Predictive maintenance control

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ABSTRACT: This paper presents a novel concept for calculation of optimal maintenance strategies for equipments that degrade over time. The concept is based on a dynamic mathematical model that is used in a look-ahead strategy to assess the effect of different future operational strategies. The optimum is given with respect to minimum costs for maintenance and the costs for not performing maintenance. The main idea is to illustrate that given these costs; the optimal strategy might be to wait a certain time before maintenance is performed. By this, considerable costs might be saved compared to performing the maintenance immediately or wait too long time before the maintenance is performed. In practice, rough estimates are often satisfactory for such decisions. Hence, the accuracy of the model behind the calculations might not be critical. The methodology is illustrated for a case of internal leakage in anti-surge valves for compressor control.

1 INTRODUCTION

Most off-shore process plants perform periodic scheduling of maintenance, also called preventive maintenance. E.g. valves are taken out for service even if no signs of fault are detected. In one case, during a maintenance shutdown about 600 valves were received for periodic maintenance. Out of these, only about 30% of them needed repair. For the remaining 420 valves it was not necessary to take them out of the plant for maintenance. The schedules are deliberately conservative because unscheduled outages are expensive. Moreover, periodic scheduling may not detect valves that will fail before their planned service date if signs of deterioration are not clearly visible in the maintenance. Consequently, in recent years, better maintenance practices of process valves have received considerable attention by the petroleum industry.

Various maintenance policies under different scenarios have been investigated in the literature. The main objective of these policies is to determine when and how to perform maintenance actions to improve the performance of the system. Extensive reviews of such research were made by Valdez-Flores & Feldman (1989), Lam & Yeh (1994) and Dekker (1996). Moustafa et al. (2004) described a maintenance model for a multi-state semi-Markovian deteriorating system. Their model allows one of three maintenance decisions; do-nothing, minimal maintenance and replacement to be taken at each state of the system. Often, the Weibull distribution for the failure rate of the system is used, see e.g. Banjevic et al. (2001). Wu & Clements-Croome (2005) developed maintenance policies for situations including time varying costs of failures, where the maintenance time influences the costs. Dynamic simulation models were discussed by van Houten et al. (1998) and applied by Hesham (2002) for a detergent-packing line.

Condition monitoring and maintenance based on justifiable priority-of-need in terms of cost-benefit calculations is expected to be cost effective. This is known as condition based maintenance. When prediction models are involved, this is known as predictive maintenance. Moubray (1997) includes this concept in what is known as reliability-centred maintenance (RCM).

In the present paper, the concept from *predictive control* is used in order to develop a novel concept for predictive maintenance. Predictive control is a scheme for real-time optimization of control actions (see e.g. Maciejowski (2002)). Adapted to maintenance, predictive maintenance control (PMC) is a scheme for real-time optimization of maintenance actions. The idea is that optimization is performed on a prediction horizon and re-optimized with a receding (i.e. moving) horizon as soon as new information is available. PMC means:

1. Provide an estimate of the current state of the equipment to be maintained. The state is described by a process condition and/or a technical condition of the equipment. An

estimate of the current state is computed using real-time data, historical data, and/or a mathematical model of the system.
2. Use a dynamic mathematical model in a look-ahead strategy to assess the effect of different future maintenance strategies.
3. Select the optimal maintenance action based on an optimization criterion.
4. Re-run the decision process, with a receding horizon, when new information is available, i.e. do *not* wait until the end of the prediction horizon.

In this paper, focus is made on formulation of the model and optimization criterion required in the PMC scheme 1-4 above. The idea is that optimal maintenance strategies are given with respect to the minimum total costs for operating equipments. The concept is in principle applicable to any degrading equipment. In order to illustrate the concept, degradation in terms of internal leakage in anti-surge valves for compressor control is examined. The work has been financed by Statoil, Norsk Hydro, ConocoPhillips and BP.

The paper is organized as follows. Section 2 describes the concept for PMC. Section 3 illustrates an application on a case with leakage in an anti-surge valve. An analysis of the sensitivity with respect to maintenance costs, leakage at the decision time instance and degradation rate is included. Section 4 contains conclusions and a discussion about the application of the concept.

2 THE CONCEPT FOR PREDICTIVE MAINTENANCE CONTROL

2.1 *System description and assumptions*

The concept of PMC is based on finding the optimal time instance for maintenance within a certain forthcoming time period, the prediction horizon. This strategy is given with respect to the minimum total costs. Basically, there are two groups of costs for this problem: (1) costs of running with degradation, and (2) costs for maintenance. Running with degradation means continuing the operation without performing maintenance. This normally leads to more degradation with time. The relative values of these costs act as weighting factors for the optimization.

The maintenance decisions include two options; do-nothing and do maintenance. Input variables for the optimization also include constraints such as acceptable time instances for maintenance and accept criterions for the performance of the equipment. The prediction horizon and the sampling time for the re-run of the decision process are other input parameters for the optimization.

The following assumptions are made:

1. When maintenance is finished, the equipment is returned in a state without degradation. This is a good assumption as long as the maintenance procedure is made properly.
2. When maintenance is finished, the equipment immediately continuous to degrade. This will happen when the equipment is re-installed in the same environment as before the maintenance was made. However, the degradation rate does not necessarily have to be same as before.
3. Maintenance is performed only once within the prediction horizon. This assumption is fulfilled when the re-run of the decision process is made immediately after maintenance is finished.
4. The direct maintenance cost rate and the lost income from production during the maintenance are considered as being fixed within the prediction horizon. This means that the prices for the product from the operation of the equipment, energy and spare parts etc. are fixed within this time period.

These assumptions are made for the sake of simplicity. The concept can be generalized to be applied without them.

2.2 *Optimal strategy decision in a prediction horizon*

First, optimization of the time instance for maintenance within the prediction horizon *P* is considered. This horizon should be chosen at least as long as it takes to exceed the accept criterion for the performance of the equipment such that maintenance is necessary. Otherwise, there is no need to plan any maintenance. In the case of a non-integrating degradation model, the horizon should also exceed the 1.order time constant for the model. The following optimization criterion (i.e. objective) is defined:

$$J = \int_{t_c}^{t_p} (C_l + C_m) dt \tag{1}$$

where C_l is the cost rate for running with degradation, C_m is the cost rate for performing maintenance, t_c is the current time instance (i.e. when the decision for the maintenance strategy is made) and t_p is the prediction time given by the prediction horizon as $t_p = t_c + P$. The optimal time instance for maintenance t_o is found by minimizing this objective.

The choice of maintenance action at time instance *k* can be regarded as a Boolean valued decision variable, u_k, i.e. with two optional feasible values 0 and 1. The decision “do maintenance” is defined by

$u_k = 1$, and "do not maintenance" (i.e. running with degradation) by $u_k = 0$. This gives a mixed integer optimization problem. By parameterizing u_k, the maintenance strategy can be described by the vector:

$$U = [u_c ... u_i ... u_n] \quad (2)$$

where u_c is the maintenance action at the current time instance t_c, u_n is the maintenance action at the last acceptable time instance where maintenance can be performed within the prediction horizon t_n, and u_i is a maintenance action at an acceptable time instance t_i between t_c and t_n. Table 1 illustrates these three maintenance strategy options.

Table 1. Three optional maintenance strategies with three acceptable time instances t_c, t_i and t_n, given by the vector element number, for performing maintenance.

Strategy	U
c	[1 0 0]
i	[0 1 0]
n	[0 0 1]

This means that e.g. $u_k = 1$ at the time instance $k = t_c$ and $u_k = 0$ at the time instances $k = t_i$ and $k = t_n$ for maintenance strategy *c*. Referring to assumption 3 above, only one of the elements in the vector *U* is always 1, i.e.:

$$\sum_{k=t_c}^{k=t_n} u_k = 1 \quad (3)$$

When the operation is continued without maintenance the degradation generally increases with time. This degradation is described by a prediction model $L_p(t)$, which gives the predicted degradation at time *t*. By including assumption 1 above, the relation between choice of maintenance action and degradation is described by a discontinuous model:

$$L(t) = \begin{cases} L_p(t), & u_k = 0 \\ 0, & u_k = 1 \end{cases} \quad (4)$$

For this optimization, it is only necessary to consider the costs relative to the case of no degradation. The cost rate for running with degradation, C_l, is the sum of the rate of lost income from lost production and the rate of lost (i.e. unnecessary) energy cost. These are time dependent functions of the prediction model $L_p(t)$ for the degradation.

The contributions from these costs to the objective *J* are given by:

$$J_l = \int_{t_c}^{t_k} C_l(L_p(t))dt + \int_{t_d}^{t_p} C_l(L_p(t))dt \quad (5)$$

where t_k is the start time instance for maintenance and t_d is the time instance when the equipment again starts to degrade, see Figure 1. According to assumption 2 above, the time period between t_k and t_d is equal to the time *M* needed for maintenance.

$C_l(L_p(t_c)) = C_c$ is the cost rate given by the current degradation. $C_l(L_p(t_d)) = 0$ is the cost rate at the start-up after maintenance, c.f. Equation 4. When t_k is the current time t_c, the first term of Equation 5 is zero. This corresponds to strategy *c* in Table 1. Note that the costs in the time period after the maintenance is performed until t_p are given by the costs of running with degradation starting from zero. This gives a benefit by increased income from production in this time period due to zero degradation at start-up after maintenance.

The cost rate in the time period when performing maintenance, C_m, is the sum of the direct maintenance cost rate and the lost income from production during the maintenance. According to assumption 4 above, these costs are constant with the maintenance time *M*. Thus, the contributions from these costs to the objective function in Equation 1 are given by:

$$J_m = C_m \cdot M \quad (6)$$

The objective *J* is given by the sum of the areas below the cost rate lines as illustrated in Figure 1:

$$J = \int_{t_c}^{t_p} (C_l(t) + C_m)dt = J_l + J_m \quad (7)$$

where:

$$C_l(t) + C_m = \begin{cases} C_l(L_p(t)), & u_k = 0 \\ C_m, & u_k = 1 \end{cases} \quad (8)$$

In Figure 1, the 0-line denotes the case of no degradation. The width of the rectangular area is equal to the maintenance time *M* and the height is the maintenance cost C_m. In the figure, it is assumed a linear integrating prediction model for the degradation as illustrated in Figure 2, and a linear relationship between the degradation and the costs. These assumptions are made only for illustration. Note, however, that the optimization problem is not linear in this case due to the integer constraint in Equation 3 and the discontinuous function in Equation 4.

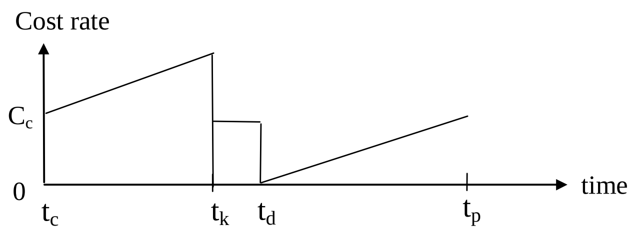

Figure 1. The costs for the strategy *i* in Table 1.

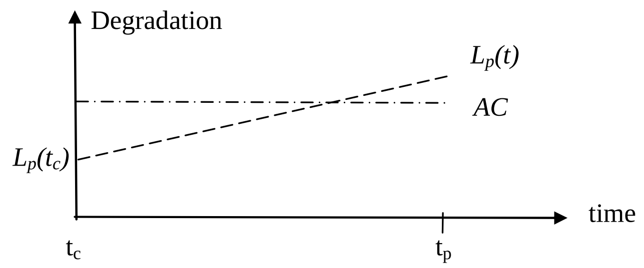

Figure 2. A linear prediction model $L_p(t)$ for the degradation. *AC* is the accept criterion.

2.3 *Periodic strategy decision by receding horizon optimization*

In the description above, the optimization is made within one time period *P*. In practice, this procedure has to be repeated periodically in order to take into account continuous process changes, like sudden severe changes in degradation and measurement noise. Thus, the prediction horizon is periodically moved and the optimal maintenance strategy is thereby updated, i.e. possibly altered. This is called *receding horizon optimization* and is an established methodology in control engineering, often denoted as model predictive control. The procedure is illustrated in Figure 3. The sampling of valve leakage and calculation of a new maintenance action are made periodically at a certain decision interval.

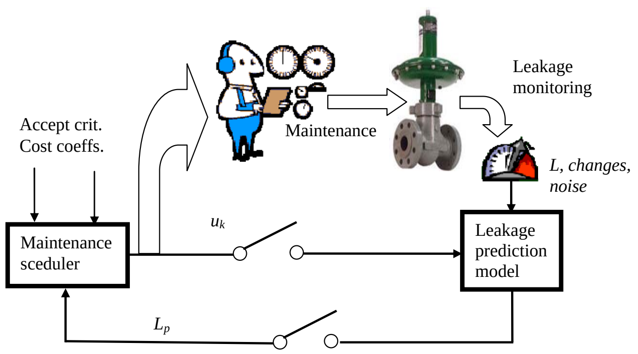

Figure 3. Receding horizon strategy decision of maintenance with respect to valve leakage.

2.4 *Solution issues and constraint handling*

Due to the integer constraint in Equation 3, the optimization problem is characterized as a so-called mixed integer dynamic optimization problem. Recent numerical optimization methods are used for such advanced applications (see e.g. Bansal et al. (2003)). Such methods are necessary in applications where there is not enough time available to calculate the whole range for the solution and finding the optimal solution by simply selecting the optimal value in this range.

As will be illustrated in the next section, a key question is what the most important constraints are. Are they the time instances for maintenance action, t_i, the accept criterion for degradation, *AC*, or are there no constraints? As illustrated, the answer influences considerably the optimal strategy and the resulting minimum costs.

Another question is whether the constraints are so-called hard or soft. Hard constraints are normally associated with the optimization decision variables, u_k. Equation 3, i.e. the sum of the decision variables is 1, and the fact that there are only the two feasible values 0 and 1 describe the hard constraints for this problem. A hard constraint related with the optimized variable is 100% degradation. This constraint is of course of little practical concern.

The degradation might, however, be subjected to soft constraints associated with the accept criterion when minimum costs are more important than the accept criterion. A soft constraint in this case means that the accept criterion might be exceeded in cases where this criterion can be relaxed in order to reduce the total maintenance costs if possible. This means that when $L_p(t) > AC$, then $u_k = 0$ for a limited period of time. In practice, it might be valuable to have a lower soft constraint for warning and an upper hard constraint for alarm calls.

3 CASE STUDY OF A SINGLE STAGE COMPRESSOR LINE

Anti-surge valves are used extensively in the petroleum industries for compressor protection. One of the main faults with these valves in offshore operations in the North Sea is internal leakage. Mechanical wear and erosion from sand and other solid particles in the flow medium is the main cause to such leakage. One consequence of increased leakage, when the anti-surge valve shall be closed, is significantly reduced gas production and profit. Another consequence of increased leakage in this case is increased energy consumption from the unnecessary circulation and cooling of the leakage gas.

Simulations have been made based on a model of a typical single-stage compressor line in off-shore operation, see Figure 4. These are documented in an internal report.

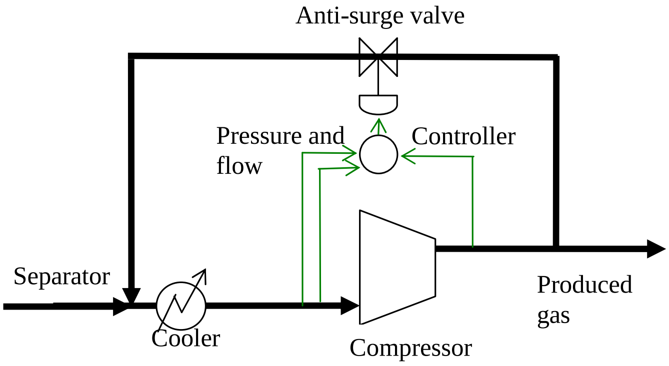

Figure 4. A single-stage compressor line.

First, optimal maintenance for nominal conditions are analyzed. Then, sensitivity of the optimal solution is examined with respect to three important conditions; (i) the relation between the maintenance costs, C_m, and the total costs, J, (ii) the current (initial) leakage, $L_p(t_c)$, and (iii) the degradation rate, i.e. characteristic of the prediction model. For simplicity, the slope of a linear model, i.e. the leakage coefficient k_L, is examined.

3.1 *Nominal conditions*

The cost of lost production corresponding to a lost income in NOK/h is given by:

$$C_{lp} = \frac{10 \cdot k_{lp} \cdot L \cdot c_g \cdot R \cdot T_S \cdot Z_S}{MW \cdot p_S} \tag{9}$$

where 10 is a conversion factor, k_{lp} is the mean gradient in the loss of produced mass flow with respect to leakage 6.3 ton gas / h / % leakage, L is the degradation [%], c_g is the gas price 1.0 NOK / Sm^3, R is the molar gas constant 8.314 J / (mol K), MW is the molar weight of the gas 17.6 g/mol, and Z_S, T_S and p_S are the compressibility [-], temperature [K] and pressure [bar a] defined as 1.0, 288.66 K and 1.01 bar a at standard gas conditions.

The cost of lost energy corresponding to a lost income in NOK / h is given by:

$$C_{le} = k_{le} \cdot L \cdot c_e \cdot k_e \tag{10}$$

where k_{le} is the mean compressor speed gradient with respect to the leakage 0.42 % rpm / % leakage, c_e is the energy price 1.0 NOK / kWh and k_e = 1.0 kW / % rpm is the coefficient for linear energy dependency with the compressor speed.

At current time t_c, the leakage is $L_p(t_c)$. Then, the leakage increases linearly by:

$$L_P(t) = L_P(t_c) + k_L \cdot (t - t_c) \tag{11}$$

where the coefficient k_L = 1 % / d and the current leakage $L_p(t_c)$ = 2 %. The associated direct costs with maintenance and the lost income from production during the maintenance are 10 NOK / h, i.e. relatively small compared to the other costs. The discretization time for the simulations is 1 day, the maintenance period M is one day and the prediction horizon P is 30 days.

The maintenance actions are restricted to be performed at the first, 10th or 22nd day. This means that the time instances for maintenance action, t_k, is a constraint for the optimization.

Figure 5 shows the total costs J and leakage L as functions of time and maintenance strategy U. Solid, dashed and dashed-dotted lines correspond respectively to the three strategies in Table 1. The dotted lines denote the case of no maintenance and the 0-lines denote the case of no leakage. The figure shows that the optimal maintenance strategy is to perform maintenance at the 9th day (dashed line). Note that the optimal strategy is neither at current time (solid line), nor at the last acceptable time instance (dashed-dotted line) within the prediction horizon.

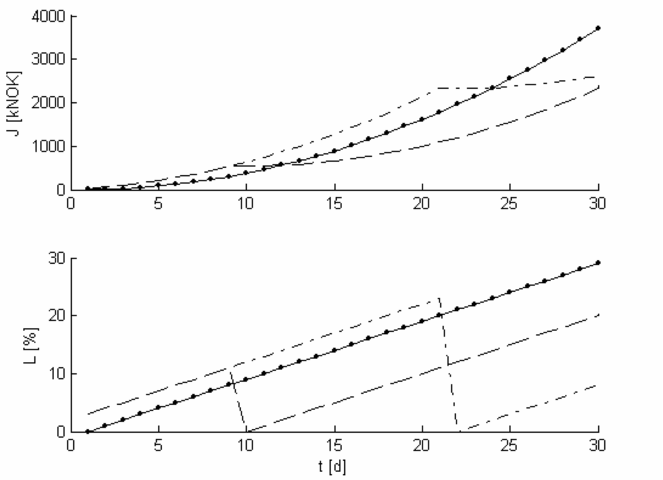

Figure 5. Total costs J and leakage L as functions of time and maintenance strategy U.

Figure 6 shows the total final costs J and leakage L at the end of the prediction horizon as functions of time for maintenance action. It appears that the optimal maintenance strategy is to perform maintenance at the 14th day when there are no constraints for the optimization. This is the unconstrained optimum (i.e. in the meaning optimum located between, and not at, the end points in the optimization region) for the costs. Since there is zero leakage immediately after maintenance, the final leakage at the end of the prediction horizon decreases with the time for the maintenance action. This means that the unconstrained optimal accept criterion for leakage, AC_o, is the leakage at the end of the time period before the optimal maintenance action is performed. The dashed lines in Figure 6 show the unconstrained optimum:

(t_o, J_o, AC_o) = (14th day, 2.2 mill NOK, 16%)

The optimum is unconstrained because (i) if there is no current leakage, it is not worth performing maintenance at current time, and (ii) the later the maintenance is performed, the higher are the total costs.

Note that when any of the optimization constraints t_k or AC is active, the optimum costs are equal or higher than in the unconstrained optimum.

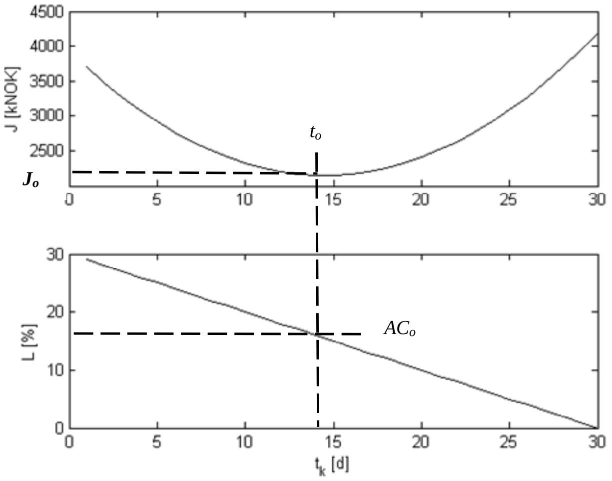

Figure 6. Total final costs J and leakage L at the end of the prediction horizon as functions of time for maintenance action.

3.2 *Sensitivity analysis*

3.2.1 *Sensitivity to the relation between the maintenance costs and the total costs*

Figure 7 shows the total final costs J at the end of the prediction horizon as function of time instance for maintenance and the maximum cost saving in the prediction horizon $J_{max} - J_{min}$ as function of the maintenance costs for C_m = 0 (solid line), 10 (dashed line) and 100 kNOK / h (dashed-dotted line). The maximum cost saving is the cost saving by choosing the optimum time instance for maintenance instead of the most unfavourable time instance, which is at the end of the prediction horizon. It appears that as long as they are constant with time, the maintenance costs do not influence the optimal time instance for maintenance or the maximum cost saving.

3.2.2 *Sensitivity to the initial leakage and the degradation rate*

Figure 8 shows the total final costs J at the end of the prediction horizon as function of time instance for maintenance for different values of the current leakage $L_p(t_c)$ and the leakage coefficient k_L. $L_p(t_c)$ = 2 % in the lower figure, and k_L = 1 % / d in the upper figure. Figure 9-11 show the optimal time instance for maintenance, t_o, the maximum cost saving in the prediction horizon $J_{max} - J_{min}$ and the leakage at the optimum L_o as functions of $L_p(t_c)$ and k_L. $L_p(t_c)$ and k_L are shown for the ranges 0-100 % and 0-25 % / d respectively.

It appears that t_o, $J_{max} - J_{min}$ and L_o are considerably influenced by $L_p(t_c)$ and k_L, and they are strongly nonlinear in these factors. This means that the sensitivities of these variables are strongly dependent of these factors.

At $L_p(t_c)$ = 2 % and k_L = 1 % / d, t_o is located at the 14th day, c.f. Figure 6. Increasing $L_p(t_c)$ at the same k_L moves t_o closer to the current time instance until the optimum becomes constrained at current time, i.e. day 1. This means, not surprisingly, that the more leakage in the valve at current time, the sooner it is recommended to perform maintenance and the larger is the cost saving for performing the maintenance at the optimum compared to at the end of the prediction horizon. In other words, the economic impact of knowing the optimal time instance for maintenance increases with $L_p(t_c)$ in this case.

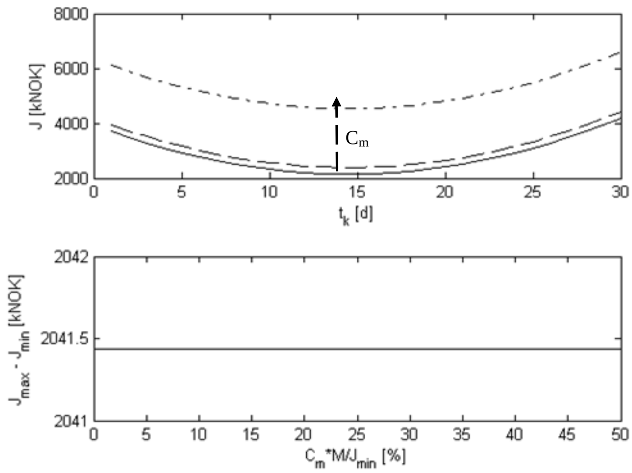

Figure 7. Total final costs J and the maximum cost saving as function of maintenance costs.

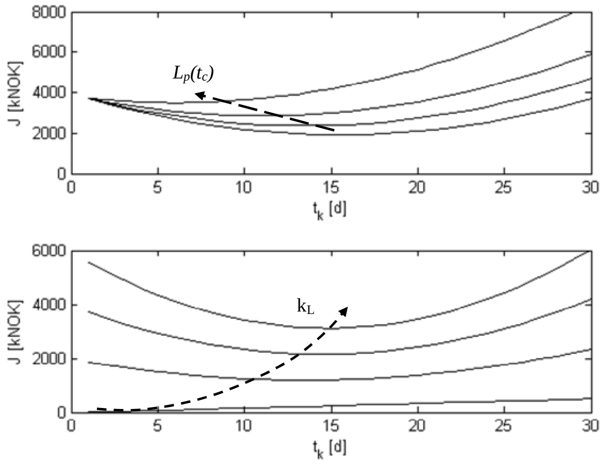

Figure 8. Total final costs J as function of current leakage and leakage rate.

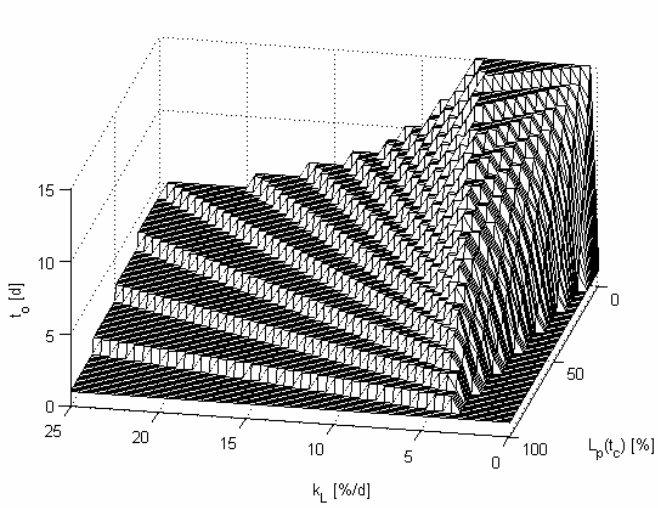

Figure 9. Optimal time instance for maintenance as function of current leakage and leakage rate.

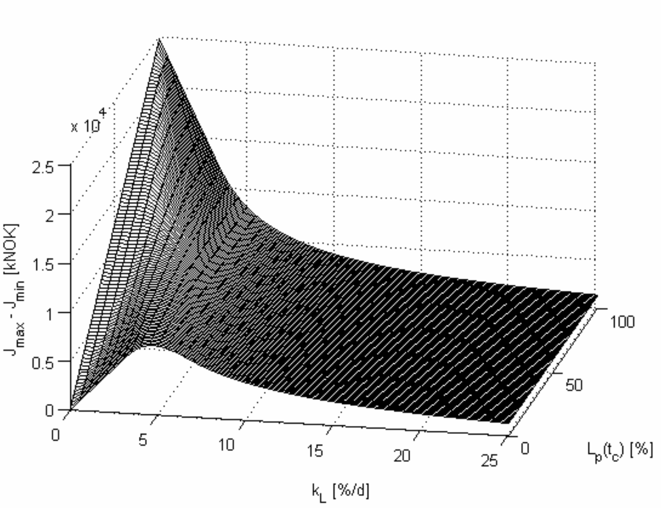

Figure 10. Maximum cost saving at the optimum as function of current leakage and leakage rate.

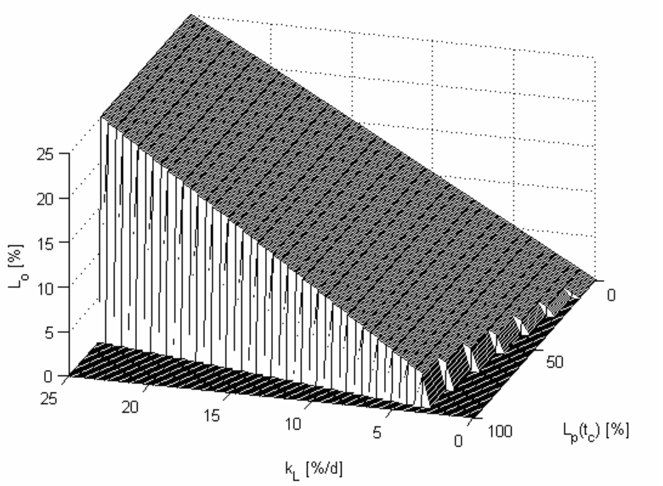

Figure 11. Leakage at the optimum as function of current leakage and leakage rate.

However, the leakage at the optima at the same k_L is independent on $L_p(t_c)$ below a certain value of $L_p(t_c)$, which depends on k_L (Figure 11). Figure 9 shows that the triangular area given by ($L_p(t_c)$, k_L) = (0, 0), (100 %, 0) and (100 %, 3 %) covers a range of $L_p(t_c)$ and k_L where it is recommended to perform maintenance immediately.

Interestingly, as opposed to the sensitivity to $L_p(t_c)$, for $L_p(t_c)$ = 2 %, t_o *increases* from t_c, i.e. constrained at day 1, with k_L in the range 0 to 1.5 % /d. For k_L in the range 1.5 to 7.0 % /d, t_o is constant at the 15th day, while for larger values of k_L, t_o decreases stepwise down to day 1 at very high values. This means that, within a certain lower range for k_L (i.e. 0 to 1.5 % /d in this case), the faster the valve degrades with respect to leakage, the later it is recommended to perform maintenance. Further, above a certain value of k_L (i.e. 7.0 % /d in this case), the faster the valve degrades with respect to leakage, the sooner it is recommended to perform maintenance. While the leakage at the optimum increases linearly with k_L (below a certain value of $L_p(t_c)$, which depends on k_L), the maximum cost saving increases for k_L in the range 0 to 5 % /d, and decreases for lager k_L. Figure 10 shows that the range of k_L, in which the cost saving is increasing with $L_p(t_c)$, decreases with $L_p(t_c)$.

Not shown here, the surfaces in Figures 9 and 10 flatten out for k_L larger than 25 % / d such that t_o is day 1 at ($L_p(t_c)$, k_L) = (0, 100 %).

4 CONCLUSIONS

The example with increasing valve leakage illustrates that by considering costs for maintenance and the costs for not performing maintenance, the optimal strategy might be to wait a certain time before maintenance is performed. By this, considerable costs might be saved compared to performing the maintenance immediately or wait too long time before the maintenance is performed. When time instance for maintenance or accept criterion for degradation constraints the optimal solution, the optimum costs are equal or higher than at the unconstrained optimum.

The calculations also show that as long as maintenance costs are constant with time, they do not influence the optimal time instance for maintenance. Further, the larger degradation at current time, the sooner it is recommended to perform the maintenance. Interestingly, within a certain lower range for the degradation rate, the faster the equipment degrades, the later it is recommended to perform the maintenance. At higher degradation rates, however, the faster the equipment degrades, the sooner it is recommended to perform maintenance. This means that the economic impact of knowing the optimal time instance for maintenance is largest at a certain degradation rate depending on the current degradation. In the example, the latest optimal time instance for performing maintenance is in the middle of the prediction horizon. Hence, the prediction horizon is also an important parameter which must be chosen in accordance with the time scale in question.

The simple illustrative example with valve leakage is meant to illustrate the concept. A more relevant business case will include a set of costs that is considerably more complex than in this example. Some main contributions may include reduced income from oil production, when this is a part of the processing plant, environmental costs with flaring off the reduced gas production, and high shut down costs outside regular stops.

As a model based application, the precision of the proposed method is sensitive to the accuracy of the prediction model for degradation. For illustration in this study, a linear deterministic model is used, and the degradation continues immediately from zero at start-up after maintenance. The accuracy of the estimates of the initial degradation and the degradation rate should be determined by the worst case in the relevant range of these factors. Hence, methods for deriving appropriate models might be critical. In many applications, development of degradation models is a comprehensive research topic. Often, a statistical (typically the Weibull) distribution for the lifetime of equipments is used. A stochastic model might improve the accuracy of the model in cases where stochastic phenomena are relevant. Improved degradation models might be useful as model errors might lead to corrective maintenance actions when an excess of the accept criterion is not predicted by the model.

In practice, however, maintenance decisions are often made conservative meaning that wide limits and rough estimates are applied. Hence, the sensitivity of the proposed method to the accuracy of the model may not be critical. In some cases when degradation is detected, maintenance is made in any case at the next opportunity. In these cases the maintenance engineers find no need to calculate such decisions with high accuracy. This practice is acceptable as long as it is based on a reasonable cost-benefit analysis similar to that described by the proposed method.

The method is, however, fairly generic. It may be valuable for other types of equipment than anti-surge valves, and it can be extended for a set of equipments like multistage compressor lines as well as for multiple process sections, e.g. several separate compressor lines.

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