Parallel Local search for the CVRP on the GPU

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Outline

1. Motivation
2. CVRP & REFs
3. Three-opt on GPU
4. Summary

Motivation

Vehicle Routing Problem

- Still gap between requirements and performance
- Variants of large neighborhood search, variable neighborhood search, iterated local search proven effective

Why parallelize local search

- Local search is an essential part of more advanced strategies such as metaheuristics
- Embarrassingly parallel: Moves independent from each other

⇒ Potential for significant speed up

Why GPU

- High computational power and memory bandwidth
- Cheap

Model

CVRP

- Given: depot & customer nodes, travelling costs, vehicle capacity, customer demands
- Wanted: Feasible route(s) with minimal length

Model

- Based on paper "A Unified Modeling and Solution Framework for Vehicle Routing and Local Search-based Metaheuristics" by Stefan Irnich, INFORMS JOURNAL ON COMPUTING, Vol. 20, No. 2, Spring 2008, pp. 270-287
- Solution represented as a giant tour
- Use of classical resource extension functions to model capacity constraint ⇒ Constant time move evaluation

Classical Resource extension function

- Resource vector $\mathbf{T} \in \mathbb{R}^n$
- Each node has a associated resource interval $[\mathbf{a}_i, \mathbf{b}_i]$
- A classical REF models change in resource from i to j:
  $$\mathbf{f}_{ij}(\mathbf{T}) = \mathbf{T} + \mathbf{t}_{ij} \quad \text{or} \quad \mathbf{f}_{ij}(\mathbf{T}) = \max(\mathbf{a}_j, \mathbf{T} + \mathbf{t}_{ij})$$
- A path is feasible if for each node i there exists a resource vector $\mathbf{T}_i \in [\mathbf{a}_i, \mathbf{b}_i]$ s.th.
  $$\mathbf{f}_{i,i+1}(\mathbf{T}_i) \leq \mathbf{T}_{i+1}$$

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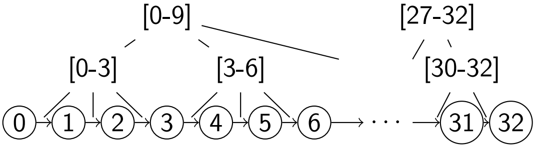
Segment hierarchy ⇒ Constant time move evaluation

Aggregation:

[3-6]: $3 \rightarrow 5$, $3 \rightarrow 6$,
$4 \rightarrow 6$, inverse

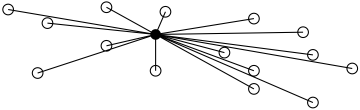
[0-9]: $0 \rightarrow 6$, $0 \rightarrow 9$,
$3 \rightarrow 9$, inverse



Method

Initial solution

- Star solution: A single route to each customer



Simple method: Local search with 3-opt move on giant tour

- Remove 3 connections/edges ⇒ 4 parts
- Reconnect parts in all possible (new) ways ⇒ 7 possibilities

⇒ $(7/6)(n-1)(n-2)(n-3)$ moves (n: #nodes in solution)

What we do on the GPU

- Once
  - Create neighborhood
- Each iteration
  - Create hierarchy
  - Evaluation of capacity constraint and length objective for each move
  - Choosing best move

⇒ Neighborhood and hierarchy live whole time on GPU

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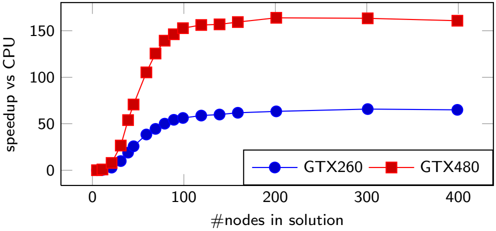
Both codes not optimized!



What we do on the GPU

- Once
    - Create neighborhood
- Each iteration
    - Create hierarchy
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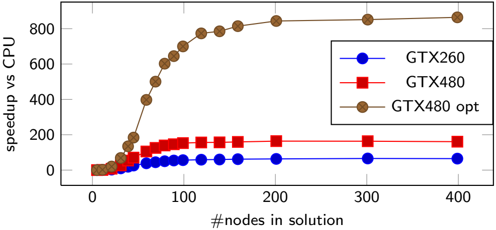
⇒ Neighborhood and hierarchy live whole time on GPU

Unfair comparison!
GPU is fast is known
Real task: Efficient usage
of GPU hardware



GPU analysis

Look at data for largest available solution (399 nodes)

	Time (ms)	Time % (%)	Bandwidth (Gbyte/sec)	L1 hit (%)	Ipc ≤ 2
First try	1069	42.5	12.2	75.4	0.73
	1410	56.1	33.5	80.8	0.68

GPU analysis

Average number of instructions per cycle on a multiprocessor (Fermi can execute 2 instructions on each multiprocessor)

Implementational approach for criterion/objective evaluation

% of neighborhood evaluation

Hit on L1-cache for global reads

Runtime on GPU

$\leq$ 177.4 Gbyte/sec

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Data for evaluation of objective (tour length)

Data for evaluation of criterion (demands)

GPU analysis

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First try	1069	42.5	12.2	75.4	0.73
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- Number of registers per thread limited to 32 as compile option $\Rightarrow$ Set to 64
- Only 32 threads per block, increase
- Default 16k Cache, change to 48k

GPU analysis

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First try	1069	42.5	12.2	75.4	0.73
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Max 64 registers,	475	40.8	68.8	86.2	1.64
128 threads, 48k Cache	657	56.3	119.6	93.3	1.39

- Currently use of array for 4 parts in 3-opt
  $\Rightarrow$ In local memory (slow)
  $\Rightarrow$ Store in registers
  (Registers per thread: before: 32/39, after: 32/37)

GPU analysis

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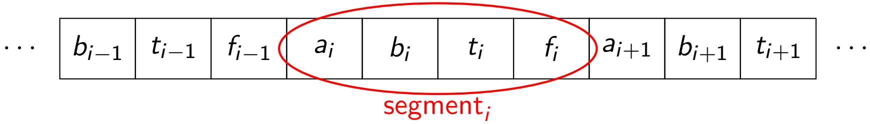
128 threads, 48k Cache	657	56.3	119.6	93.3	1.39
Parts in registers	479	45.3	24.6	89.2	1.60
	544	51.1	49.6	95.5	1.60

Array of Structures or Structure of Arrays

A hierarchy segment has 4 entries:

- Interval $[a, b]$
- Cost t
- Feasible information f

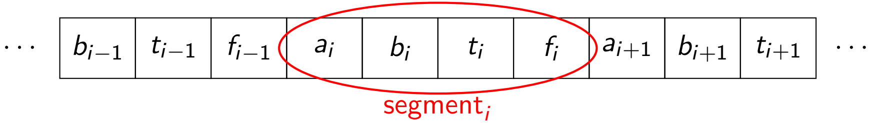
Array of Structures



$\cdots$	b_{i-1}	t_{i-1}	f_{i-1}	a_i	b_i	t_i	f_i	a_{i+1}	b_{i+1}	t_{i+1}	$\cdots$

Array of Structures or Structure of Arrays

Array of Structures

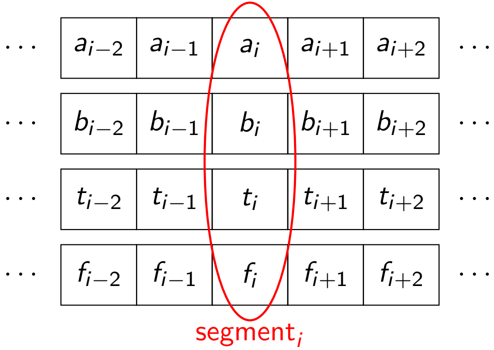


…	b_{i-1}	t_{i-1}	f_{i-1}	a_i	b_i	t_i	f_i	a_{i+1}	b_{i+1}	t_{i+1}	…

Structure of Arrays

Normally:

- Neighboring threads access neighboring entries
- Better coalescing
- Fewer transactions
- Faster



…	a_{i-2}	a_{i-1}	a_i	a_{i+1}	a_{i+2}	…
…	b_{i-2}	b_{i-1}	b_i	b_{i+1}	b_{i+2}	…
…	t_{i-2}	t_{i-1}	t_i	t_{i+1}	t_{i+2}	…
…	f_{i-2}	f_{i-1}	f_i	f_{i+1}	f_{i+2}	…

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Structure of arrays	479	43.6	24.6	89.2	1.60
	584	53.3	46.7	94.1	1.62

- Most accessed hierarchy segments identical
- All data from a segment needed to compute part
- Array of structure: Data cached!

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- So far: Complicated order to ensure access of neighboring structures (most of the times)
- But: Most accessed hierarchy segments identical, reduced coalescing due to array of structures

⇒ Use simpler order

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	544	51.1	49.6	95.5	1.60
Simpler order	295	42.3	38.4	86.6	1.59
(array of structures)	369	53.0	86.2	92.7	1.54

- Modulo operations expensive
- Integer division expensive
- Both can be replaced by bitwise operations for powers of 2

GPU analysis

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(array of structures)	369	53.0	86.2	92.7	1.54
Modulo computations	213	39.4	52.8	87.8	1.60
switched to base 2 op.	295	54.5	104.1	93.1	1.52

- So far single precision
- What about double precision

GPU analysis

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Simpler order (array of structures)	295	42.3	38.4	86.6	1.59
	369	53.0	86.2	92.7	1.54
Modulo computations switched to base 2 op.	213	39.4	52.8	87.8	1.60
	295	54.5	104.1	93.1	1.52
Double precision for tour length	215	38.9	69.3	87.8	1.60
	295	53.2	104.1	93.1	1.52

Summary & Future Work

Summary

- Local search suited for data parallelism
- Use of GPU can lead to significant speed ups
- Challenge to get full performance of GPU

Future Work

- Larger solutions: memory limit
- More advanced strategies such as metaheuristics
- Keep CPU and GPU busy
- Richer problems

Thank you for your attention!