Valuation of stored energy in dynamic optimal power flow of distribution systems with energy storage

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Abstract—Dynamic optimal power flow (DOPF) models are needed to optimize the operation of a power system with energy storage systems (ESSs) over an extended planning horizon. The optimal storage level at the end of each planning horizon depends on the possible realization of uncertainties in future planning horizons. However, most DOPF models simply require that the storage levels at the end and at the beginning of the planning horizon should be equal. In this paper we consider an AC DOPF model for a distribution system with ESS that explicitly takes into account the expected future value of stored energy. We illustrate the evaluation of the future value function for a system with a wind power plant and demonstrate the use of this value function in the operation of the ESS. The results show that such an operational strategy can be effective compared to not considering the value of stored energy.

Keywords—energy storage; battery storage; distribution systems; distribution networks; wind power; optimal power flow; stochastic optimization; dynamic programming; future cost; future value; profit-to-go; cost-to-go

I. INTRODUCTION

Deployment of energy storage systems (ESSs) is quickly gaining momentum for a number of different applications in the electric power system [1-3]. For instance, installing ESSs is increasingly relevant as an alternative to grid reinforcements, e.g. for improving the hosting capacity for distributed generation (DG) such as wind power [4, 5]. ESSs also introduce new challenges to the power system analysis that have attracted much attention from both the grid companies and the research community. Conventional optimal power flow (OPF) methods consider each time step separately, but ESSs introduce dynamics to the problem that that transforms the OPF problem to a multiperiod (or dynamic) OPF problem (DOPF). In other words, in the presence of ESSs, the optimization problem contains interdependencies (couplings) between different time steps, making it unfavourable to optimize each time step in isolation. A review of DOPF methods, although mostly in the context of economic dispatch under ramp rate constraints, is given in [6]. To compound the challenges, DG is typically introduced to distribution systems at relatively low voltage levels and often with relatively weak connections, where intermittent DG may cause voltage problems. Hence, one can generally not resort to simplifying the problem by linearizing the power flow, but has to solve a full AC DOPF.

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In this paper, we consider an OPF model for distribution systems including the following aspects: Energy storage dynamics, AC power flow, and intermittent distributed generation. A number of similar models have been reported in the literature in recent years; see e.g. [7-26]. A typical problem these models are set out to solve is determining the optimal scheduling of energy storage charging/discharging over a planning horizon, e.g. 24 hours. In the model presented in the present paper, we consider an additional aspect that to the best of our knowledge is not explicitly considered in the literature, namely the value of stored energy at the end of the planning horizon. The conventional approach is to require periodic boundary conditions for the planning horizon so that the amount of stored energy after the last time step has to be equal to, or larger than, the initial amount of stored energy [7-17]. Reference [18] instead includes a penalty function in the objective function that is proportional to the deviation of the amount of stored energy from the maximal energy capacity, promoting a fully charged energy storage at the final time step. In [19], on the other hand, a linear penalty function applies to all time steps. A similar model is used in [20], which includes a term in the objective function that is a linear combination of charged and discharged power for all time steps. Reference [21] sets the (fictitious) marginal operation cost of the ESS in the DOPF to be quadratic function of the amount of stored energy to promote charging when the generation costs in the power system and/or the storage level is low, and vice versa. A quadratic function is also used in [27] to penalize the deviation from a reference storage level, where both the penalty coefficient and the reference storage level varies over the day.

In [28, 29], the marginal operation cost of storage is given as a static function of the storage level, resulting in a decoupling of the time steps, but no guarantee for optimality. A more analytical approach is taken in [22], which relates the marginal value of storage capacity to the marginal value of electricity generation for a simple DC system under stochastic demand. Reference [23] shows that a policy that aims to keep the storage level balanced (at half the capacity) is suboptimal to a more dynamic policy obtained based on stochastic dynamic programming, but they do not consider network constraints in their work. Reference [24] presents a scheduling method based on stochastic dynamic programming that takes the future value of stored energy into account within each daily planning horizon. However, it does not explicitly discuss the storage level

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at the end of the planning horizon or network constraints in the optimization problem. Other references on energy storage scheduling that does not consider power flow or network restrictions are excluded from this brief literature review.

The main contribution of the present paper is to include explicit consideration of the value of stored energy at the end of the planning horizon when finding the optimal ESS charging/discharging schedule. The objectives of this paper are to investigate a) the expected value of energy stored at the end of the planning horizon and b) using this value in the objective function as a control signal in the operational strategy (scheduling of charging and discharging) of the ESSs. The main question we are asking is whether explicitly including the value of stored energy in the objective function could improve the performance of the operational strategy over using periodic boundary conditions as conventionally assumed in the literature.

The outline of the paper is as follows. In Sec. II we present the model and the methodology we use for determining the value function and compare it to other methods of taking into account the energy stored at the end of the planning horizon. Section III describes the test case for which we present results illustrating the methodology in Sec. IV, and the implications and limitations of our results are discussed in Sec. IV. Although the purpose of this paper emphatically is not to present a detailed decision support tool, we also discuss in Sec. V possible extensions of such a model to provide more useful decision support for reallife problems. In Sec. VI we summarize the main findings and the most interesting directions for future research.

II. METHODOLOGY

In this section, we first formulate the DOPF model we have used before describing the methodology for considering the value of stored energy and using this value in the optimization of the distribution system operation.

A. Dynamic optimal power flow model with energy storage

The problem we consider is to optimize the operation over a given planning horizon of a distribution system with a single energy storage system connected to one of the buses and a wind power plant connected to another bus. The planning horizon is defined as the time steps $t \in \{1, 2, ..., T\}$ each of duration Δt . For the case study carried out for this work we assume a planning horizon of one day with hourly resolution, i.e. T = 24 and $\Delta t = 1$ h, and we are primarily concerned with determining the schedule of charging and discharging the ESS over this planning horizon. The amount of energy stored in the ESS at each time step is denoted by E_t .

The distribution system is assumed to have a single Point of Common Coupling (PCC) to the transmission system through which power is imported or exported. We assume that the distribution system is to be operated to optimize social welfare, which corresponds to minimizing the total cost given by the objective function

$$f = \sum_{t=1}^{T} \left(c_t^{\text{imp}} \cdot P_t^{\text{imp}} + c^{\text{rat}} \sum_i P_{i,t}^{\text{rat}} \right) \Delta t \qquad (1)$$
$$- \alpha(E_T, v_T).$$

In this objective function, the first term is the cost or revenue associated with importing or exporting electric power, respectively. The second term represents the cost associated with rationing or shedding load. The last term represents the expected future value associated with having an amount of energy E_T stored in the ESS at the end of the planning horizon, given that the wind speed at that time is v_T . This term will be treated in more detail below, and it is henceforth simply referred to as the value function. The PCC is modelled as a generator with real power P_t^{imp} injected to the grid at time step t. $P_t^{\text{imp}} > 0$ when power is imported and $P_t^{\text{imp}} < 0$ when power is exported. Slack generators with non-negative power outputs $P_{i,t}^{\text{rat}}$ are introduced to all load buses *i* to represent rationing. The power prices c_t^{imp} and c^{rat} are associated with importing and rationing power, respectively.

The decision variables in the OPF problem are, in addition to the stored energy E_t for all time steps, all real and reactive bus power injections and voltage magnitudes and angles for all time steps. These variables are related through the full AC power flow equations. Other grid constraints that are considered in the model are voltage magnitude limits for all buses and time steps and apparent power flow limits for all branches and time steps.

The ESS is modelled as separate fictitious generators for charging (c) and discharging (d), with real power injection to the grid given by

$$P_t^{\text{ESS}} = P_t^{\text{ESS,d}} + P_t^{\text{ESS,c}}.$$
 (2)

Energy storage dynamics, i.e. the evolution in time of the energy stored in the ESS, is expressed by the energy balance equation

$$E_t = E_{t-1} - \frac{1}{\eta_{\text{out}}} P_t^{\text{ESS,d}} - \eta_{\text{in}} P_t^{\text{ESS,c}}, \qquad (3)$$

where η_{in} and η_{out} is the efficiency of charging and discharging the ESS, respectively. In principle, there can be solutions to this DOPF problem in which charging and discharging occurs simultaneously, but such solutions are typically disfavored by the optimization problem for $\eta_{in} < 1$ and/or $\eta_{out} < 1$.

The operation of the ESS is constrained by the real power capacity as given by the inequalities

$$0 \le P_t^{\text{ESS,d}} \le \frac{1}{\eta_{\text{out}}} P_{\text{max}}^{\text{ESS}},\tag{4}$$

$$P_{\min}^{\text{ESS}} \le P_t^{\text{ESS,c}} \le 0.$$
 (5)

The amount of energy in the ESS can never be negative or above the energy capacity E_{max} of the system:

$$0 \le E_t \le E_{\max}.\tag{6}$$

In addition, we include the constraint

$$E_T \ge E_{\min}$$
, (7)

i.e. that the amount of energy stored at the end of the planning horizon should be at least at a minimum value E_{min} .

The DOPF model is implemented using MATPOWER's extensible optimal power flow architecture [30, 31]. Energy storage dynamics is implemented in the OPF by making T duplicates of the MATPOWER network model and coupling them by the energy balance equality constraint (3).

B. Future value function for stored energy

Including the future value function $\alpha(E_T, v_T)$ in the objective function can be understood as a control measure for avoiding myopic operation of the ESS. Taking into account the estimated expected future value will ensure (on average) optimal operation also beyond the current planning horizon. The functional form of the value function is in general unknown and must be determined for each problem, but for this work, we have chosen to parameterize the value function in the objective function as a quadratic function of stored energy E_T . The motivation for this choice is threefold: 1) It generalizes the more restrictive linear value or penalty functions employed by some works in the literature, 2) using a simple two-parameter function allows for more convenient and instructive presentation than using functions depending on even more parameters, and 3) it makes implementation in MATPOWER straightforward. Another alternative that could be quite conveniently implemented is a piecewise linear function, which is an interesting option for possible improvements of this model. For the purposes of this paper, however, a quadratic function seems to approximate the actual value function sufficiently accurately.

In parameterizing the chosen quadratic value function, we have chosen to express it on the form

$$\alpha(E, v) = \gamma(v)\beta(v)E - \frac{\gamma(v)[\beta(v) - 1]}{E_{\max}}E^2 \qquad (8)$$

as a function of stored energy *E* and wind speed *v*. In this parameterization, the dependence on wind speed *v* is contained in the parameters $\gamma(v)$ and $\beta(v)$. Although these parameters are to be understood as functions of *v*, we will for notational convenience simply write γ and β in the rest of this section. The parameter γ can be interpreted as the unit value of stored energy of a fully charged ESS, as $\alpha(E = E_{\text{max}}, v) = \gamma E_{\text{max}}$. β is a parameter determining the curvature of the value function, with $\beta = 1$ giving a function where the value varies linearly with the amount of stored energy.



Fig. 1. Example of the quadratic form chosen for the value function for a set of parameter values.

We then introduce $e = E/E_{\text{max}}$ to denote the amount of stored energy normalized on the energy capacity of the ESS, such that the ESS is fully charged when e = 1. For ease of presentation we can express the value function normalized on the energy capacity of the ESS by defining the normalized value function $\tilde{\alpha} = \alpha/E_{\text{max}}$:

$$\tilde{\alpha}(e,v) = \gamma \beta e - \gamma (\beta - 1) e^2.$$
(9)

This form of the value function is illustrated as a function of energy for a number of different values of γ and β in Fig. 1. Here we have used dimensionless, constant electric energy prices equal to 1.

C. Determining the value function

We determine the value function by considering the dynamic OPF problem for the current planning horizon as the first stage of a multi-stage decision problem. In our case, the objective is to optimize social welfare when operating the distribution system, not only for the current planning horizon p, but also for future planning horizons $\{p + 1, p + 2, ...\}$. Our approach follows the stochastic dynamic programming approach presented in [32] in the context of hydro power scheduling, where a recursive Bellman equation on a form similar to

$$\alpha_{p+1}(E_T, v_T) = \mathop{\mathbb{E}}_k \{ f_{p+1}^*(E_0, v_0) \}$$
(10)

is solved iteratively. Here a_{p+1} denotes the future expected value function at the end of the current planning horizon p, $f_{p+1}^*(E_0, v_0)$ is the optimal objective value for the next planning horizon p + 1 with initial values E_0 and v_0 , and the expectation is taken over a set of possible realizations of stochastic wind speed time series, $k \in \{1, 2, ..., k_{max}\}$. The stored energy E_T and wind speed v_T are the state variables taken into account in this multi-stage decision problem, and they are being passed from one stage to the next. This means that the initial values E_0 and v_0 of the next planning horizon p + 1 equal the values E_T and v_T , respectively, for the last time step of the current planning horizon p.

To evaluate how the expected future value function depends on E_T and v_T , we must solve the optimization problem for the next planning horizon p + 1 repeatedly for a set of initial values E_0 and v_0 and for different possible realizations of wind speed uncertainty. The iterative procedure is shown schematically in Fig. 2. For the first iteration, the value function term in the objective function is initialized to zero by setting $\gamma = 0$. Each subsequent iteration will update the values of $\gamma(v_0)$ and $\beta(v_0)$ for all the values of the initial wind speed v_0 that are considered. The iterative procedure is continued until convergence of the values of $\gamma(v_0)$ and $\beta(v_0)$ is reached for all values of v_0 . To represent the stochasticity of the wind speed, a discrete-state Markov chain model is used to generate a number k_{max} of synthetic wind time series for each value of v_0 based on transition statistics from historic data.

For each iteration and each value of v_0 and E_0 , the optimization problem is solved for all the synthetic wind speed time series $k \in \{1, 2, ..., k_{max}\}$ to estimate the average optimal objective value f^* as a function of E_0 . This estimated function $f^*(E_0)$ is used to estimate the value function, and the chosen quadratic functional form (8) is fitted to the data, determining values of the parameters $\gamma(v_0)$ and $\beta(v_0)$ to be used in the next iteration. Since $\alpha(E, v)$ is determined as a continuous function of E, but for a discrete set of values of v, linear interpolation or

extrapolation is used to determine the parameter values $\gamma(v_T)$ and $\beta(v_T)$ for the actual value of v_T to use in the objective function term $\alpha(E_T, v_T)$ when solving the optimization problem.



Fig. 2. Flow chart of the procedure used to determine the value function.

For our case, we restrict ourselves to an infinite horizon problem where all planning horizons are assumed to be identical on average, and the function α_{p+1} is independent of p. The expected value function $\alpha(E)$ is meant to represent the additional value of having stored the energy amount E, and $\alpha(0) = 0$ by definition. In practice, we therefore choose to calculate the function $\alpha(E_0) = \underset{k}{\mathbb{E}} \{f^*(E_0) - f^*(0)\}$, subtracting for each iteration the objective value for $E_0 = 0$, and use this to construct the estimate for the value function $\alpha(E)$. The slope of the resulting value function, the expected marginal value $\alpha'(E)$ of adding one unit of energy to the ESS, corresponds to the so-called water values of hydropower scheduling [32].

D. Energy storage operational strategies

To evaluate the use of the value function as a control signal for the operation of the ESS, we carry out simulations of the distribution system over an extended simulation period spanning multiple 24-hour planning horizons. After finding the solution to the DOPF for one planning horizon, the value of E_T is used to initialize E_0 for the next planning horizon. We use historic wind speed time series covering the full simulation period to capture the time correlations of wind speed both within each planning horizon and between subsequent planning horizons. The effectiveness of the distribution system operation is evaluated by the sum of the objective function (the total cost) for all planning horizons that are simulated.

This use of the energy value function in the operation of the ESS is evaluated with this simulation approach by comparing it with two more commonly used ways of considering the amount of energy stored at the end of the planning horizon. We define the following operational strategies of the ESS:

a) Constraining the minimal amount of stored energy E_T at the end of the planning horizon, $0 \le E_{\min} \le E_{\max}$. This is the strategy most commonly considered in the literature, but its effectiveness is likely dependent on the system at hand and on the choice made for E_{\min} .

b) Adding a linear value function $\alpha(E_T) = \gamma E_T$ to the objective function to promote a higher amount of stored energy at the end of the planning horizon. This strategy is also assumed by a number of works in the literature, but its effectiveness is likely dependent on the choice made for the marginal energy value $\gamma > 0$.

c) The operational strategy described in this work: Adding a value function $\alpha(E_T, \nu_T)$ as given in (8) to the objective function that is determined by the procedure described in Sec. II. This strategy takes into account the actual expected future value of stored energy, considering the stochasticity and time correlations of future wind power beyond the current planning horizon.

III. TEST SYSTEM

To illustrate our model and the evaluation of the future value of stored energy, we consider a modified version of the test system presented in [5]. This is a simplified representation of a real distribution system on an island at the Norwegian coastline, shown in the single line diagram in Fig. 3. We refer to [5] for detailed grid data for the test system and present here only what are for our purposes the key characteristics. As in [5], the motivation for choosing this test system is that the location has wind conditions suitable for installing a wind power plant but limited grid capacity. A wind power plant with rated power of 10 MW is represented by a generator at bus 10. The difference from [5] is that we consider the possibility of installing an ESS in the system connected to bus 11, whereas the possibility investigated with a different method in [5] was the installation of an electrolyzer for H₂ production. We make no specific assumption of what energy storage technology is being used for the case study, but to be able to illustrate storage dynamics and the future value of stored energy for time scales of the order of one day, we choose a relatively large energy capacity of $E_{\text{max}} = 20$ MWh. The power capacity is chosen to be $P_{\text{max}}^{\text{ESS}} = -P_{\text{min}}^{\text{ESS}} = 5$ MW, and the charging and discharging efficiency is chosen to be $\eta_{\text{in}} = \eta_{\text{out}} = 0.9$. We assume for this case that the ESS does not provide reactive power support to the grid, and hence $Q_t^{\text{ESS}} = 0$. However, the model can also include reactive power Q_t^{ESS} as a decision variable, subject to constraints depending on the power electronics interface of the ESS.

The wind power generator has a real power output given by $0 \le P_t^{\text{wind}} \le P_t^{\text{wind,max}}$, with $P_t^{\text{wind,max}}$ being the maximal potential wind power generation for time step *t* as given by the wind speed and the power curve. We are using the same power curve as given in [5] and the same wind speed time series, measured for a nearby location. For our analyses, we have only used data for January and February to be consistent with the assumption that expected values for each planning horizon are identical. In the solutions of the DOPF the decision variable P_t^{wind} typically takes the value $P_t^{\text{wind,max}}$ to maximize the power exported to the transmission grid, but the possibility of curtailment is represented by solutions $P_t^{\text{wind}} < P_t^{\text{wind,max}}$ for time steps where grid and/or ESS constraints would have to be violated to export all the available wind power. The wind power generator has a constant power factor that is set to 0.96 lagging.

The load buses in the test system all have an identical profile for electric load variations during the day, with load distribution and load variations as presented in [5]. To represent typical load conditions for January and February, the average load over a day is set to 0.97 MW. All load buses have a power factor set to 0.95 lagging. The electricity prices we use are taken from the Nord Pool day-ahead area prices for Monday 2016-02-01 for the same region of Norway as the distribution system [33], augmented with a variable tariff term to account for marginal losses in the transmission system [34]. All cost and value results in this paper are given as dimensionless values measured relative to the average electric energy cost over the period (168.7 EUR/MWh). In these units, the cost of rationing is chosen to be $c^{rat} = 1000$.



Fig. 3. Single line diagram of the test system, based on [5].

Voltage levels for all buses are restricted to lie between 0.94 p.u. and 1.06 p.u. As in [5], this typically implies that voltage limits at bus 9 are violated as wind power generation is approaching the rated power of 10 MW, depending on the load level in the system. For simplicity, we assume that the voltage value at the PCC is restricted within the same interval as the other buses and that there are no restrictions on the reactive

power exported or imported at the PCC as long as the restrictions in the rest of the grid are not violated. We make no specific assumptions of how the voltage at the PCC is controlled in practice, whether it is by tap changers at the transformer or by the installation of separate devices for reactive compensation.

IV. RESULTS

Using the procedure described in Sec. II for the case described in Sec. III, the function $a(E_0)$ is evaluated using six values of E_0 for five different values of v_0 . In our analyses, we have generated $k_{\text{max}} = 50$ synthetic wind time series for each value of v_0 for determining the value function for stored energy, and the same time series are used across all initial values E_0 . In the calculations, the fmincon solver of the MATLAB Optimization Toolbox with the default interior point algorithm [35] is used to solve the OPF problem.

A. The value of stored energy



Fig. 4. Estimated future value of stored energy as a function of the amount of stored energy for different values of the wind speed.

The resulting estimate for the value function after 5 iterations is shown in Fig. 4. The data points are the calculated values a/E_{max} and the dashed lines show the function $\tilde{\alpha}$ fitted to the data. The chosen quadratic parametrization of α is seen to be a good approximation ($R^2 \gtrsim 0.9999$) in our test case. The future value of stored energy at the end of the planning horizon decreases for decreasing wind speed v_T at the end of the planning horizon. This is expected as high values of v_T are correlated with high average wind speeds in the next planning horizon (with correlation coefficient 0.67 in our test case). High wind power generation in the next planning horizon in turn increases the probability of wind power curtailment if the ESS does not have the energy capacity to accommodate the part of this intermittent generation that cannot be exported. We also observe that the value functions have a weak but negative curvature that increases somewhat for higher wind speed values. This corresponds to an expected marginal value of stored energy that decreases as more energy is stored in the ESS and can again be explained by the increasing risk of wind power

curtailment as the room for storing additional energy decreases. Fitting the chosen quadratic value function to the data yields values of the parameter γ (i.e. the unit value of a fully charged ESS) in the range 0.29 to 0.90 and values of the parameter β (indicating curvature) in the range 1.06 to 1.43. As the wind speed v_T increases, γ decreases and β increases. The values of the parameters converge rapidly after a few iterations.

B. Comparing different energy storage operation strategies

We now evaluate the use of the energy value function determined above in the operation of the ESS by carrying out simulations as described in Sec. II.D. The wind data used are the same historic wind speed time series for January and February as were used to generate the synthetic wind time series. This means that the simulation period over which the operation of the system is evaluated consists of 59 24-hour planning horizons. The use of the determined value function (strategy c) is compared with the two other strategies described in Sec. II.D.

For the first planning horizon of the simulation, we choose to initialize $E_0 = E_{min}$ for strategy (a) and $E_0 = 0$ for strategies (b) and (c) in order to have an unbiased comparison of the strategies. To this end, we furthermore ensure that the ESS is emptied at the end of the final planning horizon by setting $\gamma = 0$ before running the DOPF model for that final planning horizon for strategies (b) and (c). For strategies (b) and (c), $E_{min} = 0$.

The results for the total cost over the simulation period are shown in Fig. 5. Because strategies (a) and (b) depend on the choice of the parameters E_{\min}/E_{\max} and γ , respectively, these strategies are evaluated for a set of different values of these parameters. For strategy (c), there is no varying parameter. The values are negative for all strategies because the distribution system is a net energy exporter during the simulation period due to the large wind power generation. A lower value of the total cost implies a more effective strategy for operating the ESS. It is clear that taking into account the expected value of stored energy at the end of each planning horizon (strategy c) is better than the alternative strategies. This result holds irrespective of the choice of values for the parameters of strategies (a) and (b). If constraining the energy stored E_T by a lower bound E_{\min} (strategy a), it is advantageous to choose $E_{\min} \approx E_{\max}/2$, but even then this strategy is inferior to including a value function in the objective function. The effectiveness of strategy (b) is almost independent of what unit value γ is assigned to the stored energy as long as it lies in the range $0.3 \le \gamma \le 0.75$, but strategy (b) becomes comparable to strategy (a) if a value of γ is chosen that lies outside this interval. It should be noted that the difference between strategies (b) and (c) is small relative to the total cost for each planning horizon.

To better understand the mechanisms behind these results, we also present in Fig. 6 the fraction of the potential wind power generation that is curtailed for the three strategies for operating the ESS. Generally, more wind power is curtailed if operating the ESS with a lower bound on E_T (strategy a) than if assigning a value to E_T (strategies b and c). However, for some values of the marginal energy value γ , a linear value function not depending on wind speed (strategy b) seems to result in less curtailment than strategy (c). The reason why strategy (b) is nevertheless slightly less effective than strategy (c) also for these values of γ is that although the wind power generation is higher, this effect is offset by higher grid losses in the distribution system.



Fig. 5. Comparison of different ESS operational strategies in terms of the total cost of the distribution system over the simulation period.



Fig. 6. Comparison of different ESS operational strategies in terms of the fraction of potential wind power over the simulation period that is curtailed.

V. DISCUSSION

The main purpose of the DOPF model presented in this paper was to investigate the future value of stored energy in an ESS in a distribution system and to demonstrate in principle how this information can be taken into account in the operation of the ESS. Viewed as decision support tools for actual ESS operation, such models have a number of shortcomings. Firstly, such models are often demonstrated on test cases where a fixed 24-hour schedule is determined once a day [8-13, 18, 21, 25, 26, 36]. This choice of a 24-hour planning horizon is not a limitation to our approach, but is chosen to illustrate the benefits of looking beyond such a daily planning horizon. Second, it is assumed that the problem is deterministic within each planning horizon. In our case, this implies a perfect 24-hour wind speed forecast, which is clearly not realistic. Although the actual wind speed v_T at the end of the planning horizon will be correlated with the forecasted value, the uncertainty is substantial. One could therefore consider reducing the length of the planning horizon to a time interval over which the forecast is more accurate [27]. Another possible extension of such models to take into account stochasticity within each day would be to retain the 24-hour planning horizon as a rolling horizon, updating the schedule for the operation of the ESS e.g. each hour [14, 27, 37].

The results presented in Fig. 5 demonstrated a clear benefit of explicitly associating an value with stored energy in the objective function (strategy c) compared to simply restricting from below the amount of energy stored at the end of the planning horizon (strategy a). For strategy (b), such a benefit compared to strategy (a) only holds if the marginal energy value parameter γ in the linear value function is judiciously chosen. However, as long as this is the case, the advantage of strategy (c) over strategy (b) is relatively modest, at least for the test case we have considered. Using a corresponding method to determine the value function $\alpha(E_T) = \gamma E_T$ assumed in strategy (b) as the function $\alpha(E_T, v_T)$ used for strategy (c) indicates that the marginal value of stored energy in strategy (b) should optimally be set to $\gamma \approx 0.62$. There are two differences between operational strategies (b) and (c) that can explain the difference (albeit small) between their performance: i) Strategy (b) does not take into account the time correlations in wind speed between planning horizons, and ii) strategy (b) does not capture the fact that the risk of wind power curtailment depends on the storage level. By fixing the curvature parameter $\beta = 1$ when determining the value functions used for strategy (c), we find that the difference in performance can be explained just as much by difference (ii) as by (i). In other words, the benefit of taking time correlations in wind speed into account in optimizing the operation of the ESS is very limited in our case.

Fig. 6 showed that whereas strategy (c) was more effective than strategy (b) when measured by social welfare, measuring the effectiveness by the amount of wind power curtailment would give the opposite conclusion. This demonstrates how the criteria of minimizing the DG curtailment or maximizing the utilization of DG, which is sometimes used in the literature as a performance measure [7, 23, 36], is not necessarily giving the best operational strategy from the perspective of the distribution system [13]. The implication of this result is that a using the amount of wind power curtailment may be misleading as metric for the effectiveness of the ESS operational strategy.

In this work we have for simplicity chosen a quadratic form of the value function. As we remarked in Sec. II.B, there is no reason to expect that the actual future value function is a quadratic function. In fact, since the nonlinear AC power flow equalities makes the OPF a nonlinear and non-convex optimization problem, there are not even reasons to expect the value function to be concave. However, for the results presented for the test case considered, the value function appears to be both concave and fairly well approximated by a quadratic function. Nevertheless, since a non-concave value function is possible for this type of problem, care should be taken when fitting a specific function to the estimated future value function.

In our test case, the energy storage system was only considered for one primary application, namely improving the hosting capacity of intermittent DG. It is likely that energy storage systems in power distribution systems can reach their full economic potential only when the same ESS is deployed and used for multiple applications [1], which is an interesting direction for future research. Neither has the purpose of the paper been to consider which energy capacity is economically viable for this specific application. The ESS assumed in the test case had a large energy capacity relative to the distributed generation, but tests with smaller energy capacities show similar results although with smaller benefits of optimized operation. We also considered a single ESS at one specific location in the grid relatively close to the distributed generation. Optimization of the location and capacity of multiple ESSs in a power system is a large and interesting set of problems in its own right, and the capital expenditure assumed for installing a given ESS would have a major impact on the results. However, when the ESS is in fact already installed, obtaining even a small improvement in the performance of the operational strategy of the ESS would improve the economic benefit of the project at almost no additional cost.

VI. CONCLUSIONS AND FURTHER WORK

In this paper, we have presented a dynamical optimal power flow model for a distribution system with an energy storage system and a wind power plant where the expected future value of stored energy is explicitly taken into account. We have determined this future value for a realistic test case, taking into account both the stochasticity and time correlations of the wind power generation. Furthermore, we have demonstrated that using this information as a control signal in the operation of the energy storage system is an effective operational strategy compared to requiring that the amount of stored energy at the end of each planning horizon is above a certain level. We have also demonstrated how the criteria of minimal wind power curtailment does not necessarily lead to an operational strategy that maximises social welfare.

However, the differences in effectiveness between different operational strategies were not very large for the test case we considered. Taking into account time correlations of the wind power between planning horizons when determining the value of stored energy offered only relatively modest improvements. Approximating the value function to be a linear function of stored energy also turned out to be a fair approximation for the operational strategy of the ESS. It should be noted that the above findings are most likely system dependent. Therefore, it would be interesting to investigate whether detailed modelling of the value function has greater impact for other systems and other applications of energy storage.

Another interesting direction for further work would be to augment such a DOPF model to take into account stochasticity and forecast uncertainty also within each (daily) planning horizon. The present paper has demonstrated the value of taking into account the uncertainty associated with next planning horizon when making decisions for the current planning horizon, but a practical decision support tool should be able to support decision making under uncertainty for both for the current and future planning horizons.

ACKNOWLEDGMENT

The authors would like to thank Birger Mo, Håkon Marthinsen, Oddbjørn Gjerde and Leif Warland for many fruitful discussions during the execution of this work.

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