OPTIMIZED ELECTRIC POWER PRODUCTION PLANNING BY DIGITAL COMPUTERS IN SWEDEN

by

John Lindqvist and Lennart Tyrén
Swedish State Power Board
Stockholm, Sweden

This report is devoted to the development work at mathematical models as regards optimized economic planning of the production for the total system and this particularly within the Swedish State Power Board. Similarly work is also carried out by the private power companies and by their co-operative association, Private Power Producers' Research and Development Organization (Vast) [1,2,3]. These efforts may gradually lead forward to an all-automatic economic dispatch system. An important link in this work is the optimization of the production planning where in the first phase the digital computers are working off line.

PRODUCTION SYSTEM OF THE STATE POWER BOARD

The State Power Board is responsible for a little less than half of the total production, that is 19000 GWh/yr. The water power production, divided among some thirty major plants, has at present 95-100 % of the total production. The rest of it comes from three major thermal plants. Thermal power will, however, take a greater share of the production in the future. Almost all of the water power comes from five major rivers (incl. tributaries). Long term storage reservoirs hold a maximum of 8000 GWh. These reservoirs are used for saving water from summer to winter and from wet years to dry. The long term reservoirs are often situated near where the rivers have their sources and the power plants lying at a distance have their own pondage reservoirs. These reservoirs are utilized for saving water from holidays to weekdays and from nights to daytime when the demand for power is greater. Pure run of river plants hardly do not exist. The power is broadly speaking at 90 % produced in northern Sweden and consumed at 80 % in the south. This implies a continuous transmission of power at our trunk line system from north to south. Over this network the private companies also transfer their power to the south which, by the way, gives rise to a big daily deduction of the transmission losses between the undertakings. This work is all computerized. Data are transferred by telex.

The consumption consists at 85 % of so-called firm deliveries which must not be interrupted but in case of extremely dry years, forced outages and so on and at 15 % of deliveries to interruptible customers. Among these last customers one can find Denmark, Finland and Norway and the big private power companies. Sometimes the Board is buying power from these countries and companies.

THE PROBLEM TO BE OPTIMIZED

The problem is to operate the existing system in such a manner that the variable costs (minus income from secondary power deliveries) will be as small as possible for a longer period of time. Hereby one can neglect the variable costs for the water power stations but not those for the thermal plants. Nor can the costs or income of secondary power be neglected. The problem is a big one and one could possibly never solve it in an "exactly" manner. One must always try to find acceptable approximations so the new solutions are hardly not perfect but better than the earlier ones.

A possible approximation which we have made use of, is deviding the problem in a long-range planning problem (operating the long-term storage reservoirs) and a short-range planning problem. This is acceptable among other things because the long-term reservoirs are often situated relatively far away from the power stations, so the sinking of the storages will not influence the heads of the stations. Our problem swarms with restrictions (mathematically inequalities) of physical and judicial kind such as swell and drop limitations of the reservoirs, conditions about maximum and minimum discharge for timber floating, fishery, natural scenery and so on. This fact has of cource influenced our choise of solution methods so we have used the mathematical programming technique to a large extent.

As regards the long-term planning problem one can not neglect the variability of the inflows. As regards the short-term planning problem one may more successfully regard it as deterministic, expecially if one makes frequent replannings. In the long-term problem one must also regard the variability of the total inflow to the system. It is not "equally dry" in all rivers at the same time. One can also approximatively regard the inflows as stochastically independent from period to period since the moments of decision occur with longer intervals.

The Long-Term Planning Problem

For some time we have at the State Power Board a long-term planning model in operation. This model has been described earlier[4]. Since that presentation must be unknown to most of the readers of this report, we here want to give a brief account of the model in question. It has since then been completed to meet the demands from the despatching. An important part of the model is the computation of the incremental water value of the long-term stored water and we will mainly deal with this problem here.

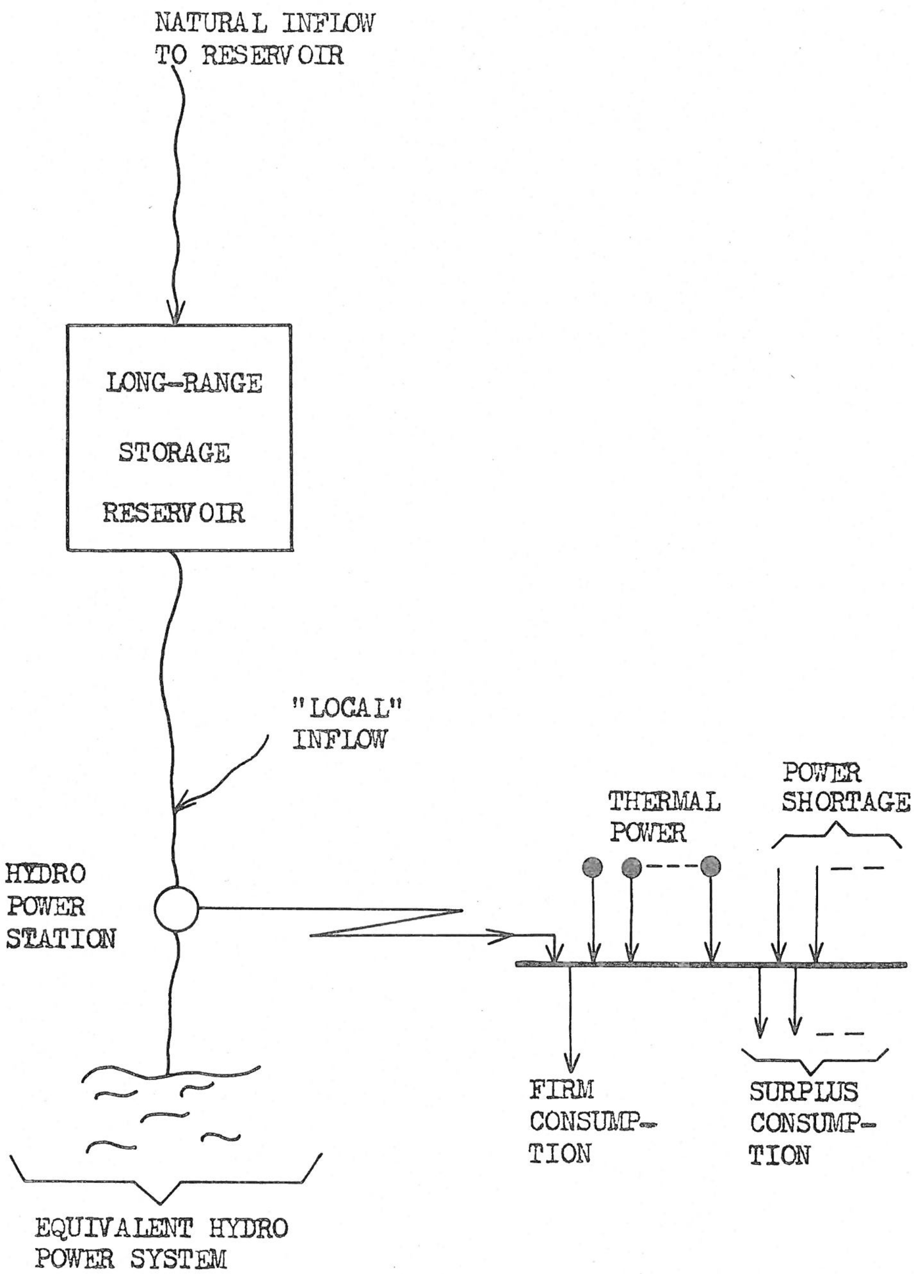
Preparation of Data. - We use a long record of inflow data for each river, namely the 30 year-series from 1930 to 1960. Inflow is expressed in GWh/period. First the water power components of the system (infloes, reservoirs, power plants and so on) are put together to an equivalent water power system consisting of one inflow to one reservoir and one inflow to one hydro-power station (figure 1). We hereby assume the stations input-output curves to be linear. This lumping the hydro-power system into one equivalent reservoir and one power plant is of cource a problem of its own. Especially the addition of the inflows to one equivalent inflow and the division of it into one part which can be stored and another which can not (called local in figure 1, but not necessarily local in the individual river) is rather intricate and calls for special computations. This division of the inflow is necessary to avoid a utilization which is too good. Our Planning Division (system expansions) has developed a special simulation model where the total water power production obtained as a result of the model here described, is for each period divided between the rivers as far as possible according to the restrictions previously mentioned here. In this model (computerized) one uses some semi-empirical rules such that the division in question is optimized. A result from this production distribution model is among other things for each chronological period a more correct computation of the total inflow (storable and local) than the one one started with.

Firm Consumption. - The magnitude of this consumption is assumed to be given for each period as a mean average. Transmission losses are included in the firm consumption.

The Incremental Cost Curves. - The year is divided in 2-52 periods. For each period is an incremental cost curve computed (figure 2). The maximum possible thermal production includes also the purchase. The curve is ended to the right with some special "production resources", namely rationing (with penalty costs) in case of an extremely dry year. These curves are computed in a special preparatory programme. These incremental costs and income constitute the incremental value of the stored water.

The Incremental Water Value Curves. - At the beginning of each period an incremental water value curve (figure 3) is computed. The curve is a function of the actual long-term storage. As soon as one is informed about the actual amount of storage one can take the corresponding water value to the incremental cost curve for the period in question and there get the optimum selling, purchase or thermal production. Thus the total optimum hydro power production can also be calculated since total power production equalizes total consumption. The way of calculation of an incremental water value curve is in just a few words the following. We start with an attempt (guess) of the curve at the beginning of a certain period (no. i) and have as the first problem to calculate the incremental water value curve for the beginning of period i-1. We hereby use our guess for period i and the incremental cost curve for period i-1. This means that for any given amount of storage on period i, and for each of the 30 possible different inflows (both the controllable and the local inflows) during period i-1, one can determine for the period an optimum thermal, or surplus, production with regard to the amount of storage and the incremental value one thereby obtains at the beginning of period no. i. (Due consideration is to be given to the previously stated restrictions.) For example, all the thermal power whose incremental cost is lower than the incremental water value arrived at at the beginning of period no. i is produced.

Since the incremental value of a stated quantity of storage at the beginning of period i-1 is the same as for the beginning of period no. 1, ignoring the interest question, one can easily determine the optimum incremental water value for the beginning of period no. i-1. In practice, 30 such optimum values for the selected quantity of storage


NATURAL INFLOW
TO RESERVOIR
LONG-RANGE
STORAGE
RESERVOIR
"LOCAL"
INFLOW
HYDRO
POWER
STATION
THERMAL
POWER
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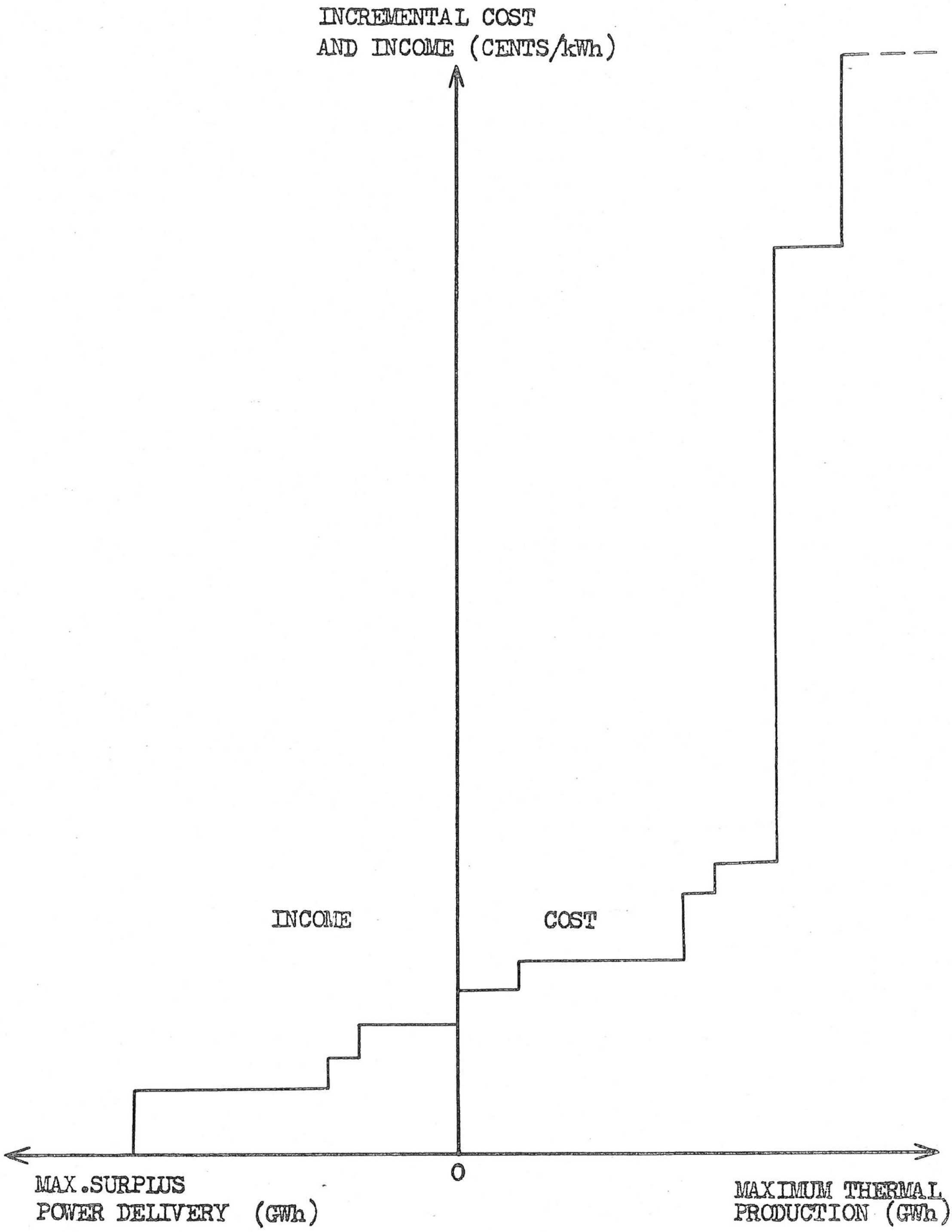
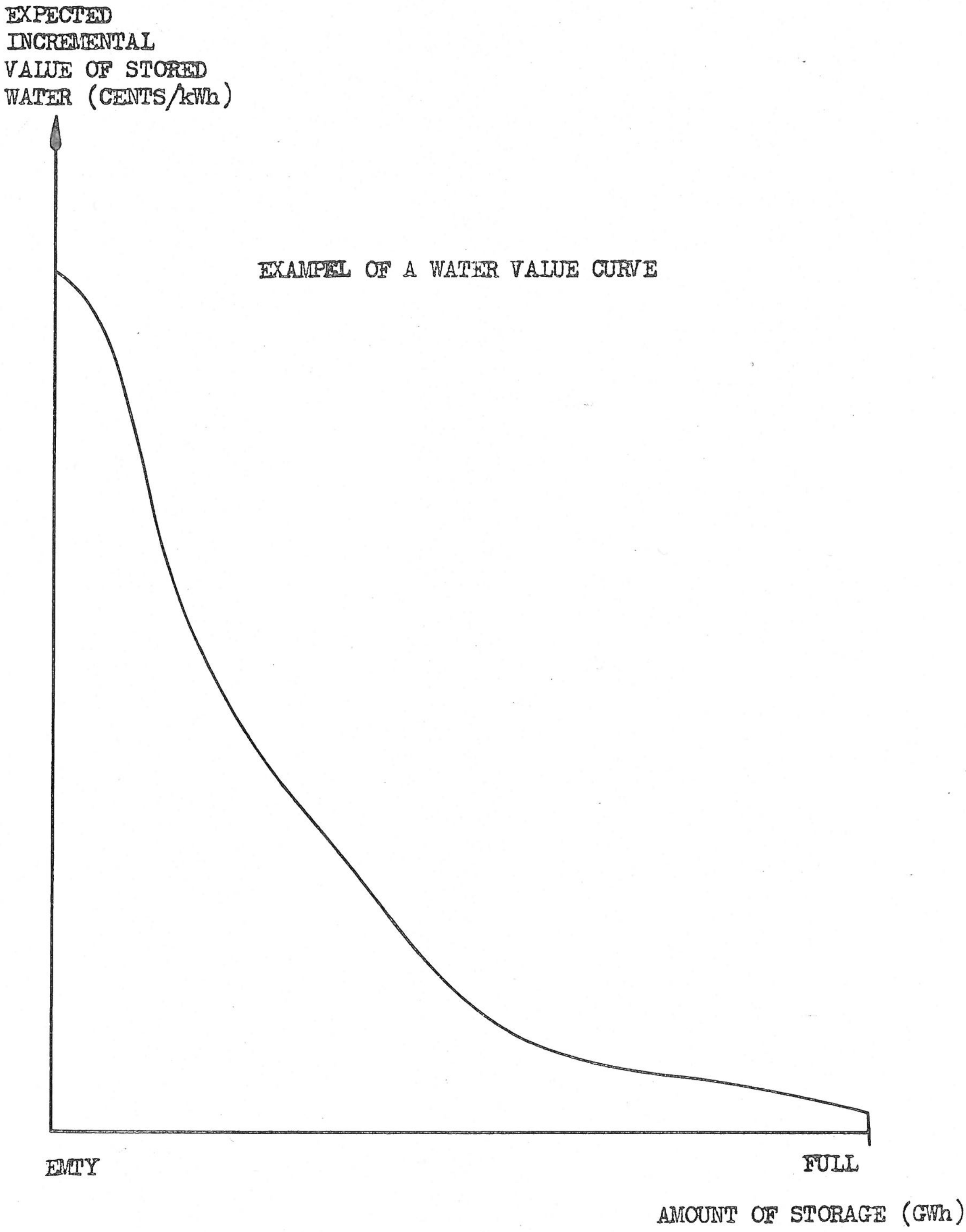
EQUIVALENT HYDRO
POWER SYSTEM

FIGURE 1



PERIOD NO. i

FIGURE 2



BEGINNING OF PERIOD NO. i

FIGURE 3

at the beginning of period i-1 are obtained, and the arithmetical mean of these values gives the expected incremental value. This is one point on the curve for the beginning of period i-1, and a similar calculation, starting from another assumed amount of storage, and assuming the same inflows as before, gives a further point on the curve. In this manner the whole form of the water value curve for the beginning of the period i-1 is developed.

Once the form of the curve is known for the beginning of period i-1, it can be calculated for the beginning of period i-2 in an analogous fashion. When one then returns to the beginning of period i, the form of the curve will probably deviate from the first assumption, whereupon a new cycle is calculated, starting from the new curve. After one or more additional cycles the form of the curve no longer deviates from that obtained for the corresponding period in the previous calculation cycle. The calculation is therefore complete, since a set of 2-52 decision functions has been obtained which are independent of the first assumption for the form of the curve for the beginning of period i. The curves have been determined solely by the requirement that all such thermal power is to be produced whose cost is less than the expected incremental water value, and that the expected incremental value of a certain quantity of storage today as well as in one period is to be the same (ignoring the interest).

We have recently extended the above described model now comprising two similar systems shown in figure 1. These two systems are connected to each other with a transmission line (loss-less as yet) limited as regards the capacity in both directions. This model gives rise to water value curves which not only are functions of the storage in the system of its own but also functions of the storage at the other system. The calculations briefly are carried out as follows. We start with a guess of water value curves for the two systems calculating then backwards in time untill the set of curves as before has converged. Also in other respects the way of computing is analogous to the previous one. Thus for each pair of inflows (to each system) during the period i-1 and for every chosen combination of the storages we optimize the hydro power production in each system and the transfer of energy between the two systems. The transfer which is regarded as an independent variable is calculated by iteration. So it is kept constant while optimum hydro power production is calculated in the two systems. Then, from the optimized incremental costs one can see how to change the transmission energy to get this variable optimized too.

The optimum hydro power production in the two systems is also obtained by relaxation.

A calculation of 13 water value curves takes 2 to 3 minutes in the one system-case at an IBM 7090 computer but about 10 times longer computer time in the two system-case.

SHORT-RANGE SCHEDULING PROBLEM

We next describe a model that has been developed by the Board to achieve a short-range optimum economic scheduling of a combined thermal and hydro-electric power system. The term "short-range" indicates that a time period of a few days, usually one week, is considered.

The method applied can be characterized as a gradient method combined with linear programming.

The program is capable of handling 40 hydro-electric plants in 10 different rivers, and takes account of station efficiencies, head variations, maximum and minimum restrictions of plant discharges and reservoirs, and in a somewhat rough manner, of transmission losses. The program is written in the programming language Fortran II for the IBM computer 7090 and has been run once or twice a week since August 1963.

Model

For computing purposes it is necessary to imitate the very complex physical system by means of a mathematical model, comprising a set of equations describing all the essential features of the real system. In constructing this model one is also forced to compromise between accuracy and simplicity, and the choice of model structure is of course influenced by the method intended to be used for optimization.

In our short-range scheduling program the combined thermal and hydro-system model is built up in the following way.

Time. - The period of time considered is divided into 2-24 intervals, not necessarily of equal length. (In our computations the time period is usually one week. Each day is divided into 14 daytime hours and 10 nighttime hours; the total number of intervals is thus 14.)

During any time interval the load, the plant discharges, output power etc. are supposed to be constant. They can also, more realistically, be thought of as mean values for the interval.

Hydro-Power. - The hydro-electric power is produced in 1 to 10 rivers, each river containing 1 to 10 stations. The total number of stations, however, must not be greater than 40.

Each station has a reservoir, supplied with water from the station above, if any, and from a non-regulated local inflow that is allowed to vary from one interval to another in a predistinated way.

A river may consist of one or two branches. Fig. 4 shows an example of a permissible river-configuration.

Maximum and minimum limits are given for the plant discharges and the reservoir contents. These restrictions are assumed to be unaltered during the whole period.

The program does not take into consideration the fact that the water flow may be delayed between stations.

Fig. 5 shows, in principle, how plant discharges and reservoir contents are supposed to vary.

Station output power is computed from the following expression

$$P_{n,t} = P^{o}_{n,t} (Q_{n,t}) - (c_n \cdot \frac{1}{2} (H_{n,t} + H_{n,t+1}) + c'_n \cdot$$

$$\cdot \frac{1}{2} (H_{n+1,t} + H_{n+1,t+1})) \cdot Q_{n,t}$$

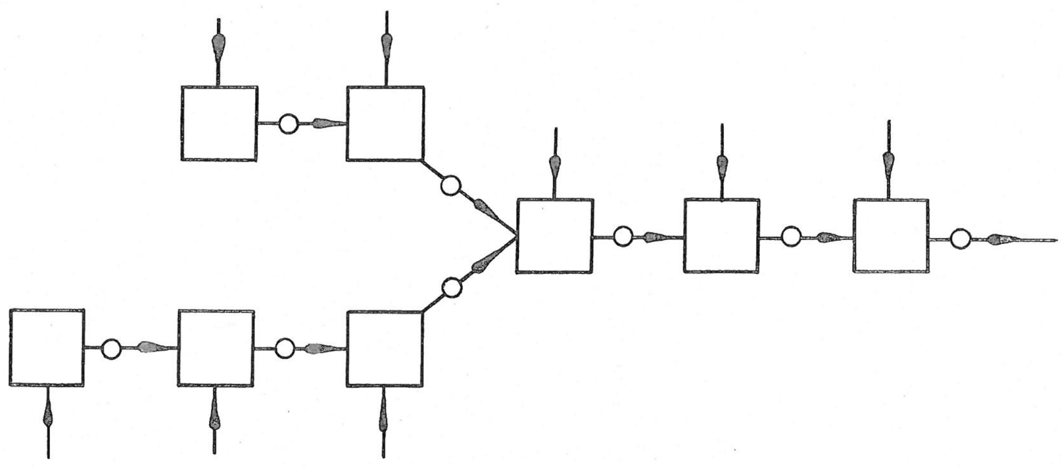
where

$P_{n,t}$ = output power (MW) from station number n in time interval no. t

$Q_{n,t}$ = discharge (m^3/s) through station number n in time interval no. t

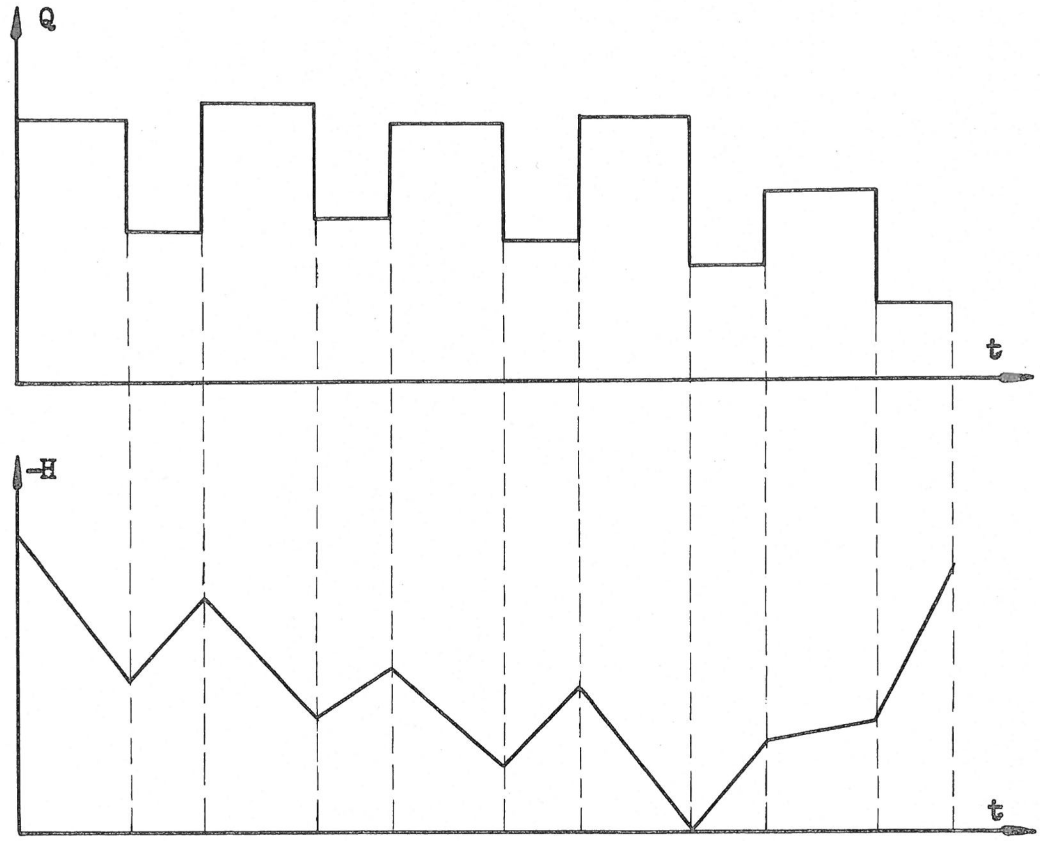
$H_{n,t}$ = lack of water in the reservoir above the station at the beginning of interval t

$H_{n+1,t}$ = the same for the reservoir below the station



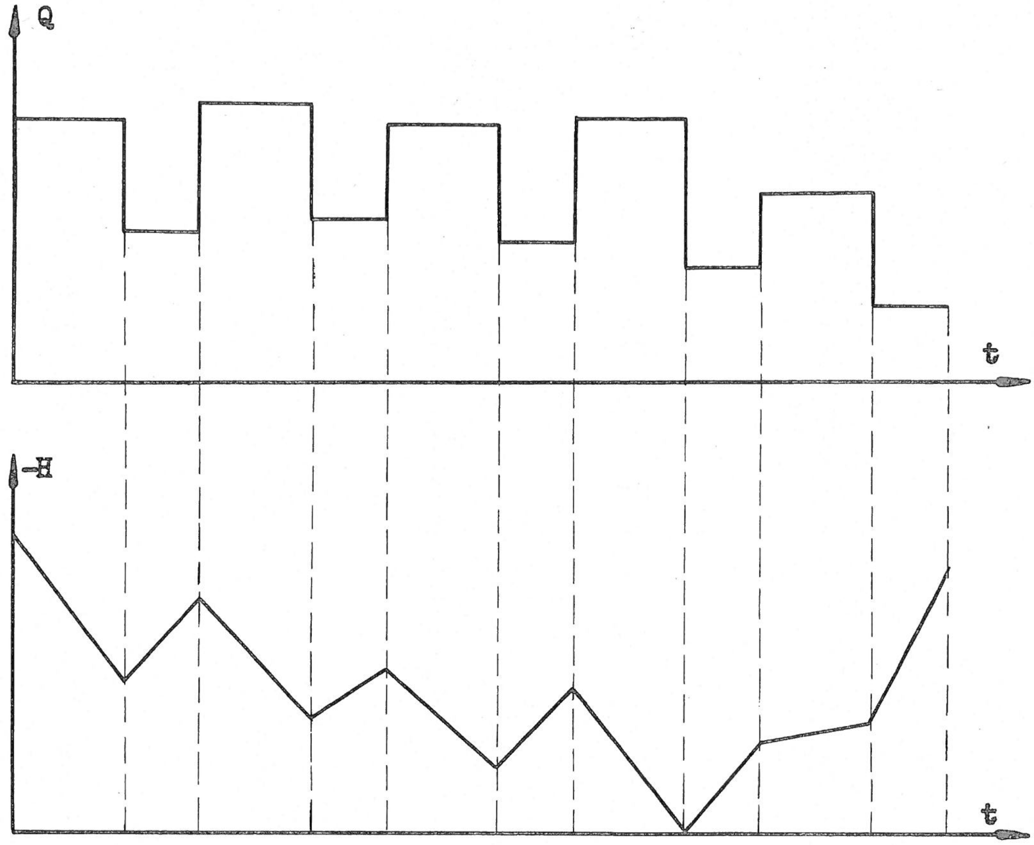
RIVER CONFIGURATION

FIGURE 4



PLANT DISCHARGE AND RESERVOIR CONTENT

FIGURE 5



PLANT DISCHARGE AND RESERVOIR CONTENT

FIGURE 5

c_n and c'_n are loss-coefficients, accounting for head variations due to reservoir contents above and below the station.

$P^o_{n,t}(Q_{n,t})$ is an input-output curve for station no. n and interval no. t. It is possible to have more than one curve for each station, and this may sometimes be desirable. The total number of curves must not exceed 80.

Each curve is represented by 1-8 parabolic segments, forming a continuous curve with a continuous and decreasing (or constant) derivative. The program includes a special subroutine that calculates such a curve from 2-9 given points by the method of least squares. This relatively time-consuming subroutine, however, need not be taken advantage of every time, since the curves thus found will be punched on cards for future use. Fig. 6 shows an example of a curve and its derivative.

The curve ought to be determined statistically. P(Q) should be the expected mean power (head losses not included) when Q is the mean discharge during the time interval.

Thermal and Surplus Power. - If the total output from all the hydro plants should be less than the power demand, one has to make up for the deficiency by purchase or by thermal power production.

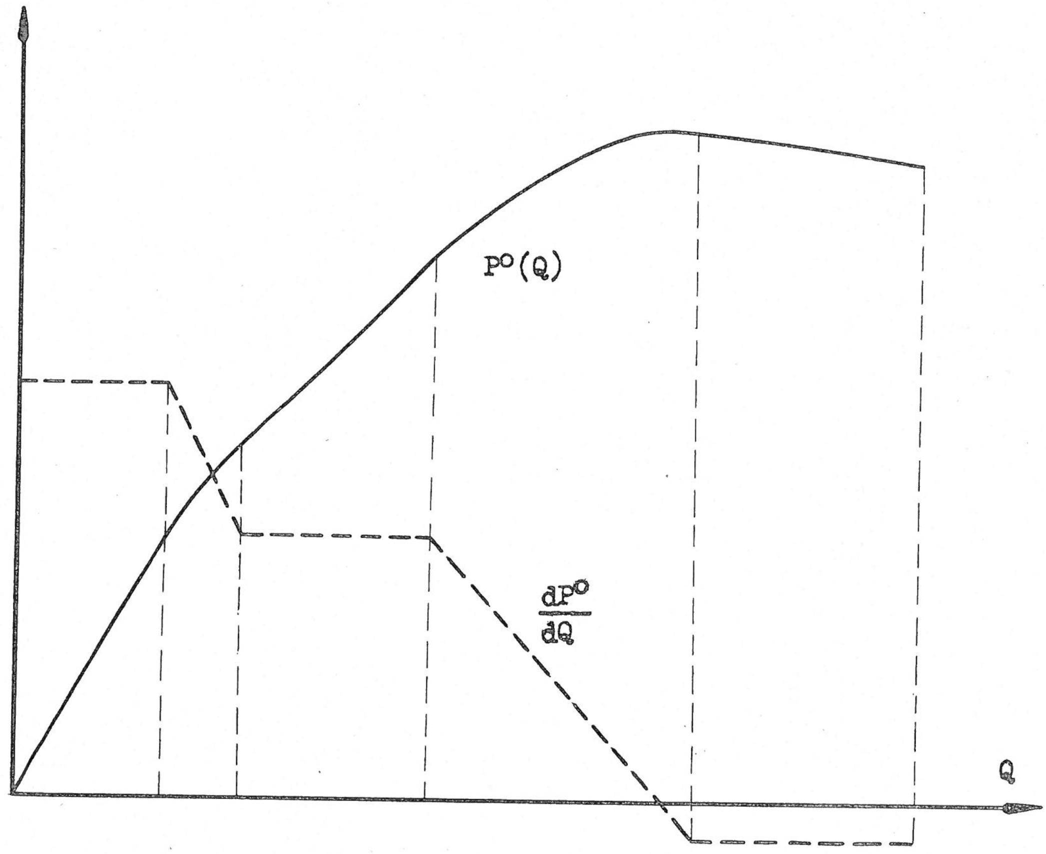
If there is, on the contrary, a surplus of power, this may be sold as secondary power.

The prices, valid for such selling and purchase (or thermal production) should be available in the form of a step curve, as shown in fig. 7.

20 price-steps can be included in the curve, and each interval of time may have a price curve of its own.

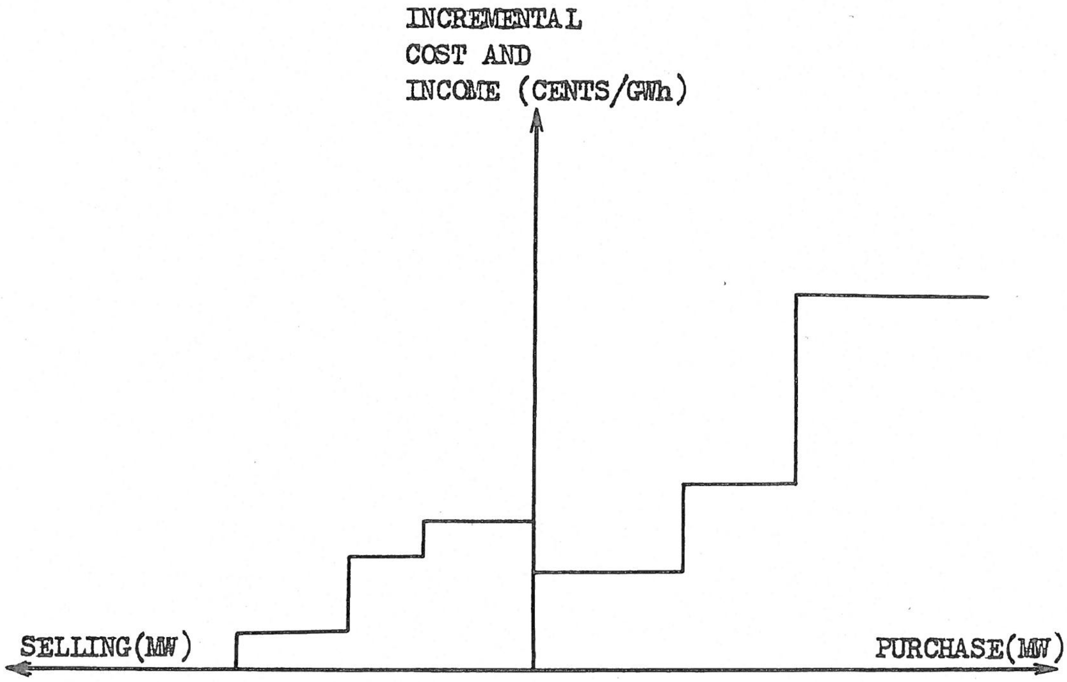
Of course purchase and selling may sometimes occur simultaneously, as might be understood from the example in fig. 7. (The steps are rearranged by the program in an increasing sequency at the beginning of the calculations. When the optimization is finished, one returns to the original curves in order to separate purchase and selling.)

Starting-up costs for thermal plants are not taken into account.



STATION INPUT-OUTPUT CURVE

FIGURE 6



PRICE-CURVE

FIGURE 7

Transmission Losses. - All important power producers in this country use as mentioned one common transmission net. The transmission losses are measured every hour and are divided among the producers according to some intricate rules.

In the program we use the following simplified formula

$$F_t = A_t \sum_i \sum_j B_{ij} (k_i P_{it} - L_{it})(k_j P_{jt} - L_{jt})$$

where F_t = our share of the losses during time interval no. t

A_t = a constant

B_{ij} = loss coefficient

P_{it} = our own power production in river no. i

$k_i P_{it}$ = total power production in river no. i

L_{it} = local load, that is not transmitted over the common net

The factor k_i is put in because of the fact that in some rivers there are foreign stations inserted between our own stations.

The formula above is of course a rather rough approximation, because the losses due to thermal production are not taken into consideration. We believe, however, that this approximation may still be applicable to the State Power Board's case since most of our hydro-power comes from central and northern Sweden, while the thermal power is produced in the southern part of the country, where the load has also its centre of gravity.

The Problem

We are now ready to formulate our optimization problem. It is assumed that the following data are given.

a. Number of time intervals and their length

b. Maximum and minimum restrictions for stations and reservoirs

c. Station coefficients for head-losses

d. Station input-output curves

e. Price curves for all intervals

f. Coefficients and other data for the transmission-loss-formula

g. Prognosis of power demand for each interval

h. Reservoir contents at the beginning of the period

i. Desired reservoir contents at the end of the period

j. Magnitude of the non-regulated local inflows to the reservoirs

Points h-j imply that each station has to consume a specified amount of water during the given period of time.

The problem is now to calculate the discharge values of all hydro plants for all intervals in such a manner that an optimum economy is achieved. The criterion of such an economy is that the cost, over the whole period, for thermal production and power purchase, reduced by the income from secondary power disposal, is minimized.

The reservoir contents at the beginning of the intervals are preferably chosen as independent variables. The contents at the beginning and the end of the period are given, thus the number of really independent variables is $N(T-1)$, where N is the number of reservoirs (stations) and T is the number of time intervals.

These variables are subject to $2N(T-1)$ constraints due to the reservoir restrictions and $2 \cdot N \cdot T$ constraints due to the discharge restrictions, i.e. $2N(2T-1)$ constraints in all.

If we, as a realistic example, have 35 stations and 14 time intervals, our optimization problem consists of minimizing a non-linear cost function of 455 independent variables, subject to 1890 linear constraints.

Method

Linearization. - In view of the large number of linear constraints, one is of course very much inclined to try the well-known linear programming methods. These, however, are not immediately applicable, owing to the non-linearity of the cost function.

We have surmounted this obstacle by the following artifice. Suppose we have found, in one way or other, a feasible solution of the problem, i.e. some point in the $N(T-1)$-dimensional variable space that satisfies all the constraints. The cost function value at that point is easily calculated. If we move to another feasible point in the vicinity, the value of the cost function will generally be changed a little. Now, if the changes of the independent variables are small enough, they will affect the cost function value in an approximately linear manner. The coefficients of this linear function are determined by partial derivation.

Accordingly we can use linear programming to find out the most profitable direction of movement. These calculations are repeated over and over again, until no further cost reduction is obtained.

This linear programming procedure assures, in a very natural way, that no restriction is ever violated.

Owing to the concavity of the cost function our final solution will very likely coincide with the desired "global" optimum. An additional condition for a satisfactory convergence is, however, that the cost function has a continuous derivative at every point, which is not the case if we use the original, discontinuous price-curves, described earlier at page 8. A point of discontinuity creates a sharp-edged furrow in the cost function surface. (The cost function can be envisaged as a surface, especially in the simple two-dimensional case.) If we are in the vicinity of such a furrow, our optimization method will only lead us nearer and nearer the edge, not observing that it may be more profitable to move along the furrow. This situation is avoided if the vertical parts of the price-curves are replaced by sloping lines.

<u>Decomposition</u>. - Let us now look at the linear programming problem that we have to solve for each step of the iteration. It still has N(T-1) independent variables and 2N(2T-1) linear constraints (455 variables and 1890 constraints in our example). This is a problem of a magnitude inspiring respect, and it has to be solved not only once, but perhaps 20 or 50 times. That calls for some sort of decomposition.

To begin with, we notice that the constraints have a special property: none of them deals with more than one river. That makes it possible for us to apply this first principle of decomposition: We always handle only one river at a time, and in doing so we let the discharges of all other rivers remain unchanged, i.e. their production is considered as fixed and can be subtracted from the load.

When each river has been calculated in this manner, the same procedure is repeated over and over again until convergence has been obtained.

Our linear programming problem now has n(T-1) variables and 2n(2T-1) constraints, n being the number of stations in the river, which is assumed to be 10 at most. Thus each sub-problem is considerably simplified, but we must not forget that there are many more of them instead.

For that reason we are forced also to adopt this additional principle of decomposition: The discharge values are varied for only two intervals at a time, so that the reservoir contents for all intervals, lying between, are altered by one and the same amount. The calculations are repeated for a lot of different combinations of time intervals, which makes it possible to transfer hydro power from one interval to any other.

Now the most extensive problem we can meet has only 10 independent variables and 60 linear constraints, and many of them have much less than that. It is true that there are now even more problems to solve, but each of them is very simple indeed, so the total computer time will still be quite reasonable.

The sub-problem is solved by means of the well-known Simplex method. It is not necessary, however, to use the original, general algorithm. A rather simplified version is possible, due to another nice property of the constraints: their coefficients are always either +1, -1, or 0.

We are anxious to point out here that the same linear programming methods and decomposition principles have previously been described in a Japanese article [5].

Further Details. - When computing the most profitable change by means of the simplex routine, we consider the cost function to be linear within the range of interest. This being only approximately true, there is a certain risk that our computations will enter a state of oscillation, jumping back and forth between two adjacent points.

This eventuality is avoided by the following precaution. Every time the simplex computation is completed, its solution is considered only as a piece of advice. Before any changes are executed, it is carefully examined if they would really yield a profit when applying the more precise, non-linear cost function. If this is not the case, we reduce the step length (step-direction being unchanged) and make a new examination. This is repeated until a cost decrease is observed or a prescribed number of step-reductions is made.

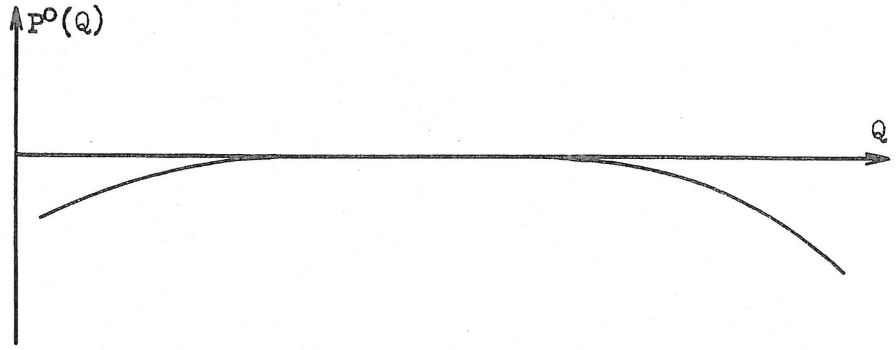
Some measures are adopted in order to accelerate the convergence. To begin with, it is obvious that the step-length ought to be greater at the beginning of the optimization than at the end of it. For that reason the program provides a possibility of breaking up the process into several phases with decreasing step-lengths. (Not to be mixed up with the step-reductions, mentioned above.)

Secondly, different rivers may need different numbers of iterations for converging. The program keeps account of the yields of each river, and when a river is not likely to make any more contributions to the cost reduction, it is taken out of the process. (When a new phase begins, however, all the rivers are included again.)

Before the real optimization can start, the program has to find out some initial, feasible solution. The first attempt is to let all stations discharge their mean values. If this solution is found out to violate any of the restrictions, there are special subroutines which perform the adequate corrections.

When one optimization is completed, the final solution of that problem can be used as the initial feasible solution for a new optimization problem that has slightly altered input data, another load, for instance, or some other price curves. Thus the program provides a possibility of calculating such alternative cases without restarting the whole procedure from the very beginning. Much computer time can be saved in this way.

When the program is applied to the hydro power system of the Swedish State Power Board, a special complication arises: in some rivers, stations belonging to other companies are inserted between our own stations. We are not allowed to make use of their power production, nevertheless we must pay some consideration to them, owing to the flow interconnection. For these stations we use particular input-output curves which make account of the efficiency losses only. Fig. 8 shows an example.



INPUT-OUTPUT CURVE FOR AN INSERTED FOREIGN STATION.

FIGURE 8

Computer Program

The program has been coded in Fortran II for the IBM computer 7090. It is so arranged that (after having been read in) all data are stored in the core memory during the whole process. The instructions, however, are divided into three separate program-links, stored on a magnetic tape and transfered to the core memory one at a time, when needed.

The task of the first two links is to read in and check all input data and to make a lot of preparatory computations upon them. Much attention has been devoted to the checking, in view of the great number of input data of different kinds.

The task of the second link is to accomplish the optimization and to write out the result.

Each link consists of 7000-11000 object program instructions. The data occupy about 18000 cells of the core memory. (There are 32768 core memory cells, in all.)

The necessary computer time will vary considerably, of course, depending on number and quality of the input data.

It can be mentioned, as an example, that the calculation of a system, consisting of 9 rivers, 31 hydro plants and 14 time intervals, has taken up about 4-6 minutes of computer time.

Discussion

The program was taken in regular operation in August 1963 and has been run once or twice a week since then. The program seems to be fairly well adapted to its main purpose: economic scheduling of our hydro power system on a weekly basis. We count upon our making considerable money savings by these calculations, even if it is impossible, for many reasons, to determine those savings more precisely.

However, various improvements and extensions of the program are conseivable. It would be desirable, for instance, to generalize the discharge and reservoir restrictions by prescribing one individual set of restrictions for each interval of time. This is often needful for those rivers, where timber-floating is going on.

An application for the program, lying close to hand, is the calculation of 24-hours schedules (24-hours period, divided into one or two-hours intervals). This is, formally, possible with the program being as it is now. In practice, however, certain complications come from the fact that our model does not take into consideration either the time lag of flow between the hydro stations or the starting-up-costs of the thermal power. These factors will be much more significant when the time interval is only one or two hours instead of 10 or 14 hours.

EXPANSIONS OF THE COMPUTER-APPLICATIONS ANTICIPATED IN THE NEAR FUTURE

Long-Range Planning Problem

As has been intimated the "dimension-barrier" is a heavy obstacle in especially the long-range planning problem. This depends partly on the complete mapping out the whole functional which takes place at the dynamic programming approach of the problem. Partly another way attacking the problem is using the algoritm developed for short-range planning for long-range planning purposes. This may be done in the following way. From a relatively long series of inflow data we form a number of for instance 5- or 10-year series of chronological inflow records for each river. From for instance a 34-year series we get 30 5-year series for each moment of the year. We take then the 5-year series one at a time optimizing the interval-production in each station during the 5 years. However, we must be able to determine in beforehand the input and output values of the long-term storages in each river. As regards the input values this is no problem. We take the actual values at the moment for the new planning period. As regards the output values in 5 years one can see that these values need not be known exactly. We think it should be possible to calculate these end-values approximatively in a tolerably simplified way. As to the lengths of interval within the 5-year period one can lengthen them farther off from the actual moment. In the way outlined here it would be possible to calculate in a reasonable time the expected incremental water value for, by way of example, the next 2-week interval and the corresponding discharge in each river.

Short-Range Scheduling Problem

Another version of the short-range program, particularly intended for a 24-hours schedule, is now being drawn up. It takes account of the water delay, but the starting-up-costs in the thermal plants will be left on one side for the present.

The period (24 hours) will be divided into 12 two-hours-intervals, and it is postulated that the water travelling time between two stations is an integer multiple of such intervals. It is also necessary to let each hydro station have predetermined discharge values for so many intervals, at the end of the period, that corresponds to the water running time. As these discharge values will also affect the economy of the next day, they can not be optimized in the same way as the other values. The discharge values from the last intervals of the preceding day must, of course, be given too.

There is another complication connected with the special state of things in this country. For each interval of time we must check that our power transmission (MW) over the common net is not greater than a certain, subscribed value. These restrictions will also be included in the new program. They can, approximately, be formulated as linear inequalities and be treated in much the same way as the other constraints of the system.

DISCUSSION

From what has been recorded above one can see that we of course not have been able to solve our optimizing problem in an "exact" manner and we will never succeed in this. However, we think we have reason to believe that a combination of theoretically correct methods and tentative ones is a good way to go ahead.

A question which often appears is how much money one can save by building a mathematical model for a certain purpose. To answer this in the case of hydro-thermal power production planning is almost impossible. Comparing different models developed for the same purpose with the aid of simulation may in many respects be a suspect method. All models distort reality in their own way and one will never get the joint reference point. Furthermore we have in our case the fact that a certain model hardly can be better than another but in the long run. There is, however, no hesitation about the paying of the programmes developed up to now. To this must be added that our production system is more and more troublesome to plan manually. Soon one can not even find permissible plans still less optimizing them without a computer. The value of a computerizing will also increase with increasing demand for electricity.

Both the long-term and short-range planning can of course be made more or less frequently. We hereby can predict a strong increase. This will of cource need faster forecasts, better data transmission and processing and also better and faster information exchange between the individual power companies including those in the neighbouring countries.

SUMMARY

In this report is given an account of the development work concerning optimized planning of the total power production within the Swedish State Power Board. The work which does not affect system expansions may be regarded as a stage on the way to the complete automation of the economic load despatch. Mathematical programming combined with tentative methods has turned out to be a good tool hereby.

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