.0	120.0	143.1	67.6	30.0	0.0	112.6	40.6
107	<11>	200.0	170.0	120.0	143.0	67.6	30.0	0.0	112.7	40.6
108	<12>	200.0	170.0	120.0	142.9	67.6	30.0	0.0	112.7	40.6
109	<13>	200.0	170.0	120.0	143.0	67.6	30.0	0.0	112.7	40.6
110	<14>	200.0	170.0	120.0	143.1	67.6	30.0	0.0	112.6	40.6
111	<15>	200.0	170.0	120.0	143.1	67.6	30.0	0.0	112.6	40.6
112	<16>	200.0	170.0	120.0	143.1	67.6	30.0	0.0	112.6	40.6
113	<17>	200.0	170.0	120.0	143.2	67.6	30.0	0.0	112.5	40.6
114	<18>	200.0	170.0	120.0	118.2	33.4	0.0	0.0	101.8	36.4
115	<19>	200.0	170.0	120.0	121.2	37.4	0.0	0.0	103.1	36.9
116	<20>	200.0	170.0	120.0	125.6	43.4	0.0	0.0	105.0	37.7
117	<21>	200.0	170.0	120.0	125.6	43.4	0.0	0.0	105.0	37.7
118	<22>	200.0	170.0	120.0	122.6	39.4	0.0	0.0	103.7	37.2
119	<23>	200.0	170.0	120.0	126.3	44.4	0.0	0.0	0.0	37.8
120	<24>	200.0	170.0	80.8	94.9	30.0	0.0	0.0	0.0	32.5

c) Generator production Friday

t		1	2	3	4	5	6	7	8	9
121	< 1>	200.0	128.7	40.0	77.8	0.0	0.0	0.0	0.0	29.6
122	< 2>	200.0	111.4	40.0	77.2	0.0	0.0	0.0	0.0	29.5
123	< 3>	200.0	102.7	40.0	76.9	0.0	0.0	0.0	0.0	29.5
124	< 4>	200.0	102.7	40.0	76.9	0.0	0.0	0.0	0.0	29.5
125	< 5>	200.0	154.6	40.0	78.6	0.0	0.0	0.0	0.0	29.8
126	< 6>	200.0	170.0	59.9	87.6	0.0	0.0	0.0	0.0	31.3
127	< 7>	200.0	170.0	96.6	100.5	0.0	0.0	0.0	0.0	33.5
128	< 8>	200.0	170.0	91.6	98.7	0.0	0.0	0.0	93.4	33.2
129	< 9>	200.0	170.0	98.6	101.2	0.0	0.0	0.0	94.5	33.6
130	<10>	200.0	170.0	120.0	117.4	0.0	0.0	0.0	101.5	36.3
131	<11>	200.0	170.0	120.0	117.4	0.0	0.0	0.0	101.5	36.3
132	<12>	200.0	170.0	120.0	117.4	0.0	0.0	0.0	101.5	36.3
133	<13>	200.0	170.0	120.0	117.4	0.0	0.0	0.0	101.5	36.3
134	<14>	200.0	170.0	120.0	117.5	0.0	0.0	0.0	101.5	36.3
135	<15>	200.0	170.0	120.0	117.4	0.0	0.0	0.0	101.4	36.3
136	<16>	200.0	170.0	120.0	111.1	0.0	0.0	0.0	98.7	35.2
137	<17>	200.0	170.0	120.0	111.1	0.0	0.0	0.0	98.7	35.2
138	<18>	200.0	170.0	67.3	90.2	0.0	0.0	0.0	89.8	31.7
139	<19>	200.0	170.0	70.8	91.4	0.0	0.0	0.0	90.3	31.9
140	<20>	200.0	170.0	74.2	92.7	0.0	0.0	0.0	90.8	32.1
141	<21>	200.0	170.0	63.8	89.0	0.0	0.0	0.0	89.2	31.5
142	<22>	200.0	170.0	70.9	91.5	0.0	0.0	0.0	0.0	31.9
143	<23>	200.0	170.0	48.9	83.8	0.0	0.0	0.0	0.0	30.7
144	<24>	200.0	163.2	40.0	78.9	0.0	0.0	0.0	0.0	29.8

d) Generator production Saturday

Figure 5.14 Summary print of generator production in fourth example analysis : 'Market Driven Hydro-Thermal Scheduling'.

MW reserve from resp. units 1 to 9

t		1	2	3	4	5	6	7	8	9
1	< 1>	0.0	0.0	42.9	56.4	0.0	0.0	0.0	0.0	37.7
2	< 2>	0.0	0.0	50.2	58.9	0.0	0.0	0.0	0.0	38.2
3	< 3>	0.0	0.0	53.9	60.2	0.0	0.0	0.0	0.0	38.4
4	< 4>	0.0	0.0	53.9	60.2	0.0	0.0	0.0	0.0	38.4
5	< 5>	0.0	0.0	32.0	52.5	0.0	0.0	0.0	0.0	37.1
6	< 6>	0.0	0.0	12.9	45.9	0.0	0.0	0.0	0.0	35.9
7	< 7>	0.0	0.0	0.0	28.3	62.0	0.0	0.0	0.0	33.0
8	< 8>	0.0	0.0	0.0	28.8	62.6	0.0	0.0	46.9	33.1
9	< 9>	0.0	0.0	0.0	25.9	58.6	0.0	0.0	45.6	32.6
10	<10>	0.0	0.0	0.0	6.9	32.4	30.0	0.0	37.4	29.4
11	<11>	0.0	0.0	0.0	7.0	32.4	30.0	0.0	37.3	29.4
12	<12>	0.0	0.0	0.0	7.1	32.4	30.0	0.0	37.3	29.4
13	<13>	0.0	0.0	0.0	7.0	32.4	30.0	0.0	37.3	29.4
14	<14>	0.0	0.0	0.0	6.9	32.4	30.0	0.0	37.4	29.4
15	<15>	0.0	0.0	0.0	6.9	32.4	30.0	0.0	37.4	29.4
16	<16>	0.0	0.0	0.0	6.9	32.4	30.0	0.0	37.4	29.4
17	<17>	0.0	0.0	0.0	6.8	32.4	30.0	0.0	37.5	29.4
18	<18>	0.0	0.0	0.0	31.8	66.6	0.0	0.0	48.2	33.6
19	<19>	0.0	0.0	0.0	28.8	62.6	0.0	0.0	46.9	33.1
20	<20>	0.0	0.0	0.0	24.4	56.6	0.0	0.0	45.0	32.3
21	<21>	0.0	0.0	0.0	24.4	56.6	0.0	0.0	45.0	32.3
22	<22>	0.0	0.0	0.0	27.4	60.6	0.0	0.0	46.3	32.8
23	<23>	0.0	0.0	0.0	23.7	55.6	0.0	0.0	0.0	32.2
24	<24>	0.0	0.0	39.2	55.1	70.0	0.0	0.0	0.0	37.5

Figure 5.15 Distribution of spinning reserve on Monday. Fourth example analysis: 'Market Driven Hydro-Thermal Scheduling'.

t		System Lambda <Kr/Mwh>	Reserve<Mw> Required	Reserve<Mw> Actual	Power reserve Lambda<Kr/Mw & h>	Deliveries<MW> Firm	Deliveries<MW> Flexible
1	< 1>	93.1	88.2	137.0	0.0	260.0	312.0
2	< 2>	91.9	86.7	147.3	0.0	240.0	321.7
3	< 3>	91.3	85.9	152.5	0.0	230.0	326.6
4	< 4>	91.3	85.9	152.5	0.0	230.0	326.6
5	< 5>	95.0	90.6	121.6	0.0	290.0	297.4
6	< 6>	101.9	94.7	94.8	3.6	370.0	244.0
7	< 7>	106.9	105.7	123.3	0.0	480.0	205.2
8	< 8>	106.6	121.4	171.5	0.0	580.0	207.2
9	< 9>	108.1	122.8	162.7	0.0	600.0	195.9
10	<10>	137.8	136.1	136.1	20.3	640.0	242.5
11	<11>	145.8	136.1	136.1	28.3	690.0	192.4
12	<12>	150.7	136.1	136.1	33.1	720.0	162.3
13	<13>	147.4	136.1	136.1	29.9	700.0	182.3
14	<14>	136.2	136.1	136.1	18.7	630.0	252.5
15	<15>	134.5	136.1	136.1	17.1	620.0	262.6
16	<16>	131.3	136.1	136.1	13.9	600.0	282.6
17	<17>	123.3	136.1	136.1	5.9	550.0	332.7
18	<18>	105.2	120.1	180.2	0.0	560.0	218.5
19	<19>	106.6	121.4	171.5	0.0	580.0	207.2
20	<20>	108.8	123.5	158.3	0.0	610.0	190.3
21	<21>	108.8	123.5	158.3	0.0	610.0	190.3
22	<22>	107.4	122.1	167.1	0.0	590.0	201.6
23	<23>	109.3	107.6	111.5	0.1	510.0	187.0
24	<24>	93.8	93.7	201.8	0.0	300.0	307.2

Figure 5.16 Summary results on the market solution for Monday. Fourth example analysis: 'Market Driven Hydro-Thermal Scheduling'.

)PERATIONAL DATA - REGION 8:
Jolume constraint cost<Kr/Mwh> : 127.7
Specified fuel/resource cost<Kr/Mwh>: 0.0
Resulting production cost <Kr/Mwh> : 127.7

	Generation<Mw>			Spot delivery<Gwh>			Local lambdas<Kr/Mwh>		
t	Prod	Pmin	Pmax	Load	Smin	Smax	P-balance	Spot	Prod
1 < 1>	0.0	0.0	150.0	100.0	0.0	100.0	93.4	1300.0	0.0
2 < 2>	0.0	0.0	150.0	100.0	0.0	100.0	92.1	1300.0	0.0
3 < 3>	0.0	0.0	150.0	100.0	0.0	100.0	91.5	1300.0	0.0
4 < 4>	0.0	0.0	150.0	100.0	0.0	100.0	91.5	1300.0	0.0
5 < 5>	0.0	0.0	150.0	100.0	0.0	100.0	95.3	1300.0	0.0
6 < 6>	0.0	0.0	150.0	100.0	0.0	100.0	102.1	1303.0	0.0
7 < 7>	0.0	0.0	150.0	100.0	0.0	100.0	107.2	1300.0	0.0
8 < 8>	103.1	50.0	150.0	100.0	0.0	100.0	106.6	1300.0	106.6
9 < 9>	104.4	50.0	150.0	100.0	0.0	100.0	108.1	1300.0	108.1
10 <10>	112.6	50.0	150.0	100.0	0.0	100.0	137.7	1300.0	117.4
11 <11>	112.7	50.0	150.0	100.0	0.0	100.0	145.8	1300.0	117.5
12 <12>	112.7	50.0	150.0	100.0	0.0	100.0	150.6	1300.0	117.5
13 <13>	112.7	50.0	150.0	100.0	0.0	100.0	147.4	1300.0	117.5
14 <14>	112.6	50.0	150.0	100.0	0.0	100.0	136.1	1300.0	117.4
15 <15>	112.6	50.0	150.0	100.0	0.0	100.0	134.5	1300.0	117.4
16 <16>	112.6	50.0	150.0	100.0	0.0	100.0	131.3	1300.0	117.4
17 <17>	112.5	50.0	150.0	100.0	0.0	100.0	123.3	1300.0	117.3
18 <18>	101.8	50.0	150.0	100.0	0.0	100.0	105.2	1300.0	105.2
19 <19>	103.1	50.0	150.0	100.0	0.0	100.0	106.6	1300.0	106.6
20 <20>	105.0	50.0	150.0	100.0	0.0	100.0	108.8	1300.0	108.8
21 <21>	105.0	50.0	150.0	100.0	0.0	100.0	108.8	1300.0	108.8
22 <22>	103.7	50.0	150.0	100.0	0.0	100.0	107.4	1300.0	107.3
23 <23>	0.0	0.0	150.0	100.0	0.0	100.0	109.5	1300.0	0.0
24 <24>	0.0	0.0	150.0	100.0	0.0	100.0	94.0	1300.0	0.0

Figure 5.17 Detailed results from Monday's clearing of one of the regional markets; here Region '8' (which is one of the two 'hydro production regions'.) Fourth example Analysis: 'Market Driven Hydro-Thermal Scheduling'

MARKET SOLUTION , summary overview:

Contractual load<Gwh>	:	81.71
Spot power delivery<Gwh>	:	38.80
Transmission losses<Gwh>	:	0.20
Total generation<Gwh>	:	120.72

TOTAL PLANT RESOURCE USE, PRODUCTIONS, AND PRODUCTION CONSTRAINTS :

Plant <No>	Resource use <Gwh>	Production <Gwh>	Constraint <Gwh>
1	78.14	33.60	600.00
2	70.20	28.05	500.00
3	45.34	16.70	200.00
4	57.60	18.96	200.00
5	18.14	5.36	200.00
6	2.52	1.20	200.00
7	0.00	0.00	200.00
8	12.73	10.84	12.73
9	7.49	6.00	7.49

Figure 5.18 Summary results on power generation and power delivery

APPENDIX 1

Modelling aspects

APPENDIX 1.1 : Modelling of price-sensitive demand

Price sensitive demand for given hour and location is presumed described in terms of a linear incremental price vs. volume curve. See Figure A1.1.

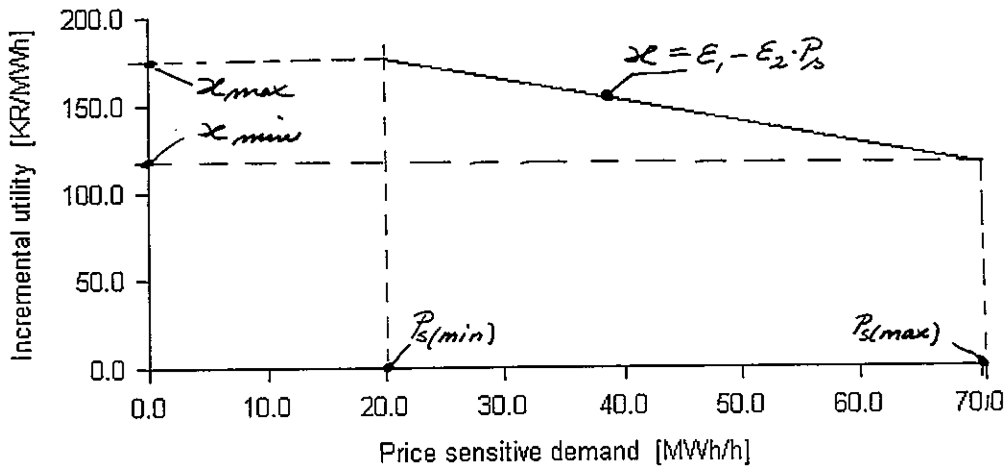


Figure A1.1 : Incremental utility as function of delivery. Presumed linear form in description of price sensitive demand

We see that incremental cost – or incremental utility – in that case can be expressed by (A1.1):

$$\kappa = \varepsilon_1 - \varepsilon_2 \cdot Ps \qquad [\text{NOK/MWh}] \qquad (A1.1)$$

A segment of price-sensitive demand as illustrated in Figure A1.1 , is defined by the four parameters (κ_{max}, κ_{min}, Ps(min), Ps(max)). From these input values, the coefficients ε_1 and ε_2 of (A1.1) , are given as follows:

$$\varepsilon_1 = (Ps(max) \cdot \kappa_{max} - Ps(min) \cdot \kappa_{min})/(Ps(max) - Ps(min)) \qquad (A1.2)$$

$$\varepsilon_2 = (\kappa_{max} - \kappa_{min})/(Ps(max) - Ps(min)) \qquad (A1.3)$$

The utility of price-sensitive delivery is found by integrating incremental utility:

$$U = \int_{Ps(min)}^{Ps} (\varepsilon_1 - \varepsilon_2 \cdot Ps) \cdot dPs + U_{Ps(min)} \qquad [\text{KR/h}]$$

Where;

$U_{Ps(min)}$ = Utility when delivery is at its minimum [KR/h]

Following integration we get:

$$U = \varepsilon_1 \cdot Ps - 0.5 \cdot \varepsilon_2 \cdot Ps^2 + [\, U_{Ps(min)} - (\varepsilon_1 \cdot Psmin - 0.5 \cdot \varepsilon_2 \cdot Psmin^2)\,]$$

Or in compact/summary notation:

$$\underline{U = \alpha_0 + \alpha_1 \cdot Ps - \alpha_2 \cdot Ps^2} \quad \text{(A1.4)}$$

Where;

$$\alpha_0 = U_{Ps(min)} - (\varepsilon_1 \cdot Psmin - 0.5 \cdot \varepsilon_2 \cdot Psmin^2) \quad \text{(A1.5)}$$

In the general case Upsmin must be specified separately. Our premises here is that the linear form of (A1.1) is valid from Ps=0. This yields $\alpha_0=0$, since then U=0 for Psmin=0

$$\alpha_1 = \varepsilon_1 \quad \text{(A1.6)}$$

$$\alpha_2 = 0.5 \cdot \varepsilon_2 \quad \text{(A1.7)}$$

Utility (U) of price-sensitive delivery (Ps) at given bus in specified time interval , when segment of market is described in terms of parameters [κ_{max}, κ_{min},Ps(min), Ps(max)]

Illustration:
At given bus in specified hour, the price-sensitive part of demand is characterized by the following parameters:

Ps(min) = 20 MWh/h
Ps(max)= 120 MWh/h
κ_{max} = 200 KR/MWh
κ_{min} = 100 " "

From (A1.2) – (A1.3):

$\varepsilon_1 = (120 \cdot 200 - 20 \cdot 100)/(120 - 100) = 220$
$\varepsilon_2 = (200 - 100)/100 = 1.0$

giving this description of incremental utility vs. delivery:

$\underline{\kappa = 220 - Ps}$

From (A1.5) – (A1.7), and our assumption that (A1.1) is valid from Ps=0:

$\alpha_0 = 0$
$\alpha_1 = 220$
$\alpha_2 = 0.5$

giving this sought description of utility vs. price-sensitive delivery at the bus;

$\underline{U = 220 \cdot Ps - 0.5 \cdot Ps^2}$ [KR/h] (A1.8)

Check of utility polynomial (A1.8) :

1) For Ps = Ps(min) = 20 MWh/h, we see from Figure A1.1 that
   $U=0.5\cdot(20.0-0.0)\cdot(200+220)=4200$

   Inserted in (A1.8):
   $U = 220\cdot20-0.5\cdot20^2 = 4200$ QED!

2) For Ps = Ps(max)=120 MWh/h, we expect according to Figure A1:
   Umax= $(220+100)\cdot0.5\cdot(120-0)=19200$ KR/h

   Inserted in (A1.8) we get:
   Umax = $220\cdot120-0.5\cdot120^2 = 19200$ KR/h QED!

Illustration of practical forecast of price-sensitive power market.

A best 168h estimate of price-sensitive demand for a power system comprising 8 load regions, is sought illustrated by Tables A1.1 – A1.2:

For region 1 - 7, the market description is forecasted the same Monday through Friday. The weekend market is different with somewhat reduced valuation of electrical power. We notice that the utility of electrical power is forecasted highest during daytime hours from 10 to 17 o'clock; in these hours on workdays, the marginal utility varies in the range 160 to 110 KR/MWh, for delivery from zero to 45 MWh/h.

Table A1.1 : Price-sensitive power demand.
168h forecast for region 1-7

Description of power spot markets for given week for example hydro-thermal power system

Day	Hours	Price range (NOK/MWh)	Volume range (MWh/h)
Monday through Friday	01 - 09	120 - 80	0 - 45
	10 - 17	160 - 110	0 - 45
	18 - 24	120 - 80	0 - 45
Saturday through Sunday	01 - 09	100 - 60	0 - 45
	10 - 17	120 - 90	0 - 45
	18 - 24	100 - 60	0 - 45

The price sensitive market of region 8 is defined as special. Valuation is here set very high to demonstrate an aspect of importance when internalizing security constraints in the process of market clearing and pricing. Namely:

At (rare) times, part of firm or contractual delivery may have to be curtailed to sustain system security constraints. To properly deal with this, part of contractual demand can formally be treated as price-sensitive, with valuation set high to reflect the inconveniance of having interrupt of firm power delivery. To illustrate this, the formal market of region 8 represents the "last" 100MW of contractual delivery. The marginal valuation is set high for all workdays hours, with slightly reduced level during the weekend.

Table A1.2 : Price sensitive power demand.
Special demand of Region 8

Formal spotmarket to equivalence the 'last' delivered 100 MWh/h of systemwide price-insensitive demand

Day	Hours	Price range (NOK/MWh)	Volume range (MWh/h)
Monday through Friday	01 - 24	300 - 200	0 - 100
Saturday through Sunday	01 - 24	250 - 180	0 - 100

APPENDIX 1.2 : Modelling of generator units

Main component related aspects to deal with are *Continous Cost* , *Startup Cost* , *Minimum Up- and Downtimes*, and *Ramping* :

CONTINOUS COST
A second order polynomial is used to describe the unit's continous (or running) cost C_c (index 'c' for 'continous') , when production is in the range Pg(min) to Pg(max) :

$$\underline{C_c = \gamma_0 + \gamma_1 \cdot Pg + \gamma_2 \cdot Pg^2} \qquad \text{(A1.9)}$$

where;

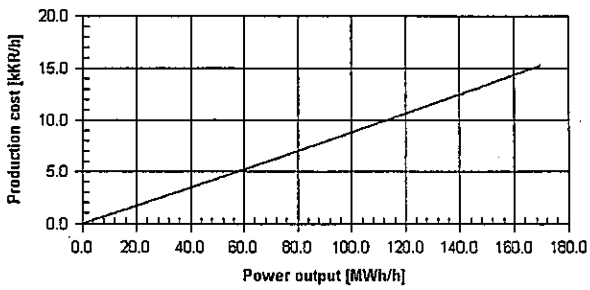
C_c : the unit's running cost; fuel+other variable cost [KR/h]
($\gamma_0, \gamma_1, \gamma_2$) : cost coefficients of generator unit
Pg : power output from unit [MWh/h]
Pg(min) : minimum production > 0 , if connected [MWh/h]
Pg(max): maximum production [MWh/h]

The present scheme of analysis applies two different 'versions' of (A1.9) for a given unit, during the process of clearing the power market and scheduling of production : A **'Specific Cost-model'** ('SC-model') valid for a limited output range (and thus a target for updating during the process of deciding on an initial committment plan on Level I analysis) , - and an **'Instantaneous model'** ('I-model') valid for the entire output range for the unit, once it has been decided committed.

The 'Specific Cost'- resp. 'Instantaneous' model version for describing *continous cost* is exemplified in (A1.10), resp. (A1.11) :

$$\underline{C_c = \gamma_0 + \gamma_1 \cdot Pg + \gamma_2 \cdot Pg^2}$$

Where ;
C_c = running cost (kNOK/h)
γ_0 = 0 , per definition
(γ_1, γ_2) = (0.0857, 2.52e-5) ;parameters to give best cost fit over anticipated production range
Pg(min) = 0 , per definition
Pg(max)= max. production [MWh/h]



(A1.10)

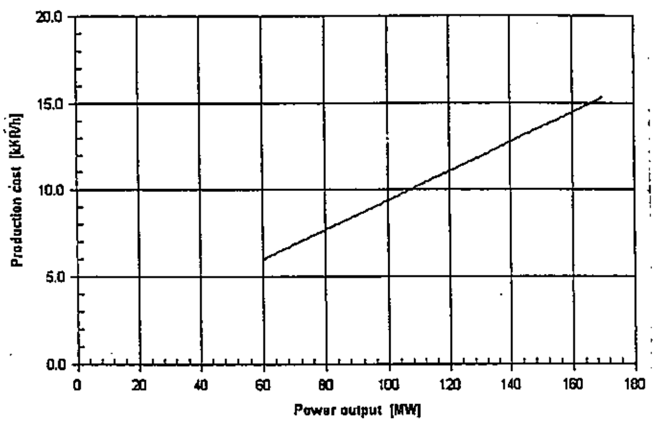
' Specific Cost model' to describe continous cost of a 170 MW high effiency, coal fired production unit over (anticipated) production range 120 – 170 MWh/h.

The *'Specific Cost'* modelling of generators is applied on Level I analysis, where a main task is to arrive at the best possible *continous variable* production schedule for the week (as basis for definition of an initial committment plan). By allowing for generator output over the theoretical range from zero to Pg(max), the 'Specific Cost' modelling may provide for systematic handling of the tradeoff between cost of start and costs related to operation and constraints, in the process of trying to approach the optimal matching of power supply and demand.

$\underline{C_c = \gamma_0 + \gamma_1 \cdot Pg + \gamma_2 \cdot Pg^2}$

where ;

C_c = running cost (kNOK/h)
$(\gamma_0, \gamma_1, \gamma_2)$ = (0.918, 8.349e-2, 6.555e-6); parameters to give best cost fit over total production range
Pg(min) = 0 if offline, and nominal value, if online
Pg(max) =max. prod. [MWh/h]



(A1.11)

'Instantaneous model' to describe continous cost of the above 170 MW generator unit over its entire production range 60 – 170 MWh/h.

The '*instantaneous*' model description is always to apply on Level II analysis, where the task is to 'slim' a current committment plan until a best possible/final scheduling and market solution scheme is found

Table A1.1 and A1.2 exemplifies polynomial coefficients for the *'Specific Cost'*- resp. '*Instantaneous*' model description of fuel consumption as function of power output, for the 7 thermal generators. The cost of fuel is also given. Table A1.3 and A1.4 summarizes the corresponding description for the two hydro units, here compactly denoted as unit '8' and '9'. Table A1.5 presents the cost of start for all nine production units.

Table A1.1 Example polynomial description of thermal units. *'Specific Cost'* models

Gen. No	$P[MWh_t/h]=C_0 + C_1 \cdot P[MWh_{el}/h] + C_2 \cdot P^2[MWh_{el}/h]$ C_0	C_1	C_2	Fuel cost [NOK/MWh$_t$	Output range P_{low} [MW]	P_{high} [MW]
1	0.0	2.2222	5.1680e-4	36.0/coal	150	200
2	0.0	2.3809	7.0028e-4	36.0/coal	120	170
3	0.0	2.5641	1.1550e-3	36.0/coal	90	120
4	0.0	2.9851	1.2322e-3	36.0/coal	70	110
5	0.0	3.2258	2.1505e-3	36.0/coal	35	65
6	0.0	1.9825	1.3526e-3	64.0/gas	30	50
7	0.0	3.6390	5.3911e-3	64.0/gas	10	15

Table A1.2 Example polynomial description of hydro units. *'Specific Cost'* models

Gen. No	$P[MWh_{nat}/h]=C_0+C_1 \cdot P[MWh_{el}/h]+C_2 \cdot P^2[MWh_{el}/h]$ C_0	C_1	C_2	Output range P_{low} [MW]	P_{high} [MW]
8	0.0	1.1718	1.8676e-4	80	120
9	0.0	1.1405	3.7548e-4	35	55

Table A1.3 Polynomial description of thermal units. 'Instantaneous' models

Gen. No	$P[MWh_t/h]=C_0+C_1 \cdot P[MWh_{el}/h]+C_2 \cdot P^2[MWh_{el}/h]$ C_0	C_1	C_2	Fuel cost [NOK/MWh$_t$	Output limits Pmin [MW]	Pmax [MW]
1	20.0	2.1784	2.3573e-4	36.0/coal	50	200
2	25.5	2.3191	1.8207e-4	36.0/coal	60	170
3	24.0	2.2276	2.2923e-3	36.0/coal	40	120
4	105.0	1.3273	6.6869e-3	36.0/coal	50	150
5	25.0	2.5952	4.8809e-3	36.0/coal	30	100
6	9.0	1.7392	1.8464e-3	64.0/gas	30	60
7	10.0	2.8197	2.1061e-3	64.0/gas	10	50

Table A1.4 Polynomial description of hydro units. 'Instantaneous' models

Gen. No	$P[MWh_{nat}/h]=C_0+C_1 \cdot P[MWh_{el}/h]+C_2 \cdot P^2[MWh_{el}/h]$ C_0	C_1	C_2	Output limits Pmin [MW]	Pmax [MW]
8	82.5	-0.0773	4.4242e-3	50	150
9	31.5	-0.0030	1.0225e-2	20	70

Table A1.5 Cost of start of generating units

Gen No	Cost of start [NOK/start]
1	58605
2	53550
3	46703
4	65455
5	54000
6	4608
7	1000
8	500
9	125

STARTUP COST

The unit startup cost can in many cases be approximated by the following equation:

$$S = S_0 \cdot (1 - a \cdot e^{(-T_{down}/T_0)}) \qquad (A1,12)$$

where;

S = cost of starting considered unit (NOK)
S_0= cost of starting cold unit (NOK)
a= pu cost coefficient
T_{down}= downtime of considered unit (h)
T_0 = boiler cool-down time constant (h)

Internalizing the time-dependency of startcost in the optimization is a complex matter that (per today) cannot be afforded unless major system simplifications are made. To illustrate : The dependency can be included in UNIT COMMITTMENT analyses of the day, provided the power transmission network is reduced to one or a very few buses, and volume constraints (on e.g. the use of fuel, emissions to air, water discharge) are absent or only very few in number.

The present scheme of analysis neglects at the outset the temperature -dependency of the startcosts and treats the costs as constant values. This is deemed appropriate in view of

1) the need for giving priority to other features such as modelling of power transmission, clearing of dispersed power markets, and volume constraints over time,

2) the presumed capability of (many) reservoir hydro units to provide for 'final tuning' of the power market balance, and thus opening up for harvesting part of the benefit that may accrue from optimally observing time-dependency of startcosts.

To demonstrate the relevance of the general algorithms (A1.15) that are applied to model the cost of start of generator units, the operation of an arbitrary unit over two consecutive time intervals t and (t+1) – and next over the period of analysis - is discussed in the following:

A continous variable $\delta t(t+1)$ in the range 0.0 – 1.0 is introduced to describe the 'part of the start process' that – in a continous frame of reference - may accrue from time interval t to interval (t+1). The corresponding incremental cost of start is defined by the product [$\delta t(t+1) \cdot S$], and added to the criterion of performance as a cost element.

To govern the variation of $\delta t(t+1)$ over the period of analysis, eqns. (A1.13) - (A1.14) are introduced:

$$pt - p(t+1) + \delta t(t+1) \geq 0 \qquad (A1.13)$$

$$\delta t(t+1) \geq 0 \qquad \text{(A1.14)}$$

where;

pt = Pt/Pt(max) = pu production in time interval t
p(t+1) = pu production in time interval (t+1)
δt(t+1) = real variable, range 0.0 – 1.0

(A1.14) is included to prevent the product (δ·S) from becoming an artificial income, as it will be if δ turns negative.

Main consequences to observe from (A1.13) – (A1.14) :

- If pt = 0.0 and p(t+1) = 1.0 , corresponding to startup and subsequent operation at full output, δt(t+1) will have to equal 1.0 in order to make (A1.13) valid. Full start cost (1.0·S) will then contribute to the criterion, - which is correct.

- If pt > p(t+1), any value of δt(t+1) ≥ 0.0 will make (A1.13) valid. The criterion itself will 'see to' that the value δt(t+1) = 0.0 is chosen, - leading to zero start cost. This is correct as no startup is implemented

- If pt = 0.0 and p(t+1) = (say) 0.65, δt(t+1) will take on the value 0.65 to secure feasibility of (A1.13). Criterionwise, an amount (0.65·S) will be added to the cost.

  If later in that production cycle, p increases further so that the accumulated δ-values amount to 1.0 , full start cost is correctly included by way of incrementally adding up to full cost of start.

- If – in general- the accumulated value of δ over the characteristic cycle of production is above zero but less than 1.0, the solution found is infeasible with respect to handling of the cost of start of the considered generator unit. Feasibility can then be attained via an iterative solution process as follows:

  1) Solution of the formal problem with current value of cost of start of units. The first time, this means nominal cost S

  2) Check of feasibility with respect to handling of cost of start. If ok: exit. If not ok, a new formal cost of start is defined so that the product $(\Sigma\delta)\cdot S_{new}=S$, Then return to 1)

The process is terminated when consistency of solution is attained. For a given generator, three characteristic 'exit' situations can occur with respect to accumulated deltas :

- Accumulated deltas = 0.0 , implying that the unit is offline, or is operated at constant or diminishing output.

- Accumulated deltas = 1.0 , implying start of unit and loading it up to nominal output in the cource of its characteristic production cycle.

- Accumulated deltas < 1.0 , and greater than zero , implying start of unit, but loading it up to less than maximum capacity. In the course of the iterative solution process, a 'new' or equivalent cost of start (Startcost(new)) is found such that the accumulated product $[(\Sigma\delta)\cdot \text{Startcost}_{new}]$ just equals the actual/ given cost of start.

Eqn. (A1.13) defines a linear relationship between increment of pu power output p and associated pu increment δ of cost of start. This relationship is illustrated by the straight line in the diagram of Figure A1.2 . A nonlinearity can be introduced in such a way that a considerable part of the cost of start is incurred even for small 'trigging' increments of pu power output. This will in principle tend to fit with practise, where full cost of start is suffered once production – however small – is initiated from the unit. Replacing the term pu power production p in (A1.13) by the modified pu expression p^{μ} , where μ is a constant less or equal to 1.0, the sought nonlinear effect can be achieved. Figure A1.2 illustrates how the modified term p^{μ} varies as a function of p, for $\mu = 1.0$, 0.7 and 0.1, respectively. We see e.g. that for $\mu = 0.1$, ca. 80% of the cost of start would be incurred by 10% increase of power output. The nonlinearity caused by applying the exponential form p^{μ} , is – for reasons of further research – introduced into the main scheme of analysis.

Eqns. (A1.15) show the set of constraints that – for a given generator - replaces (A1.13)-(A1.14), when introducing the exponent :

$p_t^{\mu} - p_{(t+1)}^{\mu} + \delta_{t(t+1)} \geq 0.0$	t=0,1,2,...nt	a)	
$\delta_{t(t+1)} \geq 0.0$	t=0,1,2,...nt	b)	
$0 < \mu \leq 1.0$		c)	(A1.15)
Contribution to cost of start:	$\sum_{t=0}^{nt} C\cdot\delta_{t(t+1)}$	d)	

Equations for handling of the cost of start of generator unit in a computational regime comprising only continous variables

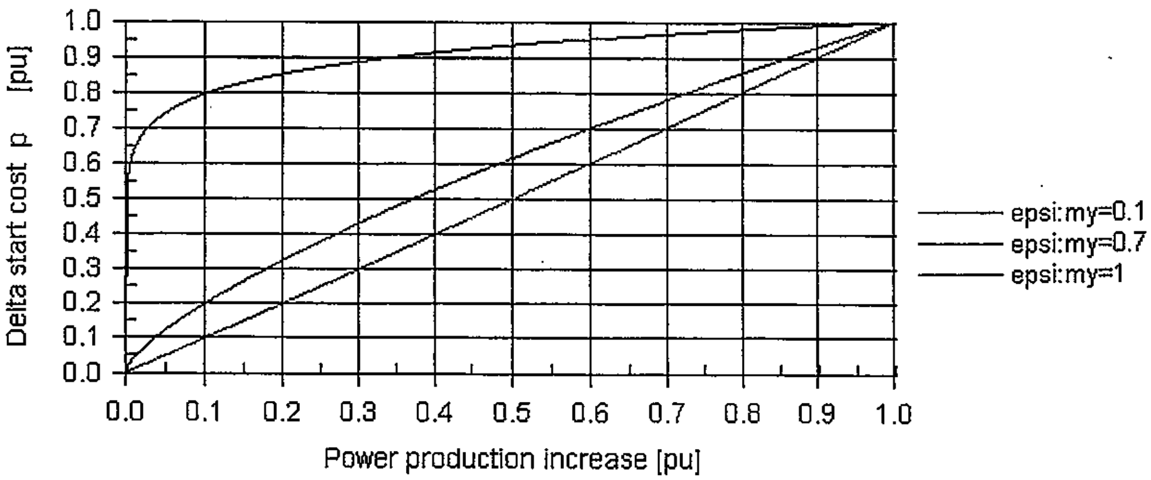


Figure A1.2 : Illustration of different relationships between increment of power output p and increment of term p^{μ}

MINIMUM UP- AND DOWNTIMES

If a unit (for thermal stress-, staffing-, logistic-, or other reason) must be 'ON' a minimum no. of hours before it can be shut off, then a minimum uptime specification must be observed. Minimum downtime is similarly the no. of hours a unit must be off-line before it can be brought online again.

The minimum uptime constraint for a generating unit can formally be expressed as follows:

$$[\ Ton_{(t-1)} - Tup]\cdot[X_{(t-1)} - X_t\,] \geq 0 \qquad \text{(A1.16)}$$

where;

- Tup : minimum uptime of unit (h)
- $Ton_{(t-1)}$: continous uptime of unit, including time interval (t-1)
- X_t : 0 –1 (integer) decision variable of unit , time interval t :
  "0" means "unit is offline"
  "1" means "unit is online"
- $X_{(t-1)}$: 0 – 1 (integer) decision variable of unit, time interval (t-1)

The minimum downtime constraint:

$$[Toff_{(t-1)} - Tdown]\cdot[X_t - X_{(t-1)}\,] \geq 0 \qquad \text{(A1.17)}$$

where;

- $Tdown$: minimum downtime of unit (h)
- $Toff_{(t-1)}$: continous downtime of unit, including time interval (t-1)

Restrictions on up- and downtimes are generally difficult/expensive to handle rigorously, as integer variables have to be introduced. The computational consequences are about as complex as those indicated above for time-dependent startcosts.

The present scheme of analysis neglects requirements to minimum up- and down-times. This is done in view of

1) *the tendency of solutions to fulfill these requirements in any case (as dictated by the criterion of economy of operation)*

2) *the need for giving priority to other features such as modelling of power trans-mission, distributed market clearing and volume constraints over time .*

3) *the presumed capability of (many) reservoir hydro units to provide for 'final tuning' of the power market balance, and thus also secure feasibilty with respect to mini-mum up- and downtimes.*

RAMPING

Ramp rate limits restrict the change of generator power output from one time period to the next. The limits are given by physical limitations.

The ramping constraint may compactly be expressed in this way:

$$|Pg_t - Pg_{(t-1)}| \leq \Delta Pg_{max} \qquad \text{(A1.18)}$$

where;

Pg_t, $Pg_{(t-1)}$: power output from considered unit in time interval t and (t-1), respectively

ΔPg_{max} : rate limit for chosen time interval size

(1.18) says that ramping constraints may apply both to increase and decrease of output from generating units. For thermal units the constraint will in many cases apply only to regimes of increase of production.

For many thermal units the rate limit is in the range 2 – 4 % of nominal power output per minute. For such units (A1.18) will be non-binding if a time resolution of e.g. 1 hour is chosen in UNIT COMMITTMENT (UC), since power output may reach any desired value within nominal range, in less than an hour. Restrictions that a priori are known to be non-binding, can always be ignored.

In the present scheme of analysis time resolution is chosen to be one hour. Hence it is presumed that ramping constraints need not be observed.

Ramping constraints may however conveniantly be implemented in the present scheme of optimization. For the (thermal) case of limitations only on the rate of increase of output, the necessary problem variables are already there in terms of Delta(i,j). See eqns. (4.10)- (4.13). It only remains to individualize the limitation of Delta(i,j) which presently is set to default value 1.0 for all units in all time intervals. (The latter setting is the relevant value when ramp rate constraints per definition are non-binding, and the only function of Delta( ) is monitoring/controlling the cost of start)

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