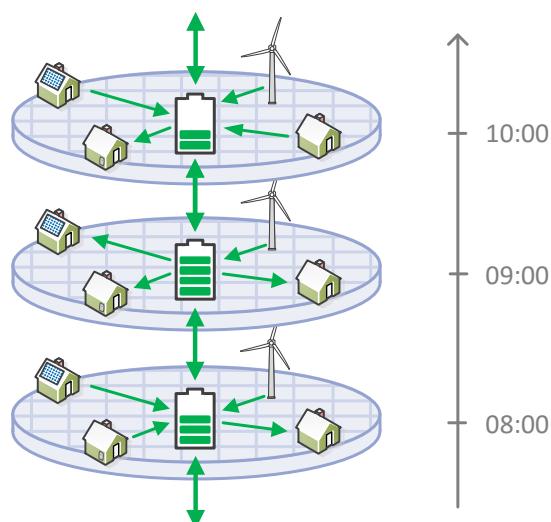# Report

Optimal power flow methods and their application to distribution systems with energy storage

A survey of available tools and methods

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ABSTRACT
Optimal power flow (OPF) problems is a large class of generally nonlinear, nonconvex, large-scale optimization problems for power system applications. A vast number of different methods for solving these problems have been developed over the years, and some of these methods are implemented in software tools that are either commercially of freely available. This report presents a survey of available methods and tools for solving OPF problems, having particular emphasis on new applications in distribution systems with energy storage. The expected large-scale integration of energy storage in distribution systems gives rise to new, multi-period OPF problems that require efficient and robust solution methods as well as practical considerations. Surveying both the scientific and industrial state of the art, the report concludes with possible implications for the development of new methods for optimizing the use of energy storage in distribution systems.

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1   Introduction

Optimal power flow (OPF) problems is a large class of optimization problems for power system applications where one requires that any solutions to the problem respect the power flow equations for the system (Frank and Rebennack 2012, p. 2). Broadly speaking, the purpose of solving OPF problems is to optimize the operation or planning of a power system according to a defined criterion while meeting all system constraints, including the power flow constraints. Generally, these problem are nonlinear, nonconvex, large-scale optimization problems, covering a wide range of possible objectives, decision variables and constraints (Capitanescu, Glavic et al. 2007). Over the course of 50 years, thousands of research works have been carried out on the solution of such problems (Stott and Alsaç 2012). There is a vast amount of different solution methods, and some of these methods are implemented in commercially available software tools. Technological developments and trends in the power system can drive the need for new models and solution methods for OPF. One recent development is the increasing deployment of energy storage systems (e.g. batteries) in power distribution systems. This report surveys the available methods and tools for solving OPF problems. Special emphasis is put on the application where the use of energy storage in distribution systems is optimized.

The report is organized as follows. In the remainder of Section 1, we introduce the background for and the scope of the survey. The rest of the report can be divided in two main parts: Sections 2 and 3 focus on methods available in the scientific literature, and Sections 4 to 6 focus on the application of such methods and commercially available tools in which they are implemented. More specifically, Section 2 presents the main categories of available methods for solving OPF problems generally, and Section 3 surveys OPF models and solution methods applied to grid-integrated energy storage systems. Section 4 presents a survey of available software tools implementing OPF methods. In Section 5, we analyse and discuss the application of OPF methods by synthesizing the findings for methods available in the scientific literature and implemented in software tools. Section 6 summarizes the findings of the previous sections and remarks on what implications these findings could have on the development of improved methods for optimizing the use of energy storage in distribution systems. This final section is also intended to read as an executive summary of the report.

1.1   Background and motivation

This survey was carried out as part of the research activity "Market simulator with storage and AC load flow" in the strategic institute project (I-SIP) "New optimization methods for production and grid planning". The purpose of this project was to build competence believed to be strategically important for the research institute in the future. In addition to building new competence on emerging methods, this also includes consolidating and rebuilding competence from past activities within the research institute. This follows from the realization that competence that was generated years or decades ago is at risk of withering unless it is transferred to more junior research scientists. As documentation of the competence building activity, the hope is that this report will serve as a useful introduction to new research scientists and students working with the research institute.

The emerging methods alluded to in the previous paragraph are related to the use of energy storage systems. The rapid decrease in the cost of energy storage technologies such as batteries has made them increasingly relevant for a number of distribution system applications. They can e.g. be used as an alternative to grid reinforcements for improving the hosting capacity for distributed generation or new and challenging loads (Luo, Wang et al. 2015, Rocky Mountain Institute 2015, Global Smart Grid Federation 2016, Malhotra, Battke et al. 2016). This is part of a wider trend in the transition towards more flexible, "smarter" distribution systems, involving also the introduction of other flexible resources such as demand response and voltage regulation.

One part of this research project was to consider what requirements these trends are expected to put on methods for optimizing the operation and planning of distribution systems in the future. From the optimal power flow perspective, the presence of energy storage requires that OPF methods somehow capture the time dependencies that are introduced to the problem. That is, energy storages couple the different time steps through intertemporal constraints, making it unfavourable to consider each time step separately as is conventionally done in OPF. The OPF problem is thus transformed to a multi-period, or dynamic, OPF problem. Furthermore, distributed generation and new forms of loads are often introduced to distribution systems at relatively low voltage levels. Hence, one can generally not resort to linearizing and thus simplifying the power flow problem, but has to take into account AC power flow in the optimization.

The background of this survey of OPF methods and tools was a desire to have a better overview of what software tools are available (commercially or openly) for performing optimal power flow analyses. The motivation was twofold: 1) To put the work carried out in the research activity in context with respect to both industry applications and the scientific state of the art, and 2) to create the basis for developing new research activities and projects on topics related to the optimization of distribution systems with energy storage. Specifically, one objective was to assess which methods and approaches one should use and what practical considerations one should make in developing new methods and tools for such applications.

The industrial state of the art was surveyed to assess which OPF methods are efficient and reliable enough to be commercially applicable to practical OPF problems. To which extent methods described in the research literature had been implemented and utilized industrially is not clear from academic sources alone. Furthermore, it was also of interest to see if available software tools already include OPF functionality tailored to distribution systems with energy storage. As potential new research projects in this case primarily are intended to benefit Norwegian grid companies and the Norwegian society, the perspective taken is mostly that of the Norwegian power system. The scientific state of the art was surveyed to better be able to assess the advantages and disadvantages of the algorithms implemented in available OPF software tools. Surveying the research literature was also necessary to better understand the principles behind state-of-the-art OPF methods.

1.2   Definitions and delimitation

There seems to be little general agreement on the terminology in the OPF research literature (Capitanescu 2016), and confusion may arise since different terms are often used for the same methods and different classifications are used to group different methods in categories. As is also emphasized by the MATLAB Optimization Toolbox documentation[1], "there exist many different methods that are very similar in structure but that are described in widely different terms". Therefore, we start by defining different terms that are being used for OPF problems and discuss which variants of OPF are and which are not included in the scope of the survey. Furthermore, throughout this survey, we try to include alternative terms used for the methods that are presented, but we do not claim that this nomenclature is complete.

In this document, we define *optimal power flow* as any optimization problem where the set of equality constraints includes power flow equations (Wood and Wollenberg 1996, p. 514, Frank and Rebennack 2012, p. 2). This distinguishes OPF from the problem of *optimal economic dispatch* (ED) of power generation, which does not include such power flow constraints, and of which OPF therefore can be seen to be an extension. Indeed, in many of its applications, the OPF problem is framed as an optimal *unit dispatch* problem. Another class of optimization problems is optimal *unit commitment* (UC), which includes binary (i.e. integer) decision variables for whether generators are started up (being "on" and "committed" to

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[1] http://se.mathworks.com/help/optim/ug/constrained-nonlinear-optimization-algorithms.html#f26684

dispatching power). In the literature, there does not seem to be universal agreement on whether UC problems are defined to also include optimal power flow or whether OPF problems are defined to also include unit commitment. In this report, we discuss neither unit commitment, nor any other OPF problems with integer decision variables or any *mixed-integer* nonlinear programming (MINLP) methods. *Security-constrained OPF* (SC OPF) is a class of OPF problems including a set of constraints for each of a set of contingencies. Some define OPF to include such contingency constraints (Stott and Alsaç 2012, Capitanescu 2016), however in this report, SC OPF will not be treated specifically, and neither will we discuss other related problems such as *security-constrained economic dispatch* (SC ED) or *security-constrained unit commitment* (SC UC).

In this document, we will focus on AC OPF, i.e. OPF formulations where the full AC power flow equations are used for the constraints and not approximations as in e.g. DC OPF. As long as there is no risk of ambiguity, we will refer to AC OPF as simply OPF. Some references in the literature also distinguish between *optimal real power flow* and *optimal reactive power flow*, where reactive power injections are excluded from the decision variables in the former and vice versa. Alternative terms used for optimal reactive power flow are *reactive power dispatch*, *VAR control* (Frank and Rebennack 2012, Section 5.2.3), *voltage-VAR optimization* (Wood and Wollenberg 1996, p. 516) or Volt/VAR optimization (CYME International T&D Inc. 2006). In this document, we will not consider any distinction between or decoupling in real and reactive subproblems, but we consider the general case where both real and reactive power can be included among the decision variables. This coupled problem is sometimes (Gabash and Pu 2012) also referred to as *active-reactive optimal power flow* (A-R OPF). *Reactive power planning*, an extension of optimal reactive power flow (Frank and Rebennack 2012, Section 5.2.4), will not be discussed explicitly.

We use the term *dynamic OPF* (DOPF) to refer to multi-period OPF problems where the operation of the power system is to be optimized over a time horizon of several time steps (periods) simultaneously. A DOPF problem arises when one has some kind of time-dependent behaviour of the power system and intertemporal constraints coupling the OPF problems for the individual time steps. This is the case e.g. for grid-integrated energy storage systems with a state of charge that must be held within a certain range during the entire time horizon. DOPF models for energy storage applications are treated in more detail in Section 3. Note that we use the term "dynamic" to refer to time dependence more generally rather than to power system dynamics phenomena on shorter time scales. In our case, the time scales of interest are typically hours rather than milliseconds.

What we refer to as the *conventional power flow* (PF) problem is the solution of the power flow equations without including any element of optimization. As indicated by the term, the prevalent solution methods for these power flow equations are conventional (i.e. typically based on the Newton method, also known as the Newton–Raphson method) and are not covered in this report. We will only discuss different approximations and solution approaches in passing where relevant.

The general methodology used for this survey has been to use the large number of already available OPF surveys and literature reviews as a starting point. The survey of solution methods reported in the research literature does not aim to be complete, but does aim to give an overview over the state of the art of solving OPF problems. For this reason, references of more historical interest (e.g. review papers older than 20 years) are only covered to a limited extent. For the more practical aspects of the development and application of OPF methods that are not often treated by the research literature, the survey is supplemented by interviews with experts at SINTEF Energy Research and NTNU (Norwegian University of Science and Technology). Since many aspects of OPF are well covered by existing surveys and literature reviews, some parts of this report serve more as a reference list than as a review. The main purpose of these sections is to direct the reader to references where the topic is covered in detail. Furthermore, the aim is to provide brief and intuitive rather than mathematically complete and rigorous explanations.

2 Survey of OPF methods

The survey of optimization methods for solving OPF problems is organized as follows. First, we provide a general overview of the research literature in Section 2.1. To further set the scene, we then discuss the different formulations of the OPF problems that the methods are designed to solve in Section 2.2. To present the different methods, we have chosen to make a distinction between how the methods handle the overall optimization problem, particularly the inequality constraints, and what types of search algorithms are included within the optimization framework. To this aim, we first introduce some basic concepts of unconstrained and constrained nonlinear optimization in Section 2.3 and discuss the general classification of OPF methods in Section 2.4. Building on this, we first present search algorithms used as part of OPF methods in Section 2.5, and then in Section 2.6, we present the main classes of OPF methods that these search algorithms are used in. Although the survey focuses on conventional, local, deterministic OPF methods, we briefly discuss a number of other methods in Section 2.7. The methods for OPF are compared and the state of the art is summarized in Section 2.8.

2.1 Overview of the literature

There exist a large number of literature surveys, reviews and state-of-the-art articles about optimal power problems and their solution methods. The survey in this report is in part based on a subset of these, focusing on the most recent. A chronological list of the review articles that have been surveyed is given in Table 1. (Where two references are listed as one entry, this is done because they are two-part review papers.) For other, older references that were not surveyed for this report we refer to the overview given in (Frank, Steponavice et al. 2012a, p. 224).

Table 1. Overview of surveys and reviews of literature on OPF methods.

Year	Reference
1991	(Glavitsch and Bacher 1991)
1999	(Momoh, Adapa et al. 1999, Momoh, El-Hawary et al. 1999)
2008	(Pandya and Joshi 2008)
2011	(Capitanescu, Martinez Ramos et al. 2011)
2012	(Frank, Steponavice et al. 2012a, Frank, Steponavice et al. 2012b)
2013	(Castillo and O'Neill 2013)
2016	(Capitanescu 2016)

We will not provide a historical overview of the development of OPF methods in this report, but some interesting accounts of the historical development can be found e.g. in (Glavitsch and Bacher 1991, Chapter 2, Cain, O'Neill et al. 2012, Chapter 2). Neither are we citing all the references that are usually regarded as the "standard" references on the inception of OPF as a field of research, but we confine ourselves to recommending the 1979 review by Carpentier (1979). For a thorough text-book introduction to optimal power flow, (Wood and Wollenberg 1996, Chapter 13) is recommended. The OPF chapter of the revised version of the same book (Sheblé, Wood et al. 2013) also gives a good introduction, but for the purposes of this survey, it does not add much information. A "primer" on optimal power flow intended to give an introduction to a reader without any background in electrical engineering can be found in (Frank and Rebennack 2012).

2.2 Formulations of the OPF problem

The "classical" example of an optimal power flow problem is the problem of dispatching electricity generation in a power system so as to minimize the total generation costs while keeping the system within its operational limits. Mathematically speaking, an OPF problem is formulated as an optimization problem defined by an objective function, a set of decision variables (also called control variables), and a set of constraints that include the power flow equality constraints. In the "classical" example, the objective function is the total generation costs in the system, the decision variables are the real power outputs of the generators, and operational limits typically include technical constraints such as real and reactive generation limits, bus voltage limits, and branch flow limits. The class of OPF problems comprises a wide set of different choices of objective functions, decision variables, and constraints. In this section, we will not review these in detail but rather summarize the most important features and refer to other OPF reviews and tutorials for more information.

The objective function represents the objective one seeks to achieve by solving the OPF problem. In addition to total generation costs, other commonly used objectives are to minimize the network losses, to minimize the total operating costs of the power system, and to maximize social welfare (and thus include the end users in the objective). Mathematically, we will formulate the OPF problem as an optimization problem with an objective function $f(x)$, $f : \mathbb{R}^n \to \mathbb{R}$, where the decision variable $x \in \mathbb{R}^n$ forms possible solutions to the power flow problem. In this report, we will assume that the OPF problem is formulated as a minimization problem so that an optimal power flow solution is one that minimizes $f(x)$ under a given set of constraints. Possible choices for and formulations of the objective function is discussed in (Glavitsch and Bacher 1991, Chapter 1.4, Cain, O'Neill et al. 2012, Chapter 5, pp. 22–23, Frank, Steponavice et al. 2012a, p. 228). Although the majority of OPF research work has focused on minimizing the power system operation cost due to real power dispatch, a number of practical problems faced by system operators also involve optimization of reactive power dispatch (Capitanescu 2016). For reactive power OPF problems, it is not obvious how the objective function should be formulated and the cost of reactive power dispatch quantified (Stott and Alsaç 2012).

Possible decision variables typically include real and reactive bus power injection and bus voltage magnitude and angle. Although all these variables (four for each bus in the system) can be regarded as decision variables from a mathematical point of view, one should keep in mind that only a subset are regarded as controllable by the system operator from a more practical point of view. Depending on the solution method, the remaining variables can be regarded as determined by the power flow equality constraints, thus reducing the number of degrees of freedom. Common decision variables controllable by the system operator include real and reactive power output of generators, the voltage magnitude of buses with voltage regulation, transformer tap settings, and load shedding. Other possible choices for the decision variables are reviewed in (Frank, Steponavice et al. 2012a, p. 227).

In addition to power flow equality constraints, OPF problems typically include inequality constraints such as real and reactive generation limits, bus voltage limits, and branch flow limits (either in terms of current, real or apparent power). Different constraints of the problem are presented in (Cain, O'Neill et al. 2012, p. 10, pp. 18–22) and (Glavitsch and Bacher 1991, Chapter 1.3), and (Frank, Steponavice et al. 2012a, p. 228) review different choices made in the literature.

The coordinates used to represent the complex decision variables (i.e. polar or rectangular coordinates for complex voltage and power) is another choice in formulating OPF as well as conventional power flow problems. Capitanescu, Glavic et al. (2007) prefer using rectangular coordinates for AC OPF since this makes the Hessian matrix elements for power flow equations, branch flow constraints and voltage constraints constant. However, they also point out that, empirically, the conventionally used polar coordinates for

voltages can result in equally efficient OPF algorithms. Although (Frank and Rebennack 2012, p. 20) agrees that rectangular voltage coordinates can be advantageous for some methods, according to them and (Frank, Steponavice et al. 2012a, p. 229), polar voltage coordinates are preferred for most OPF formulations. (Cain, O'Neill et al. 2012, p. 27) discusses possible reasons why rectangular voltage coordinates may be advantageous for OPF. The choice between rectangular and polar coordinates for the admittance seems to be of little importance for OPF applications (Frank and Rebennack 2012, p. 20). The advantages and disadvantages of different coordinate choices are also investigated for conventional power flow in (Tate 2005).

In conventional power flow, it is common to classify buses in three bus types: PV buses, PQ buses, and slack buses (Cain, O'Neill et al. 2012, p. 23, Frank and Rebennack 2012, p. 23). Real power and voltage magnitude are specified for PV buses. Real and reactive power is specified for PQ buses, and voltage magnitude and angle are specified for the slack bus. A set of power flow equations for the real power at the PV and PQ buses and the reactive power at the PQ buses is then solved to find the voltage angles at the PQ and PV buses and the voltage magnitudes at PQ buses. A slack bus is needed to have a reference point for the voltage angles. In addition, designating one bus as the slack bus is usually needed to account for network losses: When the real power generation is assumed known and specified for the PQ and PV buses, having one bus with unspecified real power generation is needed for allocating the (a priori unknown) power losses. (We do not consider here power flow formulations with distributed slack.)

Some OPF formulations maintain this distinction between PV, PQ and slack buses (Frank and Rebennack 2012, p. 24), but for many OPF methods this is not necessary. In contrast to conventional power flow, specifying a slack bus is not necessary in optimal power flow (Cain, O'Neill et al. 2012, Chapter 7, p. 25). The reason is that whereas real power at the PV and PQ buses and the reactive power at the PQ buses need to be specified before solving a conventional PF problem, these variables are generally decision variables when solving an OPF problem. Thus, allocation of losses to all generator buses is implicitly taken into account when the AC power flow equations are included as constraints to the OPF problem. Likewise, the slack bus voltage magnitude usually needs to be specified in conventional PF, but in OPF all bus voltage magnitudes may be treated as decision variables. However, one may also still have to choose one of the buses as giving (an arbitrary) reference point for the bus voltage angles. One may also still have to designate a slack bus within the OPF solution method if conventional PF calculations are part of the method.

Although we in this report only consider the full AC power flow equation, one should note that OPF problems are also solved using different approximations to the power flow equations. In the context of OPF, decoupled power flow is discussed in (Castillo and O'Neill 2013, pp. 26–27) and (Frank and Rebennack 2012, pp. 24–25). DC power flow is discussed in (Castillo and O'Neill 2013, p. 27), (Frank and Rebennack 2012, pp. 21–22) and (Frank, Steponavice et al. 2012a, pp. 229–230). The distribution factor model is also mentioned in (Castillo and O'Neill 2013, p. 27). For other linearizations of the DC power flow model, see e.g. (Coffrin, Van Hentenryck et al. 2012, Dorfler and Bullo 2013, Coffrin and Van Hentenryck 2014, Bolognani and Zampieri 2016). A method for treating active power losses in a DC OPF model is presented by Helseth (2012).

2.3   Basic concepts of nonlinear optimization

The methods used to solve OPF problems are the same kinds of methods that are used to solve other constrained nonlinear optimization problems, although substantial degrees of customization may be necessary to create an effective and efficient solution method for a particular OPF problem (Stott and Alsaç 2012). In turn, methods for solving constrained nonlinear optimization problems are typically based on

solution methods for *unconstrained* nonlinear optimization problems. Therefore, we start by outlining some of the basic concepts of unconstrained nonlinear optimization.

2.3.1 Basic concepts of unconstrained nonlinear optimization

Most deterministic, iterative solution methods for nonlinear and multi-dimensional optimization problems make steps in the solution space so that the step from iteration k to iteration $k + 1$ can be given by $\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + s_k \boldsymbol{d}_k$. Here, $\boldsymbol{d}_k$ denotes a direction in the multi-dimensional solution space, and s_k denotes the step length along this direction. Different methods differ primarily in how s_k and $\boldsymbol{d}_k$ are chosen for each iteration.

One can also distinguish two main approaches to choosing the step length: a) Approaches using line search and b) trust-region methods. Approach (a) is based on first choosing the step direction and then defining a *merit function* of the step length s_k in this direction, i.e. a function that should be reduced to approach the optimal solution. The chosen merit function can be e.g. the objective function itself, as is often done in unconstrained optimization, but is not required to be so. In some algorithms, we take steps that sometimes increase the value of the objective function, but the merit function is nevertheless decreased. A line search, using e.g. classical methods as those described in (Press, Teukolsky et al. 1992, Chapter 9–10, Nocedal and Wright 2006, Chapter 3), are then used to find a value of s_k that gives a suitable reduction in this merit function. An intuitive but potentially computationally demanding realization of this approach is to seek the step length that minimizes the objective function in the search direction. The *optimum gradient method* presented e.g. in (Dommel and Tinney 1968, Appendix III) is an example of such a line search algorithm applied to OPF. In modern line search methods it is required that the step length chosen in the line-search algorithm provides a reduction in the merit function that is sufficient according to the Wolfe conditions (Nocedal and Wright 2006, pp. 33–36). The Wolfe conditions are sufficient to guarantee that the line-search algorithm converges to a solution. An example of such an algorithm is presented together with references for further reading in (Nocedal and Wright 2006, Chapter 3.5).

Trust-region approaches (b) assumes that the solution method at each iteration is based on an approximation of the objective function. The key feature of the trust-region approach is first finding a region in the solution space where this approximation is in some sense trusted. For each iteration, such methods typically first update the trust region based on the difference between the actual improvement in the objection value for the previous step and the improvement estimated from the approximation for the objective function. This in turn determines the "trusted" step size for different step directions (Castillo and O'Neill 2013). For more information, see (Nocedal and Wright 2006, Chapter 4).

2.3.2 Basic concepts of constrained nonlinear optimization

When extending unconstrained optimization methods as described in the previous section to constrained nonlinear optimization problems, the differences between different methods are in how the constraints are taken into account. The most straightforward approach is to use various heuristics to explicitly restrict or reject the steps $s_k \boldsymbol{d}_k$ to keep within the feasible region of the solution space. More commonly, however, modern optimization methods consider constraints more implicitly. This can be done by reformulating the procedure for finding steps $s_k \boldsymbol{d}_k$ for each iteration such that one automatically keeps within the feasible region, cf. the description of reduced gradient methods in Section 2.6.1 and active set methods in Section 2.6.2. For some methods, the objective function is augmented by penalty or barrier terms that aid to avoid violation of the constraints, cf. e.g. the description of interior-point methods in Section 2.6.3. This approach to handling constraints is referred to as the sequential unconstrained minimization technique in (Hillier and Lieberman 2015, Chapter 13.9, p. 590).

$$\min f(\boldsymbol{x}), \tag{1}$$

$$\boldsymbol{g}(\boldsymbol{x}) = \boldsymbol{0}, \tag{2}$$

$$\boldsymbol{h}(\boldsymbol{x}) \leq \boldsymbol{0}. \tag{3}$$

Often, constraints are embedded in the solution methods by formulating a Lagrange function for the constrained optimization problem and thus transforming it into an unconstrained optimization problem:

$$\mathcal{L}(\boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = f(\boldsymbol{x}) + \boldsymbol{\lambda}^T \boldsymbol{g}(\boldsymbol{x}) + \boldsymbol{\mu}^T \boldsymbol{h}(\boldsymbol{x}). \tag{4}$$

Locally optimal solutions to this transformed optimization problem are however subject to the so-called Karush–Kuhn–Tucker (KKT) optimality conditions (Glavitsch and Bacher 1991, p. 19, Castillo and O'Neill 2013, p. 15):

$$\nabla f(\boldsymbol{x}) + \left(\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{x}}\right)^T \boldsymbol{\lambda} + \left(\frac{\partial \boldsymbol{h}}{\partial \boldsymbol{x}}\right)^T \boldsymbol{\mu} = \boldsymbol{0}, \tag{5}$$

$$\boldsymbol{g}(\boldsymbol{x}) = \boldsymbol{0}, \tag{6}$$

$$\boldsymbol{h}(\boldsymbol{x}) \leq \boldsymbol{0}, \tag{7}$$

$$\mathrm{diag}(\boldsymbol{\mu})\boldsymbol{h}(\boldsymbol{x}) = \boldsymbol{0}, \tag{8}$$

$$\boldsymbol{\mu} \geq \boldsymbol{0}. \tag{9}$$

See also (Nocedal and Wright 2006, Chapter 12.3) for more details. As explained later in this section, the majority of modern OPF algorithms are actually solving these KKT equations, considering constraints implicitly rather than minimizing the original objective function and considering constraints explicitly.

2.4   Classification of OPF methods

In this report, we will distinguish between *local* and *global* OPF methods. In contrast to local optimization methods, what we refer to as global optimization methods are methods that attempt to converge to the globally optimal solution of the problem rather than some locally optimal solution. Note that we are using a loose definition of global optimization where a guarantee that a global method will in fact converge to a globally optimal solution (or certify that the problem is infeasible) is *not* implied. We will focus on local OPF methods since they still appear to be most used for practical OPF applications (cf. the discussion in Section 5.1).

The classification used by Frank, Steponavice et al. (2012a) distinguish between *deterministic* optimization methods and *non-deterministic* or heuristic optimization methods. The methods referred to as deterministic methods can also be described as methods of local optimization. Non-deterministic methods are sometimes also referred to as stochastic methods, but to avoid ambiguity, one should point out that the stochasticity then refers to the solution method rather than the optimization problem. In other words, the optimization methods employ different heuristics based on stochastic processes, but the optimization problem is *not* necessarily a stochastic optimization problem, having stochastic input variables. Since the motivation for introducing stochasticity in the solution method is to make convergence to the globally optimal solution more likely, non-deterministic and heuristic methods can also be described as methods of global optimization. The review by Frank, Steponavice et al. (2012b) is fully devoted to non-deterministic, heuristic and hybrid methods applied to OPF, and we briefly mention different "global" approaches in Section 2.7.

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Focusing on local, deterministic optimization methods, there are different ways to classify the OPF solution methods. (Glavitsch and Bacher 1991, Chapter 3) distinguishes between two classes of OPF methods: Class A) "Power flow solved separately from optimization algorithm", and class B) "Power flow integrated in optimization algorithm". Although one should keep in mind that (Glavitsch and Bacher 1991) was written 25 years prior to the present survey, we still find this classification useful. In class A, conventional power flow calculations are performed iteratively through the solution procedure, and the actual optimization algorithms using a power flow solution (i.e. an operating point of the power system) as a starting point for its iterations. The sensitivity relations around the operating point one has solved the power flow for, i.e. the Jacobian and potentially the Hessian, can be used to formulate LP or QP optimization problems. In class B, on the other hand, the power flow is not solved explicitly, but rather implicitly by solving the equations for the KKT optimality conditions. In this way, the power flow equations are "embedded" in the solution methods as a subset of the constraints included in the Lagrangian formulation underlying the KKT equations.

Another classification is given by Capitanescu, Glavic et al. (2007), who distinguish between the following categories: 1) gradient methods, 2) sequential quadratic programming methods, 3) sequential linear programming methods, 4) Newton methods to the solution of the KKT optimality conditions, and 5) interior-point methods. The categories 1–3 of Capitanescu, Glavic et al. (2007) seem to overlap with class A of Glavitsch and Bacher (1991); class B roughly corresponds to category 4 since interior-point methods (category 5) did not seem to have gained popularity for OPF applications by the time (Glavitsch and Bacher 1991) was written. (Wood and Wollenberg 1996, p. 517) lists the following groups of OPF methods: lambda iteration method, gradient method, Newton's method, linear programming method, and interior point method. The lambda iteration method is the term used for standard methods of economic dispatch and will not be discussed further in this survey. See also (Nocedal and Wright 2006, Chapter 15.1) for an alternative categorization of methods for constrained nonlinear optimization.

From the above it is clear that different classifications are possible, and how one chooses to group the different methods becomes somewhat arbitrary. We have chosen to follow a similar classification as the one in (Capitanescu, Glavic et al. 2007), but following Frank, Steponavice et al. (2012a), we consider categories 2–4 together as "active set" methods due to the similarity in how constraints are handled. However, one should keep in mind that active set methods can be both based on explicit solution of power flow equations (class A) or on embedding the power flow equations in the KKT equations (class B).

Table 2 summarizes the classifications discussed in the paragraphs above. Each row corresponds to a general approach to handling constraints in the OPF problem and each column corresponds to a general approach to taking into account the power flow equations.

Table 2. Classification of main categories of local OPF methods.

Method of handling constraints	Method of taking into account power flow equations	
	Class A: Solved separately from optimization algorithm	Class B: Integrated in optimization algorithm
1) Gradient methods	Category 1A	Category 1B (not common)
2) Active set methods	Category 2A	Category 2B
3) Interior-point methods	Category 3A (not common)	Category 3B

The classification chosen in Table 2 gives six main categories of local OPF methods. As indicated, gradient methods applied to OPF methods based on solving the KKT equations (category 1B) does not appear to be a common combination. Likewise, interior-point methods are usually not based on solving the power flow equations separately and explicitly (category 3A). For each of the other categories, different types of algorithms have been considered for searching the solution space. Section 2.5 describes different search algorithms for nonlinear optimization problems (without considering the handling of constraints): i) gradient methods (e.g. steepest descent and conjugate gradient methods), ii) algorithms based on Newton's method, and iii) sequential approximation methods (based on linear or quadratic programming). Section 2.6 subsequently describes different approaches to handling inequality constraints: 1) gradient methods augmented to take inequality constraints into account, 2) active set methods, and 3) interior point methods.

For a given method for a particular OPF problem, one may also distinguish between a number of different "layers" of the method. Somewhat simplified, the distinction between the different layers can be described as follows:

- Often there is a distinct "central" or "core" "optimizer" (Stott and Alsaç 2012, Capitanescu 2016), which for instance could be a general-purpose solver (e.g. a commercial optimization package) for NLP problems. Depending on the OPF method, "central" solvers can also be used e.g. for LP subproblems if some part of the OPF problem is linearized at some point in the solution procedure.
- If there is a "central optimizer", there must be a layer outside of it that is responsible for formulating, modifying, approximating or decomposing the OPF problem, as appropriate, for the central optimizer. We will focus on this layer when describing the different OPF methods in this survey. The general solution method employed in any "central optimizer" is also included in the description, but we will not discuss possible implementations of the solver.
- For many OPF methods one can find additional layers outside of these two inner layers, for instance in cases where the actual problem is composed of multiple OPF subproblems (e.g. for security-constrained OPF problems) or when various power-system-specific heuristics are employed to improve the reliability of the method and the usefulness of the solution (Stott and Alsaç 2012). Such features and issues are not considered in this report.

2.5 Search algorithms for nonlinear optimization problems applied in OPF methods

This section reviews different groups of optimization methods for solving unconstrained nonlinear optimization problems. Here we also take such problems to include optimization problems with equality constraints that have been transformed to unconstrained optimization problems by the method of Lagrange multipliers. In OPF methods, these search algorithms are typically combined with some approach for

handling the inequality constraints typically included in OPF problems. These approaches will be described in Section 2.6.

2.5.1 Gradient methods

The simplest optimization method for nonlinear optimization problems is arguably the *steepest descent* method. Also called the *gradient descent* or *gradient search* method, the step direction is simply given by the gradient of the objective function: $\boldsymbol{d}_k = -\nabla f(\boldsymbol{x}_k)$.

The steepest descent method is often plagued by a characteristic "zig-zagging" behaviour in the solution space as one approaches the (locally) optimal solution (Press, Teukolsky et al. 1992, Chapter 10.6). This problem is in part avoided by augmenting the direction vector by a *conjugate* direction vector:

$$\boldsymbol{d}_k = -\nabla f(\boldsymbol{x}_k) + \beta_k \boldsymbol{d}_{k-1}. \tag{10}$$

Intuitively speaking, in this way one incorporates information about the previously used search directions in the next iterations of the search and avoids a repeating "zig-zagging" pattern in the search path. This *conjugate gradient* (CG) method reduces to the steepest descent method for $\beta_k = 0$. There are a number of different methods for choosing a value of β_k based on the gradient evaluated for the current and the previous iteration. The original method is called *Fletcher–Reeves*, but the *Polak–Ribière* method is the one that is usually recommended (Press, Teukolsky et al. 1992, Chapter 10.6, p. 416). For more information, see (Nocedal and Wright 2006, Chapter 5).

The gradient descent-based OPF method of Dommel and Tinney (1968) is regarded as the classical, archetypical example of an OPF method (Frank and Rebennack 2012, p. 25). It bases the gradient search on linear sensitivities around solutions to the power flow equations for each iteration. The power flow equality constraints are here implicitly taken into account by using the standard method of Lagrange multipliers, where a Lagrange function is formed by adding the power flow equations $\boldsymbol{g}(\boldsymbol{x}, \boldsymbol{u}) = \boldsymbol{0}$ multiplied by Lagrange multipliers $\boldsymbol{\lambda}^T$ to the objective function $f(\boldsymbol{x}, \boldsymbol{u})$:

$$\mathcal{L}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\lambda}) = f(\boldsymbol{x}, \boldsymbol{u}) + \boldsymbol{\lambda}^T \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{u}). \tag{11}$$

Here, $\boldsymbol{x}$ are dependent variables, while $\boldsymbol{u}$ are independent variables. That is, the method updates $\boldsymbol{u}$, and then $\boldsymbol{x}$ is calculated by solving $\boldsymbol{g}(\boldsymbol{x}, \boldsymbol{u}) = \boldsymbol{0}$, e.g. by applying Newton's method.

$$\frac{\partial \mathcal{L}}{\partial \boldsymbol{x}} = \frac{\partial f}{\partial \boldsymbol{x}} + \left(\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{x}}\right)^T \boldsymbol{\lambda} = \boldsymbol{0}, \tag{12}$$

$$\frac{\partial \mathcal{L}}{\partial \boldsymbol{u}} = \frac{\partial f}{\partial \boldsymbol{u}} + \left(\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{u}}\right)^T \boldsymbol{\lambda} = \boldsymbol{0}, \tag{13}$$

$$\frac{\partial \mathcal{L}}{\partial \boldsymbol{\lambda}} = \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{u}) = 0. \tag{14}$$

It is assumed that $\dim \boldsymbol{x} = \dim \boldsymbol{g}$, so that $\partial \boldsymbol{g}/\partial \boldsymbol{x}$ is square. It is furthermore assumed that $\partial \boldsymbol{g}/\partial \boldsymbol{x}$ is invertible. Then one can solve (12) for $\boldsymbol{\lambda}$ and insert the solution into (13) to get an expression for $\partial \mathcal{L}/\partial \boldsymbol{u}$, i.e. the gradient that is to be used as the search direction (with negative sign) in a gradient descent method. The

step length s_k can be chosen in several different ways. For the optimum gradient method of Dommel and Tinney (1968), the step length is chosen so that the value of f is minimized along the search direction.

It is possible to deduce the same method without using Lagrange multipliers, by letting x be a function of u, i.e. setting $x = x(u)$. The Lagrange function becomes $\mathcal{L}(u) = f(x(u), u)$, and the ordinary gradient descent method can be applied. When calculating $\partial \mathcal{L}/\partial u$, we use the chain rule, and get an expression involving $\partial x/\partial u$ which we can find by differentiating $g(x(u), u) = 0$ with respect to u.

2.5.2 Newton-based methods

Whereas gradient methods utilize only information about first-order derivatives of the objective function in determining the search direction, variants of *Newton's method* also utilize second-order derivatives. In other words, it is based on second-order approximations of the objective function:

$$f(x + \Delta x) \approx f(x) + \Delta x^T \nabla f(x) + \frac{1}{2} \Delta x^T H(x) \Delta x, \tag{15}$$

where the *Hessian* matrix $H(x)$ contains second-order partial derivatives of the objective function at the point x. In Newton's method, the step direction is given by $d_k = -B_k \nabla f(x_k)$, where $B_k = [H(x_k)]^{-1}$. In other words, one uses the Hessian to adjust the search direction so that one not only takes into account the slope (gradient) of the objective function but also takes into account information about the curvature.

The *quasi-Newton* class of methods also have a step direction that can be expressed as $d_k = -B_k \nabla f(x_k)$, but in this case only an approximation of the Hessian is used, i.e. $B_k \approx [H(x_k)]^{-1}$. In the context of root-finding, what can be understood as a finite-difference version of Newton's method is known as the *secant method* (Press, Teukolsky et al. 1992, p. 382). *Broyden's method* is a multi-dimensional generalization of the one-dimensional secant method (Press, Teukolsky et al. 1992, Chapter 9.7, p. 382). Quasi-Newton methods are also sometimes referred to as *variable metric methods* (Press, Teukolsky et al. 1992, Chapter 10.7).

There are two main algorithms of calculating the matrix B_k from information about gradients and step sizes for previous iterations: Using the Davidon–Fletcher–Powell (DFP) algorithm or the Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm. (Press, Teukolsky et al. 1992, Chapter 10.7, p. 418) claims, "It has become generally recognized that, empirically, the BFGS scheme is superior [in details of round-off errors, convergence tolerances, etc.]". For more information on quasi-Newton methods in general, see (Nocedal and Wright 2006, Chapter 6).

According to (Frank, Steponavice et al. 2012a, Section 4.2.1), quasi-Newton methods have received less attention in the OPF community recently (i.e. before 2012) partly because of improvements in numerical linear algebra algorithms for solving the system of linear equations involving the Hessian matrix. Therefore quasi-Newton methods are not widely used for OPF applications (Frank, Steponavice et al. 2012a, p. 250). They also point out that one for most OPF formulations can derive the Hessian matrix analytically, obviating the need for a numerical approximation of $[H(x_k)]^{-1}$ and a quasi-Newton approach; this was also pointed out already by Sun, Ashley et al. (1984).

2.5.3 Sequential approximation algorithms

The category *sequential approximation algorithm* is used in (Hillier and Lieberman 2015, Chapter 13.9, p. 590) to comprise *sequential linear programming* (SLP) algorithms and *sequential quadratic programming*

(SQP) algorithms. Equivalent terms that are also used are *successive linear programming* and *iterative linear programming*, etc. Such methods can solve nonlinear optimization problems iteratively by approximating the original problem with a linear or quadratic problem at each iteration and sequentially solve such approximated problems by ordinary LP or QP solution techniques. For instance, LP problems are typically solved by the *simplex* method or its variants (Hillier and Lieberman 2015, Chapter 4, Chapter 13.7), and the *modified simplex method* (Wolfe 1959, Hillier and Lieberman 2015, Chapter 13.7, p. 581) can be used to solve QP problems. For more information on SQP methods in general, see (Nocedal and Wright 2006, Chapter 18).

The application of SLP methods and SQP methods to OPF problems is briefly introduced in (Frank, Steponavice et al. 2012a, pp. 232–233) and (Castillo and O'Neill 2013, pp. 19–20). (Wood and Wollenberg 1996, p. 517) only includes the "linear programming method (LPOPF)" in its classification, but treat this class of methods in detail in (Wood and Wollenberg 1996, Chapter 13.4). A fairly detailed primary source on the development of SLP-based OPF methods is found in (Alsaç, Bright et al. 1990).

It is not always clear in the literature whether or not quasi-Newton methods are also included in the group of SQP methods (Hillier and Lieberman 2015, Chapter 13.9, p. 594), since these methods also solve approximated quadratic subproblems as explained in Section 2.5.2. However, in contrast to SQP methods, quasi-Newton methods do not necessarily consider constraints. One should also note that although the terms SLP and SQP usually imply methods for optimization problems with inequality constraints, LP or QP methods for approximated subproblems do not necessarily handle the inequality constraints of the original nonlinear problem. However, linearized constraints are typically added to the subproblems as discussed in more detail in Section 2.6.2, and handling of inequality constraints is pointed out as one of the strengths of the SLP approach by (Wood and Wollenberg 1996, p. 517). The same would also apply to SQP, but SQP methods are not treated explicitly by Wood and Wollenberg (1996).

Furthermore, the term "LP method" is also used in the literature and can be somewhat ambiguous by itself. Depending on the context, it can both refer to a method for directly solving a DC OPF problem (Frank, Steponavice et al. 2012a, Section 2.6.2) or it could refer to methods within an SLP method for solving the linear subproblems of a nonlinear AC OPF problem (Wood and Wollenberg 1996, p. 517). As an example of the potential confusion that this may cause, the description of SLP methods in (Capitanescu, Glavic et al. 2007) can be read to imply that these methods are only applicable to optimal real (and not reactive) power flow.

According to (Hillier and Lieberman 2015, Chapter 13.9, p. 594), SQP methods "are generally preferred [over SLP methods] in actual applications", although we should point out that the context of that reference is nonlinear programming in general (including constrained nonlinear programming). Furthermore, in the MATLAB Optimization Toolbox documentation[2] it is stated that SQP methods "represent the state of the art in nonlinear programming methods". According to Frank, Steponavice et al. (2012a), SQP has been "successfully applied in a number of research and commercial OPF algorithms", and in (Wood and Wollenberg 1996, p. 517), the SLP method is summarized as "[o]ne of the fully developed methods now in common use". (Sheblé, Wood et al. 2013, Chapter 8.10) introduces the iterative LP method in relatively much detail but does not discuss its prevalence or merits. One notable commercial application of the SLP method based on (Alsaç, Bright et al. 1990) is described in Section 4.2.10.

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[2] http://se.mathworks.com/help/optim/ug/constrained-nonlinear-optimization-algorithms.html#f26633

2.6 Methods for handling inequality constraints in solving OPF problems

The previous section reviewed optimization algorithms used in OPF methods without taking into account how inequality constraints are handled. Typically, the optimal power flow solution needs to respect technical constraints for quantities such as bus voltage and branch flows that need to be kept within certain limits to ensure the secure operation of the power system. These considerations are formulated as inequality constraints that are include in the OPF problem. This section reviews methods for solving the full AC OPF problem, including inequality constraints. The section is organized according to the categorization in Table 2 for how the inequality constraints are handled.

2.6.1 Gradient methods

Gradient methods were described in Section 2.5.1 in the context of unconstrained optimization. There are different approaches to considering constraints in such gradient methods. The most straightforward approach is to use various heuristics to explicitly restrict or reject the steps $s_k \boldsymbol{d}_k$ so that the iterates are kept within the feasible region of the solution space. More commonly, however, modern optimization methods generally seem to consider constraints more implicitly.

Reduced gradient methods (Wolfe 1963) is a class of gradient methods where the procedure for finding steps $s_k \boldsymbol{d}_k$ is modified so that one automatically keeps within the feasible region (Hillier and Lieberman 2015, Chapter 13.9, p. 590). Typically, for OPF applications this is done by introducing a penalty function to the objective function and using the so-called reduced gradient obtained from this augmented objective function in determining the step direction (Frank, Steponavice et al. 2012a, Section 4.1.1).

The method introduced by Dommel and Tinney (1968) was the first application of a reduced gradient method to OPF applications (Frank, Steponavice et al. 2012a, Section 4.1.1). They do not themselves refer to the method as a reduced gradient method in their paper, but in Section V they introduce quadratic penalty terms to the objective function for the functional constraints. For linear inequality constraints and variable bounds, they project the step onto the hyperplanes these constraints form in the solution space and refer to this as the *gradient projection technique* (Dommel and Tinney 1968, Section IV).

The reduced gradient method is also referred to as the *Frank–Wolfe* method, which in (Hillier and Lieberman 2015, Chapter 13.9, p. 591) is presented as a *sequential linear approximation technique*. It should be noted that the original Frank–Wolfe method was proposed as a method for solving quadratic programming problems (i.e. with quadratic objective functions and linear constraints) by iteratively solving linear programming subproblems (Hillier and Lieberman 2015, Chapter 13.9, p. 591). Although there does not seem to be full agreement in the literature of what exactly constitutes an RG algorithm, it appears that for OPF applications they are generally defined by the inclusion of a penalty term and the inability to directly treat inequality and functional constraints.

Generalized reduced gradient (GRG) methods are extensions of the reduced gradient method where inequality constraints are tackled by linearization and introducing slack variables (Frank, Steponavice et al. 2012a, Section 4.1.3, Castillo and O'Neill 2013, p. 20). According to (Frank, Steponavice et al. 2012a, Section 4.1.3), GRG methods also differ from RG methods in that penalty terms in the effective objective function is avoided in the former. However, this definition does not seem to be shared by (Castillo and O'Neill 2013, p. 20). The benefits of GRG methods are that they are more robust than comparable methods and better at ensuring that the iterates remain feasible through the iteration procedure (Castillo and O'Neill 2013, p. 22). For a more detailed discussion of different RG and GRG methods we refer to (Frank, Steponavice et al. 2012a, Section 4.1).

2.6.2 Active set methods

Active set methods, also known as the *active set and penalty* (ASP) methods (Frank, Steponavice et al. 2012a, Section 4.2), is a large and relatively diverse class of constrained optimization methods. What all these methods have in common is that they include some means of keeping track of the set of active inequality constraints, or the *active set*. Simplex can be regarded as an active-set algorithm (Gill, Murray et al. 1984). Furthermore, most optimization methods for constrained nonlinear problems that are based on SLP or SQP can be regarded as active set algorithms since the active set of inequality constraints is handled in the solution of the LP or QP subproblems in these methods. The MATLAB Optimization Toolbox documentation[3] states that "Virtually all QP algorithms are active set methods", and (Nocedal and Wright 2006, Chapter 18) even state in its chapter on SQP methods that "a more descriptive title for this chapter would perhaps be 'Active-Set Methods for Nonlinear Programming' ". For both LP subproblems and (linearly constrained) QP subproblems, linear constraints have to be generated somehow through linearization of the original constrained nonlinear optimization problem.

Active set methods applied to OPF problems seem to first have gained prominence through (Sun, Ashley et al. 1984), which apparently is based on the more general active set methodology described e.g. in (Gill, Murray et al. 1984). This method can be regarded as an SQP approach (Alsaç, Bright et al. 1990). The novelty of the method of Sun, Ashley et al. (1984) was to use Newton's method to solve the KKT equations (Capitanescu, Glavic et al. 2007). In fact, methods based on this approach are in the context of OPF sometimes simply referred to as *Newton methods* (Alsaç, Bright et al. 1990, Wood and Wollenberg 1996, Chapter 13.2.2). Note that this Newton-based approach to solving OPF problems should be distinguished from the direct minimization approach using Newton-based optimization methods described in Section 2.5.2.

The method of Sun, Ashley et al. (1984) constituted a breakthrough in OPF research (Capitanescu, Glavic et al. 2007) as Newton's method converges very rapidly to the optimal solution, as long as one starts in a feasible region of the solution space relatively close to the optimum. (Wood and Wollenberg 1996, Chapter 13.2.2) also states that Newton's method for solving OPF has very fast convergence and can be the best choice for OPF problems where there is not an issue of handling the constraints. It is now generally recognized that methods based on solving the KKT equations are more efficient than earlier active set methods that instead were based on directly minimizing an augmented objective function[4] (Capitanescu, Glavic et al. 2007).

Although the solution of the KKT equations by Newton's method was the main contribution of Sun, Ashley et al. (1984), what makes it an active set method are the techniques it describes for keeping track of the inequality constraints. The main challenge of all active set methods is identifying precisely which inequality constraints that should be included in the active set for the optimal solution. This was recognized already by Sun, Ashley et al. (1984), and this is a challenge that also must be addressed by SLP- and SQP-based active set methods. An approach to monitoring of active constraints for an SLP-based OPF method is outlined in (Alsaç, Bright et al. 1990). In Newton-based active set methods, inequality constraints are typically represented by adding quadratic penalty terms to the Lagrangian. However, as described in (Wood and Wollenberg 1996, Chapter 13.2.2), this approach can be potentially troublesome.

Identifying active inequality constraints for the optimal solution still seems to be the main weakness of active set methods (Capitanescu, Glavic et al. 2007). However, in the descriptions of SLP-based active set methods and their applications (Alsaç, Bright et al. 1990, López, Sadikovic et al. 2015), this potential problem is not

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[3] http://se.mathworks.com/help/optim/ug/constrained-nonlinear-optimization-algorithms.html#f26633

[4] http://se.mathworks.com/help/optim/ug/constrained-nonlinear-optimization-algorithms.html#brnox01

in any way described as being a weakness or insurmountable challenge. In general, not all constraints can be considered simultaneously in such methods, and a number of different techniques exist for considering which constraints to include or omit at each iteration. A typical problem for the Newton method is that the step length sometimes turns out to be too large, so that the Newton step brings the iterate outside the feasible region because of some constraint that was not included in that specific iteration.

2.6.3 Interior-point methods

Interior-point methods (IPMs) or *barrier methods* is a class of methods for solving constrained optimization problems where the original problem is transformed to an unconstrained problem by a barrier function that "pushes" the decision variables into the *interior* of the feasible region of the solution space. The first application of interior-point methods to OPF were SLP methods where the simplex method was replaced with a linear IPM due to Karmarkar (1984) to solve the linearized subproblems (Frank, Steponavice et al. 2012a, Section 4.6). For this historical reason, IPMs more generally are also sometimes seen to be referred to as *Karmarkar's method* (Wood and Wollenberg 1996, p. 551). For most modern IP methods, the barrier terms are logarithmic (Frank, Steponavice et al. 2012a, Castillo and O'Neill 2013) and added to the objective function for each slack variable corresponding to an inequality constraint:

$$f_\gamma(\boldsymbol{x}) = f(\boldsymbol{x}) - \gamma \sum_{i=1}^{m} \ln(z_i). \tag{16}$$

Here γ has been introduced as a parameter that controls the barrier strength. The barrier parameter is typically reduced towards zero during the iteration procedure. In this formula, the sum goes over all m inequality constraints $\boldsymbol{h}(\boldsymbol{x}) \leq \boldsymbol{0}$, which have been transformed to equality constraints by introducing slack variables $z_i > 0$ so that $\boldsymbol{h}(\boldsymbol{x}) + \boldsymbol{z} = \boldsymbol{0}$. Capitanescu, Glavic et al. (2007) note as advantages of interior-point methods that they generally converge relatively quickly, and that the introduction of logarithmic barrier functions is a straightforward way of handling inequality constraints. A variant was introduced by Polyak (1992) that does not require the initial point to be strictly feasible. For more information on IPM in general, see (Nocedal and Wright 2006, Chapter 19).

When applied to OPF and other nonlinear problems, the method typically solves the KKT equations rather than minimizing the (augmented) objective function (Frank, Steponavice et al. 2012a, Castillo and O'Neill 2013). The standard form of the class of algorithms that we here regard as interior-point OPF algorithms is often referred to as the (pure) *primal-dual* interior-point algorithms (PDIPM), as search steps are made in the combined space of primal and dual variables (including slack variables) when solving the KKT equations iteratively (Castillo and O'Neill 2013, p. 18). The search steps are often made using Newton-based methods. According to (Frank, Steponavice et al. 2012a, Chapter 4.6.1), primal-dual interior-point algorithms are "perhaps the most popular deterministic algorithm discussed in recent OPF research". (Wood and Wollenberg 1996, p. 517) summarizes the method as "[a]nother of the fully developed and widely used methods for OPF" and points out its strengths in handling inequality constraints. (Sheblé, Wood et al. 2013, Appendix 8A) introduces IPMs in relatively much detail but does not discuss their merits.

Other variants of (higher-order) interior-point algorithms have also been introduced, such as the predictor-corrector (PC) and the multiple centrality corrections (MCC) IPMs that are treated in more detail in (Capitanescu, Glavic et al. 2007). According to Capitanescu, Glavic et al. (2007), the PC algorithms "is regarded to be the benchmark of [interior-point] based algorithms". They also demonstrate how the PC method generally is superior to the pure primal-dual method for large-scale power systems. The advantages of the predictor-corrector method is that it estimates terms of higher order in the step variables ($\Delta \boldsymbol{x}$) to improve the search direction and that it uses an improved heuristic to choose the barrier parameter γ

(Capitanescu, Glavic et al. 2007). However, according to the experience of some of Capitanescu and Wehenkel (2013), the MCC method is the most reliable IPM for OPF. A more detailed comparison of different variants of IPM, including also trust-region based IPMs, is found in (Frank, Steponavice et al. 2012a, Chapter 4.6) and references therein.

2.7 OPF methods for global optimization

In addition to the local, deterministic optimization methods discussed in the previous subchapters, we also discuss a few selected classes of other OPF methods that are designed for global optimization. Note that as we define the term, the methods described in this subchapter generally do not guarantee to converge to a globally optimal solution of a (feasible) nonlinear (AC OPF) optimization problem, although some do in certain cases.

2.7.1 Heuristic and non-deterministic methods

In addition to the local methods covered in the previous sections, a large and very diverse set of non-deterministic and hybrid methods can be found in the research literature. The general motivation of developing these is to obtain a global optimization method to solve the OPF problem. In contrast, the deterministic methods described in the previous sections are local optimization methods that may well converge to a local minimum of the objective function rather than a global minimum. The heuristic and non-deterministic methods typically combine some local search algorithm of the type described in Section 2.5 with some random search algorithm and/or metaheuristic to avoid being trapped in local minima. Some, but not all, of these approaches guarantee asymptotic convergence to the global minimum (Frank, Steponavice et al. 2012b, p. 261). This section very briefly describes a selection of groups of heuristic and non-deterministic methods applied to OPF and refers to other OPF surveys for more details.

Ant colony optimization is a class of probabilistic, parallel optimization methods inspired by the behaviour of ants when finding the "optimal" path between different locations. For more information, see (Pandya and Joshi 2008, Section 8.6), and (Frank, Steponavice et al. 2012b, p. 261).

Artificial neural networks (ANN) is a wider group of methods in artificial intelligence (AI), computer science and statistics inspired by biological networks of neurons (e.g. in human brains). Very broadly speaking, the main application of ANN is for machine learning, i.e. identifying and approximating unknown relations based on large amounts of input data. For OPF application, the general underlying concept seems to be to distribute the handling of objective function (penalty) terms to different artificial neurons. For more information, see (Pandya and Joshi 2008, Section 8.1), and (Frank, Steponavice et al. 2012b, p. 262).

Bacterial foraging algorithms are inspired by the manner in which bacteria search for nutrients, similarly to how the concept of ant colony optimization is based on the behaviour of ants. For more information, see (Frank, Steponavice et al. 2012b, p. 263).

Chaos optimization algorithms are optimization algorithms including chaotic variables that may help the algorithm "escape" from local optima and thus aid the algorithm in obtaining a globally optimal solution. For more information, see (Frank, Steponavice et al. 2012b, p. 263).

Genetic algorithms (GA) and *evolutionary programming* (EP) are classes of so-called *evolutionary optimization* algorithms containing an element of random search inspired by processes of evolution and natural selection in biology. The main distinction between the two classes seems to be that EP focuses on mutation processes whereas GA focuses on crossover processes. Other examples of evolutionary algorithms

are *differential evolution* and *artificial immune systems*, but the categorization does not seem to be well defined (Frank, Steponavice et al. 2012b, p. 264). Evolutionary optimization algorithms are generally based on a "population" of possible solutions and are therefore well suited for parallelization. Evolutionary algorithms in general and genetic algorithms in particular seem to be widely used for academic OPF studies, but according to (Frank, Steponavice et al. 2012b, p. 267), few EP OPF algorithms have been used for industrial applications. For more information, see (Pandya and Joshi 2008, Section 8.3, 8.5) and (Frank, Steponavice et al. 2012b, pp. 264–269).

Tabu search is a search meta-heuristic based on a "tabu list" of possible solutions not to visit during the next iterations of the search. Allowing also iterations increasing the objective value, this may avoid getting "trapped" in local minima or oscillating between solutions. For more information, see (Frank, Steponavice et al. 2012b, p. 272).

Simulated annealing (SA) is a class of probabilistic meta-heuristics for global optimization inspired by the physical phenomenon of annealing of a material as the temperature is lowered. For a random search algorithm, a fictitious "temperature" parameter is introduced that determines the probability of searching in a direction that increases the objective value. Starting at higher temperatures one avoids being "trapped" in local minima, and as the temperature is decreased one hopefully terminates the search at a global minimum. SA appears to have been applied to OPF problems relatively early, but has since been considered inferior to other methods (Frank, Steponavice et al. 2012b, p. 271).

Particle swarm optimization is a probabilistic optimization algorithm inspired by the social, collective behaviour of populations of organisms, with different "individuals" searching different parts of the solution space. It seems to be relatively widely used for (academic) OPF applications, and being population-based, it is relatively well suited to parallelization. For more information, see (Pandya and Joshi 2008, Section 8.7) and (Frank, Steponavice et al. 2012b, p. 269).

Hybrid methods combining different deterministic and/or non-deterministic methods are described in detail in (Frank, Steponavice et al. 2012b, Section 3). Both (Pandya and Joshi 2008) and (Frank, Steponavice et al. 2012b, p. 280) also treat methods of *fuzzy logic* applied to OPF, even though fuzzy logic is by itself not a method for or approach to optimization.

2.7.2 Convex relaxation

Since OPF problems are nonconvex, one cannot guarantee that one obtains a globally optimal solution using a local solution method. One approach to global optimization is to "convexify" the problem by relaxing the constraints that make it nonconvex. This approach is usually referred to as *convex relaxation*.Only a very simplified representation of the concept of convex relaxation will be given, following (Low 2014a, Molzahn 2014, Ghaddar, Marecek et al. 2015, Molzahn and Hiskens 2016).

The basis for the convex relaxation approach is to observe that using a rectangular voltage coordinate formulation of OPF, the problem can be written as a quadratic optimization problem with real and imaginary parts of the bus voltages as decision variables. Since the problem is quadratic, the feasible part of the solution space for the complex voltage is nonconvex. By expressing the voltage decision variables by a matrix W, the problem can in turn be formulated on a linear form together with the constraint that the matrix W has rank one and is a semidefinite positive matrix. The nonconvexity of the problem in this formulation is entirely due to the rank constraint, assuming that the objective function and the functional inequality constraints are given by convex functions. The so-called *semidefinite programming* (SDP) relaxation amounts to relaxing the rank constraint and only demand that the matrix W is semidefinite positive. SDP relaxation is the lowest-order variant in a larger class of so-called *moment-based relaxations* (Capitanescu

2016) for formulations of the OPF problem as a polynomial (rather than just a quadratic) optimization problem. Another related convex relaxation approach is called *second-order cone programming* (SOCP) relaxation, referring to the algebraic structures on which the polynomials are defined.

Under certain conditions, the convex relaxation is exact (the so-called duality gap is zero). Precise sufficient conditions will not be discussed here, but convex relaxation is shown to be exact for radial networks for instance if there are no reverse power flows and for meshed networks for instance if there are phase shifters at certain locations (Low 2014b). There are also claims to numerical or empirical evidence supporting the exactness of convex relaxation for most practical power systems, see e.g. (Low 2014b) and the other references in this chapters above for more discussion. Here, exactness means that one can guarantee that the solution found for the convex relaxation of the problem is indeed the globally optimal solution of the original non-convex problem. See, however, (Molzahn and Hiskens 2016) and references within for a discussion of cases where this does not hold.

Furthermore, some possible drawbacks and challenges of convex relaxation are reviewed in (Capitanescu 2016): For a great number of cases, the global solution cannot be recovered from the convex relaxation solution procedure (since the duality gap for the solution is greater than zero). There are also potential issues with convergence precision more generally. Furthermore, for large systems, the computation time of convex relaxations is typically at least a few times larger and potentially many times larger than the computation time of conventional solution methods. However, according to Capitanescu (2016), convex relaxation should be regarded as a complement to conventional (local) solution methods rather than as a replacement. Castillo, Lipka et al. (2016) also review recent developments in convexification techniques and concur that there is still a trade-off to be made between convergence precision and computation time.

2.8   Comparison of solution methods

From the comparisons and assessments found in the literature, which OPF method is the best one seems to depend both on the application of OPF, the OPF problem formulation, and which criteria (e.g. computational speed, accuracy, or robustness) is most important. Another challenge in comparing the performance of different solution methods is that various review articles sometimes state general merits and attributes of the methods without giving references to primary sources such as benchmarking studies. It is also not always stated for which applications the relative merits of different methods apply. In the following, we will to a large extent base ourselves on the relatively thorough and clear comparison and summary in (Frank, Steponavice et al. 2012b, Chapter 5).

Both IPMs and sequential approximation active set methods are generally computationally efficient for most problems, and it is commonly agreed that state-of-the-art OPF methods are most suitably based on one of these approaches (Capitanescu, Martinez Ramos et al. 2011, Frank, Steponavice et al. 2012b). (Wood and Wollenberg 1996, p. 517) also refers to IPM and SLP as "fully developed" and widely used methods. Both approaches come with disadvantages: Identification of the set of active constraints is still a major challenge for modern active-set methods, especially for highly constrained problems (Frank, Steponavice et al. 2012b, p. 251); interior-point algorithms may depend on tuning a set of algorithmic parameters and may in some cases become unreliable and diverge. The main advantage of IPM compared with active set methods is that inequality constraints can be handled relatively straightforwardly (Wood and Wollenberg 1996, p. 517). Momoh, El-Hawary et al. (1999) also point out that IPMs have the advantage that they provide a good initial solution iterate, but a drawback according to Capitanescu and Wehenkel (2013) is that they do not benefit very much from warm starting the optimization (i.e. initializing it with a solution obtained for a similar OPF problem). This point is also confirmed by SINTEF experience as a drawback of IPMs for some applications, cf. also Chapter 6.1.

Which method is the most accurate, reliable and efficient generally appears to vary from OPF problem to OPF problem. SLP methods can be very fast and fairly accurate (Momoh, El-Hawary et al. 1999), especially for real power dispatch problems, which may make them suitable even for real-time applications (Capitanescu 2016). SQP methods may perform better when the problem involves reactive power dispatch and is also stated to be faster than SLP for many OPF problem formulations (Frank, Steponavice et al. 2012b, p. 250). There seems to be some agreement that interior-point methods overall is the class of methods that provides the quickest convergence to a solution (Momoh, El-Hawary et al. 1999, Frank, Steponavice et al. 2012b, p. 250). In case of convergence difficulties for IPMs, it may be advisable to use a trust-region IPM (Frank, Steponavice et al. 2012b, p. 250). Ensuring convergence and handling of (apparent) problem infeasibility for AC OPF still appears to be an under-addressed issue in the research literature (Stott and Alsaç 2012, Capitanescu 2016). NLP optimizers based on either general-purpose commercial solvers or customized state-of-the-art academic solvers are sometimes unable to converge even if the problem is feasible (Capitanescu and Wehenkel 2013, Capitanescu 2016).

(Glavitsch and Bacher 1991, Chapter 6) evaluates and compares the methods it classifies as class A (power flow solved separately from optimization algorithm) and class B (power flow integrated in optimization algorithm). It regards using a solved conventional power flow as a starting point as a convenient feature of class A methods. However, at least at the time of writing of (Glavitsch and Bacher 1991), accuracy could be an issue for class A methods, whereas the main issue for class B methods was the heuristics needed to handle the active set of inequality constraints. Although they present a balanced view on the relative merits of the two classes of methods, they appear to finally deem class A methods to be more promising. A possible reason for this conclusion is that much of the development of effective IPMs for OPF applications (which would typically belong to class B) was made in the early 1990s (Momoh, El-Hawary et al. 1999), i.e. in the years after Glavitsch and Bacher (1991) wrote their paper.

For almost any kind of method, technical and numerical aspects of the software implementation are key in determining its efficiency in practice (Stott and Alsaç 2012, Castillo and O'Neill 2013, pp. 13–14). These aspects include e.g. the use of matrix factorization techniques, algorithms for solving sets of linear equation, techniques for utilizing the matrix structure and sparsity particular to each problem formulation, and methods for reducing storage needs. Such issues are discussed in more detail in earlier work, see e.g. (Dommel and Tinney 1968) and (Sun, Ashley et al. 1984), but do not seem to be given much focus in more recent research. This can possibly be explained by the increased use of powerful third-party general-purpose linear algebra packages for solving numerical subproblems. Capitanescu and Wehenkel (2013) highlight the importance of using a suitable solver for very sparse and very ill-conditioned linear equation systems in implementing state-of-the-art IPMs.

3 OPF models and methods for distribution systems with energy storage systems

As explained in Section 1.1, this report was motivated by the need for a better overview of OPF methods and tools in the context of problems relating to energy storage systems (ESS) in distribution systems. For illustration, one typical example of such a problem is to optimize the charge/discharge schedule of an ESS in a distribution system over a given planning horizon, considering network constraints.

First, it is reasonable to assume that a pure DC power flow formulation would be insufficient for such distribution system applications. Including AC power flow is particularly important to capture potential violations of voltage constraints, e.g. for problems caused by intermittent distributed generation in weak grids where energy storage systems is considered as a possible remedy. The DC power flow approximation also introduces inaccuracies when the R/X ratio is too high. For instance, Purchala, Meeus et al. (2005) suggest that the error in active power flow can become unacceptable above around $R/X = 0.25$, which is lower than typical values for distribution systems. However, this conclusion has some qualifications that are discussed in more detail in their paper (Purchala, Meeus et al. 2005), and the accuracy and applicability of DC power flow and alternative linearizations is also discussed by e.g. Stott, Jardim et al. (2009) and Coffrin, Van Hentenryck et al. (2012).

Next, it is reasonable to assume that energy storage applications require the OPF method to take into account the multi-period nature of the problem. That is, charging/discharging decisions during one period affect the optimal decisions for other periods. Thus, the already challenging problem of creating robust and efficient AC OPF methods is compounded by the introduction of dynamics to the problem.

There has lately been an international upsurge of research activity on DOPF for distribution systems considering energy storage. Some of this literature was reviewed in (Sperstad, Helseth et al. 2016), the preparation of which was carried out in parallel with the work for the present report. The literature review in (Sperstad, Helseth et al. 2016) focused on how the value of stored energy at the end of the planning horizon was being considered in different methods. The present chapter builds on this literature review but characterizes more broadly the DOPF models reported in the literature. The survey below lists 46 references but should nevertheless not be regarded as exhaustive. The general methodology for reviewing the literature has been to trace the network of references and citing articles, focusing on the most cited references where it was necessary to discriminate. A systematic search in IEEEXplore for "dynamic optimal power flow" and "multi-period optimal power flow" was also carried out. In addition, the review for (Sperstad, Helseth et al. 2016) included systematic searches for terms such as "value function", "dynamic programming" and "marginal value/price/cost" in combination with "energy storage". The present survey therefore also includes some methods that are not proper DOPF methods for distribution systems but which were nevertheless relevant for the research questions of (Sperstad, Helseth et al. 2016).

We are not aware of any similar survey of the literature on OPF methods for ESS applications. An earlier review of DOPF methods in the context of economic dispatch under ramp rate constraints is given in (Xia and Elaiw 2010). The review below also includes a selection of older references on DOPF methods where the context was demand response. As for the literature review in (Sperstad, Helseth et al. 2016), the references included in the present review primarily cover OPF models for the optimal *operation* of distribution systems with ESS. For the extension of OPF problems for the optimal *planning* of distribution systems with ESS we refer to a recent review (Zidar, Georgilakis et al. 2016), although several references are included below for methods for optimal planning that include relevant DOPF methods.

3.1 Survey of dynamic optimal power flow methods

An overview of the surveyed literature on OPF for energy storage applications is given in Table 3. The reference of the work is given in the first column. The second column briefly states both the OPF model formulation and the method proposed for solving it. The default model formulation for a multi-period OPF with full AC power flow formulation is "AC DOPF", which means that AC OPF models for individual time steps are coupled with temporal constraints without any decomposition of the problem. Power flow constraints, i.e. power flow equations, branch flow constraints and voltage constraints, are included if not stated otherwise. We have primarily surveyed models that include AC power flow constraints, but we have also included a small selection of relevant references that do not. When some approximation to AC power flow is employed this is stated explicitly in the table. When not otherwise stated, the model formulation is deterministic (not stochastic) and not security constrained (SC, i.e. including constraints for contingencies). The modelling language and/or solver used for solving the model is stated when information on this is available in the reference.

The second column briefly states the application of the OPF and whether it optimizes siting and sizing of ESS as well as scheduling. For most of the references, the context of the study is grid-integrated ESS, but it is also mentioned when the context is e.g. electric vehicles (EV). The application (i.e. the service or services it is uses for) of the ESS and the objective considered is also indicated where possible. If the OPF is used for sizing and siting, capital costs of ESS are typically included. In several of the references, the applications are related to grid integration of distributed generation (DG).

The fourth column states the test system of the case study to which the model and solution method is applied. This includes the number of buses of the system and the number of time steps or periods of the multi-period optimization. Time steps have a duration of 1 hour if not otherwise stated. When common test systems (e.g. IEEE test systems) are used, they are typically modified to include a representation of energy storage systems etc., but this is not stated explicitly in the table unless more major modifications are made.

The fifth column states whether the energy stored in the ESS at the end of the planning horizon is being valuated explicitly or implicitly in the optimization. If not, depending on the application of ESS considered, there is usually no incentive for the model to avoid emptying the ESS at the end of each planning horizon. One common form of implicit valuation is referred to in the table as "periodic boundary conditions", which means that the at the end of the planning horizon, at least as much energy is required to be stored in the ESS as at the beginning of the planning horizon.

Table 3. Overview of literature on OPF models and methods for energy storage applications.

Reference	OPF model and solution method	Application	Test system	Valuation of stored energy
(Alguacil and Conejo 2000)	DC DOPF (with cosine loss representation) as a subproblem to a start-up and shut-down scheduling master problem in a Benders decomposition scheme, solved using MINOS under GAMS	Optimized generation scheduling of hydrothermal transmission power system	Based on the Spanish transmission system, with 104 buses over 24 hours	n/a
(Allard 2013)	ESS scheduling model minimizing total system costs; representing voltage constraints through local penalty terms pre-calculated using Simpow®; solved with genetic algorithm in MATLAB's Global Optimization Toolbox	Smart charging of EV in microgrid with wind power	10-bus microgrid connected to regional grid; represented as aggregated load in optimization	n/a
(Allard, Phen Chiak et al. 2013)	ESS scheduling model minimizing total system costs; power flow not taken into account; solved with genetic algorithm in MATLAB's Global Optimization Toolbox. (Although this is not stated in the paper; see (Allard 2013) for more details.)	Smart charging of EV in microgrid with wind power	Microgrid consisting of 38 households connected to regional grid; represented as aggregated load in optimization	n/a
(Atwa and El-Saadany 2010)	AC OPF for each hour, optimizing ESS size and location by first minimizing wind energy spillage and then maximizing utility benefits	Siting and sizing of ESS in distribution system with wind power (DG), maximizing benefits for both DG owner and DSO	41-node rural distribution system, simulated over 8760 hours	Periods of charging and discharging determined within each day, initial SOC for each day is set to final SOC from previous day.
(Baker, Zhu et al. 2013)	AC DOPF; solved using Unlimited Point Algorithm; rolling horizon	Minimizing generation cost for power system with wind power generation and battery storage system	IEEE 14-bus system over 8 time steps of duration 5 minutes	n/a

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Reference	OPF model and solution method	Application	Test system	Valuation of stored energy
(Bose and Bitar 2014)	Analytical treatment of DC DOPF as a stochastic control problem	Minimizing power system generation costs	Two-node network	Analytically relating the marginal value of storage capacity to the marginal value of electricity generation
(Bozchalui and Sharma 2014)	AC DOPF for unbalanced 3-phase distribution network, implementing in Python using Pyomo and solved by the Ipopt package, comparing different objective functions and operation strategies	Optimizing ESS schedule to minimize power losses in distribution system with renewable energy	IEEE 13-bus distribution test feeder over 24 hours	Periodic boundary conditions
(Cao, Du et al. 2016)	3-period preventive-corrective AC SC OPF, solved with hybrid algorithm combining primal-dual IP method based on MATPOWER and differential evolution	ESS for short-term post-contingency fast corrective actions	Modified IEEE 6-bus, IEEE 57-bus, and IEEE 118-bus systems	Not a constraint, but effect of initial SOC on the effectiveness of corrective ESS dispatch is evaluated
(Carpinelli, Celli et al. 2013)	SQP-based AC DOPF with linearized power flow constraints; genetic algorithm for sizing and siting problem as an outer loop	Siting, sizing and scheduling ESS for multiple applications to DSO and TSO (e.g. price arbitrage, loss reduction, balancing DG, provision of reactive power to TSO).	17-bus MV balanced 3-phase system	Periodic boundary conditions
(Celli, Mocci et al. 2009)	Dynamic Programming for solving scheduling of ESS (not including power flow constraints); genetic algorithm for sizing and siting problem as an outer loop (including checking of power flow constraints)	Siting, sizing and scheduling ESS in distribution system with DG, minimizing overall network cost (incl. loss reduction)	17-bus MV/LV distribution system	Periodic boundary conditions

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(Celli, Pilo et al. 2013)	Dynamic Programming for solving scheduling of ESS (not including power flow constraints); optimal network planning as outer loop, including probabilistic power flow	Optimization of expansion plan of a distribution network (incl. ESS), minimizing the generalized network cost (incl. losses)	Real 140-bus MV/LV Italian distribution system scheduled over 24 hours	Periodic boundary conditions
(Chandy, Low et al. 2010)	AC DOPF with small voltage angle approximation (convex problem), formulated as a finite-horizon optimal control problem	Minimizing total generation costs	Test system with two generator buses and 20 load buses over 24 hours	Linear penalty function in the objective function (proportional to the deviation of the amount of stored energy at the end of the planning horizon from the maximal energy capacity)
(Chengquan and Peng 2015)	DC DOPF, solved using an interior point solver with MATPOWER	Minimizing total generation costs (incl. ESS operating costs)	IEEE 14-bus system	Setting (fictitious) marginal operation cost of the ESS to be quadratic function of the amount of stored energy to promote charging when the generation costs in the power system and/or the storage level is low, and vice versa
(Costa and Costa 2007)	DOPF with a nonlinear power flow (including loss representation but neglecting reactive power flow)	Market clearing for both energy dispatch and spinning reserves (reserve allocation, ancillary service) jointly in the day-ahead markets including demand response	20-bus transmission system over 24 hours	n/a
(Eshghi and Patil 2015)	Continuous-time optimal control approach based on Pontryagin's Maximum Principle, neglecting power flow	Optimal operation of microgrid incl. conventional and renewable generation, ESS and power import	Macrogrid with aggregated load and storage representation over 24 hours	Periodic boundary conditions

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Reference	OPF model and solution method	Application	Test system	Valuation of stored energy
(Fortenbacher, Zellner et al. 2016)	Forward backward sweep DOPF with a linear AC power flow formulation; solved with MATPOWER using IPOPT with PARDISO, compared with full AC DOPF solved using CPLEX with GUROBI	Prosumers with PV generation, minimizing their energy costs	CIGRE 19-bus test system over 744 hours	n/a
(Gabash and Pu 2012)	AC DOPF (coupled real-reactive); implemented in GAMS and solved using CONOPT3 NLP solver	Optimal ESS real and reactive power dispatch for reducing losses, improving voltage profiles etc. for distribution system with DG	Real 41-bus MV distribution system over up to 120 hours	Periodic boundary conditions
(Gast, Tomozei et al. 2014)	Constructing heuristic scheduling policy (based on Markov Decision Process) using stochastic dynamic programming; neglecting power flow constraints	ESS used by TSO to balance wind power generation, optimizing trade-off between power losses and use of secondary reserves	n/a	Dynamic policy for storage level management obtained based on stochastic dynamic programming compared to policy aiming to keep the storage level balanced (at half the capacity)
(Gayme and Topcu 2011)	Convex SDP obtained as the Lagrangian dual to the rank relaxation of an equivalent reformulation of an AC DOPF	ESS to minimize total generation costs and flattening generation profiles	IEEE 14-bus system over 24 hours	n/a
(Gayme and Topcu 2013)	Convex SDP obtained as the Lagrangian dual to the rank relaxation of an equivalent reformulation of an AC DOPF; solved using the YALMIP parser and the solver SeDuMi in MATLAB	ESS to minimize total generation costs and flattening generation profiles	IEEE 14-bus system over 24 hours	n/a

Reference	OPF model and solution method	Application	Test system	Valuation of stored energy
(Gemine, Ernst et al. 2014)	AC DOPF including discrete decision variables, a) with Lagrangian relaxation decomposed into one dynamic mixed-integer subproblem for power injections and a set of static voltage subproblems, or b) with network-flow relaxation as a convex SDP problem; implemented in C++ and solved with a) IPOPT and ConicBundle or b) MOSEK	Flexible loads in distribution system, minimizing generation costs or minimizing curtailment of renewable generation (incl. losses)	Various, with 6, 9 or 14 buses, over 8 time steps	n/a
(Geth, Leyder et al. 2016)	AC DOPF for radial distribution systems based on convex relaxation and the unit modelling in (Geth, Marmol et al. 2016); OPF solver developed by Tractebel in the PlanGridEV project	Simulating the impact of different EV charging modes in distribution systems with flexible units	Real-world two-feeder LV network with 18 + 32 housholds over 24 hours (Ramaswamy, Chardonnet et al. 2016)	Requiring fully charged EV at the end of the planning horizon (Ramaswamy, Chardonnet et al. 2016)
(Geth, Marmol et al. 2016)	Mixed-integer SOCP convexification of AC DOPF, based on modelling framework in (Heussen, Koch et al. 2012)	Extended modelling framework for reactive and active power control of distributed energy resources	n/a	n/a
(Gill, Kockar et al. 2014)	AC DOPF; solved using MIPS	Optimizing import/export (or cost/revenue from import/export) from distribution systems with DG, ESS, and flexible demand	16-bus radial network with some meshing typical of a rural distribution network (based on the U.K. Generic Distribution System)	Periodic boundary conditions
(Gopalakrishnan, Raghunathan et al. 2013)	SDP relaxation of AC DOPF; solved using a branch-and-bound approach with upper and lower bounding problems solved using CONOPT and YALMIP, respectively	Minimizing total generation costs for power system with ESS and ramp rate constraints	IEEE 57 bus-system over 8 hours	Periodic boundary conditions

Reference	OPF model and solution method	Application	Test system	Valuation of stored energy
(Grillo, Pievatolo et al. 2016)	Finite-horizon optimal policy problem for Markov decision process solved by stochastic dynamic programming; power flow constraints checked using MATPOWER	ESS in distribution system with photovoltaic generation, minimizing cost of energy and power losses	European MV benchmark network of the CIGRÉ: two distribution feeders with 14 buses combined; over 24 hours with 15-minute time steps	Taken into account within each daily planning horizon through stochastic dynamic programming approach (not explicitly discussing the storage level at the end of the planning horizon)
(Heussen, Koch et al. 2010)	Model predictive control approach (rolling horizon multi-period optimization) illustrated with a DC network representation	General modelling framework ("Power Nodes") for controlling different forms of energy storage in power systems, represented as more abstract units; see also (Heussen, Koch et al. 2012)	1-node system over 16 15-minute time steps	Quadratic penalty function penalizing the deviation from a reference storage level
(Khanabadi, Moghadasi et al. 2013)	AC DOPF; rolling horizon ("Receding Horizon Control" method)	Real-time optimization of ESS charging/discharging in distribution system company (DISCO) with price forecasting	32-bus radial test system over 24 hours	n/a
(Lamadrid, Mount et al. 2011)	AC DOPF; solved with MATPOWER using the PDIPM solver; rolling horizon with 7 updates per day	Minimizing total operational costs of power systems with wind power and ESS	30-bus system over 24 hours	Periodic boundary conditions
(Levron, Guerrero et al. 2013)	Dynamic programming search in the time domain combined with conventional PF solver in the network domain	Minimize cost of energy import for a grid-connected microgrid with ESS and DG	12-bus system with 2 ESS, over 72 hours with 0.5 hour resolution	n/a
(Liu, Quilumba et al. 2014)	Day-ahead ESS scheduling problem, neglecting power flow constraints	Optimizing day-ahead production schedule of wind farm with ESS, incl. secondary ESS handling forecast errors	n/a	Periodic boundary conditions

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(Maffei, Meola et al. 2014)	AC DOPF; solved with IPOPT in GAMS together with InterPSS	Microgrid with DG and ESS, minimizing cost of energy imported from utility grid	83-bus system representing feeder supplying residential loads in Steinkjer, Norway	Linear penalty function in the objective function for each time step (proportional to the deviation of the amount of stored energy from the maximal energy capacity)
(Marley and Hiskens 2016)	AC-QP DOPF; initialized using SOCP relaxation; using trust-region step to check linearization of AC power flow	Minimizing total generation costs for power system with ESS and renewable generation	Polish 3012-bus winter peak (3012wp) system over up to 8 hours with 30-minute time steps	A quadratic penalty function penalizing the deviation from a reference storage level (with penalty coefficient and reference storage level varying over the day)
(Moghadasi and Kamalasadan 2014)	AC DOPF with SOCP relaxation; implemented in GAMS and solved with an SOCP solver; rolling horizon ("Receding Horizon Control" method)	Real-time optimization of ESS charging/discharging in a smart distribution system with price and wind forecasting	32-bus radial test system over 24 hours	n/a
(Mégel, Mathieu et al. 2015)	SDDP approach, no power flow constraints	PV plant with ESS and flexible load providing both PV generation and frequency control service	n/a	Taken into account within each daily planning horizon through SDDP approach
(Murillo-Sanchez, Zimmerman et al. 2013)	Stochastic SC AC DOPF; decomposition into AC OPF subproblems for each time step, scenario and contingency and central coordination QP problem; see also Chapter 4.3.4 for its implementation in the MATPOWER Optimal Scheduling Tool	Power systems with variable generation, ESS and flexible demand	n/a (Illustrates with IEEE 118-bus test system over 24 hours with 4 stochastic scenarios and 7 contingencies per scenario, but case studies are relegated to a follow-up paper)	Linear penalty function (that is a linear combination of charged and discharged power for all time steps); see also Chapter 4.3.4 and (Zimmerman and Murillo-Sanchez 2016, Ch. 4.4)

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Reference	OPF model and solution method	Application	Test system	Valuation of stored energy
(Nejdawi, Clements et al. 2000)	AC DOPF; solved using SQP with an IPM solver; decomposing so that one in each iteration first solves each time step in isolation and then solves QP including intertemporal constraints	Scheduling of flexible demand for large energy-intensive end users	IEEE 30-bus test system over 24 hours; price-responsive load (not storage)	n/a
(Nguyen, Le et al. 2015)	AC DOPF; comparing with solving each time step in isolation; solved using MATLAB	Minimizing generation cost for power system with wind power generation and battery storage system	IEEE 14-bus test system and IEEE 57-bus test system over 24 hours	Periodic boundary conditions
(Paudyal, Canizares et al. 2011a)	Three-phase AC DOPF; solved using GAMS with MINOS; heuristic for local search within each time period	OPF for distribution system, minimizing energy imported from substation, and the number of switching operations of load tap changers and capacitors	Hydro One test feeder and IEEE 13-node test feeder over 24 hours	n/a
(Paudyal, Canizares et al. 2011b)	Three-phase AC DOPF; solved with genetic algorithm using GAMS, with MINOS for solving OPF within each time period	OPF for distribution system, minimizing energy imported from substation, and the number of switching operations of load tap changers and capacitors	Hydro One test feeder and IEEE 13-node test feeder over 24 hours	n/a
(Sojoudi and Low 2011)	AC DOPF solved through a convex dual SDP problem	Optimal charging of EVs (in a power system with conventional generators)	IEEE 14 bus test system over 11 time steps	
(Sperstad, Helseth et al. 2016)	AC DOPF; solved using MATPOWER and default fmincon IPM solver in MATLAB Optimization Toolbox	Minimizing total costs of distribution system with wind power generation and ESS	11-bus test system representing of a real distribution system on an island (Leka) at the Norwegian coastline	Explicit valuation of the future value of stored energy

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Reference	OPF model and solution method	Application	Test system	Valuation of stored energy
(Tant, Geth et al. 2013)	Combined ESS scheduling and sizing problem, no power flow constraints but including linearized voltage constraints from base case power flow sensitivities; solved using SQP with ILOG CPLEX IPM for subproblems; implemented in MATLAB using YALMIP	Multi-objective optimization of voltage regulation, peak shaving and annual cost (including arbitrage option) for a residential distribution system with PV and ESS with 3-phase inverter; models for related applications are described in (Geth 2014)	71-bus radial system over 10752 15-minute time steps	Periodic boundary conditions
(Uturbey and Simoes Costa 2002)	DC DOPF solved with a primal-dual interior point method	Optimizing demand side management by minimizing total generation costs minus consumer benefits	IEEE 14-bus system over 24 hours	n/a
(Uturbey and Costa 2003)	DC DOPF including active losses by simplified non-linear representation of branch flows	Optimizing interruptible load management by minimizing total generation costs minus consumer benefits plus utility incentives	IEEE 30-bus system over 12 hours	n/a
(Yamin, Al-Tallaq et al. 2003)	DC SC DOPF; solved with Benders decomposition with base case DOPF without network constraints as master problem and base case and contingency cases for each period with network constraints as subproblems	Generation scheduling in a deregulated power market, taking into account transmission and ramp rate constraints	IEEE 118-bus test system over 24 hours	n/a

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4 Survey of available OPF tools

The second part of the survey presented in this report focuses on software tools, either commercial or freely available, that include OPF functionality. Section 4.1 gives an overview of possible applications and types of users of such tools. Section 4.2 lists the commercial software tools that have been surveyed and briefly describes the OPF functionalities. Section 4.2.11 focuses on a particularly relevant freely available OPF tool, namely MATPOWER, and describes different the OPF methods it supports.

4.1 Applications of OPF tools

Possible application of OPF tools include all forms of optimization of the operation of a power system according to one or more criteria, given that a set of restrictions is met that include the power flow restrictions. Applications of OPF listed in (Wood and Wollenberg 1996, p. 516) include optimal generation dispatch to minimize generation costs, preventively dispatching generation to avoid violating security constraints, correctively dispatching generation to relieve the violation of technical constraints, or voltage-VAR optimization (by optimally setting e.g. generation voltages, transformer taps, capacitors and static VAR compensators). In addition, OPF is used as a part of power system planning studies and for general power system economics studies, e.g. evaluating the locational marginal cost of power generation (Wood and Wollenberg 1996, p. 516). Both Stott and Alsaç (2012) and Capitanescu (2016) emphasize that security constraints is an essential part of most practical OPF applications and therefore implicitly defines the term OPF as meaning security-constrained OPF.

Both Wang, Murillo-Sanchez et al. (2007) and Rau (2003) argue for the relevance of AC OPF in pricing and market clearing in deregulated electricity markets. They state that, as of the early 2000s, most or all US system operators were instead using DC-based OPF methods, and Capitanescu, Martinez Ramos et al. (2011) list PJM, New-England and California as examples of US electricity markets using DC SC OPF to calculate locational marginal prices. According to Wang, Murillo-Sanchez et al. (2007), "the full AC OPF has not been widely adopted in real-time operations of large-scale power systems". More recently, Castillo, Lipka et al. (2016) indicate that US system operators still commonly use approximate and linearized OPF solution techniques in the market clearing and that they frequently use DC OPF models. Alternatively, a decoupled real-reactive OPF model is also used by some system operators (Castillo, Lipka et al. 2016). As for applications in Europe, López, Sadikovic et al. (2015) report on how the Swiss TSO is applying AC OPF in their near-real-time security management. According to Capitanescu (2016), OPF is currently primarily used in day-ahead operational planning, but some utilities also use fast AC OPF (i.e. preventive security-constrained based on a customized core optimizer) for real-time applications.

It is relevant to mention the PEGASE project,[5] which aimed to improve algorithms (including SC OPF) for real-time control and operational planning of the Pan-European transmission system. The consortium included nine TSOs in Europe and Turkey, and the initial activities of the project included mapping and clarification the SC OPF requirements for European TSOs. However, the deliverable reporting on the results from interviews and questionnaires (D1.1) it not be publicly available.

Whereas OPF methods are in operational use for generation dispatch and system operation e.g. in the US and continental Europe, as described above, applications in the Norwegian power system appear to be limited. One likely reason is that generation dispatch in Norway mainly is determined by long- and short-term hydropower scheduling models (Fosso, Gjelsvik et al. 1999). This leaves relatively few degrees of freedom relevant for optimization by OPF models on the transmission system level (Botnen and Støa 1987).

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[5] EU FP7, 2008–2012, http://www.fp7-pegase.com/

Excluding the optimization of real power decision variables, other (AC) OPF applications that have been considered as relevant for the Norwegian system include: optimal regulation of real power dispatch in case of load imbalance (Ognedal, Flatabø et al. 1986, Botnen and Støa 1987), minimization of losses by controlling reactive power resources (Botnen and Støa 1987), the placement of reactive power reserves (Flatabø and Johannesen 1980, Botnen and Støa 1987), and selecting corrective actions for contingencies and situations with voltage problems (Botnen and Støa 1987). According to SINTEF/NTNU experts, the most relevant application in the Norwegian power system is selecting a voltage profile that reduces system losses. Furthermore, OPF is used by at least some Nordic TSOs for modelling corrective actions in offline reliability analyses, but not for selecting corrective actions in near-real-time security management (Sperstad, Jakobsen et al. 2014, Sperstad, Jakobsen et al. 2015) . This impression has been confirmed through interviews and informal discussions with Nordic TSOs and regulators.

On the distribution system level, the impression of SINTEF/NTNU experts is that the use of OPF among Norwegian grid companies is very limited. This impression is corroborated by informal discussions with the Norwegian regulator, and developers and distributors of software for Norwegian grid companies. One potentially relevant application for distribution networks (or generally networks that are operated radially) that is pointed out by SINTEF/NTNU experts is optimal network reconfiguration or switching. This view is also supported by a questionnaire-based survey of the state of the art of distribution system planning and optimization methods carried out by a CIGRE Working Group (CIGRE WG C6.19 2014, Appendix A). Here, network reconfiguration is one of the applications of "optimization" that are most frequently mentioned, although this might not actually imply the use of OPF. Likewise, voltage "optimization" in the sense of conservation voltage reduction (CVR) is also mentioned. At least for the case of Western Europe, they conclude, "Sophisticated optimization tools […] tend not to be used in practice by power system planners."

4.2 Commercial software tools including OPF

This section surveys commercially available software tools that include some OPF functionality. The motivation for the survey is to get an overview of the state of the art of OPF methods for commercial and industrial (as opposed to academic) applications. Our assumption is that which methods are implemented in commercial software tools would be indicative of which methods are efficient, robust and generally well proven and suited for real-world applications.

However, it is challenging to get a good overview of what OPF solution methods are prevalent in commercial software tools. Developers of OPF tools rarely publish any details on the solution methods in the scientific literature (Capitanescu, Martinez Ramos et al. 2011), and to the best knowledge of the authors, no extensive survey of solution methods in commercial OPF tools is available. A deliverable from the PEGASE project summarizes publicly available information about several of the commercially available OPF tools (PEGASE Consortium, Chapter 14.6). There does exist some web pages with overviews of relevant software tools. The Open Electrical web site contains a comprehensive list of power systems analysis and simulation software tools with some information for each.[6] The web site of the Power System Simulation Lab of the University of Queensland contains a page listing commercially and freely available software tools for power system analysis tools.[7] The latter also states whether each of the tools listed have OPF capabilities or not. A list of tools used for distribution system analysis are also found in a report by the CIGRE Working Group on "Planning and optimization methods for active distribution systems" (CIGRE WG C6.19 2014, Chapter 3.2.3), but any OPF functionalities are not discussed.

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[6] http://www.openelectrical.org/wiki/index.php?title=Power_Systems_Analysis_Software

[7] http://www.itee.uq.edu.au/pssl/drupal7_with_innTheme/?q=node/34

The methodology of the survey was to search the web sites of the software products, including available datasheets, brochures, release notes, whitepapers, and manuals, for any information indicating OPF functionalities or some form of optimization algorithms. The list of software tools potentially including OPF that were surveyed was compiled from the references listed in the previous paragraph as well as from software tools known to SINTEF/NTNU experts. The information from the software developers is complemented by a few scientific papers cited by the review papers listed in Section 2.1 on the methods implemented in commercial tools. The information was also crosschecked with descriptions in the references listed in the previous paragraph. Where the findings about the methods implemented in the tools were ambitious, questions were sent to the developers through the e-mail addresses announced for inquiries about the tools on the company web sites. Still, as we have not had access to all the information about the tools, we can only claim to reproduce the information that is available, and some of the descriptions will necessarily be incomplete.

The following software tools were surveyed: PSS®E, DIgSILENT PowerFactory, PowerWorld Simulator, NEPLAN, SMART FLOW, ETAP, CYME, PASHA, Aspen, SPARD Power, EasyPower, PSLF, Nexant Grid360, Paladin DesignBase, Schneider Electric ADMS, DINIS, ANAREDE, ANATEM, ANAFAS, PSCAD, PSS®SINCAL, SKM POWER*TOOLS, NETBAS, CAPE, Milsoft WindMil, IPSA, TROPIC. Those that are known to include OPF functionalities are listed in Table 4 together with the OPF solution method they employ. They are described in more detail in the following subchapters. One software tool in Table 4 that is not described in one of the following subchapters is NETBAS, as the OPF module of NETBAS (Optlast) is more naturally described in Section 4.4.1 together with other relevant software tools originating from the SINTEF/NTNU research community. For the tools where the information refers to a specific version of the tool, the version is given in Table 4.

Table 4. List of commercial software tools including OPF

Tool	Version	Solution method
PSS®E	33.3	Interior point method
PowerWorld Simulator	19	SLP active set method
NEPLAN	n/a	n/a
SMART FLOW (IPSO)	n/a	Interior point method
ETAP	16	Interior point method
CYME (CYMOPF)	7.2	Interior point method
PASHA	2000	n/a
Nexant Grid360 (SCOPE)	n/a	SLP active set method
Paladin DesignBase (EDSA)	6.0	n/a
Schneider Electric ADMS	n/a	n/a
DIgSILENT PowerFactory	2016	Interior point method
NETBAS (Optlast)	11	Conjugate gradient method
SPARD Power	2012	n/a
PSLF	20	n/a
TROPIC	n/a	Interior point method

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The following of the surveyed software tools do not seem to include any OPF or related optimization functionality and are therefore not described any further:

- ANAREDE
- ANATEM
- ANAFAS
- PSCAD
- PSS®SINCAL
- SKM POWER*TOOLS
- CAPE
- Milsoft WindMil
- IPSA
- Aspen
- EasyPower
- DINIS

However, it should be mentioned that the available description of some of these tools is ambiguous, and functionalities referred to as "optimization" features may or may not include OPF calculations. EasyPower is also supposed to include OPF according to the Open Electrical web site,[8] but we have confirmed that this is not the case. The web site also lists CAPE, IPSA Power, Milsoft WindMil and SKP POWER*TOOLS as having OPF although we have not found indications that this is the case. On the other hand, ETAP and NEPLAN are listed on the Power Systems Simulations Laboratory web site as *not* having OPF capabilities,[9] whereas we have confirmed that they do.

4.2.1 PSS®E

PSS®E is a software package for electric transmission system analysis and planning developed by Siemens Power Technologies International (Siemens PTI). It contains OPF as an analytical add-on module that can also be used as an integrated part of the PSS®E for conventional power flow calculations. The OPF method employed by the software tool is not mentioned in the publicly available information about the OPF module but it is regarded as fairly widely known that it is an interior point method. The method is furthermore described in some detail in the proprietary Program Operation Manual of PSS®E that SINTEF Energy Research has access to (v. 33.3). The objectives stated in the datasheet for the OPF module (Siemens PTI 2014) include minimizing generation costs, minimizing real or reactive power losses, minimizing load adjustments, minimizing or maximizing active power transfers or interface flows, and minimizing or maximizing reactive generation reserves. Constraints include generator real and reactive power output, branch flow limits (real, reactive or apparent power, or current), voltage magnitude limits, and limits for adjustable bus shunts or branch reactances.

4.2.2 DIgSILENT PowerFactory

DIgSILENT PowerFactory is a software solution for power system modelling, analysis and simulation developed by the German software and consultancy company DIgSILENT GmbH. According to their web site,[10] it is applicable to generation grids, transmission grids, distribution grids, industrial grids and

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[8] http://www.openelectrical.org/wiki/index.php?title=Power_Systems_Analysis_Software

[9] http://www.itee.uq.edu.au/pssl/drupal7_with_innTheme/?q=node/34

[10] http://www.digsilent.de/index.php/products-powerfactory.html

applications related to the integration of renewable energy (in particular wind energy). This section is based on information about the tool is available on the web site and in the product brochure (DIgSILENT 2016). SINTEF Energy Research also has access to the user's manual (PowerFactory 2016), which contains more details.

There are two OPF tools included with DIgSILENT PowerFactory: Reactive Power Optimisation (OPF I) and Economic Dispatch (OPF II). The first tool is a reactive optimal power flow tool based on an interior point method and can be used for minimizing network losses or maximizing reactive power reserves. The economic dispatch tool is available either as an AC OPF based on an interior point method or as a DC OPF based on linear programming. Objectives for (AC) economic dispatch include minimization of losses, minimization of costs, minimization of load shedding, and minimization of corrective actions. In addition to real and reactive generator output, the controls include transformer and shunt tap positions. The tool also includes functionality for security-constrained OPF, but the contingency constraints can only be included with DC OPF.

The product brochure (DIgSILENT 2016) also lists a number of other application involving some form of optimisation: Cable reinforcement optimization, optimization of separation points for radially operated distribution networks, voltage profile optimization by optimally choosing transformer tap positions, phase balance optimization, optimal capacitor placement, optimal power restoration, and optimal remote control switch placement (for improving reliability). None of these functionalities appears to be based on mathematical optimization but rather on complete enumeration of possible solutions or on different heuristics, possibly in combination with power flow calculations. For phase balance optimization there is an option to use an approach based on simulated annealing, so this optimization method could be regarded as an actual OPF method.

4.2.3 PowerWorld Simulator

PowerWorld Simulator is a power system simulation package designed to simulate high-voltage power systems. It is developed by PowerWorld Corporation, which was founded in 1996 by Professor Thomas Overbye at University of Illinois.[11] The types of customers listed on their web page include utilities, independent system operators, government agencies and power market traders, consultants and academia. The package includes add-ons for OPF, SC OPF and OPF Reserves.

The OPF that is implemented is an SLP algorithm based on their own revised simplex code.[12] Many of the details of the implementation are described in the on-line manual,[13] where the OPF method is referred to as Primal LP. For instance, the manual describes how the algorithm determines the set of active inequality constraints.[14] The objectives that are supported by the OPF add-on are minimizing the total generation and control cost and minimizing the change of controls from a certain operating point. Supported control variables include real generator power output, load shedding, FACTS devices and phase shifting transformers, but not generator voltage set points. Constraints that are supported include generator real and reactive power limits, area real power exchange, apparent power limits, but not bus voltage limits.

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[11] http://www.powerworld.com/company/history

[12] http://www.powerworld.com/knowledge-base/what-method-does-the-opfscopf-solution-use, dated 2012-07-05, retrieved 2016-09-29

[13] http://www.powerworld.com/WebHelp, retrieved 2016-09-29

[14] http://www.powerworld.com/WebHelp/#MainDocumentation_HTML/ Determining_Set_of_Active_Inequality_Constraints.htm

The SC OPF add-on extends the OPF algorithm with contingency constraints, and the manual describes some of the details for managing these constraints. The OPF Reserves add-on extends the OPF algorithm with controls for reserves (regulating, spinning and supplemental reserves) and associated constraints. It simulates the operation of an Ancillary Services Reserves Market and co-optimizes it with the operation of the market for energy.

4.2.4 NEPLAN

NEPLAN, also referred to as NEPLAN Electricity, is a power system analysis tool developed by the Swiss software company NEPLAN AG (NEPLAN AG 2015). It is described as a planning, optimization and simulation tool for transmission, distribution, generation and industrial networks and is stated as being particularly applicable to renewable energy systems and SmartGrid applications.[15] The software includes an Optimisation/Savings package containing an $N-1$ security-constrained OPF module. Available objectives functions include of MW or MVAR losses, total generation costs, MW or MVAR import to the system, or MW interface flow. The user also has the option to optimize the system according to multiple objective functions weighted by user-specified weighting factors.

The Optimisation/Savings package also include a number of other modules solving different optimization problems, but it is not clear from the description, which of these modules employ OPF calculations or even actual mathematical optimization. The modules include optimization of separation points for radially operated distribution networks, optimal placement of capacitors, optimal balancing of phases, and optimal restoration (switching) under contingencies. The objectives available various between the different modules, but typical objectives include minimizing real power losses, optimizing transformer and generator settings, minimizing phase unbalance, and minimizing component loading.

4.2.5 SMART FLOW

SMART FLOW is a software tool for simulating, analysing and optimizing transmission or industrial power systems. It is developed by the Belgian-based Power System Consulting (PSC) group belonging to Tractebel Engineering S.A. (GDF SUEZ),[16] and it is an associated product of the EUROSTAG power systems dynamics software by the same developers. IPSO (Integrated Power Systems Optimizer) is an OPF add-on to SMART FLOW described briefly on the company web site.[17] It is also described in a report from the PEGASE project (PEGASE Consortium) where it is referred to as the Tractebel OPF package. (Tractebel was the coordinator of the PEGASE project.) The OPF method is described in (Karoui, Platbrood et al. 2008) and is based on an IPM with a logarithmic barrier function where the optimization problem is solved by the KNITRO solver. Judging by the description of the algorithm in (Karoui, Platbrood et al. 2008) and the list of algorithms in the KNITRO User's manual (Artelys 2016, Chapter 2.17), the IPM that is implemented is the "Interior/Direct" method that is described in more detail by Waltz, Morales et al. (2006).

As presented in (Karoui, Platbrood et al. 2008), the novelties of IPSO are that it is a preventive-corrective security-constrained OPF tool with a so-called successive continuous optimization approach for discrete variables and support for modelling of the primary frequency control. Available control variables include real and reactive power generation, load shedding, transformer taps and steps of capacitor or reactor shunt compensation banks. Objectives that are supported include maximization of real power transfer,

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[15] http://www.neplan.ch/neplanproduct/en-electricity/#pa

[16] http://www.eurostag.be/en/about-us/about-us/leading-consulting-group/

[17] http://www.eurostag.be/en/products/smart-flow/static-functions/network-optimisation/optimum-power-flow-ipso/

minimization of real power generation or import, real power minimization, and minimization of the voltage difference between two buses. The constraints include bus voltage limits, limits on angle differences, branch flow limits (current or real, reactive or apparent power) capability curves for real and reactive power generation. Quadratic penalty functions can be used to represent these constraints in the objective function as soft constraints. Corrective security constraints are included as limits on the possible post-contingency rescheduling of controls.

4.2.6 ETAP

ETAP is an electrical power system analysis tool developed by the U.S. company ETAP/Operation Technology, Inc. The website[18] states that it is used by tens of thousands of companies, electric utilities, and government agencies for the design, analysis, maintenance, and operation of their electrical power systems. The application domains listed include transmission systems, distribution systems, power generation plants, and power systems of a number of industries.

ETAP includes an OPF module that is based on an interior point method with logarithmic barrier functions and functionalities for handling infeasibility.[19] Possible objectives include minimizing generation fuel costs, minimizing active and reactive power losses, optimizing power exchange with other systems, minimizing load shedding, and maximizing security indices (with respect to voltages and power flow). In addition to real power generation, controls also include reactive power generation, generator voltage controls, load shed, and capacitor bank or SVC controls. Reactive power optimization, sizing of capacitators and voltage regulation by optimizing transformer tap positions are some of the applications. Similar applications are also presented as Volt/Var Optimization,[20] which can include the control of generators, capacitors and transformer tap changers.

The analysis tool also includes a number of other optimization modules. The Optimal Capacitor Placement software tool uses a genetic algorithm including power flow calculations to optimize the location and size of capacitor banks to minimize the sum of installation and operation costs.[21] The objective of installing capacitors can be set to be either voltage support, power factor correction, or both. Switching Optimization is a tool for minimizing real power losses, minimizing overloading, minimizing voltage limit violations, maximizing reliability and maximizing the spare capacity of distribution feeders.[22] However, it is not specified if OPF or mathematical optimization techniques are used in the optimization. There also exists a module for security-constrained OPF that extends the ordinary OPF module with contingency constraints.[23] The Cable Sizing software and Transformer Tap Optimization Software is stated to calculate the optimal cable size and transformer turns ratio, respectively, without indicating whether OPF calculations are involved.

OPF also seems to be integrated in a number of the functionalities of the ETAP Real-Time Platform for power system operation. The ETAP Integrated DMS (also referred to as ETAP Smart Grid and ETAP

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[18] http://etap.com/electrical-power-system-software/etap-products.htm

[19] http://etap.com/distribution-systems/optimal-power-flow.htm

[20] http://etap.com/smart-grid/volt-var-optimization.htm?lang=en-US

[21] http://etap.com/downloads/etap-ocp-article.pdf

[22] http://etap.com/smart-grid/smart-grid-switching-optimization.htm

[23] http://etap.com/transmission/security-constrained-optimal-power-flow.htm?lang=en-US

ADMS)[24] includes Switching Optimization, Optimal Capacitor Placement, Volt/Var Optimization, capabilities for, optimal load shedding, and economic dispatch. Judging from the presentation, these functionalities can also be integrated with the operator's SCADA system. For transmission system applications, the ETAP EMS includes security-constrained OPF, and the Power Management System software includes optimal load shedding functionalities.

ETAP also includes a Battery Discharge Analysis module[25] and a Battery Sizing module[26], but these seem to be software modules for the technical design of a single battery storage system rather than for its integration in a power system. No functionality were found for multi-period OPF or other analyses of power systems with grid-integrated energy storage systems.

4.2.7   CYME

CYME Power Engineering software is a suite of power system analysis tools developed by the Canadian company CYME International T&D Inc., which is now a part of Cooper Power Systems, which in turn is a part of Eaton. The software suite is also referred to as PSAF (Power System Analysis Framework). It contains a set of modules for distribution systems (CYMDIST) and another set of transmission systems and industrial power systems. The Optimal Power Flow module (CYMOPF) is available for transmission and industrial power system applications. It implements an interior point method with logarithmic barrier functions (CYME International T&D Inc. 2006). The web site[27] describes the methods employed as "robust barrier-method based nonlinear optimization techniques". The web site furthermore states that CYMOPF handles infeasibility through "automatic relaxation of immediate binding constraints", and the user's guide explains how the hard constraint that made the iterate infeasible is replaced with a soft constraint represented by a quadratic penalty function (CYME International T&D Inc. 2006).

The CYMOPF tool supports controls including real and reactive power generation, load shedding, transformer tap settings, settings of phase shifting transformers, and adjustments of shunt or series compensation. The list of possible objectives includes minimizing fuel costs, minimizing real power slack generation, minimizing active or reactive power loss, minimizing reactive compensation (shunt or series), minimizing control changes, flattening voltage profiles, minimizing load shedding, or maximizing security indices (voltage or branch flow). Among the possible applications listed on the web site are the scheduling of ancillary services (real and reactive power), analysis of voltage collapse, investigation of transfer capabilities, and calculation of locational marginal costs.

Several of the modules for distribution systems also involve optimization in some sense. The Volt/VAR Optimization finds the optimal switching of capacitors and changing of tap positions of transformers and voltage regulators in order to minimize real power demand (CVR) or minimize active power losses. There are also modules for optimal recloser (auto-reclosing circuit breaker) placement, optimal voltage regulator placement and optimal network reconfiguration. The methods for optimization are not specified for any of these modules, but the placement of reclosers and voltage regulators seem to be optimized by complete enumeration of the possible solutions ("iterative search"), with an alternative to find the optimal placement

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[24] http://etap.com/smart-grid/smart-grid.htm

[25] http://etap.com/dc-systems/dc-battery-discharge.htm

[26] http://etap.com/dc-systems/dc-battery-sizing.htm

[27] http://cyme.com/software/cymopf/

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of one device at a time ("sequential search"). According to CYME user support, all modules are based on OPF calculations.

The brochure for CYME 7.2 lists new functionality for the modelling of battery energy storage systems in power flow analyses, studies of long-term dynamics, and in short-circuit analyses. As is described more clearly in the manual (CYME International T&D Inc. 2015, Chapter 12), the software tool implements a number of modes for the control settings of the ESS, e.g. Volt/VAR control through regulating the voltage for a network element by modifying the reactive power generation of the ESS. However, dynamic OPF in the sense defined in Section 1.2 is not implemented, and the ESS functionality seems to be designed for simulation studies rather than mathematical optimization studies.

4.2.8 PASHA

PASHA (Power Apparatus & System Homological Analysis) is a power system software developed by the Iranian company TOM Industrial Consultant CAD/CAM. According to their web site, it designed for the planning and operation of electric utilities and industrial power systems.[28] It includes an OPF tool for optimal real and reactive power flow. Although few details about the tool are available, it is stated to support objectives as cost and/or loss minimization and control variables including transformer tap settings. It also appears that OPF can be included with the Geographical Information System PASHA Distribution by the same developer.

4.2.9 SPARD Power

SPARD is a software solution for electric utilities developed by the Columbian company Energy Computer Systems Ltd./Inc. (ECS). It includes SPARD POWER, which is described as a tool for simulating, analysing and optimizing electric transmission and distribution systems and industrial power systems.[29] Optimal power flow is listed among the optimization applications, but no information about the OPF method is available on the company web site. Other optimization applications that are listed are optimal network reconfiguration (for radial networks), optimal substation placement, and optimal capacitor placement and sizing. Their Graphical Information System (GIS) SPARD DISTRIBUTION is also stated to include the same optimization functionalities, but it is not clear whether these involve OPF. SPARD POWER furthermore includes an optimal protection coordination tool that uses a genetic algorithm to optimize the settings of the protection system devices, but neither is it clear whether this optimization tool involves power flow calculations.

4.2.10 Nexant Grid360

Brian Stott and Ongun Alsaç founded Power Computer Applications (PCA) in 1984 (Alsaç, Bright et al. 1990), a company developing software for power system analysis, including security-constrained optimal power flow. In (Alsaç, Bright et al. 1990), they and other co-authors affiliated with PCA describe the development of an OPF solver and production software including security constraints. The SC OPF developed by PCA is also referred to as SCOPE. The company was acquired by Nexant in 2000,[30] but Stott and Alsaç continued in Nexant in various roles for the following decade, and were as late as 2012 apparently

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[28] http://www.tomcad.com/software_d1.htm

[29] http://energyco.com/english/soluciones/spard-power

[30] http://www.nexant.com/about, 2016-05-10

still consultants to the company.[31] Their white paper (Stott and Alsaç 2012) appears to be based on their experience in PCA/Nexant. In (López, Sadikovic et al. 2015), co-authored by Nexant employees, the application of a SC OPF based on (Alsaç, Bright et al. 1990) is further described.

Today, Nexant offers the software packages Grid360 Transmission Analytics and Grid360 Distribution Analytics (DA) for transmission and distribution system operation, respectively.[32] Although not explicitly stated on their home page or in any of the available product brochures, the Nexant Security Constrained Optimal Power Flow software (Nexant Inc. 2012) seems to be included in the power flow engines of both these software packages. This SC OPF tool is based on sequential linear programming, is stated to be applicable to real-time operational planning, and supports AC power flow and reactive as well as real OPF. The examples of objective functions that are mentioned in (Nexant Inc. 2012) include minimizing the cost of real power generation and maximizing real power transfers. Minimum shift of dispatched power is mentioned as another possible objective in (López, Sadikovic et al. 2015). All the usual system operating limits are included as both pre- and post-contingency constraints for all included contingencies, and the post-contingency constraints are enforced by preventive actions in the SC OPF. According to Nexant, their OPF is "the industry's most advanced and proven power flow software". This SLP-based general-purpose OPF solver is likely to be based on the one described in (Alsaç, Bright et al. 1990), who state that this solver is "fully implemented in the production OPF software" and that most of the features "have already been utilized within the industry". In (López, Sadikovic et al. 2015), co-authored by Swissgrid (the Swiss TSO) and Nexant employees, the experience from implementing a SC OPF tool based on (Alsaç, Bright et al. 1990) in the Swiss TSO security management for near-real-time decision support is presented.

Although the description in (Nexant Inc. 2012) is given in the context of transmission system operation, the same underlying OPF capabilities seems to be included in Nexant's distribution system power flow software package (Nexant Inc. 2015). The product description of Grid360 DA (Nexant Inc. 2015) mentions optimization applications such as conservation voltage reduction (CVR, i.e. optimal lowering of substation voltage as well as any feeder capacitors) and generally optimizing the voltage and reactive power flow profiles to minimize system losses etc. However, it is not clear which of these applications involve use of actual OPF. For transmission system applications, López, Sadikovic et al. (2015) list active OPF decision variables "generator MW output and phase shifting transformer taps" and the following reactive OPF decision variables: "bus voltages regulated by generators, transformers or Static VAR controllers, in-phase transformer taps, shunt capacitors and reactors and static VAR controllers". Inclusion of switching or network reconfiguration as possible decisions are considered by a "systematic search among a list of switching scenarios, usually substation reconfigurations" (López, Sadikovic et al. 2015).

4.2.11 Paladin DesignBase

Paladin DesignBase, formerly known as EDSA, is a power systems modelling and analysis tool developed by Power Analytics Corporation (formerly EDSA Micro Corporation). The primary application of the software tool seems to be the design of power systems such as industrial facilities or microgrids.

It includes a "Power Systems Optimization platform" that is a security-constrained OPF software tool.[33] Applications listed for this tool are the optimal sizing and placement of reactive power resources such as reactors, capacitors and Static VAR Compensators. The same web page also lists sizing of other power

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[31] https://meetings.vtools.ieee.org/m/13897, 2016-05-10

[32] http://www.nexant.com/software/grid360

[33] http://www.poweranalytics.com/paladin-software/paladin-designbase-power-systems-optimization-and-sizing/

system components (batteries, transformers, etc.) as application, but this appears to be applications of other programs than the OPF program. Their web site claims that the OPF software "is capable of handling power systems comprising thousands of buses, and has been used to optimize the planning and operation of some of the world's largest and most complex power systems". It is also mentioned[34] that the Quasi-Dynamic Simulation program for time series analysis can be "used for planning and sizing [...] and/or energy storage", but no optimization seems to be involved in this functionality.

4.2.12 Schneider Electric ADMS

Schneider Electric's Advanced Distribution Management System (ADMS)[35] is a software platform for grid companies integrating energy management systems (EMS) and other control room systems and functionalities. According to the information available on their web site, the Schneider Electric EMS includes functionalities for OPF as well as for optimally changing the network topology. As presented, the EMS is primarily designed for transmission system and "subtransmission" system applications. In (Schneider Electric 2012) it is stated that the ADMS "provides a host of analytical tools that recommend the most optimal device operations" and which includes "Network operation optimization – including Volt/VAR Control to manage load tap changers, capacitors, and voltage regulators [...]". The term "Volt/Var Optimization (VVO)" is used in (Dirkman 2012). Furthermore, in (Schneider Electric 2015) it is stated that the ADMS "manages and optimizes related network assets, including [...] Energy storage systems", but no further details are given. Both loss minimization and reliability maximization are generally mentioned as applications for the grid companies.

4.2.13 PSLF

PSLF (Positive Sequence Load Flow) is a part of the Concorda software suite for power system analysis developed by GE Energy Consulting. Based on the descriptions at the web site[36], the main areas of applications are large-scale contingency analysis and simulation of the transfer of power across the transmission systems. PSLF includes OPF functionality,[37] and it has been confirmed by GE Energy Consulting (private communication) that PSLF does in fact include both AC and DC OPF capabilities. However, which OPF method that is implemented in PSLF is confidential, and further details about the OPF capabilities are not publicly available. Dynamic OPF in the sense defined in Section 1.2 is not included, but can be implemented through the PLSF customized scripting capabilities (private communication).

4.2.14 TROPIC

TROPIC is an optimal power flow tool developed by the French TSO RTE. It is a module in the ASSESS software platform for probabilistic power system security analysis developed by RTE and National Grid Transco (Paul and Bell 2004, RTE 2006). The TROPIC module is based on an AC OPF model that is solved by a primal-dual IPM. ASSESS also includes a DC security-constrained OPF tool called METRIX. Security constraints are also supported by TROPIC either in a DC representation or in a full AC representation for the contingency cases (Paul and Bell 2004). For further details about the TROPIC OPF tool, Paul and Bell

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[34] http://www.poweranalytics.com/paladin-designbase-6-0-new-features/

[35] http://www.schneider-electric.com/b2b/en/solutions/for-business/s4/electric-utilities-advanced-distribution-management-system-adms/index.jsp

[36] http://www.geenergyconsulting.com/practice-area/software-products/pslf

[37] http://www.openelectrical.org/wiki/ index.php?title=Power_Systems_Analysis_Software#GE_Energy_.28PSLF.29

(2004) cite another paper that we have not been able to find and review (Blanchon, Boukir et al. 2000). As presented in (RTE 2006, PEGASE Consortium 2011), the functionalities of the TROPIC tool include economic dispatch, voltage profile optimization and VAR (reactive) planning. Congestion management and assessment of exchange capacities to different systems are mentioned as possible applications of the economic dispatch functionalities. In the description of reactive planning, it is stated that TROPIC uses "a decomposition coordination technique of the generalized Benders type" (RTE 2006, PEGASE Consortium 2011), but it is not clear whether this applies to the handling of contingency constraints in TROPIC in general or for the reactive planning functionality in particular.

4.3   Optimal power flow using MATPOWER

MATPOWER is a freely available (under a BSD licence) MATLAB code base for power flow and optimal power flow calculations (Zimmerman, Murillo-Sanchez et al. 2011). In the standard formulation of the AC OPF, the objective function is a sum of polynomial cost functions for real and reactive power injection for each generator. The inequality constraints include voltage limits, branch flow limits and limits on real and reactive power injection. In addition, the so-called extensible OPF architecture of MATPOWER (Zimmerman, Murillo-Sanchez et al. 2009) allows the user a large degree of freedom in formulating the OPF problem. This includes user-specified decision variables, cost terms and linear constraints.

The main reason for giving MATPOWER relatively much attention in this report is that this is the OPF tool that has primarily been used for the modelling work in the project (Sperstad 2016). Furthermore, MATPOWER supports a large number of external solvers for solving the optimization problem as formulated by MATPOWER, and surveying these solvers sheds some light on the state-of-the art of general-purpose nonlinear solvers that can be used for solving OPF problems. However, it should be noted that MATPOWER primarily seems to be used for academic purposes, and the OPF methods implemented in MATPOWER may therefore not reflect the state of the art of commercial, industrial and practical OPF applications.

4.3.1   MATLAB Optimization Toolbox

The MATLAB Optimization Toolbox `fmincon` function is a solver function for constrained nonlinear problems that can be applied to OPF problems, and it is one of the possible solvers in MATPOWER. There are four possible options for the solution algorithm used by the `fmincon` function (the `Algorithm` option in the function call): Interior-point optimization (option `interior-point`), sequential quadratic programming (SQP) optimization (option `sqp`), active set optimization (option `active-set`), and trust-region-reflective optimization (option `trust-region-reflective`). In the following subchapters each of the algorithms are briefly described, based on the documentation of the MATLAB Optimization Toolbox.[38]

4.3.1.1 Interior-point optimization

The default algorithm is the option `interior-point`, an interior-point algorithm with a logarithmic barrier term that solves the KKT equations for the constrained nonlinear problem. The algorithm includes possible steps at each iteration: 1) A "direct" (Newton) step in the space of (primal) decision variables, slack variables and dual variables by solving the KKT equations using Newton's method. 2) A conjugate gradient (CG) step in the space of decision variables and slack variables using a CG algorithm to minimize a quadratic subproblem formed from approximating the Lagrange function of the original problem (Byrd, Hribar et al. 1999). The method is described in more detail in (Waltz, Morales et al. 2006) and is also implemented in the KNITRO solver (cf. Section 4.3.3). It is the combination of a line search step (1) and a trust region step (2).

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[38] http://se.mathworks.com/help/optim/ug/constrained-nonlinear-optimization-algorithms.html

The trust region step is a "safeguarding" step that is taken when the Hessian used in the Newton step (1) becomes non-positive definite, which is done to improve robustness compared to a method based solely on Newton steps.

4.3.1.2 Active set optimization

The standard active set algorithm implemented as the option `active-set` is based on the method described in (Gill, Murray et al. 1984) and is also referred to as a *projection method*. At each iteration, a quadratic programming subproblem is formed by approximating the KKT equations, the Hessian is estimated using the BFGS formula (in a way that positive definiteness is ensured), and the QP subproblem is solved by a quasi-Newton method. Constraints are taken into account by projecting the step direction onto a feasible subspace based on information about the active set (i.e. the boundaries in the solution space given by those constraints assumed to be active in the current iteration). The step length is determined by a line search approach. The MATPOWER documentation states that because it is not using sparse matrices, the algorithm is not applicable for large-scale systems (Zimmerman and Murillo-Sanchez 2015, p. 125).

4.3.1.3 SQP optimization

The `sqp` algorithm is similar to the `active-set` algorithm described above. The differences are that the `sqp` algorithm has a number of additional features making it more robust and that it is using more efficient linear algebra routines. For these reasons, the MATLAB Optimization Toolbox documentation recommends trying the `sqp` option before the `active-set` option, but adds that none of them are large-scale algorithms.[39] According to the MATLAB Optimization Toolbox documentation,[40] the "basic `sqp` algorithm" is described in (Nocedal and Wright 2006, Chapter 18). Still, as for the `active-set` algorithm, the MATPOWER documentation does not recommend using `sqp` for large-scale systems (Zimmerman and Murillo-Sanchez 2015, p. 125).

4.3.1.4 Trust-region-reflective optimization

The `trust-region-reflective` algorithm implemented in the MATLAB Optimization Toolbox is not applicable to AC OPF since it does not support general nonlinear constraints, but only linear constraints or bounds. Therefore, it is not being described in more detail here.

4.3.2 MATLAB Interior Point Solver

The MATLAB Interior Point Solver (MIPS) is included with all newer versions of MATPOWER (from version 4 and newer) and can be used as the nonlinear programming solver in the MATPOWER OPF. The method is a primal-dual interior point method and is described in detail in the MATPOWER User's Manual (Zimmerman and Murillo-Sanchez 2015, Appendix A). The method is implemented in pure MATLAB code and is based on the step-controlled primal-dual interior point method presented by Wang, Murillo-Sanchez et al. (2007). The method solves what is in (Capitanescu and Wehenkel 2013) referred to as the "reduced KKT system", which makes the size of the equation system independent of the number of inequality constraints and increases computational efficiency. The efficiency of MIPS compares favourably with an MCC IPM in some of the numerical experiments of Capitanescu and Wehenkel (2013), although it comes out as less reliable in some of the other test cases considered there.

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[39] http://se.mathworks.com/help/optim/ug/choosing-the-algorithm.html#bsbwxm7

[40] http://se.mathworks.com/help/optim/ug/constrained-nonlinear-optimization-algorithms.html#bsgppl4

4.3.3 Other nonlinear programming solvers that can be used with MATPOWER

MATPOWER is also able to interface with a number of other linear, quadratic and nonlinear solvers than MIPS and the MATLAB Optimization Toolbox solvers described above. Descriptions of all these solvers are given in the MATPOWER User's Manual (Zimmerman and Murillo-Sanchez 2015, Appendix G) and are here only mentioned briefly in the rest of this section.

TSPOPF is a collection of three AC OPF solvers implemented in C based on the methods presented by Wang, Murillo-Sanchez et al. (2007). Two of the solvers are variants of the same primal-dual interior point method as implemented in MIPS. In addition, a trust-region-based augmented Lagrangian method is also included. See (Nocedal and Wright 2006, Chapter 17) for details on these methods. (Zimmerman and Murillo-Sanchez 2015, Appendix G.12) claims that the primal-dual interior point method is the fastest of all solvers for AC OPF available for MATPOWER.

MINOPF is an AC OPF solver based on the general-purpose nonlinear programming software package MINOS.[41] It is implemented in Fortran, and according to (Zimmerman and Murillo-Sanchez 2015, Appendix G.8) it is often the fastest solver available to MATPOWER for smaller OPF problems.

IPOPT is a general-purpose interior point software package for large-scale nonlinear optimization problems. It is implemented in C++ and is freely available under the Common Public License. According to (Zimmerman and Murillo-Sanchez 2015, Appendix G.6), IPOPT with the PARDISO linear solver is the fastest OPF solver available to MATPOWER for very large-scale OPF problems.

KNITRO is a commercial, general-purpose optimization software package, including a nonlinear programming solver (Artelys 2016). However, the solvers should also be available through a free academic license. Algorithms included in KNITRO is the "Interior/Direct" IPM algorithm, the "Interior/CG" IPM algorithm, an SQP algorithm, and a similar algorithm called "Active Set" based on a combination of SLP and SQP. The "Interior/Direct" IPM algorithm, due to Waltz, Morales et al. (2006), is the same as the IPM algorithm that is implemented in the MATLAB Optimization Toolbox, cf. Section 4.3.1.1.

SDP_PF is an implementation of semidefinite programming relaxation applied to the optimal power flow problem as described more generally in Section 2.7.2. The implementation is documented by Molzahn (2014).

4.3.4 MATPOWER Optimal Scheduling Tool (MOST)

The MATPOWER Optimal Scheduling Tool (MOST) is a MATPOWER feature which was released with MATPOWER version 6.0 (Zimmerman and Murillo-Sanchez 2016). It is a framework for solving a range of electric power scheduling problems, including dynamic (multi-period) optimal power flow for scheduling energy storage systems. However, as of version 1.0 of MOST, it does not support AC power flow but only DC power flow. MOST is an implementation of the general framework described in (Murillo-Sanchez, Zimmerman et al. 2013) which according to (Zimmerman and Murillo-Sanchez 2016) can be used to solve as complex problems as "a stochastic, security-constrained, combined unit-commitment and multiperiod optimal power flow problem with locational contingency and load-following reserves, ramping costs and constraints, deferrable demands, lossy storage resources and uncertain renewable generation". However, it should be noted that including all these features (or even several of them) in the problem formulation makes the problem very computationally demanding (Murillo-Sanchez, Zimmerman et al. 2013, Zimmerman and

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[41] http://www.sbsi-sol-optimize.com/asp/sol_products_minos_desc.htm

Murillo-Sanchez 2016). As mentioned in Chapter 3.1, the framework includes valuation of the energy stored in the ESS at the end of the planning horizon through penalty (or energy cost) terms. The MOST user's manual (Zimmerman and Murillo-Sanchez 2016, Ch. 4.4) expands on the formulation of the penalty terms introduced in (Murillo-Sanchez, Zimmerman et al. 2013) in the general stochastic and security-constrained framework. In addition, MOST also includes optional constraints for the expected energy stored at the end of the planning horizon to be within a given range or to be equal to a given target value.

4.4 Relevant methods implemented by the SINTEF/NTNU research community

4.4.1 Optlast

Optlast is an OPF tool that was developed by EFI (Norwegian abbreviation for "Elektrisitetsforsyningens Forskningsinstitutt") from around 1980 to around 1990. EFI is one of the precursors of SINTEF Energy Research. The first description of the Optlast tool is found in the 1980 user guide (Flatabø and Johannesen 1980), the updated user guide from 1984 (Flatabø, Johannesen et al. 1984) is nearly identical, but more details on the actual OPF algorithm implemented is found in later versions of the user guide (Ognedal, Flatabø et al. 1986). Around 1991, the OPF algorithm of Optlast was included in a module in the Network Information System NETBAS that had also been developed by EFI. In 1996, Powel was founded as a spin-off company of what was originally EFI, and further development of NETBAS has since been done by Powel. There is still a module called Optlast in new distributions of NETBAS from Powel, but the stand-alone Optlast program is on the other hand owned by SINTEF Energy Research.

Among the applications of the program Optlast, as described in (Flatabø, Johannesen et al. 1984) is finding the amount of reactive compensation at different, preselected locations and finding how to best utilize the existing generation capacity (including capacity for generating or consuming reactive power) of a system. The typical objective function for such application typically includes the value of reduced energy losses, the value of reducing the amount of reactive power imported to the system, and the costs associated with reactive generation (for generators where this is relevant and for reactive compensation devices). Generation of real power can be included as decision variable, but typically this is not done because the program was intended for the Norwegian power system where the vast majority of generators were (and still are) hydropower generators; for these the generation schedule is assumed to already be fixed for the planning horizon of the OPF problem. Energy conversion losses in generators can also be taken into account. Constraints to the OPF problem are voltage limits and real and reactive power generation; branch flow limits are not explicitly described in (Flatabø, Johannesen et al. 1984).

The optimization method that is implemented in the model described in (Flatabø, Johannesen et al. 1984) is a steepest descent-type method with an optimum step length algorithm along the negative gradient direction. However, there is also an option to disable the automatic adjustment of the step length for objective functions where this could potentially cause problems (Flatabø, Johannesen et al. 1984). Although it is difficult to tell from the relatively brief description, the general methodology appears to be very similar to the one of Dommel and Tinney (1968). For the model described in (Ognedal, Flatabø et al. 1986), conjugate gradient methods are also included as alternative solution methods, with a Fletcher–Reeves variant being the default algorithm. In both cases, the optimization method is intended to use a previously solved power flow as a starting point, and it apparently solves a conventional power flow calculation for each iteration by a standard Newton–Raphson algorithm (Hornnes 1990).

Inequality constraints are handled by checking each variable for constraint violation after each step and bringing them back to the feasible region by different heuristics (Flatabø, Johannesen et al. 1984, pp. 15–16, p. 25, Ognedal, Flatabø et al. 1986, p. 22). E.g. in the case of voltage limit violations for load buses, the

voltage is turned into a decision variable and the reactive power generation in any neighbouring buses with reactive compensation devices is "released" instead. This corresponds to swapping which buses are PV and PQ buses. In the case of violation of the real power generation capacity for the reference bus, the value is fixed to the upper limit and another generator is assigned the status of reference bus.

In addition to those applications described in (Flatabø, Johannesen et al. 1984), some additional features and functionalities are described in later reports. For instance, Ognedal, Flatabø et al. (1986) mention the routine OPTREG which can be used to find the optimal control strategy in case the generation needs to be increased at one or several nodes in the system. Ognedal, Flatabø et al. (1986) also describe routines that use sensitivity information from the OPF solution to calculate optimal control choices for reactive power generation to support reactive load changes, real load changes, or load bus voltage changes. Furthermore, Botnen and Støa (1987) describe how Optlast can be used to optimize the choice of corrective actions in case of voltage problems or deviations between forecasted and actual load (e.g. due to contingencies). Hornnes (1990) describes possibilities for integrating Optlast in models used for transmission system and generation expansion analyses. There also exist other project memos and reports on Optlast (Ognedal 1988, Flatabø 1989), but these deal with technical details and developments of less general interest.

To the best of our knowledge, there has been no development of the Optlast program or on the Optlast OPF algorithm in NETBAS after 1991. Some of the power flow functionality of the Optlast program has since been implemented in the Samlast model (cf. Section 4.4.3), but the OPF functionality has not been used to a significant extent. The Optlast module of NETBAS appears to have a small number of users, but it is not clear whether any of them is actually an active user of the OPF functionality.

4.4.2 OPF code implemented by Olav Bjarte Fosso

Olav Bjarte Fosso, Professor at NTNU and formerly employed as Scientific Advisor and Senior Research Scientist at SINTEF Energy Research, implemented an OPF code base during the years around 1992/1993. The OPF method underlying the implementation was based on (Sun, Ashley et al. 1984), solving an equation set using Newton's method and forward and backward substitution. The implementation is reported to be robust and has been used on systems up to some thousand buses (including a version of the Nordic power system model). It is implemented in FORTRAN with data structures specified to be suited for data exchange with the Siemens PSS®E and TPLAN data file formats. In addition to personal research projects at NTNU, the code has also been considered, tested or used in a few projects by research scientists at SINTEF Energy Research. Some information for one particular project was found in a restricted SINTEF Energy Research project memo (AN 09.12.101). Apart from this, no documentation of the implementation is available.

4.4.3 SINTEF power market models

The EMPS model (EFI's Multiarea Power Market Simulator), which is owned and maintained by SINTEF Energy Research, is a fundamental power market model that can be applied for long-term hydropower scheduling, price forecasting and general system studies (Wolfgang, Haugstad et al. 2009). It finds the generation schedule for each price area that minimizes the expected sum of all system costs over the planning horizon for a hydro-thermal power system. This multi-period optimization problem is solved using the water value method (Gjelsvik, Røtting et al. 1992), which is an application of stochastic dynamic programming. It is a transportation model where power exchange between price areas described by a connection and a capacity, and hence it is not a (dynamic) optimal power flow model in the sense used in the rest of this report. The two models Samlast and Samnett, also owned and maintained by SINTEF Energy, build on the EMPS model by integrating the market clearing with detailed power flow studies.

In Samlast, a variant of the AC or DC power flow (optional; a description is given in a SINTEF Energy Research project memo (AN 11.12.87) that is not publicly available) study is performed after knowing the

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outcome of the market clearing for one week. In case power flow constraints on single lines or flow gates are violated, Samlast applies a heuristic for reducing the transport capacities used in the market clearing in a subsequent iteration. In this way overloads are alleviated by iteratively limiting transport capacities. Thus, the market clearing will implicitly take into account the power-flow constraints of the transmission grid.

In Samnett, a DC power flow study is performed after knowing the outcome of the market clearing for one week (Helseth, Warland et al. 2011, Helseth, Warland et al. 2013). In case power flow constraints on single lines or flow gates are violated, Samnett adds linear constraints indicating which market areas (or price zones) that should adjust their net position in order to most economically efficient alleviate the overload(s). Samnett builds a power grid equivalent for each base case power flow, in which each price area represents a node (bus). A user-defined weighting scheme is needed when aggregating power transfer distribution factors (PTDF) in the detailed transmission grid to an equivalent model. In contrast to Samlast, Samnett explicitly represents the physical properties of the grid in the market clearing. This scheme resembles the flow-based market clearing which is likely to be introduced in the Nordic power market in the near future.

Samlast and Samnett are fundamental hydro-thermal market models including transmission grid analyses. This type of models is typically used by TSOs, regulators and other market participants that see the need for emphasizing on the transmission grid bottlenecks in their price forecasts or system studies. The multi-period aspect is treated through a combination of water value computation by use of stochastic dynamic programming (SDP) and simulation, governed by a heuristic layer. This heuristic approach distinguishes these models from the majority of dynamic optimal power flow models described in Chapter 3. One could also consider these models as a multi-period scheduling model extended with power flow constraints, in contrast to the models in Chapter 3, which are generally optimal power flow models extended to multi-period scheduling. Multi-period optimization is essential in the SINTEF power market models due to the dominance of hydropower generation with long-term energy storage (reservoirs) in the Nordic power system.

Prodnett is another type of model combining hydrothermal market modelling and physical power flow. Prodnett was developed by SINTEF Energy Research and is documented in (Helseth, Gjelsvik et al. 2012). Unlike Samlast and Samnett, it is entirely based on formal optimization through stochastic dual dynamic programming (SDDP). The market clearing is done per transmission grid bus, and linear power flow constraints are added iteratively (when needed) in the model.

The mentioned models lack the degree of detail needed in operative short-term scheduling, as they do not treat binary variables (as e.g. needed when in exact unit commitment) or non-convex relationships (which often appear in hydropower generator efficiency curves). A linearized representation of start-up costs is included in the models as described in (Helseth, Gjelsvik et al. 2012, Ch. 5) and references within. The SDDP algorithm applied in Prodnett requires convexity; thus, adding binary variables and non-convex relationships will pose a challenge. However, several recent approaches have been presented in the literature trying to deal with non-convexities in the SDDP algorithm (Zou, Ahmed et al. 2016).

5 Application of OPF methods

This chapter discusses the application of OPF methods from three different perspectives. In Section 5.1 we discuss the available OPF methods described in Section 2 in light of the findings from the survey of commercial OPF tools in Section 4. In Section 5.2 we discuss the application of OPF methods in the studies of grid-integrated energy storage systems surveyed in Section 3. In Section 5.3 we discuss the application of OPF methods from a practical perspective, drawing on the views of SINTEF/NTNU experts as well as the published views of international OPF experts.

5.1 Application of OPF methods in commercial OPF tools

To summarize the findings in Section 4.2 on the solution methods employed in commercially available OPF tools, interior point methods and (active set) sequential linear programming methods appear to be most prevalent. Among the 15 tools surveyed that did include OPF functionality, Table 4 shows six tools using IPM and two tools using SLP. Counting also those tools where the OPF method is known to the authors but could not be represented due to confidentiality, six tools are using IPM, three tools are using SLP, one tool is using a conjugate gradient method, and for five tools we have not been able to find out which OPF solution method is employed. Although at least some information about the OPF method is publicly available for most of the commercial OPF tools, for most of them the details still appear to be regarded as a trade secret.

The conclusions on the state of the art of OPF methods for commercial applications are generally the same as the conclusions of more academic papers reviewing the state of the art of OPF methods (cf. Section 2.8): Gradient methods are almost completely superseded by more modern OPF methods. Also Newton-based optimization methods for direct minimization are largely superseded except when employed for the local solver in more sophisticated methods. Both active-set methods (based on sequential approximation algorithms, i.e. SLP or SQP) and interior-point methods appear to be popular both in OPF software tools and as topics for current research. Capitanescu (2016) indicates that OPF tools used by utilities and grid operators traditionally have been based on customized solvers (typically SLP, SQP, Newton's method, IPM), but that general-purpose solvers are now increasingly being used for the core optimizer. According to Capitanescu (2016), IPM appears to be the most popular class of solution methods for such applications. Regarding the prevalence of SQP-based OPF methods, Frank, Steponavice et al. (2012b) states that SQP has been "successfully applied in a number of research and commercial OPF algorithms" and is "used in several commercial OPF packages", but no specific references are given here. Our survey has not been able to confirm this claim that the use of SQP is prevalent among commercial and industrial OPF applications, unless it is taken to also include SQP used as part of an IPM as e.g. in (Waltz, Morales et al. 2006). Those of the surveyed tools indicating that a sequential approximation method is being used states this as an SLP method.

As mentioned in Section 4.1, DC OPF or decoupled real-reactive OPF seem to still be preferred, at least for US market clearing applications. (Cain, O'Neill et al. 2012, Chapter 7) even claims that there does not exist a "commercially viable full AC OPF", i.e. a solver for the AC OPF problem not relying on linearization and other approximations. This is in contrast with our findings that commercial OPF tools typically are based on full AC power flow. Most of the tools are furthermore presented either explicitly or implicitly as solving the coupled real-reactive OPF problem. Still, we cannot conclude from our survey whether these OPF implementations are suitable for solving all relevant practical OPF problems, let alone whether they are sufficiently reliable and efficient for OPF solutions for near-real-time operation. As Castillo, Lipka et al. (2016) conclude in their survey of the state of the art of OPF methods, there are still trade-offs that must be made between convergence properties and computational performance.

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None of the OPF tools that were surveyed state that they are using any of the global optimization methods listed in Section 2.7 (i.e. non-deterministic methods and other more unconventional OPF methods). (Stott and Alsaç 2012, p. 17) also expresses scepticism over the usefulness of stochastic OPF formulation, fuzzy optimization, multiple objectives, etc. for practical OPF applications. According to (Frank, Steponavice et al. 2012b, p. 267), few EP OPF algorithms have been used for industrial applications. In our survey, we did find a few cases where metaheuristics were used for commercial power system optimization applications: DIgSILENT PowerFactory can use simulated annealing for phase balance optimization, ETAP uses a genetic algorithm for optimizing the location of capacitors, and SPARD POWER uses a genetic algorithm to optimize protection system settings. However, it is not clear from the descriptions whether the metaheuristics are used in conjunction with (optimal) power flow calculations or whether they are stand-alone optimization methods not considering power flow constraints.

Finally, none of the OPF tools that were surveyed support multi-period optimization (DOPF), i.e. including temporal constraints to represent e.g. energy storage systems. However, the developers of some of the tools indicate that this could be handled by customized scripting.

5.2   Application of OPF methods for distribution systems with energy storage

This section summarizes the survey in Section 3.1 of OPF methods used for optimizing the use of energy storage systems in distributions and puts the findings in context. It should be noted that the work surveyed here were all academic studies, as none of the commercial tools surveyed in Section 4.2 seem to support such applications. Therefore, which methods are implemented for these studies is probably not indicative of which methods are efficient, robust and suited for real-world applications for analysing grid-integrated energy storage systems.

As shown in Table 3, the survey of the literature shows that the majority of references use a full AC DOPF model and that only a few references apply special methods for solving the resulting multi-period problem. Different decomposition or decoupling approaches are suggested in (Nejdawi, Clements et al. 2000, Murillo-Sanchez, Zimmerman et al. 2013, Gemine, Ernst et al. 2014). There appears to be no compelling arguments why the application of convex relaxation techniques as in (Gayme and Topcu 2011, Gayme and Topcu 2013, Gopalakrishnan, Raghunathan et al. 2013, Gemine, Ernst et al. 2014) should be particularly necessary for dynamic AC OPF *per se*, more than it may be for regular static AC OPF. There has also been work approaching the scheduling problem of grid-integrated energy storage from an optimal control perspective (Chandy, Low et al. 2010, Bose and Bitar 2014, Eshghi and Patil 2015), but these references do not explicitly discuss which (if any) advantages such techniques bring to the problem.

Where the solution method is stated, IPMs seem to be predominant, but the reason for this may also be that several of the implementations are based on MATPOWER, where the recommended solvers are based on IPM. Some of the references that in the contrary employ apparently custom-made OPF solvers are basing these on SQP methods (Carpinelli, Celli et al. 2013, Marley and Hiskens 2016). A few OPF methods are based on alternative linearizations of AC power flow (i.e. other than DC power flow) (Carpinelli, Celli et al. 2013, Tant, Geth et al. 2013, Fortenbacher, Zellner et al. 2016, Marley and Hiskens 2016), but the linearized power flow formulation proposed in (Bolognani and Zampieri 2016) does not yet seem to be used directly in any DOPF method. Only a few references consider stochasticity in their problems (Murillo-Sanchez, Zimmerman et al. 2013, Grillo, Pievatolo et al. 2016, Sperstad, Helseth et al. 2016). Furthermore, the majority of the references assume a 24-hour planning horizon, and only a few AC DOPF models consider rolling horizon scheduling e.g. to mitigate effects from forecast inaccuracy (Lamadrid, Mount et al. 2011, Baker, Zhu et al. 2013, Khanabadi, Moghadasi et al. 2013, Moghadasi and Kamalasadan 2014, Marley and Hiskens 2016).

It is evident from the overview in Table 3 of the case studies used in the literature that most test systems are relatively small. The exception is (Marley and Hiskens 2016), showing results for a 3012-bus test system, which effectively gives 48 192 buses in the multi-period optimization problem when multiplying by the number of periods. Apart from this reference, the test systems contain at most 140 buses, effectively corresponding to 3360 buses in the multi-period problem. It is not clear from the overview whether the computational efficiency of the predominant "brute-force" AC DOPF solution methods is sufficient for real distribution systems that could easily have more than 140 buses. However, knowing also the computation time for solving the case studies would be necessary in order to make a comparison of their efficiency, but since this is typically not stated in the references that were surveyed, no such information was included in the overview in Table 3. It should also be noted that many of the case studies are based on IEEE test systems representing transmission grids, but some of the references consider test systems representing distribution systems. Those test systems listed as distribution test systems are radial if not otherwise stated. The linearized AC DOPF method of Fortenbacher, Zellner et al. (2016) is only applicable to radial systems, but they claim that, where applicable, it is computationally superior to nonlinear AC DOPF methods.

Most of the references surveyed in Section 3 do not discuss what power system actor is owning and/or operating the ESS, nor do they discuss regulatory constraints related to the ESS. Carpinelli, Celli et al. (2013) briefly discuss a Smart Grid Operator acting as an ESS aggregator, and Celli, Pilo et al. (2013) assume that a distribution network operator operating the ESS cannot use it for price arbitrage. In microgrid applications, the ESS owner/operator is usually assumed to be the microgrid operator. Likewise, references studying power plants with intermittent generation and ESS typically assume that the same actor is owning/operating the ESS as the power plant, and the objective usually is to maximize power generation revenues. Power flow constraints are often not taken into account in these references. Other business models for third-party ESS owners are rarely considered in the DOPF references we have surveyed, but Carpinelli, Celli et al. (2013) consider DSOs using ESS to provide reactive power to the TSO.

Furthermore, which decision variables that are considered is rarely discussed explicitly. Usually real and reactive power dispatch is optimized simultaneously. If conventional generators and points of common coupling to a transmission grid are included in the system under study, their real and reactive power usually seem to be optimized simultaneously with ESS decision variables. For distribution system applications, none of the references discusses in any detail what decision variables the distribution system operator would be realistically able to control or how the OPF is to be implemented in the operation of a real distribution system.

5.3 Practical use of OPF

In the literature surveyed in the preceding sections, one rarely finds any mention of what is needed to be able to apply the OPF models and methods that are presented to real-life problems. For any future research activities by SINTEF Energy Research in collaboration with industry that involves OPF, such practical considerations are important. A few references do discuss the practical aspects around OPF, and of particular interest is the white paper by Stott and Alsaç (2012). There, a large number of comments and considerations on what is referred to as "real-life" or "practical" OPF problems is listed. Although the white paper may be seen as somewhat subjective, having few references and not being peer-reviewed, it nevertheless carries authority: The authors, Brian Stott and Ongun Alsaç, have published early and widely cited work on optimal power flow (Alsaç and Stott 1974) and have been heavily involved in the development of commercial OPF software (cf. Section 4.2.10). Many of considerations mentioned in (Stott and Alsaç 2012) were elaborated already in (Alsaç, Bright et al. 1990), where they and co-authors describe in some detail the methods underlying their OPF software. Their general message is that most of the research literature consider simplified OPF formulations that are not applicable to real-life problems and solution methods that are not

useful for industrial applications. This view is also echoed in (Capitanescu 2016). Here we simply reproduce the considerations given in (Stott and Alsaç 2012) that appears most relevant for this report:

- Convergence problems is a severely under-addressed issue.
- Solution is typically path dependent.
- Solution process for real-life, large-scale problems typically have to rely on a set of heuristics and rules to avoid oscillations and other convergence problems, and industry-grade OPF software cannot rely of general-purpose mathematical optimization packages alone. However, general-purpose solvers are increasingly used for the "central optimizer" of industry-grade OPF tools.
- Identification of the potentially binding constraints is an important part of the solution method that is specific to power system applications.
- For real-life problems, co-optimizing real and reactive power in the same OPF easily leads to solutions that make little sense from a practical engineering perspective (e.g. having unreasonably large reactive power flows, or not being possible to obtain in practice by the controls available to the operator.)
- Handling discrete controls by full MIP OPF formulation is unlikely to be practical unless the discrete controls are few and crucial to the problem.
- Ambitions of achieving a solution that is the guaranteed to be the global optimum of the original problem formulation are unrealistic for all practical OPF problems.
- For most practical applications, the majority of the calculations associated with the OPF take place outside of the optimization problem solver itself (the "central optimizer"), and it is usually these that are crucial to the efficiency and effectiveness of the solution process. The central optimizer is being called repeatedly throughout the solution process, and the optimization problem formulation is typically updated for each iteration.

For the work of this report, interviews were also carried out with OPF experts at SINTEF Energy Research and NTNU, and some practical recommendations are summarized below. Although the interviewees stressed that some of these considerations are somewhat generalized and subjective, they are based on the interviewees' experience on developing methods of optimal and conventional power flow and applying these on real systems and for industrial applications:

- A solution of an OPF decision support tool can have a rather limited value to the decision maker by itself, if he or she cannot see how such a solution can be obtained in practice. For instance, an operator at a power system control centre may have only a limited number of controls available, and in practice, only a subset of these can be controlled simultaneously. Human operators tend to limit themselves to a set of options and control parameters that is humanly manageable, and OPF formulations with a large number of decision variables is not compatible with such a way of thinking.
- When using a non-linear OPF formulation and there are multiple control parameters available to effect the same objective, the OPF solution tends to involve simultaneous changes in many of these control parameters. A solution that requires the operator to change several controls just a little is less convenient to the operator than a (nearly) equivalent solution involving just a single control.
- Furthermore, for distribution systems, the set of control parameters may be even more limited than for transmission systems.
- Therefore, one should be careful with formulating the OPF problem such that all possible decision variables are "let loose" and determined simultaneously, irrespective of what variables can actually be controlled in practice.
- To obtain reasonable solutions that make sense from a practical engineering perspective, one could consider decoupling the decision variables for active power and those determining the voltage profile, as these sets decision variables can often not be co-optimized in practice.

- Since OPF solutions may assume that larger number of decision variables than can be realistically controlled by the operator, the solution is likely to underestimate the value of the objective value (e.g. underestimate network losses) compared to reality.
- As a more practical and straightforward alternative to OPF for some problems, one should consider simply running conventional power flow computations for a set of different cases.
- Generally, one should focus on developing robust analysis methods, meaning that a) one does in fact get a solution, b) that this solution would make sense for a practical system, and c) that the solution does not vary (apparently) arbitrarily with initial values or small variations in other input parameters.
- One specific challenge to robust analysis is models where the objective function has a flat region around the optimal value or a set of (nearly) degenerate optima. In these cases, different solvers or the use of slightly different initial values could lead to widely different solutions, which in turn leads to rather confusing decision support as seen by a power system operator. A more detailed model of the system could help differentiating different near-optimal solutions in such cases.
- Instead of hard-constraint voltage limits, one should also consider using soft constraints to avoid an optimal voltage profile tending to lie unrealistically close to the upper limits. (This typically happens when the objective function implies minimization of network losses.)
- A common problem not directly related to the OPF implementation *per se* is inaccuracies in the network model. Ensuring the quality of the input data is a continuous process. Also Stott and Alsaç (2012) and López, Sadikovic et al. (2015) highlight the importance of input data quality and good network modelling.

6 Summary and outlook

Optimization problems for power system application is often referred to by the generic term optimal power flow (OPF) when one requires that solutions to the problem respect the power flow equations for the system (Frank and Rebennack 2012). Using the full AC power flow description, these optimization problems are generally nonlinear, nonconvex, large-scale optimization problems, covering a wide range of possible objectives, decision variables and constraints (Capitanescu, Glavic et al. 2007). Over the course of 50 years, thousands of research works have been carried out on the solution of such problems (Stott and Alsaç 2012). Nevertheless, obtaining robust and reasonable solutions to AC optimal power flow problems for general, real-life systems can still be notoriously difficult (Stott and Alsaç 2012). The intensity of research efforts on fundamental OPF problems therefore remains very high, and a number reviews of recent developments and state-of-the-art OPF methods are published during just the last five years (Frank, Steponavice et al. 2012b, Frank, Steponavice et al. 2012a, Castillo and O'Neill 2013, Capitanescu 2016).

6.1 The state of the art of OPF methods and software tools

During the last 20 years, the majority of academic research efforts have been directed at interior point methods (IPM), and it now appears to be the prevalent class of OPF methods applied in the research literature. They also seems to be the most prevalent OPF methods among the 15 commercially available OPF tools surveyed in this report, with sequential linear programming methods as a strong contender in terms of popularity. Although detailed description of methods implemented in commercial tools are generally not available, the fundamental features of these methods appear to be the same as those regarded as the state of the art in the scientific literature.

The main reasons for the popularity of IPM are the ease with which it handles most kinds of constraints as well as generally favourable convergence properties and computational efficiency. However, customized implementations of SLP appear to be equally efficient, and well-proven SLP implementations in some commercial OPF tools are possibly more efficient and reliable for practical, large-scale problems. Another issue is the drawback of IPMs for warm starting the optimization. This drawback is relevant for all OPF applications where the OPF solver is called iteratively for similar OPF problems, e.g. when solving stochastic optimization problems by decomposition.

In general, a point of view that seems to be shared by many experts is that one always needs a large degree of customization of the OPF implementations in order to effectively and robustly obtain solutions for practical problems. Which decision variables that are realistic to include in each instance of the optimization problem is one of the key considerations in this regard. The method implementation should also be able to handle potential infeasibility e.g. by relaxing constraints to be able to recover a useful solution when detecting convergence problems. However, with the development of mature general purpose solvers for optimization problems, the research efforts are today more directed towards the high-level details of the OPF methods than on numerical details of the "core" optimizer.

6.2 Application to distribution systems with energy storage

As discussed in Section 4.1, the practical application of OPF in the hydropower-dominated Norwegian power system is presently limited, partly due to the limited number of relevant degrees of freedom available to the system operator. At the distribution system level, for distribution systems in both Norway and internationally, the number of controllable degrees of freedom has traditionally also been small. As for the Norwegian transmission system, this is one likely reason why the application of OPF methods to distribution systems appears to have been very limited. However, we expect this to change in the near future with the

ongoing transition towards more flexible ("smarter") distribution systems with higher levels of complexity and controllability. The introduction of new flexible resources (e.g. energy storage systems, demand response, and voltage regulation) as well as new types of loads and generation (e.g. electrical vehicles and photovoltaics) in distribution systems may drive the need for new methods for and applications of power system optimization.

These expectations are behind the international upsurge of research activity on optimal power flow taking energy storage systems (ESS) into account. Energy storage introduces dynamics to the optimization problem, i.e. interdependencies between different time steps of the planning horizon for the operation of the distribution system. This exacerbates the computational challenges, as the size of the system under consideration effectively is multiplied manifold. A large number of dynamic OPF (DOPF) models for different applications are proposed and solved in the research literature, as described in Section 3 and discussed in Section 5.2. Most of the previous work is based on solving the full DOPF model with methods and solvers (often IPM-based) available for regular OPF problems. The majority of test cases are relatively small, so from the computational experience reported in the literature, it is not clear whether such a conventional OPF approach is sufficient for large-scale, practical optimization problems for ESS in distribution systems. Furthermore, methods and case studies often appear somewhat disconnected from real-life applications, control options, business models, and regulatory constraints. To ensure the relevance and applicability of the methods, they should consider realistic current constraints as well as those that may be realized in distribution system in the future.

Battery storage systems and other short-term energy storage systems in distributions systems have a natural analogy in hydropower reservoirs, being much longer-term energy storage systems in the power system at large. In this context, powerful optimization methods have been in use for a few decades to optimize production planning for the hydropower industry (Gjelsvik, Mo et al. 2010) and analyse the power system (Wolfgang, Haugstad et al. 2009). However, such methods do not typically incorporate full non-linear AC power flow (Fosso 2010, Mo, Helseth et al. 2012), and similar methods have only very recently been applied to energy storage systems in distribution systems (Grillo, Pievatolo et al. 2016, Sperstad, Helseth et al. 2016). We consider this as a direction of future work, but the applicability of optimization concepts from the hydropower domain applied to distribution systems largely remains to be investigated.

6.3 Remarks on continued development

This chapter will close by considering what implications the findings in the previous chapters could have on the development of improved methods for optimizing the use of energy storage in distribution systems. For analogous methods for hydropower scheduling, as mentioned in the previous paragraph, capturing stochasticity is key a key point. Stochastic Dual Dynamic Programming (SDDP) is a well-proven method for handling the management of multiple reservoirs (or ESSs) under stochasticity. Taking into account the expected future value of stored energy is a central challenge, but it remains to be investigated how this problem should best be solved for distribution-level applications. Moreover, SDDP was designed for decomposing stochastic linear programs, and application to nonlinear programs such as an AC DOPF problem should be made with great care. Handling non-linearities in conjunction with the future value of stored energy (Thome, Pereira et al. 2013) or in conjunction with non-convex multistage stochastic optimization problems more generally (Zou, Ahmed et al. 2016) are areas of ongoing research. More specifically, the focus in application and development of OPF methods in the future may also be influenced by recent developments in the solution of mixed-integer optimization problems.

Only a few of the previously proposed AC DOPF problems considered in the literature are solved by decomposition, and most computational studies are restricted to relatively small test systems, usually excluding stochasticity. Leaving aside the challenges due to stochasticity and focusing on the challenge due

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to the sheer amount of effective buses in a large-scale time-coupled AC DOPF problem, we outline two possible approaches for decomposing this problem: a) To solve AC OPF problems for individual time steps and then coordinate these iteratively, or b) solve a linearized DOPF problem and add constraints to this linear problem iteratively through AC power flow calculations for individual time steps. Based on computational SINTEF/NTNU experience with approaches similar to (a), such decomposition schemes can cause convergence troubles when binding constraints are "jumping back and forth" between time steps during the iterative coordination procedure. Solving the full dynamic problem in one go is therefore preferable to solving for the individual time steps in isolation. The linearization approach (b) could for instance be based on a Dantzig–Wolfe decomposition as was proposed in (Pereira and Pinto 1982) in the context of hydrothermal scheduling. Although one should keep in mind that several OPF formulations and approaches proposed since around 1990 should be considered in any new tool development, there are also many ideas and considerations from previous SINTEF/NTNU development (as described in Chapters 5.3 and 4.4) that still are relevant.

When it is computationally tractable to solve full AC DOPF problem directly, or when doing so for validation of a decomposition-based solution method, an IPM could be an acceptable solution method. IPMs are used in several commercial and industrial OPF applications, and well-proved IPM implementations are available in general-purpose nonlinear programming solvers. The reliability of IPM-based OPF methods usually depend on implementation details, but this is also the case for SLP-based methods. Implementing an SLP solver for AC DOPF is more demanding than employing a general-purpose IPM solver, but employing an SLP solver that is well proven for OPF problems is likely to be an equally good alternative. Furthermore, if considering linearized power approximations such as DC power flow or other formulations (Fortenbacher, Zellner et al. 2016), SLP would be natural approach. Irrespective of solution method, one should also consider decoupled real-reactive optimal power flow if one is concerned that simultaneous co-optimization of all relevant (real and reactive) controls could lead to unreliable and unrealistic solutions. Generally, with a view to eventual practical applications, it is important to ensure that the methods can form the basis of robust analysis and decision support. Therefore, one should also consider alternatives to nonlinear programming for validation, where the problem allows it, such as LP-based approaches, or simply comparing multiple (conventional) power flow solutions.

Strategic planning problems (siting and sizing) have also been considered in the literature, but a review (Zidar, Georgilakis et al. 2016) shows that these to a smaller extent than the operational decision problems have been addressed by advanced optimization methods (e.g. including AC power flow and stochasticity). This report focuses on OPF methods for optimizing the operation of power systems, and optimization methods for power system planning was outside the scope of the survey that was conducted. Still, strategic power system decision problems are interlinked with the more operational decision problems that have been considered here, and optimization methods to solve them typically need to somehow represent the operation of the power system. The optimal operation of the system can be taken into account by embedding OPF models for power system operation in the optimization models for power system planning. Alternatively, the planning problems could be solved by heuristics as an "outer loop" around the solution methods for operational OPF problems. One could also consider approximations and heuristics for representing typical instead of optimal ESS operational patterns. The motivation for doing this could both be to reduce the computational efforts and to avoid assuming overoptimistic operation when solving planning problems. Considering also that barriers to implementing decision support tools often are higher for operational than for strategic decision problems, there is a substantial potential for further development on methods for distribution system planning including ESS.

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